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Notation

$\mathbb{R}^n$ state space
$\mathbb{R}^p$ observation space
$\mathbb{R}^m$ ensemble space, $i = 1, \ldots, m$
t_k time, $k = 1, \ldots, K$
$\mathbf{I}$ identity matrix: $\mathbf{I}_n, \mathbf{I}_m, \mathbf{I}_p$
$\mathbf{x}$ vector
$\mathbf{x}^{\mathrm{t}}$ true state vector
$\mathbf{x}^{\mathrm{a}}$ analysis vector
$\mathbf{x}^{\mathrm{b}}$ background vector
$\mathbf{x}^{\mathrm{f}}$ forecast vector
$\mathbf{y}^{\mathrm{o}}$ observation vector
$\boldsymbol{\epsilon}^{\mathrm{a}}$ analysis error
$\boldsymbol{\epsilon}^{\mathrm{b}}$ background error
$\boldsymbol{\epsilon}^{\mathrm{f}}$ forecast error
$\boldsymbol{\epsilon}^{\mathrm{o}}$ observation error
$\boldsymbol{\epsilon}^{\mathrm{q}}$ model error
$\mathbf{M}_k$ linear model operator: $\mathbf{x}_{k+1} = \mathbf{M}_{k+1}\mathbf{x}_k$, with $\mathbf{M}_{k+1} = \mathbf{M}_{k+1:k}$ model from time step k to time step $k+1$; $\mathscr{M}$ nonlinear model operator
$\mathbf{X}_{\mathrm{a}}$ analysis perturbation matrix
$\mathbf{X}_{\mathrm{f}}$ forecast perturbation matrix
$\mathbf{P}^{\mathrm{f}}$ forecast error covariance matrix
$\mathbf{P}^{\mathrm{a}}$ analysis error covariance matrix
$\mathbf{K}$ Kalman gain matrix
$\mathbf{B}$ background error covariance matrix
$\mathbf{H}$ linearized observation operator; $\mathscr{H}$ nonlinear observation operator
$\mathbf{Q}$ model error covariance matrix
$\mathbf{R}$ observation error covariance matrix
$\mathbf{d}$ innovation vector
(j) iteration index of a variational assimilation (in parentheses).
$\mathbf{w}$ coefficients in ensemble space (ensemble transform)

Preface

This book places data assimilation (DA) into the broader context of inverse problems and the theory, methods, and algorithms that are used for their solution. It strives to provide a framework and new insight into the inverse problem nature of DA—the book emphasizes "why" and not just "how." We cover both statistical and variational approaches to DA (see Figure 1) and give an important place to the latest hybrid methods that combine the two. Since the methods and diagnostics are emphasized, readers will readily be able to apply them to their own, precise field of study. This will be greatly facilitated by numerous examples and diverse applications. The applications are taken from the following fields: geophysics and geophysical flows, environmental acoustics, medical imaging, mechanical and biomedical engineering, urban planning, economics, and finance.

In fact, this book is about *building bridges*—bridges between inverse problems and DA, bridges between variational and statistical approaches, bridges between statistics and inverse problems. These bridges will enable you to cross valleys and moats, thus avoiding the dangers that are most likely/possibly lurking down there. These bridges will allow you to fetch/go and get/retrieve different approaches and better understanding of the vast, and sometimes insular, domains of DA and inverse problems, stochastic and deterministic approaches, and direct and inverse problems. We claim that by assembling these, by reconciling these, we will be better armed to confront and tackle the grand societal challenges of today, broadly defined as "global change" issues—such as climate change, disaster prediction and mitigation, and nondestructive and noninvasive testing and imaging.

The aim of the book is thus to provide a comprehensive guide for advanced undergraduate and early graduate students and for practicing researchers and engineers engaged in (partial) differential equation–based DA, inverse problems, optimization, and optimal control—we will emphasize the close relationships among all of these. The reader will be presented with a statistical approach and a variational approach and will find pointers to all the numerical methods needed for either. Of course, the applications will furnish many case studies.

The book favours a continuous (infinite-dimensional) approach to the underlying inverse problems, and we do not make the distinction between continuous and discrete problems—every continuous problem, after discretization, yields a discrete (finite-dimensional) problem. Moreover, continuous problems admit a far richer and more extensive mathematical theory, and though DA (via the Kalman filter (KF)) is *in fine* a discrete approach, the variational analysis will be performed on the continuous model. Discrete inverse problems (finite dimensional) are very well presented in a number of excellent books, such as those of Lewis et al. [2006], Vogel [2002], and Hansen [2010], the latter of which has a strong emphasis on regularization methods.

Some advanced calculus and tools from linear algebra, real analysis, and numerical analysis are required in the presentation. We introduce and use Hadamard's well-posedness theory to explain and understand both *why things work* and *why they go wrong*. Throughout the book, we observe a maximum of mathematical rigor but with a minimum of formalism. This rigor is extremely important in practice, since it enables us to eliminate possible sources of error in the algorithmic and numerical implementations.

In summary, this is really a PDE-based book on inverse and DA modeling—readers interested in the specific application to meteorology or oceanography should additionally consult other sources, such as Lewis et al. [2006] and Evensen [2009]. Those who require a more mathematical approach to inverse problems are referred to Kirsch [1996] and Kaipio and Somersalo [2005], and for DA to the recent monographs of Law et al. [2015] and Reich and Cotter [2015].

Proposed pathways through the book are as follows (this depends on the level of the reader):

- The "debutant" reader is encouraged to study the first chapter in depth, since it will provide a basic understanding and the means to choose the most appropriate approach (variational or statistical).
- The experienced reader can jump directly to Chapter 2 or Chapter 3 according to the chosen or best-adapted approach.
- All readers are encouraged to initially skim through the examples and applications sections of Part III to be sure of the best match to their type of problem (by seeing what kind of problem is the closest to their own)—these can then be returned to later, after having mastered the basic methods and algorithms of Part I or eventually the advanced ones of Part II.
- For the most recent approaches, the reader or practitioner is referred to Part II and in particular to Chapters 4 and 7.

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[1]Centre International de Mathématiques Pures et Appliquées, Nice, France.
[2]Centre International de Rencontres Mathématiques, Marseille, France.

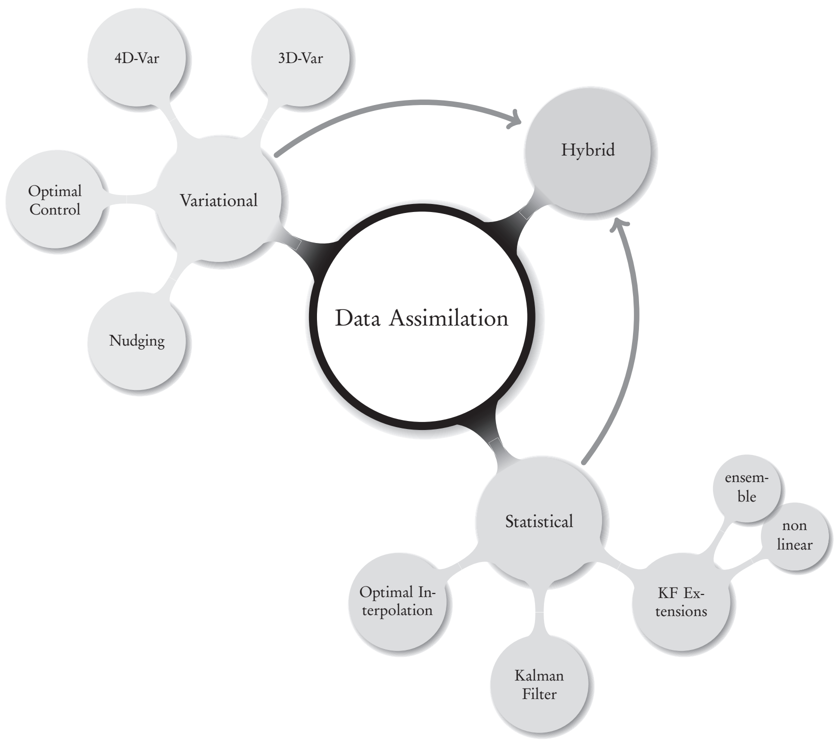

Figure 1. *The big picture for DA methods and algorithms.*