# CS 3430: S19: SciComp with Py
# Exam 1

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## Introduction

1. This exam has 6 problems worth a total of 10 points; it is open books, open notes in that it is perfectly fine for you to look at the reading handouts and/or lecture slides; it is also OK to consult online documentation at `www.python.org`; it is **not OK** to go to `stackoverflow` or any other technical forum and copy and paste someone else's code – this is plagiarism and, if discovered, will be treated as such; you have to do your own work.

2. You must use your source code for assignments 1 - 5 (`deriv.py`, `tof.py`, `const.py`, `maker.py`, `infl.py`, etc.); you may not use any third-party libraries; the imports of standard Python libraries (e.g., `math`, `numpy`, `matplotlib`) are fine.

3. You may not talk to anyone during this exam orally, digitally, or in writing.

4. You have 1 hour and 15 minutes to complete this exam; the exam is due on Canvas at 4:20pm today (02/12/19), which means that the submission will close at 4:19:59pm. I will use the formula $p(t) = 2^t$ to deduct points for late submissions, where $t$ is the number of minutes past 4:20pm. (2 points for 1 minute, 4 - for 2 minutes, etc.) This is a firm deadline. You are always better off submitting something on time for partial credit.

5. The file `cs3430_s19_exam_01.py` is the file where you will write your solutions. It has some starter code for you. You must also submit all the files needed to run your functions in `cs3430_s19_exam_01.py`. The easiest and safest thing for you to do is to zip your whole directory into `exam01.zip` and upload it to Canvas. Write your name and `A#` in the header of `cs3430_s19_exam_01.py`.

6. Do not change the names of the functions in `cs3430_s19_exam_01.py`.

7. Relax and do your best! I wish you all best of luck and, as always, Happy Hacking!

## Problem 1: (1 point)

Write the function `test_deriv(fexpr, gt, lwr, uppr, err)` that takes a function representation of a function `fexpr`, computes its derivative, converts it to a Python function, and compares it with the ground truth function `gt` on the interval `[lwr, uppr]` with the assertion testing $|drvf(x) - gt(x)| \leq err$, where `drvf` is a Python function of the derivative and $x \in$ `[lwr, uppr]`. The parameters `lwr`, `uppr`, and `err` are constant objects.

Test your implementation on the following two functions:

1. $e^{e^x} + ln(x^2 + x + 7)$;
2. $\left(\frac{x^2+5x+10}{x+5}\right)^5$.

## Problem 2: (2 points)

Write the function `max_profit(cost_fun, rev_fun)` that takes function representations of a cost function (`cost_fun`) and a revenue function (`rev_fun`) for a company. Both functions are functions of `x`, where `x` is the number of units to produce. You may assume that the cost function is a cubic polynomial and the revenue function is a line. In both polynomials all members are explicitly represented. The function returns a constant object whose value is the number of units, i.e., `x`, that maximizes the company's profit. Be careful not to return negative constants, because businesses are not interested in producing negative numbers of units. It is OK to return a float constant though, because a business can always round it up or down, depending on the production need. Use your implementation to solve the following problem.

**Test Problem:** A small one-product company estimates that its daily total cost function, in suitable units, is $C(x) = x^3 - 6x^2 + 13x + 15$ and its total revenue function is $R(x) = 28x$. Find the value of $x$ that maximizes the daily profit.

## Problem 3: (2 points)

Health officials in a Utah county use the following model to predict the size of the hive beetle population $P(t)$ at time $t$ from March 1 ($t = 0$): $P(t) = -a_3t^3 + a_2t^2 + a_3$, where $t_l <= t <= t_u$ and $a_1, a_2, a_3$ are real numbers and $t_l$ and $t_u$ are non-negative integers.

Write the function `fastest_growth_time(pm, tl, tu)` that takes a function representation of the hive beetle population model (`pm`), with the a's replaced

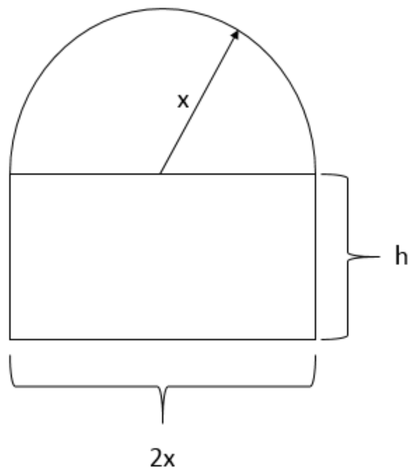


Figure 1: A norman window.

by concrete real numbers, and the lower (`tl`) and upper (`tu`) bounds of the $t$ variable in the model. The function returns a constant object whose float value is the time when the beetle population is changing at the fastest rate. In addition, this function generates the plot of the hive beetle population growth, according to the given model, whose title is "Hive Beetle Population Growth," whose $x$-axis is $t$ from `tl` to `tu`, and whose $y$-axis is $P(t)$. You do not have to save the plot to a file. Showing the plot at the end of the function is sufficient. Use your implementation to solve the following problem.

**Test Problem:** Health officials in a Utah county use the following model to predict the size of the hive beetle population $P(t)$ at time $t$ from March 1 ($t = 0$): $P(t) = -t^3 + 9t^2 + 100$, $0 \leq t \leq 9$. Determine the time when the hive beetle population is growing at the fastest rate.

## Problem 4: (2 points)

A Norman window consists of a rectangle capped by a semicircular region, as shown in Figure 1. Write the function `max_norman_window_area(p)` where $p$ is a constant encoding the window's perimeter. The function returns a float constant encoding the value of $x$ that makes the area of the window as large as possible.

## Problem 5: (2 points)

A patient is receiving radiation treatment for a tumor in his neck. The tumor has a spherical shape. The patient's doctor determined that, at the point of time when the radius $r$ of the tumor is $m$ mm, the radius is changing at a rate of $c$ mm per week. The volume of the tumor is $V = k\pi r^3$ cubic millimeters, where $k$ is a real constant. Write the function `tumor_volume_change_rate(m, c, k)` that takes the constants that encode the concrete values of $m$, $c$, and $k$ and returns a float constant whose value is the rate of change of the tumor's volume at the time when the doctor made the observation. Use your implementation to solve the following problem.

**Test Problem:** Let $m = 20$ mm. Let the radius of the tumor be decreasing at 1.5 mm per week. Let $k = 5/4$. What is the rate at which the volume of the tumor is changing?

## Problem 6 (1 point)

A person is given an injection of $m$ milligrams of penicillin at time $t = 0$. It is known that the amount of penicillin in the person's blood decays exponentially.

Write the function `penicillin_amount(p0, lmbda, t)` that takes three positive constants, `p0` (the initial amount of penicillin injected), `lmbda` (the decay constant $\lambda$) and `t` (the number of hours since injection), and returns a constant whose value is the amount of penicillin, in milligrams, that remains in the person's bloodstream $t$ hours later.

Write the function `penicillin_half_life(lmbda)` that takes the value of the constant $\lambda$ and returns a constant whose value encodes that half-life of penicillin. Use these functions to solve the following problem.

**Test Problem:** A person is given an injection of 300 milligrams of penicillin at time $t = 0$. How much penicillin will remain in the person's bloodstream 5 hours later if $\lambda = 0.6$? What is the biological half-life of the penicillin in this case?