Defence School of
Aeronautical Engineering

Aerosystems Engineer & Management Training School

Academic Principles Organisation

MATHEMATICS

BOOK 8

Proportionality

KEY LEARNING POINTS

KLP	Description
MA1.13	Distinguish between direct and inverse proportion
MA1.14	Calculate the constant of proportionality for arithmetical expressions

PROPORTIONALITY

1. Often, we can identify situations where there is a linear relationship between two variables. This means that if we increase one of the variables by a factor of, say 2 or 3 then there will be a corresponding increase in the other variable by the same amount.

2. An example of this would be when a liquid is made to flow through a pipe; the rate of flow of the liquid is related to the pressure generated by the pump. There is a linear relationship between the flow rate and the pressure; we say that the flow is directly proportional to the pressure.

The symbol for "proportional to" is "$\propto$"

3. In this case, if the rate of flow is represented by *F*, and the pressure by *P*, we can say:

$$F \propto P$$

4. At the moment there is little that can be done with this statement, but by introducing a multiplying factor, k it can be turned into an equation, as so:

$$F = k\,P$$

5. If the relationship between *F* and *P* is linear, then the value of k is constant for all values of *F* and *P*; it is known as the *constant of proportionality.* If this value can be found, it can then be used to find unknown values of *F* and *P* as the following example shows:

Example 1

6. Consider the example above, where there is a linear relationship between the pressure and the flow rate of a liquid. When the pressure is equal to 1.8 bar the flow rate is 4.5 litres per minute. What will the flow rate be if the pressure is increased to 2.8 bar?

$$F \propto P$$

7. From the information given, we know that when *F*= 4.5, *P* =1.8 so,

$4.5 = k \times 1.8$ and so,

$$k = \frac{4.5}{1.8} = 2.5$$

8. Now we have found the value of k, we can use it to subsequent unknown values of *F* or *P*, e.g. when *P* = 2.8,

$$F = 2.5 \times P = 2.5 \times 2.8 = 7$$

9. So, the new flow rate is 7 litres per min.

10. Note that when carrying the above calculation, we have made 2 "modelling" assumptions, the first being that there is a linear relationship between the 2 variables (i.e. a graph drawn of one variable against the other would be a straight line which passes through the origin) and secondly that the linearity holds when the relationship is extrapolated.

11. In practice this is unlikely to be exactly correct, but for most purposes, it would be reasonable to assume that the model will be accurate enough for our needs.

12. Sometimes one of the variables is proportional to some power of the other.

13. For example, the drag force, D on an object may be considered to be proportional to the square of its speed through a fluid, S^2.

i.e. $D \propto S^2$

14. If the drag force is 18N when the speed is 3m/s, at what speed does the drag force reach 32N?

$$D \propto S^2$$

$$\therefore \quad D = k\,S^2$$

So $k = \dfrac{D}{S^2} = \dfrac{18}{3^2} = 2$

15. In the second case,

$$D = 2 \times S^2$$

So $32 = 2 \times S^2$

And $S = \sqrt{\dfrac{32}{2}} = 4$

16. So, the drag force reaches 32N at 4m/s

Exercise 1

1. If y is proportional to $\sqrt{x}$ and $x = 5$ when $y = 9$, find a constant of proportionality and use it to find the value of x when $y = 11$

2. If y is proportional to x^3 and $x = 6$ when $y = 9$, find a constant of proportionality and use it to find the value of x when $y = 10$

3. Given that $m \propto \sqrt[3]{n}$ and $m = 15$ when $n = 3$, find a constant of proportionality and determine the value of n when $m = 18$

INVERSE PROPORTIONALITY

17. Sometimes variables are related such that an increase in one causes a decrease in the other. An example of this would be current, I and resistance, R in an electrical circuit where an increase in resistance causes a proportional decrease in current. This is called *inverse proportionality.* Another way of saying this is to say that one variable is proportional to the reciprocal of the other and is written as so:

$$I \propto \frac{1}{R}$$

18. If, for example we know that a current of 5A is produced when the resistance is 2 Ω, we could use this relationship to find what the current would be if the resistance was changed to say, 10 Ω

$$I \propto \frac{1}{R}$$

Introducing k as before, $I = \frac{k}{R}$

So $k = IR = 5 \times 2 = 10$

19. Using this value of k in the second case:

$$I = \frac{k}{R} = \frac{10}{10} = 1$$

20. So, the answer to the question is that when the resistance is increased to 10 Ω the current drops to 1A

Exercise 2

1. Given that y is inversely proportional to x^2 and y = 10 when x = 6, find a constant of proportionality and use it to find the value of x when y = 12

2. Given that y is inversely proportional to x^3 and y = 54 when x = 3, find a constant of proportionality and use it to find the value of y when x = 16

3. Given that $y \propto \frac{1}{\sqrt{x}}$ and y = 7 when x = 2, find a constant of proportionality and use it to find the value of y when x = 4

Answers

Exercise 1

1.	k = 4.025	x = 7.469
2.	k = 0.04167	x = 6.214
3.	k = 10.4	n = 5.185

Exercise 2

1.	k = 360	x = 5.477
2.	k = 1458	y = 0.3560
3.	k = 9.899	y = 4.950