Bayesian Population Reconstruction

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and
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Acknowledgements: Joint work with Adrian Raftery, Sam Clark and Patrick Gerland
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BayesPop Working Group, CSSS, and CSDE at the UW
FHES at AUT

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  - historical studies
  - many more ...

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- Contains point estimates of ...
  - vital rates (fertilty and mortality rates)
  - net migration
  - population counts
- ... over the period 1950 to the present.
- Particularly important for countries without well-resourced official statistics systems of their own.

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2. Produces estimates that can be analyst specific.
3. Does not include quantification of uncertainty.

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- *More developed countries* have vital registration systems which achieve high coverage and low bias and measurement error variance.
- Many *less developed countries* rely on surveys (e.g., DHS) for vital rate data.
  - Coverage maybe incomplete.
  - Bias and measurement error variance can be high, can vary greatly from one source to another.
  - Expert knowledge is valuable.

Fragmentary Data

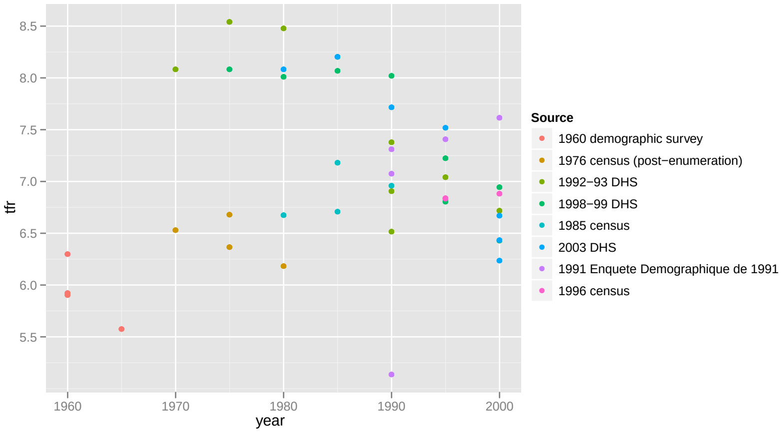

Figure: Estimates of Total Fertility Rates, Burkina Faso 1960–2000.

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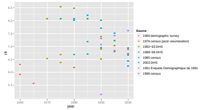

Figure: Estimates of Total Fertility Rates, Burkina Faso 1960–2000.

- This slide shows estimates of TFR for Burkina Faso.
- The data points are based on different sources and outputs of different post-processing techniques.
- Summarizing these with a single value at each time period is not telling the whole story.

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Erik De Castro / AP

Countries around the world marked the world's population reaching 7 billion Monday with lavish ceremonies for newborn infants symbolizing the milestone and warnings that there may be too many humans for the planet's resources.

While demographers are unsure exactly when the world's population will reach the 7 billion mark, the U.N. is using Monday to symbolically mark the day.

A string of festivities are being held worldwide, with a series of symbolic 7-billionth babies being born.

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7 Billionth Person Born (Or Maybe More. Or Less. Who Knows?)

Published October 31, 2011 / FoxNews.com

Oct. 31, 2011: Newborn girl Yazuri Tarmeno -- one of several people worldwide named the 7 billionth born on the planet -- cries after being born at the Maternity hospital in Lima, Peru. (AP PHOTO/KAREL NAVARRO)

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TRENDING IN SCIENCE

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4. Uses all reliable data and expert opinion.

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  - Function of $s_{a,t}$:
    $e_{0,t} = 5 \sum_{a=0}^{A} \prod_{i=0}^{a} s_{i,t} + \left(\prod_{i=0}^{A} s_{i,t}\right) (s_{A+5}/(1 - s_{A+5}))$

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- t_0 is the "baseline" year.
- We want to reconstruct the population over the period t_0 to T.

- $f_{a,t}$: babies born to women in age group a per years lived by all women in age group a in time period t.
- $s_{a,t}$: proportion of cohort in age group $a-5$ surviving to age group a between t and $t+5$.
- $g_{a,t}$: average annual net number of migrants in age group a migrating between t and $t+5$ as a proportion of $n_{a,t}$.
- I model $g_{a,t}$ instead of $n_{a,t} g_{a,t}$ because expect measurement error in $g_{a,t}$ to be less dependent on magnitude. [NOTE: var stabilizing trans for Poisson is sqrt; var stabilizing trans for any scale parameter is log, e.g., Expo(λ)]

Demographic Balancing Equation

- The fundamental relationship linking fertility, mortality, migration and population through time is:

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- Age-specific version called cohort component method of population projection (CCMPP).
- This is a discrete-time approximation to a continuous process; standard adjustments are used to improve accuracy.

Comparing Counts

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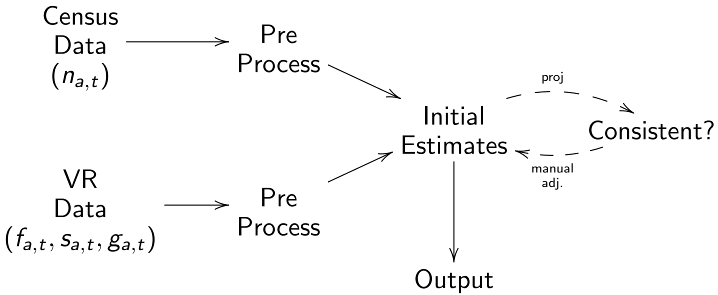
- Census counts provide a second set of estimates in census years.
- These two counts can be compared. The difference can tell us something about data accuracy.

Population Reconstruction

Current UN Practice

Current UN Practice



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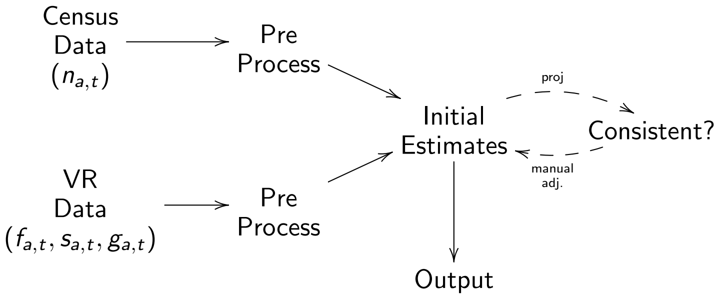

- Gather all available data
- Pre-process data to get a single initial estimate for each age and time period.

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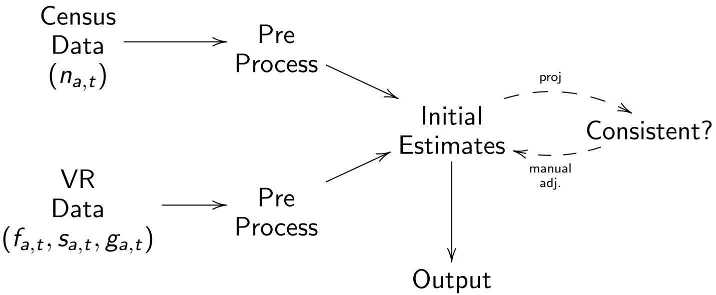
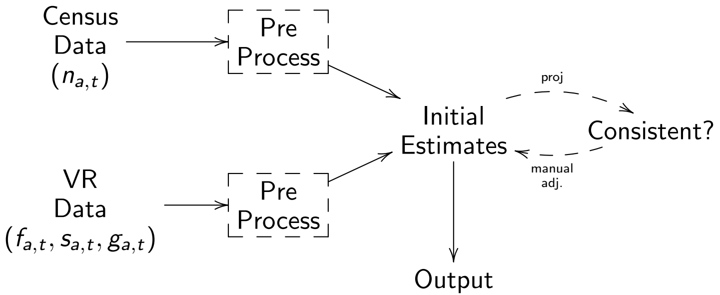


- Gather all available data
- Pre-process data to get a single initial estimate for each age and time period.
- Compare projected with census counts; are they the same?
- Manually adjust if necessary.

Current UN Practice



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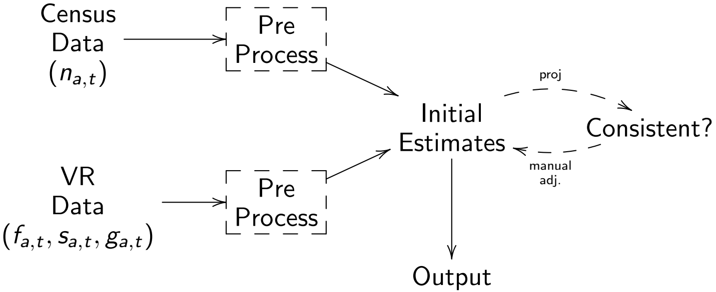
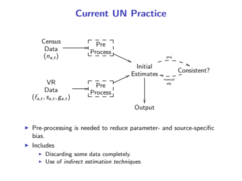


- Pre-processing is needed to reduce parameter- and source-specific bias.
- Includes
  - Discarding some data completely.
  - Use of *indirect estimation techniques*.



- UN collects all available data from surveys, vital registration and censuses.
- Pre-processes them to get a single initial estimate for each age group and time period.
- Project and manually adjust to ensure consistency.
- Pre-processing requires use of "indirect estimation" methods taken from demography literature.

Indirect Estimation

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  - Relational models for estimating mortality.
- Cause and magnitude of biases vary greatly among data sources, parameters, age groups and time.
- Bias-reduction via indirect estimation
  - Is source- and parameter-specific.
  - Requires detailed knowledge of data collection methods and possible sources of bias.
- No "one size fits all" approach: We do not attempt to replace this step.

Current UN Practice

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- Consistency among the parameters is achieved in a manual, iterative fashion.
- No quantification of uncertainty.

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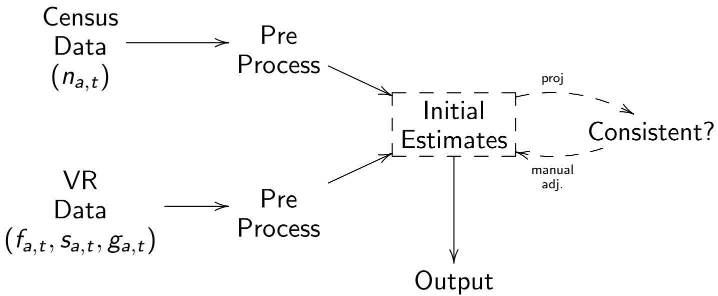
Females only (for now)

Current vs. Proposed

Current UN Practice



Proposed (Bayesian Population Reconstruction)

Hierarchical Model

(* = initial estimates and census counts which are fixed)

I. Likelihood $$\log n^*_{a,t} \,|\, n_{a,t}, \sigma^2_n \sim \text{Normal}\left(\log n_{a,t}, \sigma^2_n\right)$$

II. Projection Model

$$n_{a,t} \,|\, \mathbf{n}_{t-5}, \mathbf{f}_{t-5}, \mathbf{s}_{t-5}, \mathbf{g}_{t-5} = CCMPP(\mathbf{n}_{t-5}, \mathbf{f}_{t-5}, \mathbf{s}_{t-5}, \mathbf{g}_{t-5})$$

III. Priors on Inputs

$$\begin{aligned}
\log n_{a,t_0} \,|\, \sigma^2_n &\sim \text{Normal}\left(\log n^*_{a,t_0}, \sigma^2_n\right)\\
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$$\log n^*_{a,t} \,|\, n_{a,t}, \sigma^2_n \sim \text{Normal}\left(\log n_{a,t}, \sigma^2_n\right)$$

II. Projection Model
$$n_{a,t} \,|\, \mathbf{n}_{t-5}, \mathbf{f}_{t-5}, \mathbf{s}_{t-5}, \mathbf{g}_{t-5} = CCMPP(\mathbf{n}_{t-5}, \mathbf{f}_{t-5}, \mathbf{s}_{t-5}, \mathbf{g}_{t-5})$$

III. Priors on Inputs
$$\begin{aligned}
\log n_{a,t_0} \,|\, \sigma^2_n &\sim \text{Normal}\left(\log n^*_{a,t_0}, \sigma^2_n\right)\\
\log f_{a,t} \,|\, \sigma^2_f &\sim \text{Normal}\left(\log f^*_{a,t}, \sigma^2_f\right)\\
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g_{a,t} \,|\, \sigma^2_g &\sim \text{Normal}\left(g^*_{a,t}, \sigma^2_g\right)
\end{aligned}$$

IV. Hyperparameters
$$\begin{aligned}
\sigma^2_v &\sim \text{InvGamma}(\alpha_v, \beta_v),\\
&\quad v \in \{n, f, s, g\}
\end{aligned}$$

Hierarchical Model

(* = initial estimates and census counts which are fixed)

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II. Projection Model $$n_{a,t} \mid \mathbf{n}_{t-5}, \mathbf{f}_{t-5}, \mathbf{s}_{t-5}, \mathbf{g}_{t-5} = CCMPP(\mathbf{n}_{t-5}, \mathbf{f}_{t-5}, \mathbf{s}_{t-5}, \mathbf{g}_{t-5})$$

III. Priors on Inputs $$\begin{aligned} \log n_{a,t_0} \mid \sigma_n^2 &\sim \text{Normal}\left(\log n^*_{a,t_0}, \sigma_n^2\right) \\ \log f_{a,t} \mid \sigma_f^2 &\sim \text{Normal}\left(\log f^*_{a,t}, \sigma_f^2\right) \\ \text{logit}\, s_{a,t} \mid \sigma_s^2 &\sim \text{Normal}\left(\text{logit}\, s^*_{a,t}, \sigma_s^2\right) \\ g_{a,t} \mid \sigma_g^2 &\sim \text{Normal}\left(g^*_{a,t}, \sigma_g^2\right) \end{aligned}$$

IV. Hyperparameters $$\sigma_v^2 \sim \text{InvGamma}(\alpha_v, \beta_v), \quad v \in \{n, f, s, g\}$$

The model has four levels. I shall start at level 3:

3 The true vital rates and baseline population are given priors, conditional on the initial estimates and variance parameters which account for measurement error.

4 The variance parameters are, themselves, given distributions at level 4.

2 The priors on the vital rates and baseline counts induce a prior on subsequent counts through the deterministic projection model in level 2.

1 Level 1 is a likelihood for the census counts which is parameterized by the projected counts and a variance parameter which accounts for measurement error.

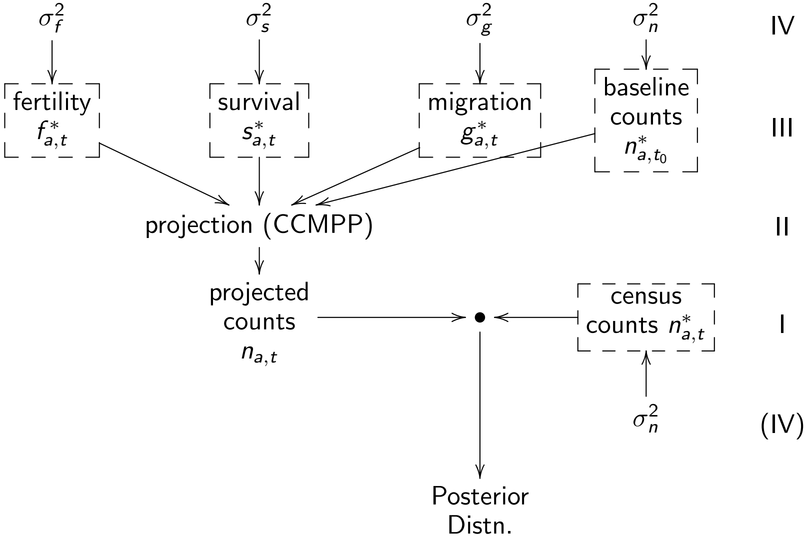
1. Here are the things we have data on
2. Here are the things we have to specify
3. Here are the things we do inference on

Inference

- Inference is based on the joint posterior distribution of the input parameters $(f_{a,t},\ s_{a,t},\ g_{a,t},\ n_{a,t_0})$.
- An MCMC algorithm is used to draw a large sample from the posterior.
- 95 percent credible intervals from the marginal posterior distributions of each

Hierarchical Model: Key Relationships

(inputs are boxed)



Measurement Error Variance

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We choose the hyperparameters using the MAE of the marginal prior distributions of the observables. For example, we use the joint marginal distribution of the vector of fertility rates, $(f_{1,1}, \dots f_{A,T})$ which is

$$t_{AT}\left(\log \mathbf{f}^*, \beta_f/\alpha_f \mathbf{I}_{AT}, 2\alpha\right), \; \alpha \geq 1/2, \; \beta > 0$$

Then

$$\begin{aligned}
\text{MAE}\left(\log \mathbf{f}\right) &\equiv \text{E}\,|\log \mathbf{f} - \log \mathbf{f}^*| \\
&= \text{E}\left(\text{E}\left[\left(|\log \mathbf{f} - \log \mathbf{f}^*|\right)\Big|\sigma_f^2\right]\right) \\
&= \mathbf{1}_{AT} \cdot \text{E}\left(\sqrt{\frac{2}{\pi}}\sigma_f\right) \\
&= \mathbf{1}_{AT} \cdot \sqrt{\frac{2\beta}{\pi}} \cdot \underbrace{\frac{\Gamma(\alpha - 1/2)}{\Gamma(\alpha)}}_{h(\alpha)}, \; \alpha \geq 1/2, \; \beta > 0
\end{aligned}$$

where $\mathbf{1}_{AT}$ is a length AT vector of 1s.

Summary

Inputs

- Bias-reduced initial estimates of
  - Age-specific fertility and mortality rates.
  - Net international migration.
  - Population counts.
- Expert opinion about measurement error variance (based on data if available).

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Outputs

- Joint posterior distribution of the inputs.
- Gives 95% credible intervals for input parameters and transformations.

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Method

- Simulate an hypothetical population.
- Generate initial estimates under the model.
- Run reconstruction many times.

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- Coverage of 95% Credible Intervals
  - Achieved coverages were close to 0.95, as desired.
- Improvement Over Initial Estimates
  - On average, the truth was closer to the posterior medians than the initial estimates.
- Further Model Checks
  - We also compared our model with a more flexible version where σ_n varied by age and time.
  - Separate variances seemed unnecessary.
  - Results were unchanged.

Developing and Developed Countries

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	1961	1985	2005
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- Data quality and availability are very different.

Initial Estimates: Laos

Parameter	Sources	Methods	Rel. Error. (%)
Pop count	Censuses 1985, 1995, 2005	Adjustments for undercount	10
Fertility	Birth history surveys	Indirect methods, smoothing	10
Survival	Birth history surveys	Indirect methods, model life tables	10
Migration	None	Centered at zero, large rel. err.	20

Initial Estimates: New Zealand

Parameter	Sources	Methods	Rel. Error. (%)
Pop count	Censuses, 5-yearly, 1961–2006	Post-enumeration survey	1
Fertility	Vital registration	Coverage study	1
Survival	Vital registration	Coverage study	1
Migration	Immigration Cards		5

Results

Total Fertility Rate

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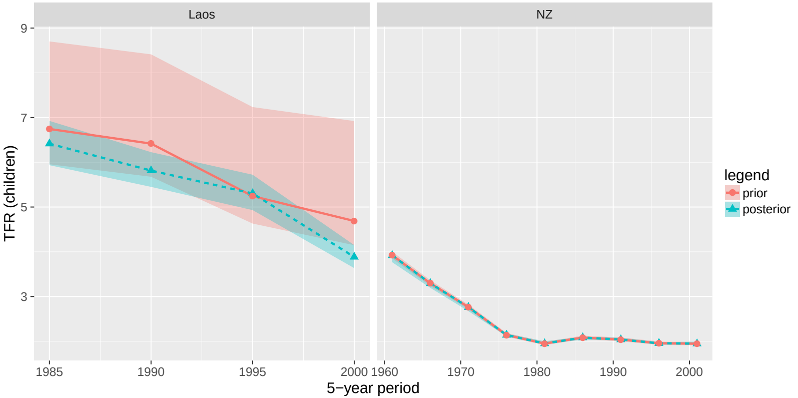

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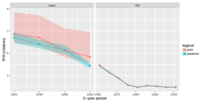
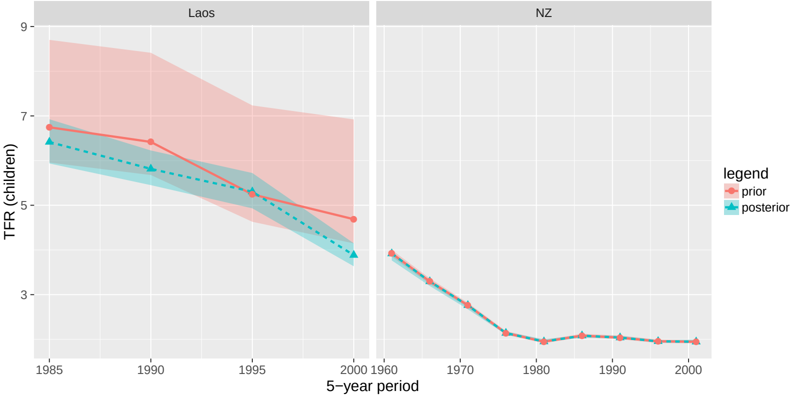


- These plots show prior and posterior medians and the limits of 95% credible intervals.
  - The prior on TFR is in red, the posterior is in blue.
  - The initial estimates correspond to the prior median, shown by the red line.
- TFR in Laos was much higher than in New Zealand.
- Posterior uncertainty is quantified using the mean half-width of the credible intervals:
  - Mean half-width for New Zealand was about 1/10th that of Laos.
- I show the WPP 2010 estimates for comparison
  - Very similar data were used but our methods are different.
  - The WPP 2010 estimates are different but not too different!

Total Fertility Rate



95% Posterior Interval Half-widths

	Mean Half-Width
Laos	0.4
NZ	0.04

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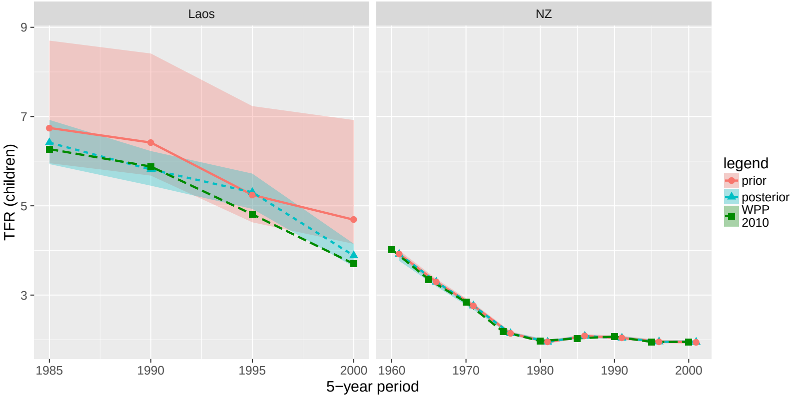

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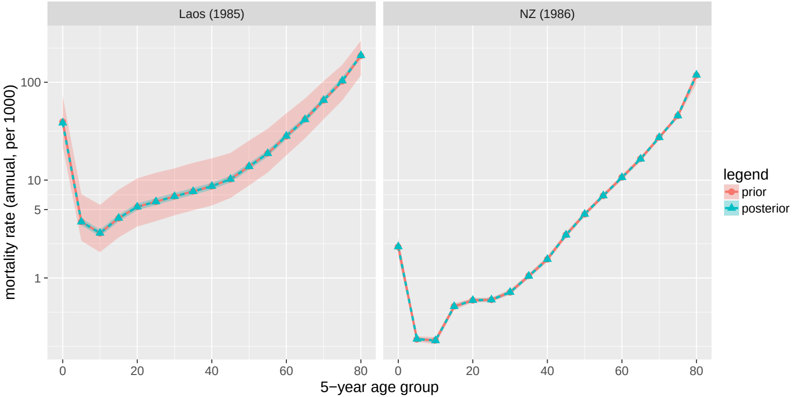
Age Specific Mortality Rate



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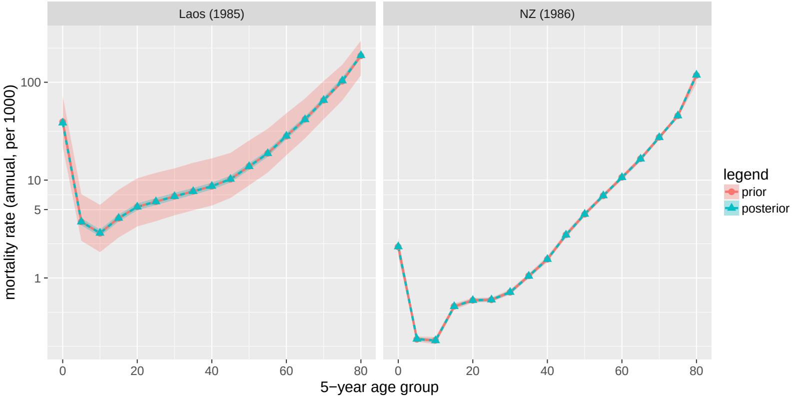
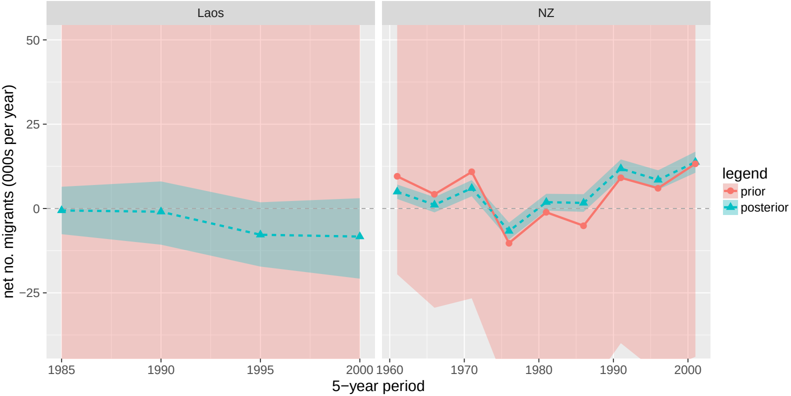


95% Posterior Interval Half-widths

	Mean Half-Width
Laos	2
NZ	0.5

Total Net Migration



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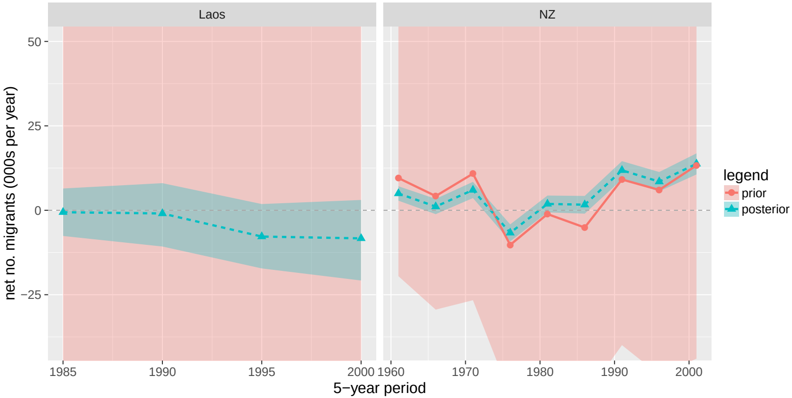

95% Posterior Interval Half-widths

	Counts (000s)
	Mean Half-Width
Laos	9.5
NZ	2.6

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Summary

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- Posterior credible intervals were much more narrow for New Zealand than Laos, reflecting higher data quality.
- The method works for countries with very good data as well as those with fragmentary and unreliable data.

Two-Sex Reconstruction

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- Requires
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  - two-sex CCMPP,
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- As a consequence of including males, two sex reconstruction allows estimation of sex ratios such as
  - sex ratio at birth (SRB),
  - sex ratios of mortality.

Two-Sex CCMPP

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- For ages 5+, projection is done separately for each sex with sex-specific mortality, migration and population counts

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- All-sex births are computed then decomposed because SRB is a parameter of interest.

Two-Sex Hierarchical Model

I. Likelihood $\log n^*_{a,t,\ell} \,|\, n_{a,t,\ell}, \sigma^2_n \sim \text{Normal}\left(\log n_{a,t,\ell}, \sigma^2_n\right)$

II. Projection Model

$$n_{a,t,\ell} \,|\, \mathbf{n}_{t-5,\cdot}, \mathbf{f}_{t-5}, \ldots, SRB_{t-5} = Proj(\mathbf{n}_{t-5,\cdot}, \mathbf{f}_{t-5,\cdot}, \mathbf{s}_{t-5,\cdot}, \mathbf{g}_{t-5,\cdot}, SRB_{t-5})$$

III. Priors on Inputs

$$\log SRB_t \,|\, SRB^*_t, \sigma^2_{SRB} \sim \text{Normal}\left(\log SRB^*_t, \sigma^2_{SRB}\right)$$

$$\log n_{a,t_0,\ell} \,|\, \sigma^2_n \sim \text{Normal}\left(\log n^*_{a,t_0,\ell}, \sigma^2_n\right)$$

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IV. Hyperparameters
$$\sigma^2_v \sim \text{InvGamma}(\alpha_v, \beta_v), \quad v \in \{n, f, s, g, SRB\}$$

- This is very similar to the female-only model. The additions are highlighted.
- SRB is now estimated and varies with time.
- SRB can be interpreted as the odds a birth is male, so this is a model for the log-odds of a male birth.
- There are separate population counts, survival proportions and migration proportions for each sex.
- There is one set of (female) fertility rates. Female and male counts are related *a priori*.

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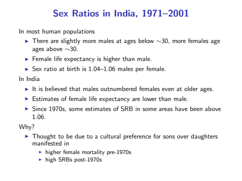
  - high SRBs post-1970s



- In the dissertation I reconstruct the full populations of Laos, Thailand and India. I will discuss only India here.
- 1971 is the first year in which estimates of vital rates independent of the census were available.
- Sex ratios in India are high at all ages. Usually, have more boys at very young ages and more women at very old ages.
- Preference for sons intensified in India by a falling fertility rate.
- Female infanticide being replaced by sex-selective abortion.
- The SRB estimates are from the NFHS-3 (2000–06) and SRS (2004–06).

Sex Ratios in India, 1971–2001

Bayesian Reconstruction will allow us to answer probabilistic questions about sex ratios.

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Research Questions

What is the probability that

i. SRB was unusually high between 1971 and 2001?

ii. The SRB increased in India since 1971?

Initial Estimates

Parameter	Sources	Methods	Rel. Error. (%)
Pop count	Censuses, 5-yearly, 1971–2001	Adjustments for undercount	10
Fertility	Indian SRS, Nat. Fam. Health Sur.	Indirect methods, smoothing	10
Survival	"	Indirect methods, model life tables	10
Migration	None	Centered at zero, large rel. err.	20
SRB	Indian SRS, Nat. Fam. Health Sur.	Indirect methods, smoothing	10

Sex Ratio at Birth: Level

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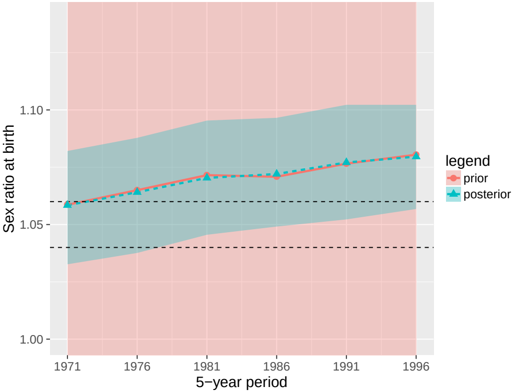

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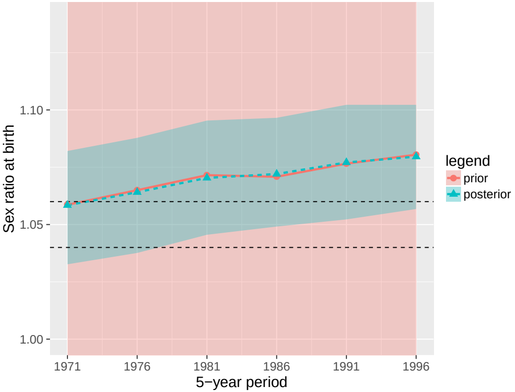
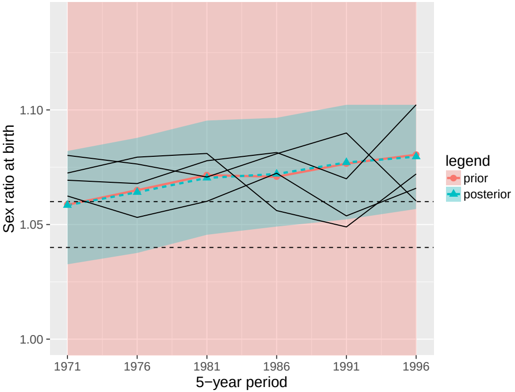


Pr(SRB > 1.06)

1971	1976	1981	1986	1991	1996
0.44	0.66	0.83	0.86	0.93	0.96

Sex Ratio at Birth: Level



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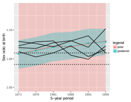

Pr(SRB > 1.06)

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0.44	0.66	0.83	0.86	0.93	0.96

- Posterior uncertainty about SRB is high.
- Typical values are contained in the posterior intervals for all time periods.
- SRB appears to be highest in the 1996 period where the probability that SRB $>$ 1.06 is 0.96.
- Pr(SRB $>$ 1.06) increases over time. To look at this more formally, I computed statistics of increase from each sample trajectory in the posterior, a few of which are shown here.

Sex Ratio at Birth: Trend

- The trend in SRB between 1971 and 2001 in the posterior was summarized by:

Sex Ratio at Birth: Trend

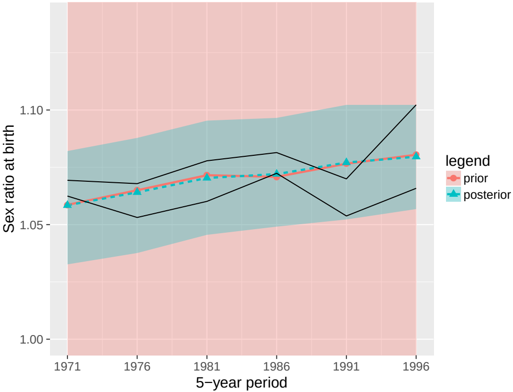
- The trend in SRB between 1971 and 2001 in the posterior was summarized by:
    1. the slope coefficient of the linear model (OLS slope)

    $$\mathrm{SRB}_i = \alpha_i + \beta_i \cdot \mathrm{time} + \epsilon_{i,\mathrm{time}}$$

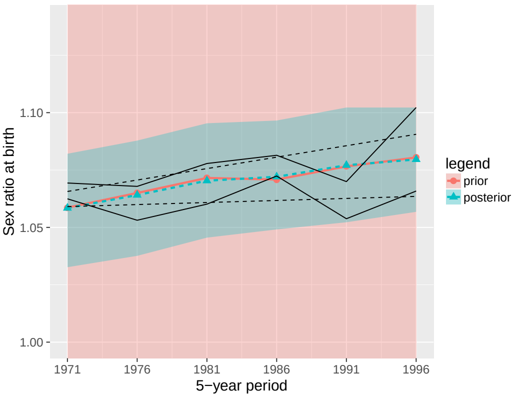
    2. the simple difference

    $$\mathrm{SRB}_{i,1996} - \mathrm{SRB}_{i,1971}$$

Sex Ratio at Birth: Trend



Sex Ratio at Birth: Trend



Sex Ratio at Birth: Trend

Pr(SRB Increased)

Statistic values greater than zero indicate an increase.

Statistic	95% CI	Prob > 0
OLS slope	$[-0.00034, 0.0021]$	0.93
$\text{SRB}_{1996} - \text{SRB}_{1971}$	$[-0.011, 0.054]$	0.92

Conclusions

Research Questions

What is the probability that

i. SRB was unusually high between 1971 and 2001?
   - $\Pr(\text{SRB} > 1.06) = \{0.93\ (1991\text{–}1996),\ 0.96\ (1996\text{–}2001).\}$

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Life Expectancy at Birth

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What is the probability that

i. Female life expectancy was lower than male life expectancy between 1971 and 2001?

ii. The gap between male and female life expectancy decreased since 1971?

Life Expectancy at Birth

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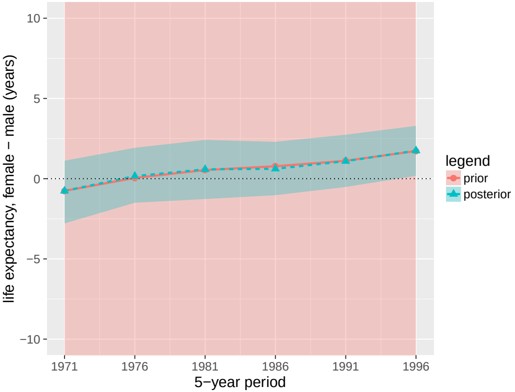

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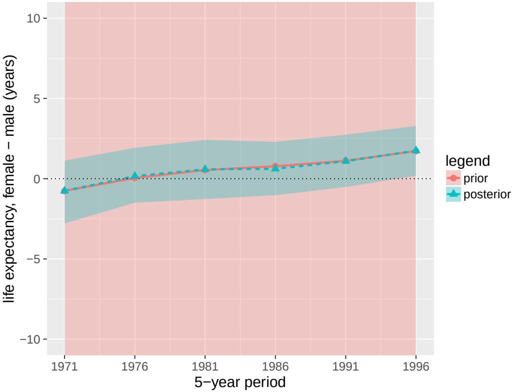
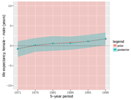


Probability female life expectancy $<$ male

1971	1976	1981	1986	1991	1996
0.79	0.42	0.26	0.22	0.09	0.01

Life Expectancy at Birth



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- This is the same plot as for SRB but shows the difference in LEB; female − male.
- The difference appears most negative in the 1971 period where the probability that fem LE $<$ male LE is 0.79.
- The probabilities decrease over time.

Life Expectancy at Birth

- Repeating the trend analysis done for SRB:
  - Statistic values greater than 0 indicate an increase in the difference over the reconstruction period.

Life Expectancy at Birth

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Statistic	95% CI	Prob > 0
OLS slope (LEB $\sim$ time)	[0.0066, 0.1721]	0.98
$\text{LEB}_{1996} - \text{LEB}_{1971}$	[0.01, 0.17]	0.98

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Summary

- Bayesian Population Reconstruction can provide probabilistic answers to meaningful questions about transformations of the input parameters, as well as the parameters themselves.

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Bayesian Population Reconstruction is a new method for reconstructing population structures of the past.

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- By "coherent" I mean that each sample trajectory in the posterior satisfies the balancing equation.

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(*Refs:* **?**, *J. Am. Stat. Assoc.*; U. of Wash. CSSS working papers 117 & 138; *popReconstruct* on CRAN)