Defence School of
Aeronautical Engineering

Aerosystems Engineer & Management
Training School

Academic Principles Organisation

Mathematics

BOOK 1
Basic Numeracy

WARNING

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BASIC NUMERACY

CONTENTS

BASIC NUMERACY

KEY LEARNING POINTS

KLP	Description
MA1.1	Explain and use the decimal counting system.
MA1.2	Estimate values for calculations
MA1.3	Solve calculations using standard operators on arithmetical expressions
MA1.4	Define and use arithmetical terms (directed numbers)
MA1.5	Express fractions in different forms
MA1.6	Solve calculations using standard operators on fractions
MA1.7	Solve calculations using standard operators on decimals
MA1.9	Define and use types of decimal numbers
MA1.10	Convert between fractions, decimals and percentages
MA1.11	Calculate percentage values for common engineering variables
MA1.12	Manipulate simple arithmetic ratios

WHOLE NUMBERS INTRODUCTION

1. Dealing with numbers is an everyday experience. Whether totting up prices at the supermarket, keeping score in a game or measuring things at work, numbers are present in almost every aspect of our lives. Aircraft Technicians are routinely faced with tasks where the ability to carry out calculations with speed and accuracy is an asset to their proficiency.

CALCULATORS

2. Electronic calculators are immensely useful in speeding up the process of solving numerical problems and removing what many feel is the drudgery of 'number crunching'. However, due to the conditions in which aircraft have to operate and the effect of aircraft high voltage electrical systems on magnetic media (including calculators) the **use of a calculator cannot always be counted on** as an option. For this reason, the Aircraft Technician needs to be capable of carrying out accurate calculations **both mentally and on paper**.

CHECKING ANSWERS

3. Errors are common in calculations such as these and **an incorrect answer wastes the time and effort used in finding it**. Worse still, an uncorrected error in an aircraft operating environment could have serious consequences. **Therefore, it is always wise to check answers for accuracy**.

ESTIMATION

4. Estimation is a very powerful tool during the process of completing mathematical calculations. If an estimate has been completed before the calculation it gives an approximation of the answer. If your answer is not close to the estimate it gives you an indication that you may have performed the calculation incorrectly. The easiest way to perform an estimation is to round up or down the numbers within the calculation then it will become a relatively easy mental arithmetic sum to perform.

CALCULATIONS

The Denary System

5. The number system most commonly used day to day is based on the number ten and consists of nine number symbols known as digits. These are 1, 2, 3, 4, 5, 6, 7, 8, and 9. The function of a digit is to denote a quantity. A tenth symbol, 0, indicates the absence of a quantity.

Place Value

6. Each place in a number contains values which are ten times greater than the place to its right (see Table 1). A digit in a number indicates how many of its place value are to be counted.

	5th Place	**4th Place**	**3rd Place**	**2nd Place**	**1st Place**
Contents	**Ten thousands**	**Thousands**	**Hundreds**	**Tens**	**Units**
Place value	$1\times10\times10\times10\times10$ 10000	$1\times10\times10\times10$ 1000	$1\times10\times10$ 100	1×10 10	1 1

Table 1 - The Contents and Place Value of the First Five Places in the Denary System

Using the system above a five-digit number e.g., 32 045 would indicate that it is made up of:

	3 lots of ten thousand	i.e.,	30 000
	2 lots of a thousand	i.e.,	2 000
	No hundreds		000
	4 lots of ten	i.e.,	40
And	5 units	i.e.,	5

Note: Without the 0 marking the absence of hundreds this number would appear as 3245. There would be no means of identifying the place value of the digits and so the number would be meaningless.

Signs, Symbols and Terminology

7. Mathematics uses a type of shorthand to express mathematical statements in a way that is economical. Digits are an example of the symbols used in the denary system and are used in conjunction with signs which give an instruction or describe something about the quantities being dealt with. These are summarised in the table below.

Sign	Meaning	Instruction	Outcome
+	Add or plus	Add one number to another	The result of addition is called the **sum**
-	Subtract or minus	Take one number away from another	The result of subtraction is called the **difference**
×	Multiply or times	Find the total of several lots of a given number	The result of multiplication is called the **product**
÷	Divide	(1) Find how many times one number fits into another (2) Find the size of each share if a quantity is shared equally among a given number	The result of division is called a **quotient**
=	Equals	Indicates that one quantity has the same value as another	

Table 2 - Summary of the Signs used in Basic Calculations

ADDITION

8. The task of addition is to combine two or more quantities into a single total.

Rules of Addition

a. The result of an addition will be the same no matter which order the numbers are placed in the problem:
e.g., 4 + 5 has the same value as 5 + 4 as they are both equal to 9.

b. Quantities can only be added together if they are of the same type:
e.g., 4 bolts + 5 bolts = 9 bolts

but 4 bolts + 5 rivets although giving a total of 9 items this quantity cannot be expressed using the name of either original quantity.

Solving Addition Problems

a. Consider the following:

$$4 + 8 + 5 + 2 + 6 + 7 + 5 + 3 =$$

The solution to this problem may be found by finding the sum of the first two numbers and adding this to the third to give the sum of the first three numbers which is added to the fourth and so on. However, the numbers may be rearranged into an order which makes the problem easier to deal with.

e.g., $8 + 2 + 6 + 4 + 7 + 3 + 5 + 5 =$

b. When placed in this order it can be seen that each pair of numbers in the problem will yield a sum of 10 and as there are four pairs in the problem it may now be simplified to 10 + 10 + 10 + 10 giving a total of 40.

c. Rearranging the numbers has made the answer easier to see and arrive at. The first approach, although sound and methodical is not necessarily the most efficient. Finding ways of simplifying problems is crucial to improving the ability to solve them.

Adding Large Numbers

9. When adding numbers greater than 9 it is customary to set the numbers out one above the other. Ensuring that the numbers form columns and that **the digits in each column have the same place value**.

Worked Example

54 + 412 + 133 = would be set out as

$$\begin{array}{r} 54\ + \\ 412 \\ \underline{133} \end{array}$$

Starting with the units, the total for each column is found by addition.

$$\begin{array}{r} 54\ + \\ 412 \\ \underline{133} \\ \mathbf{599} \end{array}$$

Notes:

a. All of the units are in line, all of the tens are in line and all of the hundreds are in line, **including those in the answer**.

b. The line separating the numbers in the problem from the numbers in the answer.

Carrying Over

10. Sometimes the sum of a column turns out to be greater than 9. In these cases, the units of the sum are written in the column being dealt with and the tens are carried over to the column to the left.

Guided Example

375 + 483 +128 = set out as

```
 375 +
 483
 128
 ---
 986
 11
```

In this problem the sum of the units column is 16.

The 6 is written in the units column of the answer and the 10 is carried into the tens column where it has a value of 1, which is indicated by the small digit placed below the tens column.

The sum of the tens column, **plus the one carried over**, is 18. This is treated in the same way as the units.

Checking Addition

11. In the case of addition, if the digits in a column were added upwards then, they should be checked by adding downwards. This presents the digits in a different order and reduces the risk of repeating an error.

EXERCISE 1.1

Find the sum of the following.

1. 47 + 33 + 65 =

2. 139 + 74 + 238 =

3. 1042 + 1503 + 1670 =

4. An aircraft log shows that it had flown 447 hours in its first year in service, 392 hours in its second year, 458 hours in the next and 505 hours in the next. How many flying hours had it completed in its first four years of service?

5. A fuel bowser was used to refuel five aircraft. The first took 875 kilograms (kg) of fuel, the second 724 kg, the third 833 kg, the fourth 756 kg and the fifth 487 kg. How much fuel did the bowser dispense in all?

SUBTRACTION

12. The aim of subtraction is to calculate the difference between two quantities. The order in which numbers are subtracted seriously affect the outcome.

e.g., 7 - 3 **is not the same as** 3 – 7

Solving Subtraction Problems

13. A subtraction problem involving numbers with values greater than 10 may set out in columns similar to those used in addition.

Worked Example

197 - 76 =

would be set out as

$$\begin{array}{r} 197 \\ \underline{-\ 76} \end{array}$$

14. As with addition the units are dealt with first and so the units on the bottom line are taken away from the units of the number above. The tens are treated in the same way and so on.

Thus

$$\begin{array}{r} 197 \\ \underline{-\ 76} \\ \mathbf{121} \end{array}$$

i.e., 197 - 76 = 121

15. Occasionally the digit on the top line is too small to take away the value of the corresponding digit on the bottom line.

Guided Example

92 - 55 =

$$\begin{array}{r} 92 \\ \underline{-55} \end{array}$$

16. In this example the 5 units on the bottom line cannot be taken away from the 2 units on the top line. This difficulty is overcome by increasing the quantity of the unit column on the top line.

To do this:

a. Take 1 away from the digit in the tens column (leaving 8 there in this example) and carried it into the units column where it has a value of ten.

b. Combined with the 2 this will now count as 12 in the top line units, permitting the 5 units on the bottom line to be taken away.

c. The resulting 7 units are written in and then the tens are calculated by taking the 5 tens on the bottom line from the 8 tens remaining on the top line.

Thus

$$\begin{array}{r} \overset{8}{\cancel{9}}\,{}^{1}2 \\ \underline{-5\ 5} \\ \mathbf{3\ 7} \end{array}$$

Checking Subtraction

17. To confirm that you have arrived at the correct answer simply add your answer to the lower number. If it is correct then the outcome should be the same as the original number.

e.g. from the example above

$$\begin{array}{r} 37\ + \\ \underline{55} \\ \mathbf{92} \\ {\scriptstyle 1} \end{array}$$

Other Methods

18. There are alternative methods of subtraction. If your method differs from the one shown here **bring it to the attention of your lecturer**, who can check its reliability.

EXERCISE 1.2

Calculate the difference for the following.

1. 247 - 74 =

2. 1762 - 848 =

3. 2091 - 365 =

4. During a stock check in a Squadron's tool store a box was found to contain 19 tubes of grease. If the box originally contained 48 tubes, how many tubes have been issued from the box?

5. The distance that the ram of a hydraulic jack is capable of moving is called the stroke. If a jack is 348 mm long when the ram is fully retracted and 603 mm long when it is fully extended, what is the length of its stroke?

MULTIPLICATION

19. Multiplication provides an efficient means of repeatedly adding a number.

e.g., $6 + 6 + 6 + 6 = 24$

20. Using the sign '$\times$', meaning times, this addition can be condensed into

$6 \times 4 = 24$ read as 6 times 4 equals 24

meaning 6 added 4 times is equal to 24

Note: $6 \times 4 = 24$ and $4 \times 6 = 24$ showing that the order in which numbers are multiplied makes no difference to the outcome.

Terminology

21. When numbers are multiplied the outcome is often referred to as the **product** of those numbers. In the example above **24** may be said to **be the product of 6 and 4**.

22. At other times **24** may be described as being a **multiple of 6** or a **multiple of 4**.

23. When a number is multiplied by another to give a product both numbers may be said to be **factors** of the product.

e.g., 6 and 4 are factors of 24

Multiplication Tables

24. A multiplication table is a list of multiples of a given factor usually up to a multiple of 12.

e.g., the table with 2 as the factor looks like this:

$1 \times 2 = 2$
$2 \times 2 = 4$
$3 \times 2 = 6$
$4 \times 2 = 8$
$5 \times 2 = 10$
$6 \times 2 = 12$
$7 \times 2 = 14$
$8 \times 2 = 16$
$9 \times 2 = 18$
$10 \times 2 = 20$
$11 \times 2 = 22$
$12 \times 2 = 24$

REFRESHER PRACTICE 1

25. Below are the multiplication tables from 3 to 9 with the products missing. Improve your familiarity with these tables by filling in the missing products.

$1 \times 3 =$	$1 \times 4 =$	$1 \times 5 =$	$1 \times 6 =$	$1 \times 7 =$
$2 \times 3 =$	$2 \times 4 =$	$2 \times 5 =$	$2 \times 6 =$	$2 \times 7 =$
$3 \times 3 =$	$3 \times 4 =$	$3 \times 5 =$	$3 \times 6 =$	$3 \times 7 =$
$4 \times 3 =$	$4 \times 4 =$	$4 \times 5 =$	$4 \times 6 =$	$4 \times 7 =$
$5 \times 3 =$	$5 \times 4 =$	$5 \times 5 =$	$5 \times 6 =$	$5 \times 7 =$
$6 \times 3 =$	$6 \times 4 =$	$6 \times 5 =$	$6 \times 6 =$	$6 \times 7 =$
$7 \times 3 =$	$7 \times 4 =$	$7 \times 5 =$	$7 \times 6 =$	$7 \times 7 =$
$8 \times 3 =$	$8 \times 4 =$	$8 \times 5 =$	$8 \times 6 =$	$8 \times 7 =$
$9 \times 3 =$	$9 \times 4 =$	$9 \times 5 =$	$9 \times 6 =$	$9 \times 7 =$
$10 \times 3 =$	$10 \times 4 =$	$10 \times 5 =$	$10 \times 6 =$	$10 \times 7 =$
$11 \times 3 =$	$11 \times 4 =$	$11 \times 5 =$	$11 \times 6 =$	$11 \times 7 =$
$12 \times 3 =$	$12 \times 4 =$	$12 \times 5 =$	$12 \times 6 =$	$12 \times 7 =$

$1 \times 8 =$	$1 \times 9 =$
$2 \times 8 =$	$2 \times 9 =$
$3 \times 8 =$	$3 \times 9 =$
$4 \times 8 =$	$4 \times 9 =$
$5 \times 8 =$	$5 \times 9 =$
$6 \times 8 =$	$6 \times 9 =$
$7 \times 8 =$	$7 \times 9 =$
$8 \times 8 =$	$8 \times 9 =$
$9 \times 8 =$	$9 \times 9 =$
$10 \times 8 =$	$10 \times 9 =$
$11 \times 8 =$	$11 \times 9 =$
$12 \times 8 =$	$12 \times 9 =$

REFRESHER PRACTICE 2

26. The grid below is another means of setting out the multiplication tables up to 9. Complete the grid by placing in each box the product of the number to the left of the row and the number at the top of the column.

×	2	3	4	5	6	7	8	9
2								
3								
4								
5								
6								
7								
8								
9								

27. When you have completed the grid explore the result to look for patterns and features in the numbers which may help you remember these multiples.

Simple Multiplication

28. Multiplication of single digit numbers is a straight forward application of the multiplication tables above. When one of the numbers in a multiplication is larger than those in the tables the problem is usually treated as follows:

Worked Example

312 × 3 =

would be set out as $\begin{array}{r} 312\ \times \\ \underline{\quad 3} \end{array}$

Note: It is customary to place the larger number on the top line.

Method:

Starting with the units, each digit in the top line is multiplied by the lower number and the product is written into the corresponding place below the line.

For the units (3 × 2 = 6)	For the tens (3 × 1 = 3)	For the hundreds (3 × 3 = 9)
$\begin{array}{r} 312\ \times \\ \underline{\quad 3} \\ \mathbf{6} \end{array}$	$\begin{array}{r} 312\ \times \\ \underline{\quad 3} \\ \mathbf{36} \end{array}$	$\begin{array}{r} 312\ \times \\ \underline{\quad 3} \\ \mathbf{936} \end{array}$

If the product of multiplying a place is greater than 9 the units of the product are written in that place in the answer and the tens are carried over to be added to the product of the place to the left.

Guided Example

$436 \times 6 =$

set out as

$$\begin{array}{r} 436 \times \\ \underline{6} \end{array}$$

For the units	For the tens	For the hundreds
($6 \times 6 = 36$)	($6 \times 3 = 18$ plus 3 carried over = 21)	($6 \times 4 = 24$ plus 2 carried over = 26)

$$\begin{array}{r} 436 \times \\ \underline{6} \\ \mathbf{6} \\ {\scriptstyle 3} \end{array} \qquad \begin{array}{r} 436 \times \\ \underline{6} \\ \mathbf{16} \\ {\scriptstyle 2} \end{array} \qquad \begin{array}{r} 436 \times \\ \underline{6} \\ \mathbf{2616} \end{array}$$

Exercise 1.3

Calculate the product of the following.

1. $362 \times 4 =$

2. $7159 \times 6 =$

3. $27 \times 3 \times 7 =$

4. There are 1760 yards in a mile and 3 feet in a yard. How many feet are there in a mile?

5. Modification kits are required for each of 273 aircraft. Each kit contains 8 special bolts. How many of these bolts are required to make up all of the kits?

Multiplying by 10 and Powers of 10

29. When a number is multiplied by 10 it has the effect of shifting all the digits in that number to the left by one place. So that the units become tens, the tens become hundreds, the hundreds become thousands and so on. This leaves the units place vacant which is marked by a 0.

e.g. $21 \times 10 = 210$

30. The product appears as the original number with a 0 on the end.

Powers of 10

31. One hundred is the product of multiplying ten by ten.

i.e. $10 \times 10 = 100$

32. When one number is multiplied another number of the same value the product is referred to as that number to a **power of 2.** Thus 100 is ten to a **power of two**, which is written as **10^2.**

Hence $10^2 = 10 \times 10 = 100$

33. Similarly, one thousand is the product of three tens multiplied together

i.e., $10 \times 10 \times 10 = 1000$

and so, may be referred to as ten to a power of three. Written as **10^3**.

Therefore $10^3 = 10 \times 10 \times 10 = 1000$

34. The powers of ten, from ten up to one million, are summarised in the table below.

Quantity	Multiple	Number	Power
Ten	10	10	10^1
One hundred	10×10	100	10^2
One thousand	$10 \times 10 \times 10$	1 000	10^3
Ten thousand	$10 \times 10 \times 10 \times 10$	10 000	10^4
One hundred thousand	$10 \times 10 \times 10 \times 10 \times 10$	100 000	10^5
One million	$10 \times 10 \times 10 \times 10 \times 10 \times 10$	1 000 000	10^6

Table 3 - Summary of the Powers of Ten from Ten to One Million

Multiples of Powers of 10

35. Consider $47 \times 30 =$

36. Here is a number which is being multiplied by a quantity of tens. The product will be a quantity of tens and so will have no units.

Worked Example

For the units

$$\begin{array}{r} 47 \times \\ \underline{30} \\ \mathbf{0} \end{array}$$

The absence of units is marked by **0** in this place.

The solution to the problem is completed by entering in the product of 47 and 3

For the tens

$$\begin{array}{r} 47 \times \\ \underline{\quad 30} \\ \mathbf{1}0 \\ {\scriptstyle \mathbf{2}\ \ \ } \end{array}$$

$3 \times 7 = 21$, the 1 is written into the tens place and the 2 is carried into the hundreds place.

For the hundreds

$\begin{array}{r} 47 \times \\ \underline{30} \\ \mathbf{14}10 \\ {\scriptstyle \mathbf{2}} \end{array}$	3 × 4 = 12, plus the 2 carried over equals 14, the 4 is written into the hundreds place and the 2 is carried into the thousands place.

37. When a multiple of 100 is a factor in a multiplication (e.g., 300) there will be no tens or units in the product, so these places will be occupied by 0s.

e.g., $\begin{array}{r} 312 \times \\ \underline{300} \\ \underline{936\mathbf{00}} \end{array}$

Long Multiplication

38. Multiplying numbers with two or three digits or more is accomplished by combining the skills illustrated above.

Worked Example

417 × 23 =

set out as $\begin{array}{r} 417 \times \\ \underline{23} \end{array}$

Step 1. Multiply the top number by the digit in the units place of the lower number.

$$\begin{array}{r} 417 \times \\ \underline{23} \\ \mathbf{1251} \end{array}$$

Step 2. Multiply the top number by the digit in the tens place of the lower number. Remembering that this is a multiple of ten and that this product will be shifted one place to the left with a 0 in the units place.

$$\begin{array}{r} 417 \times \\ \underline{23} \\ 1251 \\ \underline{\mathbf{8340}} \end{array}$$

Step 3. Complete the solution by adding the products found in the previous steps.

$$\begin{array}{r} 417 \times \\ \underline{23} \\ 1251 \\ \underline{8340} \\ \mathbf{9591} \end{array}$$

Guided Example

$407 \times 407 =$

set out as
$$\begin{array}{r} 407 \times \\ \underline{407} \end{array}$$

39. Multiplying the top number by the units of the lower number.

$$\begin{array}{r} 407 \times \\ \underline{407} \\ \mathbf{2849} \end{array}$$

$7 \times 7 = 49$, write the 9 and carry the 4
$7 \times 0 = 0$, plus the $4 = 4$, written in the tens
$7 \times 4 = 28$, write the 8 in the hundreds place and the 2 in the thousands

40. There are no tens in the lower number, therefore **no action is necessary** as any number **multiplied by 0** will have a **product of 0**.

41. Continue by multiplying the top number by the digit in the hundreds place of the lower number.

$$\begin{array}{r} 407 \times \\ \underline{407} \\ 2849 \\ \underline{\mathbf{162800}} \end{array}$$

As this is a multiple of 100, write 0s in the units and tens places.
$4 \times 7 = 28$, write in the 8 and carry the 2.
$4 \times 0 = 0$, plus the 2 carried over $= 2$
$4 \times 6 = 16$, write the 6 in ten thousands place and the 1 in the hundred thousands place.

42. Complete the solution by adding the products found in the previous steps.

$$\begin{array}{r} 407 \times \\ \underline{407} \\ 2849 \\ \underline{162800} \\ \mathbf{165649} \end{array}$$

Exercise 1.4

Calculate the product of the following:

1. $263 \times 17 =$

2. $428 \times 66 =$

3. $372 \times 248 =$

4. How many hours are there in a leap year (366 days)?

5. A man lives 18 kilometres (km) from his place of work. In one year he worked a total of 243 days. How many kilometres did he cover travelling to and from work in that year.

DIVISION

43. The process of division is used to determine the size of each part when a quantity is split into a number of equal parts. It may also give the number of times a quantity needs to be added to make another quantity.

Terminology

44. The numbers in a division are referred to as follows:

a. **Dividend** is the number to be to be divided
b. **Divisor** is the number by which the dividend is divided
c. **Quotient** is the outcome of a division

Simple Division

45. Many division problems can be solved by using the multiplication tables in reverse

e.g., $18 \div 3 =$
the multiplication table with a factor of 3 shows that
$3 \times \mathbf{6} = 18$
therefore $18 \div 3 = \mathbf{6}$

Division Brackets

46. For a simple division where the dividend is outside the scope of the multiplication table of the divisor an 'L' shaped bracket is used in solving the problem.

Worked Example

$736 \div 4 =$
would be set out as $4\underline{)\ 736}$

Note: the dividend is written inside the bracket and the divisor is written outside the bracket to the left. The quotient will be written in the space below the bracket.

47. Unlike the calculations covered so far, where the units are dealt with first and the working proceeds towards the left, division starts with the highest place value and continues to the right.

Hence:

Step 1. Determine how many times 4 will fit into 7
i.e., $7 \div 4 = \mathbf{1}$ remainder **3**
The 1 is written into the quotient in the place corresponding to the 7 and the remainder is carried one place to the right in the dividend.

$$\begin{array}{r} 4\underline{)\ 7\ {}^{\mathbf{3}}3\ 6} \\ \mathbf{1}\quad\ \ \end{array}$$

Note: The 3 carried over into the tens place is 3 hundreds therefore in the tens place it will have a value of 30. Thus, the tens place now contains 33.

Step 2. Determine how many times 4 will fit into 33

i.e., 33 ÷ 4 = **8** remainder **1**

$$\begin{array}{r} 4\overline{)\,7\;{}^{3}3\;{}^{\mathbf{1}}6} \\ 1\;\mathbf{8} \end{array}$$

The 8 is written into the tens place of the quotient and the remainder is carried over to the units of the dividend. Giving 16 in this place.

Step 3. Determine how many times 4 will fit into 16

i.e., 16 ÷ 4 = **4** exactly

Writing this quotient into the quotient of the main problem concludes the calculation.

$$\begin{array}{r} 4\overline{)\,7\;{}^{3}3\;{}^{\mathbf{1}}6} \\ 1\;8\;\mathbf{4} \end{array}$$

Checking Division

48. The accuracy of an answer to a division problem can easily be checked by carrying out the process in reverse. That is by multiplying the quotient by the divisor. If correct the product will match the dividend.

Check for the example above.

$$\begin{array}{r} 184 \times \\ \underline{4} \\ \mathbf{736} \\ \mathbf{3\,1} \end{array}$$

Exercise 1.5

1. 333 ÷ 9 =

2. 2944 ÷ 8 =

3. 4921 ÷ 7 =

4. The bill for a meal for five people comes to £85. If each person pays an equal share, how much will each pay?

5. A sheet of metal has to be cut into six equal strips. If the sheet is 432 centimetres wide, what will be the width of each strip?

Long Division

49. The principle described above for simple division also applies were the divisor is beyond the limits of the multiplication tables. However, the method for solving such problems is altered slightly. Firstly, the division bracket is inverted so that the quotient will appear above the bracket.

Worked Example

$578 \div 17 =$

Would be set out as $17\overline{)\,5\;7\;8}$

Step 1. Dividing the highest place value.
5 cannot be divided by 17 so the digit in the place to the right is brought into the calculation. This gives 57 divided by 17.

Note: This part of the calculation may call for an estimation of the value of 57 divided by 17.

$$17 \times 3 = 51$$

Thus 51 is the largest multiple of 17 which will fit into 57.
This is recorded by writing 51 underneath 57 in the dividend and placing 3 in the tens place of the quotient.

$$\begin{array}{r} \mathbf{3} \\ 17\overline{)\,5\;7\;8} \\ \underline{5\;1} \end{array}$$

Step 2. Find the remainder from $57 \div 17$ by subtracting 51 from 57.

$$\begin{array}{r} \mathbf{3} \\ 17\overline{)\,5\;7\;8} \\ \underline{5\;1} \\ 6 \end{array}$$

Step 3. Rather than carrying the remainder 6 from the tens to the units, the units are brought down to the remainder to give 68 units.

$$\begin{array}{r} \mathbf{3} \\ 17\overline{)\,5\;7\;8} \\ \underline{5\;1} \\ 6\;8 \end{array}$$

Step 4. Conclude the calculation by dividing 68 by 17.
$68 \div 17 = 4$ which is written into the quotient.

$$\begin{array}{r} 3\;\mathbf{4} \\ 17\overline{)\,5\;7\;8} \\ \underline{5\;1} \\ 68 \end{array}$$

Therefore $578 \div 17 =$ **34**

Exercise 1.6

Calculate the quotients for the following:

1. $3645 \div 15 =$

2. $7291 \div 23 =$

3. $12330 \div 18 =$

4. A lottery syndicate win £3094. If there are 14 members of the syndicate how much will each receive?

5. A credit agreement for the purchase of a computer amounts to £1152 to be repaid over 2 years. How much will the monthly repayments be?

Whole Numbers - Consolidation Test

1. $4728 + 384 + 2779 =$

2. $2337 - 888 =$

3. $84 - 59 + 32 - 26 + 47 - 75 =$

4. $328 \times 7 =$

5. $47 \times 5 \times 9 =$

6. $293 \times 64 =$

7. $169 \times 169 =$

8. $2112 \div 6 =$

9. $6175 \div 13 =$

10. $2112 \div 24 =$

FRACTIONS

50. **Proper fractions** may be defined as fractions less than 1; they are also known as Vulgar or common fractions.

For example: $\frac{1}{2}, \frac{1}{3}, \frac{3}{5}, \frac{11}{17},$ etc.

51. **Improper fractions** are fractions which are greater than 1.

For example: $\frac{7}{3}, \frac{5}{3}, \frac{17}{11}, \frac{8}{5},$ etc.

52. **Mixed numbers** include whole numbers and proper (vulgar) fractions.

For example: $1\frac{1}{2}$, $2\frac{3}{5}$, $6\frac{4}{11}$, $27\frac{6}{7}$, etc.

53. For all fractions the number above the bar is called the *numerator*. The number below the bar is called the *denominator*.

Simplest Form

54. The simplest form of $\frac{30}{60}$ is $\frac{1}{2}$. Fractions can be expressed in simplest form by dividing numerator and denominator by equal numbers until they will not divide further.

For example: $\frac{8}{12} = \frac{2}{3}$ in simplest form (after dividing numerator and denominator by 4).

55. **Cancelling**. The process of dividing numerator and denominator by equal values is called cancelling.

For example: $\frac{27}{81} = \frac{9}{27} = \frac{3}{9} = \frac{1}{3}$

56. **Converting** mixed numbers to improper fractions. Multiply whole numbers by denominator and add to numerator.

For example: $2\frac{3}{5} = \frac{13}{5}$

57. **Converting** improper fractions to mixed numbers. Divide numerator by denominator to give whole number - remainder gives new numerator.

For example: $\frac{25}{4} = 6\frac{1}{4}$

58. Cancelling improper fractions involves exactly the same process as cancelling vulgar fractions.

For example: $\frac{28}{4} = \frac{7}{1} = 7$

and $\frac{45}{6} = \frac{15}{2} = 7\frac{1}{2}$

59. For cancelling a mixed number, first convert it to an improper fraction then cancel down as before.

For example: $7\frac{3}{6}$

$= \frac{45}{6}$

$= \frac{15}{2}$

$= 7\frac{1}{2}$

60. **Addition**

a. Express all fractions as mixed numbers in lowest terms.

b. Add the whole numbers together.

c. To add vulgar fractions you must convert each fraction so that their denominators are all the same. This is done by finding the Lowest Common Multiple, (LCM) of the denominators.

(1) $\frac{1}{5} + \frac{1}{6} + \frac{3}{10}$

$= \frac{6+5+9}{30}$

$= \frac{20}{30}$

$= \frac{2}{3}$

Note: If your addition of fractions results in improper fractions, you must convert this to a mixed number as shown in example (2).

(2) $$\frac{9}{4}+\frac{5}{12}+1\frac{3}{8}$$

$$= 2\frac{1}{4}+\frac{5}{12}+1\frac{3}{8}$$

$$= 3 + \frac{1}{4}+\frac{5}{12}+\frac{3}{8}$$

$$= 3+\frac{6+10+9}{24}$$

$$= 3+\frac{25}{24} = 3+1\frac{1}{24}$$

$$= 4\frac{1}{24}$$

61. **Subtraction**. The same basic procedure should be used for subtraction as for addition.

a. $$\frac{8}{9}-\frac{2}{3}$$

$$= \frac{8-6}{9}$$

$$= \frac{2}{9}$$

b. $$\frac{8}{3}-1\frac{4}{7}$$

$$= 2\frac{2}{3}-1\frac{4}{7}$$

$$= 1+\frac{2}{3}-\frac{4}{7}$$

$$= 1+\frac{14-12}{21}$$

$$= 1\frac{2}{21}$$

c. $4\frac{1}{3} - 1\frac{3}{4}$

$= 3 + \frac{1}{3} - \frac{3}{4}$

$= 3 + \frac{4-9}{12}$

$= 2\frac{7}{12}$

Note: As numerator (4 - 9) in example c gives a negative value, one whole unit has to be converted to $\frac{12}{12}$ *before the subtraction of fractions is carried out.*

62. Mixed Addition and Subtraction can be carried out exactly as above.

Example:

a. $5\frac{7}{12} - 4\frac{1}{2} + 3\frac{3}{4}$ dealing with whole numbers first gives:

$= 4 + \frac{7-6+9}{12}$

$= 4 + \frac{10}{12}$

$= 4\frac{5}{6}$

b. $2\frac{1}{8} - 1\frac{3}{4} + 4\frac{1}{3}$

$= 5 + \frac{3-18+8}{24}$

$= 4 + \frac{24+3-18+8}{24}$

$= 4\frac{17}{24}$

63. Remember that your final step in any calculation must be to simplify (cancel fractions).

Example:

$$3\frac{3}{5}+1\frac{9}{10}-2\frac{3}{4}$$

$$= 2+\frac{12+18-15}{20}$$

$$= 2\frac{3}{4}$$

Exercise 1.7

1. Add the following fractions and mixed numbers:

   a. $\frac{1}{3}+\frac{4}{7}$ b. $\frac{4}{5}+\frac{5}{6}$ c. $\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$

   d. $\frac{1}{4}+\frac{4}{5}+\frac{7}{10}$ e. $3\frac{1}{3}+4\frac{3}{7}$ f. $2\frac{3}{4}+1\frac{1}{5}+1\frac{7}{10}$

2. Subtract the following:

   a. $\frac{1}{2}-\frac{1}{3}$ b. $\frac{3}{5}-\frac{2}{7}$ c. $4\frac{5}{6}-2\frac{2}{3}$

   d. $7\frac{2}{5}-2\frac{2}{3}$ e. $5\frac{1}{4}-1\frac{5}{6}$ f. $6\frac{3}{8}-5\frac{7}{12}$

3. Evaluate the following, simplifying as far as possible:

   a. $\frac{2}{5}+\frac{9}{10}-\frac{3}{4}$ b. $3\frac{5}{6}-1\frac{1}{2}+\frac{3}{4}$ c. $2\frac{1}{8}-1\frac{5}{15}+2\frac{1}{2}$

   d. $5\frac{5}{12}-2\frac{4}{9}+3\frac{1}{4}$ e. $4\frac{1}{4}+3\frac{1}{3}-5\frac{5}{8}$ f. $1\frac{1}{5}-3\frac{2}{3}+2\frac{1}{2}$

64. **Multiplication**

   a. Express all mixed numbers as improper fractions.

   b. Cancel vertically if possible.

   c. Cancel across the multiplication sign if possible.

   d. Multiply numerators together; multiply denominators together.

   e. If the result is an improper fraction, convert to a mixed number.

   f. Check that your answer is in simplest form.

Examples:

(1) $\frac{2}{9}\times 4$

$= \frac{2}{9}\times\frac{4}{1}$

$= \frac{8}{9}$

(2) $1\frac{4}{5}\times 2\frac{1}{3}\times\frac{5}{14}$

$= \frac{\cancel{9}^{3}}{\cancel{5}_{1}}\times\frac{\cancel{7}^{1}}{\cancel{3}_{1}}\times\frac{\cancel{5}^{1}}{\cancel{14}_{2}}$

$= \frac{3}{2}$ $= 1\frac{1}{2}$

45. **Division**. Convert all mixed numbers to improper fractions, invert the fractions you are dividing by and then proceed as for multiplication.

Examples:

a.

$$\frac{3}{4} \div 1\frac{5}{7}$$
$$= \frac{3}{4} \div \frac{12}{7}$$
$$= \frac{\cancel{3}^1}{4} \times \frac{7}{\cancel{12}_4}$$
$$= \frac{7}{16}$$

b.

$$\frac{3}{4} \div 7$$
$$= \frac{3}{4} \times \frac{1}{7}$$
$$= \frac{3}{28}$$

c.

$$\frac{3}{8} \div \frac{5}{16}$$
$$\frac{3}{\cancel{8}_1} \times \frac{\cancel{16}^2}{5}$$
$$= \frac{6}{5}$$
$$= 1\frac{1}{5}$$

46. Mixed multiplication and division can be carried out by simply inverting all the fractions preceded by a division sign and then treating the calculation as a multiplication only.

Example:

$$1\frac{3}{4} \div 4\frac{1}{2} \times 1\frac{5}{7}$$
$$= \frac{7}{4} \div \frac{9}{2} \times \frac{12}{7}$$
$$= \frac{7}{4} \times \frac{2}{9} \times \frac{12}{7}$$
$$= \frac{\cancel{7}^1}{\cancel{4}_1} \times \frac{2}{\cancel{9}_3} \times \frac{\cancel{12}^{31}}{\cancel{7}_1}$$
$$= \frac{2}{3}$$

*Note: You **only** turn upside-down the fraction you are dividing by, i.e., the fraction **after** the division sign.*

Exercise 1.8

1. Convert the following mixed numbers to improper fractions:

   a. $2\frac{3}{8}$ b. $6\frac{3}{4}$ c. $1\frac{7}{8}$ d. $3\frac{3}{16}$

2. Convert the following improper fractions to mixed numbers:

   a. $\frac{13}{4}$ b. $\frac{7}{2}$ c. $\frac{43}{8}$ d. $\frac{39}{16}$

3. Multiply the following, simplifying as far as possible:

   a. $\frac{3}{4}\times\frac{1}{2}$ b. $\frac{5}{6}\times\frac{9}{10}$ c. $\frac{1}{6}\times\frac{2}{3}$

   d. $\frac{5}{8}\times\frac{4}{15}$ e. $\frac{2}{3}\times1\frac{1}{8}$ f. $1\frac{3}{5}\times1\frac{9}{16}$

   g. $\frac{1}{3}\times\frac{6}{7}\times\frac{1}{2}$ h. $1\frac{1}{5}\times2\frac{1}{3}\times\frac{1}{21}$ i. $2\frac{3}{4}\times1\frac{5}{9}\times3\frac{6}{7}$

4. Divide the following, simplifying as far as possible:

   a. $\frac{5}{6}\div\frac{7}{8}$ b. $\frac{7}{12}\div\frac{14}{15}$ c. $4\frac{1}{8}\div2\frac{3}{4}$

5. Evaluate the following, simplifying as far as possible.

   a. $1\frac{1}{2}\times2\frac{2}{3}\div1\frac{1}{4}$ b. $2\frac{2}{15}\times3\frac{1}{8}\div2\frac{1}{7}$

   c. $2\frac{1}{4}\div1\frac{1}{2}\times3\frac{1}{3}$ d. $3\frac{1}{2}\div2\frac{1}{3}\times1\frac{1}{6}$

   e. $2\frac{2}{3}\times3\frac{6}{7}\div1\frac{1}{5}$

POSITIVE AND NEGATIVE NUMBERS (DIRECTED NUMBER)

47. Numbers in arithmetic and symbols in algebra, can be either **positive** or **negative**. A positive number is indicated by placing a + sign in front of it. A negative number is indicated by placing a - sign in front of it. If the number is not identified by a sign, we must assume that it is positive. For example:

a. Positive 12 is written + 12 or just 12.

b. Negative 12 is written - 12.

48. **Brackets** can be used to distinguish between, for example, the positive number 'plus three' and the operation 'add three'. Similarly between the negative number 'minus four' and the operation 'subtract four'. The rules for removing brackets containing directed numbers can be seen from the following.

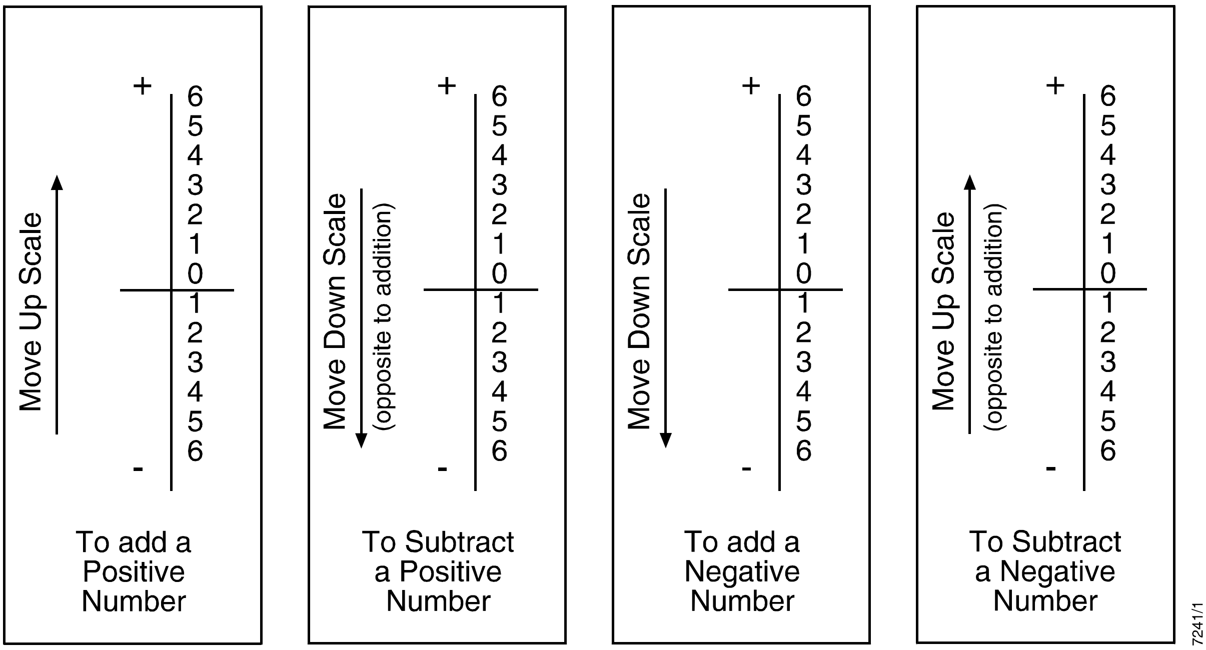
a. + (-2) = -2

b. - (+3) = -3

c. - (-4) = +4

d. + (+5) = +5

49. The logic behind the removal of the above brackets can be seen by considering the following number scales and remembering that subtraction is the opposite operation to addition.



For example:

a. $(-2) + (+4) = -2 + 4 = +2$

b. $(+1) - (+5) = 1 - 5 = -4$

c. $(+2) + (-4) = 2 - 4 = -2$

d. $(-2) - (-5) = -2 + 5 = +3$

50. When evaluating expressions which contain positive and negative numbers you should combine the numbers starting from the beginning of the expression, using the above rules. For example:

a. $3 + 4 - 7 + 6 - 1$

$= 7 - 7 + 6 - 1$

$= 0 + 6 - 1$

$= 5$

b. $7 - 4 + 1 - 6 - 3$

$= 3 + 1 - 6 - 3$

$= 4 - 6 - 3$

$= -2 - 3$

$= -5$

*Note however that we would **not** expect you to have to write down each of these lines of working.*

51. When multiplying directed numbers **like** signs give **positive** answers, **unlike** signs give **negative** answers. For example:

a. $(+5) \times (+2) = (+10)$

b. $(+5) \times (-3) = (-15)$

c. $(-5) \times (+4) = (-20)$

d. $(-5) \times (-5) = (+25)$

52. Similarly, when dividing directed numbers, **like** signs give **positive** answers, **unlike** signs give **negative** answers. For example:

a. $(+25) \div (-5) = (-5)$

b. $(-20) \div (-5) = (+4)$

c. $(-15) \div (+5) = (-3)$

d. $(+10) \div (+2) = (+5)$

Exercise 1.9

1. Remove brackets and evaluate:

   a. (+8) - (+5)

   b. (+7) + (-4)

   c. (+6) - (-2)

   d. (-2) - (+4)

   e. (-3) + (-6)

   f. (-4) - (-7)

   g. (-3) + (+3)

   h. 0 - (-4)

   i. (+2) + (-2)

   j. (+8) + (-4)

   k. (-7) + (-3)

   l. (+2) - (+4)

   m. (-2) - (-3)

   n. (-6) - (-5)

   o. (-2) - (+3)

2. Evaluate:

   a. 13 - 7 - 8

   b. 14 - 15 + 3

   c. 3 - 5 - 2

   d. 14 - 52 - 3 + 7

   e. 93 - 102 - 76

   f. 19 + 7 - 6 - 4 - 7 + 3

   g. 14 - 7 + 1 - 14 - 1 + 7

3. Evaluate:

a. $(-2) \times (+3)$

b. $(-3) \times (-4)$

c. $(+5) \times (-2)$

d. $(+5) \times (+1)$

e. $(-3) \times (+5)$

f. $(-2) \times (-1)$

g. $0 \times (-5)$

h. $(+6) \div (-2)$

i. $(-12) \div (-1)$

j. $(-8) \div (+8)$

k. $(-6) \div (-3)$

l. $\dfrac{(+12)}{(-3)}$

m. $\dfrac{(-2)}{(-4)}$

n. $0 \div (-5)$

DECIMALS

1. Decimals are an extension of the place values in the denary system and are used to express values less than one. In this respect they may be regarded as being the same as fractions.

DECIMAL POINT

2. In the same way a fraction is recognised by the use of the dividing line, a decimal is recognisable by the use of a dot separating whole numbers from values less than one, which is known as the decimal point.

3. Thus in the number 11.111 each digit is one tenth of the digit in the place to the left. Therefore the digit in the place after the decimal point (first decimal place) has a value of one tenth and the digit in the second decimal place one hundredth and so on.

ADDING AND SUBTRACTING WITH DECIMALS

4. When setting out addition or subtraction problems with decimals it is vital that the decimal points are kept in line with each other. This ensures that all of the same place values are treated together.

Worked Example

Addition 32.15 + 8.125 + 331. 4 + 0.333 =
Arranged thus

$$\begin{array}{r@{.}l}
32 & 15 \quad + \\
8 & 125 \\
331 & 4 \\
\underline{0} & \underline{333}
\end{array}$$

It is permissible to use zeros to even up the number of digits after the decimal point, as they will not alter the value. Then calculate the columns in the normal way.

$$\begin{array}{r@{.}l}
32 & 15\mathbf{0} \quad + \\
8 & 125 \\
331 & 4\mathbf{00} \\
\underline{0} & \underline{333} \\
\underline{372} & \underline{008}
\end{array}$$

Subtraction 24.32 – 8.45 =

Set out with decimal points in line.

$$\begin{array}{r@{.}l}
24 & 32 \\
\underline{8} & \underline{45}
\end{array}$$

Calculate in the normal manner.

$$\begin{array}{r@{.}l}
24 & 32 \\
\underline{8} & \underline{45} \\
\underline{15} & \underline{87}
\end{array}$$

Exercise 2.1

Calculate the following:

(1) 76.435 + 113.87 + 62.322 =

(2) 0.046 + 0.728 + 0.24 =

(3) 27.625 + 9.25 + 14.7 + 7.066 =

(4) 15.54 – 7.77 =

(5) 278 – 85.36 =

(6) 0.418 – 0.273 =

MULTIPLYING DECIMALS

5. **Multiplying Decimals by Powers of 10**. In the section dealing with whole numbers it was seen that multiplying a number by a power of ten shifted the digits in the number to the left so that they have a greater place value. When decimal values are involved this shift requires the decimal point to **move to the right.** It will do so, at the rate of **one place for every zero** in the power of ten being used.

Exercise 2.2

Calculate the product of the following.

(1) 346.874 × 100 =

(2) 0.0046 × 1000 =

(3) 2.625 × 1000 =

(4) 13.054 × 100 =

(5) 0.000278 × 10000 =

6. **Multiplying Decimals Together**. When multiplying decimal numbers it is easier to treat them as whole numbers first, then place the decimal point back in the result in the correct place.

Worked Example

12.43 × 4.3 =

The first stage of this calculation is to express the numbers as whole numbers by removing the decimal points but noting how many decimal places have been removed. In this case there are 3.

Carry out the multiplication.

$$\begin{array}{r} 1243 \times \\ \underline{\quad 43} \\ 3729 \\ \underline{49720} \\ 53449 \end{array}$$

7. **Placing the Decimal Point**. Once the correct figures have been arrived at there are two means of reinstating the decimal point in the correct place.

a. Count the number of decimal places removed in the first step and put it back into the answer.

i.e., 53.449

Thus the answer will have the same number of decimal places as the question.

b. Make a rough estimate of the answer by multiplying approximations of the original numbers. In the example above 12.5×4 would suffice to show that the answer is in the vicinity of 50.

Therefore 53.449 gives the closest fit.

Both of these methods yield the correct answer, so one may be used to check the other.

Exercise 2.3

Calculate the product of the following.

(1) $46.74 \times 1.3 =$

(2) $0.06 \times 5.12 =$

(3) $2.65 \times 10.5 =$

(4) $0.154 \times 1.44 =$

(5) $22.2 \times 2.22 \times 0.222 =$

DIVIDING DECIMALS

8. **Dividing a Decimal by Powers of Ten**. When a decimal number is divided by a power of ten the decimal point for the answer will shift to the **left one place for each zero** in the power of ten being used.

Exercise 2.4

Calculate the following.

(1) $0.0404 \div 100 =$

(2) $452.625 \div 1000 =$

(3) $130 \div 100 =$

(4) $492.874 \div 100 =$

(5) $30.15 \div 10000 =$

9. **Dividing a Decimal by a Whole Number**. The division of a decimal number by a whole number is set out the same as a normal division problem.

Worked Example

4.32 ÷ 3 =

Set out thus 3$\underline{)\ 4.3\ 2}$

The division proceeds as normal and the decimal point is placed in the answer as it is encountered in the calculation.

Thus

$$\begin{array}{l} 3\underline{)\ 4.{}^{1}3{}^{1}2} \\ \quad\ 1.\ 4\ 4 \end{array}$$

This principle also applies when long division is called for.

Worked Example

4.32 ÷ 16 =

Set out thus $16\overline{)4.32}$

4 will not divide by 16, so 0 is placed above the four and the decimal point is placed in the answer above the decimal point in the bracket.

$$\begin{array}{r} 0. \\ 16\overline{)4.32} \end{array}$$

43 will divide by 16, twice. So 2 is written above the 3 in the bracket and 32 (2×16) is subtracted from 43 to give the remainder.

$$\begin{array}{r} 0.2 \\ 16\overline{)4.32} \\ \underline{3\ 2} \\ \mathbf{1\ 1} \end{array}$$

The two in the second decimal place is brought down to the remainder, giving 112, which divided by 16 will give 7.

$$\begin{array}{r} 0.27 \\ 16\overline{)4.32} \\ \underline{3\ 2} \\ 1\ 1\ 2 \end{array}$$

Exercise 2.5

Calculate the following.

(1) $492.872 \div 4 =$

(2) $0.406 \div 8 =$

(3) $52.625 \div 25 =$

(4) $8.64 \div 12 =$

(5) $301 \div 40 =$

DIVIDING BY A DECIMAL

10. **Recall of Relevant Information**

- A division problem can be set out as a fraction.
- Equivalent fractions express the same value in different terms.
- Multiplying decimals by powers of ten shifts the decimal point to the right.

11. Presented with the problem $8.34 \div 0.3 =$ there are two ways of looking at these numbers which suggest possible solutions. Given that it is easier to divide by a whole number than a decimal, this type of problem may be solved in one of the following ways.

a. The whole division may be written as a fraction.

i.e., $\frac{8.34}{0.3}$

Multiplying each number by the same power of ten would give an equivalent fraction.

In this case if each number is multiplied by10 the denominator will become a whole number.

Therefore $8.34 \div 0.3 = 83.4 \div 3$

Which can be set out thus $3\,\overline{)\,83.4}$

b. Recognising 0.3 as being $\frac{3}{10}$.

$$8.34 \div 0.3 = 8.34 \div \frac{3}{10}$$

Using the division rule for fractions this will become

$$8.34 \div 0.3 = 8.34 \times \frac{10}{3} = 83.4 \div 3$$

Multiplying across the top line and dividing by the number on the bottom line completes the solution.

Exercise 2.6

Calculate the following using either of the methods shown.

(1) $92.872 \div 0.4 =$

(2) $0.426 \div 0.03 =$

(3) $2.625 \div 2.5 =$

(4) $8.64 \div 0.09 =$

(5) $301 \div 0.8 =$

SIGNIFICANT FIGURES

12. The value of any digit in a number is dependent upon its position. We call the first non-zero digit the first significant figure and the last non-zero digit the last significant figure. These three numbers all have 4 significant figures.

a.	3 0 9 7 | | 1st 4th	b.	3 0 . 9 7 | | 1st 4th	c.	0.00003097 | | 1st 4th

Note: Do not confuse with decimal places, sub-paragraph c has 8 decimal places.

13. Further examples:

a.	14384	(5 sig fig)	e.	1.005	(4 sig fig)
b.	0.089	(2 sig fig)	f.	72300	(3 sig fig)
c.	29.08604	(7 sig fig)	g.	10000000	(1 sig fig)
d.	72	(2 sig fig)	h.	630100	(4 sig fig)

14. To write a number to a given number of significant figures we look at one more significant figure than is required and if this digit is 5 or more, we round up the preceding figure; 4 or less we leave the preceding figure as it is. For example:

a. 187340 is 187000 to 3 significant figures 190000 to 2 significant figures.

b. 0.001239 is 0.00124 to 3 significant figures, 0.0012 to 2 significant figures.

c. 35.603 is 35.6 to 3 significant figures, 36 to 2 significant figures.

d. 0.081778 is 0.08178 to 4 significant figures, 0.0818 to 3 significant figures.

e. 1.006 is 1 to one significant figure.

ADJUSTING NUMBERS TO A SET NUMBER OF DECIMAL PLACES

15. When correcting to a certain number of decimal places (rounding off) it is only the digit immediately following the required decimal place which affects it. If the digit immediately following is 5 or more, the previous digit should be increased by 1. If it is less than 5 the previous digit is unaltered. For example:

a.	0.833	=	0.83 to 2 decimal places.
b.	0.866	=	0.87 to 2 decimal places.
c.	0.2346	=	0.23 to 2 decimal places.
d.	127.12304	=	127.1 to one decimal place
e.	0.0001449	=	0.00014 to 5 decimal places

Exercise 2.7

1. Write the following to 3 significant figures.

a.	6962	b.	70.406	c.	0.012392
d.	0.010991	e.	45.607	f.	2345

2. Write 24.86582 to the following number of significant figures.

a.	6 sig fig	b.	4 sig fig	c.	2 sig fig

3. Write the following to 3 decimal places.

a.	0.19387	b.	12.30972	c.	65.4555
d.	5.9997	e.	0.004977	f.	0.00321

4. Write 12.0938157 to the following number of decimal places.

a.	1 dp	b.	4 dp	c.	6 dp

NOTES:

DECIMALS AND FRACTIONS

1. The Imperial system of measurement makes extensive use of fractions to express partial quantities. However there are occasions where these partial quantities are expressed as decimals. For instance, the diameter of a fastener hole may be given as a fraction of an inch and the clearance to allow the fastener to pass through the hole given as thousandths of an inch expressed as a decimal to three places.

2. Choosing the correct size of drill bit to cut a clearance hole for a fastener is an example of an occasion where it will be necessary to add a fraction to a decimal. This will involve either converting a fraction into a decimal or converting a decimal into a fraction.

CONVERTING FRACTIONS TO DECIMALS

3. In the absence of an appropriate conversion chart the decimal equivalent of a fraction may be found in the following way:

Worked Example

Write $\frac{1}{4}$ as a decimal.

Obey the instruction given by the line and divide the numerator by the denominator.

i.e., $4\overline{)1.00}$

4 will not divide into 1 and so 0 is placed above it on the answer line and the 1 is carried into the first decimal place where it has a value of 10 tenths.

10 tenths divided by 4 is 2 (which is written in the first decimal place in the answer) with 2 remaining which is carried into the second decimal place where it will be worth 20 hundredths.

20 hundredths divided by 4 is 5 exactly which is entered in the second decimal place of the answer

$$\begin{array}{r} 0.25 \\ 4\overline{)1.00} \end{array}$$

*Note: When converting a **proper fraction** there will be **no whole number** before the decimal point in the answer.*

Exercise 3.1

Convert the following fractions into decimals.

a. $\frac{3}{4}$ b. $\frac{3}{5}$ c. $\frac{7}{20}$ d. $\frac{5}{8}$ e. $\frac{3}{16}$

Sometimes it is necessary to limit the number of decimal places in the answers to this type of calculation. For example, a task given to an aircraft technician may involve the accurate measurement of very small distances. The level of accuracy normally expected is no finer than a few thousandths of an inch. Therefore it is sensible to limit these numbers to three decimal places. More or less decimal places may be specified depending on the level of accuracy required.

Worked Example

(1) $\frac{5}{16}$ to 2 decimal places.

Convert the fraction into a decimal.

$\frac{5}{16} = 0.3125$

The aim here is to express $\frac{5}{16}$ to the nearest hundredth.

0.3125 is closer to 0.31 than it is to 0.32, so the numbers in the third and fourth decimal places are discarded.

Therefore $\frac{5}{16}$ to 2 decimal places = 0.31

(2) $\frac{7}{16}$ to 2 decimal places.

Convert the fraction into a decimal.

$\frac{7}{16} = 0.4375$

0.4375 is closer to 0.44 than it is to 0.43, so the number in the second decimal place is rounded up from 3 to 4.

Therefore $\frac{7}{16}$ to 2 decimal places = 0.44

If the number in either of these examples were to be expressed to 3 decimal places it can be seen that the 5 in the fourth decimal place lies exactly in the middle of the limits of the third decimal place.

The rule for dealing with the dilemma of what to put into the decimal place in question when the number in the following place is a 5 is,

When it’s five or more,
Increase the score.
When it’s four or below,
Let it go.

Exercise 3.2

Convert the following fractions into decimals. Answers to be written to **3 decimal places.**

a. $\frac{1}{3}$ b. $\frac{2}{3}$ c. $\frac{5}{9}$ d. $\frac{1}{6}$ e. $\frac{5}{12}$

CONVERTING DECIMALS TO FRACTIONS

4. To find the decimal equivalent of a fraction, the figures in the decimal represents the numerator and this needs to be placed over a **denominator** which will be a **power of 10**.

i.e., 0.25 will become $\frac{25}{100}$ which cancels down to $\frac{1}{4}$ in its lowest terms.

*Note**:** The quantity of **0**s after the **1** of the denominator is the same as the quantity of decimal places in the original number.*

Exercise 3.3

Find the fractional equivalent of the following decimals

a. 0.35 b. 0.22 c. 0.44 d. 0.375 e. 0.625

Exercise 3.4

Calculate the following, giving answers as fractions and decimals:

a. $0.23 + \frac{3}{5}$

b. $0.351 + \frac{2}{5}$

c. $\frac{1}{4} + 1.28$

NOTES

REPEATING DECIMALS

5. The decimal representation of a number is called a repeating (or recurring) decimal if at some point it becomes periodic: i.e. there is some finite sequence of digits that is repeated indefinitely. All repeating decimal numbers are rational, that is, they can be represented in the form $\frac{a}{b}$ where a and b are whole numbers.

For example the decimal representation of $\frac{1}{3}$ is 0.333… (spoken as “0.3 repeating” or “0.3 recurring”). This becomes periodic just after the decimal point, repeating the single digit sequence 3 infinitely. A more complicated example is $\frac{3227}{555}$ which, with long division becomes 5.8144144… where the decimal representation becomes periodic after the first digit, repeating the sequence 144 infinitely.

NOTATION

6. There are several accepted methods of showing repeating or recurring decimals. One method is to put a horizontal line (known as a vinculum) above the repeating digits, or place dots above the outermost numerals of the repeating digits. An alternative method is to enclose the repeating digits in brackets. Another convention is to repeat the sequence at least twice and then show further repetitions with an ellipsis (…), this method has become almost universal in printed documents, because it does not require the use of special characters. Examples of each are shown below.

Fraction	**Ellipsis**	**Vinculum**	**Dots**	**Brackets**
$^{1}/_{9}$	0.111…	$0.\overline{1}$	$0.\dot{1}$	0.(1)
$^{1}/_{3}$	0.333…	$0.\overline{3}$	$0.\dot{3}$	0.(3)
$^{2}/_{3}$	0.666…	$0.\overline{6}$	$0.\dot{6}$	0.(6)
$^{9}/_{11}$	0.818181…	$0.\overline{81}$	$0.\dot{8}\dot{1}$	0.(81)
$^{7}/_{12}$	0.58333…	$0.58\overline{3}$	$0.58\dot{3}$	0.58(3)
$^{1}/_{82}$	0.01219512195…	$0.0\overline{12195}$	$0.0\dot{1}219\dot{5}$	0.0(12195)
$^{22}/_{7}$	3.142857142857…	$3.\overline{142857}$	$3.\dot{1}4285\dot{7}$	3.(142857)

CONVERTING REPEATING DECIMALS TO FRACTIONS

7. Repeating decimals are difficult to work with (you cannot, for instance, enter them into a calculator) and their use inevitably results in an approximation. For this reason, if possible, they are best avoided. As stated previously, all repeating decimals numbers are rational, which means that they can be expressed in the form $\frac{a}{b}$ where a and b are whole numbers. Fortunately, it is a simple matter to find the numbers a and b using the “rule of nines” shown below:

There are three possible situations:

a. If the repeating numbers start directly after the decimal point (this is the most common situation), write down the number, or group of numbers that repeat, as the numerator in the fraction, with the same number of 9s in the denominator, then cancel down if necessary. For example:

$$0.444\ldots = \frac{4}{9}$$

$$0.333\ldots = \frac{3}{9} \quad \text{which cancels to } \frac{1}{3}$$

$$0.4545\ldots = \frac{45}{99} \quad \text{which cancels to } \frac{5}{11}$$

$$0.467467\ldots = \frac{467}{999} \quad \text{etc.}$$

b. If there are 0s after the decimal point, but before the start of the repeating number(s), write the same number of 0s after the 9s in the denominator. For example:

$$0.0444\ldots = \frac{4}{90} \quad \text{which cancels to } \frac{2}{45}$$

$$0.0066\ldots = \frac{6}{900} \quad \text{which cancels to } \frac{1}{150}$$

$$0.004545\ldots = \frac{45}{9900} \quad \text{which cancels to } \frac{1}{220}$$

c. A number which starts repeating after some other numbers can be dealt with as follows:

$$0.58333\ldots = 0.58 + 0.00333\ldots = \frac{58}{100} + \frac{3}{900} = \frac{522+3}{900} = \frac{525}{900} = \frac{7}{12}$$

$$0.41666\ldots = 0.41 + 0.00666\ldots = \frac{41}{100} + \frac{6}{900} = \frac{369+6}{900} = \frac{375}{900} = \frac{5}{12}$$

Any repeating decimal can be dealt with in this manner, some will cancel down into small fractions and some will not, however, it is quite feasible to enter even large fractions into a calculator, thus avoiding the need for an approximation.

Exercise 3.5

Write down equivalent common fractions for the following repeating decimals:

a. 0.555…

b. 0.444…

c. 0.3636…

d. 0.0111…

e. 0.4666…

NOTES

PERCENTAGES AND RATIOS

Percentages

1. **Definition**. - A percentage is a fraction whose denominator is 100.

Example: $3\%\ means\ \frac{3}{100}$

2. **Changing a Fraction to a Percentage**. - To change a fraction to a percentage, multiply by 100.

Example: $\frac{3}{5}\ as\ a\ percentage\ = \frac{3}{5} \times 100 = 60\%$

3. **Changing a Percentage to a Fraction**. - To change a percentage to a fraction divide by 100. For example:

a. 8% as a fraction $= \frac{8}{100} = \frac{2}{25}$

b. 12½% as a fraction $= \frac{12.5}{100} = \frac{1}{8}$

4. **Changing a Percentage to a Decimal**. - To convert a percentage to a decimal, divide by 100. (Move decimal point two places to the left). For example:

a. 65% as a decimal = 65 ÷ 100 = 0.65

b. 32.5% as a decimal = 32.5 ÷ 100 = 0.325

5. **Changing a Decimal to a Percentage**. To convert a decimal to a percentage, multiply by 100. (Move decimal point two places to the right). For example:

a. 0.21 as a percentage = 0.21 × 100 = 21%

b. 0.037 as a percentage = 0.037 × 100 = 3.7%

6. **Percentage of a Quantity**. To find the value of a percentage of a quantity, multiply by the percentage value divided by 100. For example:

a. 4% of 60 = 60 × 4 ÷ 100 = 240 ÷ 100 = 2.4

b. 3.5% of 1500 = 1500 × 3.5 ÷ 100 = 15 × 3.5 = 52.5

7. **One Quantity as a Percentage of Another**. To express one quantity as a percentage of another, make a fraction of the two quantities and multiply by 100.

For example: a. 12 as a percentage of 50 $= \frac{12}{50} \times 100$ = 24%

b. 4 as a percentage of 60 $= \frac{4}{60} \times 100$ = 6.67%

Exercise 4.1

1. Express as a proper fraction:

a.	0.6	b.	0.35	c.	0.48
d.	0.05	e.	0.325	f.	25%
g.	13%	h.	$4\frac{1}{2}$%	i.	$16^{1}/_{3}$%

2. Express as a percentage:

a.	0.43	b.	0.025	c.	1.25
d.	$^{2}/_{3}$	e.	$^{3}/_{7}$	f.	$^{1}/_{12}$
g.	$^{3}/_{8}$				

3. Calculate:

a.	4% of 30	b.	0.8% of 360	c.	1.5% of 60
d.	120% of 75	e.	80% of 90		

4. Express:

a.	30 as a percentage of 50	b.	24 as a percentage of 16
c.	0.5 as a percentage of 12.5	d.	3.2 as a percentage of 2.4
e.	0.08 as a percentage of 0.72		

RATIOS

8. In a widely used concrete mix, cement, sand and gravel are in the ratio:

1 to 2 to 4 (often written 1 : 2 : 4)

This means that for every 1 part of cement, there are 2 of sand and 4 parts of gravel.

Example 1 4 kg of cement requires 8 kg of sand and 16 kg of gravel.

20 kg of cement requires 40 kg of sand and 80 kg of gravel.

The ratio of sand to cement is always 2 to 1 (twice as much sand as cement), gravel to cement is always 4 to 1.

9. A ratio compares quantities using the same units - here kilograms. So here 1 tonne of cement would need 2 tonnes of sand and 4 tonnes of gravel.

Example 2 In a scale drawing, using the **scale** 1:50,
1 cm on the drawing stands for 50 cm on the ground.
37 mm on the drawing stands for $37 \times 50 = 1850$ mm = 1.85 metres on the ground.
3 metres on the ground, though becomes $3 \div 50 = 0.06$ m or 60 mm on the drawing.
8.5 metres on the ground becomes $8.5 \div 50 = 0.17$ m = 170 mm on the drawing.

Example 3 In a dye works, the ratio of colour A to colour B is 5:4.

So, 5 g of A requires 4 g of B
1 g of A requires $^4/_5$ g of B
And 9 g of A requires $9 \times {}^4/_5$ of B
$= {}^{36}/_5$
$= 7^1/_5 = 7.2$ g of B

DIVIDING AN AMOUNT INTO A GIVEN RATIO

10. This involves dividing the amount into equal parts and then distributing the parts according to the given ratio.

Example Divide £36 in the ratio 5:4

The ratio 5:4 tells us we need 5 + 4 = 9 parts
£36 divided by 9 means that each part is £4

So the distribution is $5 \times £4 : 4 \times £4$
or £20 : £16

Check by adding £20 + £16 = £36

Exercise 4.2

1. Divide 24 kg in the ratio 3 : 5
2. Divide 63 ml in the ratio 2 : 5
3. Divide 80 m in the ratio 1 : 3 : 4
4. Divide £48 in the ratio 2 : 3 : 7

Exercise 4.3

1. The length of an aircraft and the length of its model are in the ratio 200 : 1. If the aircraft is 30 m long, how long is its model?

2. A drawing is to be made $^1/_5$ full size. If a dimension of 740 mm is to be represented on the drawing, what size will it be?

3. Divide a line 140 mm long in the ratio 4 : 3.

4. A metal suitable for high speed bearings is made from tin and lead in the ratio 8.6 : 1.4. Find the mass of each metal in a sample of metal which has mass 15 kg?

5. A bar of metal 10.5 m long is to be cut into 3 parts in the ratio ½ : 1¾ : 3. Find the length of each part?

6. A mass is composed of 3 parts copper to 2 parts zinc. Find the mass of copper and zinc in a casting which has a mass of 80 kg?

7. How much copper is required to be melted with 40 kg zinc to make a brass so that the ratio of the copper to zinc is 7 : 3?

8. A right-angled triangle has sides in the ratio 3 : 4 : 5. If the hypotenuse (the longest side) is 70 mm long, how long are the other sides?

BODMAS

Evaluating Expressings Using BODMAS Rule

1. When evaluating numerical expressions we need to know the order in which addition, subtraction, multiplication and division are carried out.

Consider evaluating $3 + 4 \times 5$

With addition carried out first: $3 + 4 \times 5 = 7 \times 5 = 35$

and with multiplication carried out first

$3 + 4 \times 5 = 3 + 20 = 23$

So from the above it can be seen that the order in which numerical operations are carried out is very important.

2. Using the BODMAS rule it tells us the order which we must carry out these numerical operations.

BODMAS stands for:

Brackets ( )	First priority
Order	Second priority
Division ÷	Third priority
Multiplication ×	Third priority
Addition +	Fourth priority
Subtraction -	Fourth priority

3. Occasionally problems involve the term ‘of’ in these instances the ‘of’ is to be regarded as a multiplication, so ½ of 10 means ½ × 10. (Third priority)

Examples using BODMAS:

a. $2 + 3 \times 4$ multiplication first
$2 + 3 \times 4 = 2 + 12 = 14$

b. $(2 + 3) \times 4$ brackets first
$(2 + 3) \times 4 = 5 \times 4 = 20$

c. $4 + 4 \div 2$ division first
$4 + 4 \div 2 = 4 + 2 = 6$

d. $20 \div (4 \times 5)$ brackets first
$20 \div (4 \times 5) = 20 \div 20 = 1$

e. $5 + \frac{1}{2}$ of 20 × (of) first
$5 + \frac{1}{2}$ of 20 $= 5 + 10 = 15$

Exercise 5.1

Evaluate:

1. $3 + 4 \times 2 =$
2. $8 - 4 \div 2 =$
3. $5 \times 3 + 2 =$
4. $(5 + 3) \times 4 =$
5. $(8 - 4) \div 2 =$
6. $8 + 5 \times 3 - 2 =$
7. $13 - {}^{1}/_{4} \; of \; 32 =$
8. $({}^{3}/_{4} - {}^{1}/_{4}) of \; 22 =$
9. $3 + 8 \times 9 =$
10. $3 \times 7 - 2 =$
11. $8 \times 5 - 2 + 5 \times 6 =$
12. $5 - 15 \times (5 - 2) =$
13. $2(-7 - 7) =$
14. $9 \div 3 \times (-4) + (-6) \div (-2) =$
15. $-200 \div 10 \times (-4) + (-16) \div (-8) =$
16. $3 \times (-12) + (-44) \div (-2) =$
17. $\big((-12) \times (20) \div (-6) - (+4)\big) \times {}^{1}/_{2} =$
18. $15 + 6 \div 2 + 5 \times (6 \div 2) =$
19. $-21 \div (-3) \times (-4) + (-11) \times 14 \times (6 \div 2 + 4) =$
20. $108 \div 3 \times (5 + 4) \div (-6 \times 3) =$
21. $\frac{15+21}{(-2-7)} \times \frac{11\times4+3}{47} + \frac{6\times(-5)}{-2} =$
22. $\frac{3^2+4^2}{5^2} =$
23. $\sqrt{31 + 18} + 4(9 + 2) =$ (Note: Assume all roots to be positive)
24. $(2^3)^2 \div 16 + 4 =$
25. $\big((3^2 \times 9 \div 3) \div (-1)\big) \times 3 =$
26. $\frac{6^2 \div \sqrt{134+10} + 16 - \sqrt{64} + 5^2}{2^2 \times 3^2} =$ (Note: Assume all roots to be positive)

ANSWERS TO EXERCISES

Exercise 1.1

1.	145	2.	451	3.	4215
4.	1802 hours	5.	3675 kg		

Exercise 1.2

1.	173	2.	914	3.	1726
4.	29 tubes	5.	255 mm		

Exercise 1.3

1.	1448	2.	42954	3.	567
4.	5280 feet	5.	2184 bolts		

Exercise 1.4

1.	4471	2.	28248	3.	92256
4.	8784 hours	5.	8748 km (i.e., 36 km per day)		

Exercise 1.5

1.	37	2.	368	3.	703
4.	£17	5.	72 cm		

Exercise 1.6

1.	243	2.	317	3.	685
4.	£221	5.	£48		

Consolidation (Whole Numbers)

1.	7891	2.	1449	3.	3
4.	2296	5.	2115	6.	18752
7.	28561	8.	352	9.	475
10.	88				

Exercise 1.7

1	a.	$\frac{19}{21}$	b.	$1\frac{19}{30}$	c.	$\frac{47}{60}$
	d.	$1\frac{3}{4}$	e.	$7\frac{16}{21}$	f.	$5\frac{13}{20}$
2.	a.	$\frac{1}{6}$	b.	$\frac{11}{35}$	c.	$2\frac{1}{6}$
	d.	$4\frac{11}{15}$	e.	$3\frac{5}{12}$	f.	$\frac{19}{24}$
3.	a.	$\frac{11}{20}$	b.	$3\frac{1}{12}$	c.	$3\frac{7}{24}$
	d.	$6\frac{2}{9}$	e.	$1\frac{23}{24}$	f.	$\frac{1}{30}$

Exercise 1.8

1.	a.	$\frac{19}{8}$	b.	$\frac{27}{4}$	c.	$\frac{15}{8}$	d.	$\frac{51}{16}$
2.	a.	$3\frac{1}{4}$	b.	$3\frac{1}{2}$	c.	$5\frac{3}{8}$	d.	$2\frac{7}{16}$
3.	a.	$\frac{3}{8}$	b.	$\frac{3}{4}$	c.	$\frac{1}{9}$	d.	$\frac{1}{6}$
	e.	$\frac{3}{4}$	f.	$2\frac{1}{2}$	g.	$\frac{1}{7}$	h.	$\frac{2}{15}$
	j.	$16\frac{1}{2}$						
4.	a.	$\frac{20}{21}$	b	$\frac{5}{8}$	c.	$1\frac{1}{2}$		
5.	a.	$3\frac{1}{5}$	b.	$3\frac{1}{9}$	c.	5	d.	$1\frac{3}{4}$
	e.	$8\frac{4}{7}$						

Exercise 1.9

1.	a.	3	b.	3	c.	8	d.	-6
	e.	-9	f.	3	g.	0	h.	4
	i.	0	j.	4	k.	-10	l.	-2
	m.	1	n.	-1	o.	-5		
2.	a.	-2	b.	2	c.	-4	d.	-34
	e.	-85	f.	12	g.	0		
3.	a.	-6	b.	12	c.	-10	d.	5
	e.	-15	f.	2	g.	0	h.	-3
	i.	12	j.	-1	k.	2	l.	-4
	m.	$\frac{1}{2}$	n.	0				

Exercise 2.1

1.	252.627	2.	1.014	3.	58.641
4.	7.77	5.	192.64	6.	0.145

Exercise 2.2

1.	34687.4	2.	4.6	3.	2625
4.	1305.4	5.	2.78		

Exercise 2.3

1.	60.762	2.	0.3072	3.	27.825
4.	0.22176	5.	10.941048		

Exercise 2.4

1.	0.000404	2.	0.452625	3.	1.3
4.	4.92874	5.	0.003015		

Exercise 2.5

1.	123.218	2.	0.02075	3.	2.105
4.	0.72	5.	7.525		

Exercise 2.6

1.	232.18	2.	14.2	3.	1.05
4.	96	5.	376.25		

Exercise 2.7

1.	a.	6960	b.	70.4	c.	0.0124
	d.	0.0110	e.	45.6	f.	2350
2.	a.	24.8658	b.	24.87	c.	25
3.	a.	0.194	b.	12.310	c.	65.456
	d.	6.000	e.	0.005	f.	0.003
4.	a.	12.1	b.	12.0938	c.	12.093816

Exercise 3.1

a.	0.75	b.	0.6	c.	0.35
d.	0.625	e.	0.1875		

Exercise 3.2

a.	0.333	b.	0.667	c.	0.556
d.	0.167	e.	0.417		

Exercise 3.3

a.	$^{7}/_{20}$	b.	$^{11}/_{50}$	c.	$^{11}/_{25}$
d.	$^{3}/_{8}$	e.	$^{5}/_{8}$		

Exercise 3.4

a. $\frac{83}{100}$ = 0.83 b. $\frac{751}{1000}$ = 0.751 c. $\frac{153}{100}$ = 1.53

Exercise 3.5

a. $\frac{5}{9}$ b. $\frac{4}{9}$ c. $\frac{4}{11}$ d. $\frac{1}{90}$ e. $\frac{7}{15}$

Exercise 4.1

1.	a.	$^{3}/_{5}$	b.	$^{7}/_{20}$	c.	$^{12}/_{25}$
	d.	$^{1}/_{20}$	e.	$^{13}/_{40}$	f.	$^{1}/_{4}$
	g.	$^{13}/_{100}$	h.	$^{9}/_{200}$	i.	$^{49}/_{300}$
2.	a.	43%	b.	2.5%	c.	125%
	d.	66.$^{2}/_{3}$%	e.	42$^{6}/_{7}$%	f.	8$^{1}/_{3}$%
	g.	37$^{1}/_{2}$%				
3.	a.	1$^{1}/_{5}$ or 1.2	b.	2.88	c.	0.9
	d.	90	e.	72		
4.	a.	60%	b.	150%	c.	4%

d. 133.33…% (or 133$^{1}/_{3}$%) e. 11.11…% (or 11$^{1}/_{9}$%)

Exercise 4.2

1. 9 kg, 15 kg
2. 18 ml, 45 ml
3. 10 m, 30 m, 40 m
4. £8, £12, £28

Exercise 4.3

1. 0.15 m
2. 148 mm
3. 80 mm and 60 mm
4. 12.9 kg tin and 2.1 kg lead
5. 1 m, 3½ m, 6 m
6. 48 kg copper, 32 kg zinc
7. $93^1/_3$
8. 42 mm and 56 mm

Exercise 5.1

1. 11
2. 6
3. 17
4. 32
5. 2
6. 21
7. 5
8. 11
9. 75
10. 19
11. 68
12. -40
13. -28
14. -9
15. 82
16. -14
17. 18
18. 33
19. -1106
20. -18
21. 11
22. 1
23. 51
24. 8
25. -81
26. 1