Defence School of
Aeronautical Engineering

Aerosystems Engineer & Management Training School

Academic Principles Organisation

Mathematics

BOOK 6

Errors and Conversions

WARNING

These course notes are produced solely for the purpose of training. They are not subject to formal amendment action after issue and they must NOT be used for operating and maintaining the equipment described. Operation and maintenance of equipment is governed by the limitations and procedures laid down in the authorized publications and manuals which must be complied with for these purposes

KEY LEARNING POINTS

KLP	Description
MA2.5	Explain the terms ‘relative error’ and ‘absolute error’
MA2.6	Apply error arithmetic to experimental data

Consider the following:

1. Numbers representing real quantities that are obtained from measurement, such as the length of a control cable, or the current in an electrical circuit, are rarely exact but measured to within a certain resolution, depending on the measuring instruments being used.

2. Manufactured items such as electrical components or mechanical parts that are used in engineering cannot be made to an exact value but are built to a tolerance.

3. Constants such as π, g etc. that are used in calculations can rarely be quoted to an exact value, but are also rounded to a certain number of significant figures.

4. All of these things serve to introduce a level of uncertainty in the results of our calculations. It is important to be able to know exactly the size of this potential error, so that we can avoid a situation where we reach incorrect conclusions and assumptions, based on inaccurate evidence.

5. Errors can be quoted one of two ways:

Absolute Error - is the difference between the expected and the actual result:

$$E = \left(a^* - a\right)$$

where a in the expected result and a^* is the actual (or experimental) result.

Relative Error - Expresses the error as a proportion of the expected result:

$$E_r = \frac{\left(a^* - a\right)}{a}$$

6. This is often quoted as a percentage. An example of an error calculation is an experiment to confirm the position of the Centre of Gravity (C of G) of an aircraft.

For example:

Let a = distance from datum point to the calculated C of G

= 16.9 m

And, a^* = position of C of G found by experimentally balancing the aircraft

= 17.1 m

Continuing with the example on page 1:

Then,

Absolute Error

= $(a^* - a)$

= 17.1 – 16.9

= 0.2 m

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Relative Error = $\dfrac{(a^* - a)}{a}$

= $\dfrac{0.2}{16.9}$

= 0.0118 or 1.18%

It can be seen that errors can be positive (as above) or negative, so, often they are quoted as ±%. For example, a velocity could be quoted as 300 m/s ± 2%.

This means that the measured velocity could be as low as 294 m/s or as high as 306 m/s, a variation of 12 m/s.

DEALING WITH ERRORS IN EXPERIMENTAL DATA

7. When carrying out calculations involving the results of experimental data it is useful to be able to estimate the maximum possible error in the final solution by taking the worst-case situation as follows:

a. When adding or subtracting quantities that are subject to error,

The absolute error of the sum or difference is equal to the sum of the absolute errors.

For example, an electrical resistor has a marked value of 50Ω ± 10%, so has a maximum possible value of 55Ω (i.e. a maximum absolute error of 5Ω). This is then connected in series with a similar resistor of 100Ω ± 2% and therefore a maximum possible value of 102Ω (an absolute error of 2Ω).

As:

Total resistance = resistance of resistor 1 + resistance of resistor 2,

This then is a situation where we are adding quantities that are subject to error. Using the rule above we can say that the absolute error in the total resistance is the sum of the individual absolute errors, in this case, 5 + 2 = 7 Ω

b. When multiplying or dividing quantities that are subject to error, it is necessary to calculate the maximum possible relative error. The situation is more complicated, but we can still find the resulting errors empirically by considering the worst-case situation.

For example, in the formula for electrical power,

$$P = I^2 R$$

If the maximum relative error of I is 5%, this means that the actual value of I could be anywhere between $0.95 \times I$ to $1.05 \times I$, likewise there may be an error in R of, say up to 3%. Therefore, dealing with the situation where, (for example) $I = 5$ A and R = 1000Ω.

Worst case value of $I = 5 \times 1.05 = 5.25$ A
Worst case value of $R = 1000 \times 1.03 = 1030$ Ω

Therefore, the worst-case value of P is: $5.25^2 \times 1030 \approx 28389$ watts instead of 25000,

Which is a maximum possible relative error of $\dfrac{28389 - 25000}{25000}$ = 0.136

or 13.6% (approximately, to 3 sig. fig)

RELATIVE ACCURACY AND IMPLIED ACCURACY

8. Relative accuracy is another word for the maximum possible relative error in a number or quantity.

9. When numbers are rounded a relative accuracy is implied depending on the number of significant figures quoted. For example, quoting a number to 3 sig. fig. implies an accuracy of 1 part in 1000, 4 sig. fig. implies 1 part in 10000 etc.

10. It is generally accepted that:

*When multiplying and dividing numbers, the answer should **not** be given to an accuracy greater than the least accurate of the given numbers*

Consider the following calculation, which uses numbers from experimentally obtained data.

$$\frac{5.73 \times 21}{0.6243} \quad \text{the result of which is 192.74387 from a calculator}$$

11. The least accurate of the three numbers is 21, which has 2 significant figure accuracy. Hence the answer should not be stated any more accurately than this: namely 190 which is obtained by rounding 192.74387 to 2 sig. fig.

12. *Accuracy greater than this cannot be justified from the data.* Note that it is the number of significant figures that is important here, *not* the number of decimal places.

13. A consequence of this is that when multiplying or dividing by constants (π, conversion factors, etc.) which are not exact, the answer should not be quoted to a greater number of significant figures than the rounded constant.

Example

A current in an electrical circuit is measured to within one-hundredth of an ampere but has been incorrectly listed as 7 A.

What is the value of relative accuracy implied by the way the current has been listed?
How should the current have been listed and what is the true value of the maximum absolute error and the relative accuracy?

Answer

Listing implies a possible range of values of 6.5 to 7.5 A
So relative accuracy is:

$$\frac{7.5-7}{7} \approx 0.07 \quad \text{or} \quad 7\,\% \qquad\qquad \text{(1 sig fig)}$$

Current should be listed as 7.00 A; maximum possible absolute error is 0.005A and true relative accuracy is:

$$\frac{7.005 - 7.00}{7.00} \approx 0.0007 \quad \text{or } 0.07\% \qquad \text{(1 sig fig)}$$

Try the following:

Exercise 1:

1. $a = 200.5$
   $a^* = 199.5$, find E and E_r.

2. $a = 0.00352$
   $a^* = 0.00410$, find E and E_r.

3. A calculation is required, of the kinetic energy stored in a moving body. The correct formula is:

   Kinetic energy = ½mv²

   The value of m (the mass) is known to an accuracy of ± 10% and value of v (the velocity) is to be measured with an expected accuracy of ± 20%. For a calculation of the energy where m and v are supposedly 10kg and 50 m/s respectively, find the maximum possible relative error in the calculated value of the kinetic energy

(Answers on page 6)

ANSWERS TO EXERCISE 1

1. $E = -1$ $E_r \approx -0.4988$ %

2. $E = 0.00058$ $E_r \approx 16.5$ %

3. $E_r = 0.584$ or 58.4 %

KEY LEARNING POINTS

MA2.7	Convert aircraft fuel loads between US, Imperial and SI units
MA2.8	Convert system pressures between Imperial and SI units

14. Because of a lack of standardisation in the aircraft industry, there are several *systems of units,* (for example, SI, metric, imperial and US systems) in common use. For instance, for pressure, psi, bar and the standard atmosphere are all used. Aircraft fuel loads may be given in litres, gallons (US), gallons (UK), pounds or kilograms. It will be necessary to have the ability to be able to convert from one system of units to another, in order to accurately replenish aircraft systems.

15. To perform conversions a conversion factor is required. A conversion factor is the equivalent value of 1 unit of a substance expressed in a different (required) unit. Given a conversion factor, we can then perform a calculation to change, for example, pounds to kilograms.

Change 15kg into pounds (lb).

1kg = 2.2046lb

Therefore, the conversion factor is 2.2046. A calculation is to convert 15kg to lbs is performed below:

Old unit		Conversion Factor		New unit
15	x	2.2046	=	33.069lb

We can now reverse the process by dividing through by the conversion factor to get back to what we started with.

Old unit		Conversion Factor		New unit
33.069	÷	2.2046	=	15kg

16. If we look closely at the maths involved in doing this and rearranged the formula we would see a pattern.

To find a conversion factor to change lb to kg divide both sides of the equation by 2.2046

$$1(kg) = 2.2046\,(lb)$$

$$\frac{1}{2.2046}\,(kg) = \frac{2.2046}{2.2046}\,(lb)$$

$$0.4536\,(kg) = 1(lb)$$

17. What we can see is that if we divide the conversion factor from kg to pounds into 1 i.e. take the reciprocal a new conversion factor from pounds to kilograms can be generated.

18. This can be done easily with a calculator by using the $\boxed{x^{-1}}$ button that takes the reciprocal of a number.

Exercise 2

Complete the following table. (All data required is present)
The conversion factors have been rounded to an appropriate number of significant figures

1 pound	= 0.4536	kilograms
1-pound water	= 0.0998	gallons [UK]
1-pound water	= 0.1198	US gallons
1-pound water	= 0.4536	litres
1 litre water	=	pounds (4 sig fig)
1 litre	= 0.2642	US gallons
1 litre	= 0.2200	gallons [UK]
1 gallon [UK]	=	litres (4 sig fig)
1 gallon [UK]	=	US gallons (4 sig fig)
1 gallon [UK] water	= 10.02	pounds
1 US gallon	=	litres (4 sig fig)
1 US gallon	= 0.833	gallons [UK]
1 US gallon water	= 8.345	pounds
1 atmosphere [standard]	= 101325	newton/square metre [N/m^2]
1 atmosphere [standard]	= 1.0133	bar
1 atmosphere [standard]	= 14.696	pound/square inch
1 pound/square inch	=	bar (4 sig fig)
1 pound/square inch	=	atmosphere [standard] (4 sig fig)
1 pound/square inch	= 6 894.8	newton/square metre [N/m^2]
1 bar	= 14.5038	pound/square inch
1 bar	=	atmosphere [standard] (4 sig fig)
1 bar	= 100000	newton/square metre [N/m^2]
1 newton/square metre [N/m^2]	=	pound/square inch (4 sig fig)

1 newton/square metre [N/m^2]	=	atmosphere (4 sig fig)
1 newton/square metre [N/m^2]	=	bar

Try the following exercise:

Exercise 2a

1. If a system is pressured to 36 psi, what pressure is this in bar?

2. What is 200 000 newtons/square metre in psi?

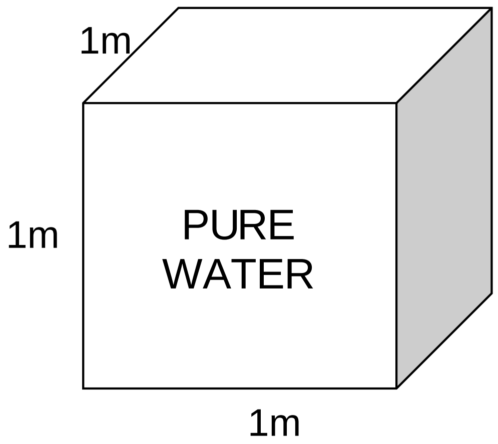
3. Convert 1.013 bar to atmospheres.

(Answers on page 14)

FINDING THE MASS OF A GIVEN VOLUME OF MATERIAL

19. To find the mass of particular volume of a substance, we must take into account the density of the substance (ρ), as follows.

20. Considering firstly the example of a given volume of pure water 1m by 1m by 1m as shown below:



21. In the SI system, a volume of $1m^3$ is equivalent to 1000 litres, and 1 litre of (pure) water has a mass of 1kg. So, for pure water, $1m^3$ of material has a mass of 1000 kg or 1 tonne.

$$\text{Density } (\rho) = \frac{\text{mass}}{\text{volume}} = \frac{1000}{1} = 1000\text{kg/m}^3$$

22. If we consider the density of a substance in relation to water we get a new value called the relative density (rd) or specific gravity. The calculation below shows the relative density of pure water.

$$\text{relative density} = \frac{\rho_{\text{Substance}}}{\rho_{\text{Water}}}$$

So, for water: $$\text{relative density} = \frac{1000}{1000} = 1$$

<u>Note</u>
Relative density (rd) is a ratio and therefore has no units

23. As we would expect when compared to itself, the value will equal one, however other substances such as aircraft fuel give a value other than one which will affect the conversion calculation.

If a substance has a relative density value of less than 1 it will float in water

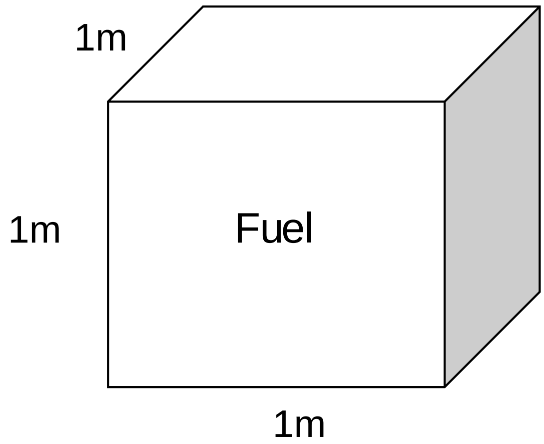
If a substance has a relative density value of greater than 1 it will sink in water

24. If we were to now convert 5000 litres of water (rd = 1) to pounds we use the calculation below

Old unit		Conversion Factor		Relative density		New unit
5000	x	2.2046	x	1	=	11023lb

25. What we can see from this calculation is that when the relative density value is equal to 1 it has no effect in the calculation.

26. If we now consider aircraft fuel (which has a density less than water) we get less mass for a given volume.



27. The mass of $1m^3$ of fuel can be found by experiment and is approximately 800 kg

(more accurately, aircraft fuel can vary between 780 and 820 kg/m^3 depending on temperature)

So,

$$\text{relative density} = \frac{\rho_{\text{Substance}}}{\rho_{\text{Water}}} = \frac{800}{1000} = 0.8$$

28. If we were to now convert 5000 litres of fuel (rd = 0.8) to pounds we use the calculation below

Old unit		Conversion Factor		Relative density		New unit
5000	x	2.2046	x	0.8	=	8818.4lb

29. Thus, we can see that, taking into account the relative density of the fuel has affected the results considerably.

Note: As the rd of aircraft fuel can vary between 0.78 and 0.82, in practice, it will be necessary to ascertain an accurate value, based on the temperature of the fuel, before the calculations are performed, or this could lead to inaccuracy.

Exercise 2b

4. Convert 1000litres of water (rd=1) in to lb.

5. Convert 1000 litres of fuel (rd=0.8) in to lb.

6. Convert 550 lb in to kg.

7. Convert 1250 gallons (UK) of fuel in to litres.

8. Convert 1000 gallons (US) of fuel (rd=0.78) in to kg.

(Answers on page 14)

Answers to exercise 2a and 2b

1 2.45 bar
2 29.008 psi
3 1 atmosphere
4 2204.6 lb
5 1763.68 lb
6 249.48 kg
7 5681.25 litres
8 2952.3 kg

ASSIGNMENT

30. You are detached to a forward operating base with other NATO allies. You will be required to assist with the refuelling and servicing of various aircraft. Operating in field conditions will also require the replenishment and daily servicing of vehicles and ground equipment. A variety of fuel supplies are used so you may be required to convert from one unit to another. A table of unit equivalents has been provided.

ALL WORKING OUT TO BE SHOWN

1. A 4-tonne truck is found to have an under-inflated tyre; the pressure is stated as 100 psi. The only tyre gauge is calibrated in bar. What will the tyre pressure be in bar?

2. A Harrier has a hydraulic problem. Number 1 system is developing 180 bar instead of the correct pressure of 230 bar. Calculate the pressure difference in psi.

3. An American helicopter arrives, the pilot requests refuelling. The fuel load is 1,500 US gallons. The fuel bowser arrives but has its instruments calibrated in litres. How many litres are required? The helicopter crewman requires the weight of fuel in pounds so that he can calculate the aircraft weight. What is the fuel load in pounds?

4. The American generators and vehicles at the location require refuelling daily. The total amount of diesel required is estimated as 350 gallons (US) a day. Diesel is supplied in 20 litre jerricans-how many will be required?

5. The RAF vehicles and generators also require daily refuelling, it is estimated that 2,200 litres of diesel will be required. These will be supplied in 20 litre jerricans at the same time as the fuel for the US equipment. The delivery trucks can carry 5000 kg. Assuming an empty jerrican weighs 2 kg; calculate the total weight of diesel required and how many trucks will be needed to supply the diesel each day?