dde_Poisson2D

September 28, 2023

1 Poisson equation over an L-shaped domain

We solve

$$-u_{xx} - u_{yy} = 1, \qquad \Omega = [-1,1]^2 \backslash [0,1]^2$$

with Dirichlet boundary conditions everywhere,

$$u(x,y) = 0, \qquad (x,y) \in \partial\Omega.$$

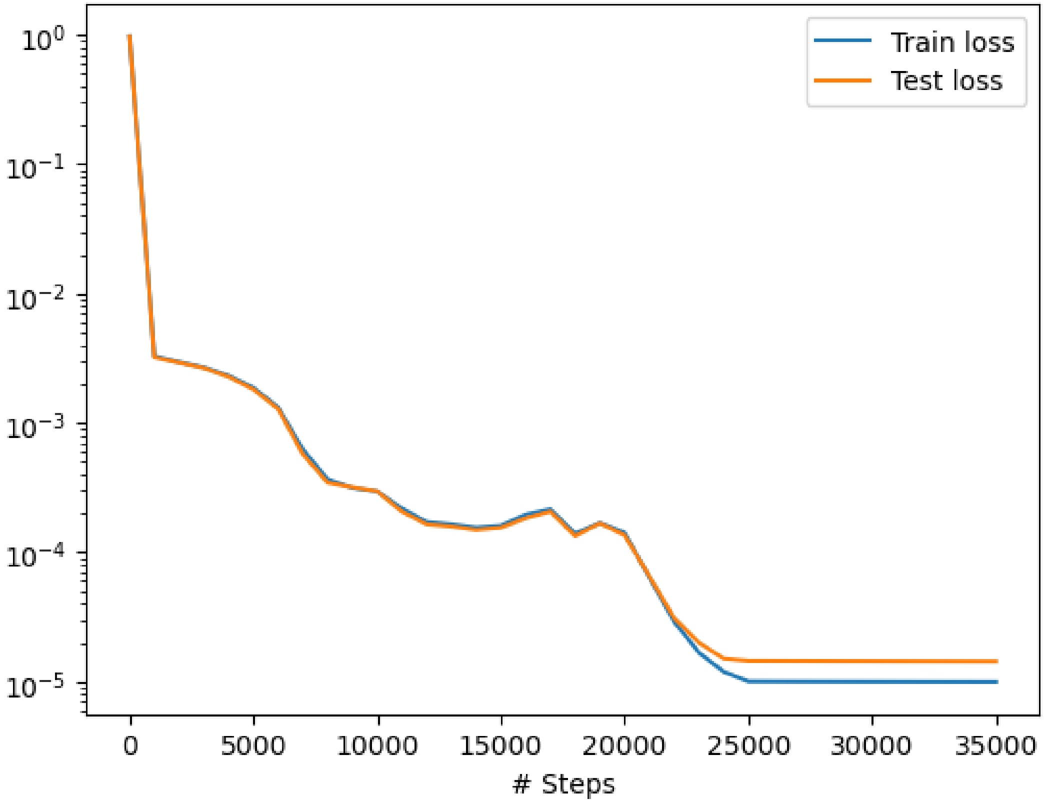
Please see DeepXDE doc for fuller details.

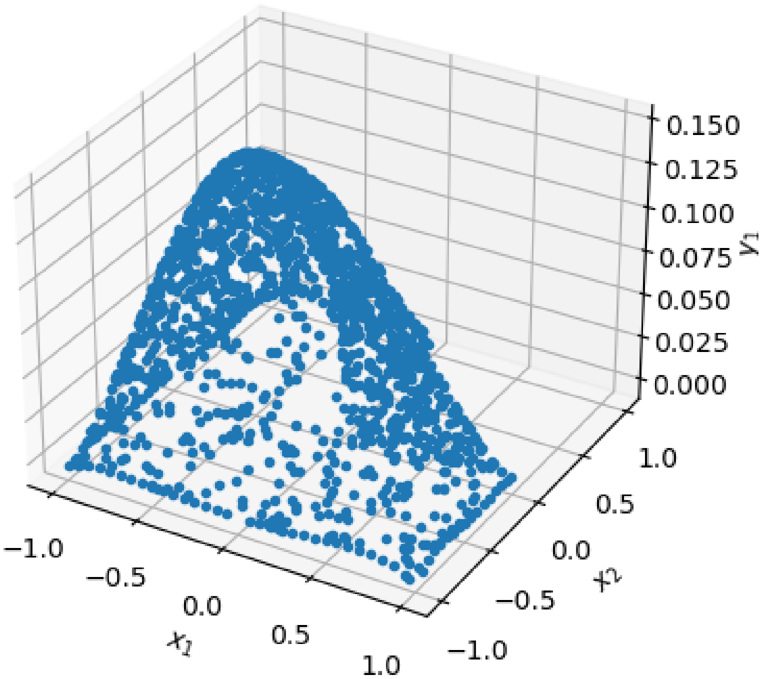
[1]:

```
import deepxde as dde


def pde(x, y):
    dy_xx = dde.grad.hessian(y, x, i=0, j=0)
    dy_yy = dde.grad.hessian(y, x, i=1, j=1)
    return -dy_xx - dy_yy - 1


def boundary(_, on_boundary):
    return on_boundary


geom = dde.geometry.Polygon([[0, 0], [1, 0], [1, -1], [-1, -1], [-1, 1], [0,␣
 ↪1]])
bc = dde.icbc.DirichletBC(geom, lambda x: 0, boundary)

data = dde.data.PDE(geom, pde, bc, num_domain=1200, num_boundary=120,␣
 ↪num_test=1500)
net = dde.nn.FNN([2] + [50] * 4 + [1], "tanh", "Glorot uniform")
model = dde.Model(data, net)

model.compile("adam", lr=0.001)
model.train(iterations=20000)
model.compile("L-BFGS")
losshistory, train_state = model.train()
dde.saveplot(losshistory, train_state, issave=True, isplot=True)
```

```
Using backend: pytorch
Other supported backends: tensorflow.compat.v1, tensorflow, jax, paddle.
paddle supports more examples now and is recommended.

Warning: Polygon.uniform_points not implemented. Use random_points instead.
Compiling model…
'compile' took 0.000060 s

Training model…

Step      Train loss                Test loss                 Test metric
0         [9.15e-01, 5.93e-02]      [9.13e-01, 5.93e-02]      []
1000      [2.79e-04, 3.00e-03]      [2.11e-04, 3.00e-03]      []
2000      [1.14e-04, 2.84e-03]      [8.72e-05, 2.84e-03]      []
3000      [1.09e-04, 2.58e-03]      [7.95e-05, 2.58e-03]      []
4000      [1.94e-04, 2.13e-03]      [1.46e-04, 2.13e-03]      []
5000      [2.70e-04, 1.60e-03]      [2.16e-04, 1.60e-03]      []
6000      [2.73e-04, 1.05e-03]      [2.37e-04, 1.05e-03]      []
7000      [1.82e-04, 4.42e-04]      [1.31e-04, 4.42e-04]      []
8000      [1.12e-04, 2.55e-04]      [9.07e-05, 2.55e-04]      []
9000      [1.19e-04, 1.95e-04]      [1.23e-04, 1.95e-04]      []
10000     [1.28e-04, 1.67e-04]      [1.29e-04, 1.67e-04]      []
11000     [7.40e-05, 1.44e-04]      [6.20e-05, 1.44e-04]      []
12000     [3.42e-05, 1.37e-04]      [2.75e-05, 1.37e-04]      []
13000     [2.77e-05, 1.37e-04]      [2.16e-05, 1.37e-04]      []
14000     [2.55e-05, 1.30e-04]      [1.92e-05, 1.30e-04]      []
15000     [2.98e-05, 1.31e-04]      [2.41e-05, 1.31e-04]      []
16000     [6.45e-05, 1.31e-04]      [5.24e-05, 1.31e-04]      []
17000     [8.38e-05, 1.31e-04]      [7.53e-05, 1.31e-04]      []
18000     [2.01e-05, 1.19e-04]      [1.42e-05, 1.19e-04]      []
19000     [4.77e-05, 1.21e-04]      [4.66e-05, 1.21e-04]      []
20000     [2.42e-05, 1.17e-04]      [1.82e-05, 1.17e-04]      []

Best model at step 18000:
  train loss: 1.40e-04
  test loss: 1.34e-04
  test metric: []

'train' took 141.981709 s

Compiling model…
'compile' took 0.000178 s

Training model…

Step      Train loss                Test loss                 Test metric
20000     [2.42e-05, 1.17e-04]      [1.82e-05, 1.17e-04]      []
21000     [1.35e-05, 5.03e-05]      [1.50e-05, 5.03e-05]      []
```

```
22000     [9.47e-06, 1.97e-05]    [1.14e-05, 1.97e-05]    []
23000     [6.20e-06, 1.06e-05]    [9.44e-06, 1.06e-05]    []
24000     [4.46e-06, 7.46e-06]    [7.63e-06, 7.46e-06]    []
25000     [3.46e-06, 6.58e-06]    [7.91e-06, 6.58e-06]    []
26000     [3.45e-06, 6.58e-06]    [7.90e-06, 6.58e-06]    []
27000     [3.44e-06, 6.58e-06]    [7.89e-06, 6.58e-06]    []
28000     [3.44e-06, 6.57e-06]    [7.88e-06, 6.57e-06]    []
29000     [3.43e-06, 6.57e-06]    [7.87e-06, 6.57e-06]    []
30000     [3.43e-06, 6.56e-06]    [7.86e-06, 6.56e-06]    []
31000     [3.43e-06, 6.56e-06]    [7.85e-06, 6.56e-06]    []
32000     [3.43e-06, 6.56e-06]    [7.85e-06, 6.56e-06]    []
33000     [3.42e-06, 6.55e-06]    [7.84e-06, 6.55e-06]    []
34000     [3.42e-06, 6.55e-06]    [7.83e-06, 6.55e-06]    []
35000     [3.42e-06, 6.55e-06]    [7.82e-06, 6.55e-06]    []

Best model at step 35000:
  train loss: 9.97e-06
  test loss: 1.44e-05
  test metric: []

'train' took 133.421110 s

Saving loss history to
/Users/markasch/Dropbox/3Teaching/Assim/Assim_ML_2023_Caraga/02course-
advanced/02Examples/01_SciML/DDE/loss.dat …
Saving training data to
/Users/markasch/Dropbox/3Teaching/Assim/Assim_ML_2023_Caraga/02course-
advanced/02Examples/01_SciML/DDE/train.dat …
Saving test data to
/Users/markasch/Dropbox/3Teaching/Assim/Assim_ML_2023_Caraga/02course-
advanced/02Examples/01_SciML/DDE/test.dat …
```


Train loss
Test loss
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
5000
10000
15000
20000
25000
30000
35000
# Steps


0.150
0.125
0.100
0.075
0.050
0.025
0.000
y_1
−1.0
−0.5
0.0
0.5
1.0
x_1
1.0
0.5
0.0
−0.5
−1.0
x_2

5

[ ]: