opt_visu

September 12, 2023

1 Optimization Visualization

We want to visualize the progress of different optimization algorithms, to better understand the difficulties an pitfalls. We will show how to create an animation.

1.1 Formulation

We consider the Himmelblau function, defined as

$$f(x, y) = (x^2 + y - 11)^2 + (x + y^2 - 7)^2.$$

It has four identical local minima

$$\begin{aligned}
f(x_*) = 0 \quad &\text{at} \quad x_* = (3, 2), \\
f(x_*) = 0 \quad &\text{at} \quad x_* = (-2.8051, 3.2832), \\
f(x_*) = 0 \quad &\text{at} \quad x_* = (-3.7793, -3.2832), \\
f(x_*) = 0 \quad &\text{at} \quad x_* = (3.5845, -1.8481),
\end{aligned}$$

and one local maximum,

$$f(x_*) = 181.167 \quad \text{at} \quad x_* = (-0.2708, -0.9230).$$

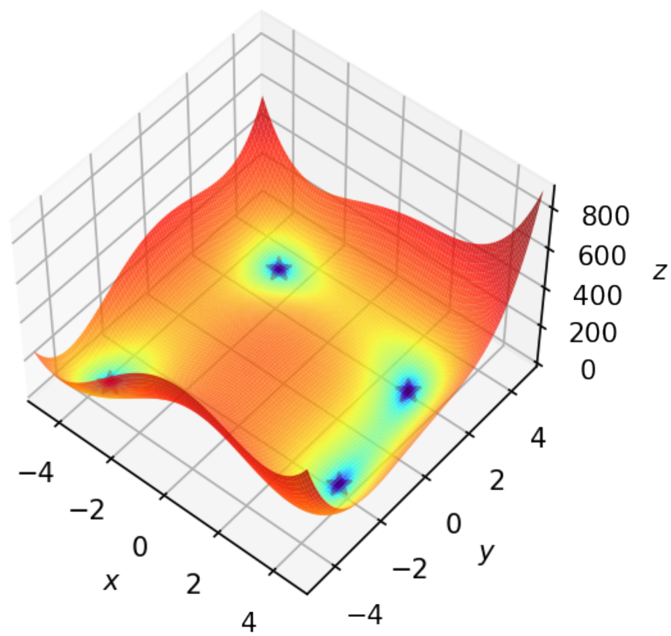
The Himmelblau Function is defined in the two dimensional space, and is used to test the performance properties of optimization algorithms such as: - Convergence rate - Precision - Robustness

The Himmelblau function has the following characteristics: - Multi-modal - Non-separable - Non-convex - Continuous

[1]:
```
# imports
import matplotlib.pyplot as plt
import numpy as np
from mpl_toolkits.mplot3d import Axes3D
from matplotlib.colors import LogNorm
from matplotlib import animation
from IPython.display import HTML
import math
#from itertools import zip_longest
```

```python
#from sklearn.datasets import make_classification
```

[33]:
```python
f  = lambda x, y: (x**2 +y -11)**2 + (x + y**2 -7)**2
xmin, xmax, xstep = -5, 5, .1
ymin, ymax, ystep = -5, 5, .1
# 3D plot
x, y = np.meshgrid(np.arange(xmin, xmax + xstep, xstep), np.arange(ymin, ymax +␣
 ↪ystep, ystep))
z = f(x, y)
# 4 minima to display on the plot
min1 = np.array([[3.], [2.]])
min2 = np.array([-2.80, 3.13])
min3 = np.array([-3.78, -3.3])
min4 = np.array([3.6, -1.85])

# plot
fig = plt.figure(dpi=150, figsize=(6, 4))
ax = plt.axes(projection='3d', elev=50, azim=-50)

ax.plot_surface(x, y, z, norm=LogNorm(), rstride=1, cstride=1,
                edgecolor='none', alpha=.8, cmap=plt.cm.jet)
ax.plot(*min1, f(*min1), 'r*', markersize=10)
ax.plot(*min2, f(*min2), 'r*', markersize=10)
ax.plot(*min3, f(*min3), 'r*', markersize=10)
ax.plot(*min4, f(*min4), 'r*', markersize=10)

ax.set_xlabel('$x$')
ax.set_ylabel('$y$')
ax.set_zlabel('$z$')

ax.set_xlim((xmin, xmax))
ax.set_ylim((ymin, ymax))

plt.show()
```



1.1.1 Gradients

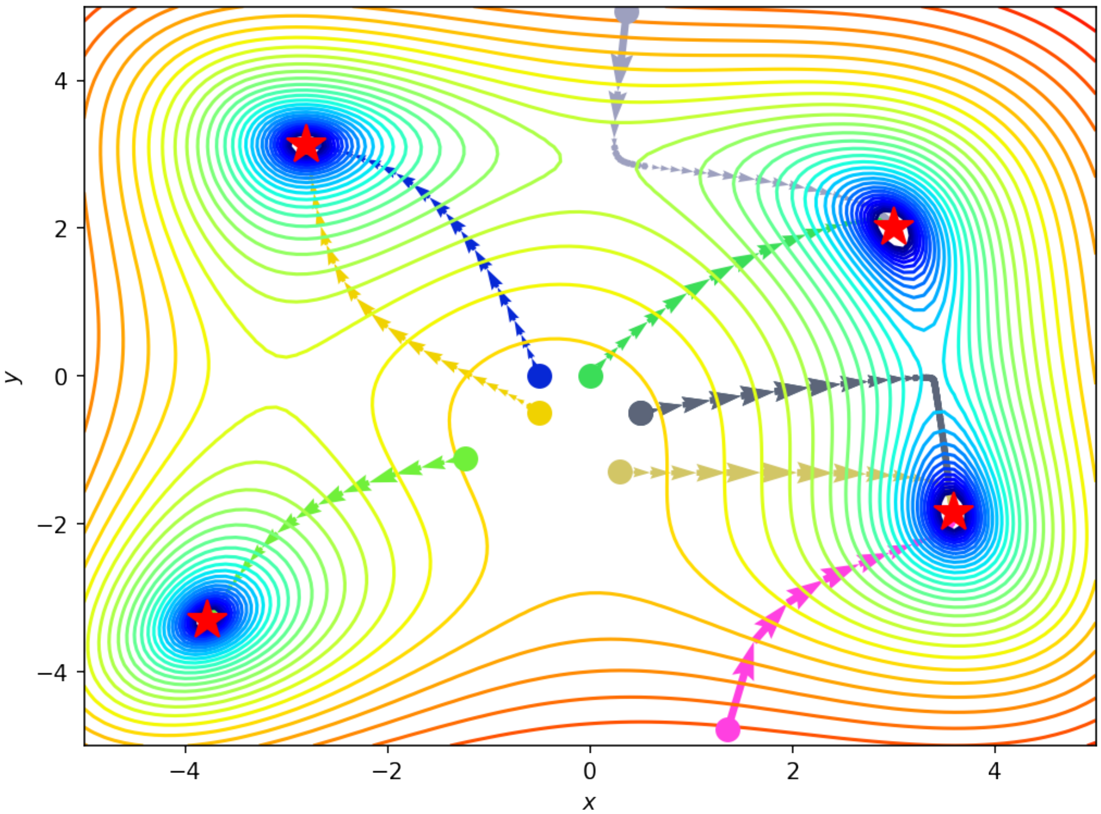
We need the gradients to manually code the descent algorithm that stores the descent path for subsequent plotting.

[11]:
```
dx = lambda x, y: 2*(x**2 +y -11)*2*x + 2*(x + y**2 -7)
dy = lambda x, y: 2*(x**2 +y -11) + 2*(x + y**2 -7)*2*y

def gradient_descent(init_point,learning_rate, num_epochs):
    x0 = init_point[0]
    y0 = init_point[1]

    path = np.zeros((2,num_epochs+1))

    path[0][0] = x0
    path[1][0] = y0

    for i in range(num_epochs):
        x_ =  learning_rate*dx(x0,y0)
        y_ =  learning_rate*dy(x0,y0)
        x0 -= x_
        y0 -= y_
        path[0][i+1] = x0
        path[1][i+1] = y0

    return (path,(x0,y0))
```

We define a few starting points that will each be used to initilize the GD, then display their paths.

[12]:
```
begin_points = [
    np.array([0.,0.]),
    np.array([-0.5,0.]),
    np.array([-0.5,-0.5]),
    np.array([0.5,-0.5]),
    np.array([0.5,-0.5]),
    np.array([-1.23633,-1.11064]),
    np.array([0.295466,-1.2946]),
    np.array([0.3616,4.9298]),
    np.array([1.362,-4.774]),
               ]

paths = []
for begin_point in begin_points:
    path,_ = gradient_descent(begin_point,learning_rate=0.001, num_epochs=300)
    path = path[:, [i for i in range(0,path.shape[1],5)]]
    paths.append(path)
```

[25]:
```
# visualize on a contour plot

fig, ax = plt.subplots(dpi=150, figsize=(8, 6))
#
ax.contour(x, y, z, levels=np.logspace(0, 3.25, 35), norm=LogNorm(), cmap=plt.
 ↪cm.jet)

for i,path in enumerate(paths):
    color = c=np.random.rand(3,)
    ax.quiver(path[0,:-1], path[1,:-1], path[0,1:]-path[0,:-1], path[1,1:
 ↪]-path[1,:-1],\
              scale_units='xy', angles='xy', scale=1, color=color)
    ax.plot(*begin_points[i], color=color ,marker='o', markersize=10)

ax.plot(*minima1_, 'r*', markersize=18)
ax.plot(*minima2_, 'r*', markersize=18)
ax.plot(*minima3_, 'r*', markersize=18)
ax.plot(*minima4_, 'r*', markersize=18)

ax.set_xlabel('$x$')
ax.set_ylabel('$y$')

ax.set_xlim((xmin, xmax))
ax.set_ylim((ymin, ymax))

plt.show()
```



Note how the starting point influences the path, and the local minimum attained.

1.1.2 Animation

We generate matplotlib animations for multiple paths as follows.

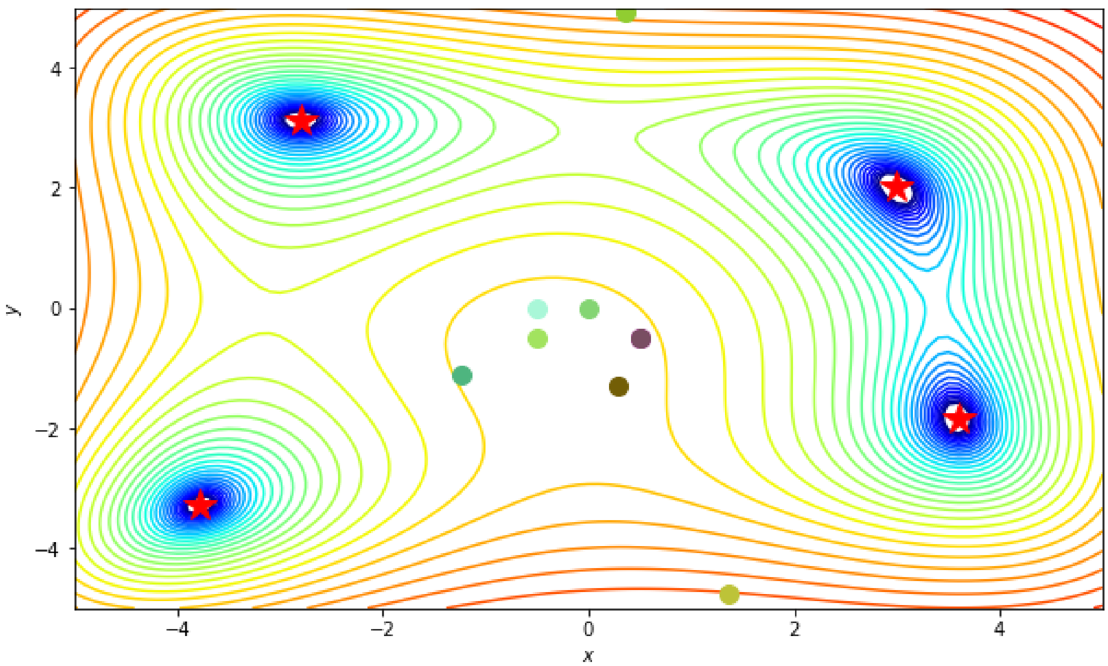
[14]:
```
class TrajectoryAnimation(animation.FuncAnimation):

    def __init__(self, *paths, labels=[], fig=None, ax=None, frames=None,
                 interval=60, repeat_delay=5, blit=True, **kwargs):

        if fig is None:
            if ax is None:
                fig, ax = plt.subplots()
            else:
                fig = ax.get_figure()
        else:
            if ax is None:
                ax = fig.gca()

        self.fig = fig
```

```
        self.ax = ax

        self.paths = paths

        if frames is None:
            frames = max(path.shape[1] for path in paths)

        self.lines = [ax.plot([], [], label=label, lw=2)[0]
                      for _, label in zip_longest(paths, labels)]
        self.points = [ax.plot([], [], 'o', color=line.get_color())[0]
                       for line in self.lines]

        super(TrajectoryAnimation, self).__init__(fig, self.animate,␣
↪init_func=self.init_anim,
                                                  frames=frames,␣
↪interval=interval, blit=blit,
                                                  repeat_delay=repeat_delay,␣
↪**kwargs)

    def init_anim(self):
        for line, point in zip(self.lines, self.points):
            line.set_data([], [])
            point.set_data([], [])
        return self.lines + self.points

    def animate(self, i):
        for line, point, path in zip(self.lines, self.points, self.paths):
            line.set_data(*path[::,:i])
            point.set_data(*path[::,i-1:i])
        return self.lines + self.points
```

[26]:

```
from itertools import zip_longest

fig, ax = plt.subplots(figsize=(10, 6))
ax.contour(x, y, z, levels=np.logspace(0, 3.25, 35), norm=LogNorm(), cmap=plt.
↪cm.jet)

for i,path in enumerate(paths):
    color = c=np.random.rand(3,)
    ax.plot(*begin_points[i].reshape(-1,1), color=color ,marker='o',␣
↪markersize=10)

ax.plot(*minima1_, 'r*', markersize=18)
ax.plot(*minima2_, 'r*', markersize=18)
ax.plot(*minima3_, 'r*', markersize=18)
ax.plot(*minima4_, 'r*', markersize=18)
```

```python
ax.set_xlabel('$x$')
ax.set_ylabel('$y$')

ax.set_xlim((xmin, xmax))
ax.set_ylim((ymin, ymax))

anim = TrajectoryAnimation(*paths, ax=ax)

# ax.legend(loc='upper left')
```



[27]:
```python
HTML(anim.to_html5_video())
```

[27]: <IPython.core.display.HTML object>

[ ]:
```python

```