Defence School of
Aeronautical Engineering

Aerosystems Engineer & Management Training School

Academic Principles Organisation

MATHEMATICS

BOOK 9

Further Transposition of Formula

WARNING

These course notes are produced solely for the purpose of training. They are not subject to formal amendment action after issue and they must NOT be used for operating and maintaining the equipment described. Operation and maintenance of equipment is governed by the limitations and procedures laid down in the authorized publications and manuals which must be complied with for these purposes

WARNING
INTELLECTUAL PROPERTY RIGHTS

KEY LEARNING POINTS

KLP	Description
MA3.11	Transpose a formula to change the subject.
MA3.12	Use formulae to obtain engineering and scientific data

To recap the subject please follow the worked examples.

Example 1

If A = LB transpose the formula to make L the subject.

Divide both sides by B: $\frac{A}{B} = \frac{L\cancel{B}}{\cancel{B}}$

Giving $L = \frac{A}{B}$

Note $\frac{B}{B} = 1$

Example 2

If $Y = \frac{X}{Z}$ transpose the formula to make X the subject.

Multiply both sides by Z: $YZ = \frac{X\cancel{Z}}{\cancel{Z}}$

Giving $X = YZ$

Example 3

If $a = \frac{b}{c}$ transpose the formula to make c the subject.

Multiply both sides by c: $ac = \frac{b\cancel{c}}{\cancel{c}}$

Divide both sides by a: $\frac{\cancel{a}c}{\cancel{a}} = \frac{b}{a}$

Giving $c = \frac{b}{a}$

Example 4

If y = x + c, transpose this formula to make x the subject.

Subtract c from both sides $y - c = x + c - c$

Giving $x = y - c$

Note $+c - c = 0$ *and* $-c + c = 0$

Example 5

If $p = \frac{q - m}{r}$ transpose the formula to make q the subject.

Multiply both sides by r: $pr = \frac{r(q - m)}{r}$

Add m to both sides: $pr + m = q - m + m$

Giving $q = pr + m$

Example 6

If y – x = m, transpose this formula to make x the subject.

Subtract y from both sides: $y - x - y = m - y$

Multiply both sides by –1: $-x = m - y$

Giving $x = y - m$

Note If, after transposition, the subject is negative it can be made positive by multiplying both sides by –1.

Example 7

If $A = \pi r^2$ transpose this formula to make r the subject.

Divide both sides by π: $\frac{A}{\pi} = \frac{\pi r^2}{\pi}$

Square root both sides: $\sqrt{\frac{A}{\pi}} = \sqrt{r^2}$

Giving $\sqrt{\frac{A}{\pi}} = r$

Example 8

If $A = \pi rl + \pi r^2$ transpose this formula to make l the subject.

Subtract πr^2 from both sides: : $A - \pi r^2 = \pi rl + \pi r^2 - \pi r^2$

Divide both sides by πr: $\frac{A - \pi r^2}{\pi r} = \frac{\pi rl}{\pi r}$

Giving $\frac{A - \pi r^2}{\pi r} = l$

TRANSPOSITION OF FORMULAE (COMMON FACTORS)

1. When the subject exists in **two or more terms** the formula can only be transposed correctly when the **subject is taken out as a common factor**. For example:

a. Transpose the formula bc + c = a to make c the subject.

Take out c as a common factor: $c(b+1) = a$

$$c = \frac{a}{b+1}$$

b. If 2r = pq + rs, make r the subject:

Subtract rs from both sides: $2r - rs = pq$

Take out r as a common factor: $r(2 - s) = pq$

$$r = \frac{pq}{2-s}$$

c. If $X = \frac{ab+c}{a+c}$ make c the subject:

Multiply both sides by (a + c): $X(a + c) = ab + c$

Remove brackets $aX + cX = ab + c$

Collect terms containing c onto one side $cX - c = ab - aX$

$c(X - 1) = ab - aX$

$$c = \frac{ab - aX}{x - 1}$$

$$c = \frac{a(b - X)}{X - 1}$$

Exercise 1

Transpose the following formulae to make the letter in brackets the subject:

1. $XY + Y = 7$ (X)

2. $ab - b = c$ (b)

3. $p = st - pq$ (p)

4. $X = \frac{Y+3}{Y}$ (Y)

5. $\frac{a-b}{a} = c$ (a)

6. $d = \frac{t-u}{u}$ (u)

7. $\frac{1}{a} = \frac{2}{b} + \frac{3}{c}$ (b)

8. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ (R_1)

9. $A_F = \frac{A_0}{1+A_0\beta}$ (A_0)

10. $f = f_0\left(\frac{c+v}{c-u}\right)$ (c)

11. $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ (u)

TRANSPOSITION OF FORMULAE (LONGER EQUATIONS)

2. In previous periods, you were shown some of the basic types of situations that can arise when changing the subject of a formula. These basic types are often combined into a single problem. Such problems can be treated in a variety of ways, but, if you are in any doubt at all, the following sequence should be followed:

a. **First**. Remove root signs.

b. **Second**. Remove fractions.

c. **Third**. Remove brackets.

These three steps may be carried out in a different order for certain problems, but all roots, fractions and brackets should be removed before carrying out the fourth step.

d. **Fourth**. Re-arrange formula, collecting all terms containing the required letters on one side of the equation and **all** other terms on the other side.

e. **Fifth**. Take the subject out as a **common factor**.

f. **Sixth**. **Divide** through by the coefficient of the subject.

g. **Seventh**. Take roots (if necessary).

3. If may not always be necessary to use all the above steps, but nevertheless, the sequence should be followed. For example:

a. Transpose $V^2 = u^2 + 2fs$ to make u the subject.

There are no root signs, fractions or brackets, therefore re-arranging:

$u^2 = V^2 - 2fs$

There are no common factors or coefficients, therefore take roots:

$u = \sqrt{V^2 - 2fs}$

Note: This does not equal $u = V - \sqrt{2fs}$

b. Transpose $T = 2\pi\sqrt{\frac{l}{g}}$, to make g the subject:

Square both sides: $T^2 = (2\pi)^2\left(\sqrt{\frac{l}{g}}\right)^2$

$= 4\pi^2\left(\sqrt{\frac{l}{g}}\right)^2$

$= 4\pi^2\frac{l}{g}$

Multiply through by g: $T^2g = 4\pi^2 l$

Divide through by coefficient T^2: $g = \frac{4\pi^2 l}{T^2}$

Exercise 2

Transpose the following formula to make the letter in brackets, the subject:
(You may assume that all variables can only take positive values)

1. $v^2 = 2gh$ (v)

2. $C = \frac{5}{9}(F - 32)$ (F)

3. $I = \frac{E}{r + R}$ (R)

4. $v = \sqrt{2gh}$ (h)

5. $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$ (C)

6. $\frac{1}{u} + \frac{1}{v} = \frac{1}{j}$ (u)

7. $T = 2\pi\sqrt{\frac{\ell}{g}}$ (ℓ)

8. $S = ut + \frac{1}{2}at^2$ (a)

9. $X = \sqrt{\frac{s - a}{s - b}}$ (s)

10. $I = \frac{PTR}{100}$ (R)

11. $V = I\sqrt{r^2 + X_L}$ (r)

12. $t = 2\pi\sqrt{\frac{h^2 + g^2}{h}}$ (g)

13. $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$ (R)

Exercise 3

Using the formula and the data given, find the required quantity below:
Give answers in engineering form.

1. if $v^2 = u^2 + 2as$ Find s, given

   $u = 20\text{m/s}$
   $a = 30\text{m/s}^2$
   $v = 32\text{m/s}$

2. if $s = ut + \frac{1}{2}at^2$ Find a, given

   $u = 17\text{m/s}$
   $t = 20\text{s}$
   $s = 1250\text{m}$

3. if $X_L = 2\pi fL$ Find f, given

   $L = 10 \times 10^{-3} H$
   $X_L = 1.88 \times 10^6 \Omega$

4. if $X_c = \frac{1}{2\pi fC}$ Find C, given

   $f = 50\text{Hz}$
   $X_C = 145\Omega$

5. if $L = \frac{1}{2}\rho v^2 S C_L$ Find C_L, given

   $L = 737.28\text{kN}$
   $\rho = 0.9\text{kg/m}^3$
   $v = 200\text{m/s}$
   $S = 51.2\text{m}^2$

6. if $D = \frac{1}{2}\rho v^2 S C_D$ Find v, given

   $D = 4896\text{N}$
   $\rho = 0.85\text{kg/m}^3$
   $S = 51.2\text{m}^2$
   $C_D = 0.01$

7. if $KE = \frac{1}{2}mv^2$ Find v, given

   $KE = 781.25\text{kJ}$
   $m = 2.5\text{tonne}$

8. if $f_0 = \frac{1}{2\pi\sqrt{LC}}$ Find L, given

$C = 470 \times 10^{-6}\text{F}$

$f_0 = 50\text{Hz}$

9. if $Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}$ Find C, given

$R = 220\Omega$

$L = 50 \times 10^{-3} H$

$Q_0 = 2.2\text{C}$

10. if $V_1 = \frac{V_2 T_1}{T_2}$ Find V_2, given

$T_1 = 524\text{K}$

$T_2 = 290\text{K}$

$V_1 = 0.75\text{m}^3$

Answers

Exercise 1

1. $X = \frac{7}{y} - 1$ or $X = \frac{7-y}{y}$

2. $b = \frac{c}{a-1}$

3. $p = \frac{st}{1+q}$

4. $Y = \frac{3}{X-1}$

5. $a = \frac{b}{1-c}$

6. $u = \frac{t}{d+1}$

7. $b = \frac{2ac}{c-3a}$

8. $R_1 = \frac{RR_2}{R_2 - R}$

9. $A_0 = \frac{A_F}{1 - A_F\beta}$

10. $c = \frac{f_0 v + fu}{f - f_0}$

11. $u = \frac{fv}{v-f}$

Exercise 2

1. $v = \sqrt{2gh}$

2. $F = \frac{9}{5}c + 32$

3. $R = \frac{E - Ir}{I} = \frac{E}{I} - r$

4. $h = \frac{v^2}{2g}$

5. $C = \frac{L}{Q^2 R^2}$

6. $u = \frac{jv}{v-j}$

7. $\ell = \frac{T^2 g}{4\pi^2}$

8. $a = \frac{2(s-ut)}{t^2}$

9. $s = \frac{X^2 b - a}{X^2 - 1}$

10. $R = \frac{100I}{PT}$

11. $r = \sqrt{\frac{V^2}{I^2} - X_L}$

12. $g = \sqrt{\left(\frac{t}{2\pi}\right)^2 h - h^2}$

13. $R = \sqrt{\frac{L}{C} - 4\pi^2 f^2 L^2}$

Exercise 3

1. 10.4m
2. 4.55m/s^2
3. 29.9 x 10^{6}Hz
4. 22 x 10^{-6} F
5. 0.8
6. 150m/s
7. 25m/s
8. 21.6 x 10^{-3}H
9. 213 x 10^{-9}F
10. 415 x 10^{-3}m^3

Notes