Defence School of
Aeronautical Engineering

# Aerosystems Engineer & Management Training School

# Academic Principles Organisation

# Mathematics

# BOOK 4
# HCF, LCM & Factorisation

WARNING

These course notes are produced solely for the purpose of training. They are not subject to formal amendment action after issue and they must NOT be used for operating and maintaining the equipment described. Operation and maintenance of equipment is governed by the limitations and procedures laid down in the authorized publications and manuals which must be complied with for these purposes.

# KEY LEARNING POINTS

| KLP | Description |
|---|---|
| MA1.15 | Calculate the HCF and LCM of numerical expressions |
| MA3.1 | Calculate the HCF and LCM of algebraic expressions |

## PART 1 - HCF AND LCM OF NUMERICAL EXPRESSIONS

1. The highest common factor (HCF) of a set of numbers is the largest number that will divide exactly into each element of the set.

The lowest common multiple (LCM) of a set of numbers is the smallest number that each element will divide into exactly, without leaving a remainder.

With small numbers it is usually possible to find the HCF and LCM by inspection, but with larger numbers a method will be needed.

2. First of all, consider the Prime numbers: (numbers with no proper divisors)

2, 3, 5, 7, 11, 13, 17, 19, 23 …etc

A number is either prime or composite, if it is composite, for this procedure, it is necessary to express it as a product of powers of prime factors.

For example:

The number 6 is composite and can be written as 3x2

The number 9 is composite and can be written as $3^2$

Likewise:

$30 = 2 \times 3 \times 5$

$25 = 5^2$

For larger numbers, try dividing the number by each of the prime numbers in turn until only primes remain, e.g.

$924 = 2 \times 462$

$= 2^2 \times 231$

$= 2^2 \times 3 \times 77$

$= 2^2 \times 3 \times 7 \times 11$ which are all prime.

$$
\begin{aligned}
240 &= 2 \times 120 \\
&= 2^2 \times 60 \\
&= 2^3 \times 30 \\
&= 2^4 \times 15 \\
&= 2^4 \times 3 \times 5 \qquad \text{which are all prime.}
\end{aligned}
$$

## Exercise 1

Express each of the following numbers as a product of powers of prime factors.

1. 12
2. 28
3. 30
4. 75
5. 39
6. 76
7. 105
8. 125
9. 63
10. 126

*Answers on page 11*

## Highest Common Factor

3. Once the numbers have been written as products of the prime factors, the HCF of two or more numbers is the product of the prime factors that are common to both numbers

For example:

Consider the set of numbers 36 and 90

First express as products of powers of prime factors

$$2^2 \times 3^2 \quad \text{and} \quad 2 \times 3^2 \times 5$$

Pick out the prime factors that are common to both, choosing the highest common order.

$$2 \text{ and } 3^2$$

Here the highest common order of 2 is $2^1$, as $2^2$ is not common to 90

The HCF is the product of these prime factors

$$2 \times 3^2 = 18$$

As a check, note that 18 goes into 36 twice and 18 goes into 90 five times, there is no number larger than 18 that divides both 36 and 90, it is therefore the highest common factor (HCF).

If there are no common factors the numbers are co-prime and the HCF is 1

## Example 1

Find the HCF of the numbers 28, 42, and 84.

$28 = 2^2 \times 7$
$42 = 2 \times 3 \times 7$
$84 = 2^2 \times 3 \times 7$

$\therefore$ HCF = 2 x 7 = 14

Here the highest common order of 2 is $2^1$, as $2^2$ is not common to 42

## Exercise 2

Find the HCFs of the following sets of numbers:

1. 6, 8
2. 12, 28
3. 18, 39
4. 8, 4, 26
5. 30, 75, 105
6. 12, 28, 44
7. 7, 14, 28, 63
8. 24, 36, 60,114,
9. 7, 23, 79, 241
10. 33, 55, 110, 242

*Answers on page 11*

## Lowest Common Multiple

4. The LCM of a set of numbers is the smallest number that all members of the set will divide into. First express the numbers as prime factors as before, then choose the *highest* order of each factor that are in *any* of the numbers.

For example:

Consider the set of numbers 36 and 90

First express as products of powers of prime factors

$2^2 \times 3^2$ and $2 \times 3^2 \times 5$

Pick out the prime numbers that are in either, choosing the highest order

$2^2$ and $3^2$ and 5

The LCM is the product of these prime factors

$2^2 \times 3^2 \times 5 = 180$

As a check, note that 36 goes into 180 five times and 90 goes into 180 twice, there is no number smaller than 180 that both 36 and 90 divide into, it is therefore the lowest common multiple (LCM).

## Exercise 3

Find the LCMs of the following sets of numbers:

1. 32, 12
2. 35, 70
3. 25, 35
4. 22, 27
5. 42, 63
6. 120, 125
7. 3, 5, 6
8. 4, 10,12,
9. 3, 5, 9
10. 10, 15, 40

*Answers on page 11*

## Exercise 4

Find the HCF and LCM for the following set of numbers:

9, 39, 78, 96, 117

5. Finally, it may be useful to know that, if *A* and *B* are 2 numbers then:

$$LCM\{A,B\} = \frac{A \times B}{HCF\{A,B\}}$$

And:

$$HCF\{A,B\} = \frac{A \times B}{LCM\{A,B\}}$$

This means that for pairs of numbers, if *either* the LCM or HCF is known, then the other can be calculated easily using the above formula.

## Exercise 5

Find the HCF and LCM of the following pairs of numbers

1. 9,3
2. 12,18
3. 45,60
4. 40,48

*Answers on page 11*

## PART 2 - HCF AND LCM OF ALGEBRAIC TERMS AND EXPRESSIONS

6. The techniques explained above for finding the HCF and LCM of numerical terms and expressions can be generalised to algebraic terms and expressions.

Consider the following set of terms:

$$ab, a^2c, a^3d$$

The individual variables $a, b, c$ etc. cannot be split further and so must be treated as prime numbers.

This means that, for the above set:

HCF $\{ab, a^2c, a^3d\}$ = $a$

And

LCM $\{ab, a^2c, a^3d\}$ = $a^3bcd$

*Note:* If the set includes any polynomial terms, (i.e. with more than one term) such as $(x-y)$ then this too cannot be expressed as a product of any smaller variables and must be treated as prime.

## Example 2

Find the HCF and LCM of the following:

$$\frac{3xyz^2}{w^2}, \frac{6xy^2z}{w^3}, \frac{9x^2yz}{w^4}$$

HCF = $\frac{3xyz}{w^2}$

LCM = $\frac{18x^2y^2z^2}{w^4}$

## Exercise 6

Find the HCF of the following sets of algebraic terms:

1. $p^3q^2, \ p^2q^3, \ p^2q$
2. $m^2n^3p^3, \ m^3n^3, \ mn^2p^2$
3. $3ab^2, \ 6abc, \ 12a^2bc^2$
4. $\frac{ac}{x}, \ \frac{bc}{x^2}, \ \frac{cd}{x^3}$
5. $5x^2yz, \ 15x^2yz, \ 30xy^2z^3, \ 5xyz^2,$

*Answers on page 11*

Factorising an expression means finding the HCF and placing it outside of a bracket with the quotients inside the bracket, such that multiplying out the bracket will result in the original expression. Note that factorising an expression does not change its value and there must be the same number of terms inside the bracket as in the original expression.

For example:

Factorise:

1. $5ab+15a^2c \quad = \quad 5a(b+3ac)$
2. $3a+9a^2-27a^3 \quad = \quad 3a(1+3a-9a^2)$ etc.

## Exercise 7

Factorise (as far as possible) the following expressions:

1. $3y - 9y^2$
2. $ax^2 + ax$
3. $ax + bx + cx$
4. $I_0 + I_0\alpha t$
5. $\frac{a}{3} - \frac{b}{6} + \frac{c}{9}$
6. $2m^2 - 3mn + b^3$
7. $\frac{a^2b^2}{15} - \frac{a^2b}{20} + \frac{a^2b^2}{10}$
8. $5a$-$10b$+$15c$
9. $mx$-$my$
10. $pb$+$2pc$
11. $ab^3$-$a^2b$
12. $5x^3$-$10x^2y$+$15xy^2$
13. $2q^2$-$8qn$
14. $2a^2$-$3ab$+$b^2$
15. $x^3y$-$x^2$+$5x$
16. $ab^3$-$a^2b$
17. $2\pi r^2 + \pi rh$
18. $2xy^2$+$6x^2y$+$8x^3y$
19. $21a^2b^2$-$28ab$
20. $\frac{\beta^2}{\delta\varepsilon} - \frac{\beta^3}{\delta\varepsilon^2} + \frac{\beta^4}{\delta\,\varepsilon^2}$

21. $\frac{m^2}{pn}$ - $\frac{m^3}{pn^2}$ + $\frac{m^4}{p^2n^2}$

22. $2x^2+10x^2+15xy^2$

*Answers on page 12*

## Exercise 8

Find the LCM of the following sets of algebraic terms and expressions:

1. A, B
2. A, B, $B^2$
3. 2j, $3j^2$, j, $j^4$
4. $3xy, xyz, 9xz^2$
5. $(x+y),(x-y)$
6. $(a-2),(a+2),(a+2)^2$

*Hint:* see note in para. 6 and questions 1 and 2, this exercise.

When adding or subtracting fractions, it is necessary to write the fraction over the lowest common denominator. The lowest common denominator is, in fact the lowest common multiple of the denominators.
Find a suitable lowest common denominator for the fractions below:

7. $\frac{1}{p}-\frac{1}{q}$
8. $\frac{m}{n}+\frac{p}{q}$
9. $\frac{x}{y}-y$
10. $\frac{1}{ab}+\frac{1}{a}+1$
11. $\frac{8}{(a+3)}+\frac{4}{(a-5)}$
12. $\frac{2}{m}+\frac{4}{m+2}$

*Answers on page 12*

## Answers:

Exercise 1

1. $2^2 \times 3$
2. $2^2 \times 7$
3. $2 \times 3 \times 5$
4. $3 \times 5^2$
5. $3 \times 13$
6. $2^2 \times 19$
7. $3 \times 5 \times 7$
8. $5^3$
9. $3^2 \times 7$
10. $2 \times 3^2 \times 7$

Exercise 2

1. 2
2. 4
3. 3
4. 2
5. 15
6. 4
7. 7
8. 6
9. 1
10. 11

Exercise 3

1. 96
2. 70
3. 175
4. 594
5. 126
6. 3000
7. 30
8. 60
9. 45
10. 120

Exercise 4

HCF = 3
LCM = 3744

Exercise 5

1. HCF = 3 LCM = 9
2. HCF = 6 LCM = 36
3. HCF = 15 LCM = 180
4. HCF = 8 LCM = 240

Exercise 6

1. $p^2q$
2. $mn^2$
3. $3ab$
4. $\frac{c}{x}$
5. $5xyz$

## Exercise 7

1. $3y(1-3y)$
2. $ax(x+1)$
3. $x(a+b+c)$
4. $I_o(1+\alpha t)$
5. $\frac{1}{3}\left(a-\frac{b}{2}+\frac{c}{3}\right)$
6. $m(2m-3n)+b^3$
7. $\frac{a^2b}{5}\left(\frac{b}{3}-\frac{1}{4}+\frac{b}{2}\right)$
8. $5(a-2b+3c)$
9. $m(x-y)$
10. $p(b+2c)$
11. $ab(b^2-a)$
12. $5x(x^2-2xy+3y^2)$
13. $2q(q-4n)$
14. $a(2a-3b)+b^2$
15. $x(x^2y-x+5)$
16. $ab(b^2-a)$
17. $\pi r(2r+h)$
18. $2xy(y+3x+4x^2)$
19. $7ab(3ab-4)$
20. $\frac{\beta^2}{\delta\varepsilon}\left(1-\frac{\beta}{\varepsilon}+\frac{\beta^2}{\varepsilon}\right)$
21. $\frac{m^2}{pn}\left(1-\frac{m}{n}+\frac{m^2}{pn}\right)$
22. $x(2x+10x+15y^2)$

## Exercise 8

1. AB
2. $AB^2$
3. $6j^4$
4. $9xyz^2$
5. $(x+y)(x-y)$ or $x^2-y^2$
6. $(a-2)(a+2)^2$
7. $pq$
8. $nq$
9. $y$
10. $ab$
11. $(a+3)(a-5)$
12. $m(m+2)$

Notes