Defence School of
Aeronautical Engineering

No.2 School of Technical Training

Academic Principles Organisation

MATHEMATICS

BOOK 8

Proportionality

KEY LEARNING POINTS

KLP	Description
MA1.13	Distinguish between direct and inverse proportion.
MA1.14	Calculate the constant of proportionality for arithmetical equations.

MA1.13, MA1.14

DIRECT PROPORTION

1. There are many situations where there is a linear relationship between two variables. This means that if one of the variables increases by a factor of, say 2 then there will be a corresponding change in the other variable, I.e. As x increases, y changes in proportional increments (*Direct Proportion*). See Figure 1 below.

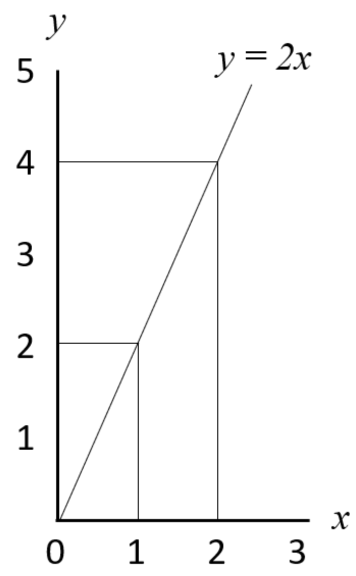

Figure 1

2. An example of this would be when a liquid is made to flow through a pipe; the rate of flow of the liquid is related to the pressure generated by the pump. There is a linear relationship between the flow rate and the pressure, so flow rate is directly proportional to the applied pressure.

The symbol for “proportional to” is “$\propto$”.

3. In this case, if the rate of flow is represented by F, and the pressure by P, so:

$$F \propto P$$

4. At the moment there is little that can be done with this statement, but by **introducing** a multiplying factor, $\boldsymbol{k}$ in place of the proportionality symbol, it can be turned into an equation, as so:

$$F = \boldsymbol{k}\, P$$

5. If the relationship between F and P is linear, then the value of $\boldsymbol{k}$ is constant for all values of F and P; it is known as the *constant of proportionality.* If this value can be found, it can then be used to find unknown values of F and P as the following example shows:

Example 1

6. Consider the example above, where there is a linear relationship between the pressure and the flow rate of a liquid. When the pressure is equal to 1.8 bar the flow rate is 4.5 litres per minute. What will the new flow rate be if the pressure is increased to 2.8 bar?

$F \propto P$ Therefore $F = kP$

From the information given, we know that F = 4.5 when P = 1.8 so,

4.5 = $\boldsymbol{k}$ ×1.8 and so,

$k = \dfrac{4.5}{1.8} = 2.5$ ***k*** has no units

Now we have found the value of k, we can use it to find subsequent unknown values of F or P. For example: Find F e.g. when P = 2.8.

$F = 2.5 \times P = 2.5 \times 2.8 = 7$

So (F), the new flow rate, is 7 litres per min.

7. Note that when carrying the above calculation, two “modelling” assumptions have been made, the first being that there is a linear relationship between the two variables (i.e. a graph drawn of one variable against the other would be a straight line which passes through the origin; (see Figure 1) and secondly that the linearity holds when the relationship is extrapolated.

8. In practice this is unlikely to be exactly correct, but for most purposes it would be reasonable to assume that the model will be accurate enough for our needs.

9. Sometimes one of the variables is proportional to some power of the other.

10. For example, the drag force, D on an object may be considered to be proportional to the square of its velocity through a fluid, v^2.

i.e. $D \propto v^2$

If the drag force is 18N when the velocity is 3m/s, at what velocity does the drag force reach 32N?

$D \propto v^2$

$\therefore$ $D = k\,v^2$

So, $k = \dfrac{D}{v^2} = \dfrac{18}{3^2} = 2$ ***k*** has no units

Using the constant of proportionality ($k = 2$) calculated above, and the general equation becomes:

$$D = 2 \times v^2$$

Transposing the equation for velocity (v), this becomes:

$$32 = 2 \times v^2$$

And $$v = \sqrt{\frac{32}{2}} = 4$$

11. So, the drag force reaches 32N at 4m/s

Exercise 1

1. The engine speed of an aircraft is directly proportional to the propeller speed.

   a. If the engine rotates at 15 750 rpm and the propeller rotates at 2 250 rpm, find the constant of proportionality for the reduction gearbox.

   b. Using the constant of proportionality calculated above; find the propeller speed when the engine speed is 16 100 rpm.

2. The drag (D) acting on an aircraft in flight is directly proportional to the square of the velocity (v^2).

   a. Find the constant of proportionality if the drag force is 1 852.2 N when the aircraft's velocity (v) is 210 m/s.

   b. Using the constant of proportionality calculated above; determine the aircraft's velocity (v) when the drag force is 2 221.8 N.

3. The Frictional Force (F) to move a container is directly proportional to the Weight (W) of the container.

   a. Determine the constant of proportionality (k) if the Frictional Force is 192 N when the Weight is 256 N.

   b. When additional weight is added to the container, the Frictional Force increases to 270 N. Use the constant of proportionality calculated above to determine the new Weight of the container.

INVERSE PROPORTION

12. Sometimes variables are related such that an increase in one causes a decrease in the other. For example: Figure 2 below shows that as x increases, y changes in decreasing increments.

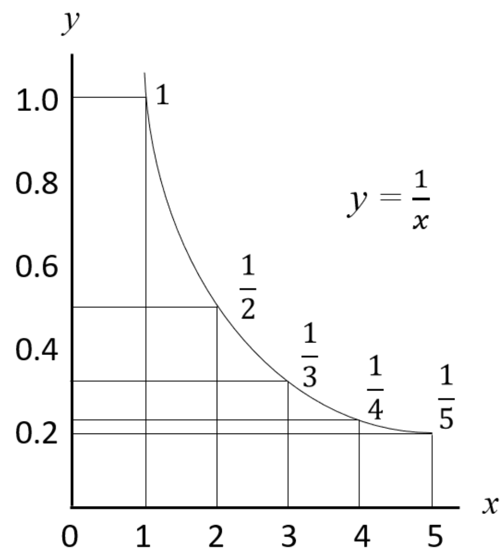

Figure 2

13. An example of this would be current intensity (I) and resistance (R) in an electrical circuit, where an increase in resistance causes a proportional decrease in current. This is called *inverse proportion.* Another way of saying this is to say that one variable is proportional to the reciprocal of the other and is written as:

$$I \propto \frac{1}{R}$$

14. If, for example a current of 6 Amps is produced when the resistance is 2 Ohms, this relationship could be used to find what the current would be if the resistance was changed to say, 4 Ohms:

$$I \propto \frac{1}{R}$$

Introducing $\boldsymbol{k}$ as before, $I = \frac{\boldsymbol{k}}{R}$

$\boldsymbol{k}$ has no units

So, $\boldsymbol{k} = IR = (6 \times 2) = 12$

Using $\boldsymbol{k} = 12$, the current at 4 Ohms would be:

$$I = \frac{k}{R} = \frac{12}{4} = 3 \text{ Amps}$$

So, when the resistance is increased to 4 Ohms the current drops to 3 Amps.

15. Another example is Light Intensity (I), which is inversely proportional to the square of the distance from the light source (d^2).

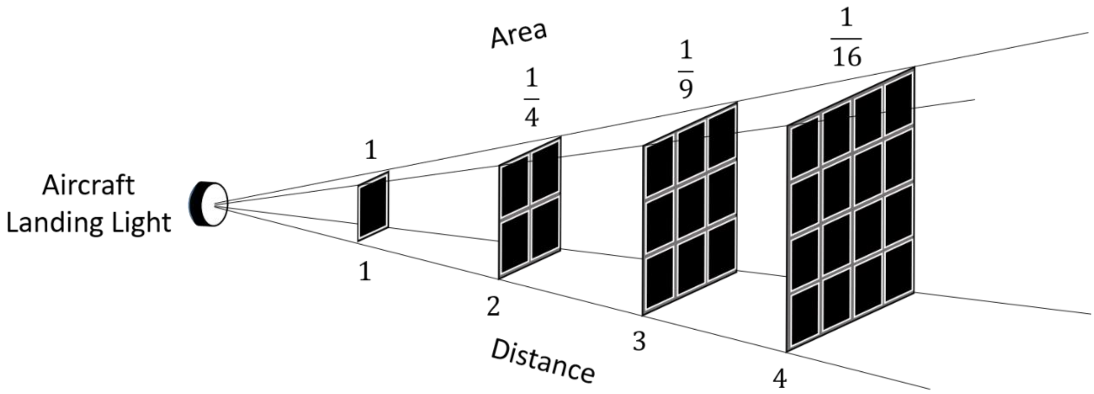

Figure 3

16. This means that as the distance increases, light intensity decreases with the square of the distance. I.e. $I \propto \frac{1}{d^2}$

a. Calculate the constant of proportionality if the intensity is 192 Candela when the distance is 100 metres.

$I \propto \frac{1}{d^2}$ So, $I = \frac{k}{d^2}$ and $k = Id^2$

$k = 192 \times 100^2 = 1\,920\,000$

b. Use the constant of proportionality calculated above to find distance (d) at which the Intensity (I) is 425 000 Candela.

$I = \frac{k}{d^2}$ So, $d^2 = \frac{k}{I}$ and $d = \sqrt{\frac{k}{I}}$

$d = \sqrt{\frac{1920\,000}{425\,000}}$ $d =$ 2.13 metres $to\ 2dp$.

Exercise 2

1. The Volume (V) of a hydraulic cylinder is inversely proportional to the Pressure (P) inside the cylinder.

   a. Determine the constant of proportionality, if the pressure is 205.6 bar when the volume is 0.035 m^3.

   b. Using the constant of proportionality calculated above; find the Pressure (P) when the Volume (V) is increased to 0.04 m^3.

2. The intensity of the current (I) in an electrical circuit is inversely proportional to the resistance (R) in the circuit.

   a. Determine the constant of proportionality if the current is 7.5 Amps when the resistance is 13 Ohms (Ω).

   b. Using the constant of proportionality calculated above; determine the resistance when the current is increased to 12.5 Amps (A).

3. The intensity (I) of an aircraft landing light is inversely proportional to the square of the distance from the light (d^2). As the distance increases, light intensity decreases.

   a. Calculate the constant of proportionality if the intensity is 850 Candela when the distance is 50 metres (m).

   b. Use the constant of proportionality calculated above to find distance (d) at which the Intensity (I) is 405 000 Candela. Express your answer to 2dp.

Answers

Exercise 1

1a. k = 7	1b. Propeller speed = 2 300 rpm
2a. k = 0.042	2b. Velocity (v) = 230 m/s
3a. k = 0.75	3b. Weight = 360 N

Exercise 2

1a. k = 7.196	1b. V = 179.9 bar
2a. k = 97.5	2b. R = 7.8 Ohms (Ω)
3a. k = 2 125 000	3b. d = 2.29 m to 2dp.