Defence School of
Aeronautical Engineering

Aerosystems Engineer & Management
Training School

Academic Principles Organisation

MATHEMATICS

BOOK 12

Areas and Volumes

WARNING

These course notes are produced solely for the purpose of training. They are not subject to formal amendment action after issue and they must NOT be used for operating and maintaining the equipment described. Operation and maintenance of equipment is governed by the limitations and procedures laid down in the authorized publications and manuals which must be complied with for these purposes

KEY LEARNING POINTS

KLP	Description
MA6.1	Define the components of a circle
MA6.2	Solve problems related to circular dimensions
MA6.3	Create geometrical constructions
MA6.4	Convert between linear, square and cubic units
MA6.5	Use formulae to calculate perimeter and area of plane figures
MA6.6	Use formulae to calculate surface area and volume of common solids

1. The final part of these notes is an assignment requiring the student to find both the volume and surface area of various composite shapes. Composite shapes look more difficult than they really are, they are simple shapes combined to make complicated looking shapes. The simple shapes have simple formulae, which can be used to find the required solutions. What this topic requires is a systematic, organised approach. The diagrams should be large enough to allow the gradual addition of information as it is attained.

The Circle

2. A circle can be defined as the path that would be taken by a point moving a constant distance from a fixed point. The complete path is called the *Circumference* of the circle. The fixed point is the *Centre* of the circle and the constant distance is called the *Radius, r.* The distance across the circle is called the *Diameter*, D, such that the diameter of the circle is twice the radius, $D = 2r$ as shown in *Fig1*:

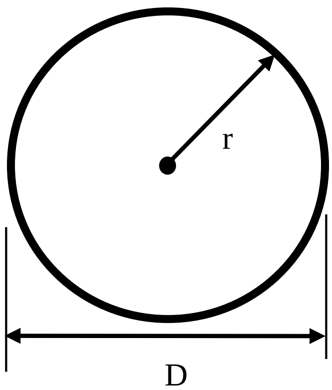


Fig 1

3. There is an important relationship between the diameter and circumference of the circle. The circumference is always a fixed number of multiples of the diameter. This fixed number is the same for all circles (a constant) and is denoted by the Greek letter π

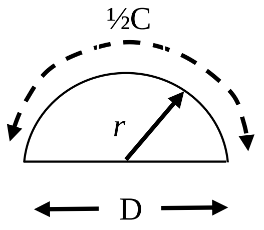
For a circle:

$$Circumference = \pi D \text{ or } 2\pi r$$

$$Area = \pi r^2$$

The Semi-circle

4. A semicircle is half of a circle, so its area is half of that for the full circle. Its perimeter is half of the circumference of a circle of the appropriate radius, plus one diameter.



$$\text{Area } = \frac{1}{2}\pi r^2$$

$$\text{Perimeter } = \frac{1}{2}\pi D + D$$
$$= \pi r + D$$

Fig 2

The Value of π and approximation

5. The number π is what is called by mathematicians, a *transcendental* number. This means it cannot be written down exactly, except as the sum of an infinite series. In practice, this is not a problem as we can approximate π to any number of significant figures we wish to achieve the level of accuracy required for our calculation.

6. The π button on the calculator gives us a value of π correct to 10 significant figures; this is more than adequate for our needs.

7. Often the number 3.142 is used for the value of π, this is correct to 4 significant figures.

8. Sometimes the fraction $\frac{22}{7}$ is convenient, this is correct to 3 significant figures.

9. It is important, however to realise that any calculations involving multiplying by the number π involve an approximation, and we must not assume a level of precision in the answer beyond the level of the original approximation of π.

10. It would be wrong, for example, to use 3.142 as a substitute for π and then give an answer correct to 6 or 7 figures. The rule is to round the final answer of any calculation to 1 significant figure less than the least accurate constant used. In this case, only the first 3 figures in the answer have any significance and the rest are meaningless and should be rounded.

11. All of the basic shapes that will be used to set questions on this course for both area and volume are listed below.

2 Dimensional (Plane) Figures

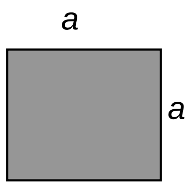


Fig 3

The Square

Area $= a^2$
Perimeter $= 4a$

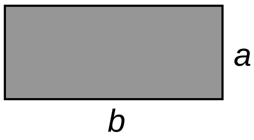


Fig 4

The Rectangle

Area $= ab$
Perimeter $= 2a + 2b$ *or* $2(a + b)$

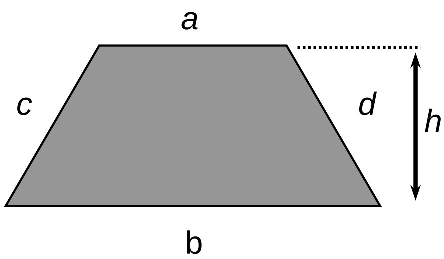


Fig 5

The Trapezium

Area $= \frac{1}{2}(a + b)h$
(Half the sum of the 2 parallel sides multiplied by the separation)

a and *b* are parallel lines and *h* the distance between them.

Perimeter = $a + b + c + d$

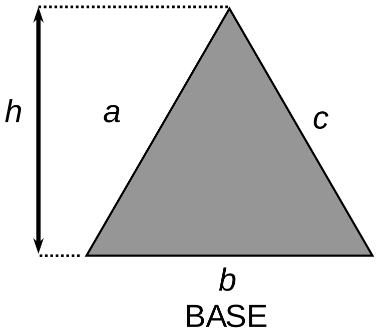


Fig 6

The Triangle

Area $= \frac{1}{2}\text{bh}$

(Use this formula if given a perpendicular height h)

Also, Area $= \sqrt{s(s-a)(s-b)(s-c)}$

Where s is the semi-perimeter, $s = \dfrac{a+b+c}{2}$.

(Use if given the length of all sides)

Also, Area = $\dfrac{1}{2}ab\sin C$

(Half the length of any 2 sides x the sin of the angle between them)

(Use if given 2 sides and the included angle)

Perimeter = $a+b+c$

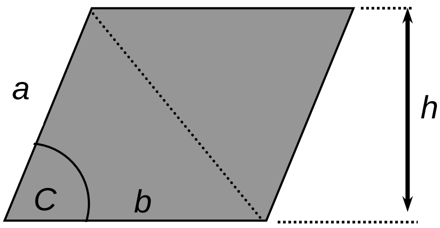


Fig 7

The Parallelogram

Area $= bh$ or,

Area $ab\sin C$

(The length of any 2 adjacent sides x the sin of the angle between them)

Note that a parallelogram is effectively two triangles

Perimeter $= 2a + 2b$

3 DIMENSIONAL SHAPES

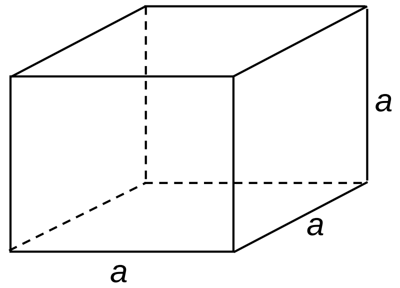


Fig 8

The Cube

Volume $= a^3$

Surface area $= 6a^2$

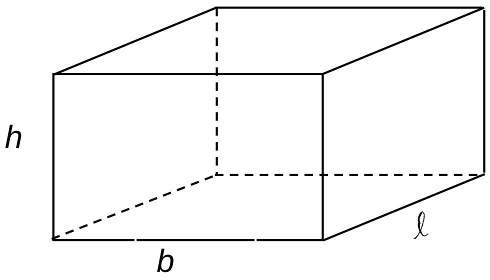


Fig 9

The Cuboid

Volume $= \ell bh$

Surface area $= 2(\ell b + \ell h + hb)$

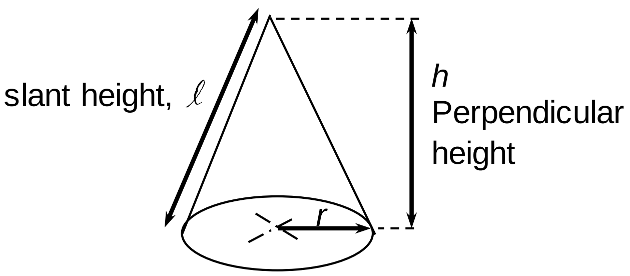


Fig 10

The Cone

Volume $= \frac{1}{3}\pi r^2 h$

Total surface area $= \pi r\ell + \pi r^2$

Where:

$\pi r\ell =$ Area of the curved surface

$\pi r^2 =$ Area of a circular (flat) base

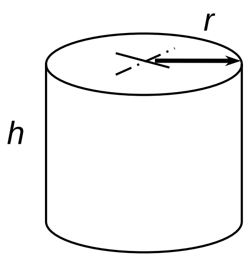


Fig 11

The Cylinder

Volume $= \pi r^2 h$

Total surface area $= 2\pi r^2 + 2\pi r h$

[top and bottom] plus [curved surface]

The curved surface is a rectangle when rolled out flat:

The total surface area formula can be simplified to: $2\pi r(r+h)$

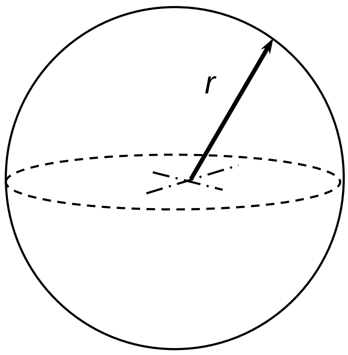


Fig 12

The Sphere

Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

(A hemisphere is half of a sphere)

Volume of hemisphere = $\frac{2}{3}\pi r^3$

Surface area of hemisphere $= 2\pi r^2 + \pi r^2$

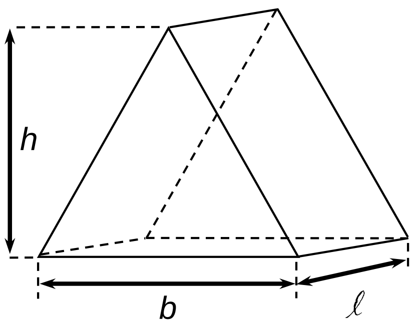


Fig 13

The Prism

A prism is any solid figure with a uniform cross-sectional area.
Fig. 13 shows an example of a prism with a triangular cross section. (i.e. a triangular prism)

Volume = cross sectional area x length (depth)

(in this particular case, Volume $=\frac{1}{2}bh\ell$)

Compare this formula with that for a cylinder, which is a type of prism with a circular cross section

<u>Note</u>

In the exam a formula sheet will be provided.

Multiples and sub-multiples of square and cubic units

As part of the calculation it may be necessary to convert between multiples of units.

It should be noted that:

$$1m = 100cm = 1000mm$$

But for area:

$$1m^2 = 10{,}000cm^2 = 1{,}000{,}000mm^2$$

And for volumes:

$$1m^3 = 1{,}000{,}000cm^3 = 1{,}000{,}000{,}000mm^3$$

Note also that 1 litre of a fluid is equivalent to a volume of $1000cm^3$

Example 1

12. An aircraft jacking pad (the dimensions and shape of which are given below) is to be electroplated with an anti-corrosive treatment. In order to calculate the amount of electrolyte needed, find the total surface area to be plated.

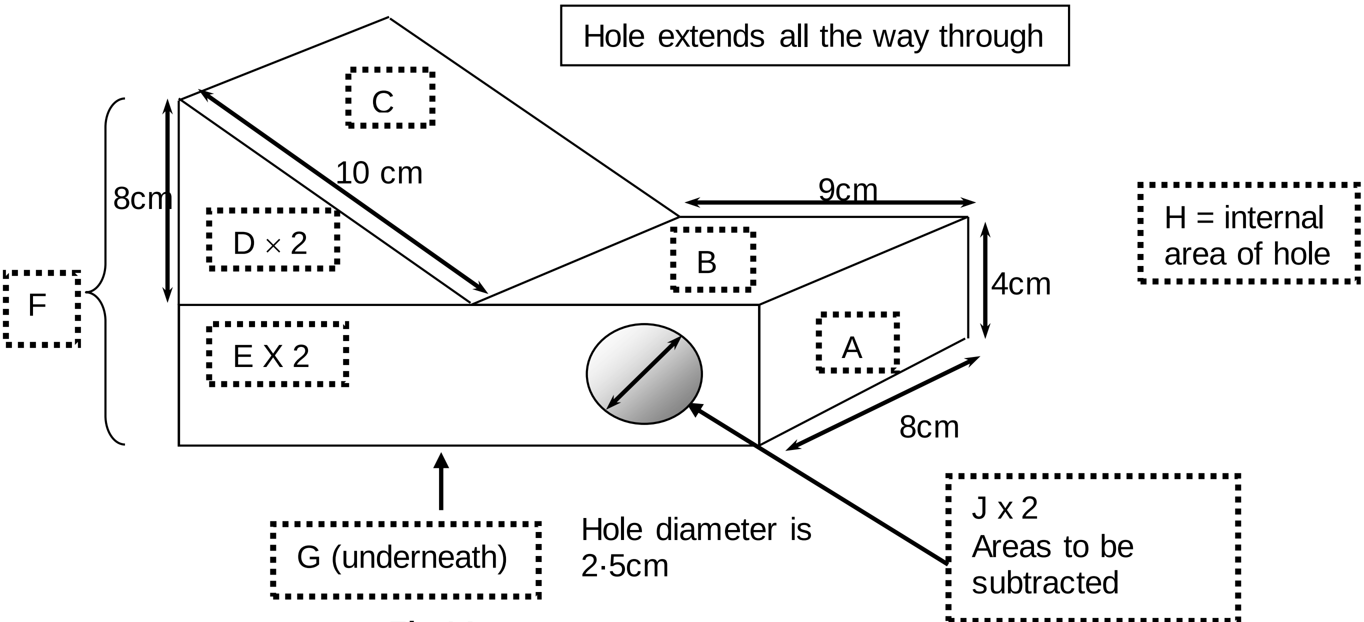


Fig 14

13. This is a compound shape. Break it down in to individual shapes; remembering there are things you cannot see from this perspective in this case the base and the back.

1. Label the bits – note we have some dimensions apparently missing but with a little thought all the "missing bits" can be found

2. Take shape D the Triangle

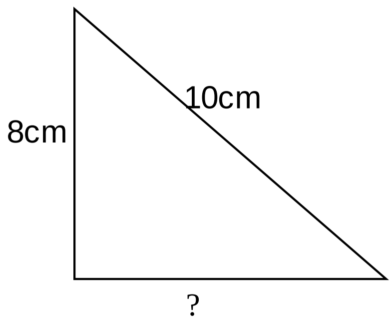


Fig 17

This is a right-angle triangle in which the dimensions of Hypotenuse and one other side are known.

Using Pythagoras' theorem, the length of the unknown side is given by:

Unknown side = $\sqrt{10^2 - 8^2}$ = 6cm

So, length of base is now 9 + 6 = 15cm

3. Now the main part of the solution can be started

Area of side	Formula	Calculation	Value
A	Rectangle a x b	8 x 4	32cm^2
B	Rectangle a x b	8 x 9	72cm^2
C	Rectangle a x b	8 x 10	80cm^2
2 x D	Triangle ½ x b x h	2 x(½ x 6 x 8)	48cm^2
2 x E	Rectangle a x b	2 x(9+6) x 4	120cm^2
F	Rectangle a x b	(8+4) x 8	96cm^2
G	Rectangle a x b	(9+6) x 8	120cm^2
H	Curved area of a cylinder 2πrh	2 x 3.142 x 1.25 x 8	62.8cm^2
2 x J	Circle πr^2	2 x 3.142 x 1.25^2	- 9.8cm^2
		Total Surface Area	621cm^2

4. Thus, the solution to our problem is that the surface area is approximately 621cm^2 (when appropriate rounding is applied)

5. Here the answer is in the units of cm^2. If we wished to provide an answer in m^2 then as there are 10000 cm^2 in 1 m^2 then the answer would be:

$$\frac{621}{10000} = 0.0621m^2$$

Example 2

Find the total volume of the jacking pad in example 1

1. It can be seen that the object has a uniform cross-sectional area and therefore is a type of prism. We can use the prism formula:

Volume = cross sectional area x length (depth)

2. Our first step will be to find the cross-sectional area, most of the work has been done here in example 1 i.e.

Cross sectional area = area D + area E – area J
= 24 +60 -4.91
= 79.09

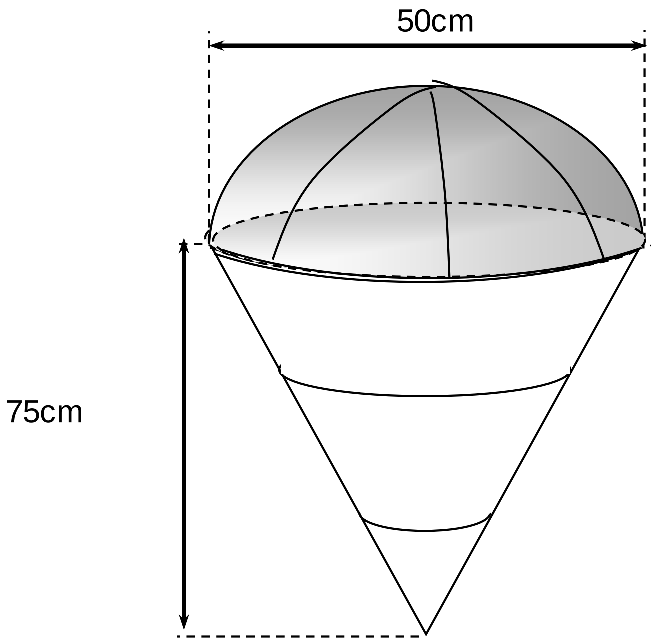
So, Total volume = 79.09 x 8 = 632.72 $\approx$ 633cm^3

If an answer is required in m^3 then as there are 1000000 (10^6) cm^3 per m^3
Then the answer is 0.000633 or 6.33 x 10^{-4}m^3

Exercise 1

1. The object drawn below consists of a hemisphere joined to the base of a cone. Calculate:

a. The surface area of the object.
b. Its volume.



Surface Area of Sphere $= 4\pi r^2$

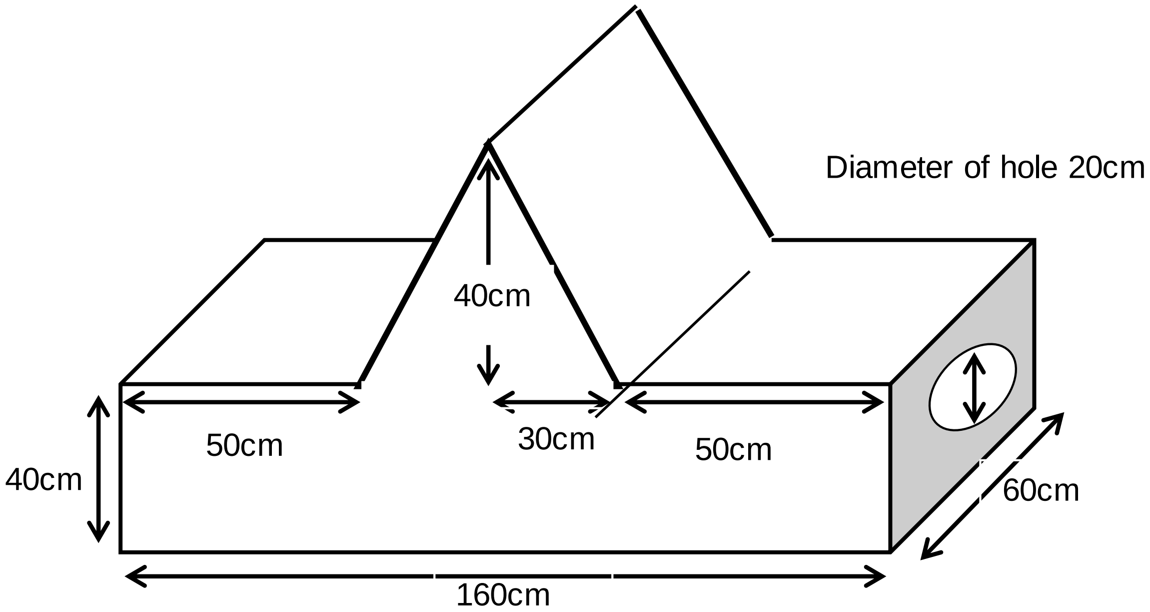
Volume of Sphere $= \dfrac{4}{3}\pi r^3$

Surface Area of Cone $= \pi r^2 + \pi r l$

Volume of Cone $= \dfrac{1}{3}\pi r^2 h$

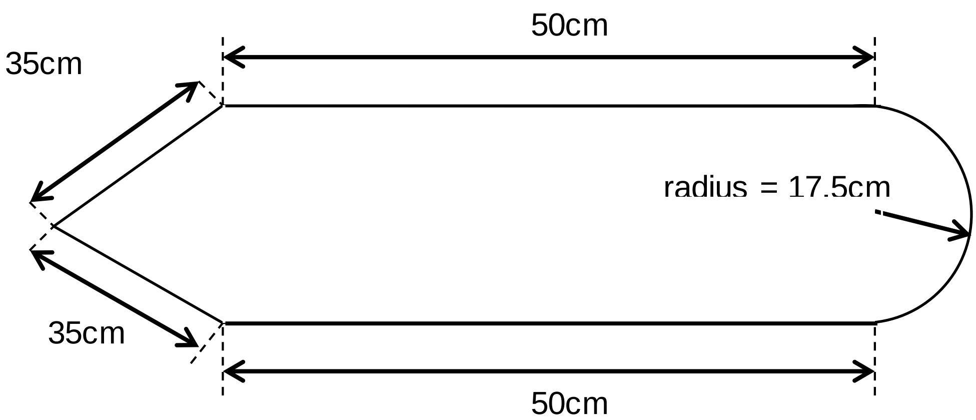
2. Using the object drawn below - Find:

a. The surface area,
b. The volume,



3. For the shape drawn below, find:

a. The perimeter.
b. The area.



c. If the shape were constructed from aluminium 5mm thick, what would be its volume?

14. The following examples are more difficult because there are no drawings to follow. The trick here is to read carefully the question, several times if necessary, and try to visualise the object being described. Make a drawing of the object, adding all the information in the question. As you work through the problem, add any further information that becomes available. Eventually you will see that the problem can be solved by application of the simple formulae as before. In the later examples you may need to transpose some of the basic formulae to find the answer required.

4. A door wedge is made from a piece of wood in the shape of a prism with a right angled, triangular cross section. It is 4cm wide, 15 cm long and 5cm high. Calculate the volume of wood used in the wedge and its surface area.

5. A hollow, thin metal, cone has a base diameter of 20cm and a vertical height of 60cm. How many litres of water can it hold?

6. A washer is made from a circular piece of metal 4mm thick and 80mm in diameter, by cutting a 30mm diameter hole completely through it. Calculate the volume of metal in the washer and its total surface area.

7. The Albert Hall can be represented as a hemi-spherical dome on top of a cylindrical base. If the hall is 200m across and has a maximum height of 200m calculate the total enclosed volume. The external wall (not dome) is to be painted to a thickness of 2mm. Calculate the area to be covered and how many 20 litre drums of paint are required to do the job.

8. A cuboid of lead 1m by 30cm by 40cm is melted down to produce spherical musket balls each 10mm in diameter. Assuming no losses how many musket balls can be made?

9. A piece of lead piping is to be cut up and melted down to produce ballast weights. The external diameter of the piping is 140cm and the internal diameter is 60cm. The length of the piping is 300cm. If $1m^3$ of lead has a mass of 11,300kg, how many 5kg ballast weights can be produced?

10. The surface area of a sphere is $6800cm^2$. Calculate the radius and volume of the sphere, expressing the volume in cubic metres.

11. A rectangular solid has a square cross-sectional area. If the length of the solid is 2.5m, and the volume is $3.6m^3$, calculate the length of side of the square face. Calculate the surface area of the solid.

12. It is required to replace two pipes of internal diameters 28mm and 70mm respectively with a single pipe that has the same cross-sectional area of flow. Find the internal diameter of this single pipe.

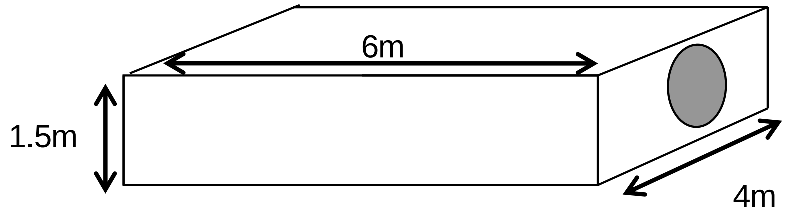
Answers

1.	a)		1.013	m^2
	b)		81810	cm^3
2.	a)		5.102	m^2
	b)		0.4057	m^3
3	a)		225	cm
	b)		2761	cm^2
	c)		1381	cm^3
4.	vol	=	150	cm^3
	area	=	218.2	cm^2 (could also be 251cm^2)
5.			6.283	litres of water
6.	vol	=	17.28	cm^3
	area	=	100	cm^2
7	vol	=	5236000	m^3
	wall area	=	62832	m^2
	paint required		6283	drums
8.			229183	musket balls
9.			8520	weights
10.	radius	=	23.26	cm
	vol	=	0.0527	m^3
11.	length	=	1.2	m
	area	=	14.88	m^2
12.			75.39	mm

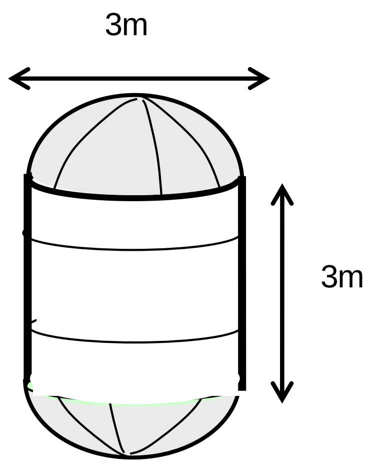
Assignment

1. Find the *surface area* of the rectangular block shown below. The hole passes all the way through the block and has a diameter of 1 m.

2. Calculate the volume of the block.



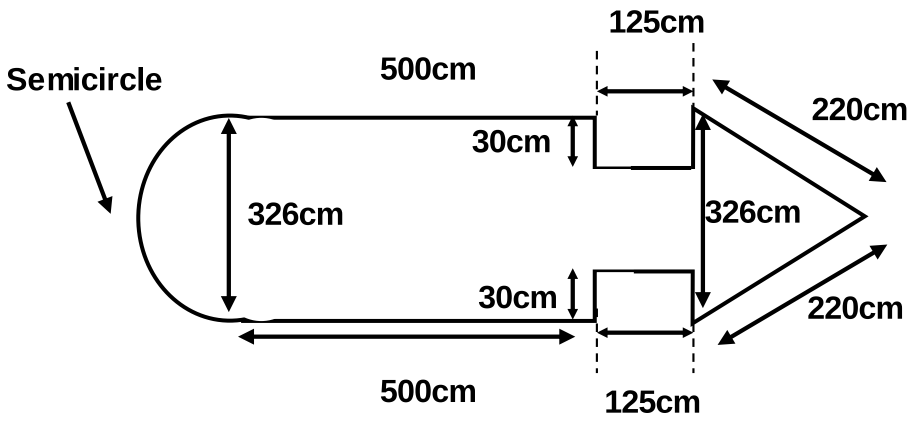
3. Find the surface area of the shape shown below
4. Calculate its volume.



Surface Area of Sphere $= 4\pi r^2$

Volume of Sphere $= \frac{4}{3}\pi r^3$

5. Find the perimeter of the shape below.
6. Find the area.



Notes