Rasterization - Theory

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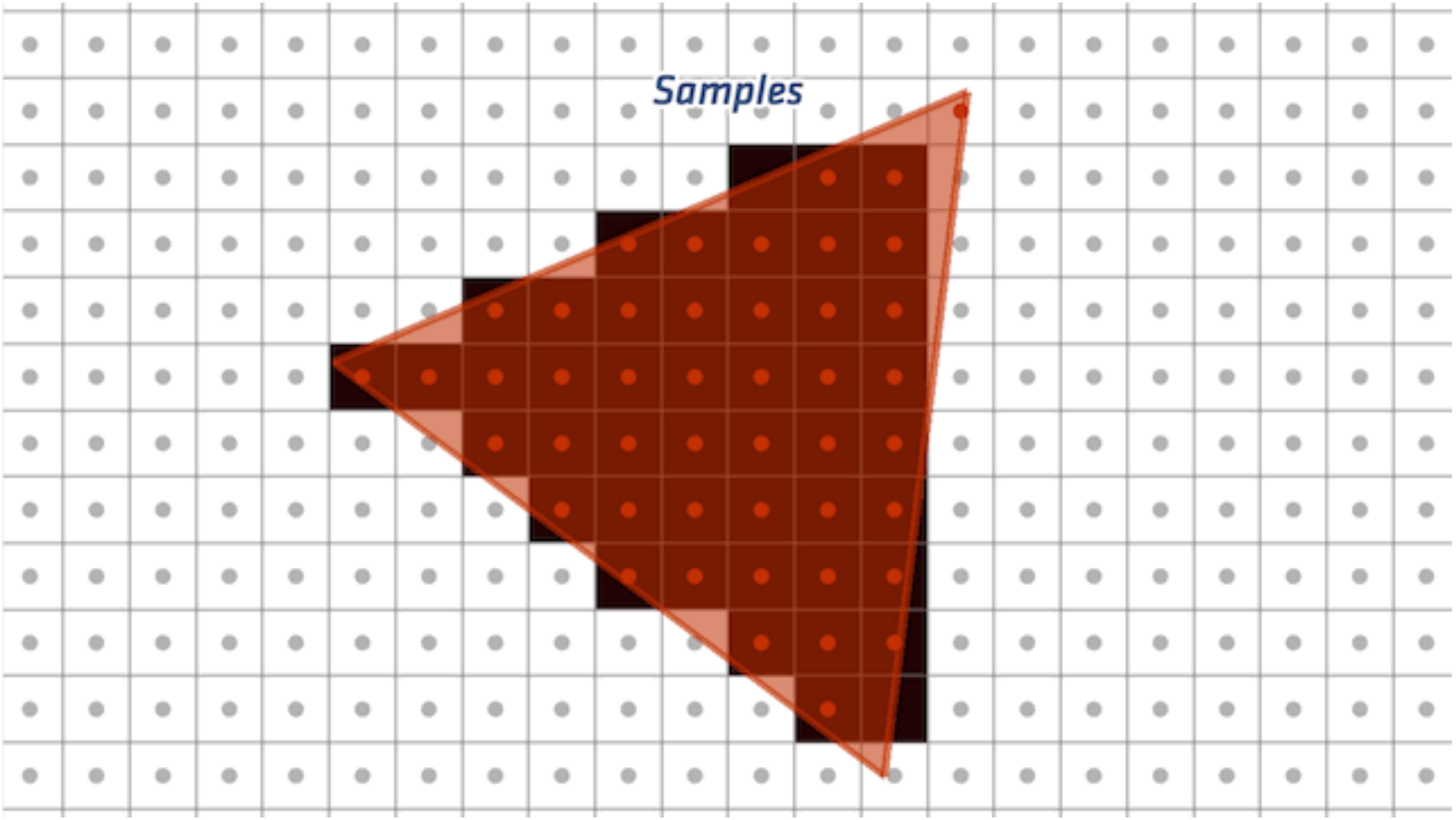
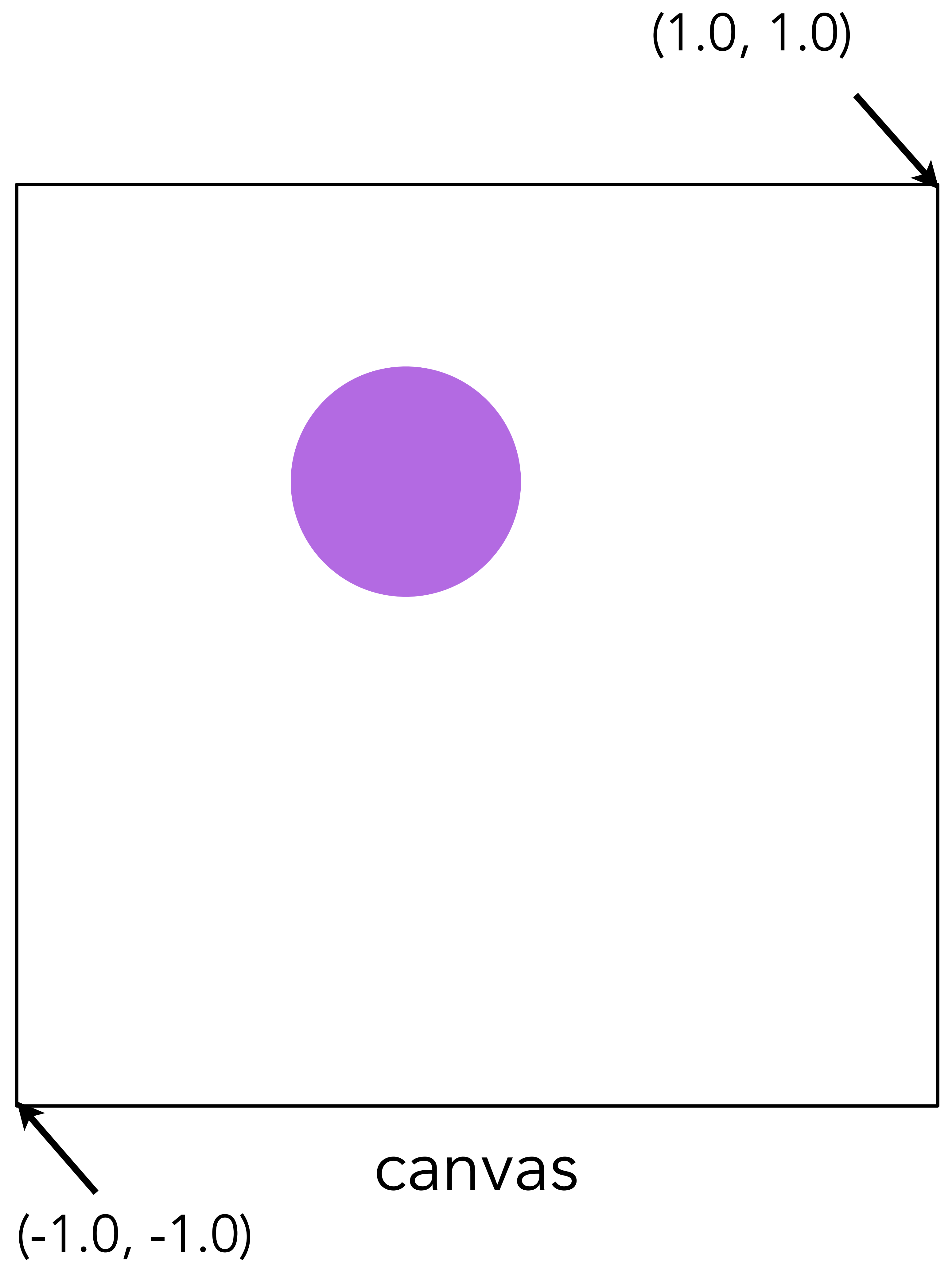
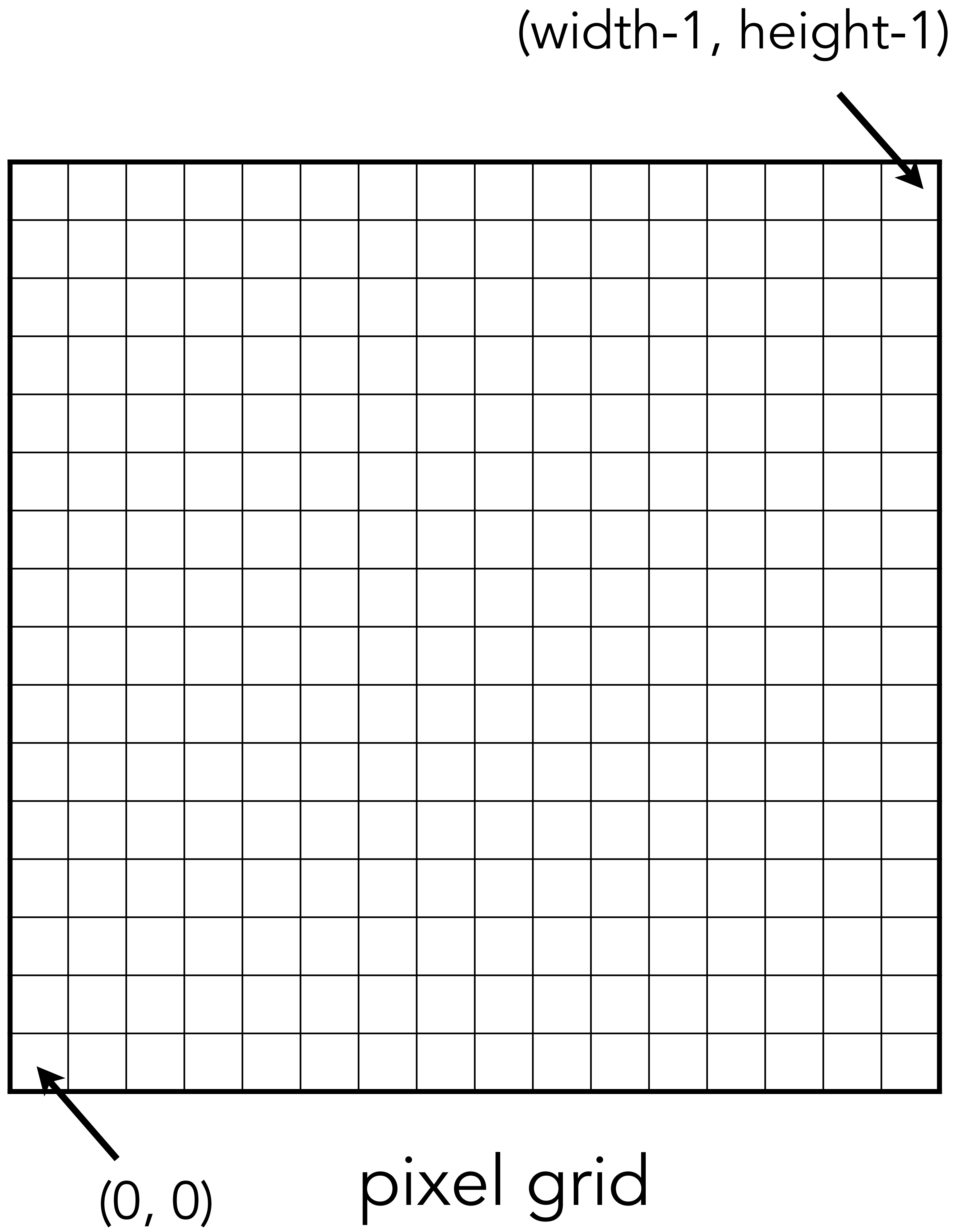

Samples

2D Canvas

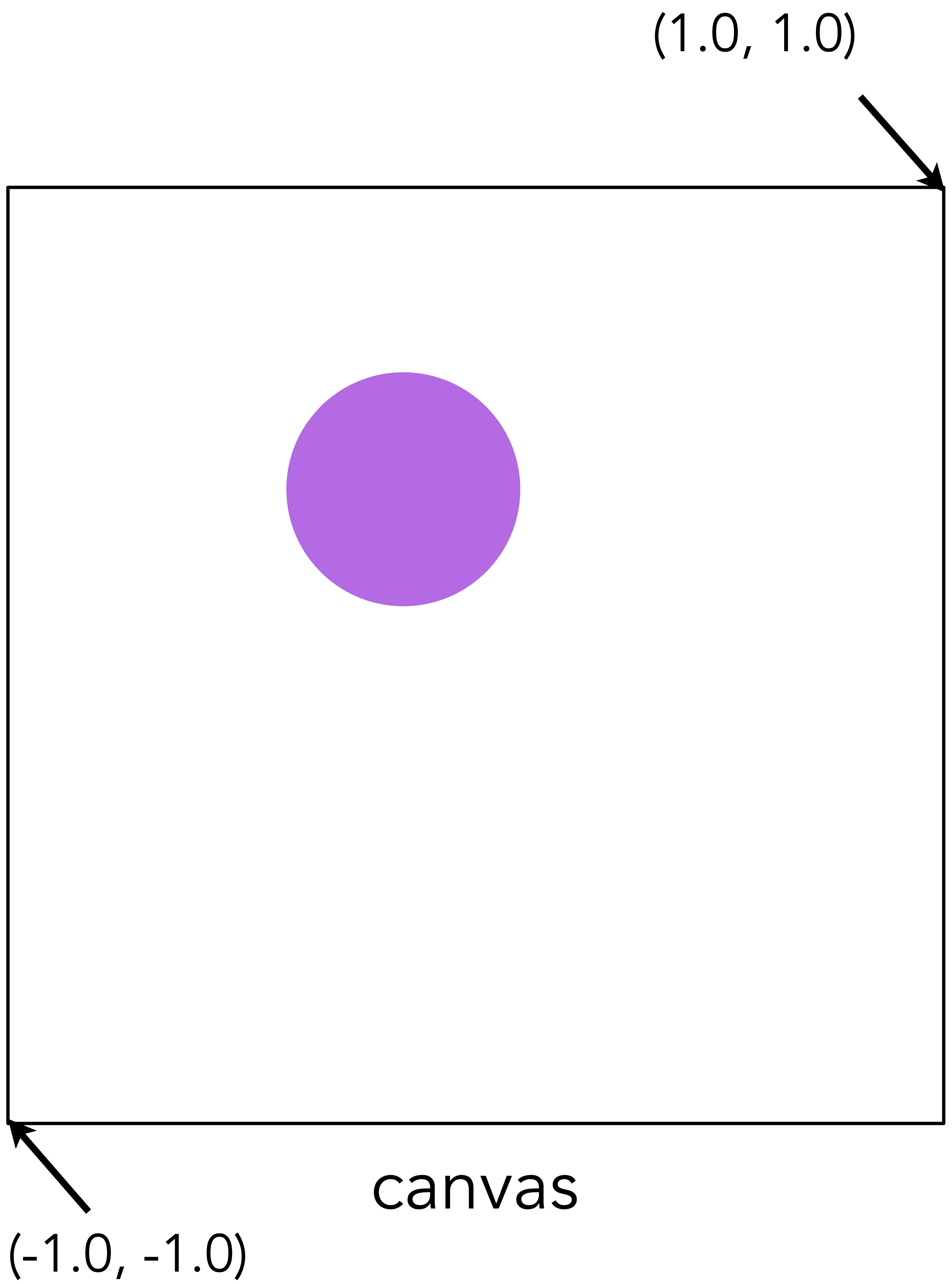


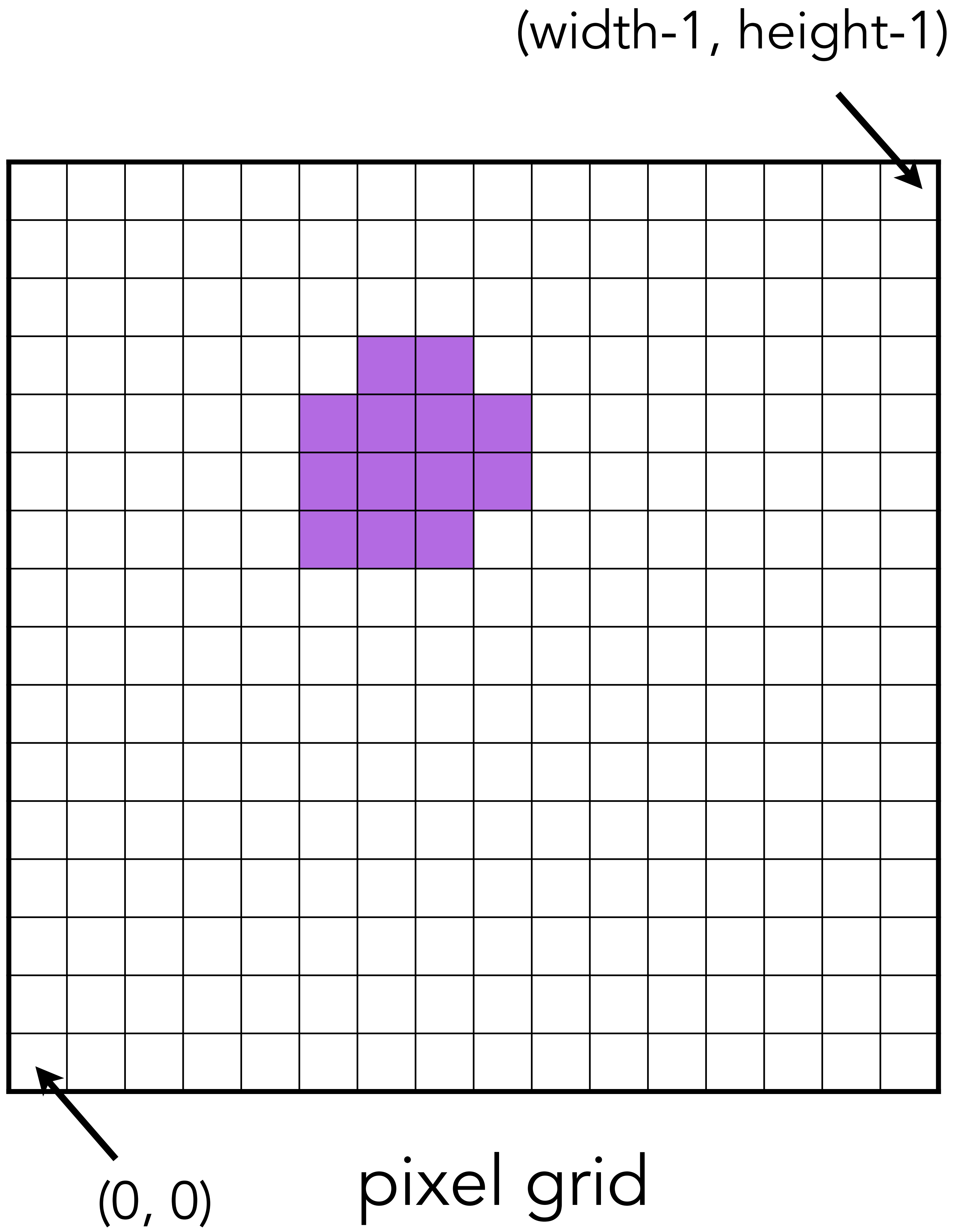
canvas



pixel grid

2D Canvas





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Implicit Geometry Representation

- Define a curve as zero set of 2D implicit function
  - *F(x,y) = 0* → on curve
  - *F(x,y) < 0* → inside curve
  - *F(x,y) > 0* → outside curve
- Example: Circle with center (c_x, c_y) and radius r

$$F(x,y) = (x - c_x)^2 + (y - c_y)^2 - r^2$$

Implicit Geometry Representation

- Define a curve as zero set of 2D implicit function
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  - $F(x,y) < 0 \rightarrow$ inside curve
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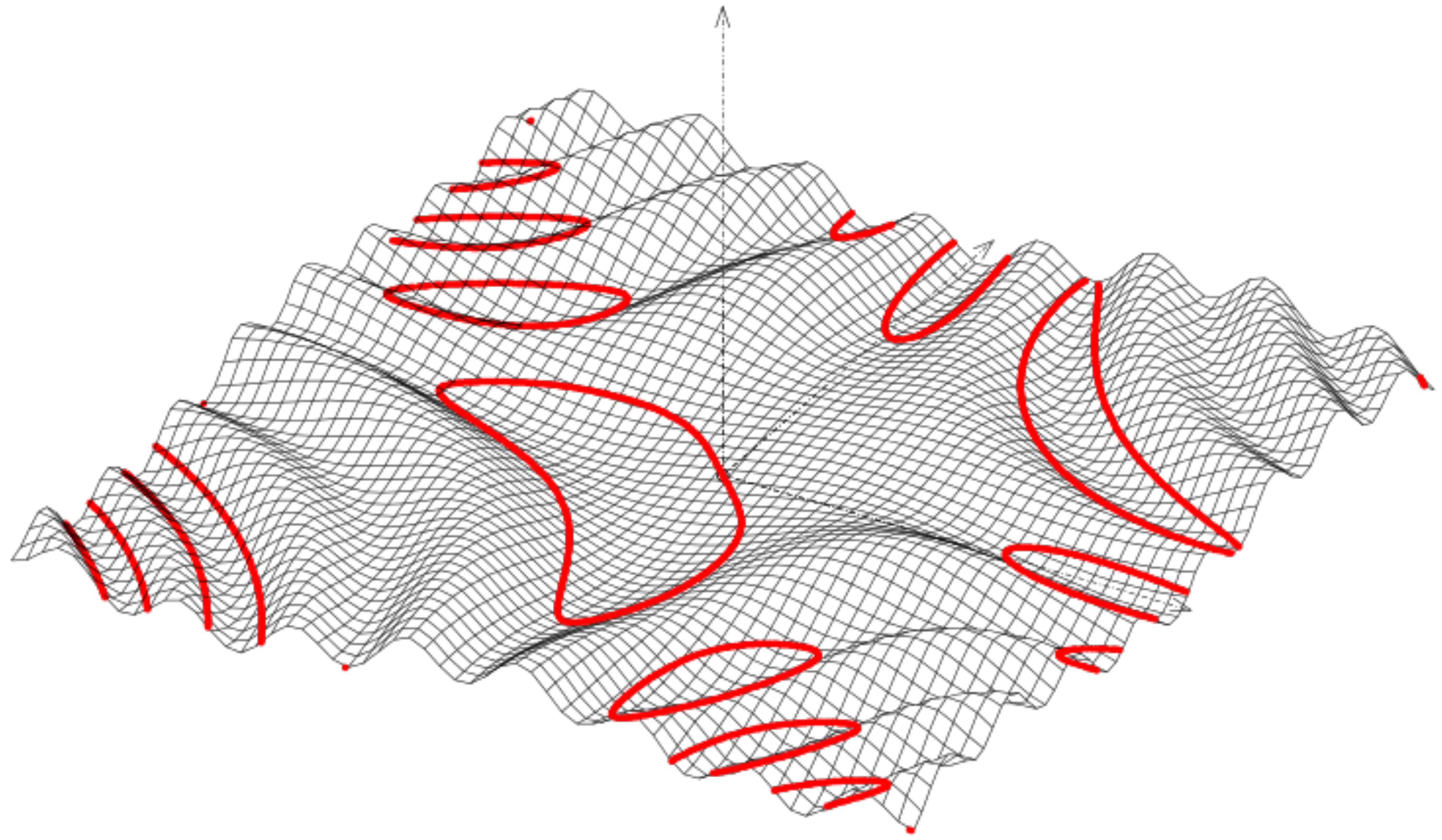


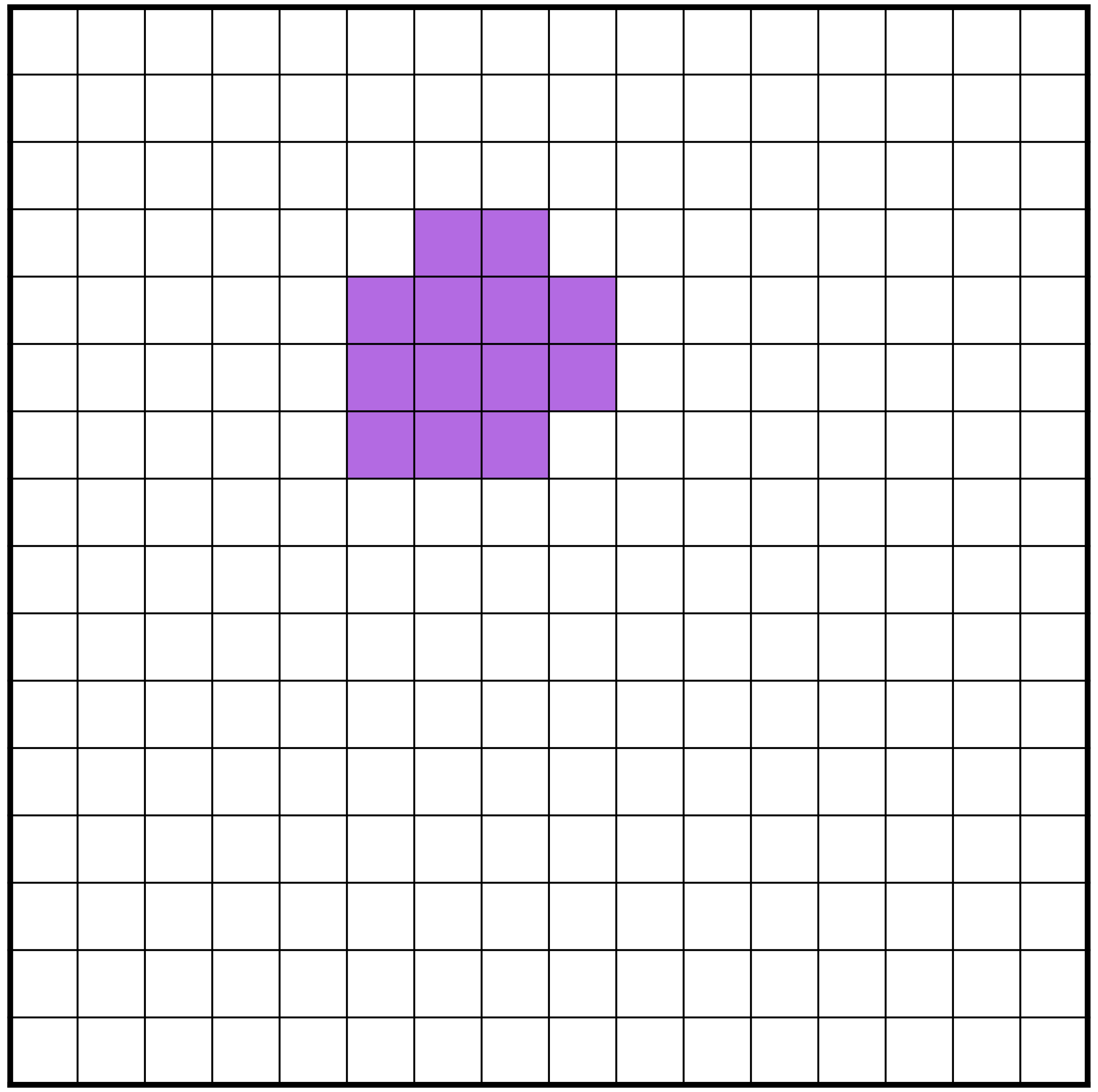
Implicit Rasterization

for all pixels (i,j)

(x,y) = map_to_canvas (i,j)

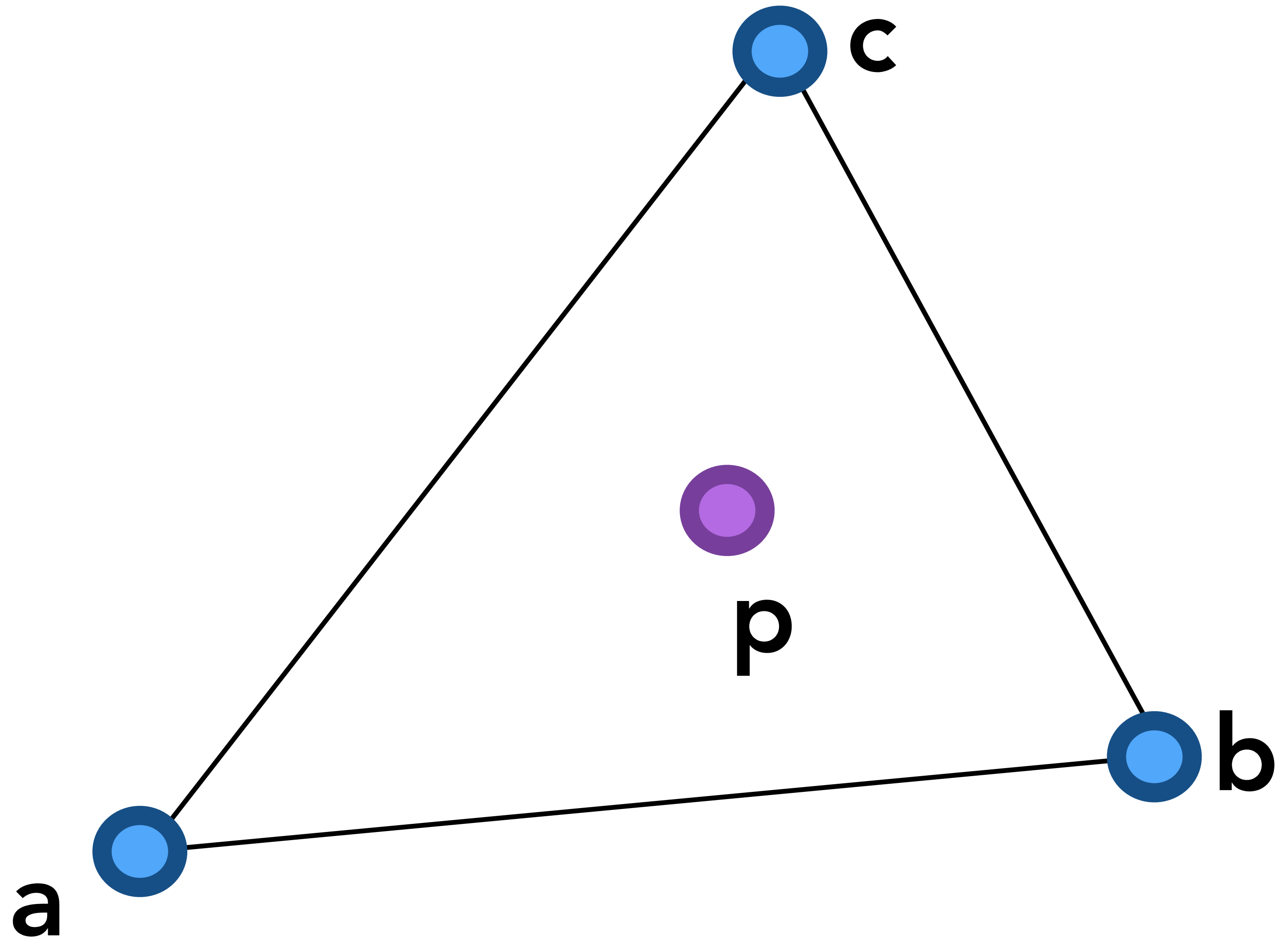
if F(x,y) < 0

set_pixel (i,j, color)



Barycentric Interpolation

- Barycentric coordinates:
  - $\mathbf{p} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}$ with $\alpha + \beta + \gamma = 1$



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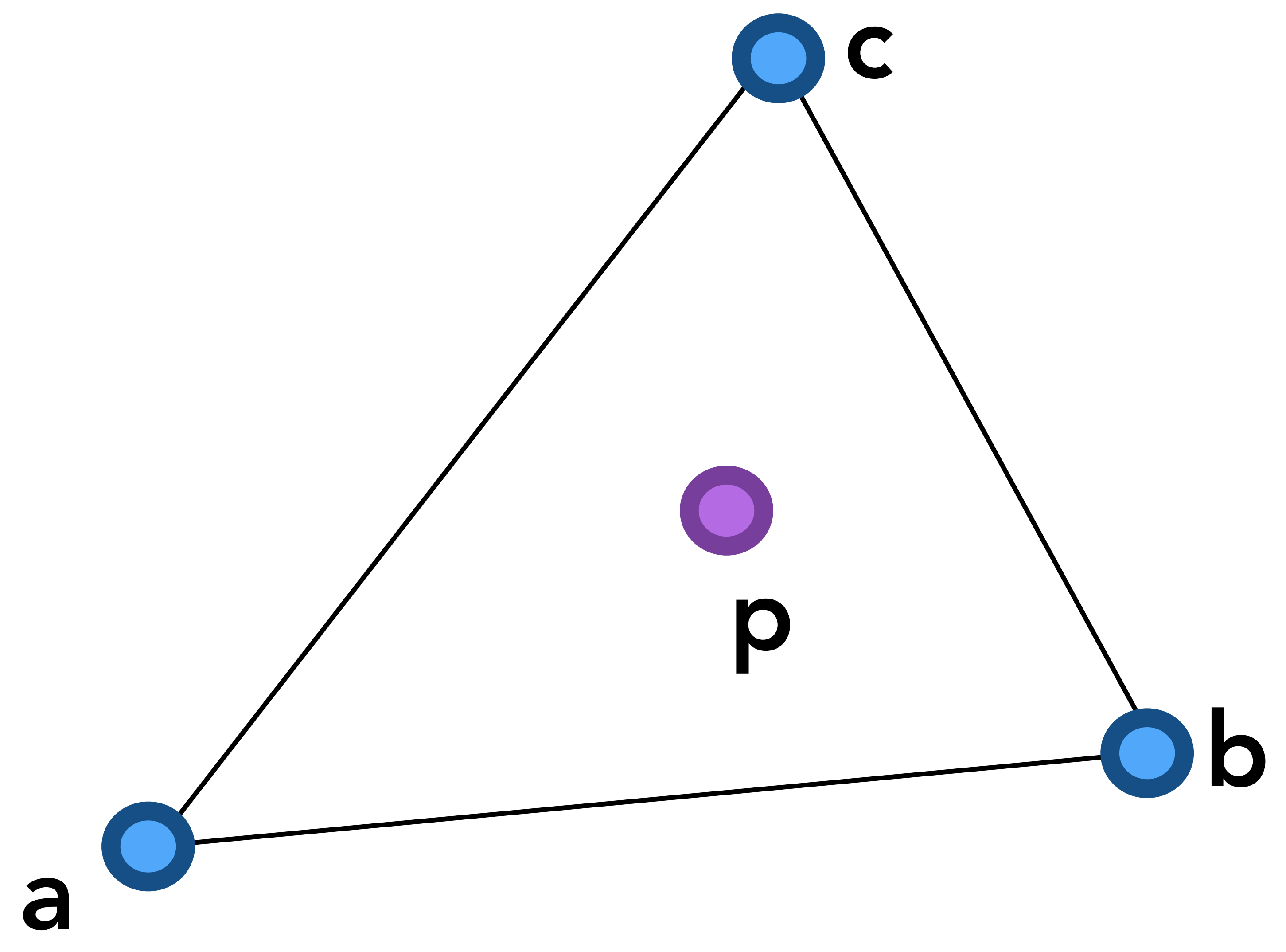
  - Unique for non-collinear **a**,**b**,**c**



$$\begin{bmatrix} \mathbf{a}_x & \mathbf{b}_x & \mathbf{c}_x \\ \mathbf{a}_y & \mathbf{b}_y & \mathbf{c}_y \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} \mathbf{p}_x \\ \mathbf{p}_y \\ 1 \end{bmatrix}$$

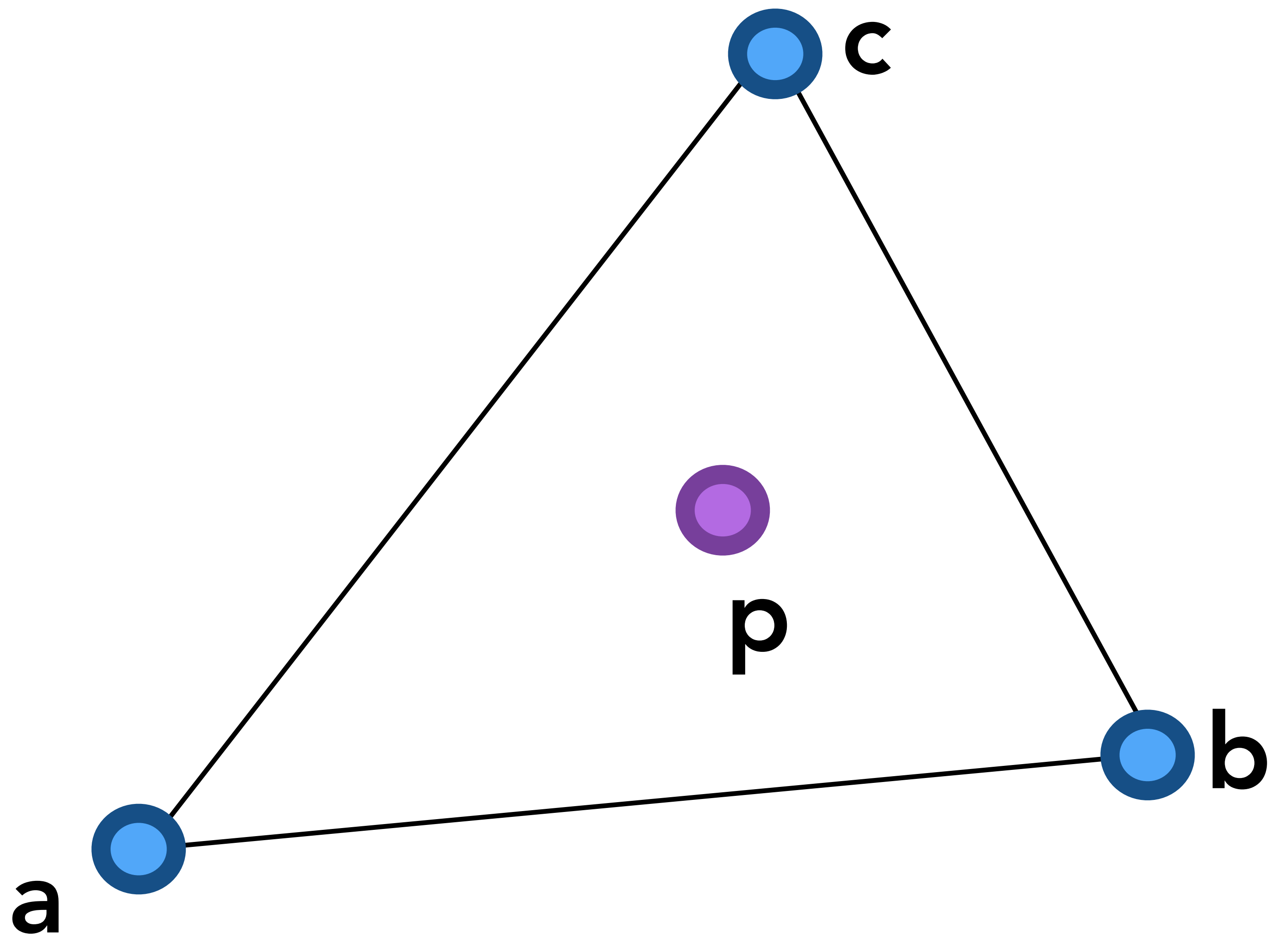
Barycentric Interpolation

- Barycentric coordinates:
  - **p** = α**a** + β**b** + γ**c**   with α + β + γ = 1
  - Unique for non-collinear **a**,**b**,**c**
  - Ratio of triangle areas

$$\alpha(\mathbf{p}) = \frac{\text{area}(\mathbf{p}, \mathbf{b}, \mathbf{c})}{\text{area}(\mathbf{a}, \mathbf{b}, \mathbf{c})}$$

$$\beta(\mathbf{p}) = \frac{\text{area}(\mathbf{p}, \mathbf{c}, \mathbf{a})}{\text{area}(\mathbf{a}, \mathbf{b}, \mathbf{c})}$$

$$\gamma(\mathbf{p}) = \frac{\text{area}(\mathbf{p}, \mathbf{a}, \mathbf{b})}{\text{area}(\mathbf{a}, \mathbf{b}, \mathbf{c})}$$



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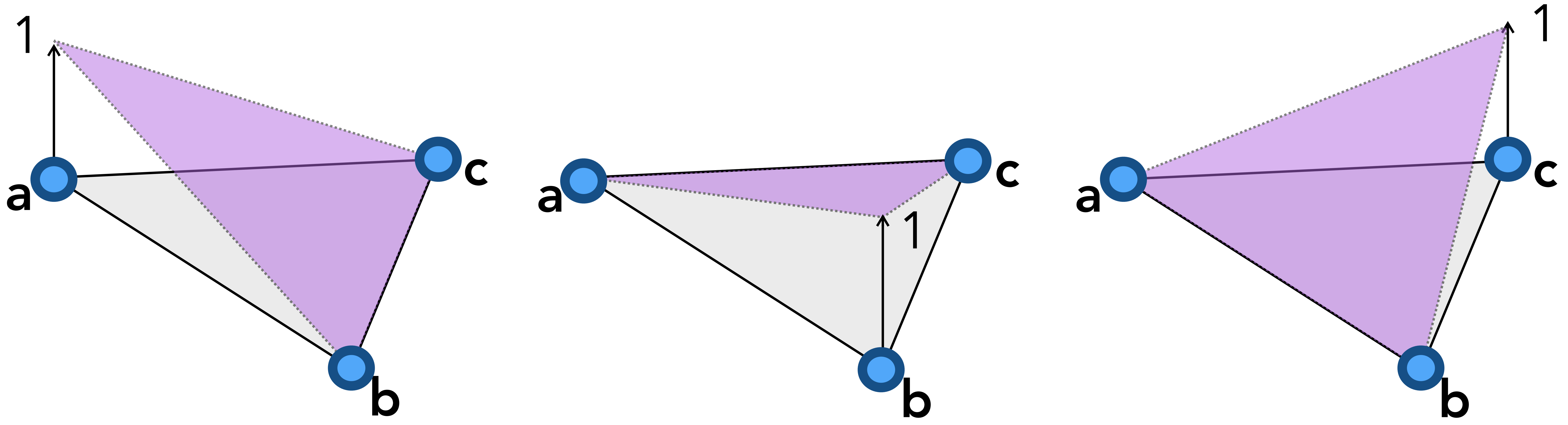
  - $\alpha(\mathbf{p})$, $\beta(\mathbf{p})$, $\gamma(\mathbf{p})$ are linear functions



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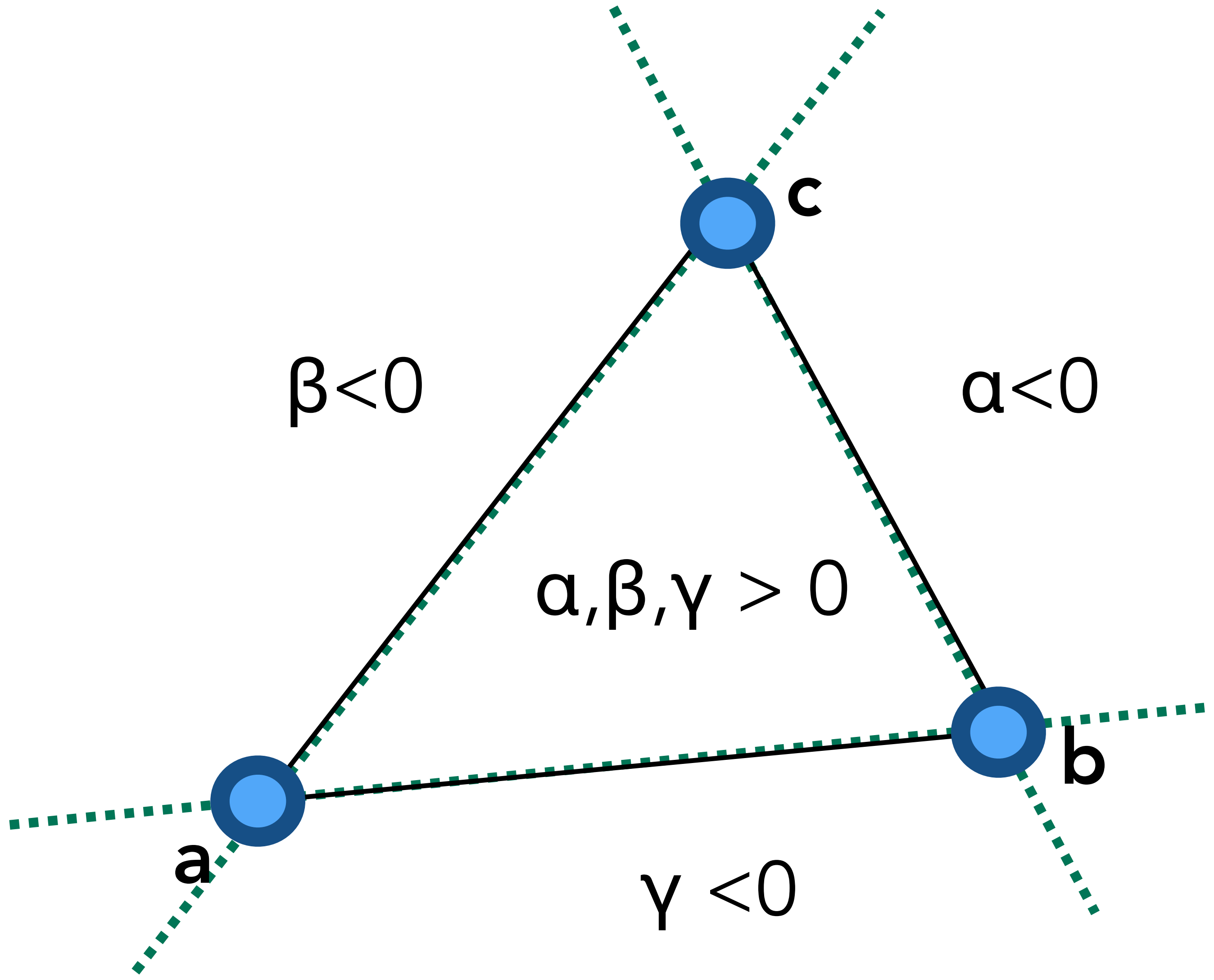
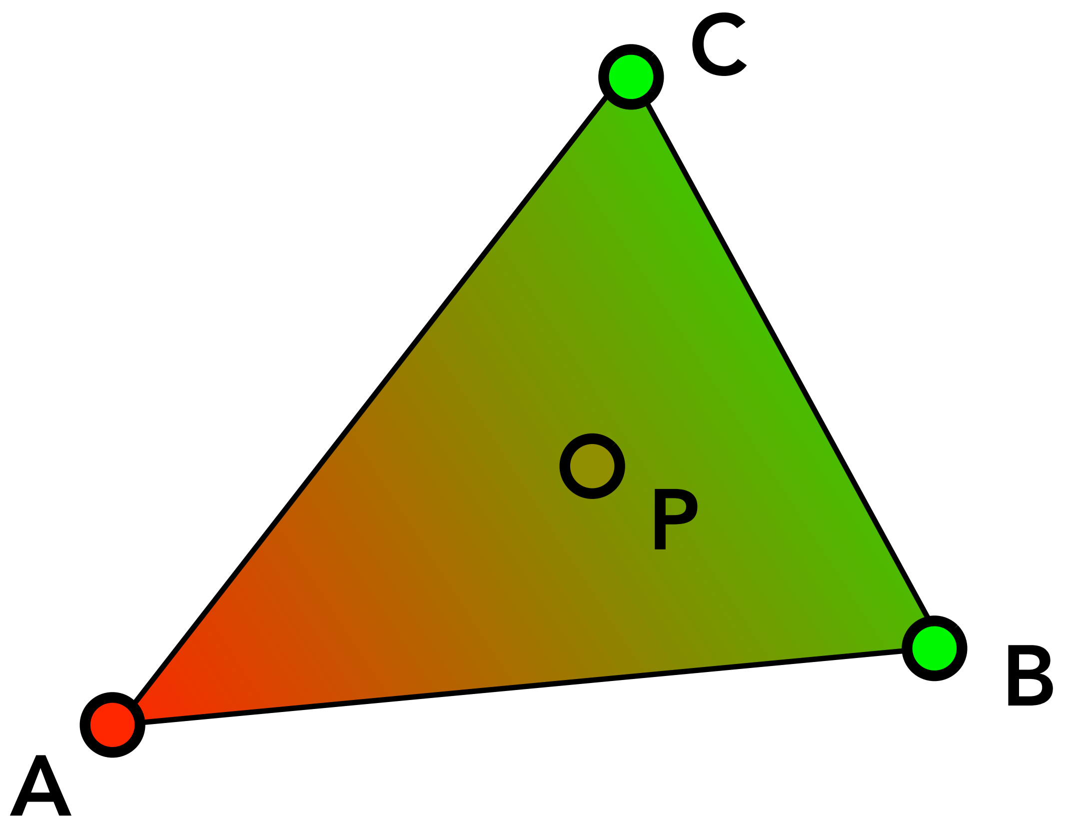
  - Gives inside/outside information



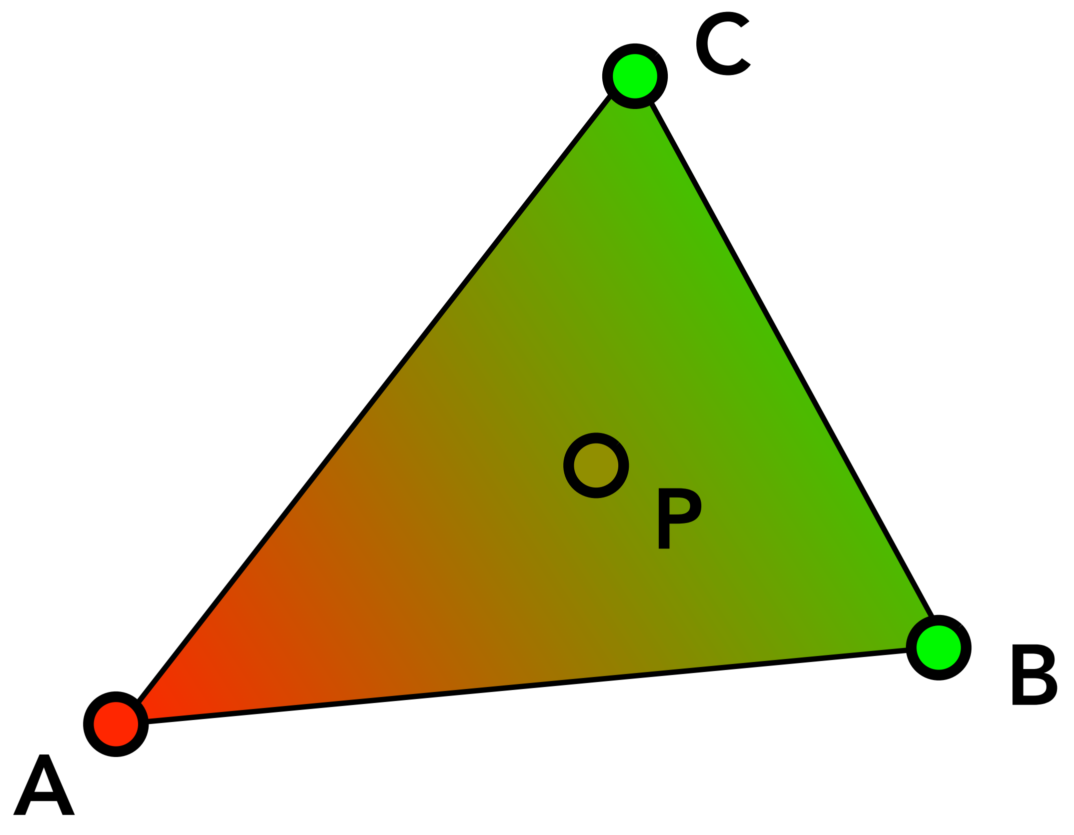
Barycentric Interpolation

- Barycentric coordinates:
  - **p** = α**a** + β**b** + γ**c** with α + β + γ = 1
  - Unique for non-collinear **a**,**b**,**c**
  - Ratio of triangle areas
  - α(**p**), β(**p**), γ(**p**) are linear functions
  - Gives inside/outside information
  - Use barycentric coordinates to interpolate vertex normals (or other data, e.g. colors)

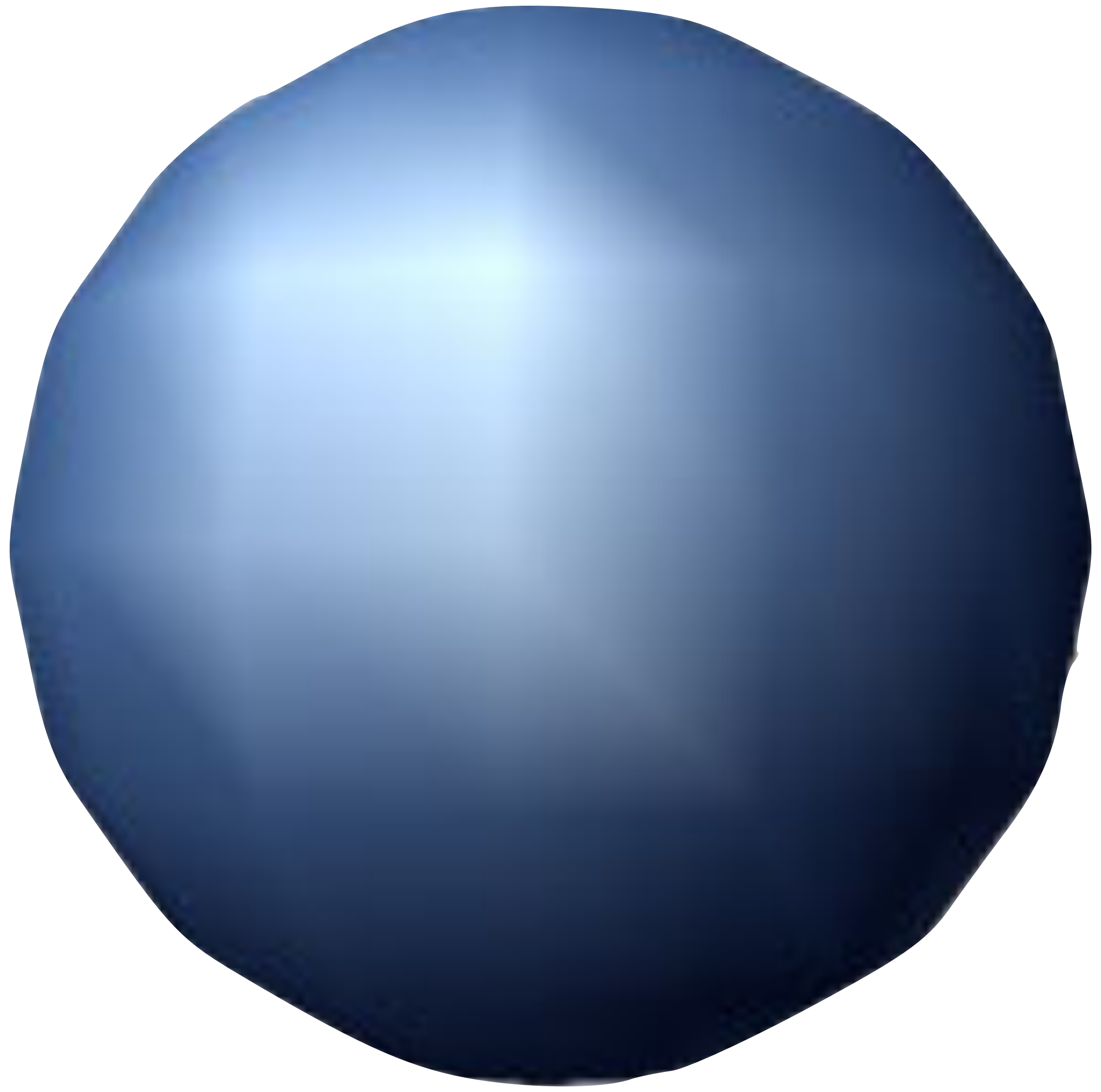
$$\mathbf{n}(\mathbf{P}) \quad = \quad \alpha \cdot \mathbf{n}(\mathbf{A}) + \beta \cdot \mathbf{n}(\mathbf{B}) + \gamma \cdot \mathbf{n}(\mathbf{C})$$



Color Interpolation

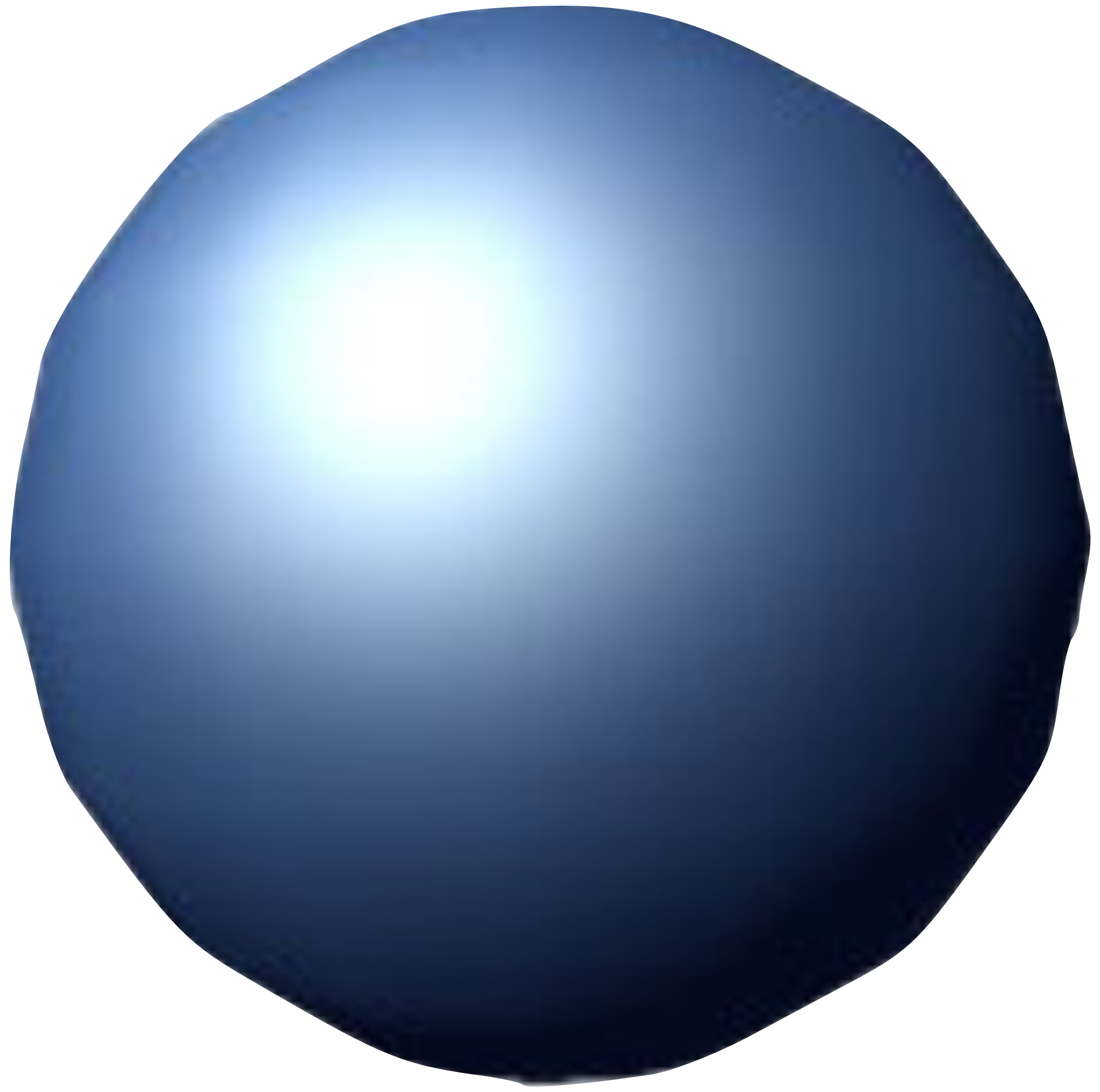


Per-vertex



Evaluate color on vertices, then interpolates it

Per-pixel



Interpolates positions and normals, then evaluate color on each pixel

http://www.cgchannel.com/2010/11/cg-science-for-artists-part-2-the-real-time-rendering-pipeline/

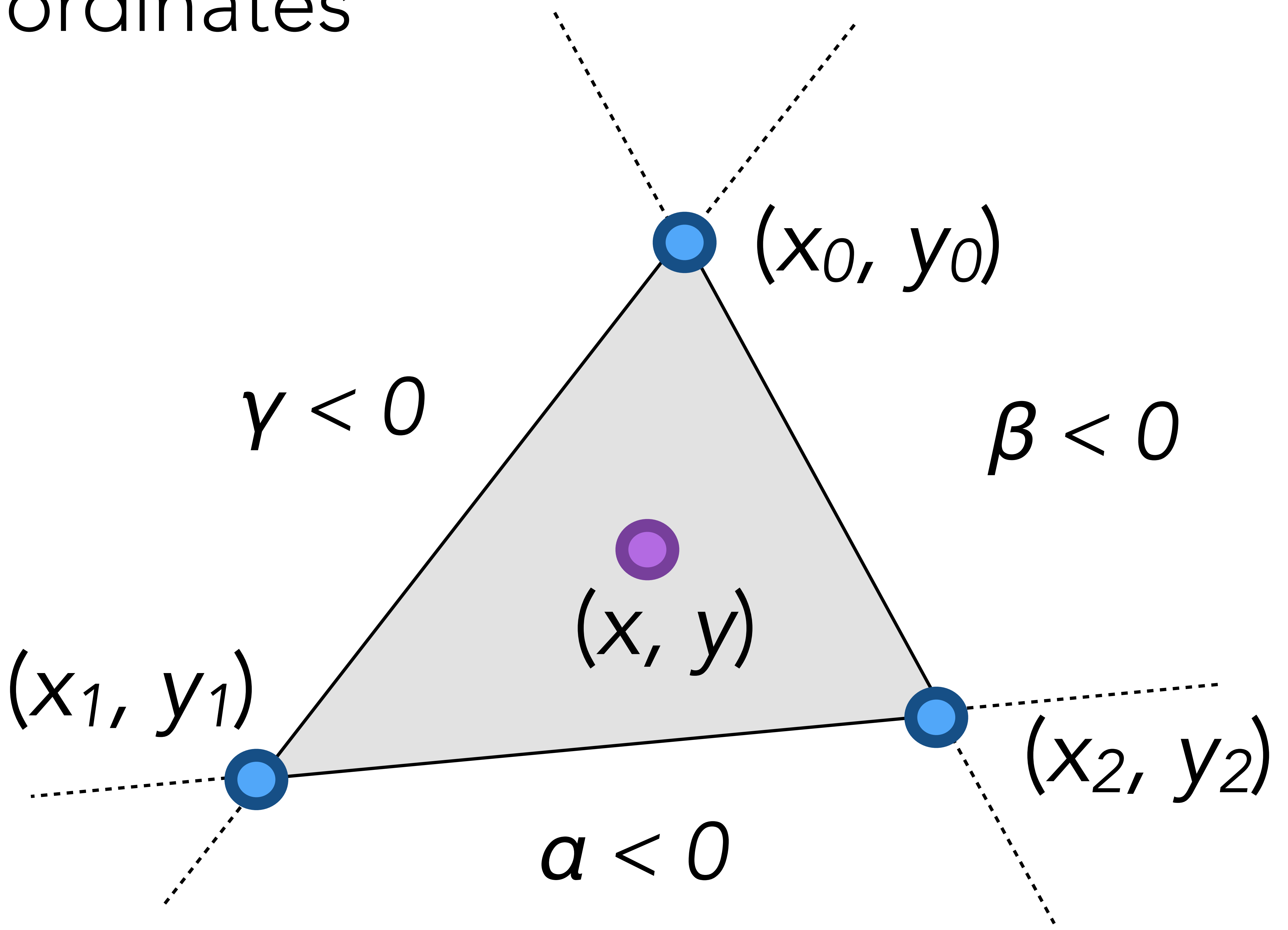
Triangle Rasterization

- Each triangle is represented as three 2D points (x_0, y_0), (x_1, y_1), (x_2, y_2)
- Rasterization using barycentric coordinates

$$x = \alpha \cdot x_0 + \beta \cdot x_1 + \gamma \cdot x_2$$

$$y = \alpha \cdot y_0 + \beta \cdot y_1 + \gamma \cdot y_2$$

$$\alpha + \beta + \gamma = 1$$



Triangle Rasterization

- Each triangle is represented as three 2D points $(x_0, y_0), (x_1, y_1), (x_2, y_2)$
- Rasterization using barycentric coordinates

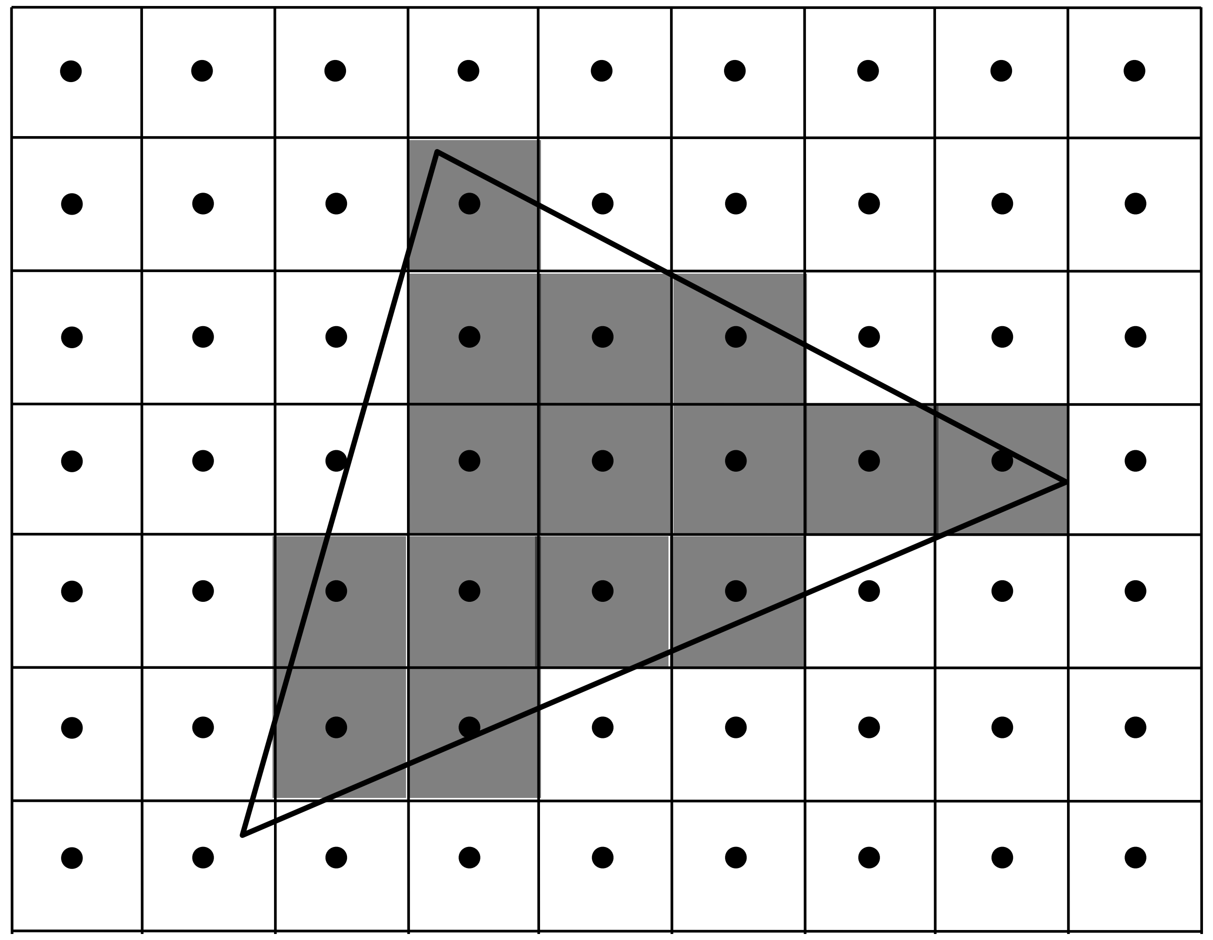
for all y **do**

  for all x **do**

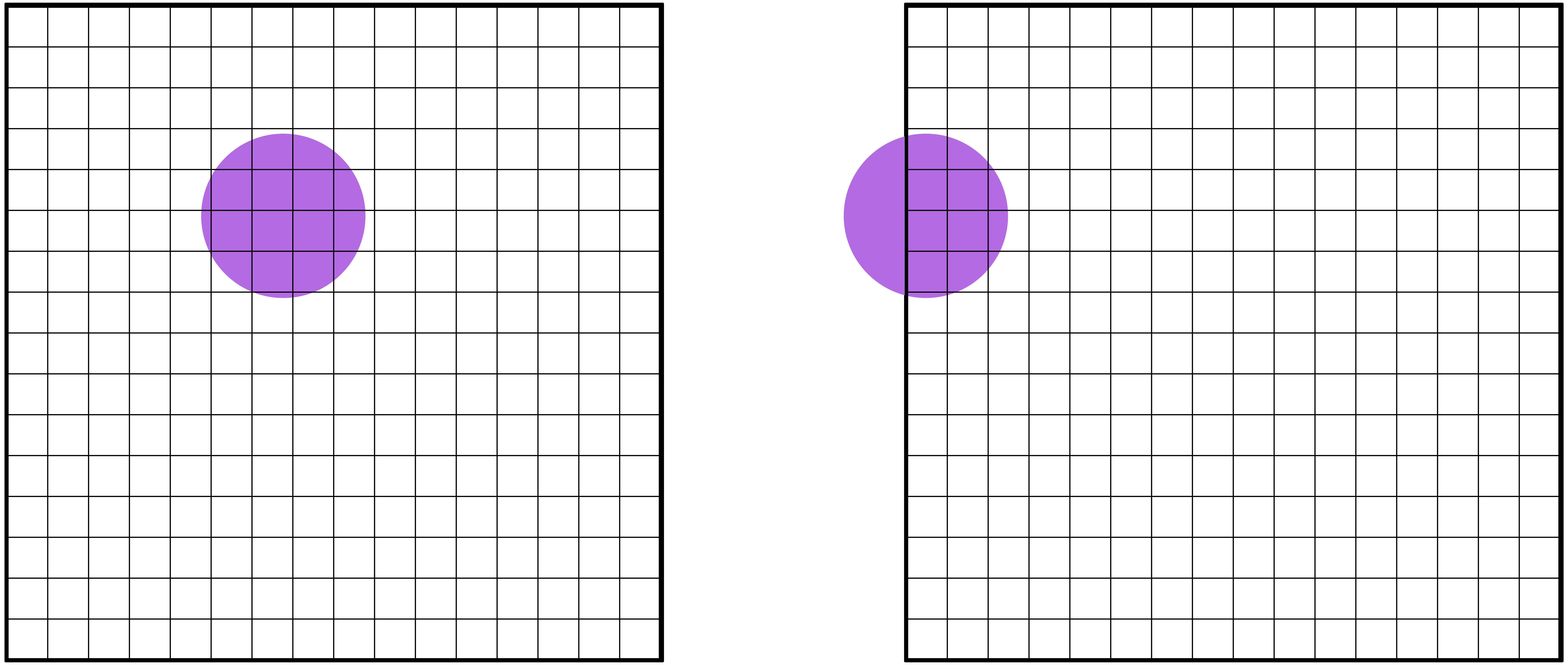
    compute (α,β,γ) for (x,y)

    if ($\alpha \in [0,1]$ and $\beta \in [0,1]$ and $\gamma \in [0,1]$]

      set_pixel (x,y)



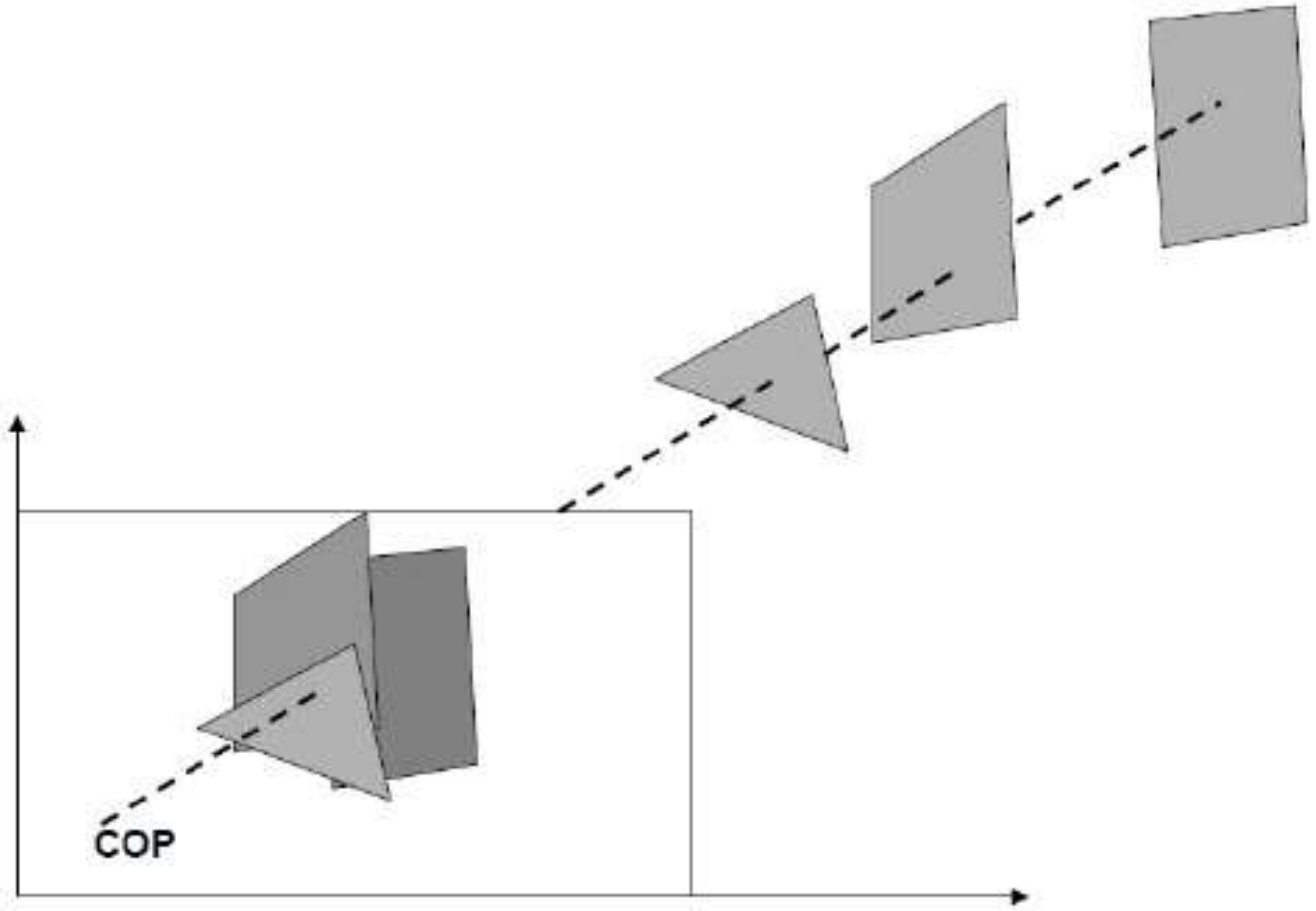
Clipping



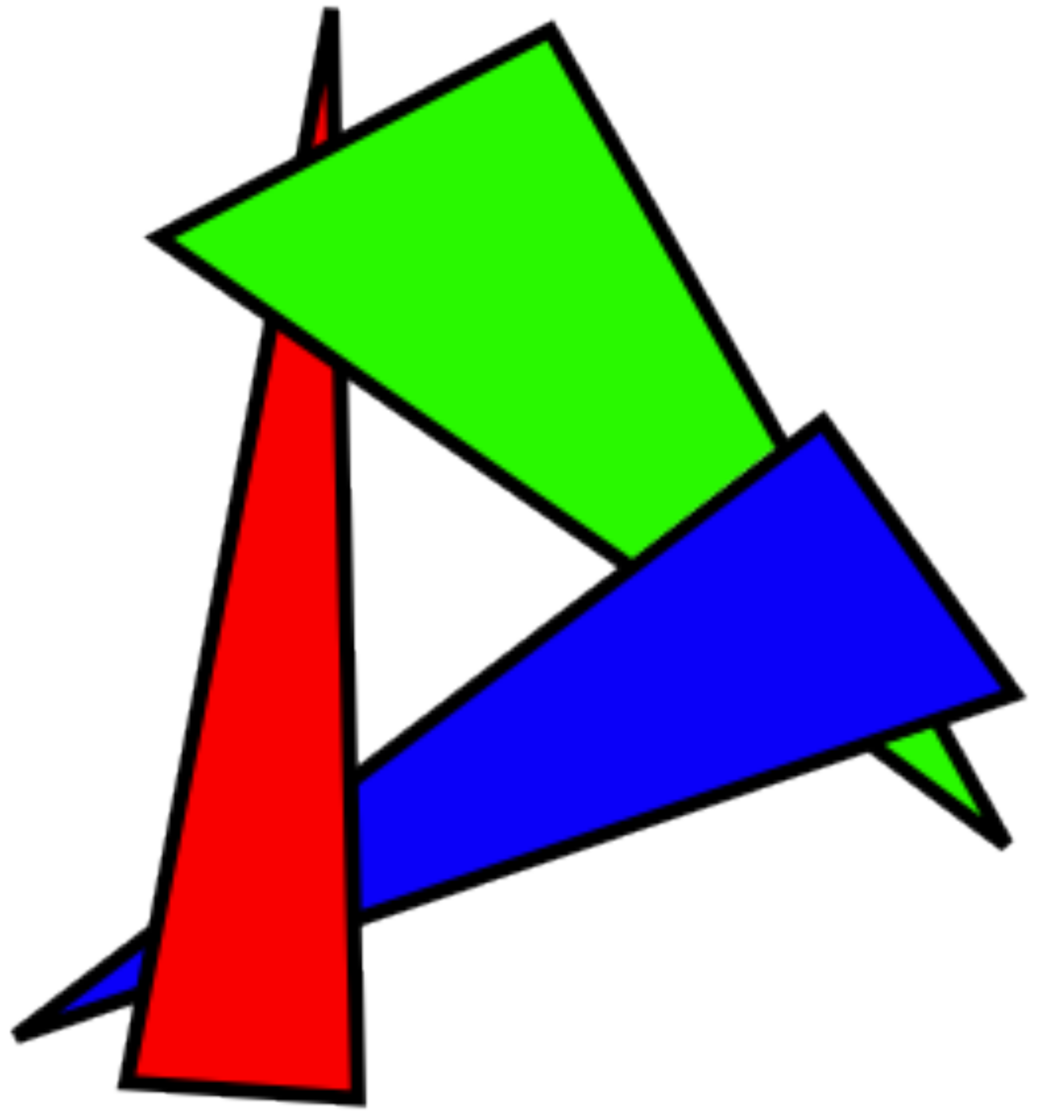
- Ok if you do it brute force
- Care is required if you are explicitly tracing the boundaries

Objects Depth Sorting

- To handle occlusion, you can sort all the objects in a scene by depth



- This is not always possible!



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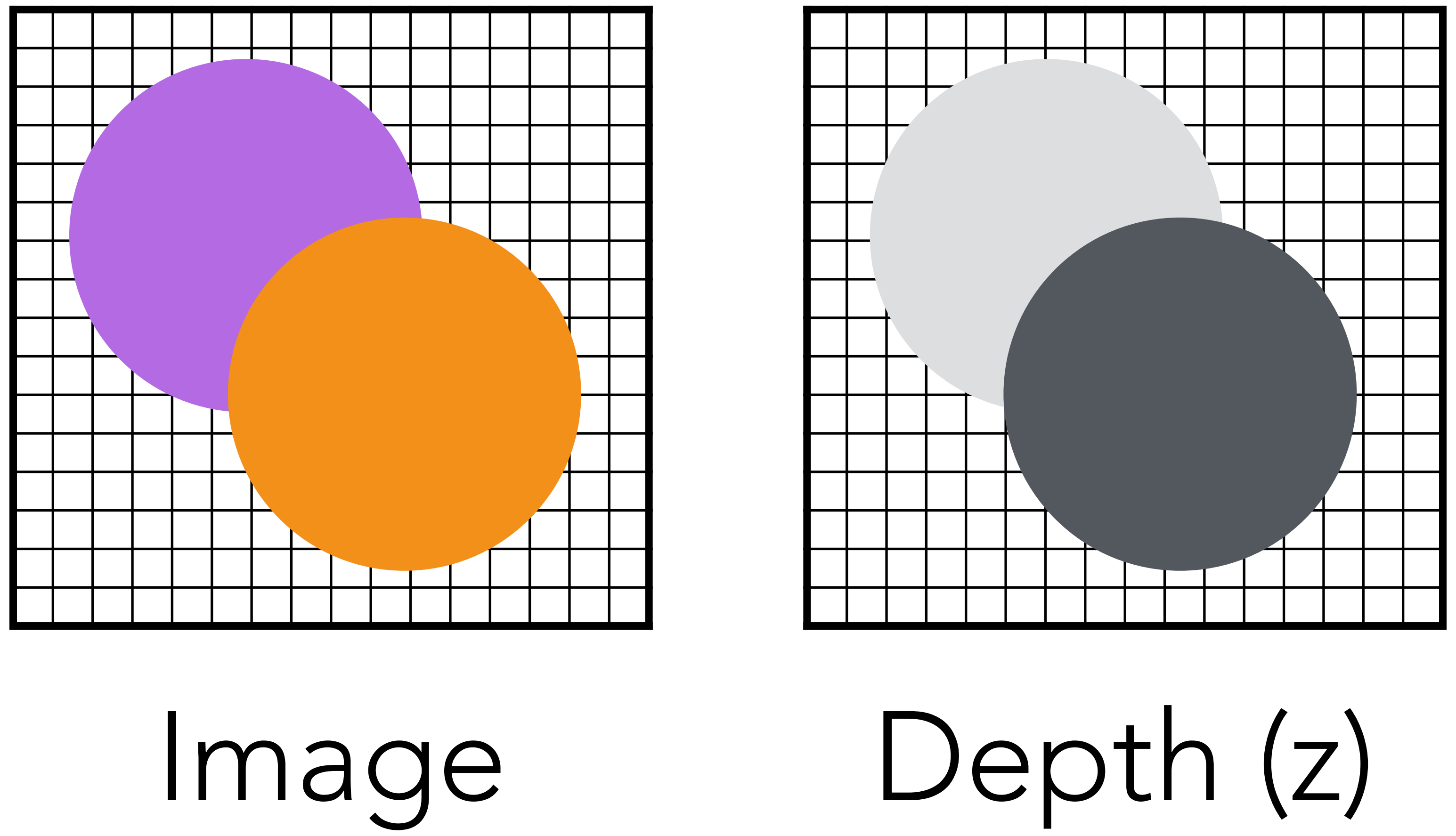
z-buffering

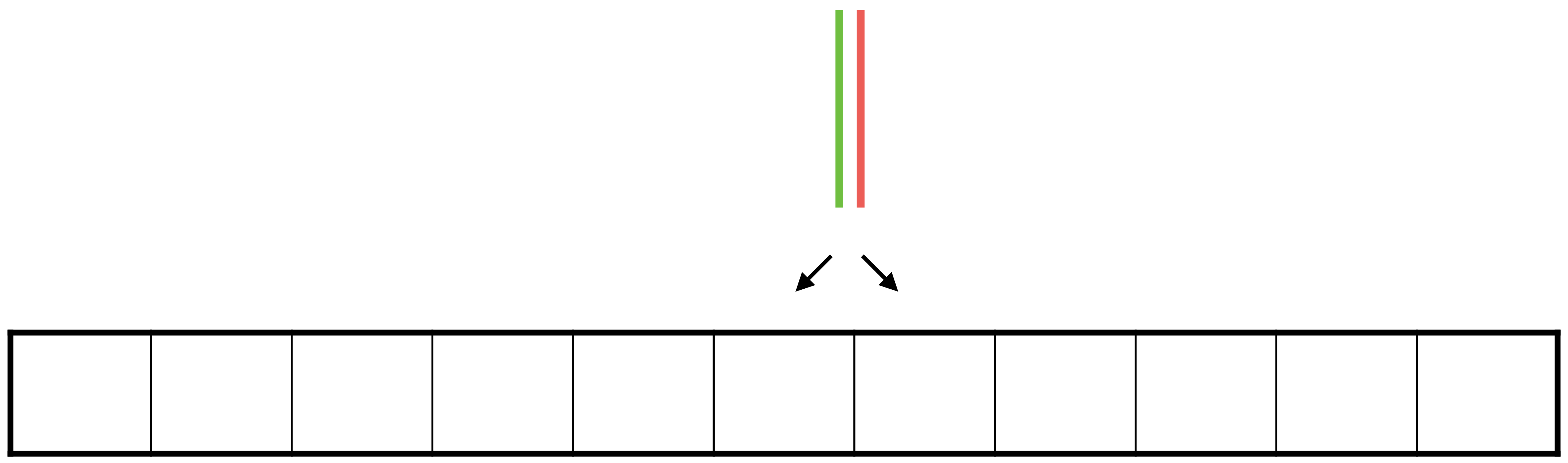


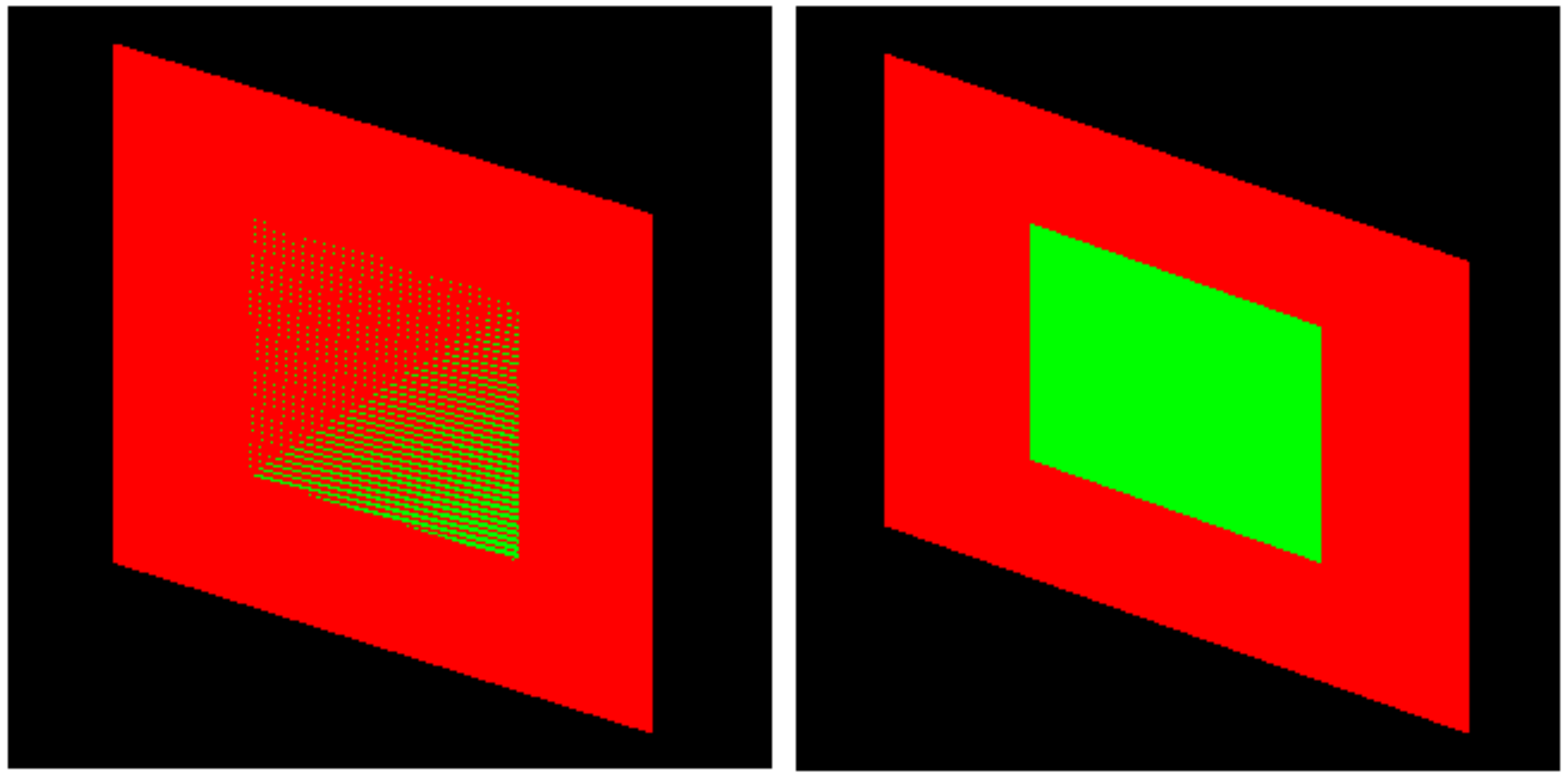
- You render the image both in the Image and in the depth buffer, where you store only the depth
- When a new fragment comes in, you draw it in the image only if it is closer
- This always work and it is cheap to evaluate! It is the default in all graphics hardware
- You still have to sort for transparency…

z-buffer quantization and “z-fighting”

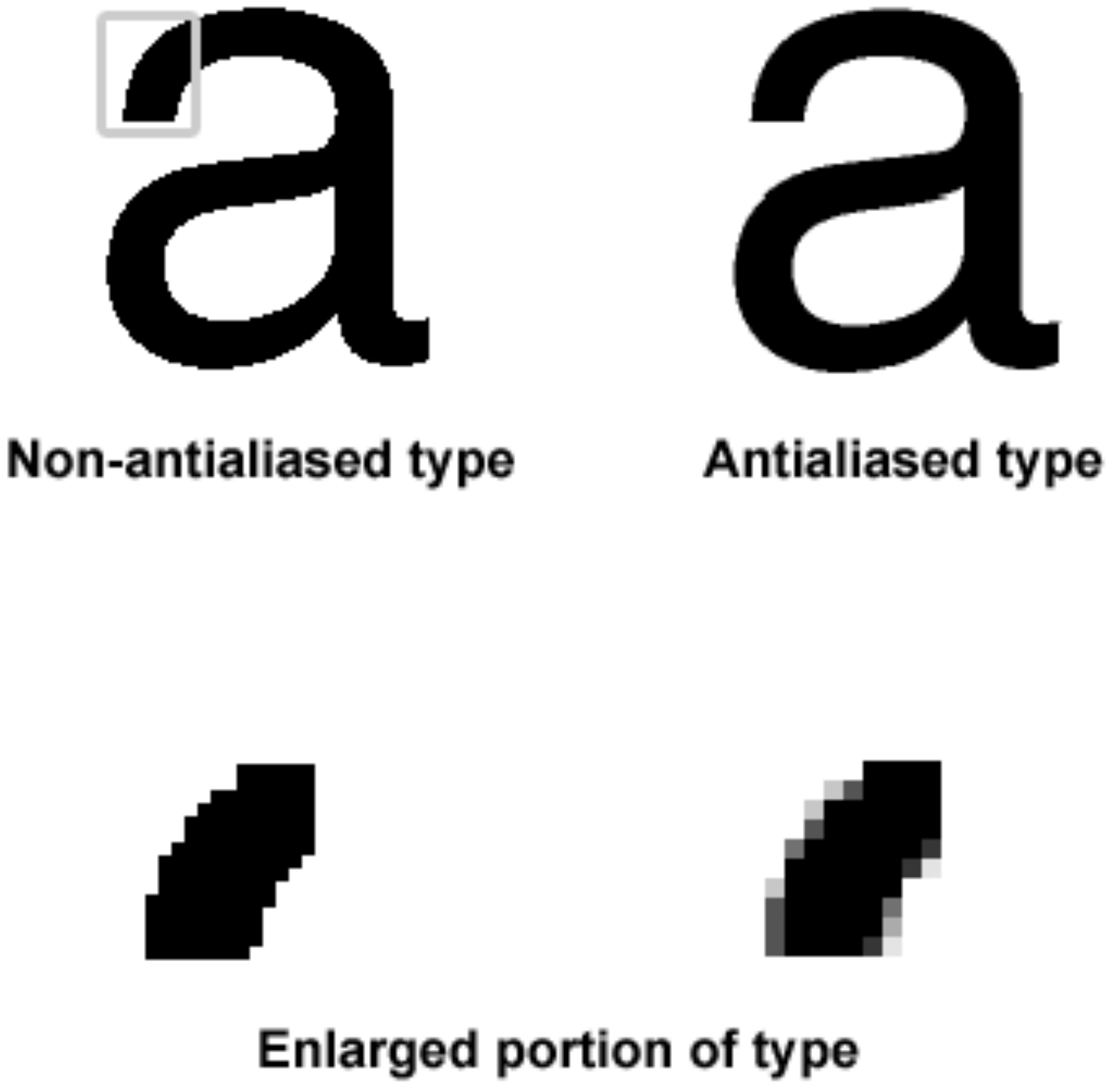
- The z-buffer is quantized (the number of bits is heavily dependent on the hardware platform)

- Two close object might be quantized differently, leading to strange artifacts, usually called “z-fighting”

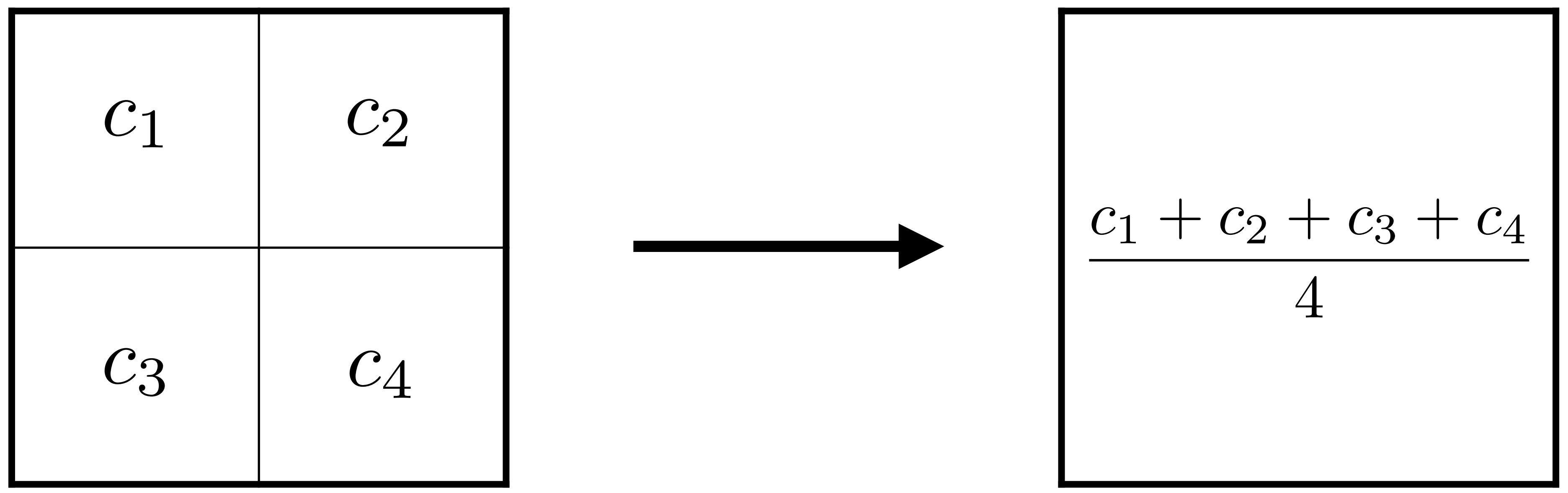




Super Sampling Anti-Aliasing

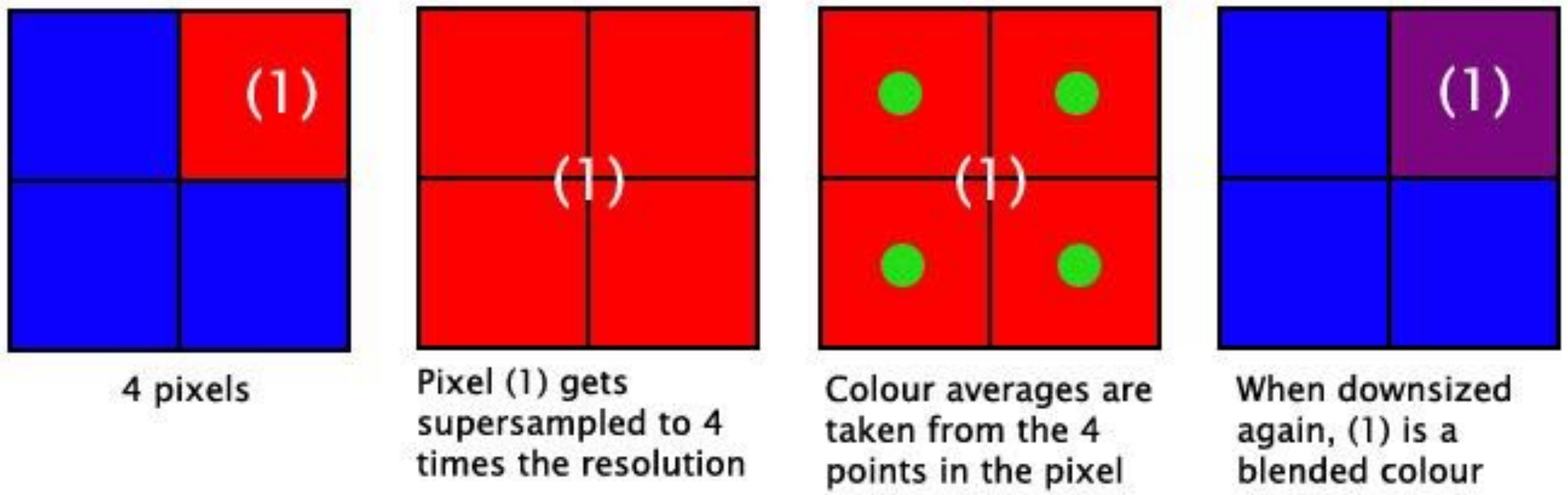


- Render nxn pixels instead of one
- Assign the average to the pixel

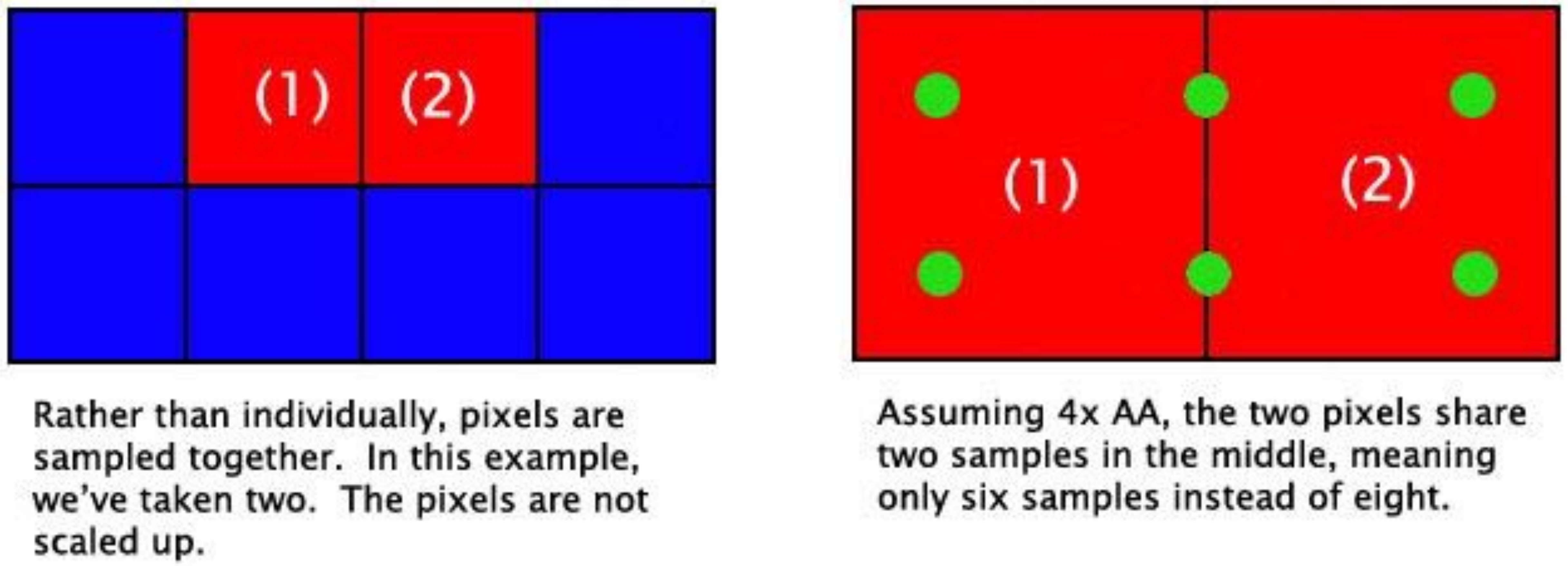


Many different names and variants

- SSAA (FSAA)
- MSAA
- CSAA
- EQAA
- FXAA
- TX AA



MSAA



References

Fundamentals of Computer Graphics, Fourth Edition
4th Edition **by Steve Marschner, Peter Shirley**

Chapter 8

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