dde_simpleODE

September 28, 2023

1 Solve a Simple ODE System

We consider the simple system of two equations,

$$\frac{dy_1}{dt} = y_2, \qquad \frac{dy_2}{dt} = -y_1, \qquad \text{where} \quad t \in [0, 10],$$

with initial conditions

$$y_1(0) = 0, \quad y_2(0) = 1.$$

The exact solution is

$$y_1 = \sin(t), \quad y_2 = \cos(t).$$

1.1 Implementation

We begin by importing the necesary modules. Note that `torch` is detected automatically.

[1]:
```
import deepxde as dde
import numpy as np
```

```
Using backend: pytorch
Other supported backends: tensorflow.compat.v1, tensorflow, jax, paddle.
paddle supports more examples now and is recommended.
```

We now define the system of ODEs, where the first argument to `ode_system` is the network input, i.e., the t-coordinate, which we represent as `x`. The second argument to ode_system is the network output, which is a 2-dimensional vector where the first component `y[:, 0:1]` is y_1 and the second component `y[:, 1:]` is y_2.

We also consider the initial condition, where we need to implement a function that should return `True` for points inside the subdomain and `False` for the points outside. In our case, the point t of the initial condition is $t = 0$. The argument `x` to boundary is the network input and is a d-dimensional vector, with $d = 1$ here. To facilitate the implementation of boundary, a boolean `on_initial` is used as the second argument. If the point $t = 0$,. then `on_initial` is `True`, otherwise, `on_initial` is `False`.

[2]:
```
def ode_system(x, y):
    """ODE system.
    dy1/dx = y2
    dy2/dx = -y1
    """
```

```
    y1, y2 = y[:, 0:1], y[:, 1:]
    dy1_x = dde.grad.jacobian(y, x, i=0)
    dy2_x = dde.grad.jacobian(y, x, i=1)
    return [dy1_x - y2, dy2_x + y1]


def boundary(_, on_initial):
    return on_initial


def func(x):
    """
    y1 = sin(x)
    y2 = cos(x)
    """
    return np.hstack((np.sin(x), np.cos(x)))
```

Now, we define the geometry, the innitial conditions and the data.

We use 35 training residuall points, 2 training points on thee boundaries, and 100 points for testing the ODE residual.

The argument `solution=func` is the reference solution to compute the error of our solution, and is defined above.

[3]:
```
geom = dde.geometry.TimeDomain(0, 10)
ic1 = dde.icbc.IC(geom, lambda x: 0, boundary, component=0)
ic2 = dde.icbc.IC(geom, lambda x: 1, boundary, component=1)
data = dde.data.PDE(geom, ode_system, [ic1, ic2], 35, 2, solution=func,␣
 ↪num_test=100)
```

We now define the network. We use a fully connected neural network of depth 4 (i.e., 3 hidden layers) and width 50.

[4]:
```
layer_size = [1] + [50] * 3 + [2]
activation = "tanh"
initializer = "Glorot uniform"
net = dde.nn.FNN(layer_size, activation, initializer)
```

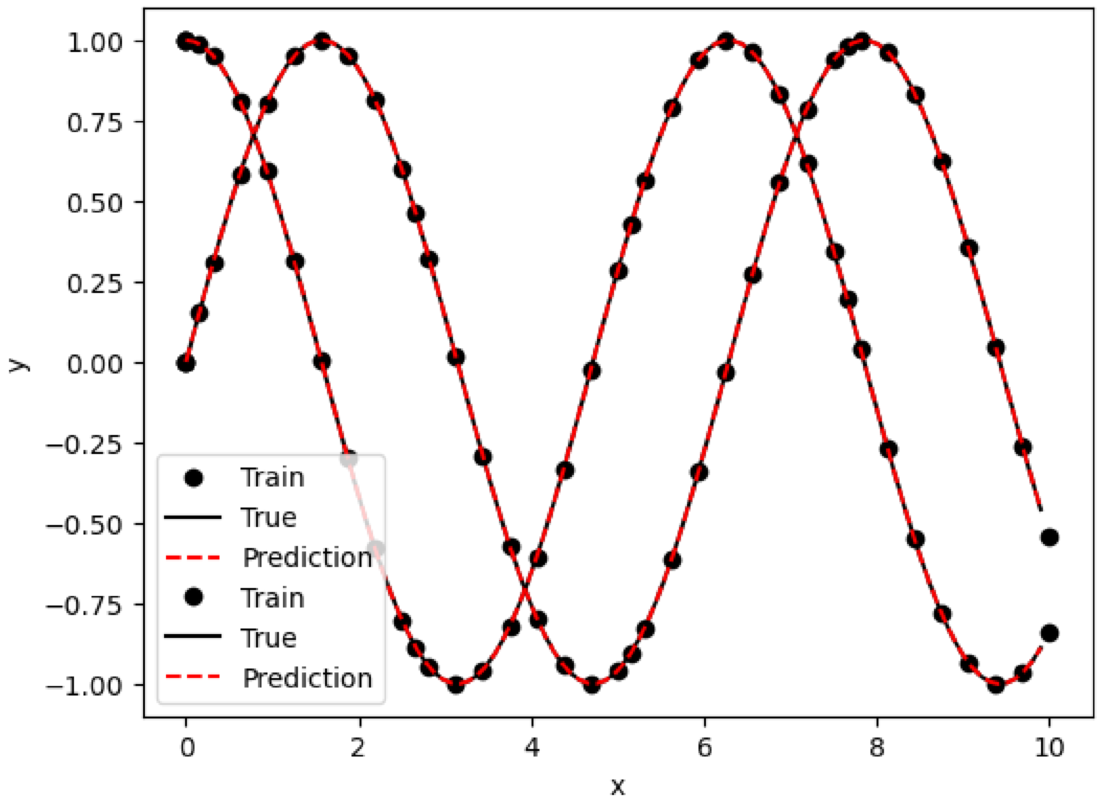
Finally, we define the model, train for 20000 iterations and plot the results.

[5]:
```
model = dde.Model(data, net)
model.compile("adam", lr=0.001, metrics=["l2 relative error"])
losshistory, train_state = model.train(iterations=20000)

dde.saveplot(losshistory, train_state, issave=True, isplot=True)
```

```
Compiling model...
'compile' took 0.000413 s
```

```
Training model…

Step      Train loss                                 Test loss
Test metric
0         [6.60e-02, 5.64e-02, 0.00e+00, 1.00e+00]    [6.57e-02, 5.83e-02,
0.00e+00, 1.00e+00]    [1.05e+00]
1000      [9.25e-03, 7.65e-03, 2.21e-07, 2.51e-04]    [1.02e-02, 7.46e-03,
2.21e-07, 2.51e-04]    [7.63e-01]
2000      [4.84e-03, 4.10e-03, 8.80e-07, 1.02e-04]    [4.99e-03, 4.36e-03,
8.80e-07, 1.02e-04]    [5.77e-01]
3000      [2.49e-03, 3.15e-03, 2.02e-06, 4.89e-05]    [2.62e-03, 3.18e-03,
2.02e-06, 4.89e-05]    [4.20e-01]
4000      [1.65e-03, 2.31e-03, 1.72e-05, 7.83e-06]    [1.72e-03, 1.91e-03,
1.72e-05, 7.83e-06]    [2.86e-01]
5000      [1.52e-04, 3.51e-04, 4.81e-08, 4.15e-06]    [1.72e-04, 2.59e-04,
4.81e-08, 4.15e-06]    [9.61e-02]
6000      [2.09e-05, 7.38e-05, 2.88e-08, 1.69e-07]    [2.35e-05, 5.54e-05,
2.88e-08, 1.69e-07]    [2.68e-02]
7000      [1.44e-04, 1.65e-04, 2.64e-05, 1.16e-06]    [1.44e-04, 1.38e-04,
2.64e-05, 1.16e-06]    [2.14e-02]
8000      [4.20e-06, 1.30e-05, 1.41e-09, 9.64e-09]    [4.33e-06, 1.03e-05,
1.41e-09, 9.64e-09]    [6.23e-03]
9000      [1.82e-06, 6.88e-06, 4.84e-10, 2.78e-09]    [2.06e-06, 5.93e-06,
4.84e-10, 2.78e-09]    [3.47e-03]
10000     [1.01e-05, 9.96e-06, 3.17e-06, 2.05e-06]    [1.11e-05, 9.92e-06,
3.17e-06, 2.05e-06]    [3.61e-03]
11000     [9.71e-07, 2.85e-06, 7.21e-12, 1.20e-09]    [1.23e-06, 2.75e-06,
7.21e-12, 1.20e-09]    [1.49e-03]
12000     [8.85e-07, 2.02e-06, 3.13e-10, 8.19e-12]    [1.14e-06, 2.02e-06,
3.13e-10, 8.19e-12]    [1.12e-03]
13000     [5.86e-06, 6.08e-06, 2.45e-06, 2.48e-06]    [6.33e-06, 6.14e-06,
2.45e-06, 2.48e-06]    [2.53e-03]
14000     [1.97e-06, 3.18e-06, 7.32e-07, 1.16e-06]    [2.01e-06, 3.23e-06,
7.32e-07, 1.16e-06]    [1.76e-03]
15000     [2.04e-05, 2.99e-05, 1.07e-05, 1.65e-05]    [2.01e-05, 3.07e-05,
1.07e-05, 1.65e-05]    [5.70e-03]
16000     [7.92e-07, 8.20e-07, 2.16e-08, 4.14e-09]    [9.82e-07, 8.64e-07,
2.16e-08, 4.14e-09]    [5.76e-04]
17000     [1.29e-04, 4.47e-05, 5.88e-05, 4.57e-06]    [1.34e-04, 3.70e-05,
5.88e-05, 4.57e-06]    [1.10e-02]
18000     [7.04e-07, 6.36e-07, 1.14e-08, 2.54e-09]    [9.04e-07, 7.15e-07,
1.14e-08, 2.54e-09]    [5.10e-04]
19000     [8.94e-07, 8.22e-07, 1.64e-07, 1.55e-07]    [1.01e-06, 8.78e-07,
1.64e-07, 1.55e-07]    [7.58e-04]
20000     [6.03e-07, 4.67e-07, 5.61e-11, 4.07e-11]    [7.71e-07, 5.53e-07,
5.61e-11, 4.07e-11]    [3.73e-04]

Best model at step 20000:
```

```
  train loss: 1.07e-06
  test loss: 1.32e-06
  test metric: [3.73e-04]

'train' took 13.001740 s

Saving loss history to
/Users/markasch/Dropbox/3Teaching/Assim/Assim_ML_2023_Caraga/02course-
advanced/02Examples/01_SciML/DDE/loss.dat …
Saving training data to
/Users/markasch/Dropbox/3Teaching/Assim/Assim_ML_2023_Caraga/02course-
advanced/02Examples/01_SciML/DDE/train.dat …
Saving test data to
/Users/markasch/Dropbox/3Teaching/Assim/Assim_ML_2023_Caraga/02course-
advanced/02Examples/01_SciML/DDE/test.dat …
```


1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
y
0
2
4
6
8
10
x
Train
True
Prediction
Train
True
Prediction

```
[ ]:
```