Bayesian Population Reconstruction

Training Course on Bayesian Population Projections: Congreso Internacional de ALAP 2018, Puebla, Mexico

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NICHD, Grants R01 HD054511 and K01 HD057246
BayesPop Working Group, CSSS, and CSDE at the UW
FHES at AUT

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  - historical studies
  - many more . . .

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- Contains point estimates of ...
  - vital rates (fertilty and mortality rates)
  - net migration
  - population counts
- ... over the period 1950 to the present.
- Particularly important for countries without well-resourced official statistics systems of their own.

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- *More developed countries* have vital registration systems which achieve high coverage and low bias and measurement error variance.
- Many *less developed countries* rely on surveys (e.g., DHS) for vital rate data.
  - Coverage maybe incomplete.
  - Bias and measurement error variance can be high, can vary greatly from one source to another.
  - Expert knowledge is valuable.

Fragmentary Data

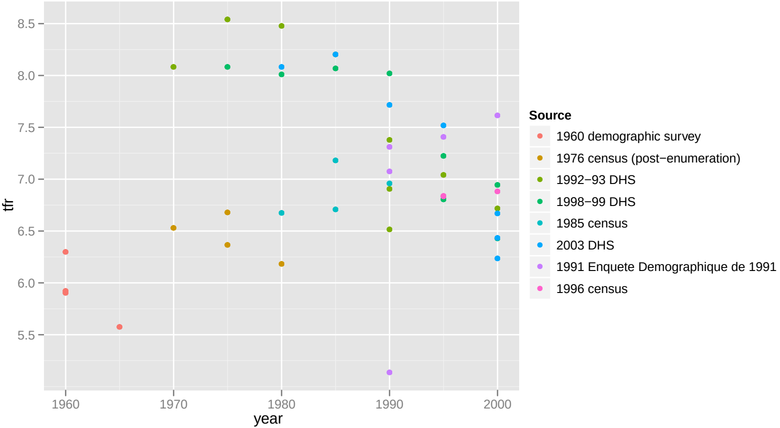

Figure: Estimates of Total Fertility Rates, Burkina Faso 1960–2000.

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4. Uses all reliable data and expert opinion.

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- $n_{a,t}$: age-specific population count.

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- $e_{0,t}$: life expectancy at birth
  - The average age at death under current age-specific mortality rates.
  - Function of $s_{a,t}$:
    $e_{0,t} = 5 \sum_{a=0}^{A} \prod_{i=0}^{a} s_{i,t} + \left(\prod_{i=0}^{A} s_{i,t} \right) (s_{A+5}/(1 - s_{A+5}))$

Demographic Balancing Equation

- The fundamental relationship linking fertility, mortality, migration and population through time is:

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- Age-specific version called cohort component method of population projection (CCMPP).
- This is a discrete-time approximation to a continuous process; standard adjustments are used to improve accuracy.

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- These two counts can be compared. The difference can tell us something about data accuracy.

Population Reconstruction

Current UN Practice

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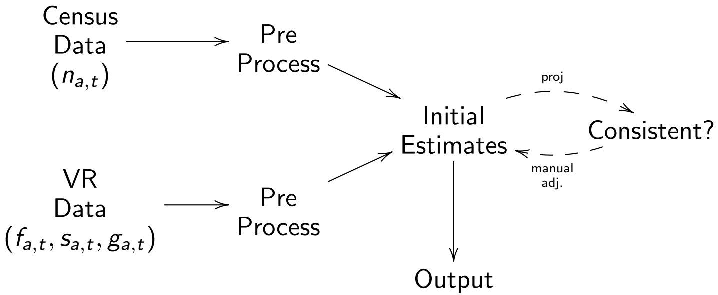

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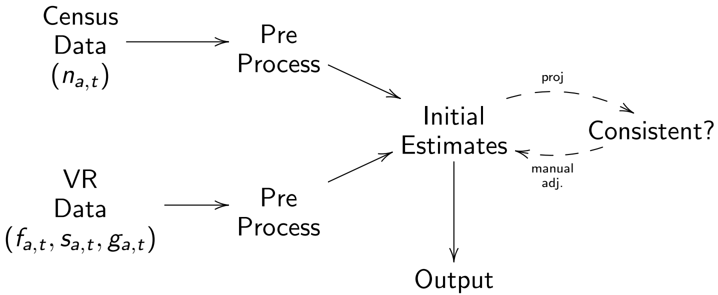

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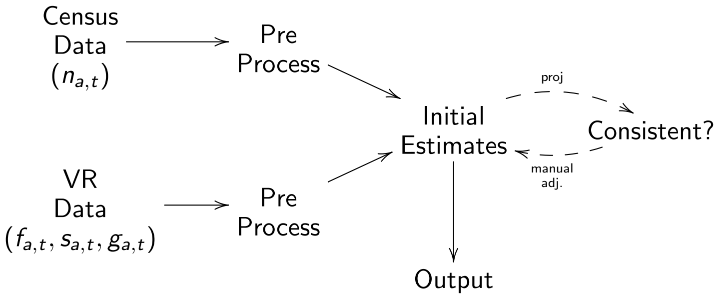

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- Pre-process data to get a single initial estimate for each age and time period.
- Compare projected with census counts; are they the same?
- Manually adjust if necessary.

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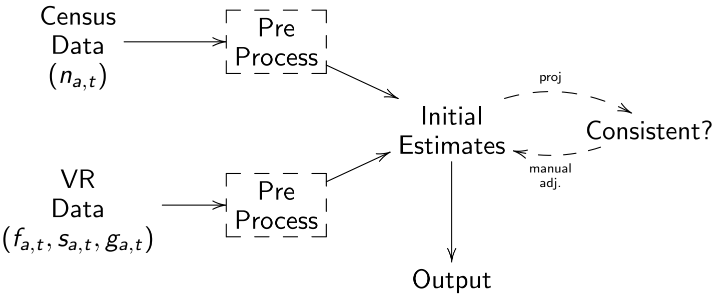

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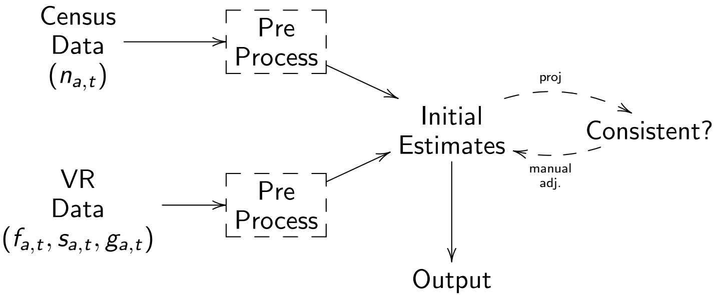

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- Includes
  - Discarding some data completely.
  - Use of *indirect estimation techniques*.

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- Cause and magnitude of biases vary greatly among data sources, parameters, age groups and time.
- Bias-reduction via indirect estimation
  - Is source- and parameter-specific.
  - Requires detailed knowledge of data collection methods and possible sources of bias.
- No "one size fits all" approach: We do not attempt to replace this step.

Current UN Practice

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- Consistency among the parameters is achieved in a manual, iterative fashion.
- No quantification of uncertainty.

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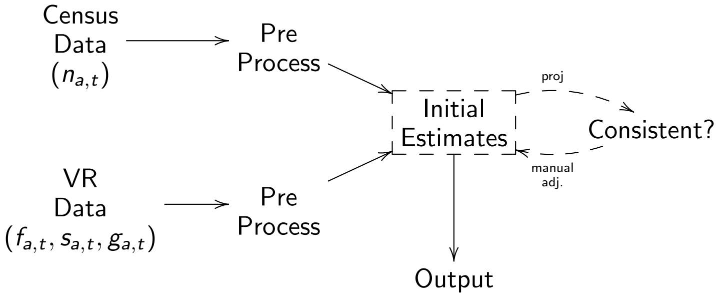
Females only (for now)

Current vs. Proposed

Current UN Practice



Proposed (Bayesian Population Reconstruction)

Hierarchical Model

(* = initial estimates and census counts which are fixed)

I. Likelihood

$$\log n^*_{a,t} \mid n_{a,t}, \sigma^2_n \sim \text{Normal}\left(\log n_{a,t}, \sigma^2_n\right)$$

II. Projection Model

$$n_{a,t} \mid \mathbf{n}_{t-5}, \mathbf{f}_{t-5}, \mathbf{s}_{t-5}, \mathbf{g}_{t-5} = CCMPP(\mathbf{n}_{t-5}, \mathbf{f}_{t-5}, \mathbf{s}_{t-5}, \mathbf{g}_{t-5})$$

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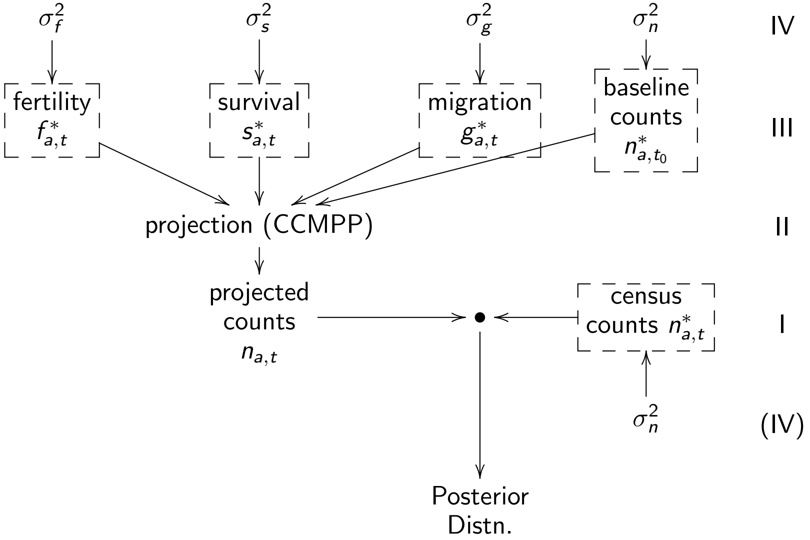
Hierarchical Model: Key Relationships

(inputs are boxed)



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- This is a flexible approach: some data sources have information about their accuracy, but others do not.

Summary

Inputs

- Bias-reduced initial estimates of
  - Age-specific fertility and mortality rates.
  - Net international migration.
  - Population counts.
- Expert opinion about measurement error variance (based on data if available).

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Outputs

- Joint posterior distribution of the inputs.
- Gives 95% credible intervals for input parameters and transformations.

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Method

- Simulate an hypothetical population.
- Generate initial estimates under the model.
- Run reconstruction many times.

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- Improvement Over Initial Estimates
  - On average, the truth was closer to the posterior medians than the initial estimates.
- Further Model Checks
  - We also compared our model with a more flexible version where σ_n varied by age and time.
  - Separate variances seemed unnecessary.
  - Results were unchanged.

Developing and Developed Countries

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	1961	1985	2005
Laos	—	1.8	2.8
New Zealand	1.2	1.7	2.1

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	1961	1985	2005
Laos	—	1.8	2.8
New Zealand	1.2	1.7	2.1

- Data quality and availability are very different.

Initial Estimates: Laos

Parameter	Sources	Methods	Rel. Error. (%)
Pop count	Censuses 1985, 1995, 2005	Adjustments for undercount	10
Fertility	Birth history surveys	Indirect methods, smoothing	10
Survival	Birth history surveys	Indirect methods, model life tables	10
Migration	None	Centered at zero, large rel. err.	20

Initial Estimates: New Zealand

Parameter	Sources	Methods	Rel. Error. (%)
Pop count	Censuses, 5-yearly, 1961–2006	Post-enumeration survey	1
Fertility	Vital registration	Coverage study	1
Survival	Vital registration	Coverage study	1
Migration	Immigration Cards		5

Results

Total Fertility Rate

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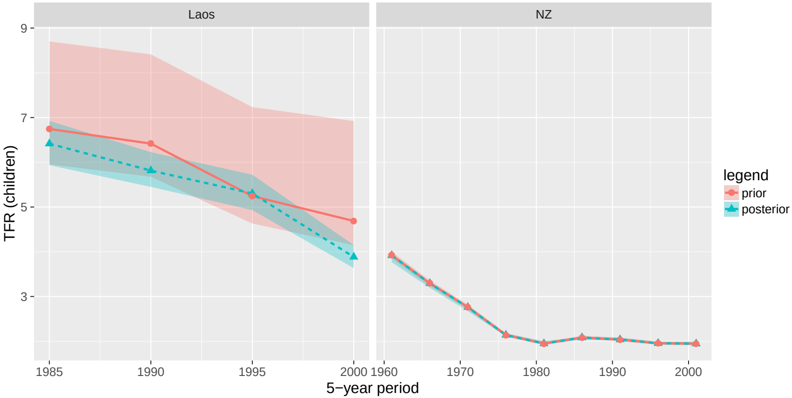

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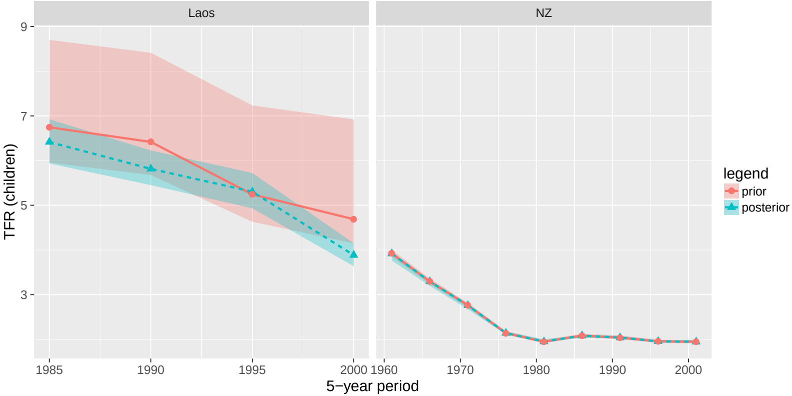

95% Posterior Interval Half-widths

	Mean Half-Width
Laos	0.4
NZ	0.04

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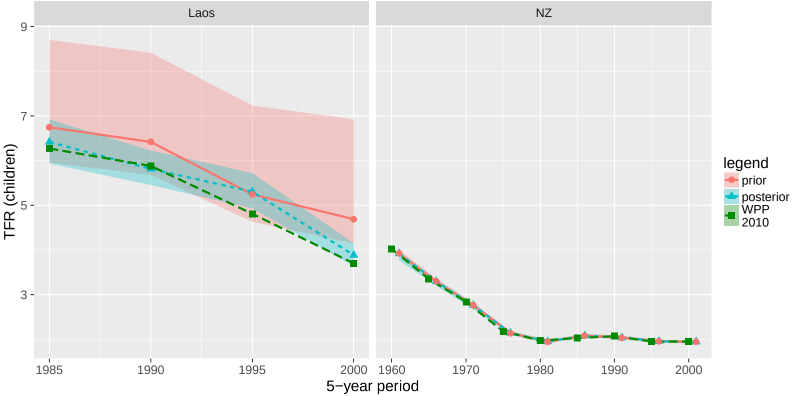

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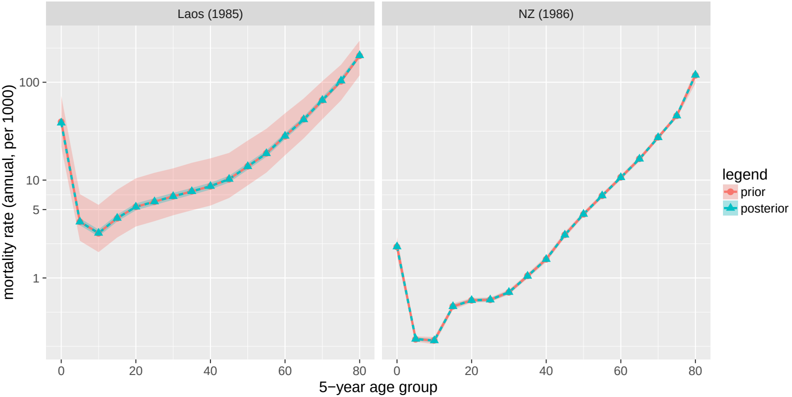
Age Specific Mortality Rate



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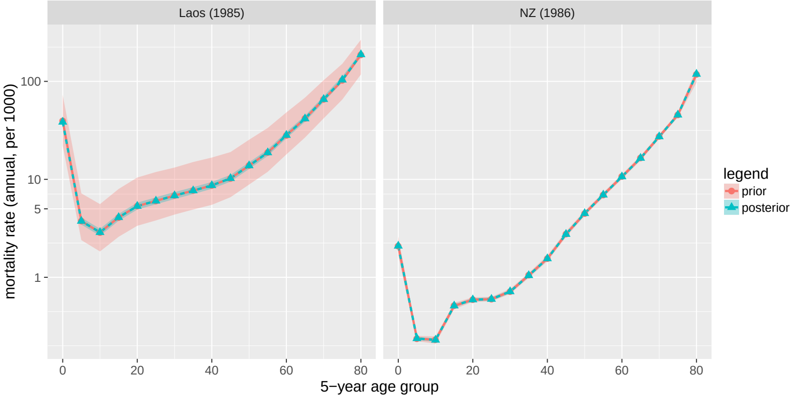
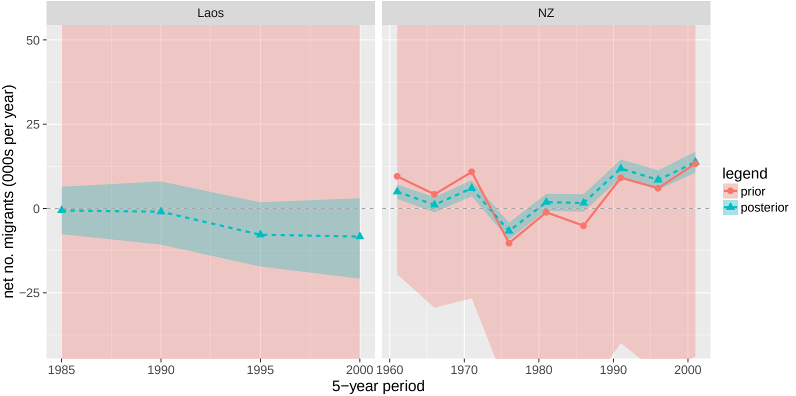


95% Posterior Interval Half-widths

	Mean Half-Width
Laos	2
NZ	0.5

Total Net Migration



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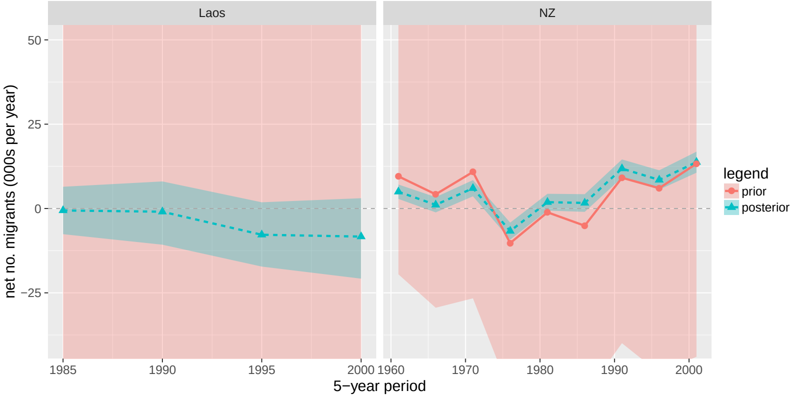

95% Posterior Interval Half-widths

	Counts (000s)
	Mean Half-Width
Laos	9.5
NZ	2.6

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Summary

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- The method works for countries with very good data as well as those with fragmentary and unreliable data.

Two-Sex Reconstruction

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- Requires
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  - two-sex CCMPP,
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- As a consequence of including males, two sex reconstruction allows estimation of sex ratios such as
  - sex ratio at birth (SRB),
  - sex ratios of mortality.

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- All-sex births are computed then decomposed because SRB is a parameter of interest.

Two-Sex Hierarchical Model

I. Likelihood $\log n^*_{a,t,\ell} \mid n_{a,t,\ell}, \sigma^2_n \sim \text{Normal}\left(\log n_{a,t,\ell}, \sigma^2_n\right)$

II. Projection Model

$$n_{a,t,\ell} \mid \mathbf{n}_{t-5,\cdot}, \mathbf{f}_{t-5}, \ldots, SRB_{t-5} = Proj(\mathbf{n}_{t-5,\cdot}, \mathbf{f}_{t-5,\cdot}, \mathbf{s}_{t-5,\cdot}, \mathbf{g}_{t-5,\cdot}, SRB_{t-5})$$

III. Priors on Inputs

$$\log SRB_t \mid SRB^*_t, \sigma^2_{SRB} \sim \text{Normal}\left(\log SRB^*_t, \sigma^2_{SRB}\right)$$

$$\log n_{a,t_0,\ell} \mid \sigma^2_n \sim \text{Normal}\left(\log n^*_{a,t_0,\ell}, \sigma^2_n\right)$$

$$\log f_{a,t} \mid \sigma^2_f \sim \text{Normal}\left(\log f^*_{a,t}, \sigma^2_f\right)$$

$$\text{logit}\, s_{a,t,\ell} \mid \sigma^2_s \sim \text{Normal}\left(\text{logit}\, s^*_{a,t,\ell}, \sigma^2_s\right)$$

$$g_{a,t,\ell} \mid \sigma^2_g \sim \text{Normal}\left(g^*_{a,t,\ell}, \sigma^2_g\right)$$

IV. Hyperparameters

$$\sigma^2_v \sim \text{InvGamma}(\alpha_v, \beta_v), \quad v \in \{n, f, s, g, SRB\}$$

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  - high SRBs post-1970s

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Research Questions

What is the probability that

i. SRB was unusually high between 1971 and 2001?

ii. The SRB increased in India since 1971?

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Fertility	Indian SRS, Nat. Fam. Health Sur.	Indirect methods, smoothing	10
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SRB	Indian SRS, Nat. Fam. Health Sur.	Indirect methods, smoothing	10

Sex Ratio at Birth: Level

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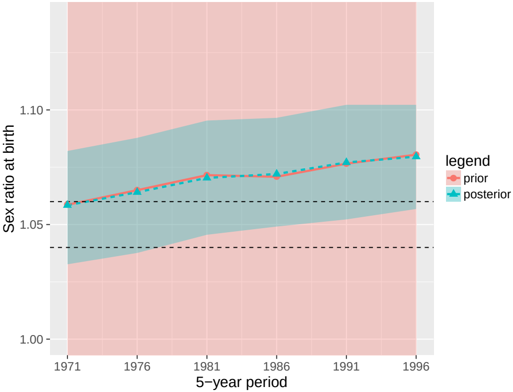

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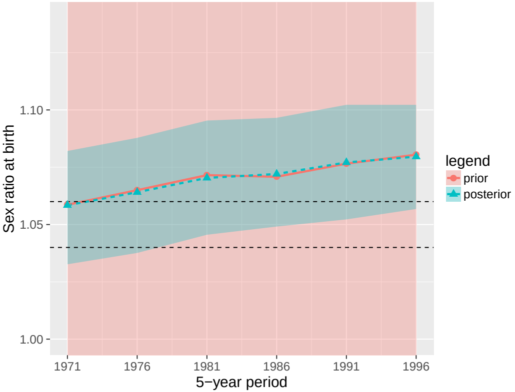

Pr(SRB > 1.06)

1971	1976	1981	1986	1991	1996
0.44	0.66	0.83	0.86	0.93	0.96

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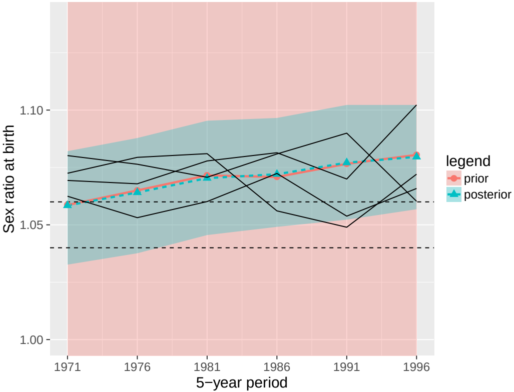

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Sex Ratio at Birth: Trend

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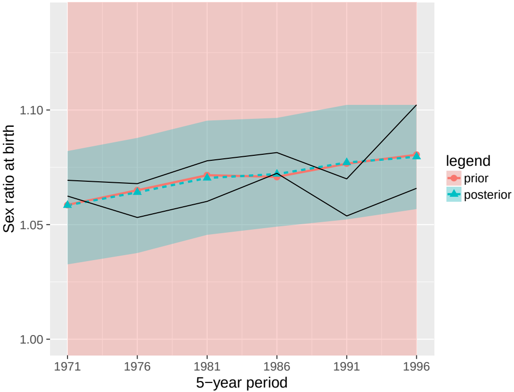
  1. the slope coefficient of the linear model (OLS slope)

     $$\mathsf{SRB}_i = \alpha_i + \beta_i \cdot \mathsf{time} + \epsilon_{i,\mathsf{time}}$$

  2. the simple difference

     $$\mathsf{SRB}_{i,1996} - \mathsf{SRB}_{i,1971}$$

Sex Ratio at Birth: Trend



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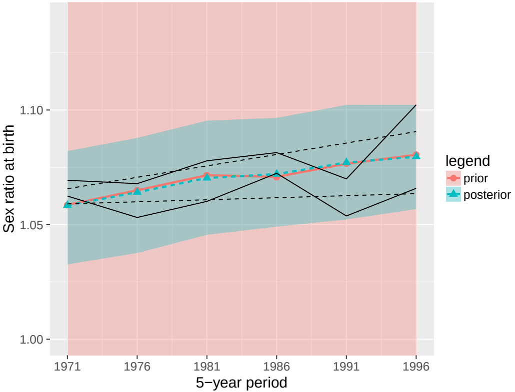

Sex Ratio at Birth: Trend

Pr(SRB Increased)

Statistic values greater than zero indicate an increase.

Statistic	95% CI	Prob > 0
OLS slope	$[-0.00034, 0.0021]$	0.93
$\text{SRB}_{1996} - \text{SRB}_{1971}$	$[-0.011, 0.054]$	0.92

Conclusions

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What is the probability that

i. SRB was unusually high between 1971 and 2001?
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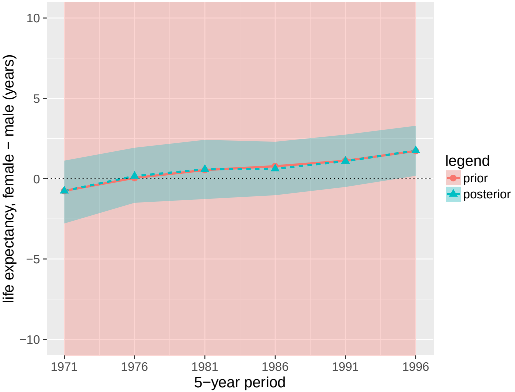

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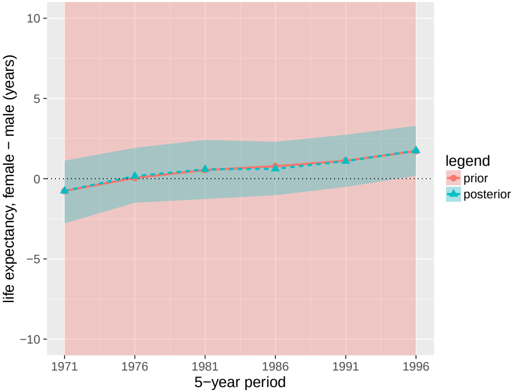

Probability female life expectancy < male

1971	1976	1981	1986	1991	1996
0.79	0.42	0.26	0.22	0.09	0.01

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Statistic	95% CI	Prob > 0
OLS slope (LEB $\sim$ time)	[0.0066, 0.1721]	0.98
$\text{LEB}_{1996} - \text{LEB}_{1971}$	[0.01, 0.17]	0.98

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Summary

- Bayesian Population Reconstruction can provide probabilistic answers to meaningful questions about transformations of the input parameters, as well as the parameters themselves.

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Bayesian Population Reconstruction is a new method for reconstructing population structures of the past.

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  - Bias-reduced initial estimates of age-specific vital rates, net international migration and population counts.
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(*Refs:* Wheldon et al. (2013, 2016, 2015); *popReconstruct* on GitHub)

References

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