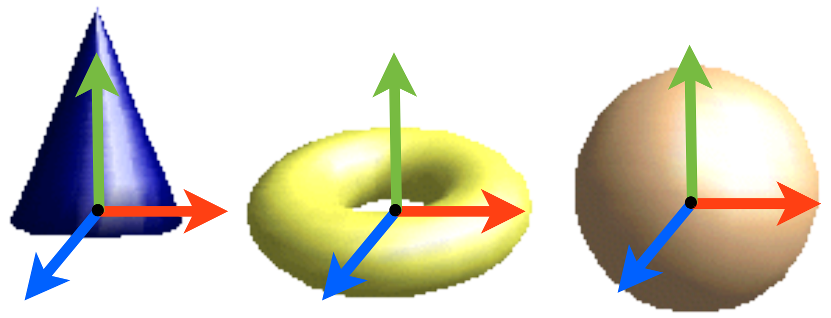
Viewing Transformations

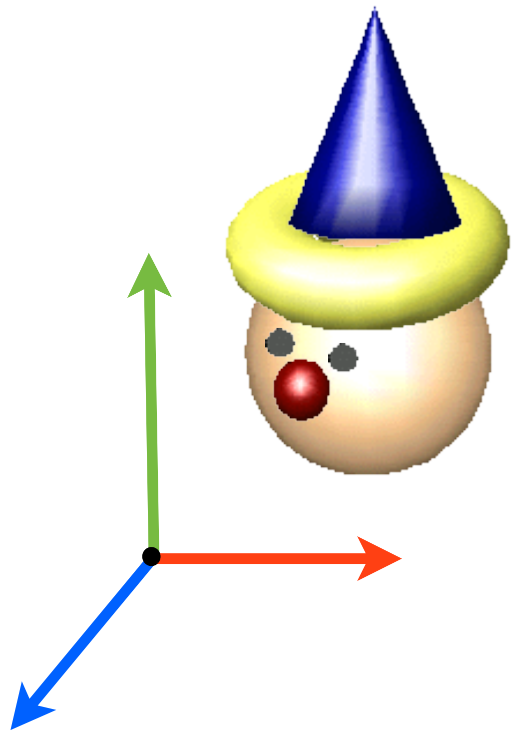
Viewing transformations

World Coordinates → [Modeling | Camera | Projection | Viewport] → Screen Space (pixels)

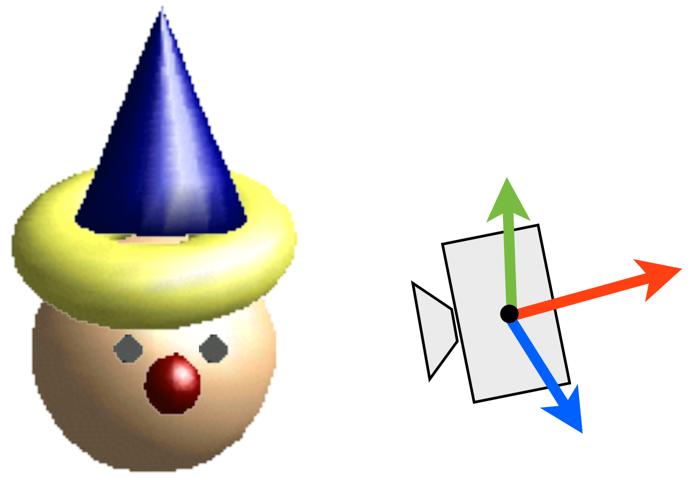
Coordinate Systems



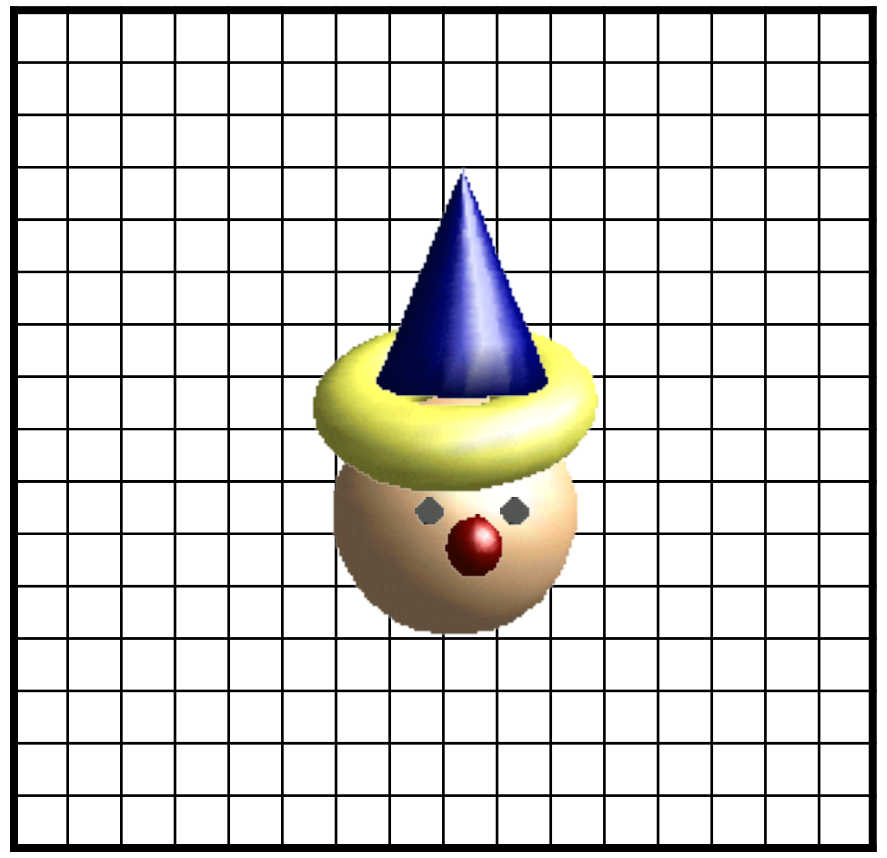
object coordinates



world coordinates



camera coordinates



screen coordinates

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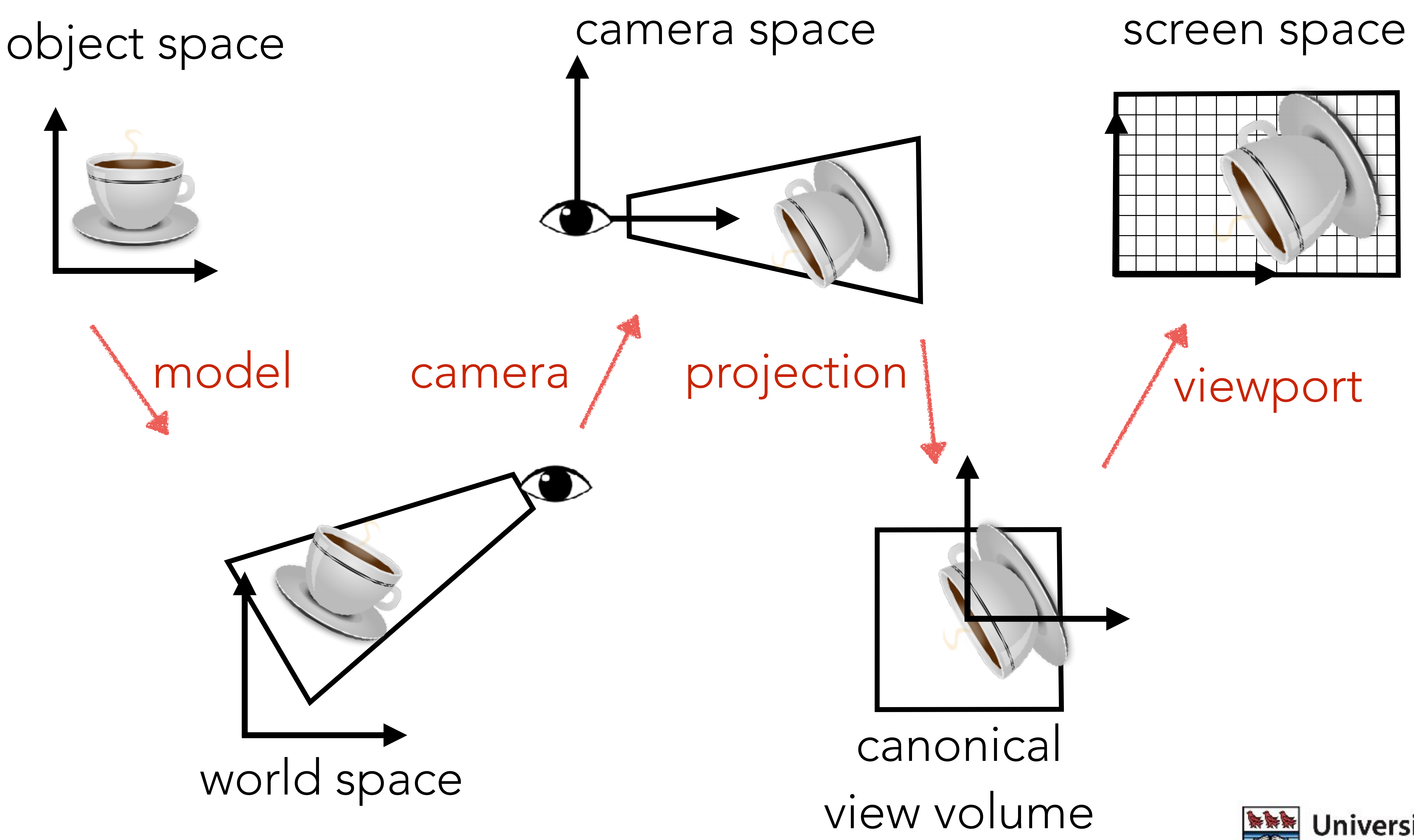
Viewing Transformation



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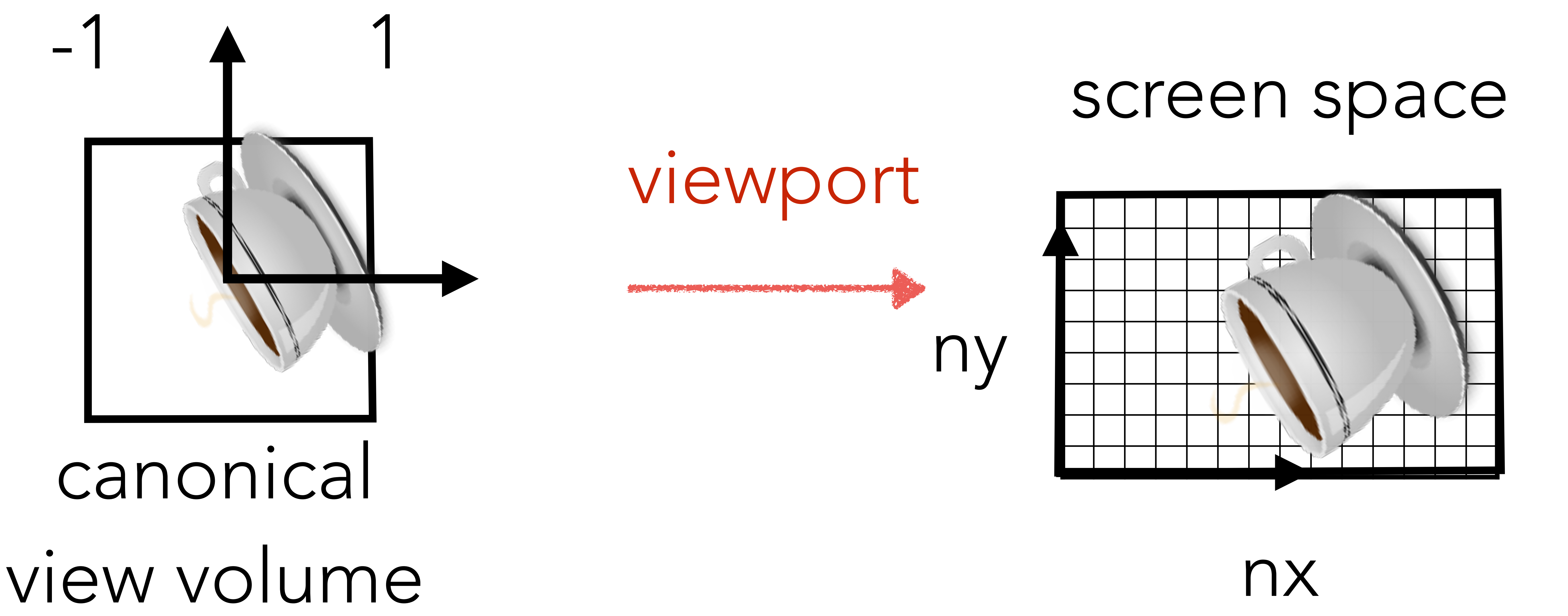
Viewport transformation



$$\begin{bmatrix} x_{screen} \\ y_{screen} \\ 1 \end{bmatrix} = \begin{bmatrix} nx/2 & 0 & \frac{n_x - 1}{2} \\ 0 & ny/2 & \frac{n_y - 1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{canonical} \\ y_{canonical} \\ 1 \end{bmatrix}$$

How does it look in 3D?

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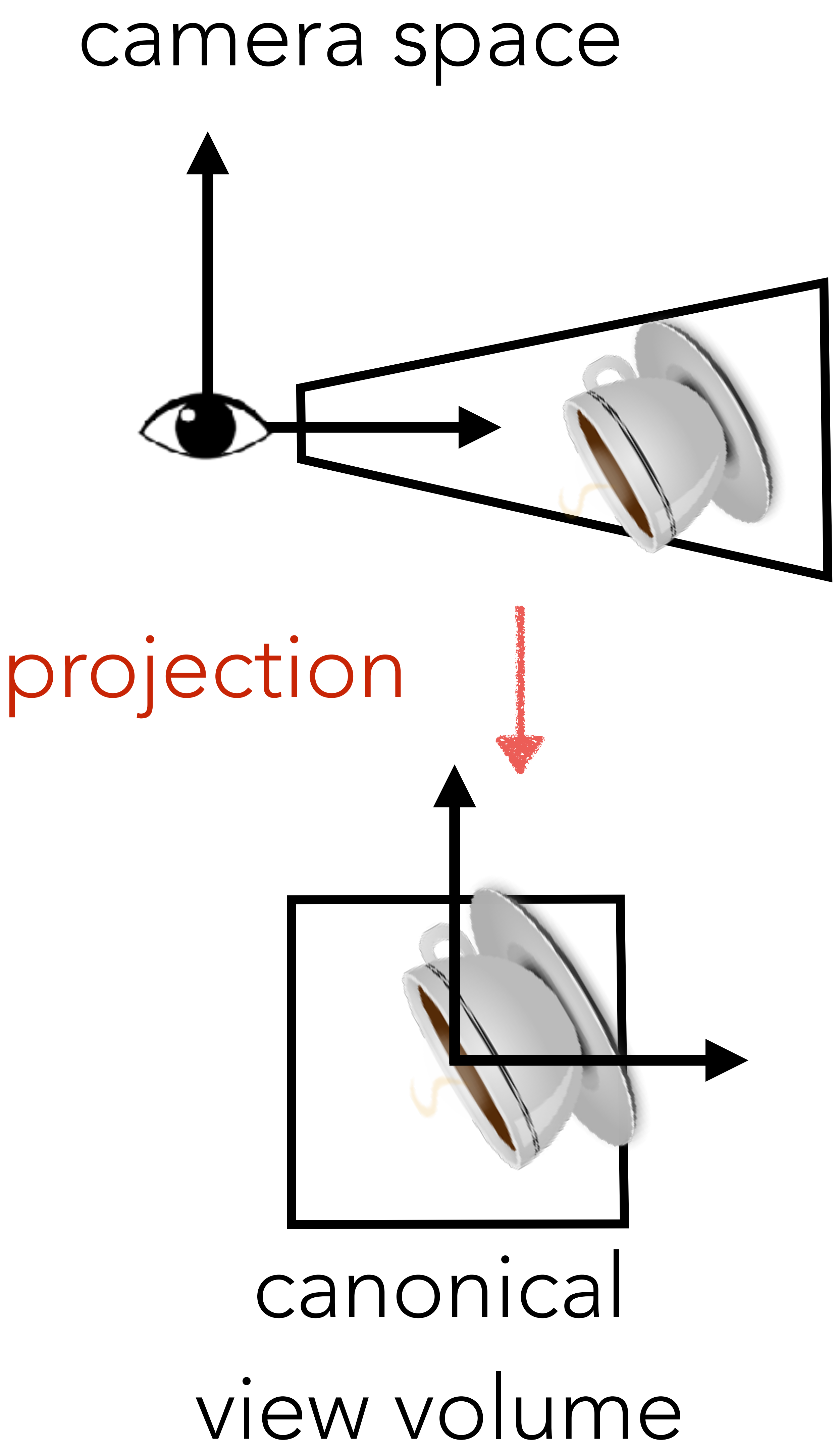
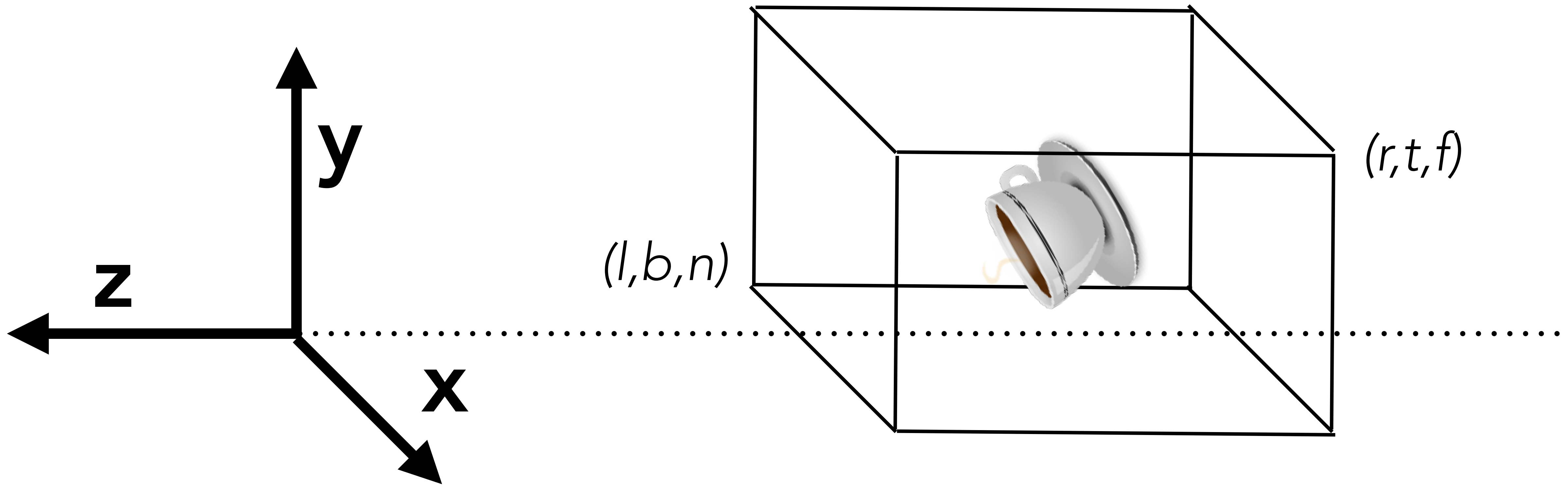
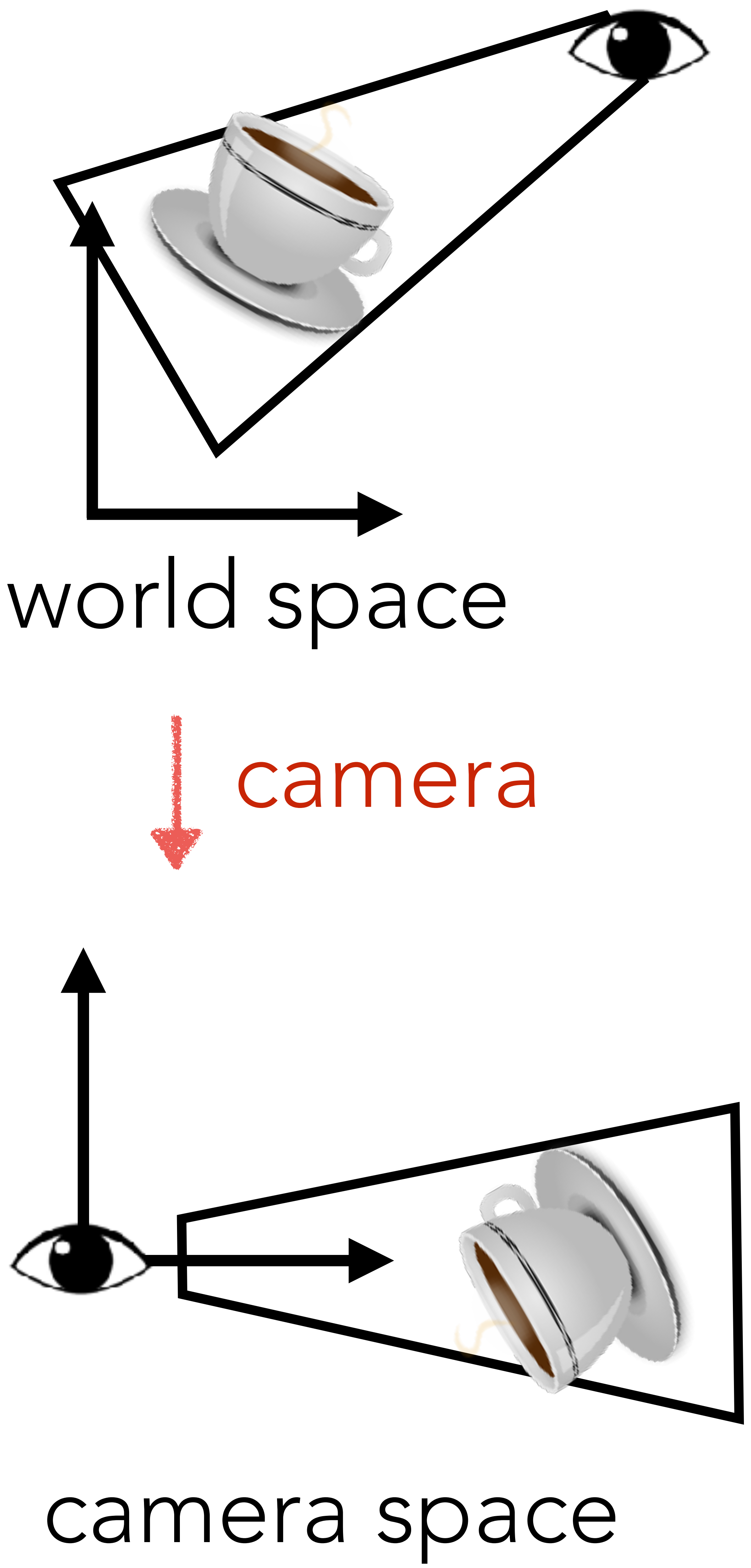
Orthographic Projection



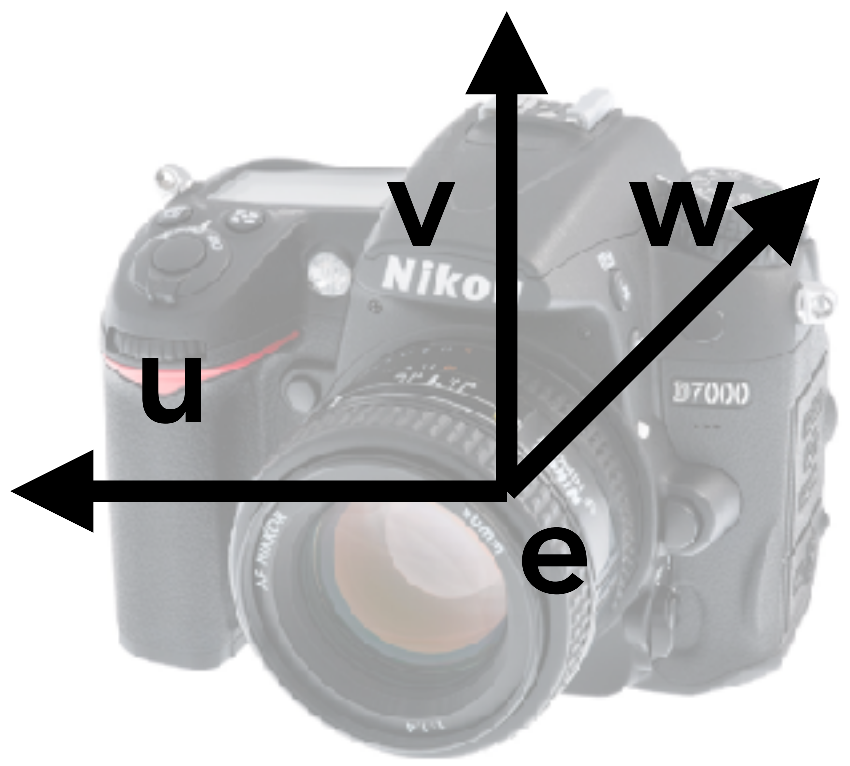


$$\mathbf{M}_{orth} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Camera Transformation



1. Construct the camera reference system given:
   1. The eye position **e**
   2. The gaze direction **g**
   3. The view-up vector **t**

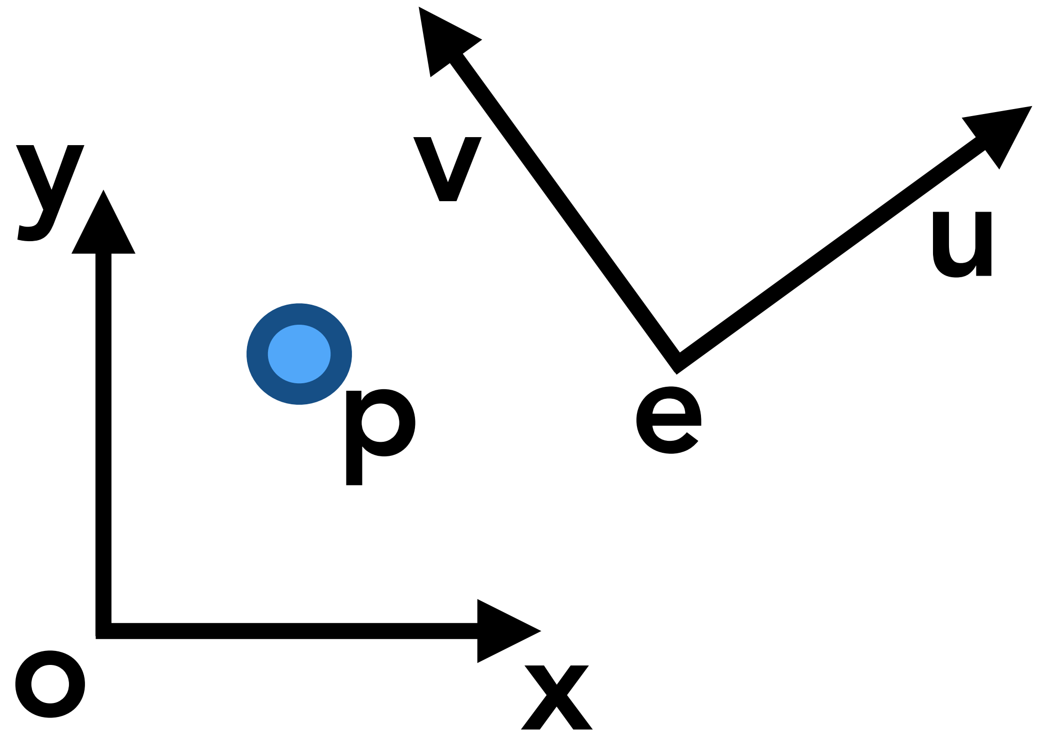


$$\mathbf{w} = -\frac{\mathbf{g}}{||\mathbf{g}||}$$

$$\mathbf{u} = \frac{\mathbf{t} \times \mathbf{w}}{||\mathbf{t} \times \mathbf{w}||}$$

$$\mathbf{v} = \mathbf{w} \times \mathbf{u}$$

Change of frame



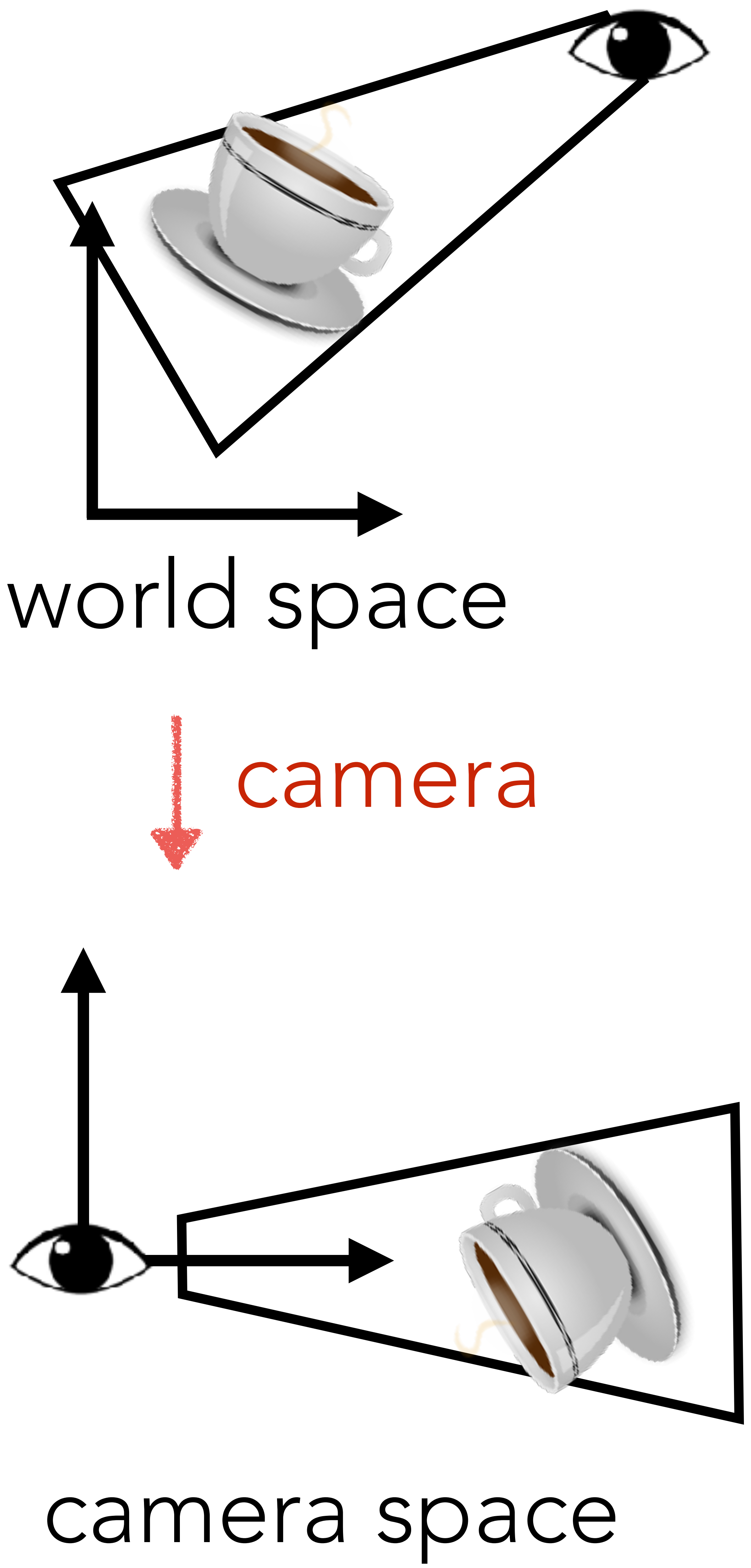
$$\mathbf{p} = (p_x, p_y) = \mathbf{o} + p_x\mathbf{x} + p_y\mathbf{y}$$
$$\mathbf{p} = (p_u, p_v) = \mathbf{e} + p_u\mathbf{u} + p_v\mathbf{v}$$

$$\begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & e_x \\ 0 & 1 & e_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_x & v_x & 0 \\ u_y & v_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_u \\ p_v \\ 1 \end{bmatrix} = \begin{bmatrix} u_x & v_x & e_x \\ u_y & v_y & e_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_u \\ p_v \\ 1 \end{bmatrix}$$

$$\mathbf{p}_{xy} = \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{e} \\ 0 & 0 & 1 \end{bmatrix} \mathbf{p}_{uv} \qquad \mathbf{p}_{uv} = \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{e} \\ 0 & 0 & 1 \end{bmatrix}^{-1} \mathbf{p}_{xy}$$

Can you write it directly without the inverse?

Camera Transformation



1. Construct the camera reference system given:
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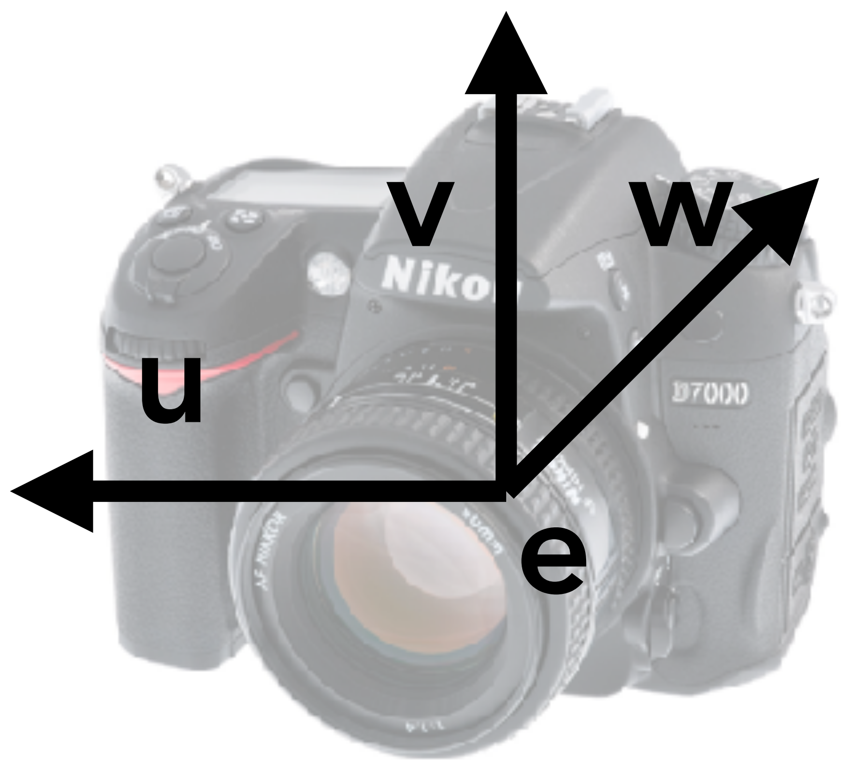

$$\mathbf{w} = -\frac{\mathbf{g}}{||\mathbf{g}||}$$

$$\mathbf{u} = \frac{\mathbf{t} \times \mathbf{w}}{||\mathbf{t} \times \mathbf{w}||}$$

$$\mathbf{v} = \mathbf{w} \times \mathbf{u}$$

2. Construct the unique transformations that converts world coordinates into camera coordinates

$$\mathbf{M}_{cam} = \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{w} & \mathbf{e} \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1}$$

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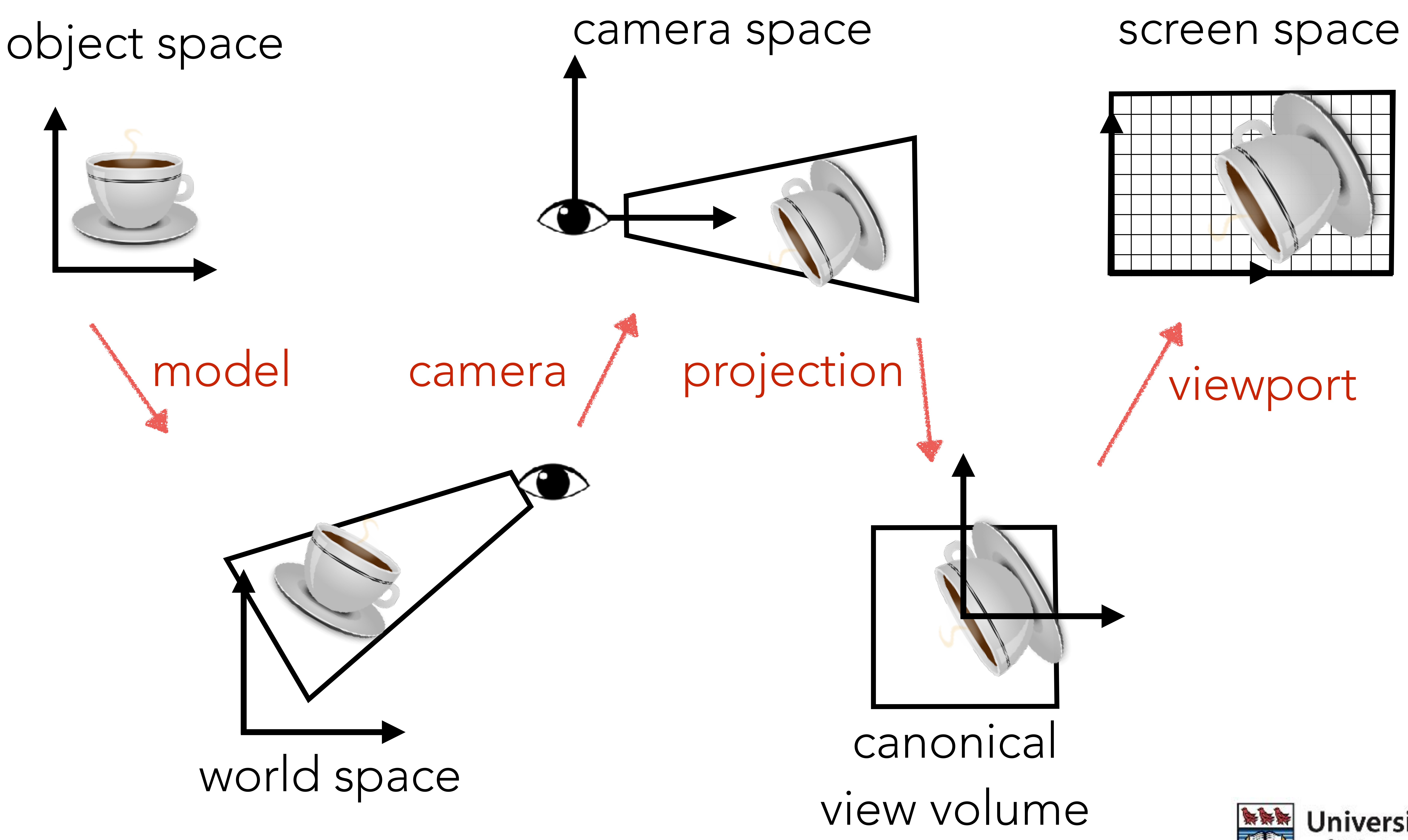
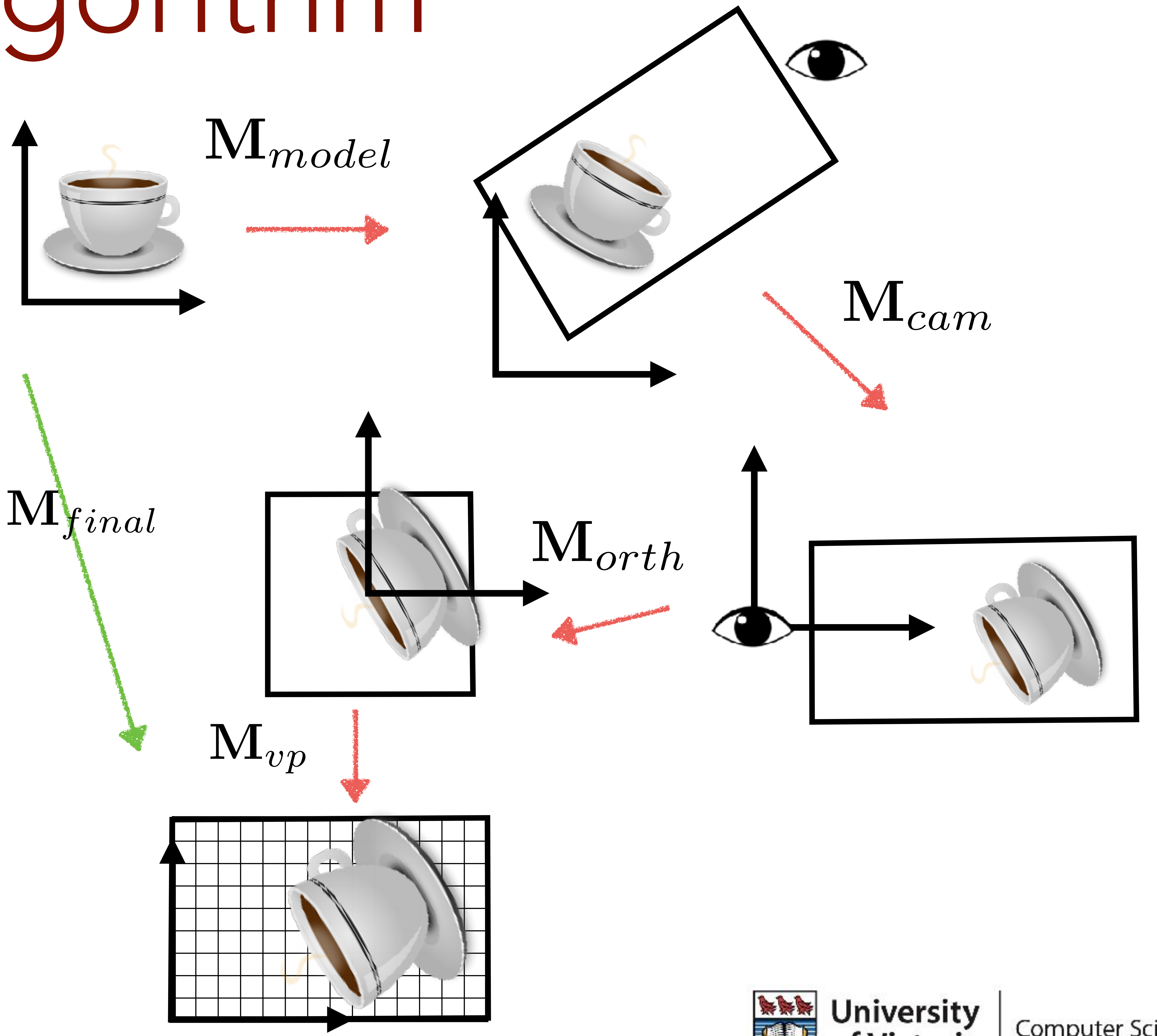
Viewing Transformation



Algorithm

- Construct Viewport Matrix $\mathbf{M}_{vp}$
- Construct Projection Matrix $\mathbf{M}_{orth}$
- Construct Camera Matrix $\mathbf{M}_{cam}$
- $\mathbf{M} = \mathbf{M}_{vp}\mathbf{M}_{orth}\mathbf{M}_{cam}$
- For each model
  - Construct Model Matrix $\mathbf{M}_{model}$
  - $\mathbf{M}_{final} = \mathbf{M}\mathbf{M}_{model}$
  - For every point **p** in each primitive of the model
    - $\mathbf{p}_{final} = \mathbf{M}_{final}\mathbf{p}$
  - Rasterize the model

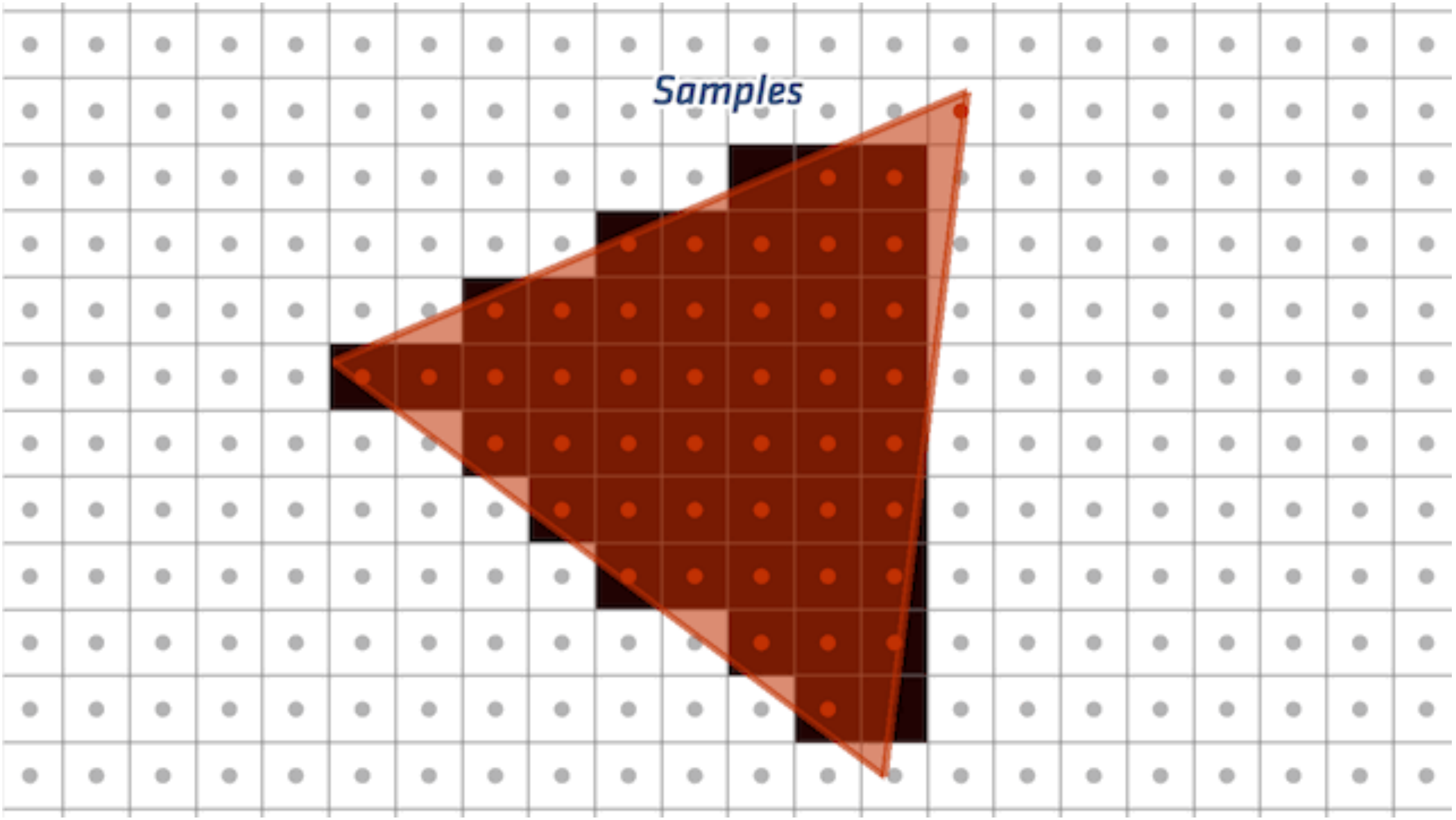


References

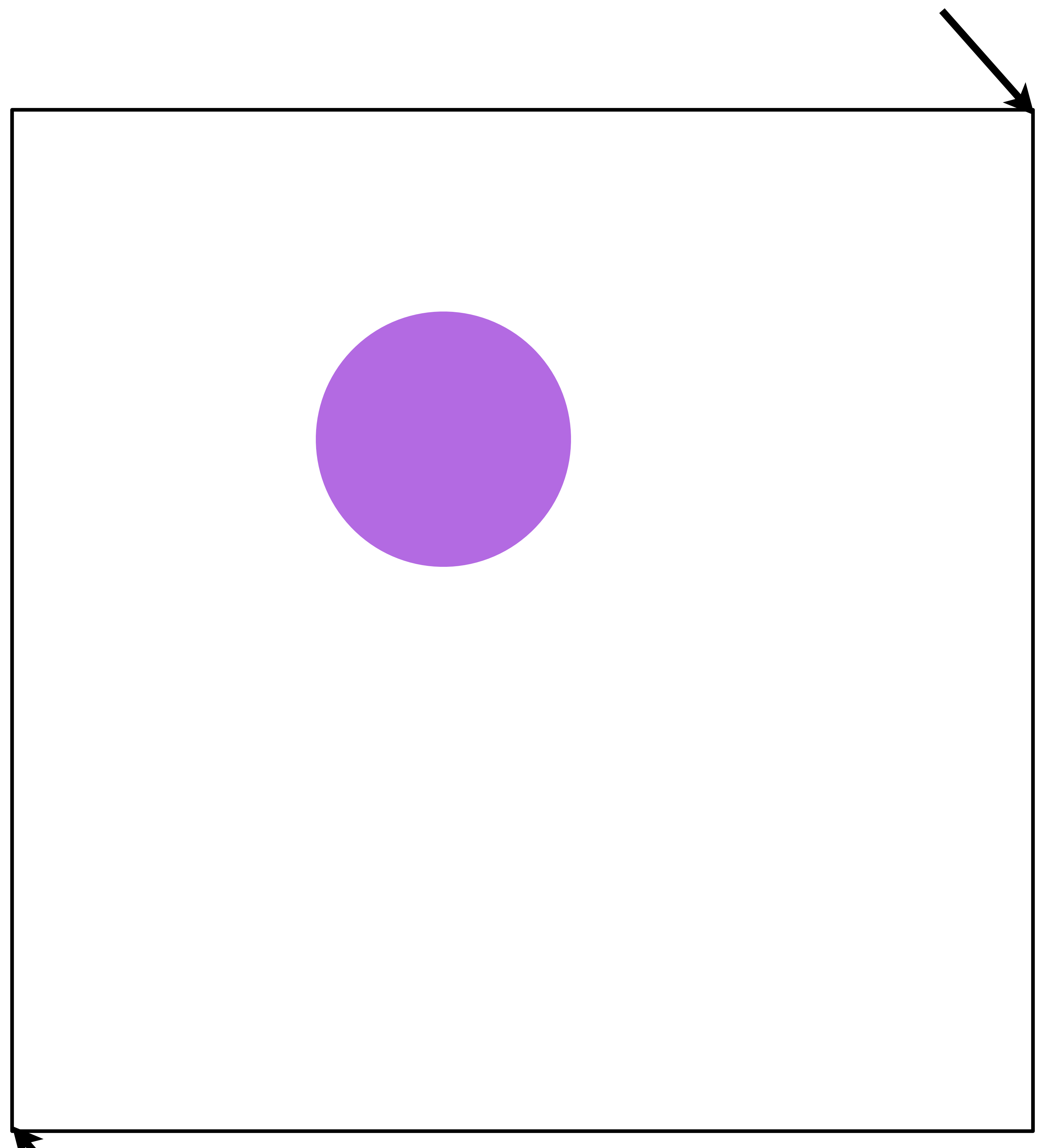
Fundamentals of Computer Graphics, Fourth Edition
4th Edition **by Steve Marschner, Peter Shirley**

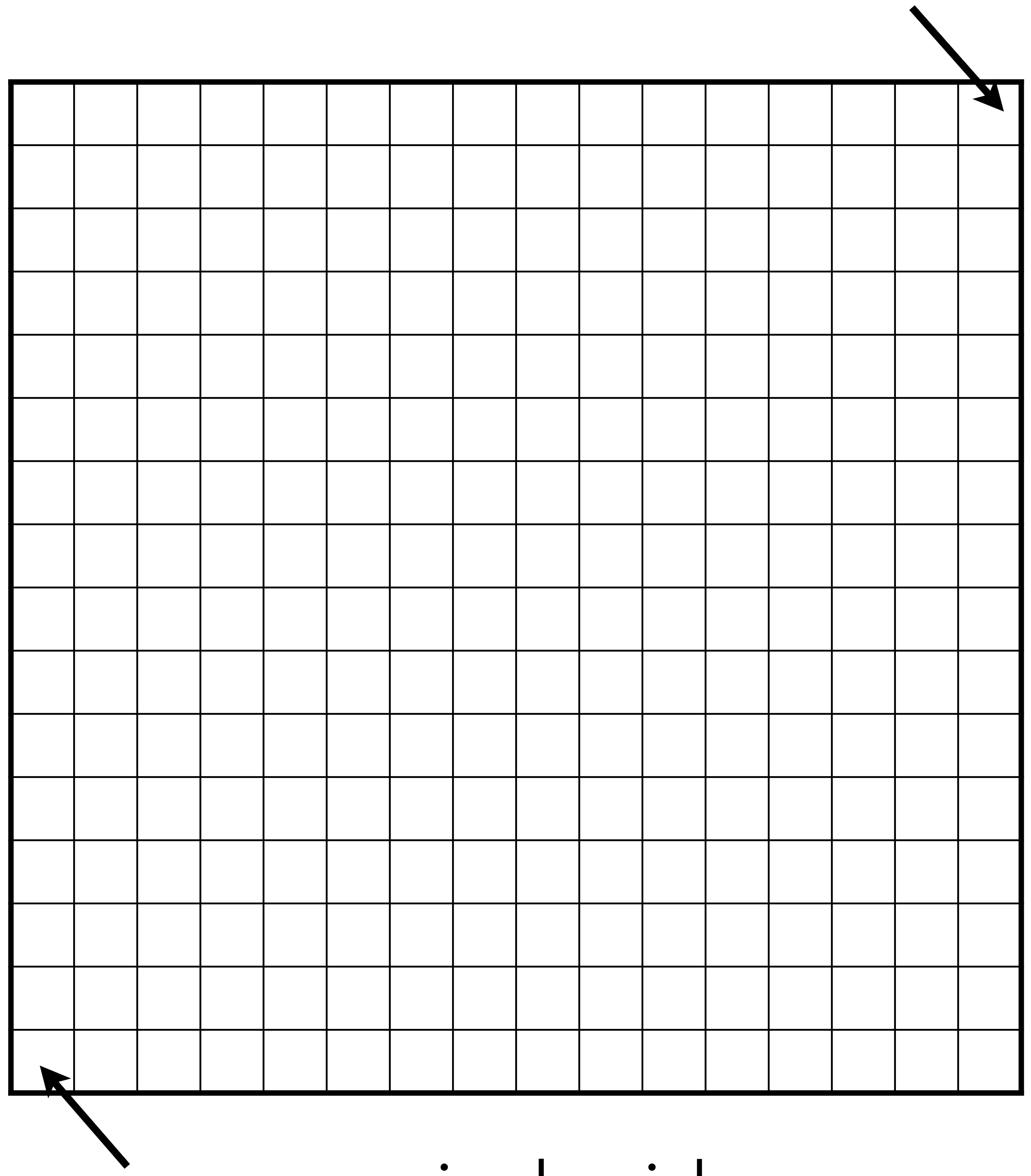
Chapter 7

Rasterization


Samples

2D Canvas





2D Canvas

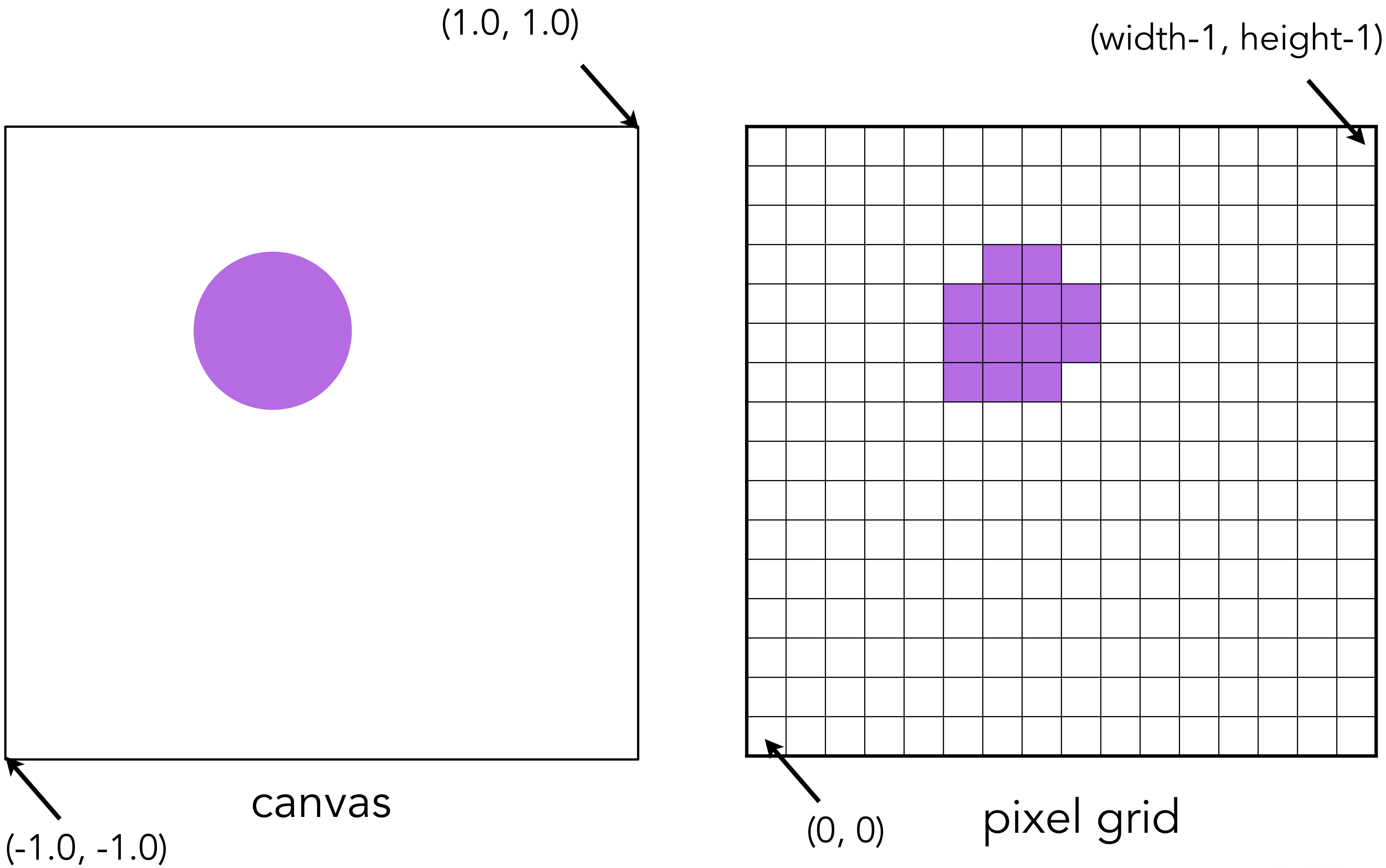


Image Copyright: Andrea Tagliasacchi

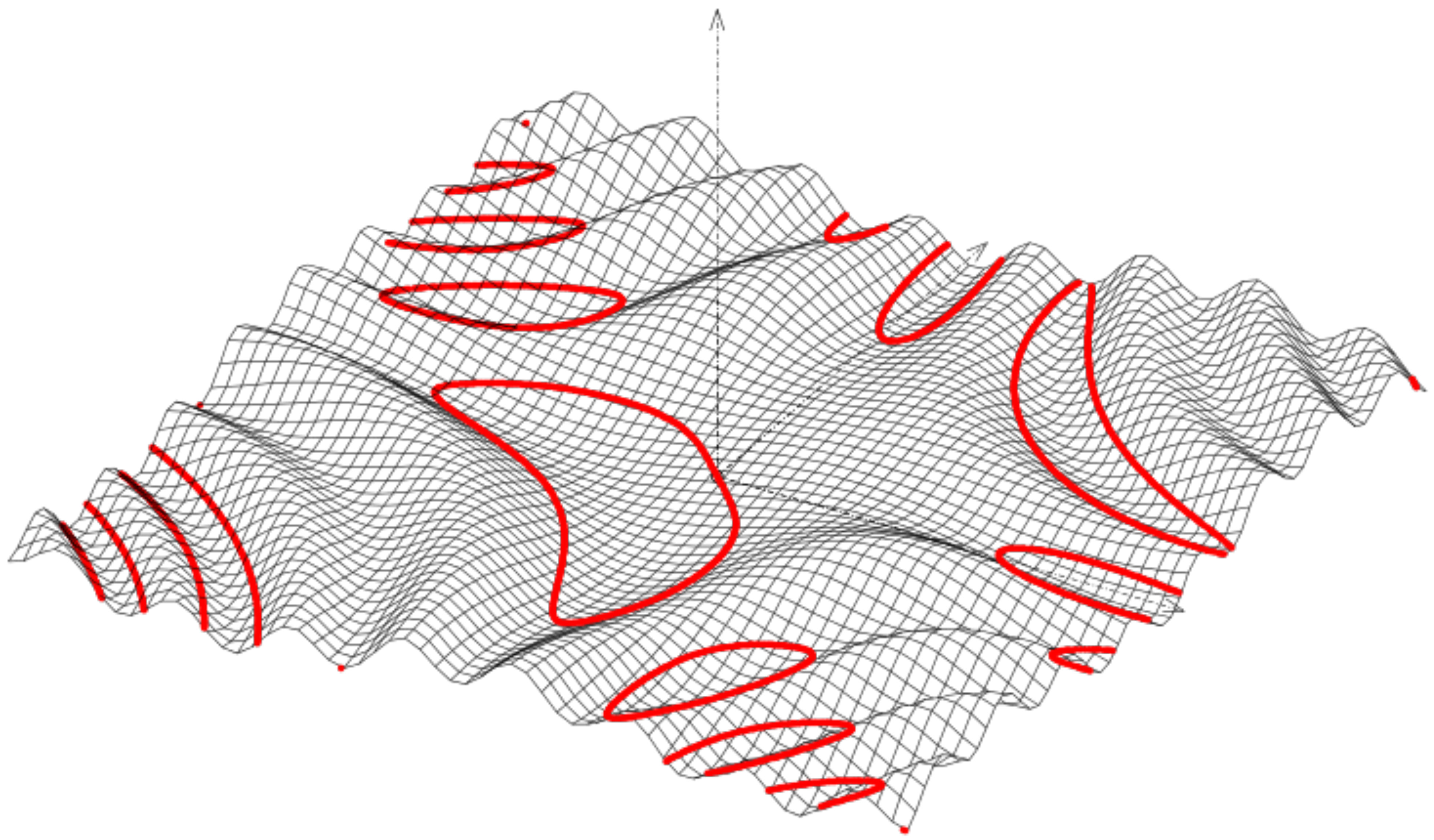
Implicit Geometry Representation

- Define a curve as zero set of 2D implicit function
  - $F(x,y) = 0 \rightarrow$ on curve
  - $F(x,y) < 0 \rightarrow$ inside curve
  - $F(x,y) > 0 \rightarrow$ outside curve
- Example: Circle with center (c_x, c_y) and radius r

$$F(x, y) = (x - c_x)^2 + (y - c_y)^2 - r^2$$

Implicit Geometry Representation

- Define a curve as zero set of 2D implicit function
  - $F(x,y) = 0 \rightarrow$ on curve
  - $F(x,y) < 0 \rightarrow$ inside curve
  - $F(x,y) > 0 \rightarrow$ outside curve



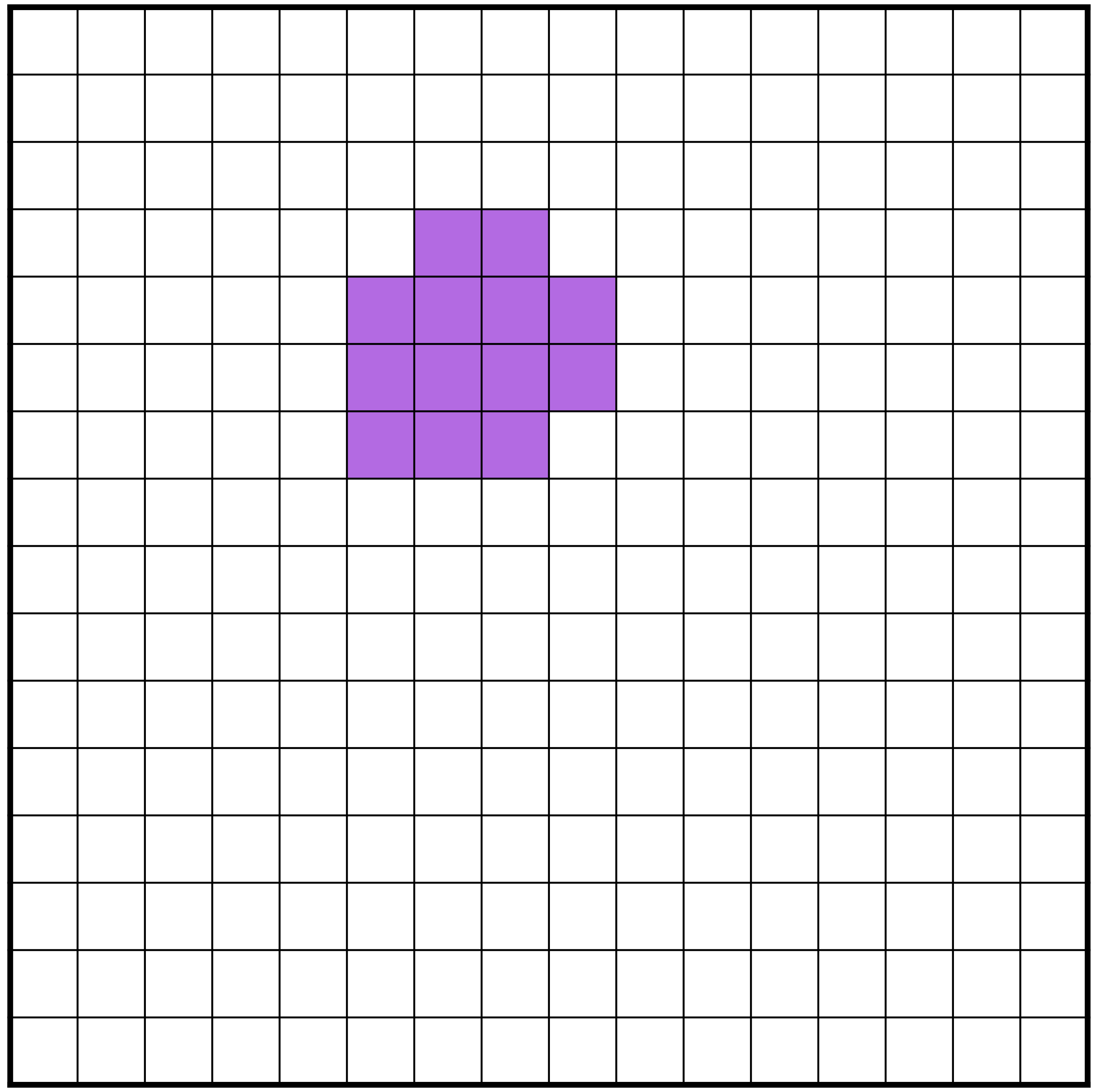
Implicit Rasterization

for all pixels (i,j)

(x,y) = map_to_canvas (i,j)

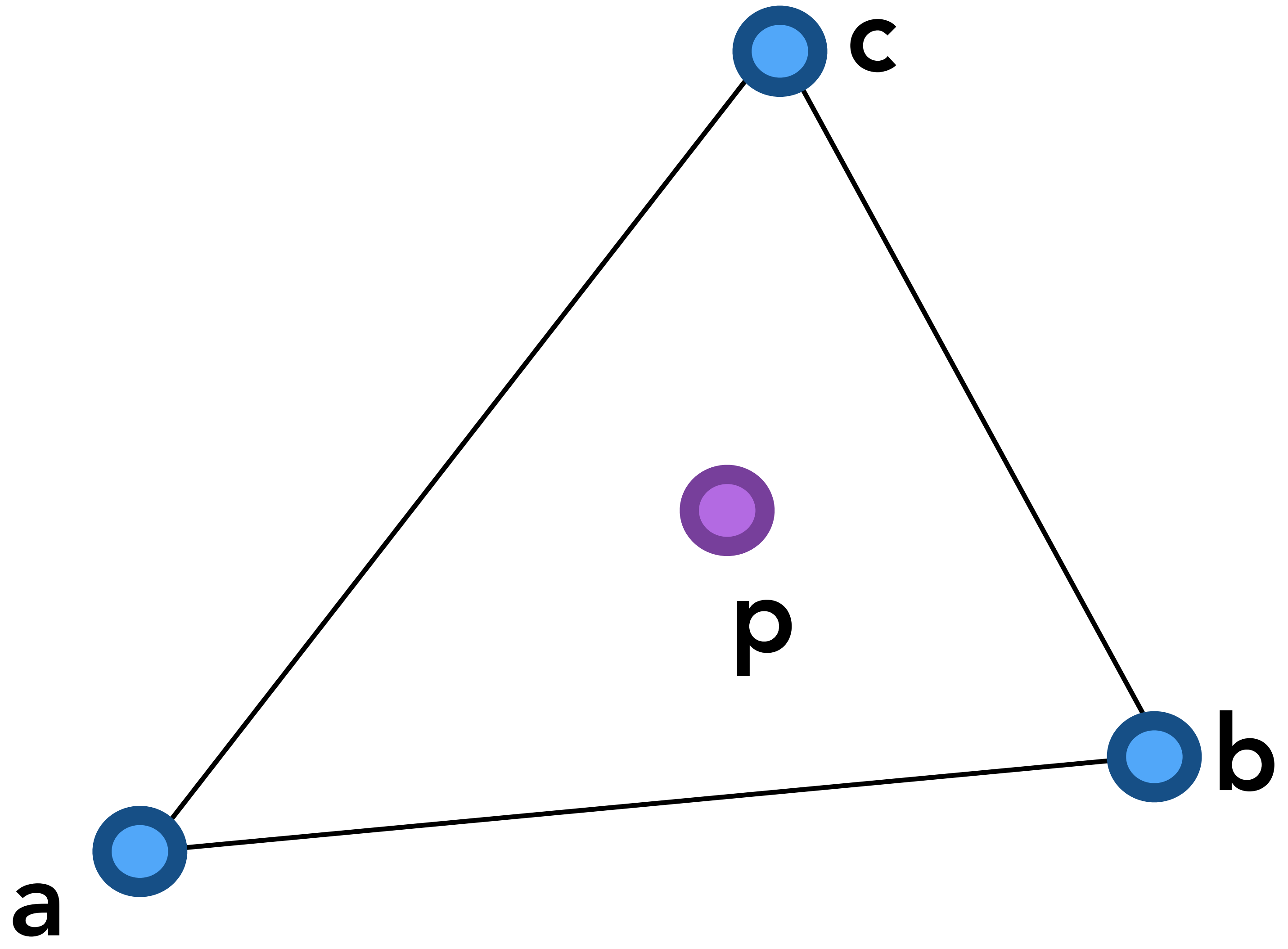
if F(x,y) < 0

set_pixel (i,j, color)



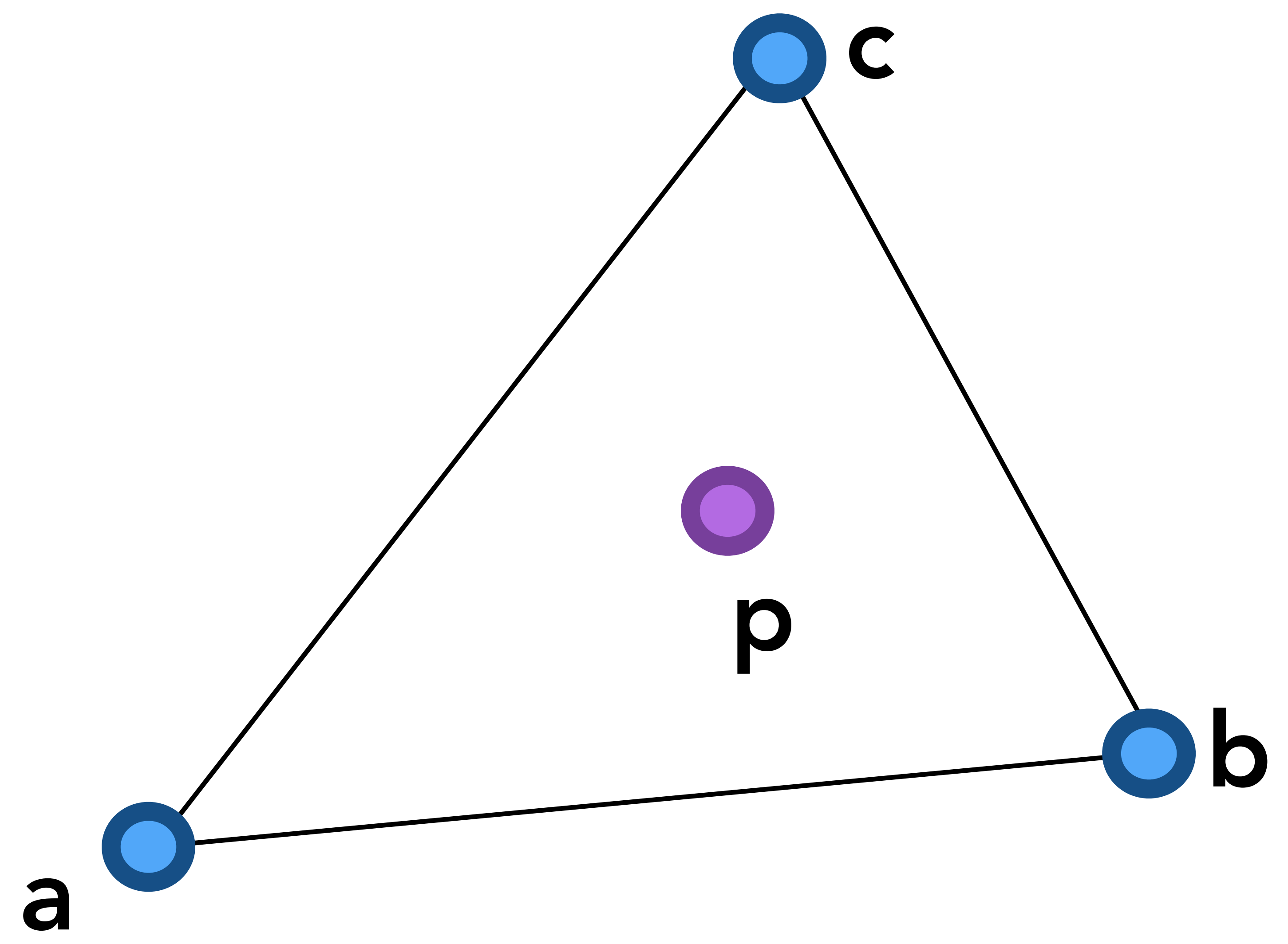
Barycentric Interpolation

- Barycentric coordinates:
  - $\mathbf{p} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}$ with $\alpha + \beta + \gamma = 1$



Barycentric Interpolation

- Barycentric coordinates:
  - $\mathbf{p} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}$ with $\alpha + \beta + \gamma = 1$
  - Unique for non-collinear **a**,**b**,**c**



$$\begin{bmatrix} \mathbf{a}_x & \mathbf{b}_x & \mathbf{c}_x \\ \mathbf{a}_y & \mathbf{b}_y & \mathbf{c}_y \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} \mathbf{p}_x \\ \mathbf{p}_y \\ 1 \end{bmatrix}$$

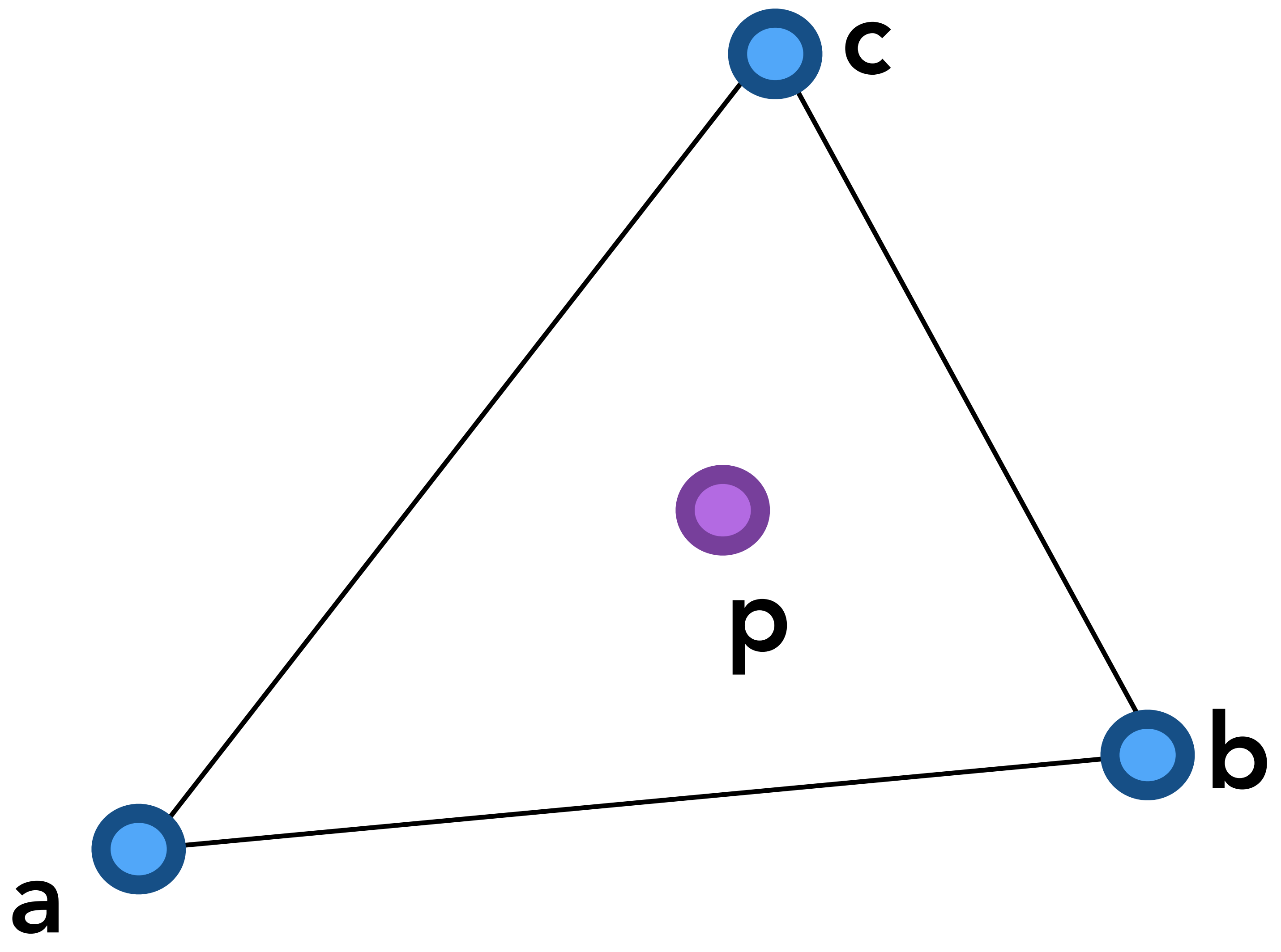
Barycentric Interpolation

- Barycentric coordinates:
  - $\mathbf{p} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}$ with $\alpha + \beta + \gamma = 1$
  - Unique for non-collinear **a**,**b**,**c**
  - Ratio of triangle areas

$$\alpha(\mathbf{p}) = \frac{\text{area}(\mathbf{p}, \mathbf{b}, \mathbf{c})}{\text{area}(\mathbf{a}, \mathbf{b}, \mathbf{c})}$$

$$\beta(\mathbf{p}) = \frac{\text{area}(\mathbf{p}, \mathbf{c}, \mathbf{a})}{\text{area}(\mathbf{a}, \mathbf{b}, \mathbf{c})}$$

$$\gamma(\mathbf{p}) = \frac{\text{area}(\mathbf{p}, \mathbf{a}, \mathbf{b})}{\text{area}(\mathbf{a}, \mathbf{b}, \mathbf{c})}$$



Barycentric Interpolation

- Barycentric coordinates:
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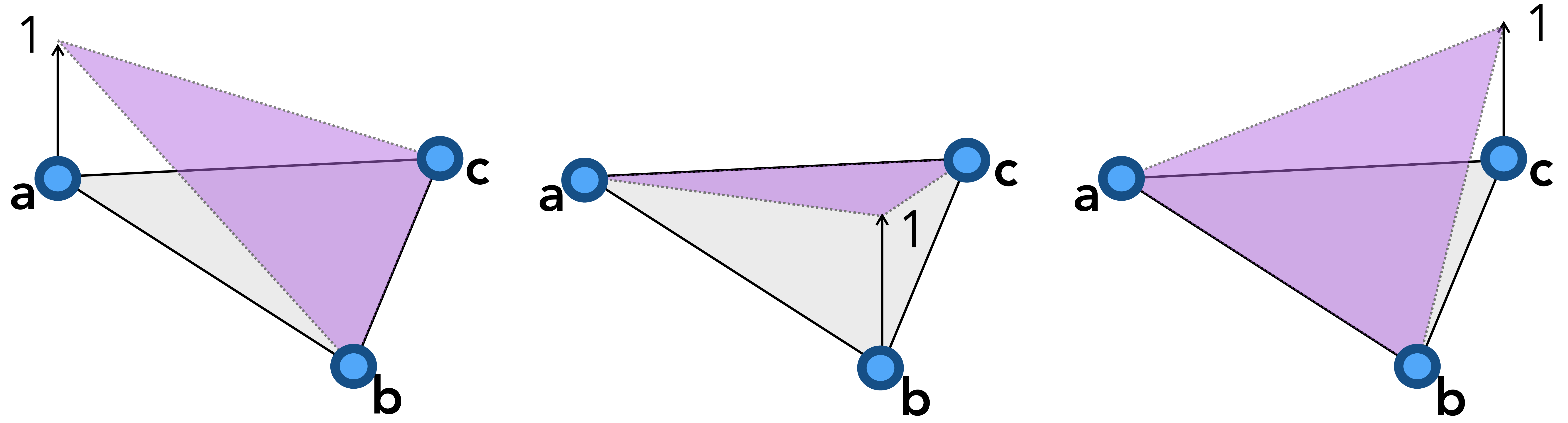
  - Unique for non-collinear **a**,**b**,**c**
  - Ratio of triangle areas
  - $\alpha(\mathbf{p})$, $\beta(\mathbf{p})$, $\gamma(\mathbf{p})$ are linear functions



Barycentric Interpolation

- Barycentric coordinates:
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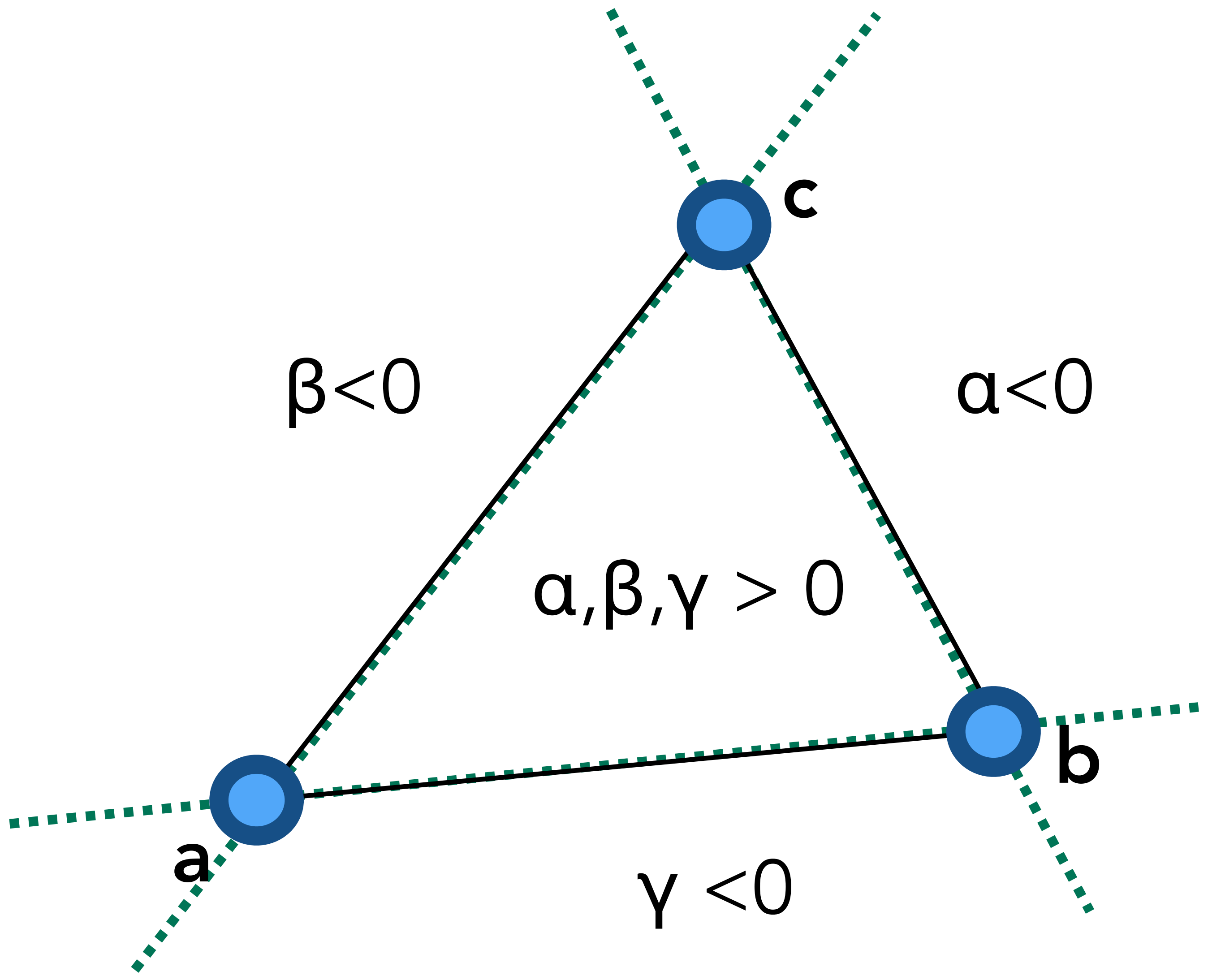
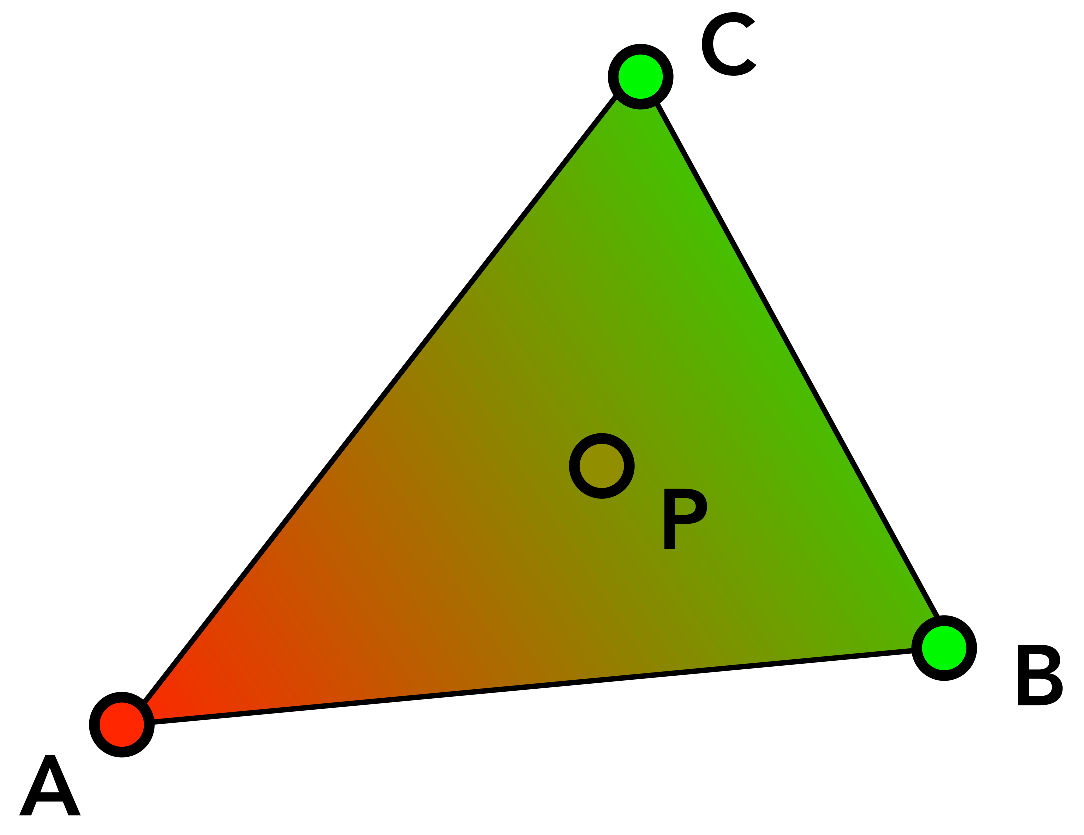
  - Unique for non-collinear **a**,**b**,**c**
  - Ratio of triangle areas
  - $\alpha(\mathbf{p})$, $\beta(\mathbf{p})$, $\gamma(\mathbf{p})$ are linear functions
  - Gives inside/outside information



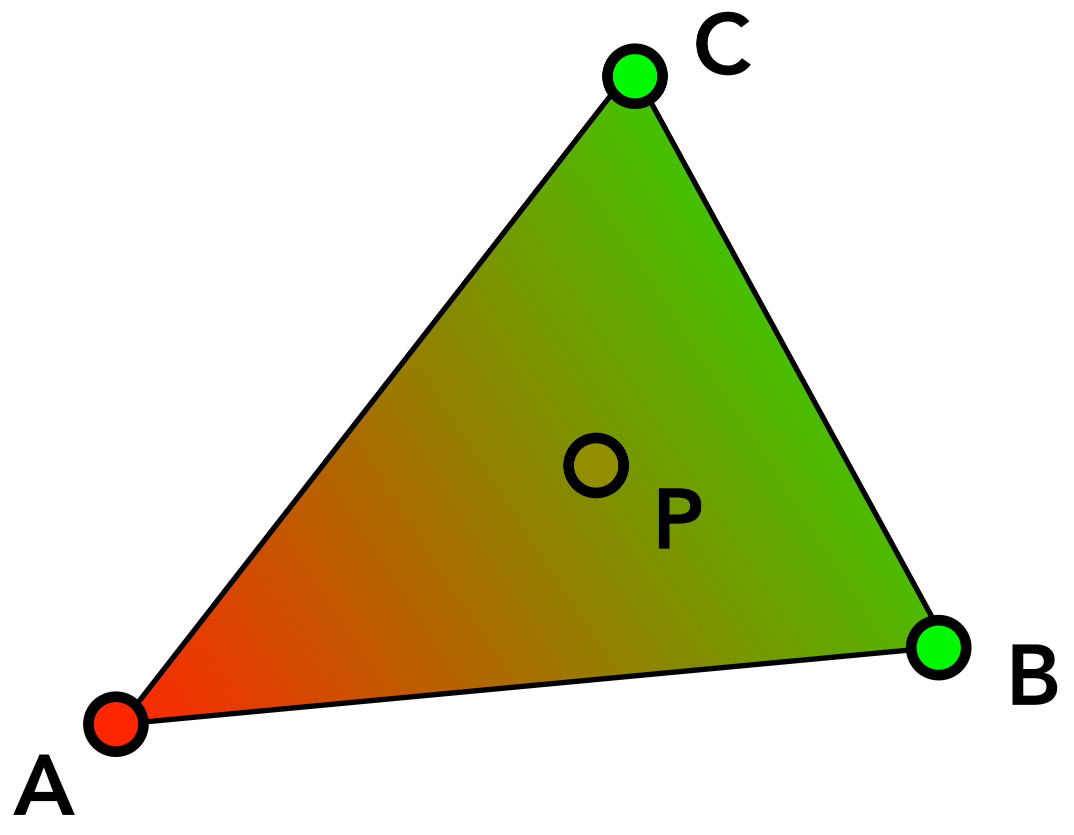
Barycentric Interpolation

- Barycentric coordinates:
  - $\mathbf{p} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}$ with $\alpha + \beta + \gamma = 1$
  - Unique for non-collinear **a**,**b**,**c**
  - Ratio of triangle areas
  - $\alpha(\mathbf{p})$, $\beta(\mathbf{p})$, $\gamma(\mathbf{p})$ are linear functions
  - Gives inside/outside information
  - Use barycentric coordinates to interpolate vertex normals (or other data, e.g. colors)

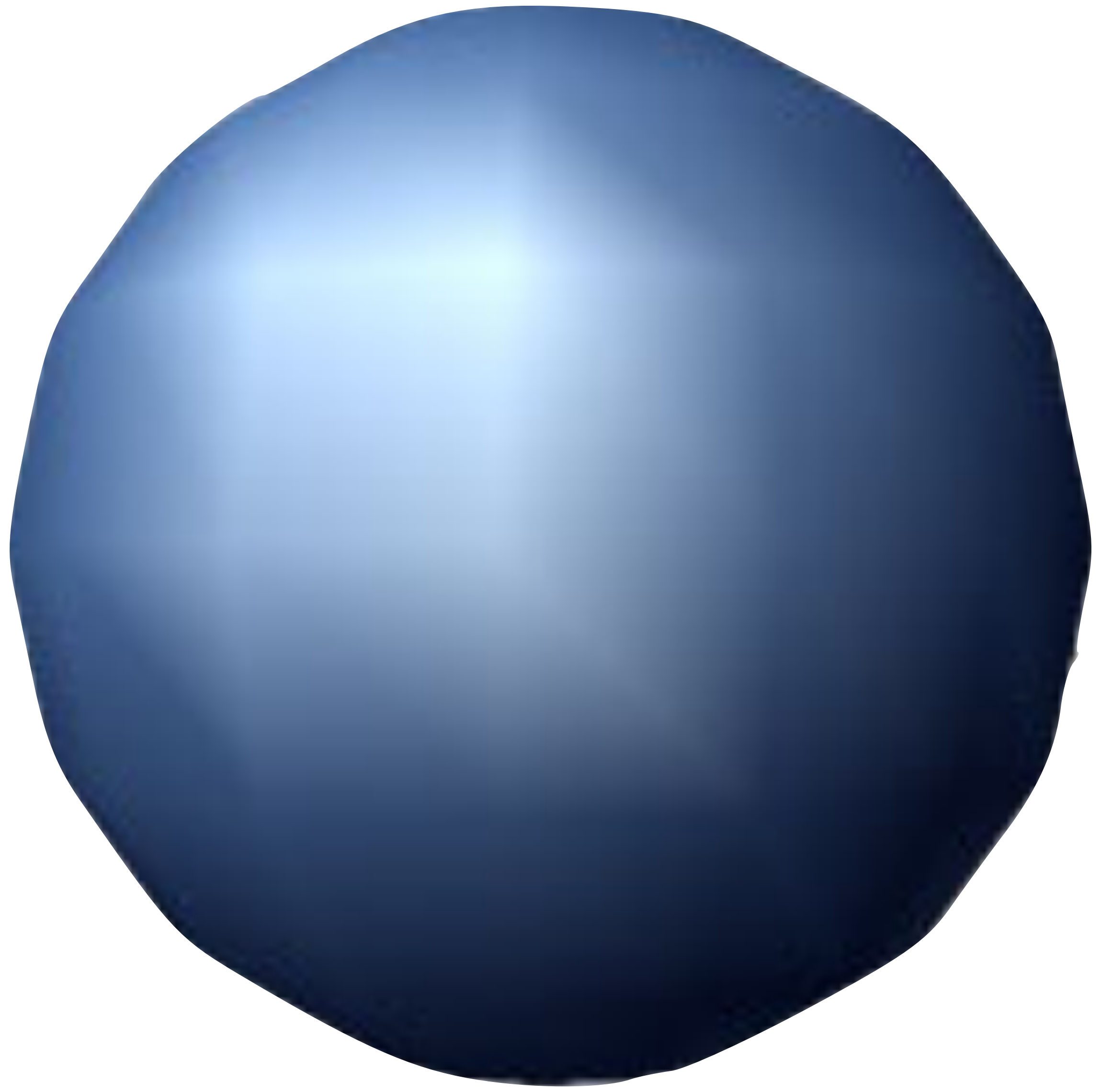
$$\mathbf{n}(\mathbf{P}) = \alpha \cdot \mathbf{n}(\mathbf{A}) + \beta \cdot \mathbf{n}(\mathbf{B}) + \gamma \cdot \mathbf{n}(\mathbf{C})$$



Color Interpolation

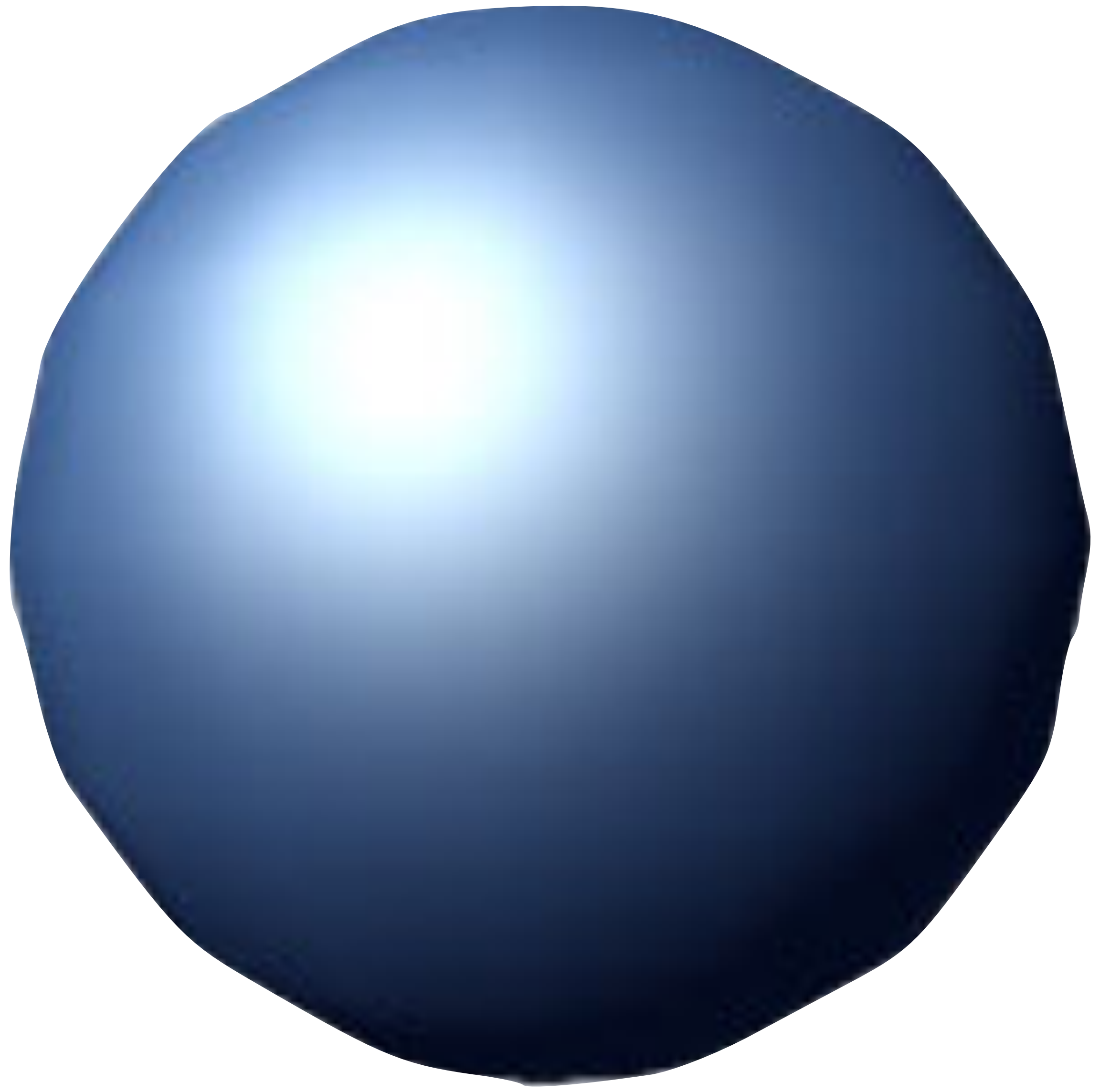


Per-vertex



Evaluate color on vertices, then interpolates it

Per-pixel



Interpolates positions and normals, then evaluate color on each pixel

http://www.cgchannel.com/2010/11/cg-science-for-artists-part-2-the-real-time-rendering-pipeline/

Triangle Rasterization

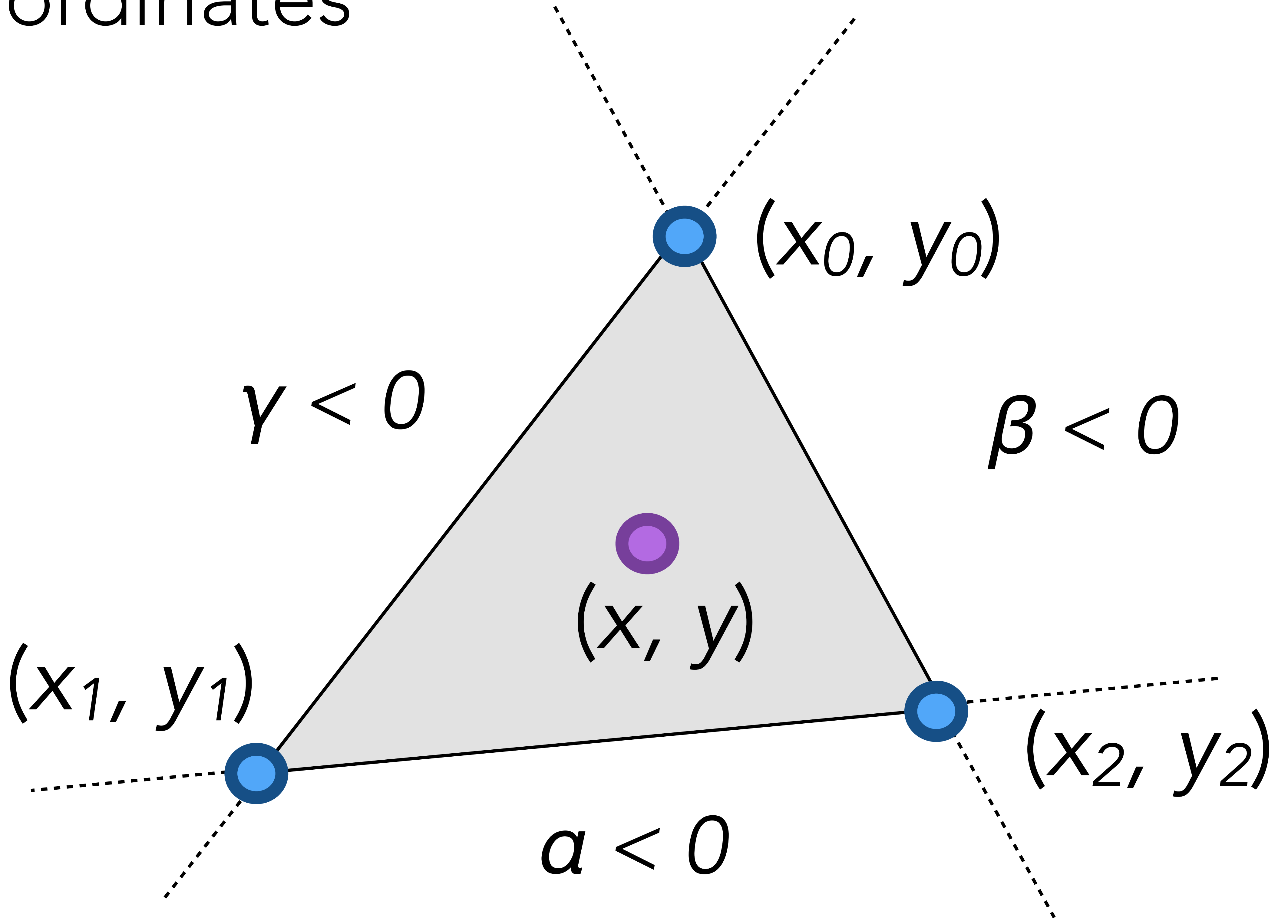
- Each triangle is represented as three 2D points (x_0, y_0), (x_1, y_1), (x_2, y_2)

- Rasterization using barycentric coordinates

$$x = \alpha \cdot x_0 + \beta \cdot x_1 + \gamma \cdot x_2$$

$$y = \alpha \cdot y_0 + \beta \cdot y_1 + \gamma \cdot y_2$$

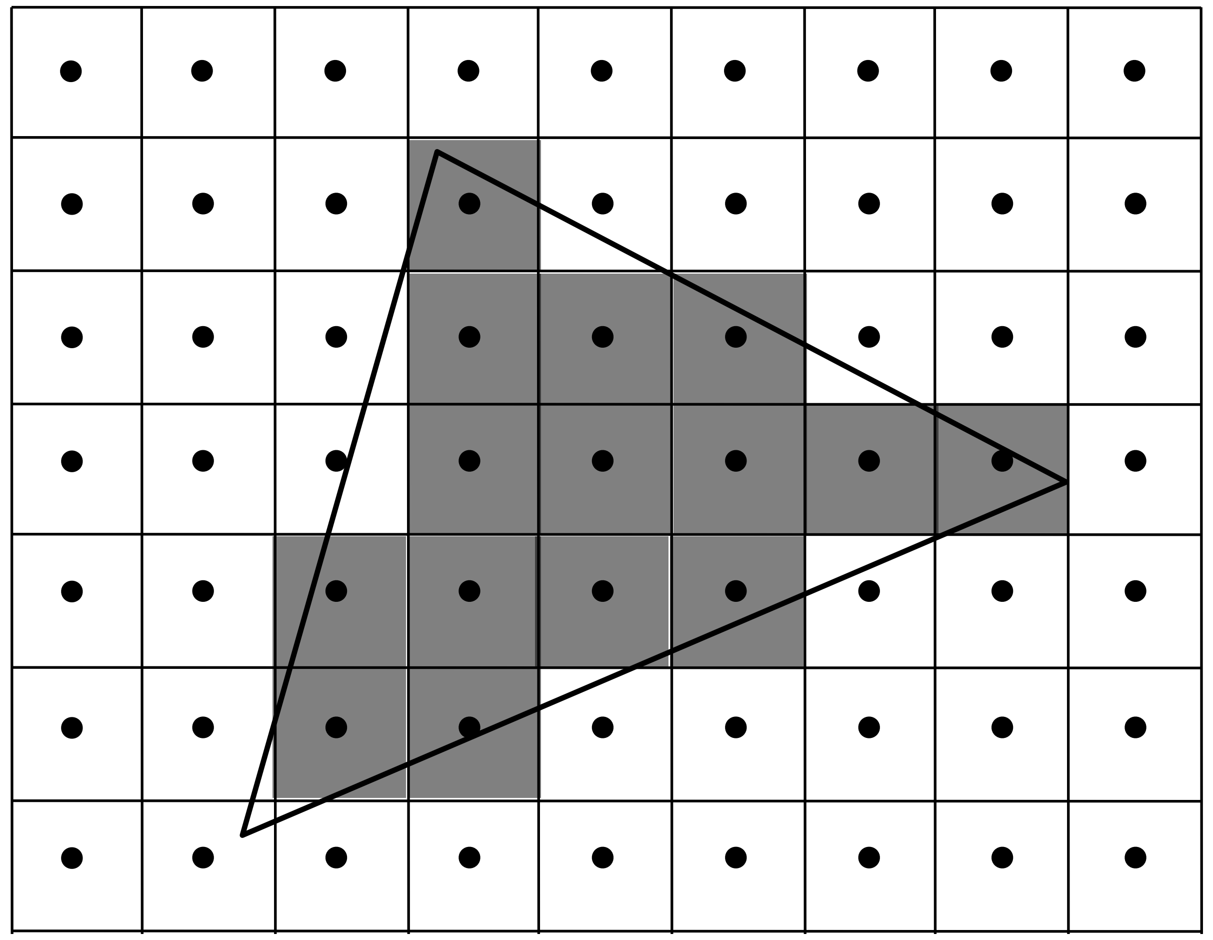
$$\alpha + \beta + \gamma = 1$$



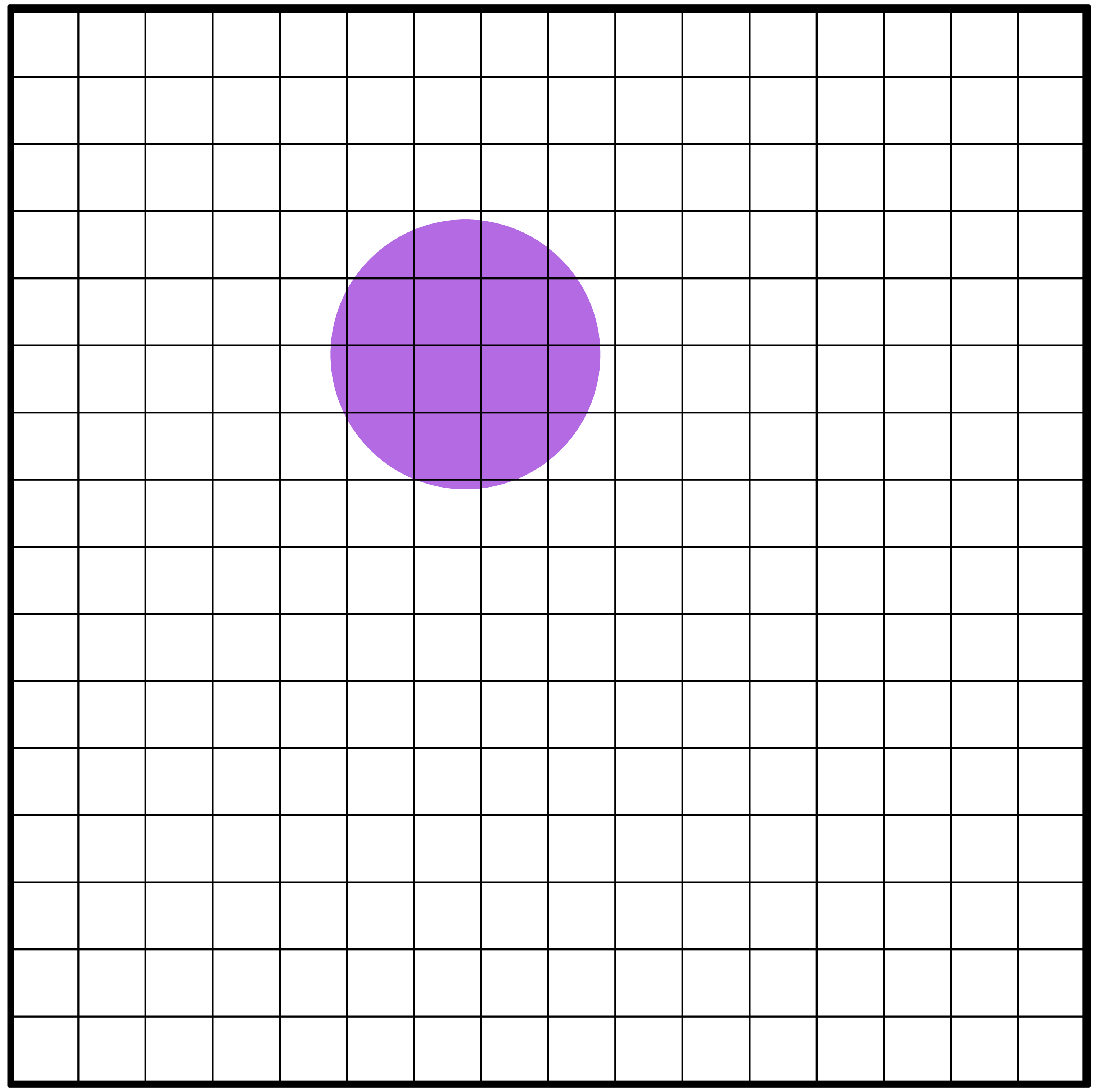
Triangle Rasterization

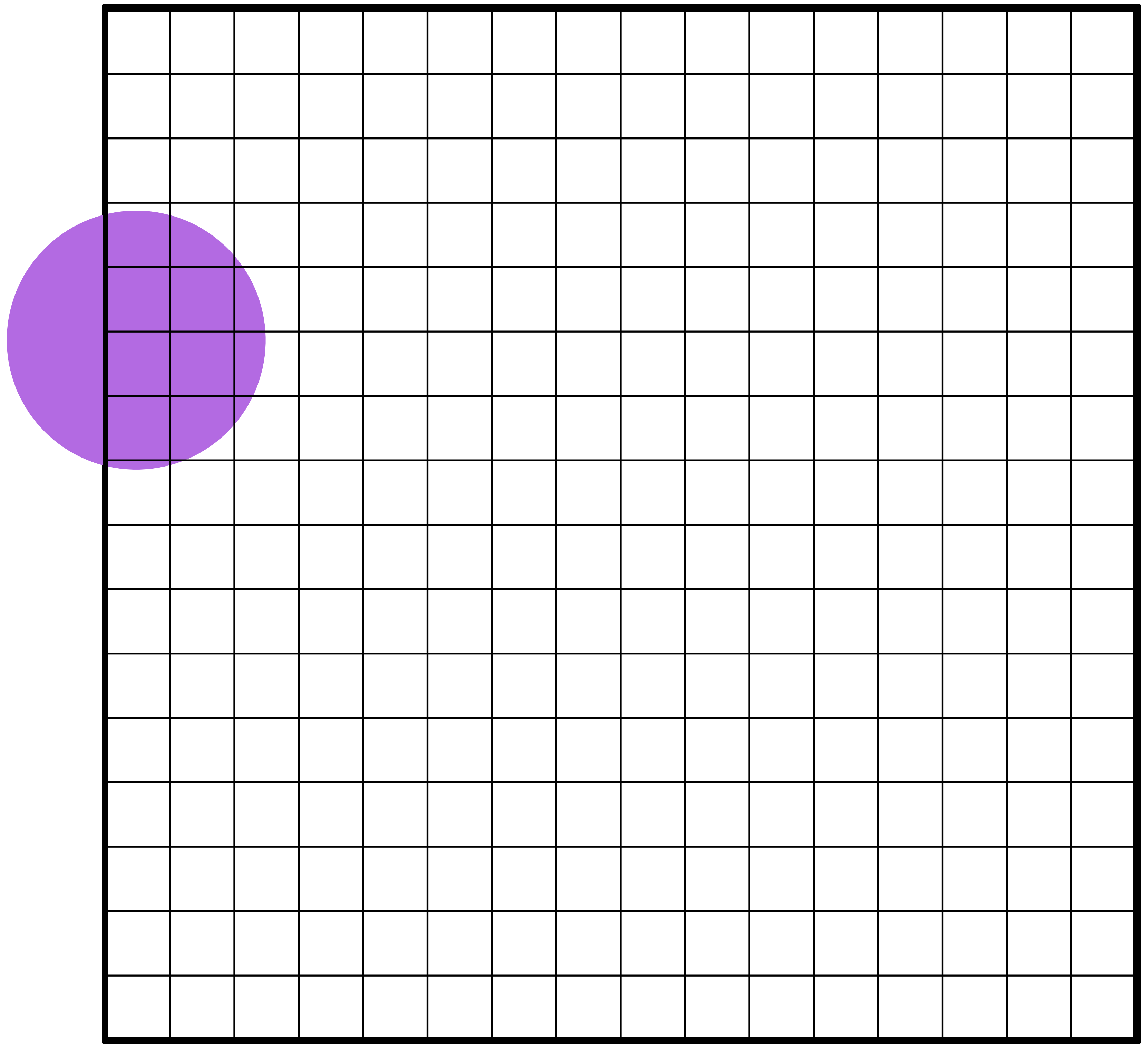
- Each triangle is represented as three 2D points
  $(x_0, y_0), (x_1, y_1), (x_2, y_2)$
- Rasterization using barycentric coordinates

for all y **do**
    for all x **do**
        compute (α,β,γ) for (x,y)
        if ($\alpha \in [0,1]$ and $\beta \in [0,1]$ and $\gamma \in [0,1]$
            set_pixel (x,y)



Clipping

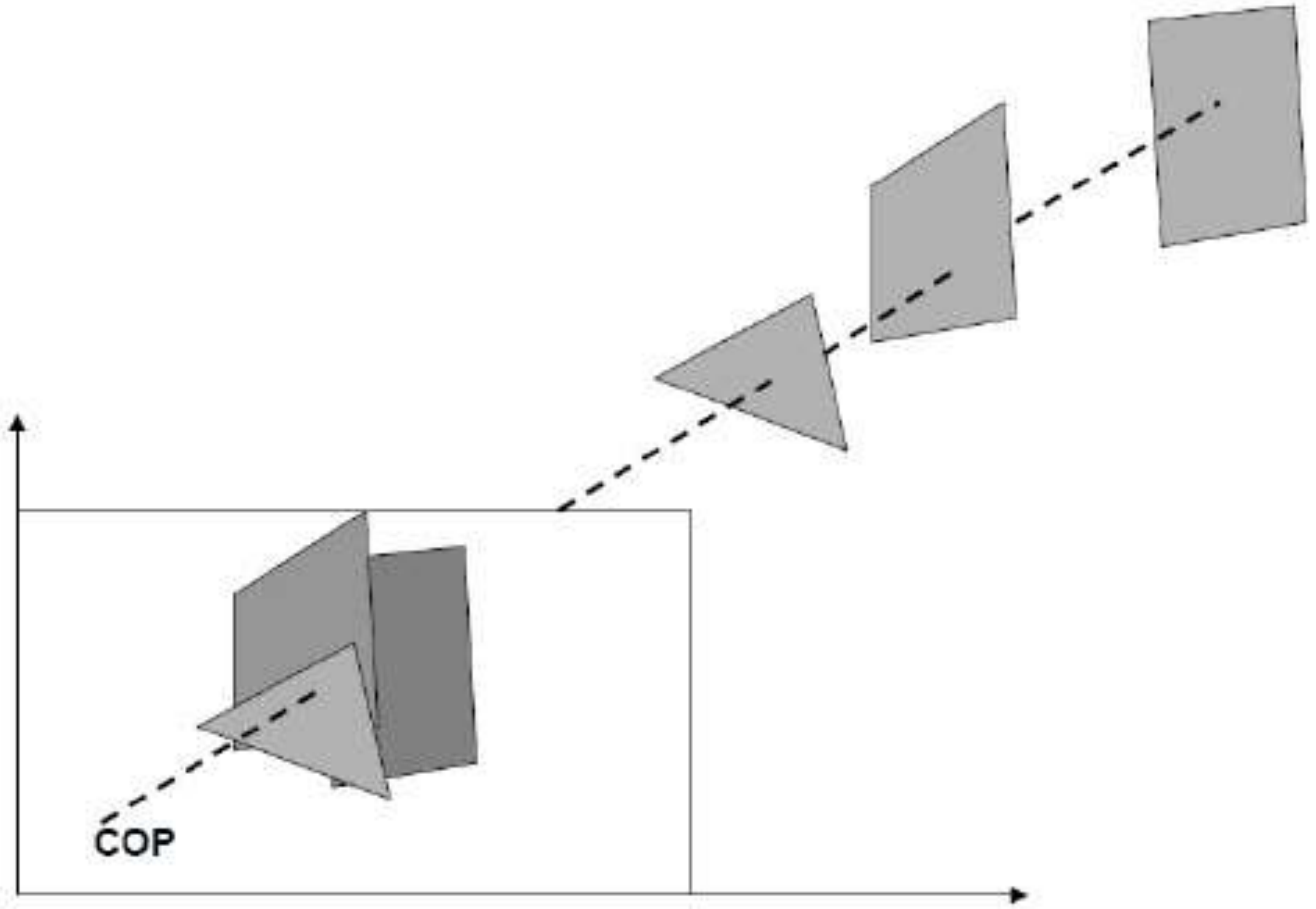




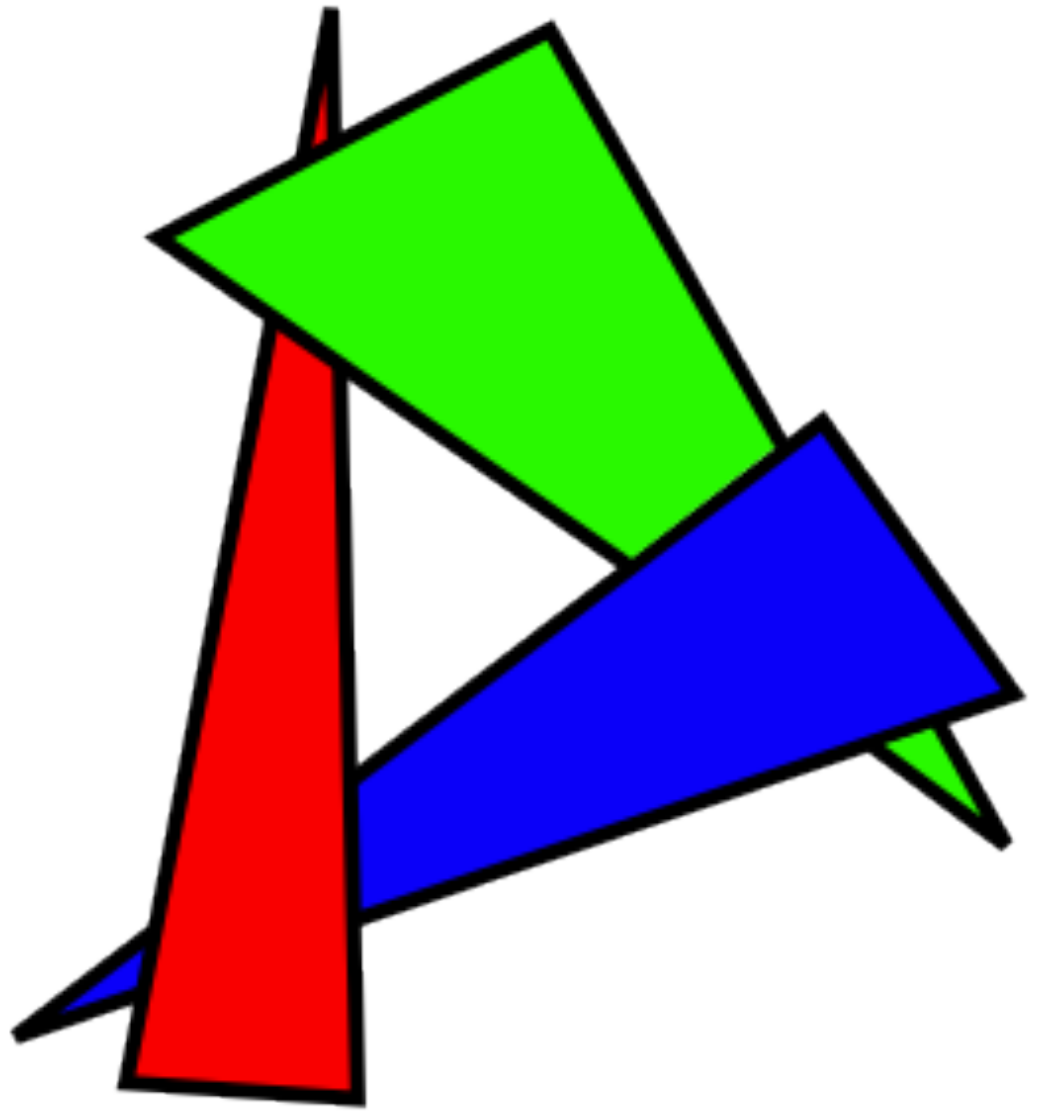
- Ok if you do it brute force
- Care is required if you are explicitly tracing the boundaries

Objects Depth Sorting

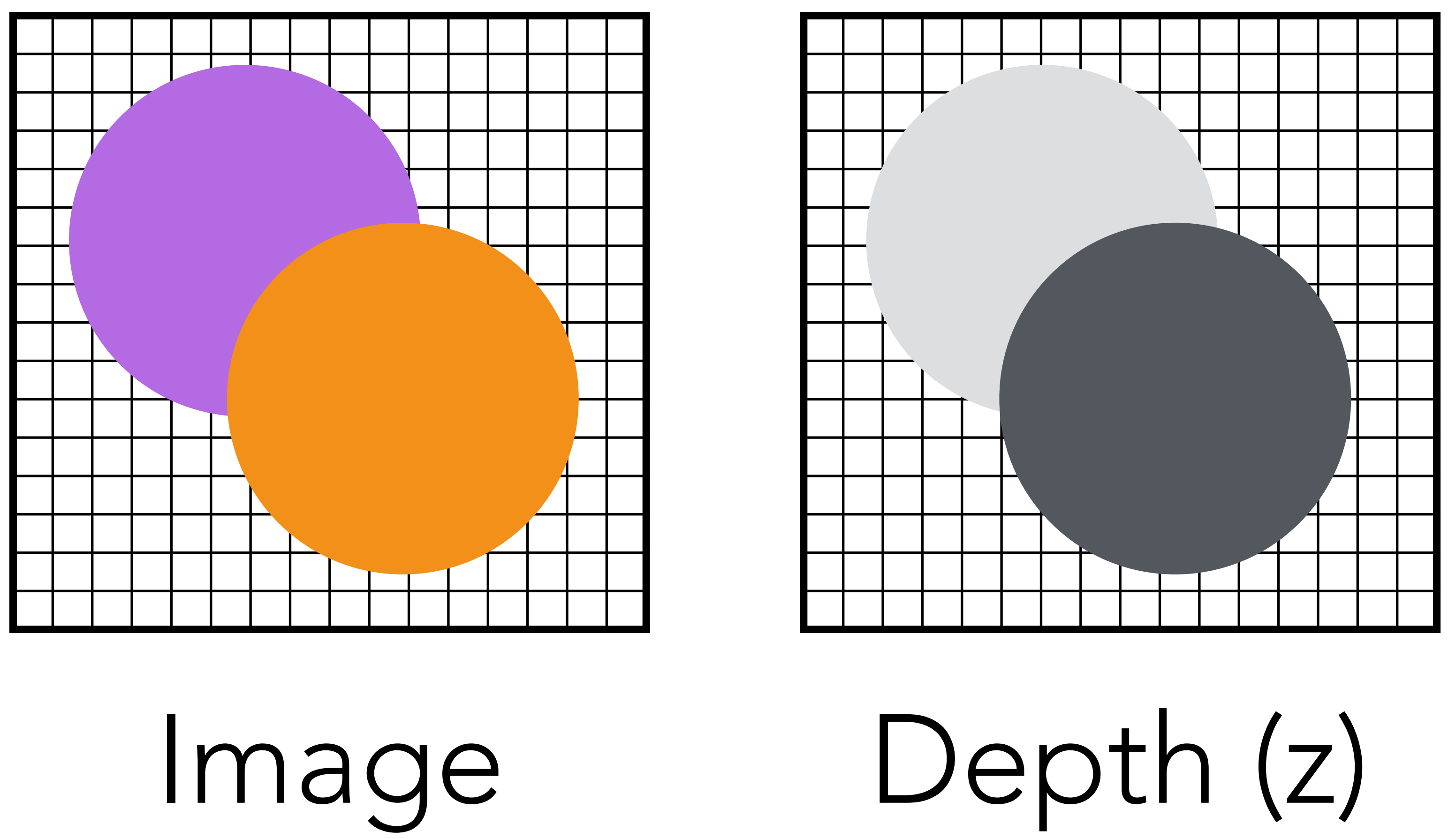
- To handle occlusion, you can sort all the objects in a scene by depth



- This is not always possible!



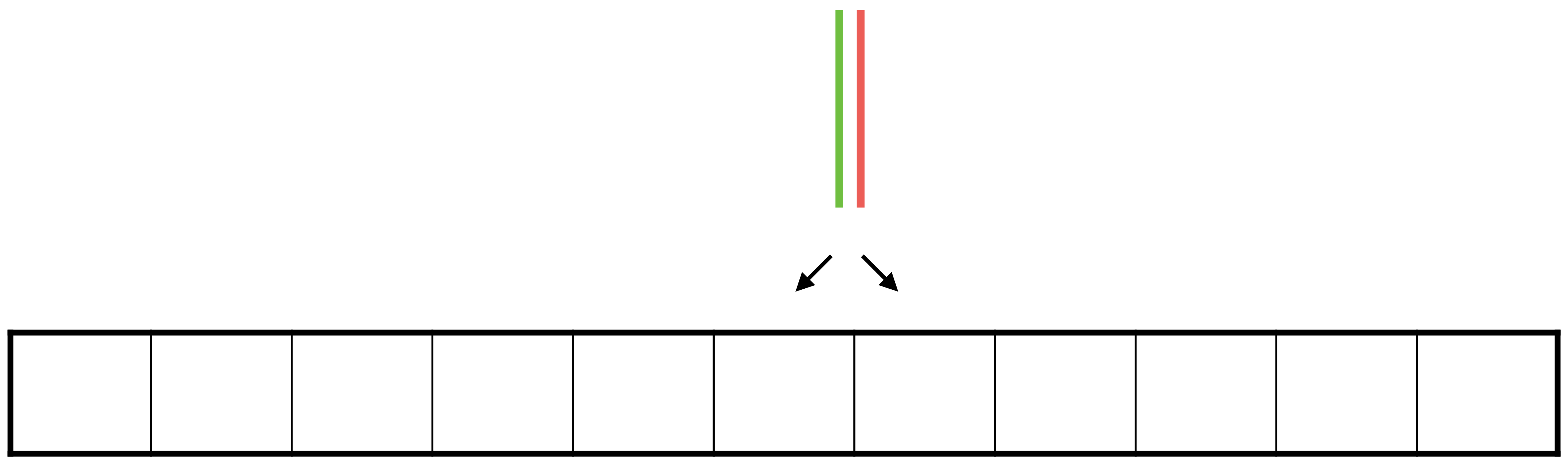
z-buffering

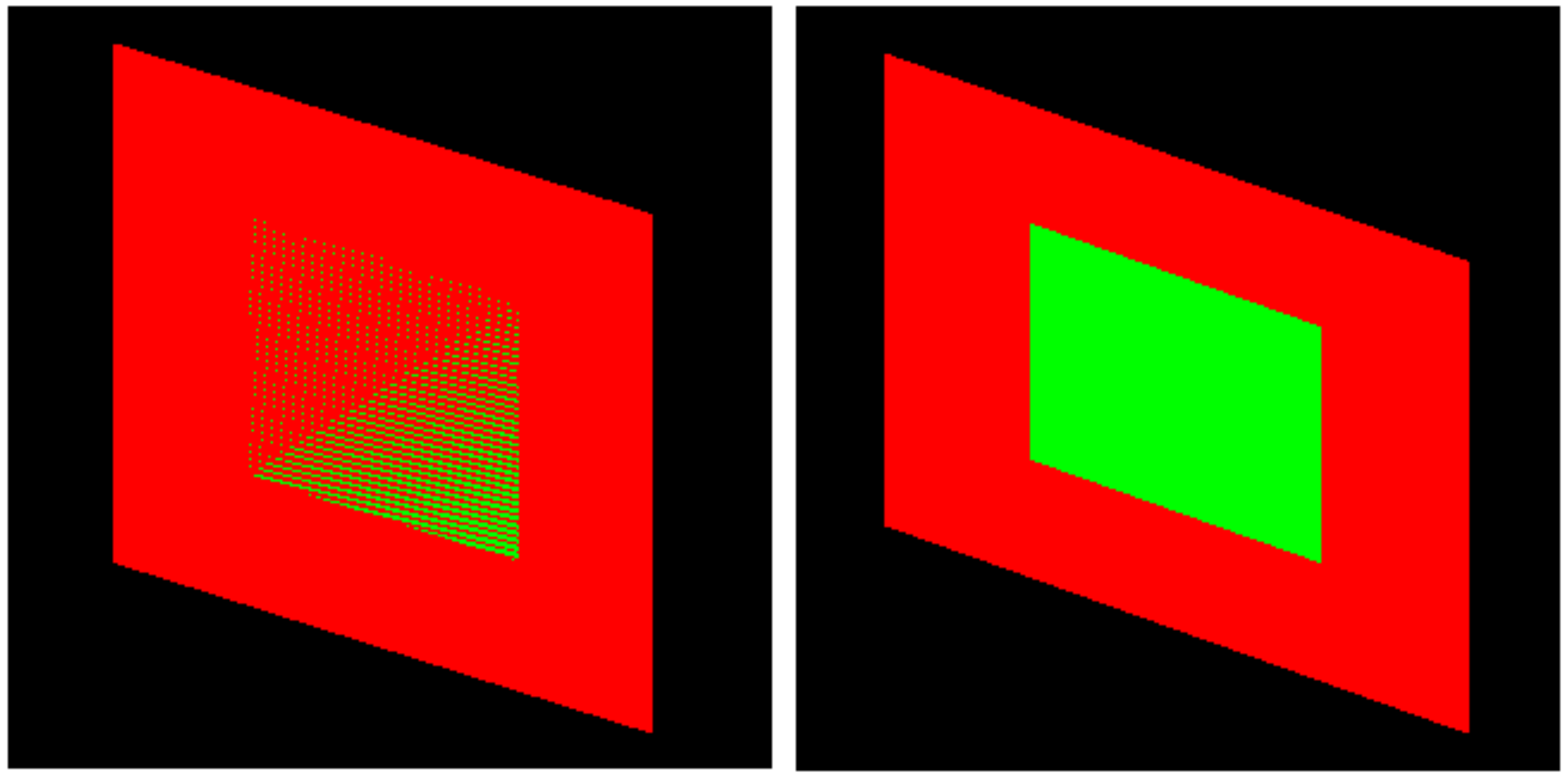


- You render the image both in the Image and in the depth buffer, where you store only the depth
- When a new fragment comes in, you draw it in the image only if it is closer
- This always work and it is cheap to evaluate! It is the default in all graphics hardware
- You still have to sort for transparency…

z-buffer quantization and “z-fighting”

- The z-buffer is quantized (the number of bits is heavily dependent on the hardware platform)
- Two close object might be quantized differently, leading to strange artifacts, usually called “z-fighting”





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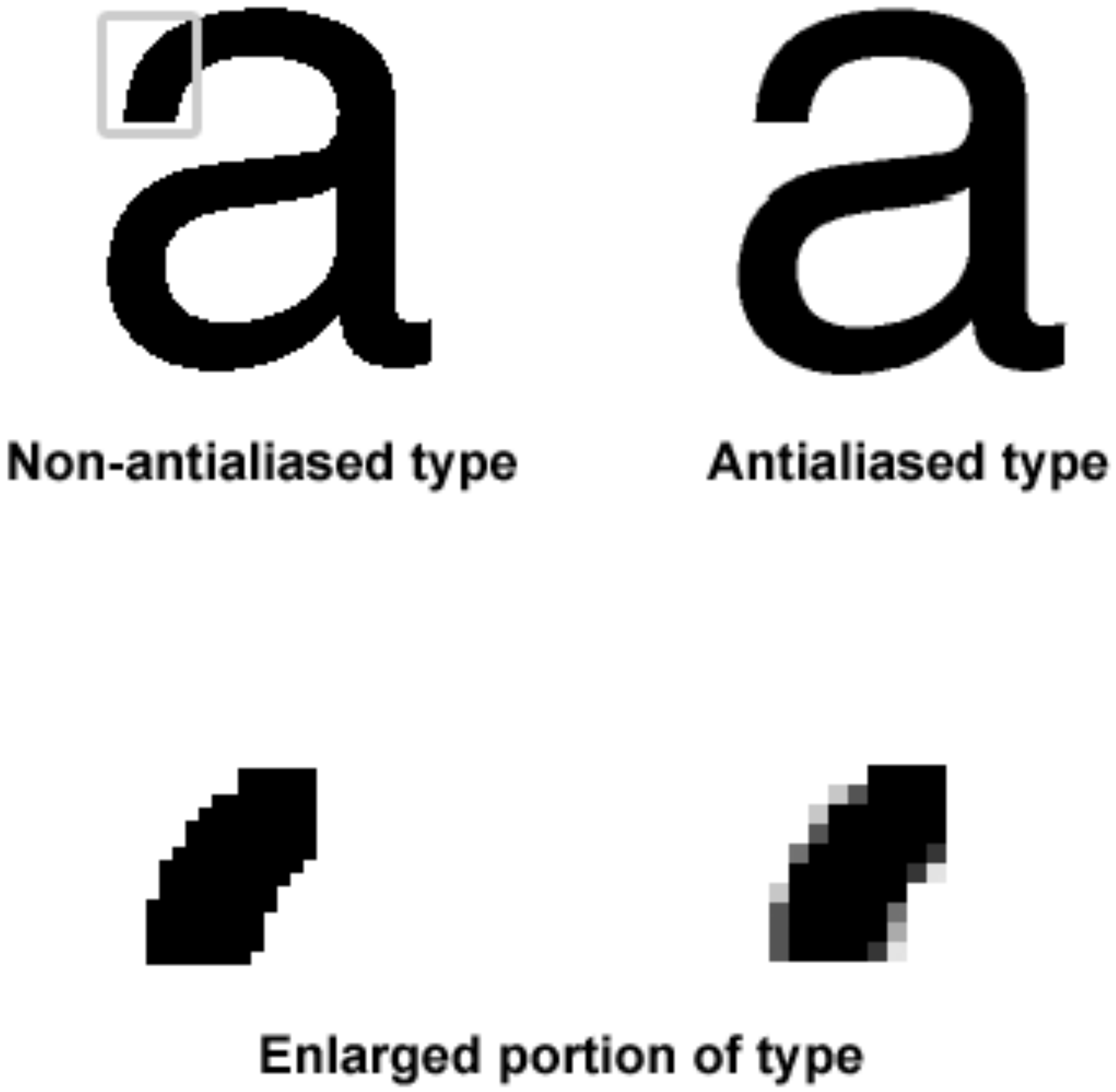
Super Sampling Anti-Aliasing

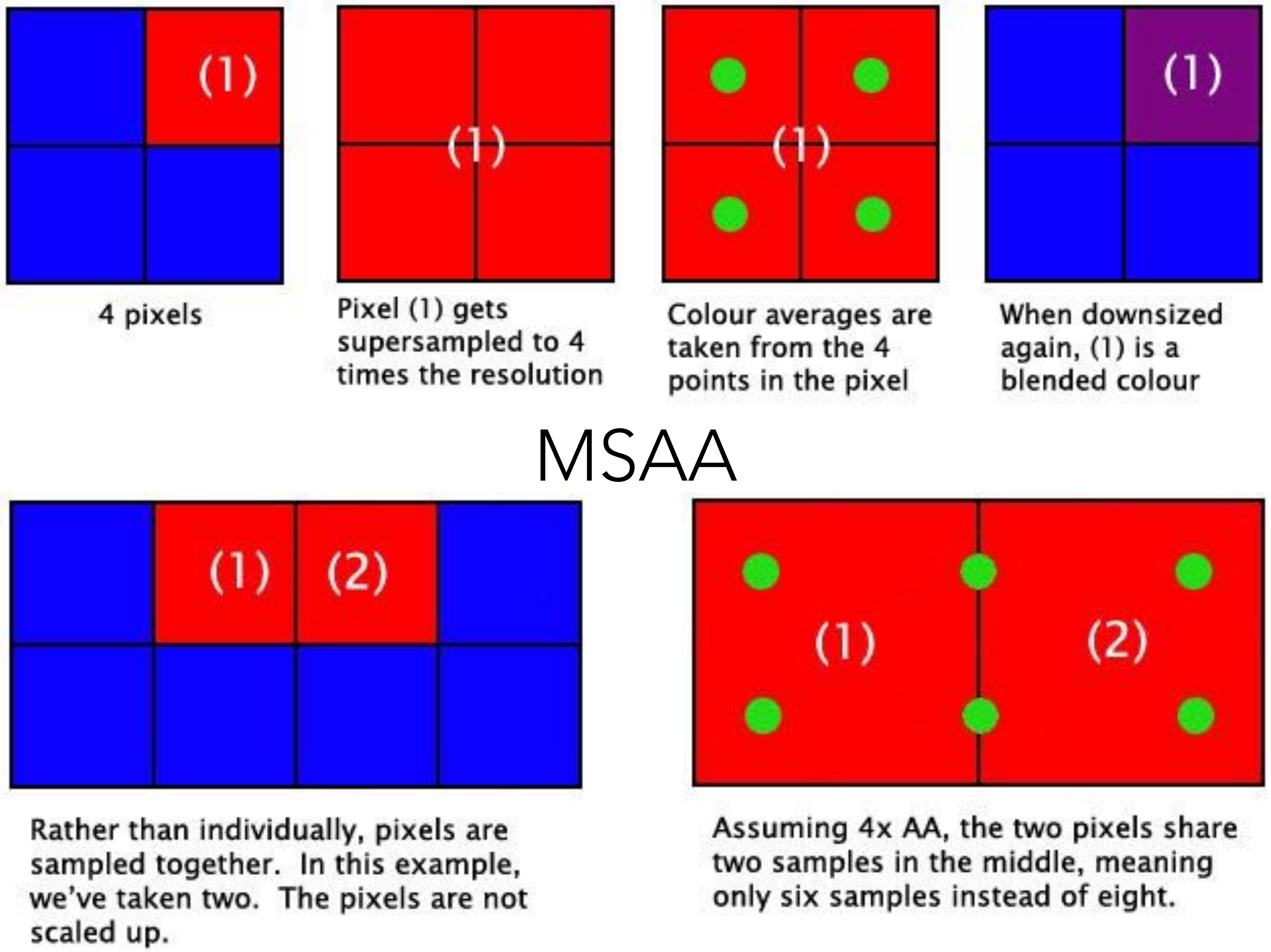


- Render nxn pixels instead of one
- Assign the average to the pixel

$$\begin{array}{|c|c|} \hline c_1 & c_2 \\ \hline c_3 & c_4 \\ \hline \end{array} \longrightarrow \boxed{\frac{c_1 + c_2 + c_3 + c_4}{4}}$$

Many different names and variants

- SSAA (FSAA)
- MSAA
- CSAA
- EQAA
- FXAA
- TX AA



References

Fundamentals of Computer Graphics, Fourth Edition
4th Edition **by Steve Marschner, Peter Shirley**

Chapter 8

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