Defence School of
Aeronautical Engineering

Aerosystems Engineer & Management Training School

Academic Principles Organisation

Science

BOOK 2

Statics

WARNING

These course notes are produced solely for the purpose of training. They are not subject to formal amendment action after issue and they must NOT be used for operating and maintaining the equipment described. Operation and maintenance of equipment is governed by the limitations and procedures laid down in the authorized publications and manuals which must be complied with for these purposes

ENABLING OBJECTIVE

SC2	Explain the basic principle of ‘statics’ in relation to aircraft loading buoyancy and hydrostatic pressure.

KEY LEARNING POINTS

SC2.1	Identify forces represented graphically as vectors.
SC2.2	Explain the concept of equilibrium
SC2.3	Solve problems graphically using the parallelogram of forces theorem.
SC2.4	Solve problems graphically using the triangle of forces theorem.
SC2.5	Define the moment of a force about a point.
SC2.6	Define centre of gravity.
SC2.7	Solve problems involving straight levers, bell cranks and aircraft loading.
SC2.8	Define pressure and its units.
SC2.9	Solve problems using the basic gas laws.
SC2.10	Explain the difference between gauge pressure and absolute pressure.
SC2.11	Solve problems involving atmospheric, gauge and absolute pressure.
SC2.12	Explain density and relative density.
SC2.13	Calculate pressures in liquids using basic physical measurement.

Exam
1 Question from SC2.1 to SC2.3
1 Question from SC2.4 to SC2.5
1 Question from SC2.6 to SC2.7
1 Question from SC2.8 to SC2.9
1 Question from SC2.10 to SC2.11
1 Question from SC2.12 to SC2.13

SC2.1 – IDENTIFY FORCES REPRESENTED GRAPHICALLY AS VECTORS

Mass, weight and the acceleration due to gravity

1. The mass of a body is the amount of matter contained in the body. The SI unit of mass is the kilogram (kg).

2. The weight of a body is the force with which it is attracted to the centre of the earth by gravity. The SI unit of weight is the Newton (N).

3. The acceleration due to gravity (g) is the acceleration with which a free-falling body will travel towards the centre of the earth. The SI unit of acceleration is metres per second squared (m/s^2). The value of g is normally 9.81 m/s^2.

4. Relationship between mass and weight. $W = mxg$

Where: W = weight in Newtons (N)
m = mass in kg
g = 9.81 m/s^2

Example:

5. What is the weight of a block of wood of mass 25 kg?

$$\text{Weight} = mg = 25 \times 9.81 \text{ N} = 245.25 \text{ N}$$

Scalar and vector quantities

6. A scalar quantity has magnitude (size) only. Examples are mass (kg), time (s), volume (m^3) and temperature (K).

7. A vector quantity has magnitude and direction. Examples are force (N), weight (N), velocity (m/s), acceleration (m/s^2) and displacement (m).

8. A force is defined as a push or pull, which changes or tends to change the state of rest of a body or its uniform motion along a straight line or to a curved path.

9. Force is a vector quantity. The effect of a force depends on the magnitude of the force, the direction and the point of application on a physical body.

10. The unit of force in the SI system is the Newton (N). In the Imperial system the unit of force is the pound force (lbf).

11. Forces, which are vector quantities, have the following characteristics:

   a. Magnitude.
   b. Direction.
   c. Point of application.

Representation of a force as a vector.

A 70 N force acting at 30 ^{0} above the horizontal going to the right can be represented by:

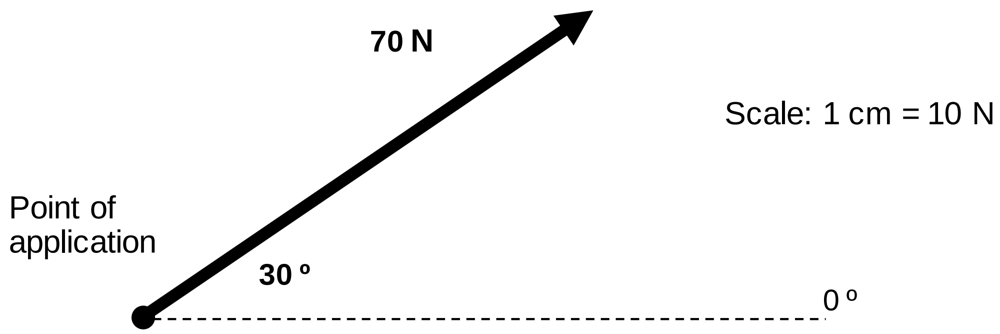


Figure 1 - Representation of force as a vector

Note: The angle is always measured from the positive x-axis.

SC2.2 – EXPLAIN THE CONCEPT OF EQUILIBRIUM

Equilibrium

12. Definition of Equilibrium:

    a. Equilibrium is defined as a body at rest or constant velocity where all the forces acting on it are balanced out and that the turning effects of those forces are in balance.

    b. A body is certain to be in equilibrium if both the forces and moments acting on it are in balance.

13. Static equilibrium:

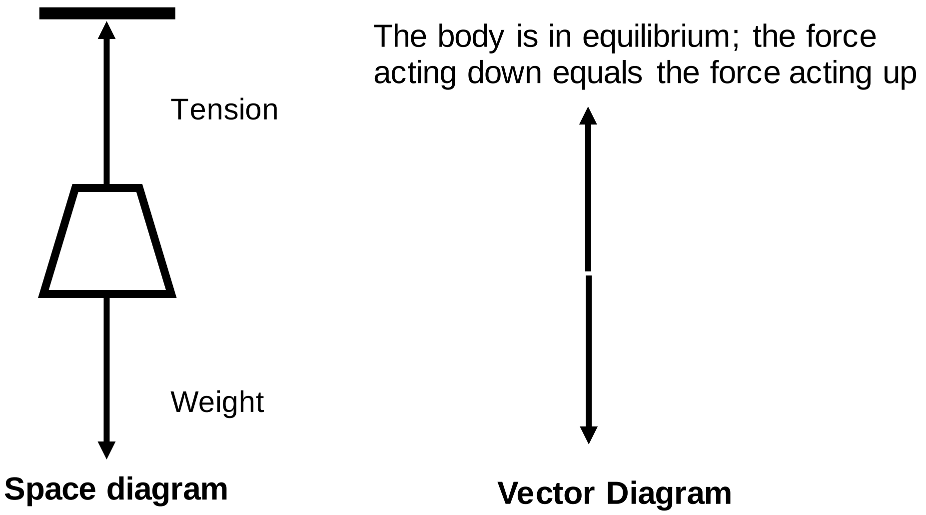
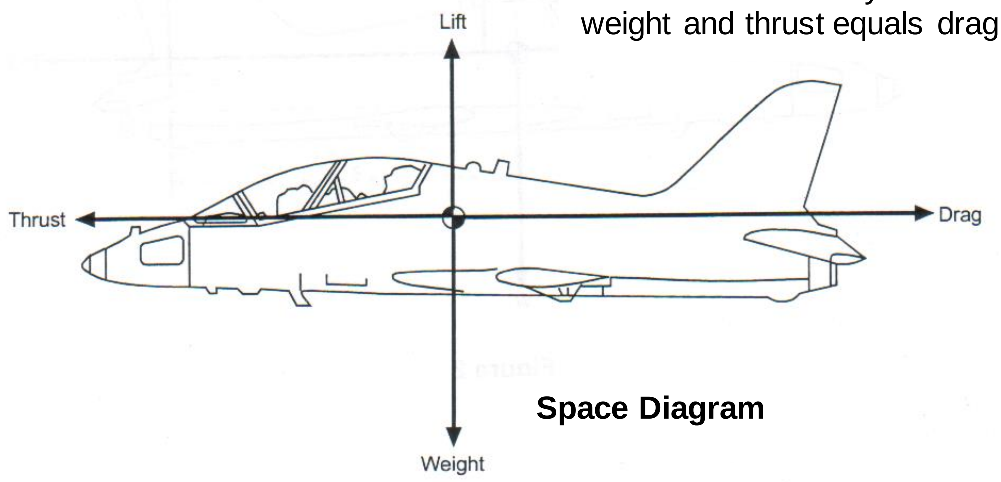


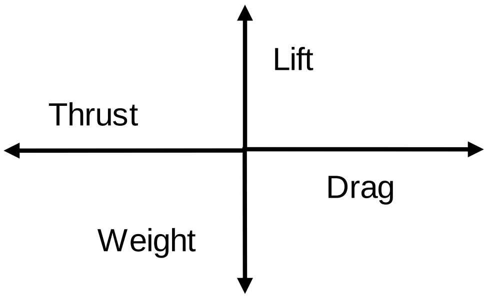
Figure 2 - Static equilibrium

14. Uniform Motion Equilibrium:

The aircraft is in equilibrium travelling with constant velocity when lift equals weight and thrust equals drag.



Space Diagram



Vector Diagram

Figure 3 - Coplanar equilibrium

SC2.3 – SOLVE PROBLEMS GRAPHICALLY USING THE PARALLELOGRAM OF FORCES THEOREM

Addition of co-linear forces

15. When forces act along the same line they can be added like numbers.

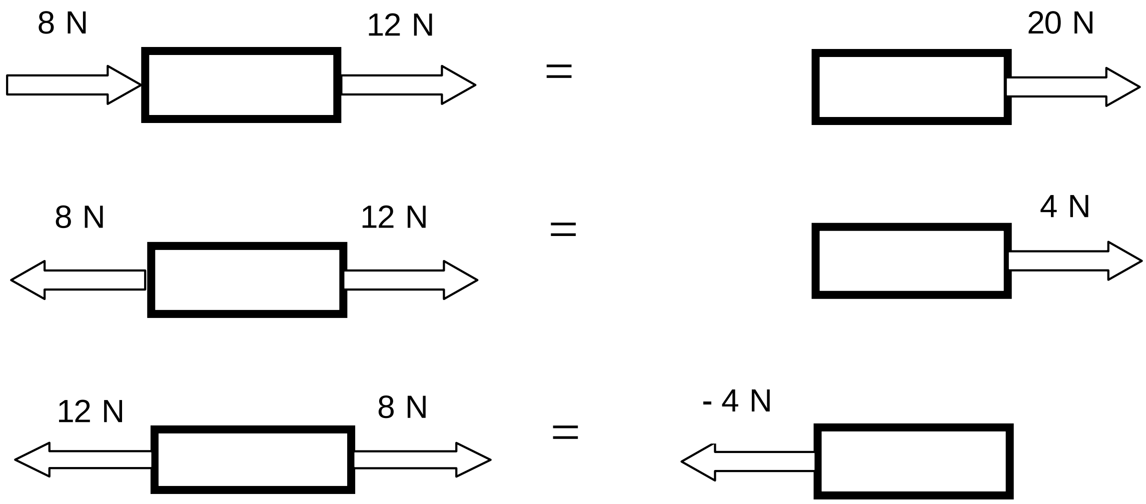


Figure 4 - Addition of co-linear forces

Coplanar Forces

16. Forces acting in the same plane are described as coplanar.

17. The resultant of a system of forces is the one force that has the same effect as the system of forces

18. Coplanar forces can be added together graphically by the use of a scale drawing.

Example:

19. Two forces at 90 º

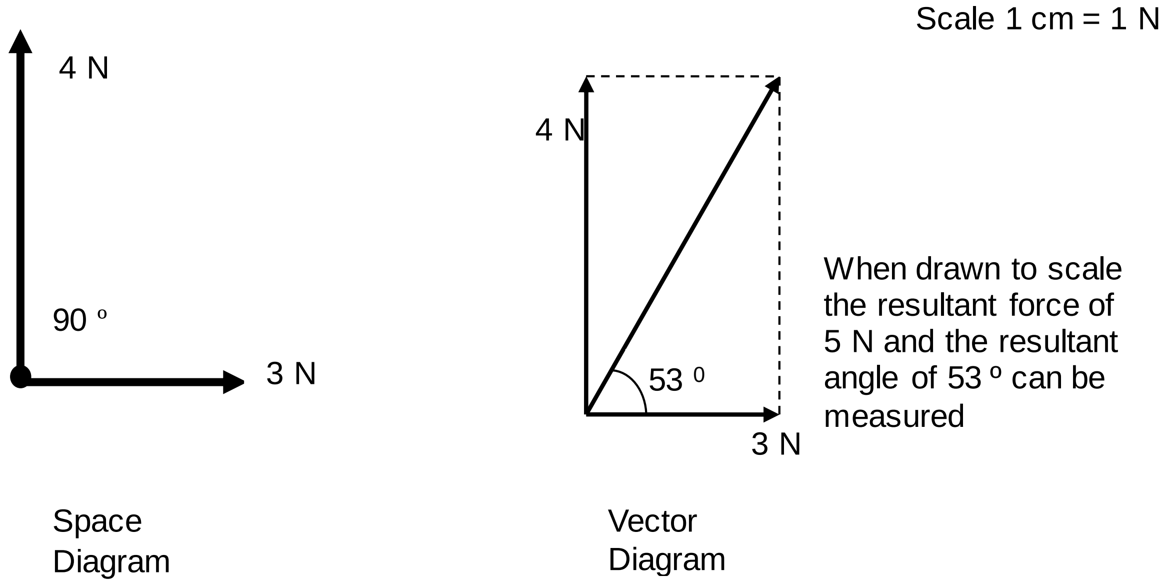


Figure 5 - Forces at right angles

20. The accuracy is dependant upon the accuracy of the vector diagram. Rulers measured correct to the nearest 1 mm, protractors to the nearest 1º.

Examples:

21. Two forces at any angle can be added graphically using the parallelogram of forces method, drawn to scale.

a.

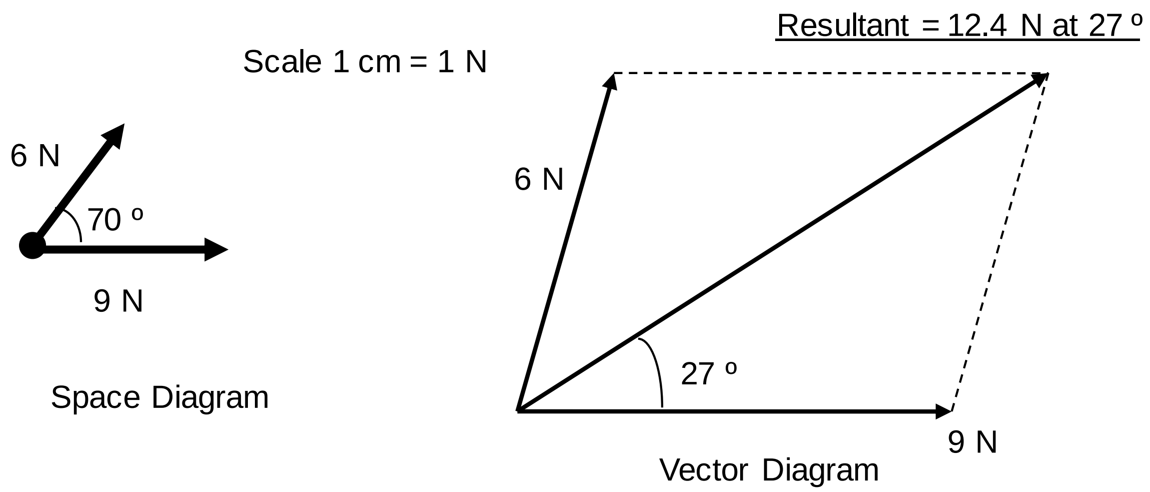


Figure 6 - General Coplanar forces

b.

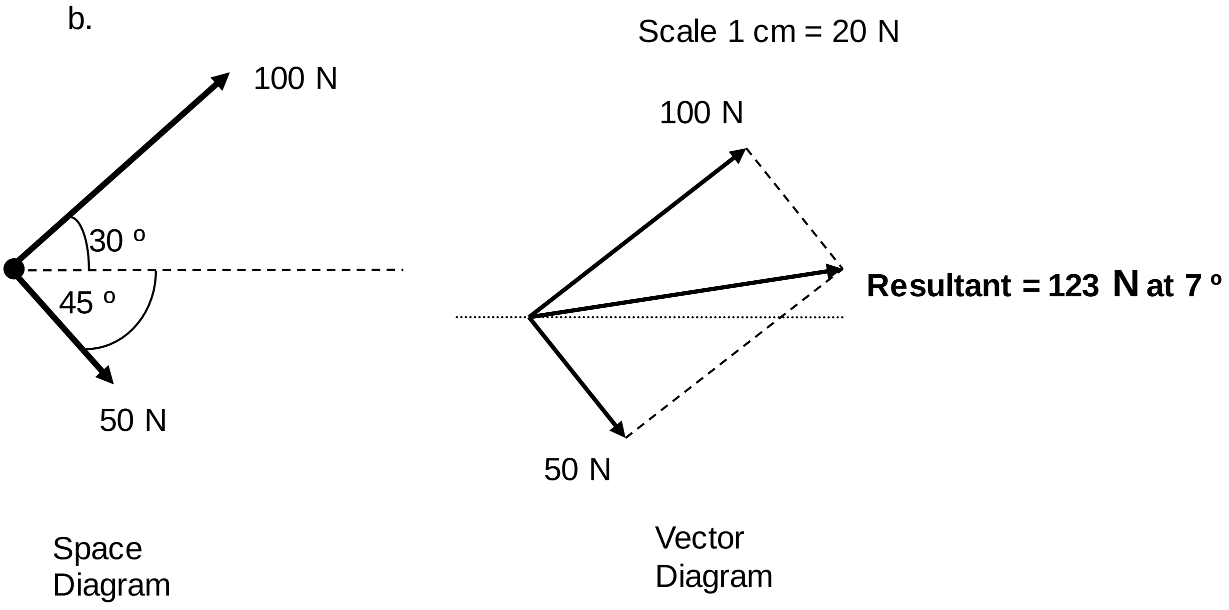


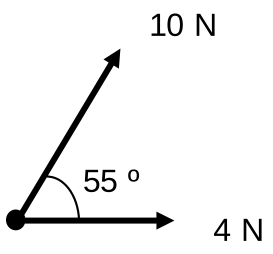
Figure 7 - Parallelogram of forces

Exercise 1

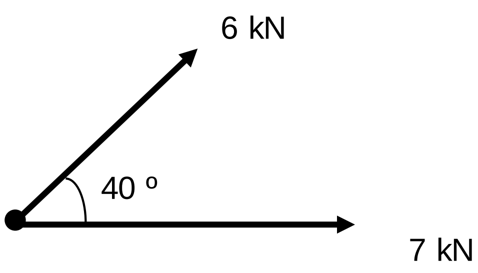
Answers can be expressed as positive or negative from the positive "*x*" axis

1. Add the following forces using the parallelogram of forces.

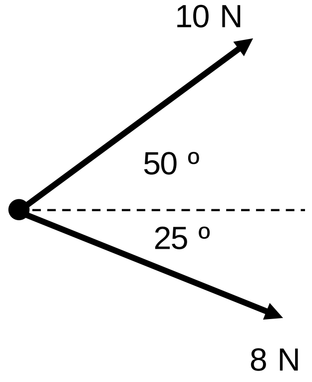
a.



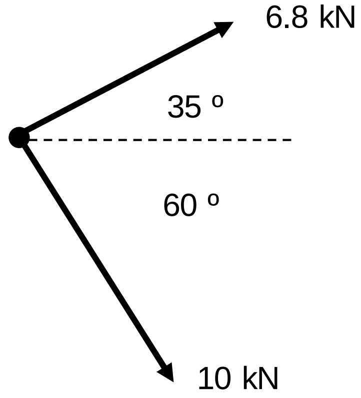
b.



c



d.



Equilibrant

23. The equilibrant of a system of forces is defined as the force that is equal in magnitude but opposite in direction to the resultant force.

24. Given a system of forces, the equilibrant can be found graphically from the resultant.

Example:

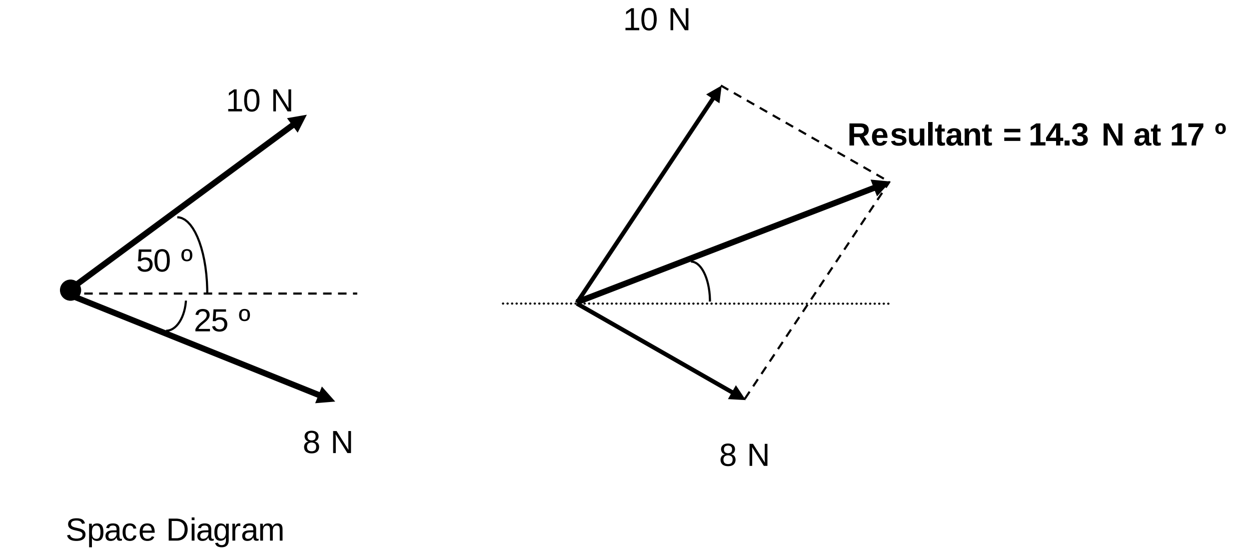


Figure 8(a) - Resultant construction.

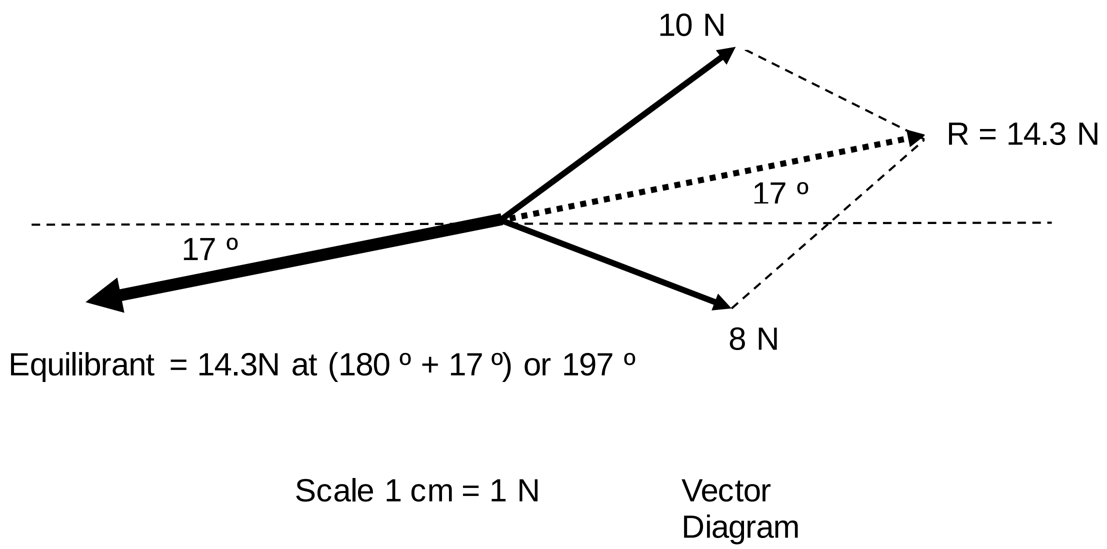


Figure 8(b) - Equilibrant determination.

25. Alternatively, to find the equilibrant of the two forces shown in the space diagram below, the equilibrant force is the one that produces equilibrium (or balance). If the forces are drawn to scale in their directions, the equilibrant force is the one that closes the triangle and it is in the opposite direction to the resultant.

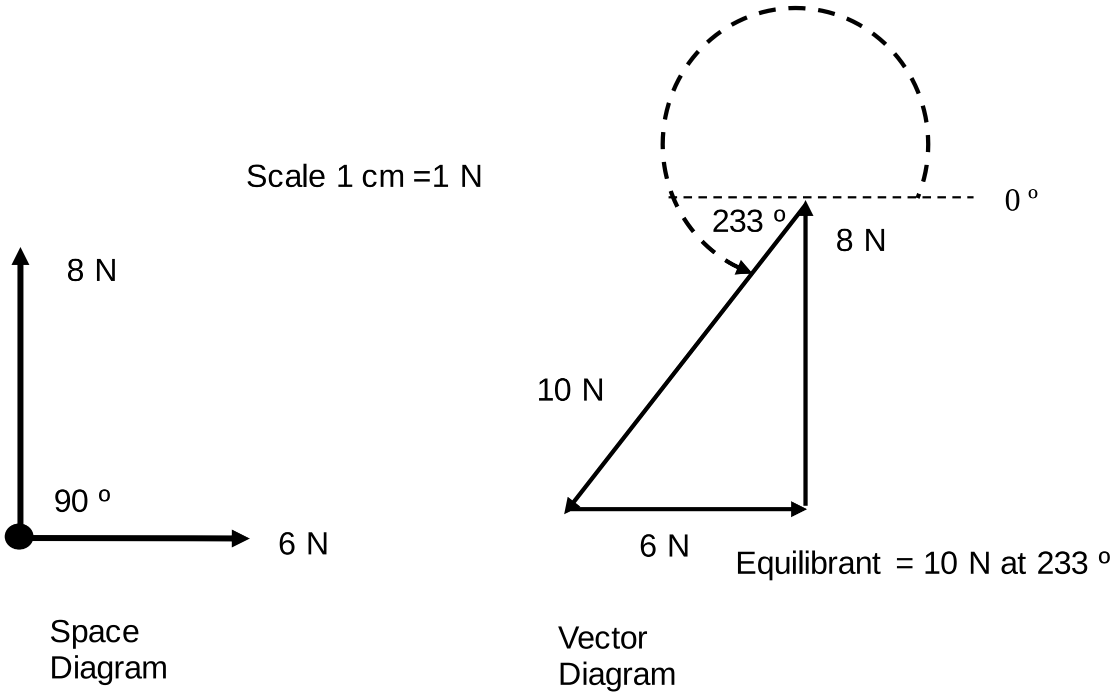
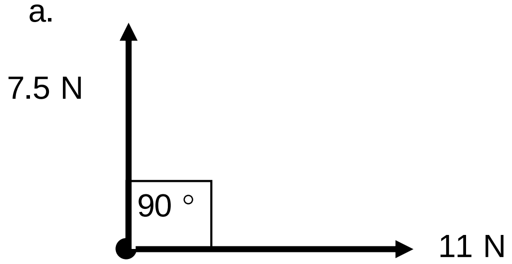


Figure 9 - Equilibrant determination by drawing.

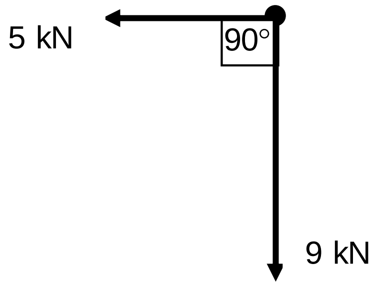
Exercise 2

1. Find the equilibrant of the following force system graphically.

a.



b.



SC2.4 – SOLVE PROBLEMS GRAPHICALLY USING THE TRIANGLE OF FORCES THEOREM

Triangle of forces

26. The triangle of forces theorem states that if three coplanar forces acting at a point are in equilibrium, they can be represented in magnitude and direction by the sides of a triangle.

27. Consider the pin-jointed structure below with two unknown forces.

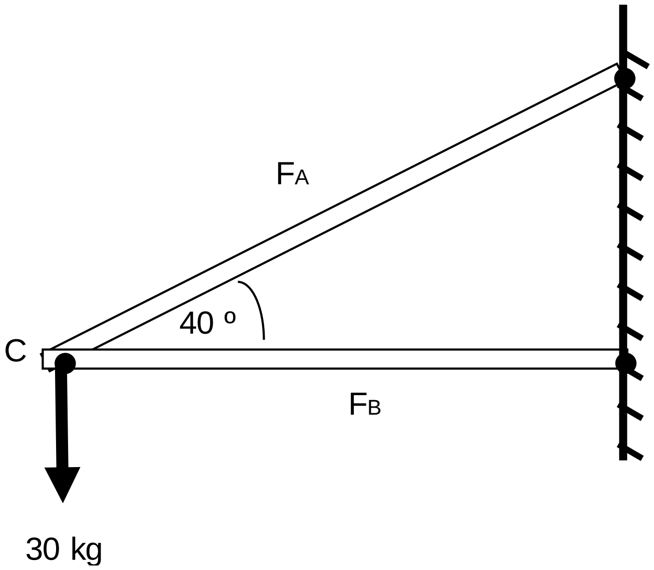
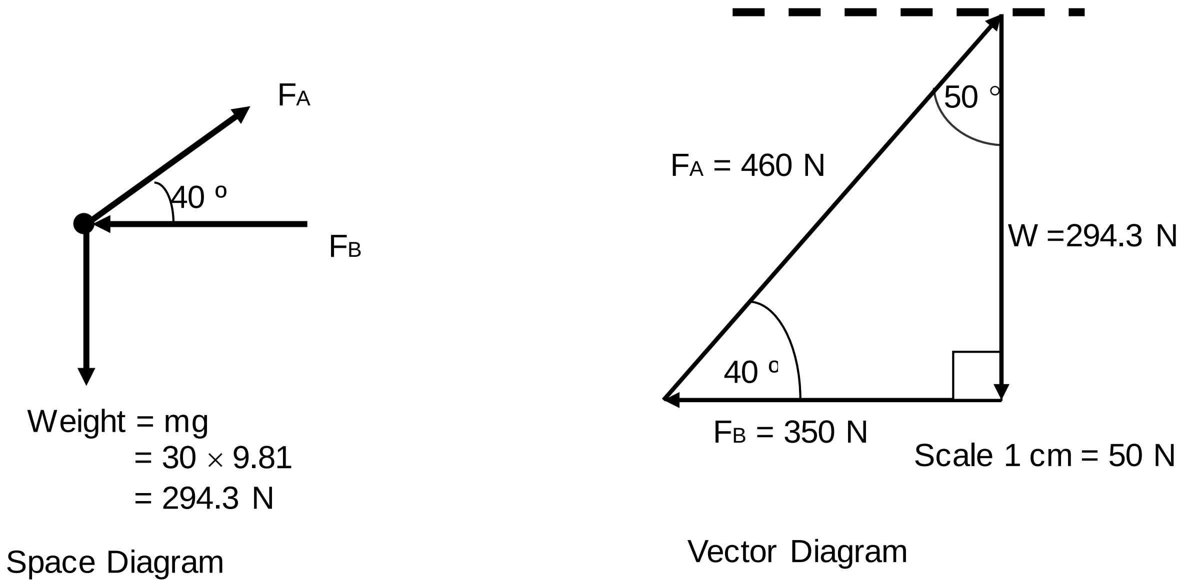


Figure 10 - Pin jointed structure.

28. The forces F_A and F_B can be found by drawing method, applying the triangle of forces to point C. From the space diagram, the vector diagram can be drawn to scale and the forces found by measurement.



Solution by drawing F_A = 460 N F_B = 350 N

Figure 11 - Space & vector diagram for a pin jointed structure.

Examples:

29. An aircraft mass 9 tonne is climbing at an angle of 20 º. Determine the net thrust and lift required at this angle of climb.

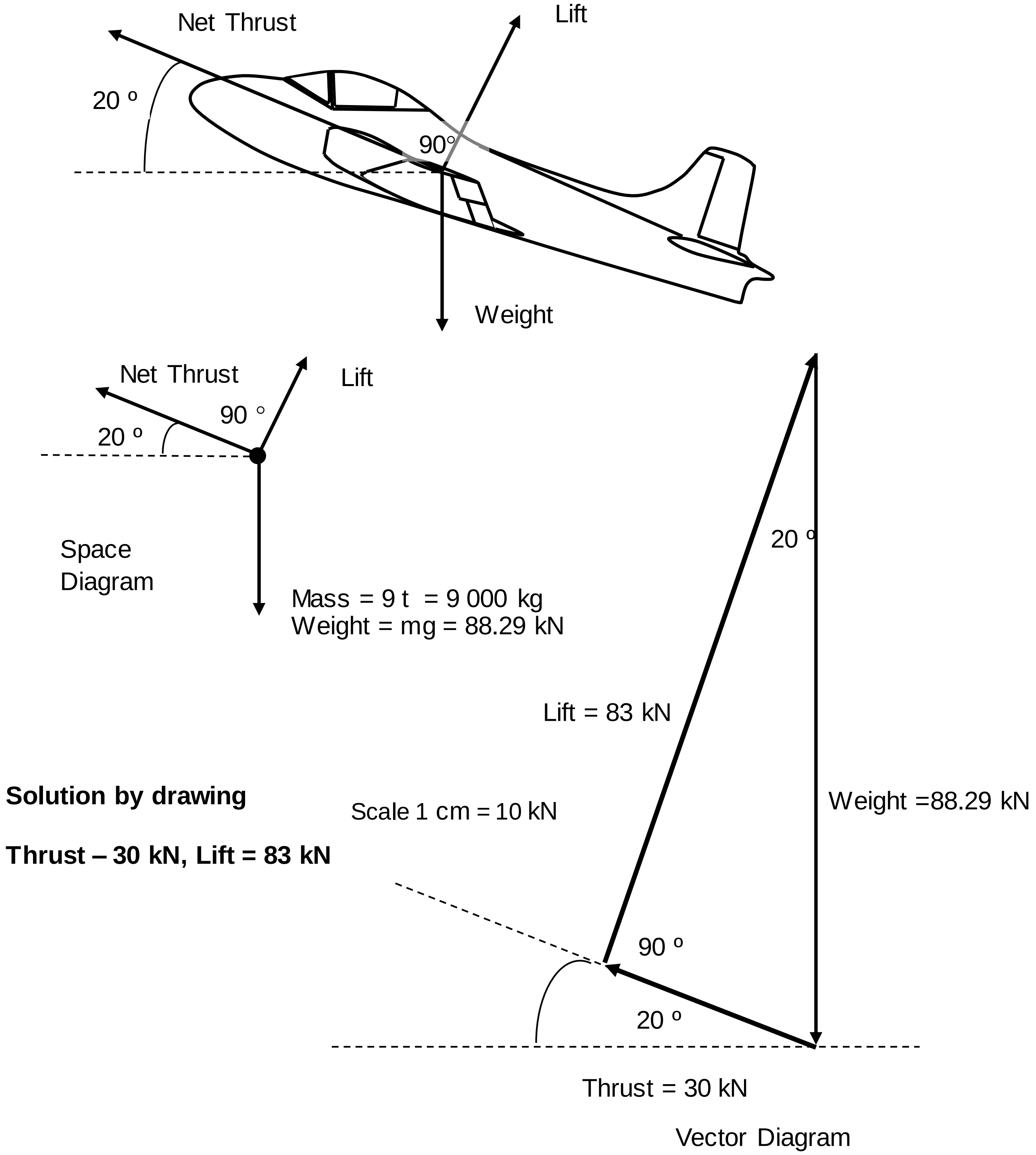
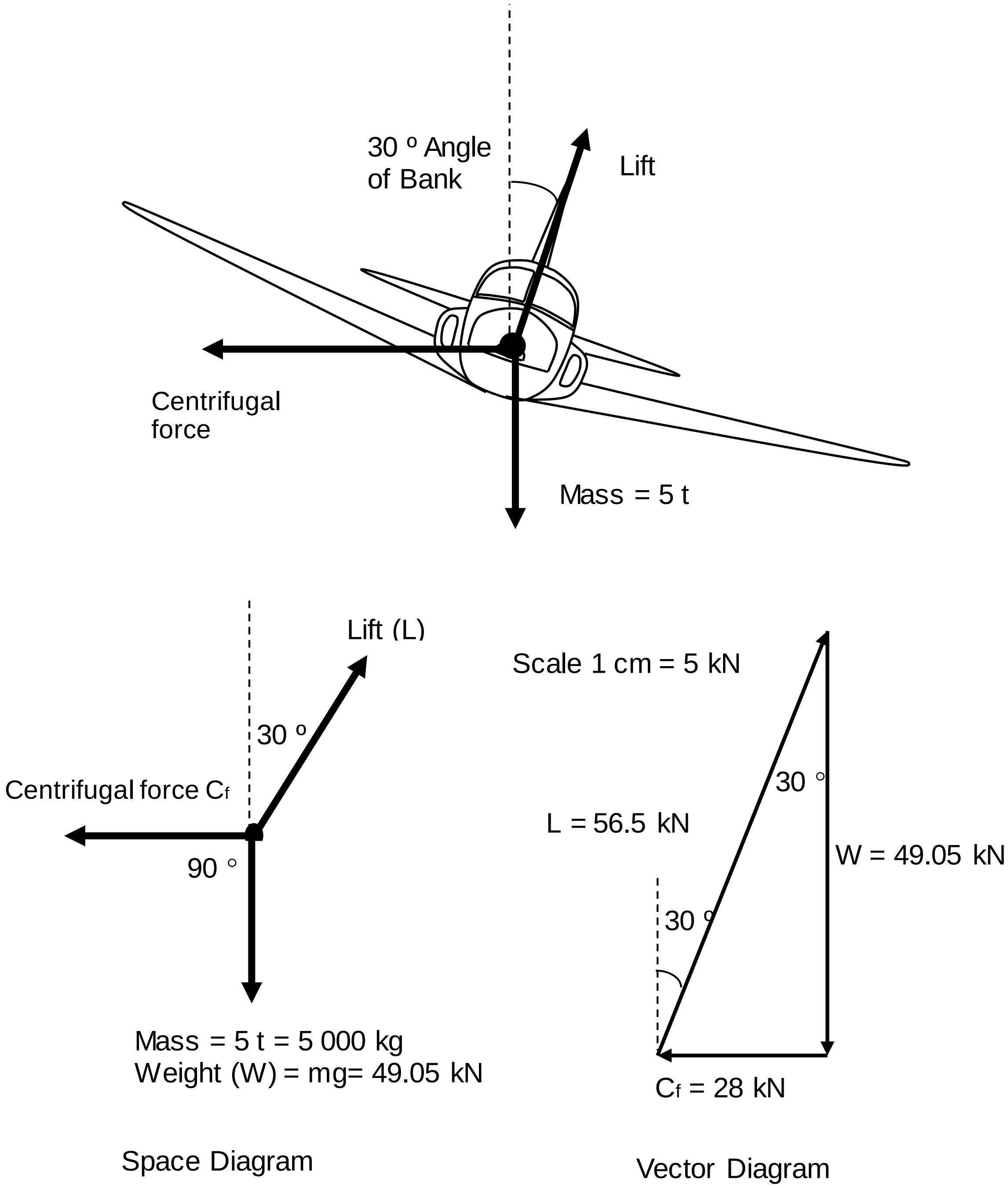


Figure 12 - Lift and Thrust determination by drawing.

30. An aircraft of mass 5 tonne enters a 30 º banked turn. Determine the lift and the centrifugal force required to balance the turn.



Solution by drawing: Lift – 56.5 kN Centrifugal force = 28 kN

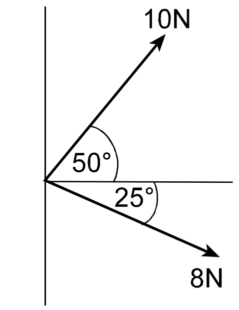
Figure 13 - Lift and centrifugal force determination by drawing.

Exercise 3

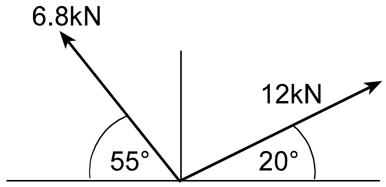
Note: Express directions of forces in degrees from x-axis

1. Use a graphical method to determine the resultant of the following forces:

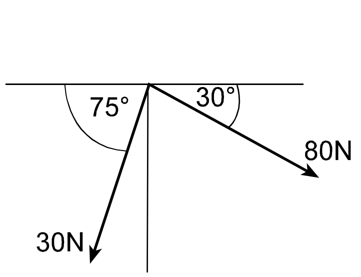
a.



b.



c.



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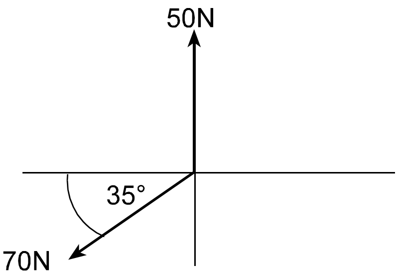

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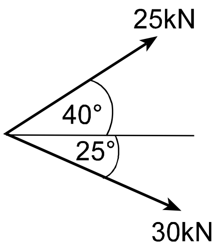
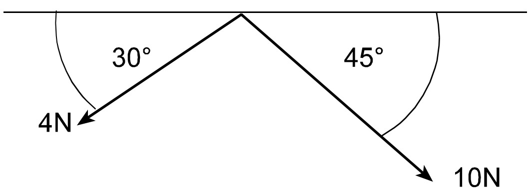
2. Determine the equilibrant of the following forces graphically:

a.



b.



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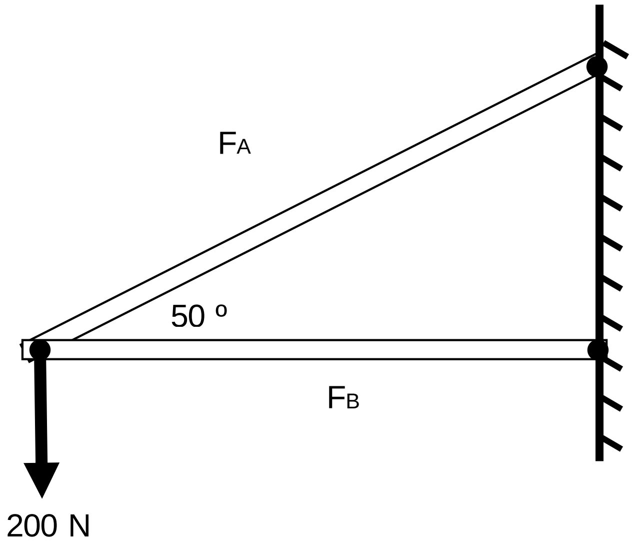
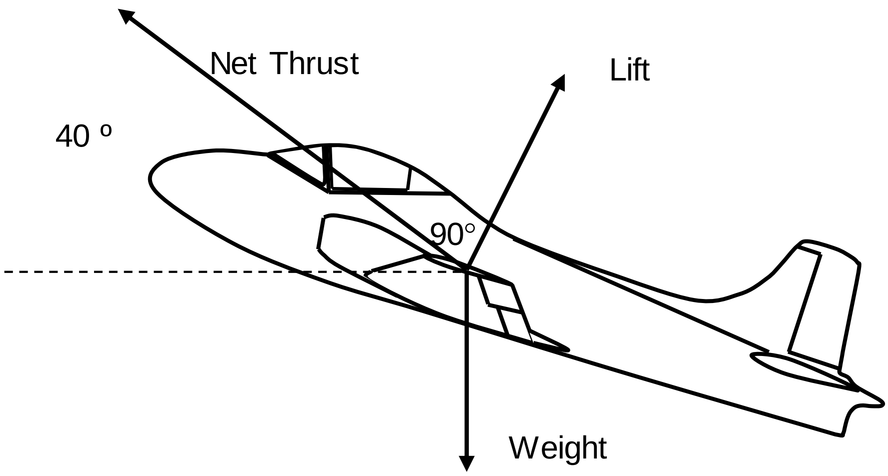
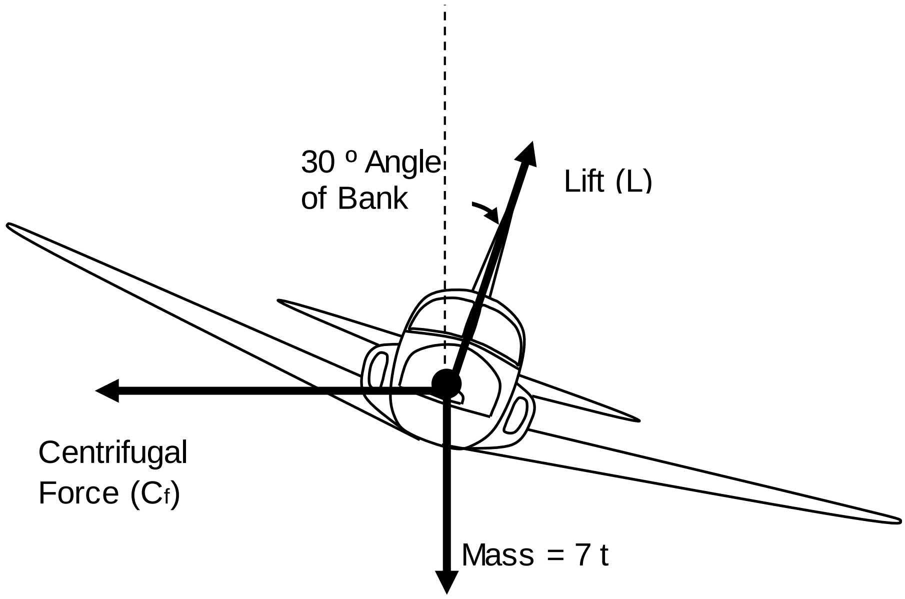
3. In the pin-jointed structure shown, determine F_A and F_B



4. An aircraft of mass 8 tonne is climbing at an angle of 40 ° as in the figure below. Determine the net thrust and lift required to maintain this angle of climb.



5. An aircraft mass 7 tonne enters a 30 º banked turn. Determine the lift and the centrifugal force required to balance the turn.



PRACTICAL EXERCISE - FORCES AND EQUILIBRIUM

Aim

To investigate forces for systems in equilibrium.

Apparatus

2 x load cells, pegboard, 2 x pulleys, long string and 1 kg mass, perspex sheet and pegs, ruler, protractor, pencil, graph paper

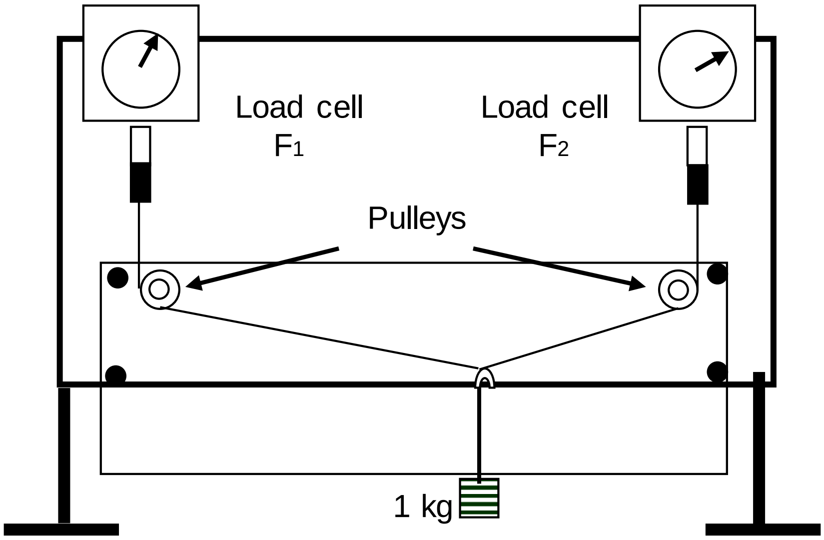


Figure 1. Forces Apparatus

Procedure

1. Ensure the position of the load cells as shown in Figure 1. The load cells and pulleys are permanently fixed in correct positions so do not attempt to move them.

2. Set the load cells to zero.

3. Suspend the 1 kg mass and string from the load cells.

4. Ensure the string is routed around the pulleys and the mass hangs freely. Avoid placing the mass directly in the centre of the string.

5. Calculate the weight force generated by the 1 kg mass and record below.

W = mg =	**N**

6. Read the load cells F_1 and F_2 and record the results in the boxes below:

7. Attach a blank page of graph paper to the perspex sheet behind the strings using the clip supplied. Make sure that the paper is positioned horizontally. One of the vertical gridlines should be directly behind the vertical string supporting the mass and

the point at which the three strings meet should be directly in front of one of the intersections of the graph paper.

8. Mark the position of the three strings using a sharp pencil by placing 2 dots, spaced well apart, on the graph paper directly behind each of the strings.

9. Detach the string from the hooks on the load cells to avoid straining the springs.

10. Remove the graph paper and extend the lines until they cross to produce a space diagram as shown in Figure 2. The three lines should meet at a single point. If they do not then repeat the procedure making sure that your dots are placed precisely behind the strings.

11. Label the lines F_1, F_2 and W and add arrows to show the direction of the forces involved.

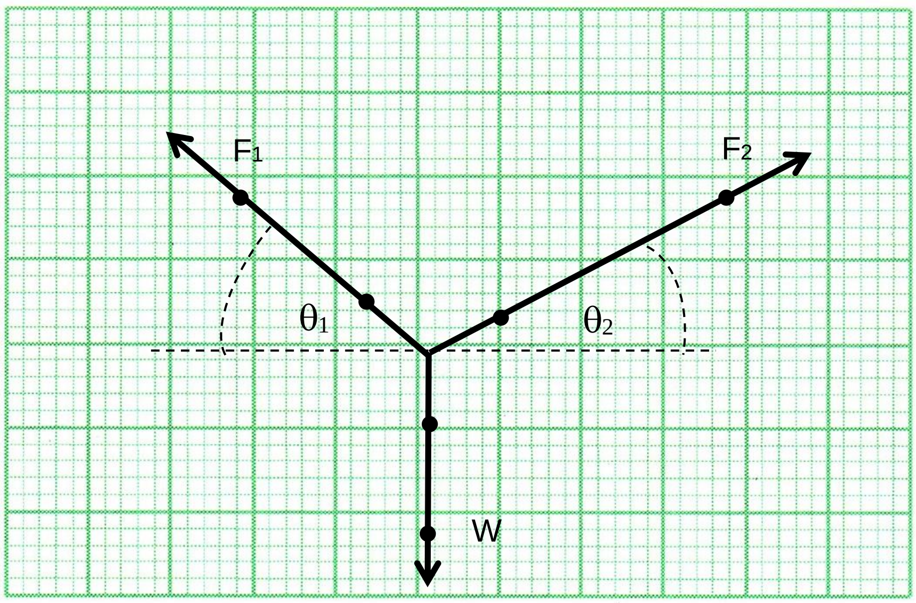


Figure 2. Space Diagram

12. Measure the angles (θ_1 and θ_2) between the forces F_1 and F_2 and the horizontal. Record the results in the table below.

θ_1	°
θ_2	°

13. Now move the 1 kg mass to a different position on the string and repeat the experiment. Record your results for the second experiment in the table below.

Second experiment values:

F_1 =	N	θ_1 =	°
F_2 =	N	θ_2 =	°

Analysis

14. Use the space diagrams and results above to construct a vector diagram of both the force systems on separate sheets of graph paper as follows: (i.e. all students to produce and submit both diagrams.)

a. Choose a suitable scale to represent the magnitude of the forces involved.
b. Draw the vector to represent the weight W.
c. Draw the vectors representing the forces F_1 and F_2 as vector additions to the weight vector. Make sure that the lengths of these vectors represent the magnitudes of the forces and that their directions reflect the directions obtained from the space diagrams.

Questions

15. "For a system to remain in equilibrium the vector sum of the forces acting on it must be …………………………." (What size in total?)

16. Bearing this in mind what type of figure (i.e. shape) would you expect your vector diagram to produce?

17. Does your vector diagram accord with this theory?

18. If not, what do you think are the reasons for the discrepancies?

First Experiment

Second Experiment

SC2.5 – DEFINE THE MOMENT OF A FORCE ABOUT A POINT

The moment of a force

31. The turning effect of a force is known as the moment of the force.

32. The moment of a force about a point is defined as the product of the force and the perpendicular distance from the point to the line of action of the force. The point is known as the pivot or fulcrum.

33. The moment of a force F, acting at a perpendicular distance d, from a pivot point A, is shown in Figure 14.

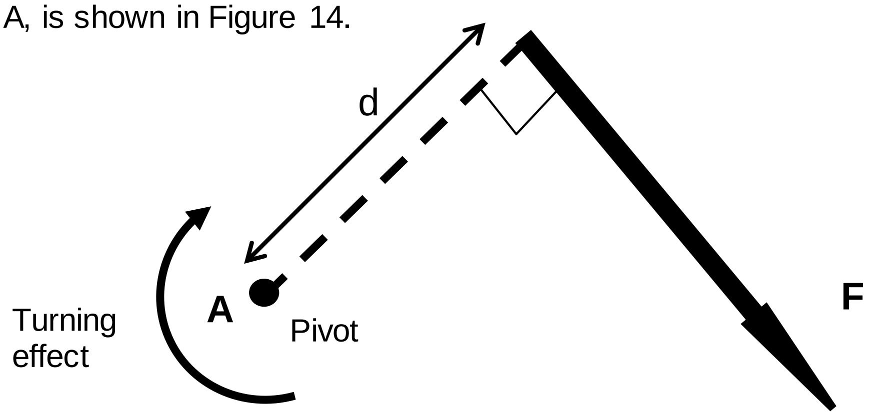


Figure 14 - Moment of a force

Moment = Force x perpendicular distance

M = F x d (Nm)

34. Moments have direction. The direction of a moment is usually expressed as clockwise or anticlockwise as shown in figure 15.

By convention:

a. Clockwise moments are positive

b. Anticlockwise moments are negative

Clockwise Moment (+ ve) **Anticlockwise Moment (-ve)**

Figure 15

Example:

35. A force of 30 N is applied at the end of a lever at a distance of 1.5 m from the pivot. Calculate the moment of the force about the pivot if

a. The force is applied at right angles to the lever.
b. The force is applied at an angle of 60° to the lever.

a. Force at right angles to lever:

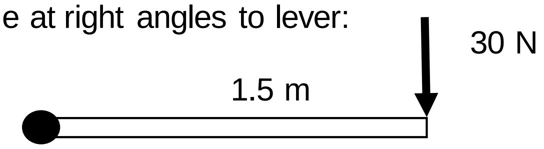


Figure 16 - Force at 90°

Moment of force = force × perpendicular distance
= 30×1.5
M = 45 Nm (+ve as in drawing)

Note . Ensure students know how to calculate the sine of an angle.

b. Force at 60° to same lever:

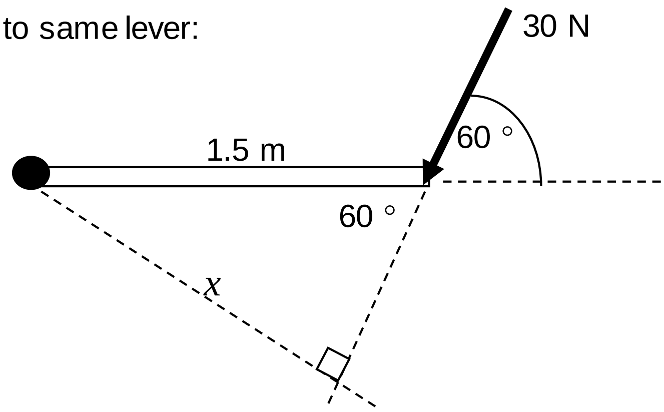


Figure 17 - Force at 60°

Moment of force = force × perpendicular distance
= 30 N × x

But trig gives:

$$\frac{x}{1.5} = \text{Sin } 60°$$

$$x = 1.5 \times \sin 60°$$

Moment of force = $30 \times 1.5 \times \sin 60°$

M = 38.97 Nm (+ve as in drawing)

Exercise 4

1. Define the term “moment of a force”.

2. A negative moment is in which direction?

3. Calculate the moment when a force of 40 N is applied to the end of a 2.5 m long lever.

4. Calculate the moment when a mass of 50 kg is placed on the end of a 600 mm long horizontal lever.

5. Calculate the moment of a 91 kg mass placed on the end of a 20 cm long horizontal lever.

6. Calculate the moment when a force of 200 N is applied to the end of a 4.25 m long lever at an angle of 55 ^{0} to the horizontal.

Resultant moment of multiple forces

36. The resultant moment of a system of forces acting on a lever can be calculated using the sign convention. The lever shown in figure 18, pivoting about A, has two forces acting on it:

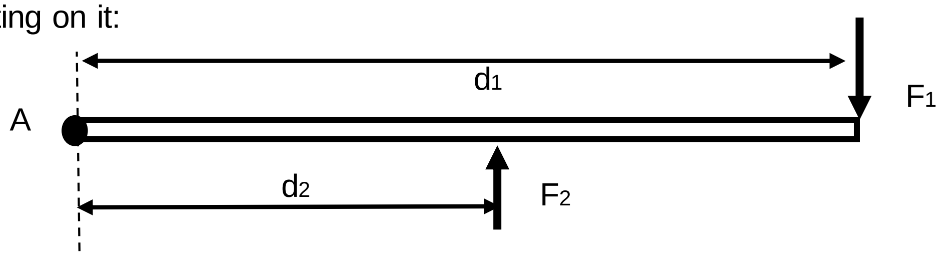


Figure 18 - Simple lever system.

37. The resultant turning force about point A is found by calculating the moment of each force about A and add them together:

Moment of force F_1 about A = $F_1 \times d_1$ (clockwise +ve)

Moment of force F_2 about A = $F_2 \times d_2$ (anticlockwise -ve)

Resultant (total) moment about A = $(F_1 \times d_1) - (F_2 \times d_2)$

Example:

38. Calculate the resultant moment about point A for the lever shown in figure 19.

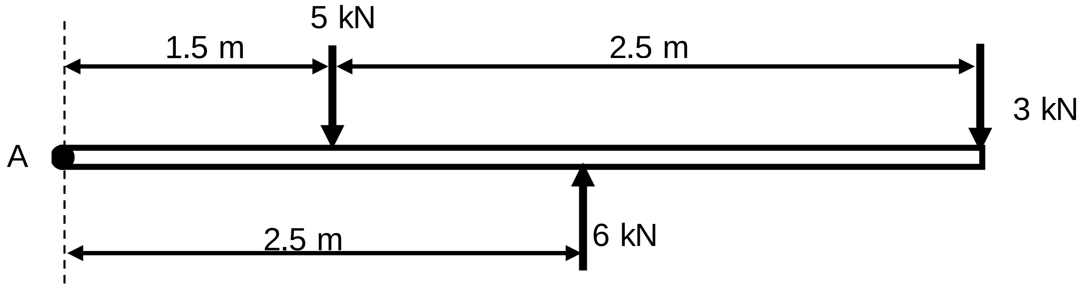


Figure 19

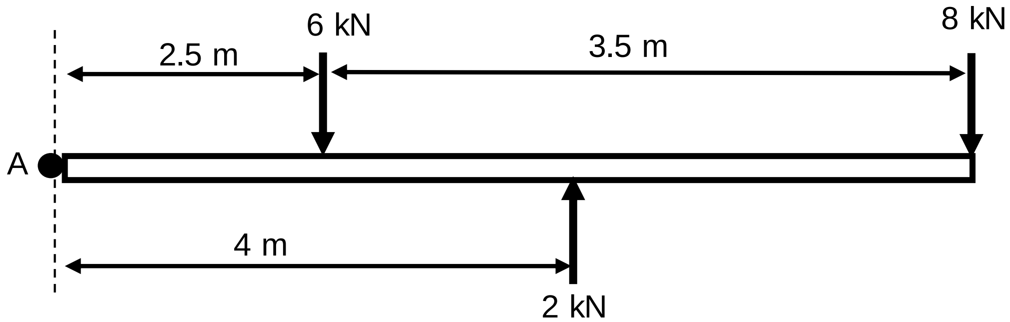
Taking moments about A:

5 kN force Moment = 5×1.5 = 7.5 kNm +ve
6 kN force Moment = 6×2.5 = 15 kNm -ve
3 kN force Moment = 3×4 = 12 kNm +ve

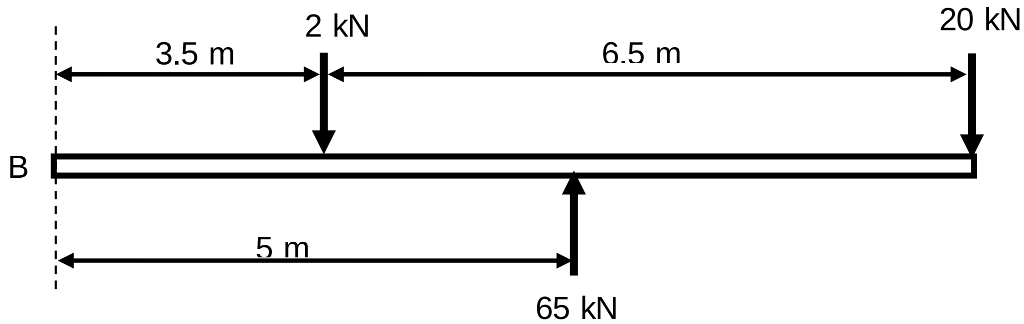
Resultant moment = 7.5 - 15 + 12 kNm
Resultant moment = +4.5 kNm (clockwise)

Exercise 5

1. Calculate the resultant moment about A for the system of forces shown below:



2. Calculate the resultant moment about B for the system of forces shown below:



Torque

39. The moment of a force is defined as the turning effect of a force about a pivot or fulcrum.

40. Torque is defined as the turning effect of a force about an axis of rotation.

41. Moments are used when static balance or equilibrium is implied. Torque is used when rotation is implied.

42. Torque is calculated using the following:

Torque = Force (N) x radius of rotation (m) = Nm

Examples:

43. Calculate the torque applied to a bolt by a 300 N force acting at the end of a 20cm spanner.

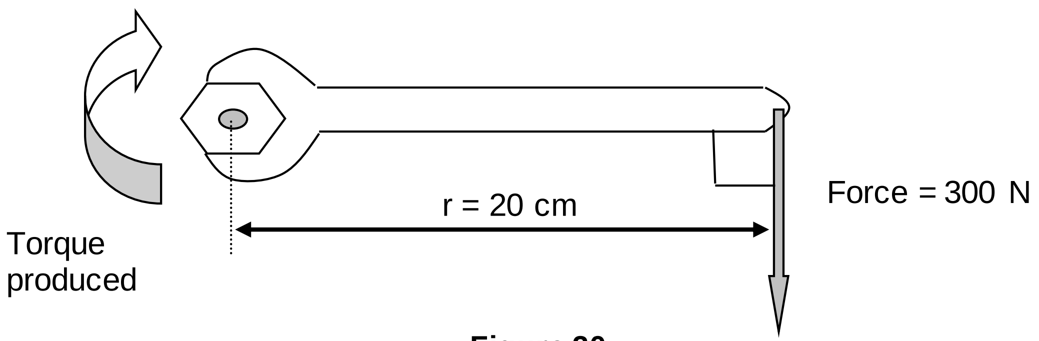


Figure 20.

Torque = Force x radius
= 300 N x 0.2 m
T = 60 Nm

44. A nut has a specified torque loading of 500 Nm. The length of the spanner to be used is 0.5 m. Calculate the force necessary to generate the specified torque.

T = 500 Nm   r = 0.5 m

$$T = Fr$$

$$F = \frac{T}{r}$$

$$= \frac{500}{0.5}$$

$$F = 1000 \text{ N}$$

Exercise 6

1. A force of 50 N is applied at right angles to a spanner of length 0.5 m. Calculate the torque generated.

2. A torque of 2 kNm is applied to a wheel of diameter 700 mm. Calculate the force generated at the rim of the wheel.

3. A bolt has a specified torque loading of 100 Nm. The spanner is 300 mm long but restricted access requires that the force be applied at an angle of 60° to the line of the spanner. Calculate the force required to torque up the bolt.

SC2.7 – SOLVE PROBLEMS INVOLVING STRAIGHT LEVERS, BELL CRANKS AND AIRCRAFT LOADING.

Equilibrium and the principle of moments

45. A body is certain to be in equilibrium if both the forces and moments acting on it are in balance. Two conditions must be satisfied for a body to be in equilibrium.

a. All the forces acting on it are balanced.

b. All the turning effects of those forces (moments) are in balance

46. The Principle of Moments states that when a body is in equilibrium the sum of the anticlockwise moments about any point is equal to the sum of the clockwise moments about that point.

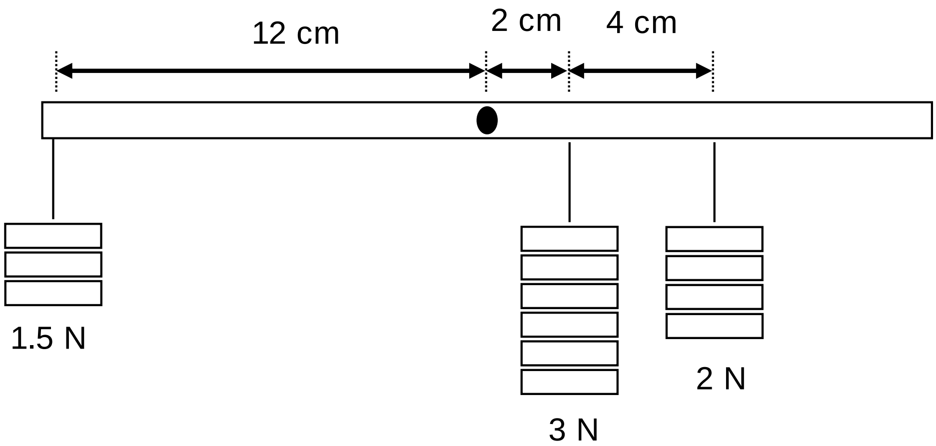


Figure 21 - Principle of moments.

47. The forces in Fig. 21 will be in equilibrium when the anticlockwise moments (ACM) are equal to the clockwise (CWM) moments and the downward forces are equal to the upward forces.

48. Calculating ACM and CWM for the system in figure 21 gives:

ACM = 1.5 x 12 Ncm	CWM = (3 x 2)+(2 x 6) Ncm
ACM = 18 Ncm	CWM = 6 + 12 Ncm
	CWM = 18 Ncm

49. The total downward force is (1.5 + 3 + 2) N = 6.5 N. The supporting pivot provides the upward force to balance the total downward force in Fig 21. A line diagram of the system is shown in Fig 22 below.

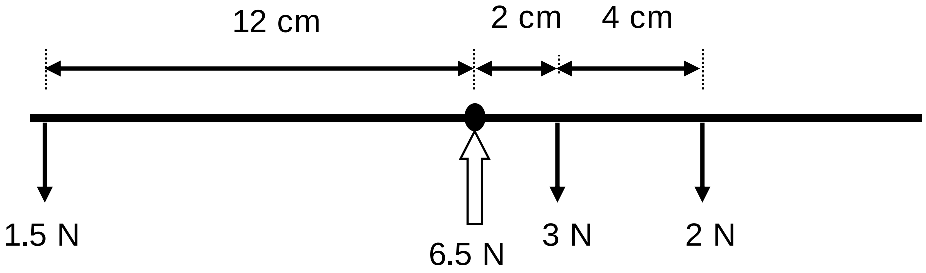


Figure 22.

Examples:

50. The beam in Fig 23 is freely pivoted about point A. Calculate the force F, necessary to maintain the beam in equilibrium.

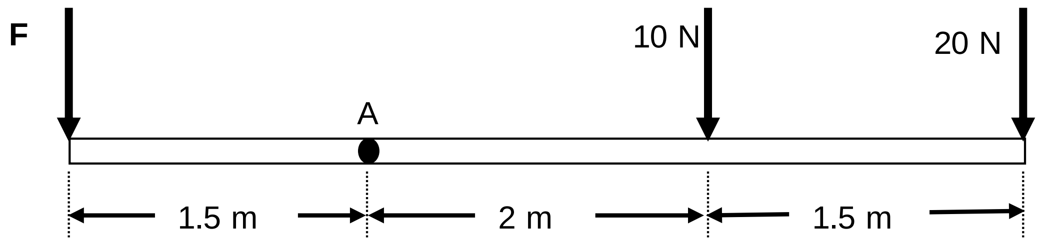


Figure 23

Taking moments about A:

Clockwise moments = $(10 \times 2) + (20 \times (2+1.5)) = 90$ Nm

Anticlockwise moments = $F \times 1.5$ Nm

By the principle of moments

$$\text{Anticlockwise Moments} = \text{Clockwise Moments}$$

$$F \times 1.5 = 90$$

$$F = \frac{90}{1.5}$$

$$F = 60 \text{ N (downwards)}$$

51. The lever in Figure 24 forms part of an aircraft control system. If the load at the output is 2 kN, calculate the force required at the input to maintain the position of the controls.

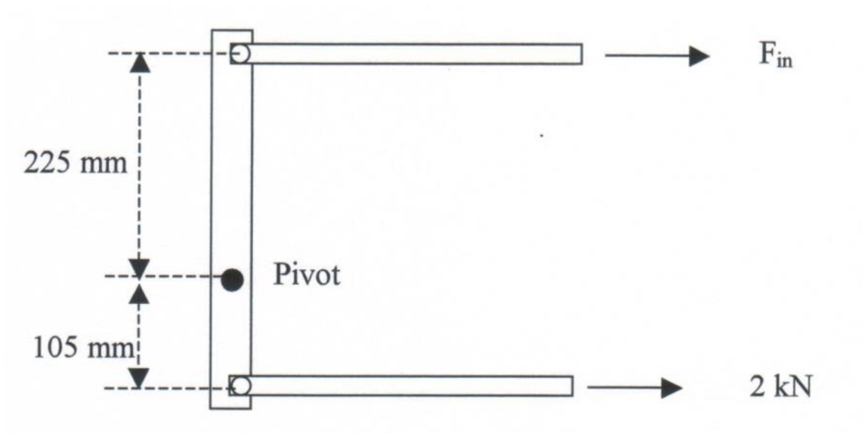


Figure 24.

Taking moments about the pivot:

Clockwise moments = $F_{in} \times 0.225$ m

Anticlockwise moments = 2 kN $\times$ 0.105 m

By principle of moments:

$$F_{in} \times 0.225 \text{ m} = 2 \text{ kN} \times 0.105 \text{ m}$$

$$F_{in} = \frac{210\ Nm}{0.225\ m}$$

$$F_{in} = 933.3 \text{ N}$$

52. Figure 25 represents a car brake pedal. If the driver exerts a force of 1 200 N, calculate the resulting force on the master cylinder piston.

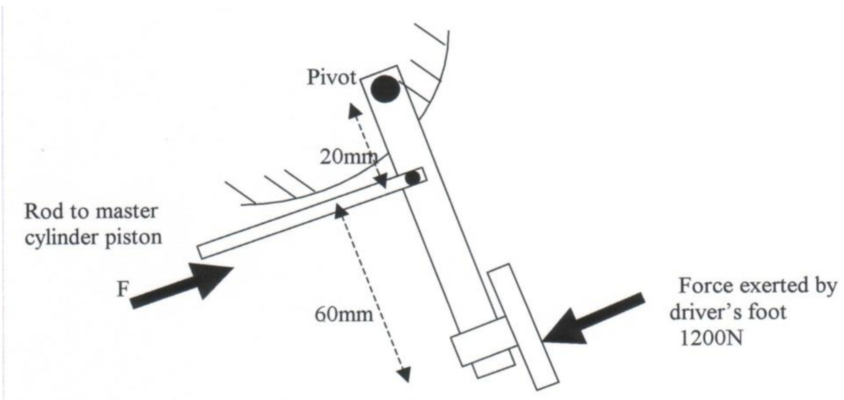


Figure 25.

The force exerted by driver on the brake pedal will be balanced by the reaction at the master cylinder.

Taking moments about the pedal pivot:

Clockwise moment $= 1\,200 \text{ N} \times (20 + 60) = 96\,000 \text{ Nmm}$

Anticlockwise moment $= F \times 20 \text{ mm}$

By principle of moments $F \times 20 \text{ mm} = 96\,000 \text{ Nmm}$

$$F = \frac{96\,000 \text{ Nmm}}{20 \text{ mm}}$$

$$F = 4\,800 \text{ N} = 4.8 \text{ kN}$$

53. The bell crank shown in Figure 26, forms part of an engine throttle control linkage. If the load on the output link is 30 N calculate the input force F_{in} necessary to maintain the throttle setting.

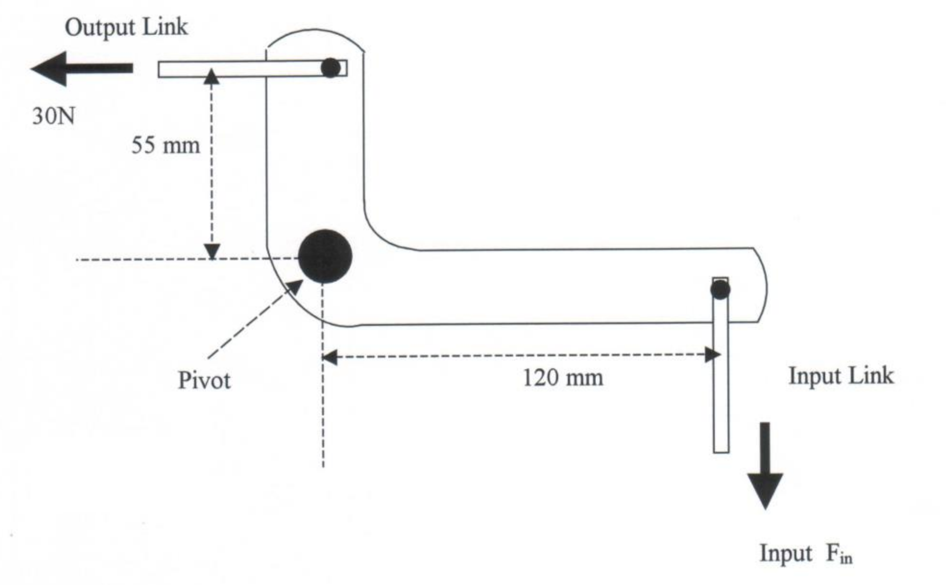


Figure 26.

Taking moments about the pivot:

Clockwise moment $= F_{in} \times 120 \text{ mm}$

Anticlockwise moment $= 30 \text{ N} \times 55 \text{ mm}$

By principle of moments:

$$F_{in} \times 120 = 30 \times 55$$

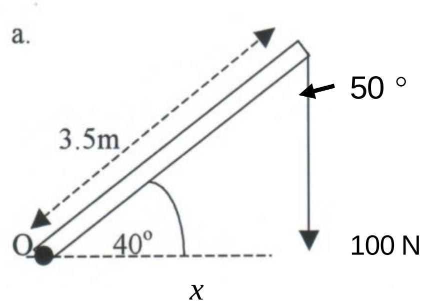
$$F_{in} = \frac{30 \times 55}{120}$$

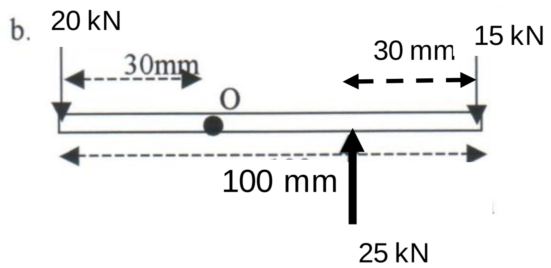
$$F_{in} = 13.75 \text{ N}$$

Exercise 7

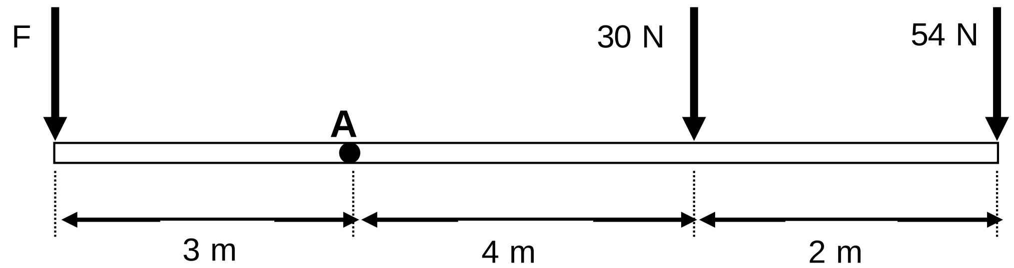
1. Describe the properties of a force.

2. Calculate the resultant moments about the pivots O for the following levers:

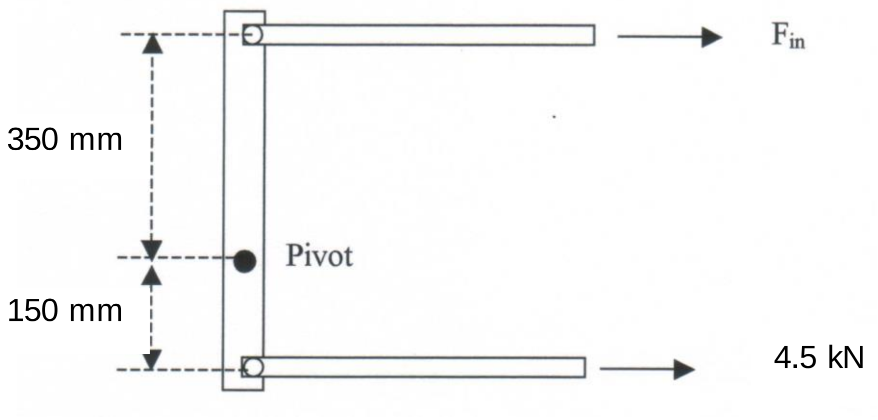




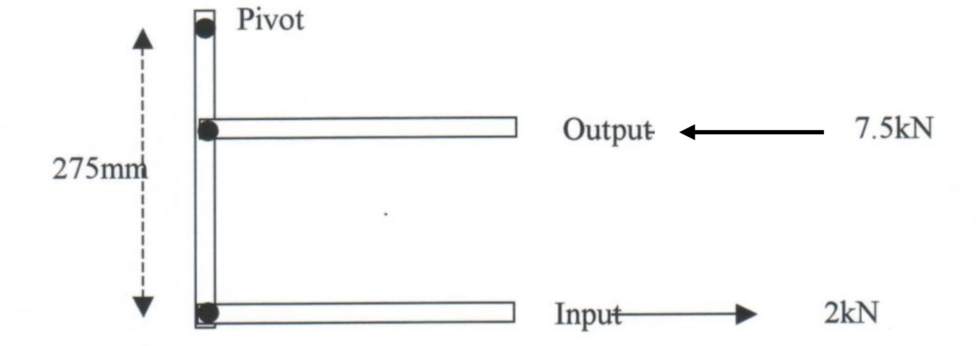
3. What value of force F will bring the beam into equilibrium about point A?



4. The lever in below forms part of an aircraft control system. If the load at the output is 4.5 kN, calculate the force required at the input to maintain the position of the controls.



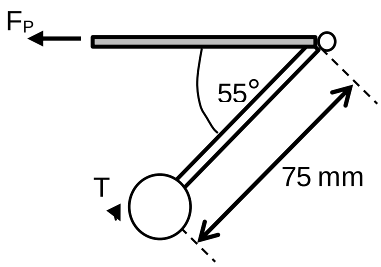
5. A spring on the input linkage shown below exerts a force of 2 kN. For the linkage to remain in equilibrium an output force of 7.5 kN must be opposed. Calculate the distance of the output rod from the pivot.



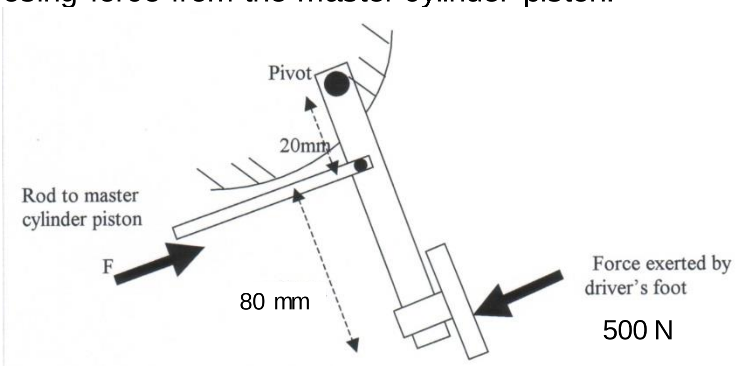
6. A cable operated engine control is connected to the operating lever as shown below.

i. Calculate the force on the cable necessary to generate a torque of 35 Nm on the operating shaft.

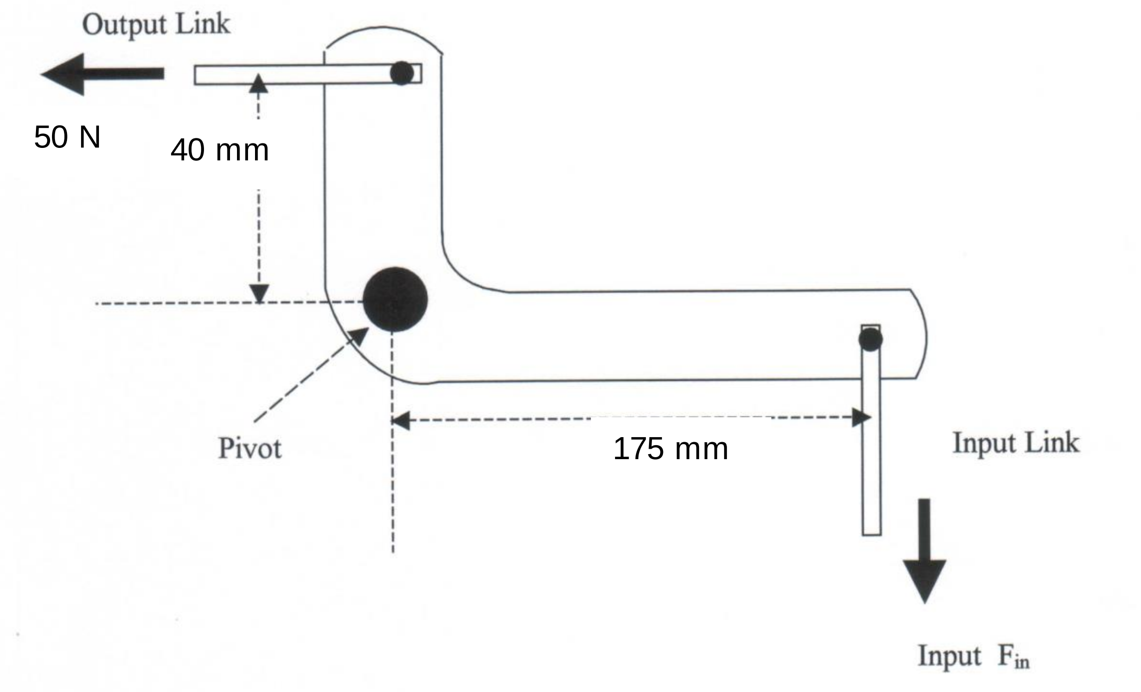
ii. State whether the torque on the shaft will increase or decrease as the control is operated.



7. A car brake pedal is shown below. If the driver exerts a force of 500 N, calculate the opposing force from the master cylinder piston.



8. The bell crank shown in below forms part of an engine throttle control linkage. If the load on the output link is 50 N calculate the input force F_{in} necessary to maintain the throttle setting.



PRACTICAL EXERCISE – MOMENTS AND EQUILIBRIUM

Aim

To investigate moments for systems in equilibrium.

Apparatus

Pegboard, balance beam, bearing, 10 N force meter, mass carrier 50 g, 100 g, 200 g masses.

Procedure

1. Ensure the equipment set up on the second peg board is as illustrated in Figure 1 below.

2. Ensure that a safety peg is positioned under the right hand side of the beam as shown, this will prevent the beam swinging to a vertical position.

3. Suspend a mass of 200 g on the right hand end of the beam, 170 mm from the pivot. Note: the pitch of the holes in the beam is 10 mm.

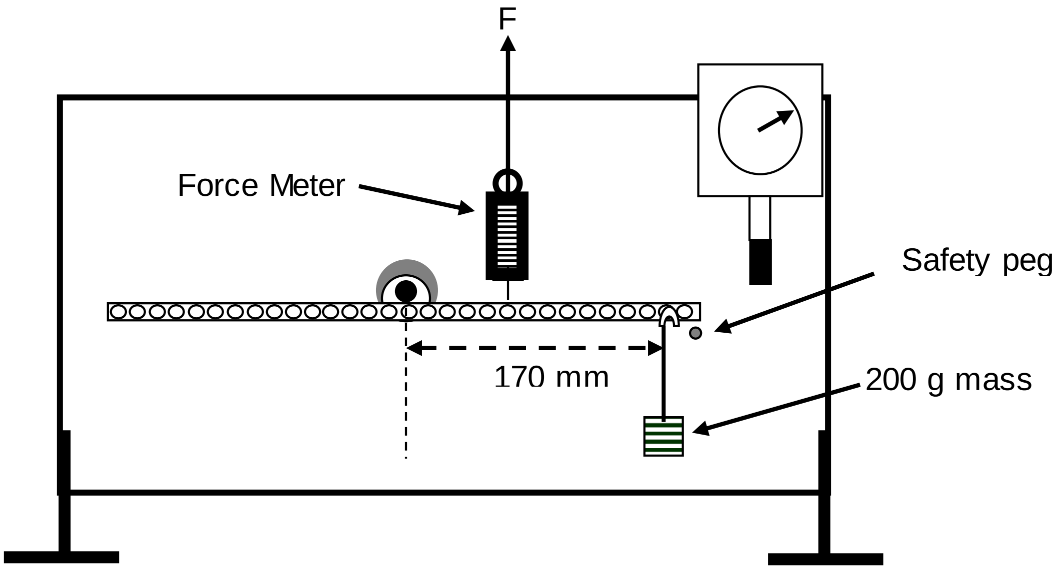


Figure 1 - Balance Beam

4. Calculate the clockwise moment generated by the 200 g mass and enter the result in the box below:

Clockwise moment = ………………… Nm

5. Set the force meter to zero and use it at the positions listed in the table below to establish the force F necessary to balance the beam. Take the readings with the beam in the horizontal position. Enter the results in the table below and calculate the anti-clockwise moment.

Distance to right of pivot mm	**Force required to balance N**	**Anticlockwise moment Nm**
50		
70		
90		
110		
130		
150		
	Average Anti-clockwise Moment	

Questions

6. What do you notice about the clockwise moment and the average anticlockwise moment?

7. Why is the weight of the beam not taken into account in the calculations?

8. Complete the following sentence:

“For a system to be in equilibrium the sum of the turning moment must be: ……………………….……………………………………………….”.

9. Taking into account the results of the forces and the moments practical exercises, describe below the 2 conditions that must be satisfied for a system to be in equilibrium?

1.

2.

SC2.6 – DEFINE CENTRE OF GRAVITY

Centre of gravity

54. The centre of gravity of a body is the point, through which the weight of the body always acts.

55. The centre of gravity of a symmetrical shaped solid is always at the 'geometric' centre.

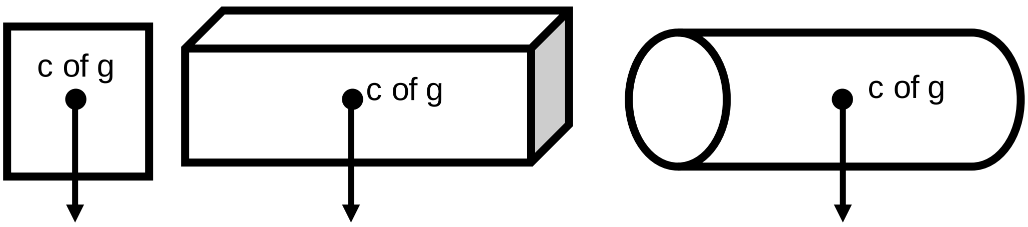


Figure 27 - Centres of gravity.

56. An aircraft also has a centre of gravity through which the weight of the aircraft always acts.

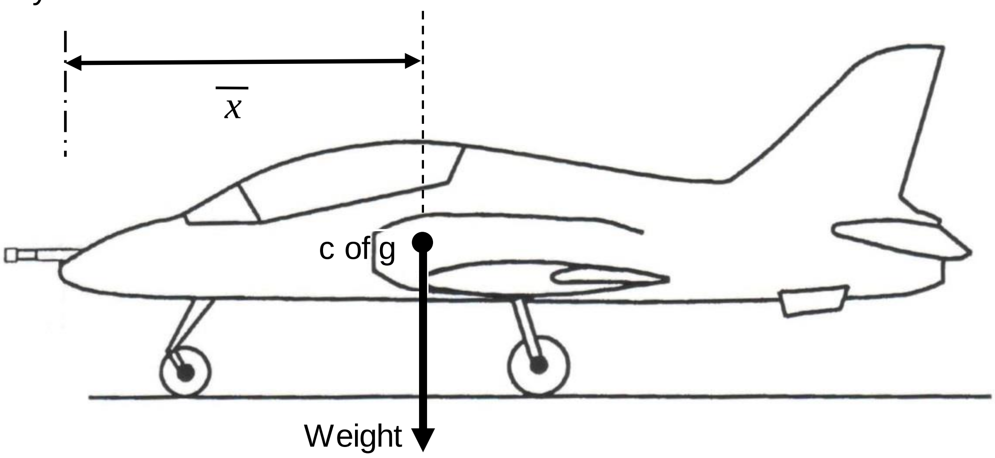


Figure 28 - Aircraft centre of gravity

57. The centre of gravity can be quoted as a distance from a fixed datum ($\overline{x}$). Its position can affect the stability of the aircraft and can move as expendable loads such as fuel or bomb load is used.

58. An aircraft with the centre of gravity in front of the centre of lift is stable in pitch. An aircraft with the centre of gravity behind the centre of lift is unstable in pitch. This is covered in aerodynamics.

Derivation of aircraft centre of gravity

59. The principle of moments is used to calculate the centre of gravity of an aircraft. All measurements are taken from a datum point which is fixed along the aircrafts longitudinal axis.

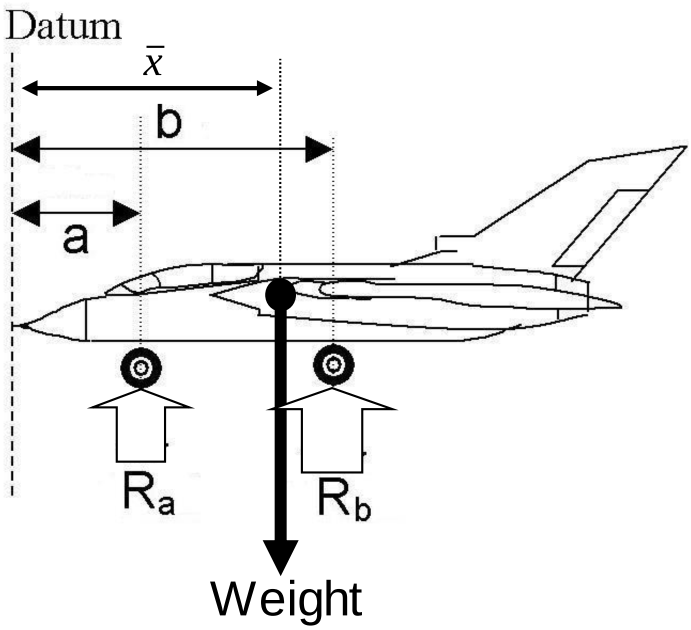


Figure 29.

60. The undercarriage supports the weight of the aircraft and the forces acting can be measured by load cell placed beneath the undercarriage.

61. The total weight must equal the total force acting through the wheels, in this case Ra + Rb. Note that most aircraft have 2 main wheels, therefore R_b will be the total reaction at the main wheels and will normally be 2x the measurement on one wheel.

62. Applying the principle of moments - the total moment exerted by the weight acting at the centre of gravity distance must equal the total moment exerted by the forces acting through the wheels.

Moment exerted by Weight = Total Moment exerted by the Undercarriage

$$Weight \times \bar{x} = (R_a \times a) + (R_b \times b)$$

$$But\ weight\ = R_a + R_b$$

$$So, (R_a + R_b) \times \bar{x} = (R_a \times a) + (Rb \times b)$$

$$Centre\ of\ gravity\ distance\ \bar{x} = \frac{(R_a \times a) + (R_b \times b)}{(R_a + R_b)}$$

Example:

63. Figure 30 shows the results of a load test on a Hawk, calculate the position of the centre of gravity from the datum**.**

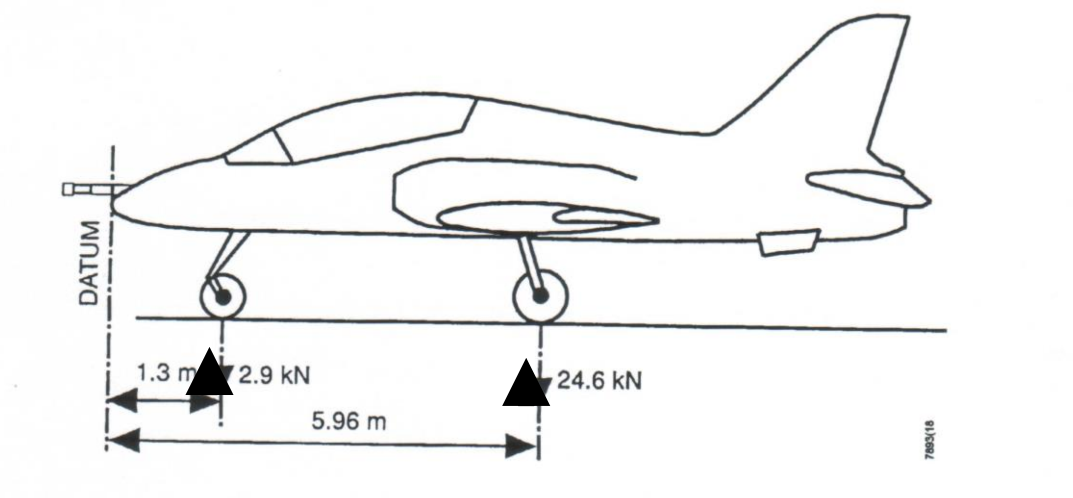


Figure 30.

Note: The load cell under each main undercarriage leg reads 24.6 kN.

R_a = 2.9 kN, a = 1.3 m, R_b = 2 × 24.6 kN, = 49.2 kN b = 5.96 m.

$$\bar{x} = \frac{(R_a \times a) + (R_b \times b)}{(R_a + R_b)}$$

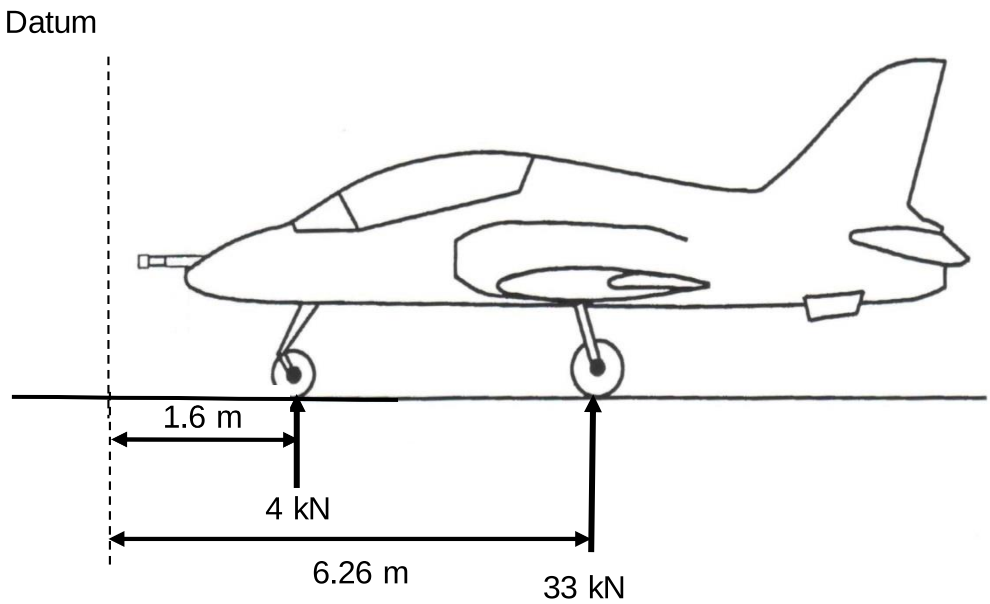
$$\frac{(2.9 \times 1.3) + (49.2 \times 5.96)}{(2.9 + 49.2)}$$

$$\bar{x} = 5.7\,m$$

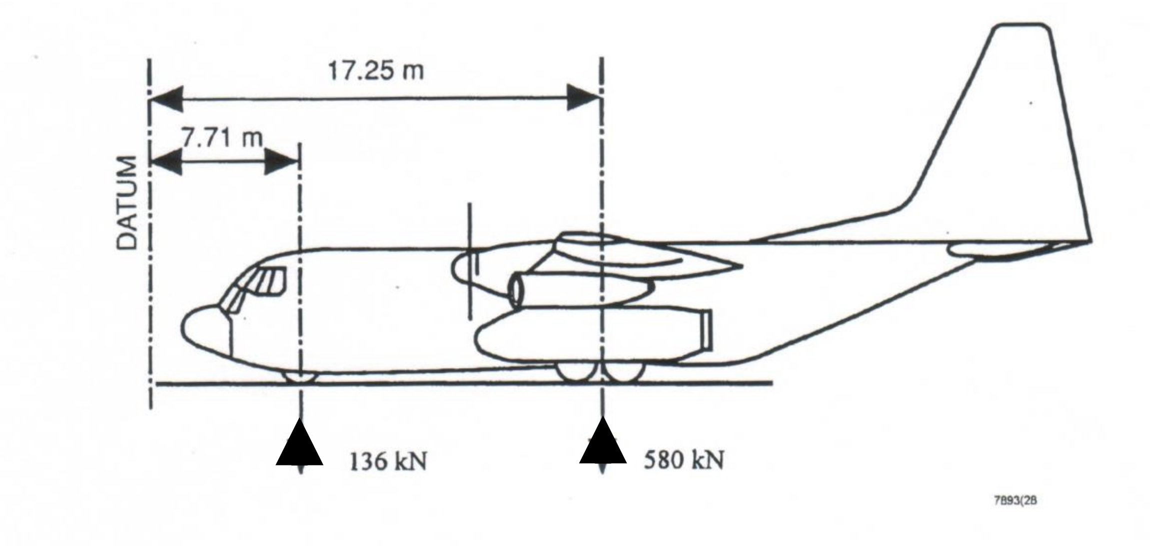
The Centre of Gravity is 5.7 m from the datum.

Exercise 8

1. A Hawk is weighed as part of its routine maintenance. The readings of the load cells are shown below. The reading of 33 kN is for each of the 2 main undercarriages. Calculate the position of the centre of gravity of the aircraft with reference to the datum given.



2. The readings of the load cells are shown below for a Hercules on routine maintenance. The reading of 580 kN is for each of the 2 main undercarriages. Calculate the position of the centre of gravity.



3. An aircraft is in a hangar and has the following measurements taken in order to determine the position of the centre of gravity. All measurements were taken from the same datum point (the hangar wall in front of the aircraft). Determine the position of the centre of gravity.

Front undercarriage load	R_a	=	12 kN
Front undercarriage distance	a	=	4 m
Rear undercarriage load (for each main wheel)	R_b	=	38 kN
Rear undercarriage distance	b	=	12 m

4. The Aircraft mentioned above is now fully fuelled and the undercarriage reactions are re-measured to be R_a = 18 kN and R_b = 45 kN.

Calculate the new position and the shift of the C of G.

PRACTICAL EXERCISE - DETERMINATION OF AIRCRAFT CENTRE OF GRAVITY

Aim

To investigate the position of the centre of gravity of an aircraft structure.

Apparatus

2 x load cells, Tornado aircraft, pegboard, mass carriers, 50 g, 100 g, 200 g masses, ruler.

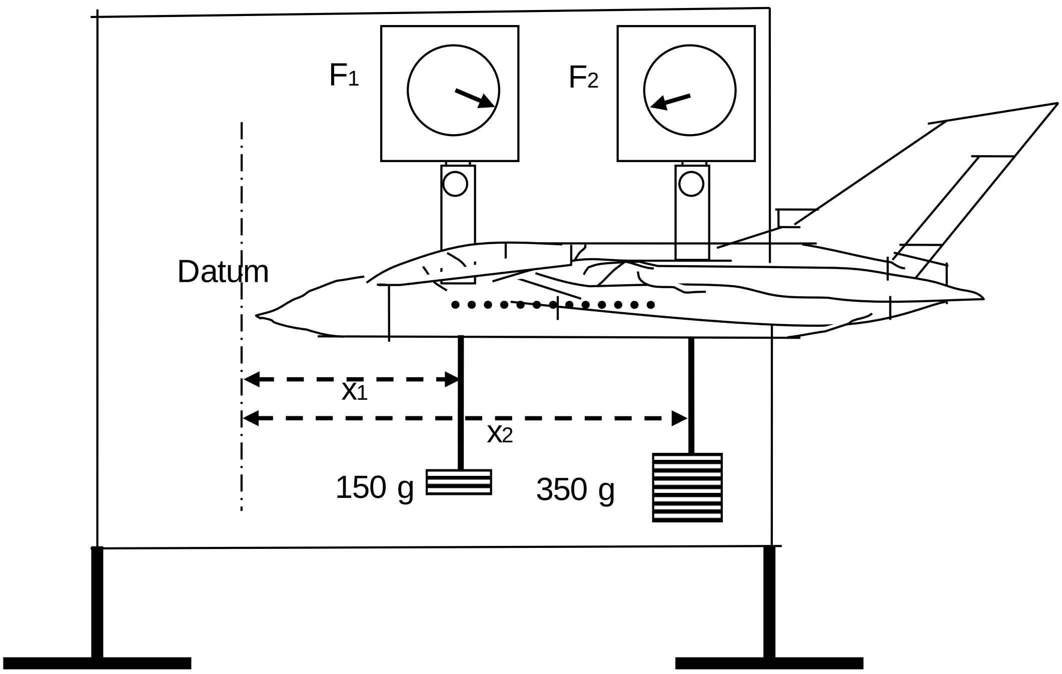


Figure 1 - Centre of Gravity Apparatus

Procedure

1. Ensure the equipment set up on the peg board is as illustrated in Figure 1 above.

2. Set both load cells to zero and then suspend the aircraft using the white plastic nuts and bolts as shown in Figure 1. The load cell readings represent the reactions F_1 and F_2 on the nose and main undercarriage legs.

3. Leave the nuts sufficiently loose to allow free movement between the aircraft suspension points and the load cells

4. Hang a 150 g mass from the front suspension point and a 350 g mass from the rear suspension point as shown in Figure 1. These masses represent the loads on the aircraft due to the mass of the engine, radar, fuel load etc.

5. Read the load cells and record the value of the forces F_1 and F_2 in the boxes below:

F_1	N
F_2	N

6. Measure the distances x_1 (nose undercarriage) and x_2 (main undercarriage) from the aircraft datum and record them in the boxes below

x_1	m
x_2	m

7. Calculate the distance, $\overline{x}$, (the aircraft centre of gravity distance) from the aircraft datum using the formula below. Show working out in the space provided.

$$\bar{x} = \frac{\lfloor (F_1 \times x_1) + (F_2 \times x_2) \rfloor}{(F_1 + F_2)}$$

Distance of aircraft c of g from datum by calculation = …………m

8. Remove the aircraft (complete with masses) from the load cells and suspend it from load cell F_2 using the slotted black cradle and a plastic nut and bolt as shown in Figure 2.

9. Slide the aircraft, within the cradle until it balances, checking that the bottom of the fuselage is level with reference to the grid lines on the pegboard. The cradle is now positioned at the c of g. Measure the distance x from the datum to the centre of the cradle.

10. Record the result in the box below:

Distance of aircraft c of g from datum by balancing =	m

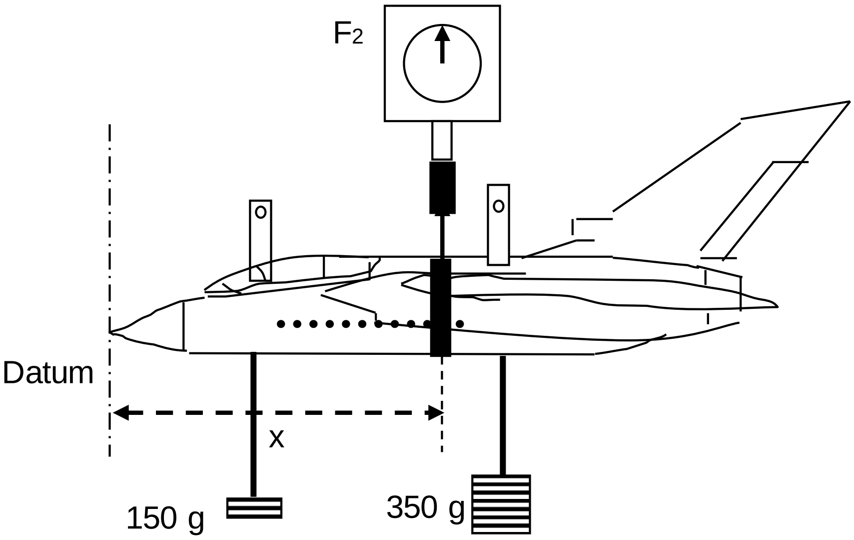


Figure 2 - Aircraft balanced

Questions

11. Does the c of g position obtained by calculation agree with that obtained by balancing? If they do not agree, what reasons may there be for the discrepancy?

12. The figures obtained in Para 5 may be used to calculate the total aircraft weight. Calculate below how you think that this may be achieved and record the total weight of the aircraft in the box below.

Aircraft total weight by calculation = N

13. Now measure the total aircraft weight by reading load cell F_2 whilst the aircraft is suspended in the cradle. Record the result below and comment on any discrepancies between the values obtained in paragraph 12 and 13.

Aircraft basic weight by balancing = N

Varying the Load Conditions

Apparatus

As for original c of g equipment but with an additional mass carrier and masses.

Procedure

14. Set up the apparatus as for original investigation, but with an additional load of 200 g suspended from the aircraft at a point 130 mm from the datum as shown in Figure 3. This load represents a variable or disposable load such as a reconnaissance pod or missile.

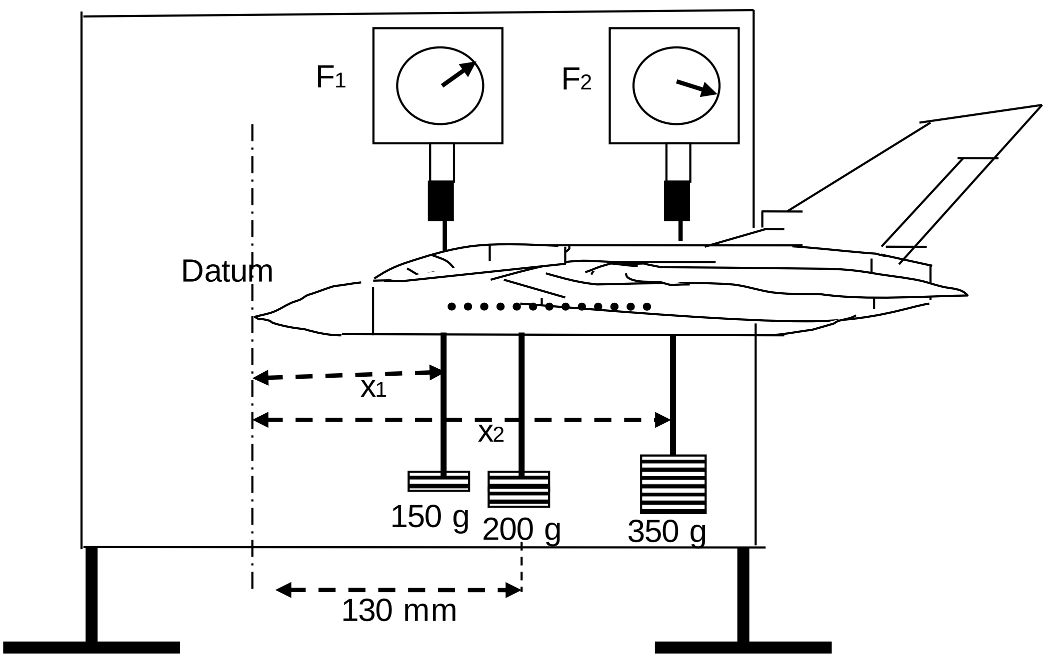


Figure 3 - Loaded aircraft

15. Read the load cells and record the value of the forces F_1 and F_2 in the boxes below: (note distances x_1 and x_2 are the same as before).

F_1	N	x_1	m
F_2	N	x_2	m

16. Calculate the distance, $\overline{x}$, (the loaded aircraft centre of gravity distance) from the aircraft datum using the formula below. Show working out in the space provided.

$$\bar{x} = \frac{\lfloor(F_1 \times x_1) + (F_2 \times x_2)\rfloor}{(F_1 + F_2)}$$

Distance of loaded aircraft c of g from datum by calculation = m

Questions

17. Compare your answer above with that obtained for the unloaded aircraft and comment on any differences between the two.

18. What do you think are the implications of these results when considering the loading of a full size aircraft?

SC2.8 – DEFINE PRESSURE AND ITS UNITS

The units of pressure

64. Force is defined as a push or pull, which changes or tends to change the state of rest of a body or uniform motion in a straight line or a curved path. It is a vector quantity with magnitude and direction and is measured in Newtons.

65. Pressure is defined as the total force per unit area.

$$P = \frac{F}{A}$$

P = Pressure in N/m^2

F = Force in Newtons

A = Area in m^2

66. The SI units of pressure are N/m^2 and Pascal (Pa).

67. The Pascal is not in common use and calculations are probably easier to understand if the units of pressure are kept in their more fundamental form of N/m^2.

Note: 1 Pa = 1 N/m^2

Example:

68. A box has dimensions 150 mm by 200 mm by 100 mm. It has a mass of 2 kg. Calculate the pressure produced if it is placed on:

a. Its largest side (150 mm $\times$ 200 mm)

b. Its smallest side (100 mm $\times$ 150 mm)

a. Largest side (150 mm $\times$ 200 mm)

Area = $0.15 \times 0.2 = 0.03$ m^2

Force = weight = mg = $2 \times 9.81 = 19.62$ N
Pressure = F/A =$19.62 \div 0.03 = 654$ N/m^2

b. Smallest side (100 mm $\times$ 150 mm)

Area = $0.15 \times 0.1 = 0.015$ m^2

Force = weight = mg = $2 \times 9.81 = 19.62$ N
Pressure = F/A =$19.62 \div 0.015 = 1\ 308$ N/m^2 or 1.31 N/m^2

69. The bar is a non-SI unit of pressure equal to 100 kPa. It is approximately equal to the atmospheric pressure at sea level.

1 bar = 100 000 N/m^2
1 bar = 10^5 N/m^2

1 mbar = 0.001 bar (10^{-3} bar)
1 mbar = 100 N/m^2 (10^2 N/m^2)

70. The Imperial units of pressure are lbf/in^2, pounds per square inch, (psi).

1bar is approximately equal to 14.5 lbf/in^2.

Examples:

71. Convert 90 kN/m^2 to bar.

Solution: 90 000 N/m^2 = 90 000 ÷ 100 000 = 0.9 bar

72. Convert 101 000 N/m^2 to mbar.

Solution: 101 000 N/m^2 = 101 000 ÷ 100 = 1 010 mbar

SC2.9 –SOLVE PROBLEMS USING THE BASIC GAS LAWS

The Gas Laws

73. Air, and other gases, used on aircraft, will not necessarily be at atmospheric pressure. Oxygen bottles, accumulators, tyres, air bottles etc. often contain pressure well above atmospheric pressure. The pressure in these containers can vary with changes in volume or temperature. Variations in pressure are predictable and can be calculated using 3 laws; Boyle's Law, Charles' Law and the Combined Gas Law.

74. The use of each of the laws will only be achieved correctly when the correct units of pressure, volume and temperature are used.

Pressure

75. Absolute pressure must be used in all gas law calculations. Absolute pressure is the sum of the gauge pressure (the pressure being measured on an aircraft skin gauge, tyre pencil gauge manometer or digital pressure gauge) and the local atmospheric pressure.

Absolute Pressure = Gauge Pressure + Atmospheric Pressure.

76. The pressures used in calculations involving the gas laws are normally expressed in SI units (Pascal). However they can be left as bar, millibar, N/m^2 or psi as long as the units are consistent and not mixed within the calculations.

Examples

77. If an aircraft's local atmospheric pressure is 99.23 kN/m^2 and the gauge pressure in the aircraft's accumulator is 1.379 MN/m^2, calculate the absolute pressure in the accumulator.

Absolute Pressure = Gauge Pressure + Atmospheric Pressure.

Therefore, Absolute Pressure = $1.379 \times 10^6 + 99.23 \times 10^3$

Absolute Pressure = 1.47823 MN/m^2

78. If the absolute pressure in an aircraft tyre is 124.8 psi and the atmospheric pressure is 14.8 psi, calculate the pressure the technician would expect to read on the tyre pressure gauge when checking it.

Absolute Pressure = Gauge Pressure + Atmospheric Pressure.

So, Gauge Pressure = Absolute Pressure - Atmospheric Pressure.

Gauge Pressure = 124.8 – 14.8 psi

Gauge Pressure = 110 psi

Volume

79. As with pressure, volume is normally expressed in SI units (cubic metre). However, as long as the units of volume are consistent throughout calculations involving the gas laws, cubic feet, inches, millimetres etc. can also be used.

Temperature

80. Kelvin is the only unit that can be used in gas law calculations. All other units must be converted to Kelvin. Normally the conversion will be from Celsius to Kelvin.

Note: When converting from Celsius to Kelvin add 273.15 to the Celsius temperature and when converting from Kelvin to Celsius subtract 273.15 from the Kelvin temperature.

e.g. 35 °C = 35 + 273.15 = 308.15 K

And 445 K = 445 – 273.15 = 171.85 °C

Boyle's Law

81. In the 1660s Robert Boyle studied the relationship between the pressure and volume of a gas where the temperature remains constant (this is known as an isothermal process). He studied a syringe of gas and altered its volume by pushing in a plunger. The volume was calculated from a graduated scale on the side of the syringe and the pressure measures using an attached pressure gauge. He found that when the volume was halved, the pressure was doubled. This relationship is called "Boyle's Law".

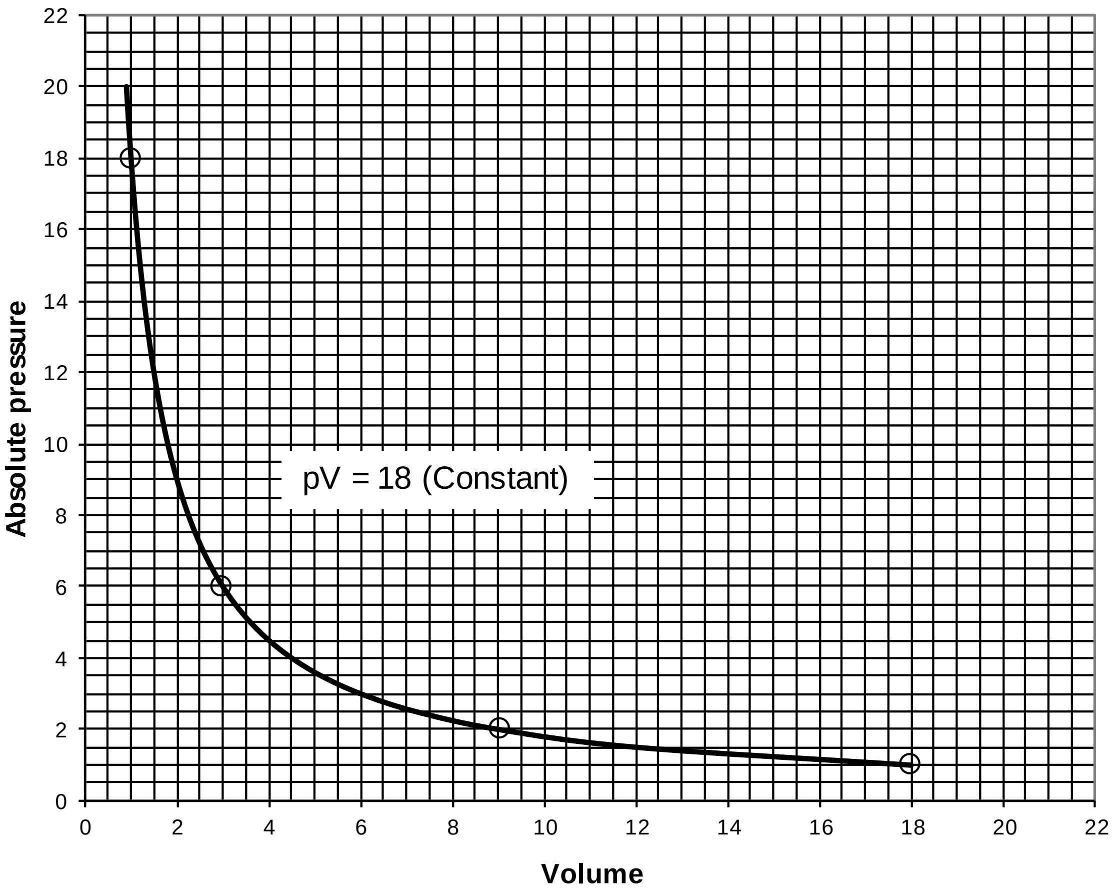


Figure 31 – Graph of pV = C (Constant)

82. If a syringe of gas has a pressure gauge attached to it, and the plunger in the syringe is moved slowly down so the temperature remains the same, the gas will be

compressed and the volume reduced. If a graph of the absolute pressure against volume were plotted from this experiment, it would be like the one shown in Fig. 31. It can be seen that all through the compression the pressure (p) times the volume (V) equals a constant (C). In this case pV = 18. All so-called perfect gases behave in this same way.

83. From this, Boyle produced a law, which states,

"The volume (V) of a fixed mass of gas is inversely proportional to the pressure (p), provided the temperature remains constant".

i.e. $p \propto \frac{1}{V}$ or pV = Constant

Or as a formula: $p_1V_1 = p_2V_2$

Example

84. Air in an accumulator chamber has an original volume of 0.025 m^3 and is compressed slowly to a volume of 0.002 m^3. If the initial absolute pressure was 2.3 bar and the temperature of the air remained constant, calculate the final absolute pressure.

p_1 = 2.3 bar V_1 = 0.025 m^3 V_2 = 0.002 m^3

$$p_1V_1 = p_2V_2$$

So, $$p_2 = \frac{p_1V_1}{V_2} \qquad p_2 = \frac{2.3\times0.025}{0.002}$$

p_2 = 28.75 bar.

Charles's Law

85. Towards the end of the 18th century, Jacques Charles investigated the relationship between the temperature and volume of a gas, while the pressure remains constant (this is known as an isobaric process). His research was prompted by an interest in flight using a hot air balloon. He found that when air was heated, its volume increased in direct proportion to the rise in absolute temperature, provided the pressure remained constant. This result, which applies to all gases, is known as Charles's Law.

86. Charles's law states,

"The volume (V) of a fixed mass of gas is proportional to the absolute temperature (T), provided the pressure remains constant."

i.e. $V \propto T$ or $\frac{V}{T}$ = Constant

Or as a formula: $$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Example

87. A balloon in a room heated to 25 °C has an original volume of 500 in^3. When taken outside the volume drops to 475 in^3. Calculate the outside temperature if the pressure in the balloon remains constant.

T_1 = 25 °C = 25 + 273.15 = 298.15 K V_1 = 500 in^3 V_2 = 475 in^3

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

So, $T_2 = \frac{V_2 T_1}{V_1}$ $T_2 = \frac{475 \times 298.15}{500}$

T_2 = 283.2 K   = 283.2 – 273.15 = 10.1 °C

Gay-Lussac's Law

88. French chemist Joseph Louis Gay-Lussac was, amongst other things, investigating the properties of gases and the law of combining volumes before and after chemical reactions. He also was credited with the law concerning pressure and temperature relationships for a sample of gas often known as Amontons' Law.

> **"The pressure of a gas of fixed mass and fixed volume is directly proportional to the gas's absolute temperature."**

89. The relationship holds true because as temperature is a measure of the average kinetic energy of a substance; if its kinetic energy was increased, the particles would collide with the walls of the container more rapidly and therefore its pressure would increase.

Combined Gas Law

90. The above 3 laws can be combined to form an equation that incorporates all 3 variables, pressure, volume and temperature:

Combined gas law gives: $\frac{pV}{T} = \text{Constant}$

Or as a formula: $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$

91. This is the most useful equation to remember because if the temperature is constant (Boyle's Law), T_1 and T_2 can be cancelled out; or if the pressure is constant (Charles' law), p_1 and p_2 can be cancelled out; or if the volume is constant (Gay-Lussacs' law) V_1 and V_2 can be cancelled.

92. This means that you can use this formula even if you have forgotten Boyle's or Charles' or Gay-Lussacs' formulae.

Examples

93. A balloon of original volume 0.12 m^3 is released from the ground where its temperature and absolute pressure of the air within it are 15 °C and 100 kN/m^2 respectively. Calculate the new pressure if it rises to a height where the volume has expanded to 0.15 m^3 and the temperature has dropped to -60 °C.

V_1 = 0.12 m^3 T_1 = 15 °C = 15 + 273.15 = 288.15 K p_1 = 100 kN/m^2

V_2 = 0.15 m3 T_2 = - 60 °C = - 60 + 273.15 = 213.15 K

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

so,

$$p_2 = \frac{p_1V_1T_2}{T_1V_2}$$

$$p_2 = \frac{100x10^3 \text{ x } 0.12 \text{ x } 213.15}{288.15 \text{ x } 0.15}$$

p_2 = 59.18 kN/m^2

94. An aircraft tyre at an absolute pressure of 90 psi and temperature of 10 °C is heat soaked until the temperature is 35 °C and then cooled until it is – 30 °C. Calculate the new absolute pressures at these temperatures if the volume is assumed to remain constant.

$V_1 = V_2 = V_3$ T_1 = 10 °C = 10 + 273.15 = 283.15 K p_1 = 90 psi

T_2 = 35 °C = 35 + 273.15 = 308.15 K T_3 = - 30 °C = -30 + 273.15 = 243.15 K

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

so,

$$p_2 = \frac{p_1V_1T_2}{T_1V_2}$$

$$p_2 = \frac{90 \text{ x } V_1 \text{ x } 308.15}{283.15 \text{ x } V_2}$$

p_2 = 97.95 psi

$$\frac{p_1V_1}{T_1} = \frac{p_3V_3}{T_3}$$

so,

$$p_3 = \frac{p_1V_1T_3}{T_1V_3}$$

$$p_3 = \frac{90 \text{ x } V_1 \text{ x } 243.15}{283.15 \text{ x } V_3}$$

p_3 = 77.29 psi

Exercise 9

1. $4\ m^3$ of gas at an absolute pressure of 100 kN/m^2 is expanded to 8 m^3 at constant temperature. Calculate its new pressure.

2. Gas at a temperature of 40 °C, and having a volume of 6 m^3, is heated to 200 °C. If the pressure remains constant, calculate its new volume.

3. A quantity of gas occupies a volume of 0.3 m^3 at an absolute pressure of 200 mb and temperature of 12 °C. The gas is compressed until the pressure and temperature are 700 mb and 135 °C respectively. If there is no loss of gas, what volume will it now occupy?

4. An accumulator with no hydraulic pressure acting on it contains nitrogen at an absolute pressure of 800 psi and temperature of 15 °C. If the hydraulics are activated, the nitrogen is compressed into a volume of 43.2 in^3 at a pressure of 3000 psi. If the temperature also increases to 38 °C, calculate the initial volume of the accumulator.

5. A tyre is inflated so it has an absolute pressure of 150 psi at 11 °C. What is the new absolute pressure if the tyre is heat soaked at 36 °C, assuming the volume remains constant.

6. A hot air balloon contains 135 m^3 of air at 23 °C, what volume will it occupy at 33 °C assuming the pressure remains constant.

7. An air compressor receives 4 m^3 of air at a pressure of 102 kN/m^2 absolute and compresses it to a pressure of 1224 kN/m^2 absolute. Calculate the final volume of the air if no change in temperature takes place.

8. If 2 m^3 of gas at 50 °C is heated at constant pressure until its volume is doubled, what will be the final temperature?

9. A volume of 5 m^3 is compressed from initial conditions of 110 kN/m^2 absolute and 12 °C to final conditions of 440 kN/m^2 absolute and 127 °C. Calculate the final volume of the air.

10. Air is compressed in a cylinder from an absolute pressure of 1.05 bar and a temperature of 20 °C to one-quarter of its original volume at constant temperature. What is the resulting pressure? The air is then heated at constant pressure until it occupies its original volume. Calculate the final temperature of the air in degrees Celsius.

SC2.10 – EXPLAIN THE DIFFERENCE BETWEEN GAUGE PRESSURE AND ABSOLUTE PRESSURE

Atmospheric, gauge and absolute pressure

95. Atmospheric pressure is the pressure exerted at a point by the weight of air above it. Atmospheric pressure decreases with altitude.

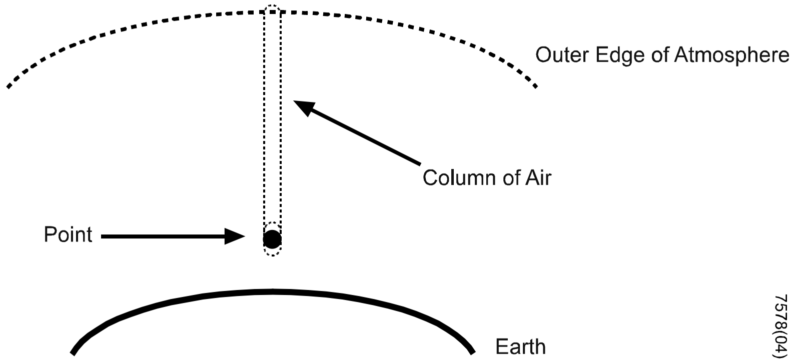


Figure 32 - Atmospheric Pressure

96. A barometer is an instrument used to measure atmospheric pressure. Figure 33(a) shows a simple barometer, where a tube is filled with mercury and inverted into a bath of mercury. The level of the mercury will fall to a height of 760 mm, leaving a vacuum above it. As the pressure of the day varies so does the column of mercury. The simple barometer forms the basis of the Fortin barometer Figure 33(b). It is more accurate and safer than the simple barometer.

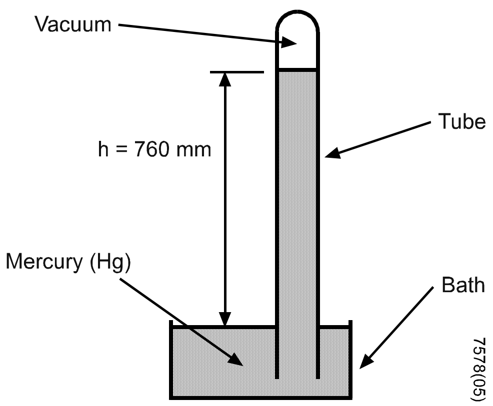


Figure 33(a) - Simple Barometer

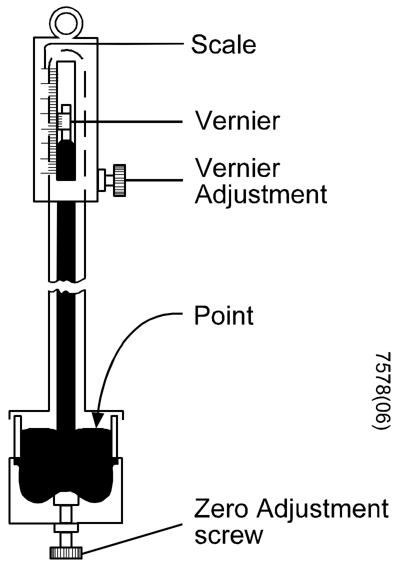


Figure 33(b) - Fortin Barometer

97. Gauge pressure is the pressure measured relative to atmospheric pressure. The pressure in a car tyre is gauge pressure. The pressure in the tyre is above atmospheric pressure.

98. In a closed vessel, gas molecules travel with a random pattern colliding with each other and the wall. The pressure exerted by the gas is the result of these collisions against the wall as in figure 34(a). Absolute zero pressure is found in a perfect vacuum due to the total absence of gas molecules as in figure 34(b) below

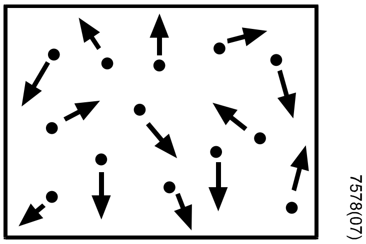


Figure 34(a) - Random Movement of Gas Molecules

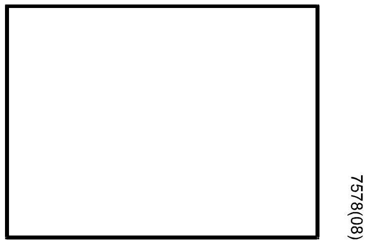


Figure 34(b) - No Molecules – No Pressure

99. Absolute pressure is the sum of the atmospheric pressure and gauge pressure. It is the pressure measured from absolute zero pressure. Figures 35(a) & (b) show positive and negative gauge pressures.

Absolute Pressure = Atmospheric Pressure + Gauge Pressure

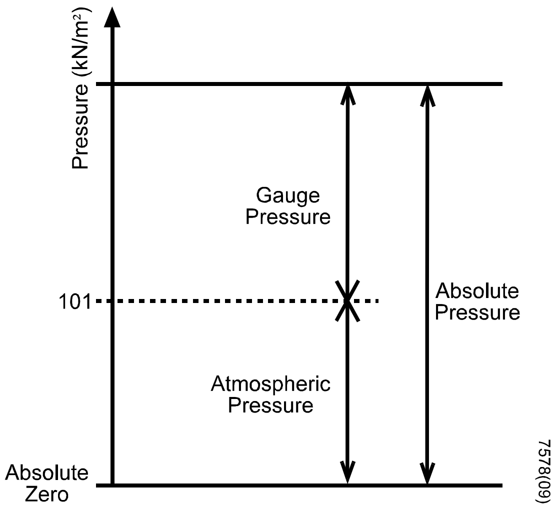


Figure 35(a) - Positive Gauge Pressure

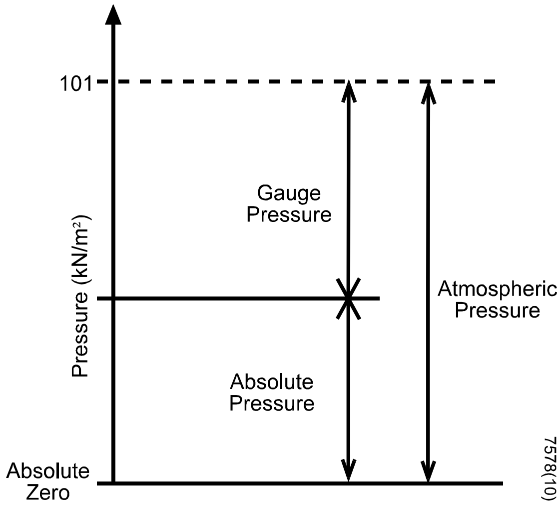


Figure 35(b) - Negative Gauge Pressure

Example:

100. Convert a gauge pressure of 534 kN/m^2 to absolute pressure. Give the answer in bar. (Atmospheric pressure = 101 300 N/m^2.)

$$P_{absolute} = P_{atmospheric} + P_{gauge}$$

$$= 101\ 300 + 534\ 000\ \text{N/m}^2$$

$$= 635\ 300\ \text{N/m}^2$$

$$= \frac{635\ 300}{100\ 000}\ bar$$

$$P_{absolute} = 6.353\ \text{bar}$$

Exercise 10

1. Define pressure and state the SI units of pressure.

2. What would be the absolute pressure in a perfect vacuum?

3. A force of 3 000 N acts uniformly over an area of 3 m^2. Calculate the pressure acting upon the area.

4. A pressure of 800 N/m^2 acts uniformly over a plate whose area is 3 m^2. What is the total force acting on the plate?

5. A force of 5 kN acts on the piston of a hydraulic cylinder. If the diameter of the cylinder is 200 mm, find the pressure of the liquid in the cylinder in bars

6. A piston operating within a hydraulic cylinder has a diameter of 50 mm. It is moved by oil operating at a pressure of 5 bar. What force is exerted on the piston?

7. Given that the atmospheric pressure is 101 300 N/m^2, convert the following gauge pressures to absolute pressures in bar.

    a. 630 kN/m^2
    b. 1.057 kN/m^2
    c. 5 kN/m^2

SC2.11 – SOLVE PROBLEMS INVOLVING ATMOSPHERIC, GAUGE AND ABSOLUTE PRESSURE

Differential and ambient pressure

101. Atmospheric pressure is the pressure exerted on a body by the atmosphere and decreases with altitude.

102. The differential pressure is the difference between the internal and external pressure of a body e.g. an aircraft.

Differential pressure = Internal pressure – External pressure

103. The ambient pressure is the pressure surrounding a body. It could be the pressure surrounding an aircraft or the internal pressure surrounding the crew.

104. The pressure, temperature and density of the air vary with altitude, prevailing climatic conditions and from day to day and point to point on the earth. There is a need therefore to standardise the average conditions at various areas/regions of the world so that aircraft can be designed and their performance assessed under these different operational conditions.

105. The International Standard Atmosphere (ISA) is an agreed set of values representing these average operational conditions at various aeronautical centres around the world. These set the sea level conditions expected and the variation of pressure, temperature, density and viscosity with altitude from observations.

106. The ISA Values for the Northern European ISA (which applies to the UK) are set at sea level values of:

Temperature	288.2 K (15.0 ºC)
Pressure	1 013.25 mbar (1.01325 bar or 101 325 N/m^2)
Density	1.225 kg/m^3

Note: An example ISA table is shown in Annex A at the end of this booklet and will be used in more detail in the Aerodynamics unit.

Examples:

107. If the ambient cabin pressure in an aircraft is 1 013 mbar, what is the differential pressure at 11 km in mbar and N/m^2?

Atmospheric pressure from ISA table = 226.32 mbar

Differential pressure = Internal pressure – External pressure

Differential pressure = 1 013 – 226.32 mbar

= 786.68 mbar = 78 668 N/m^2

108. What is the atmospheric pressure at an altitude of 5 500 m?

Atmospheric pressure at 5 000 m = 540.2 mbar

Atmospheric pressure at 6 000 m = 471.81m

Difference = 68.39 mbar

500m is half the difference = 34.195 mbar

Atmospheric pressure at 5 500 m = 540.2 – 34.195 mbar

= 506.005 mbar

Exercise 11

1. If the ambient cabin pressure in an aircraft is 1 000 mbar, what is the differential pressure at 5 km in mbar and N/m^2?

2. If the ambient cabin pressure in an aircraft is 1 000 mbar, what is the differential pressure at 2 km in mbar and N/m^2?

3. What is the atmospheric pressure at the following altitudes?

   a. 8 000 m

   b. 8 500 m

SC2.12 – EXPLAIN DENSITY AND RELATIVE DENSITY

Density and relative density

109. Density is defined as the mass per unit volume and is given the symbol ρ (rho).

$$\rho = \frac{m}{V}$$

ρ = density in kg/m^3.
m = mass in kg.
V = volume in m^3.

Example:

110. A block of steel is 300 mm $\times$ 200 mm $\times$100 mm and has a mass of 46.8 kg, Calculate the density of the block.

$$\rho = \frac{m}{V}$$

$$= \frac{46.8}{0.3 \times 0.2 \times 0.1}$$

$$\rho = 7\ 800 \text{ kg/m}^3$$

111. The density of water is 1 000 kg/m^3.

112. The relative density of a substance is the ratio of the density of the substance to the density of another reference material. The density of water (1 000 kg/m^3) is used as the reference material (often known as the specific gravity).

Formula for relative density: $RD = \frac{\rho_{substance}}{\rho_{water}}$

Example:

113. Calculate the relative density of steel given the density of steel is 7 800 kg/m^3 and the density of water is 1 000 kg/m^3.

$$RD = \frac{\rho_{substance}}{\rho_{water}}$$

$$= \frac{7\ 800}{1\ 000}$$

$$RD = 7.8$$

Exercise 12

1. Calculate the relative density of concrete that has a density of 2 400 kg/m^3.

2. A tank that measures 1.2 m × 0.8 m × 0.3 m is half full of AVGAS of density 800 kg/m^3. Calculate its weight and relative density.

3. Given that lead has a density of 11 400 kg/m^3, calculate the volume that lead having a weight of 200 N would occupy.

SC2.13 – CALCULATE PRESSURES IN LIQUIDS USING BASIC PHYSICAL MEASUREMENT

Pressure at any point in a fluid

114. The pressure at a point in a liquid is produced by the weight of liquid immediately above the point as in figure 36 below. Also, the formula for pressure at a depth is derived.

$$p = \frac{F}{A} = \frac{W}{A} = \frac{mg}{A}$$

$$p = \frac{\rho vg}{A} \qquad \left(from\ \rho = \frac{m}{v} \right)$$

$$p = \frac{\rho Ahg}{A} \qquad \text{(from v = Ah)}$$

$$p = \rho gh$$

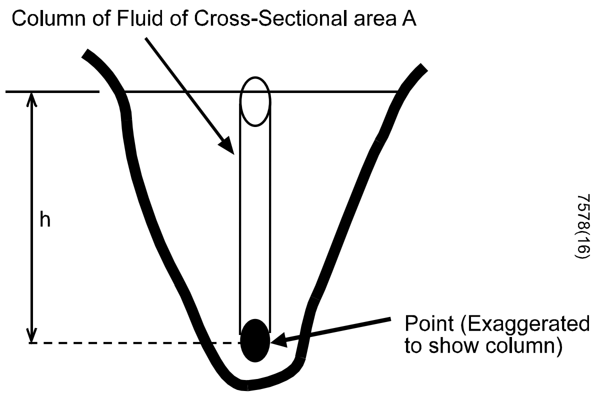


Figure 36 - Water Reservoir

$P = \rho$ gh where: P = Pressure in N/m^2
ρ = Density in kg/m^3
g = acceleration due to gravity in m/s^2
h = height or depth of fluid in metres.

115. Pressure varies with height or depth of water. The deeper a submarine submerges the greater the pressure will be on its hull.

116. When used as a gauge, the greater the height or depth of fluid the greater the pressure. This can be seen in figure 37 below where the lowest pipe emits water at the greatest pressure.

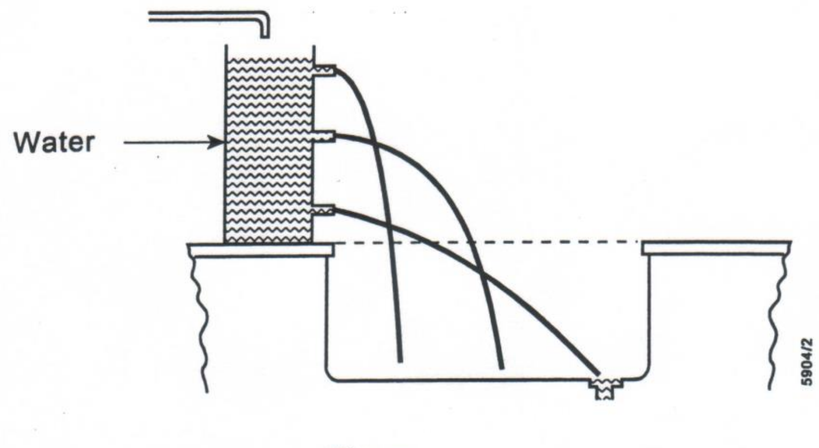


Figure 37

Examples:

117. The barometer in figure 38, reads 760 mm Hg (mercury). Given that mercury has a density of 13 600 kg/m³, calculate the atmospheric pressure of the day.

$p = \rho g h$

$p = 13\,600 \times 9.81 \times 0.760$

$p = 101\,396$ N/m²

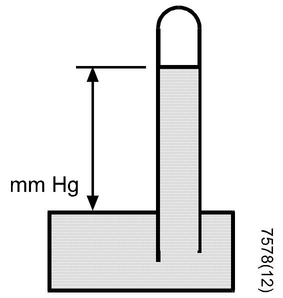


Figure 38 - Simple Barometer

118. Figure 39 shows a basic manometer tube filled with water used to measure gauge pressure in the pipe. The pressure of the gas on the surface of the water in the right hand limb is the same as that at the same level in the left hand limb; i.e. at the same level in the same fluid. This pressure is known as gauge pressure.

Gas pressure $= p_{gauge}$
$= \rho gh$
$= 1\,000 \times 9.81 \times 0.300$
$= 2\,943$ N/m²

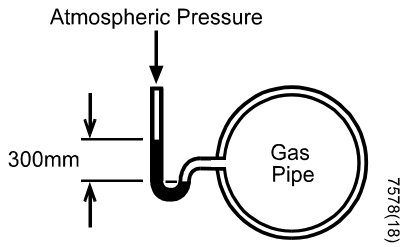


Figure 39 - Manometer

119. The gauge pressure of a gas in a cylinder measures 350 mm of oil (figure 40). The oil has a density of 960 kg/m³. The pressure of the day is measured using a barometer (figure 41), which reads 740 mm Hg. Calculate the absolute pressure of the gas.

$P_{absolute} = p_{gauge} + p_{atmospheric}$

$p_{gauge} = \rho gh$

$= 960 \times 9.81 \times 0.35$

$p_{gauge} = 3\,296$ N/m²

$p_{atmospheric} = \rho gh$

$= 13\,600 \times 9.81 \times 0.74$

$p_{atmospheric} = 98\,728$ N/m²

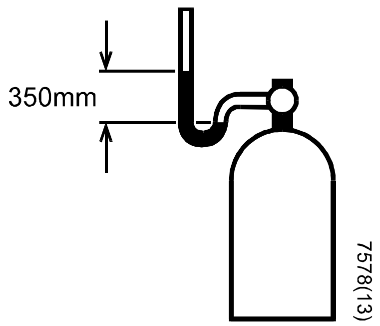


Figure 40 - Manometer

$$P_{absolute} = 3\,296 + 98\,728$$

$$= 102\,024 \text{ N/m}^2$$

$$P_{absolute} = 102 \text{ kN/m}^2$$

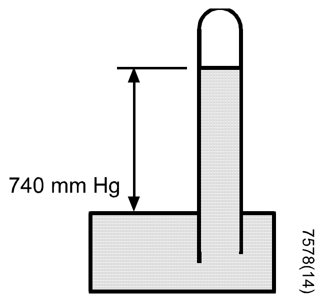


Figure 41 - Simple Barometer

Exercise 13

1. A manometer reads 2.65 m. If the working fluid has a relative density of 0.8, calculate the gauge pressure being indicated, both in N/m^2 and bar.

2. Given atmospheric pressure as 101.4 kN/m^2 and that 'sea water' has a relative density of 1.02, calculate the absolute pressure on the hull of a submarine at depth of 10 m. (Give your answer both in N/m^2 and in bar.)

3. The manometer on a gas pipe reads 240 mm of oil with a density of 1 200 kg/m^3. A barometer alongside the pipe reads 745 mmHg, density 13 600 kg/m^3. Calculate the absolute pressure of the gas both in N/m^2 and bar.

120. Consider the beaker of water shown below in figure 42. The dotted line indicates the boundary of an imaginary body of water of weight W within the main body of the water. The arrows represent the pressures exerted by the water on this imaginary body. (As seen previously, pressure increases with depth and therefore the pressures below the body will be greater than those above it).

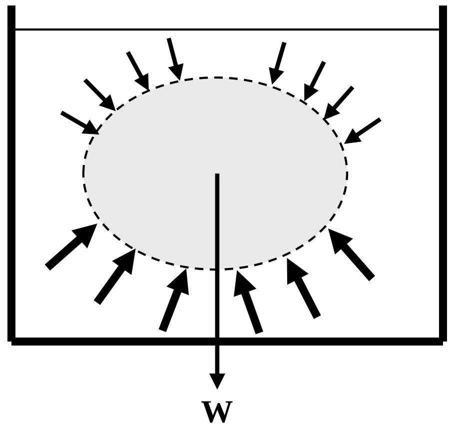


Figure 42 - Beaker of water

121. Provided there are no outside influences such as heating or stirring of the water, this imaginary body of water will be remain stationary. It is in equilibrium and the sum of the forces acting on it must therefore be zero. Its weight is exactly balanced by the differences in the pressure exerted by the water around it.

122. Now consider replacing the imaginary body of water with a solid body of exactly the same size and shape.

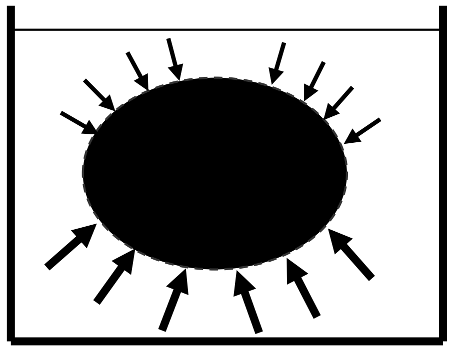


Figure 43 - Upthrust

123. The pressure changes around this body will be exactly the same as around the imaginary body of water that it has replaced and it will therefore experience the same upward force. This force, known as upthrust, will be equal to the weight of the water displaced by the body.

124. A body immersed in a fluid experiences an upthrust equal to the weight of fluid displaced.

125. A ship will float when the upthrust is equal to the weight of the ship. Consider a straight - sided flat bottomed ship of weight W and waterline area A floating in water of density ρ and floating to a depth of h metres (This is known as the draught of the ship).

Upthrust force = weight of water displaced

But force = pressure x area = weight of water displaced

and pressure in fluid = ρgh

Upthrust force = $\rho gh \times A$ = weight of water displaced

When the ship is floating the upthrust must equal the weight of the ship W.

Therefore $\rho gh \times A = W$

Example:

126. A ship weighs 1 059.5 kN and its waterline area is 72 m^2. Find its draught in:

(a) Fresh water, density 1 000 kg/m^3

(b) Sea water, density 1 025 kg/m^3.

(a) Fresh water: $\rho gh \times A = W$

$$h = \frac{W}{\rho g A}$$

$$h = \frac{1\,059.5 \times 10^3}{1\,000 \times 9.81 \times 72} \text{ m}$$

Fresh water draught (h) = 1.5 m

(b) Sea water. $\rho gh \times A = W$

$$h = \frac{W}{\rho g A}$$

$$h = \frac{1\,059.5 \times 10^3}{1\,025 \times 9.81 \times 72} \text{ m}$$

Salt water draught (h) = 1.46 m

127. A balloon is immersed in a fluid (air) and experiences a lift (upthrust) equal to the weight of air that it displaces. If this lift is greater than the total weight of the balloon (the balloon itself, the gas inside it and its payload) then the balloon will rise. If it is less then the balloon will fall.

128. Hot air and helium filled balloons rise because the gas inside them is significantly less dense than the air they displace. This allows the total weight to be reduced below the weight of the air displaced by the balloon so that it will rise.

Lift (upthrust) = Weight of air displaced

$$L = \text{Density of air} \times \text{Volume of balloon} \times g$$
$$L = \rho \times V \times g$$

Where L = Lift experienced by the balloon in N
ρ = Density of air surrounding the balloon in kg/m^3.
V = Volume of the air displaced in m^3.
g = acceleration due to gravity in m/s^2.

129. Changes in the temperature of the atmosphere will affect the density of the air in which the balloon is flying and therefore will affect the lift.

130. A decrease in the temperature of the atmosphere at any given height will increase the density of the air and the lift will increase. An increase in the temperature of the atmosphere will decrease its density and the lift (upthrust) will decrease. As with all airborne vehicles, therefore, the performance of a balloon will reduce with an increase in ambient temperature.

Example

131. Calculate the change in lift generated by balloon of volume 500 m^3, when a decrease in the temperature of the atmosphere increases the density of the air from 1.112 kg/m^3 to 1.225 kg/m^3

$$\text{Increase in lift} = (\rho_2 \times V \times g) - (\rho_1 \times V \times g)$$

$$= (1.225 \times 500 \times 9.81) - (1.112 \times 500 \times 9.81) \text{ N}$$

$$\text{Increase in lift} = 554.3 \text{ N}$$

Exercise 14

1. A ship has a weight of 2 000 kN. The area at the waterline is 50 m^2, Calculate the draught of the ship (how deeply it floats) in:

a. Fresh water of density 1 000 kg/m^3

b. Sea water of density 1 025 kg/m^3.

2. Calculate the change in lift on a balloon of volume 800 m^3, when a decrease in temperature increases the density of the air from 1.006 kg/m^3 to 1.112 kg/m^3

Statics Consolidation

1. List the 3 properties of a force.

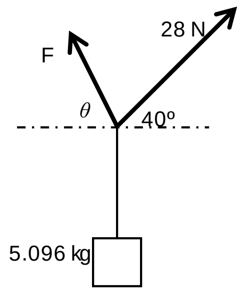
2. State the 2 conditions required for equilibrium.

3. With three examples of each, explain the difference between Scalar and Vector quantities.

4. By graphical drawing, determine the Resultant and Equilibrant (magnitude and direction) of adding the following pair of vectors.

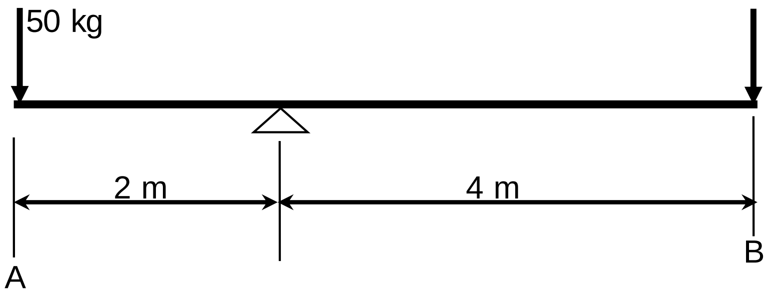
$$10\ \text{N} \angle 60^{o} \text{ and } 15\ \text{kN} \angle 40^{o}$$

5. Find the magnitude and direction of force F that would keep the 5.096 kg box suspended in the air as shown.

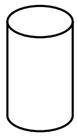


6. What is the force required at point B to keep the beam in equilibrium?

What would be the reactive force at the pivot support?
(Ignore the mass of the beam)



7. A cylindrical material that weighs 15 N, has a diameter of 60 mm and a height of 20 cm. Calulate its density, relative density and the pressure it exerts on the floor if it stands on its face.



8. Calculate the absolute pressure when the gauge pressure is 25.8 kN/m^2 and the atmospheric pressure is 1 013 mbar.

9. Find the density and relative density of a material which has a mass of 42.8 kg and dimensions of 20 cm $\times$ 10 cm $\times$ 10 cm.

10. What is the mass of 2.5 litres of water?

11. What is the weight of 2.5 litres of water?

12. An aircraft tyre pressure is measured at 85 psi, inside the hangar where the temperature is 12 ^{0}C. The aircraft is then moved out into the sun where the temperature rises to 25 ^{0}C. Assuming the volume of air inside remains constant what would the pressure now be?

Answers to Exercises

Exercise 1 p8

1.	a.	12.7 N at 40 ^{0}	b.	12.2.kN at 18.5 ^{0}
	c.	14.3 N at 17 ^{0}	d.	11.6 kN at 336 ^{0} (-24 ^{0})

Exercise 2 p10

1.	a.	13.3 N at 214 ^{0}	b.	10.3 kN at 61 ^{0}

Exercise 3 p14

1.	a.	14.4 N at 18 ^{0}	b.	12.2.kN at 53 ^{0}
	c.	92 N at 312 ^{0} (-48 ^{0})	d.	58N at 170 ^{0}
2.	a.	46 kN at 184 ^{0}	b.	9.8.N at 112 ^{0}

3. F_A = 260 N F_B = 166 N 4. Lift = 60 kN Thrust = 50 kN

5. Lift = 79 kN C_f = 40 kN

Exercise 4 p24

1. The turning effect of a force about a point. 2. Anticlockwise

3. 100 Nm 4. 294.3 Nm

5. 178.5 Nm 6. 696.3 Nm

Exercise 5 p25

1. 55 kNm 2. -118 kNm (anticlockwise)

Exercise 6 p27

1. 25 Nm 2. 5.71 kN 3. 385 N

Exercise 7 p32

1. Force is a vector with magnitude, direction and point of application. It is a push or a pull.

2. a. 268 Nm b. -550 Nm

3. 148 N downwards 4. 1.93 kN 5. 73.3 mm

6. 570 N 7. 2.5 kN 8. 11.43 N

Exercise 8 p40

1. 5.99 m 2. 16.25 m 3. 10.91 m

4. 10.67 m, shift = 0.24 m forwards

Exercise 9 p 54

1. 50 kN/m^2 2. 9.07 m^3 3. 0.123 m^3 4. 150 in^3

5. 163.2 psi 6. 139.6 m^3 7. 0.33 m^3 8. 373 ^0C

9. 1.754 m^3 10. 4.20 bar, 899 ^0C

Exercise 10 p58

1. Force per unit area. Units: N/m^2 ; Pascal or bar. 2. Zero.

3. 1 000 N/m^2 4. 2.4 kN 5. 1.59 bar

6. 982 N 7. a. 7.313 bar b. 1.024 bar c. 1.063 bar

Exercise 11 p60

1. 459.8 mbar, 45980 N/m^2

2. 205.05 mbar, 20505 N/m^2

3. a. 356 mbar b. 331.71 mbar

Exercise 12 p62

1. 2.4

2. 1 130 N, 0.8

3. 1.79×10^{-3} m^3

Exercise 13 p65

1. 20.8 kN/m^2, 0.2 bar

2. 201.5 kN/m^2, 2 bar

3. 102.2 kN/m^2, 1 bar

Exercise 14 p68

1. a. 4.08 m b. 3.98 m

2. Increase of 831.9 N

Answers to Statics Consolidation

1	Magnitude, direction and point of application
2	Sum of forces = 0 and sum of moments = 0
3	Scalar magnitude only, Vector magnitude and direction. See Statics C.M. Page 2/3
4	24.6 kN $\angle$ 48.0 ^{0}
5	38.5 N $\angle$ 123.8 ^{0} (θ = 56.2 ^{0})
6	245.25 N **;** 735.75 N
7	2 703.958 kg/m^3 **;** 2.704 **;** 5 305.165 N/m^2
8	127.1 kN/m^2 or 1 271 mbar
9	21 400 kg/m^3 ; 21.4
10	2.5 kg
11	24.525 N
12	88.88 psi

SCIENCE DEFINITIONS

Book 2 – Statics

Scalar	Quantity with magnitude only.
Vector	Quantity with magnitude, direction and point of application.
Mass	Amount of matter in a body – units are kg.
Weight	Force created when gravity acts on a mass. F = ma W = mg
Force	Push or a pull that changes a bodies state of rest or uniform motion in a straight line. Vector quantity - units are Newtons (N).
Resultant	Value of one force that has the same effect as several forces.
Equilibrium	A system where the vector sum of the forces is zero and the sum of the moments (clockwise and anti-clockwise) is zero.
Equilibrant	A force equal in magnitude but opposite in direction to the Resultant.
Moment	The turning effect of a force about a point - the product of the force and perpendicular distance from the point - units are Newton metre (Nm).
Torque	Turning effect of a force about an axis of rotation - the product of a force and perpendicular distance from the axis of rotation - units are Nm.
Centre of gravity	The point from which the weight of a body acts. If body is a symmetrical solid then its C of G is at geometric centre.
Pressure or Pascal (Pa)	Force per unit area - units are Newton per metre squared (N/m^2)
Density	Mass per unit volume - units are kg per metres cubed (kg/m^3).
Relative density	Density of a substance with relation to pure water - No units.
Absolute pressure	Pressure measured relative to a vacuum. Atmospheric pressure + Gauge pressure.
Atmospheric pressure	Caused by the column of air above.
Gauge pressure	Pressure measured relative to atmospheric pressure. Using a gauge.

Differential pressure	A difference of 2 pressures. Normally internal – external.
Ambient pressure	Pressure surrounding a body.
Barometer	Device to measure atmospheric pressure.
Manometer	Device to measure low pressure gas in pipelines.
Boyle's Law	Constant temperature. Volume inversely proportional to Pressure.
Charles's Law	Constant pressure. Volume proportional to Temperature.
Gay Lussac's Law	Pressure and Volume is proportional to Absolute Temperature.
Combined Gas Law	Combination of above : $P_1V_1 / T_1 = P_2V_2 / T_2$ must use absolute Pressure and Kelvin temp

ANNEX A

INTERNATIONAL STANDARD ATMOSPHERE

Height km	Pressure mbar	Density kg/m^3	Temperature K	Viscosity kg/m/s
0	1013.25	1.225	288.2	1.789×10^{-5}
1	898.75	1.112	281.7	1.758×10^{-5}
2	794.95	1.006	275.2	1.736×10^{-5}
3	701.09	0.909	268.7	1.694×10^{-5}
4	616.40	0.819	262.2	1.661×10^{-5}
5	540.20	0.736	255.7	1.628×10^{-5}
6	471.81	0.660	249.2	1.595×10^{-5}
7	410.61	0.589	242.7	1.561×10^{-5}
8	356.00	0.525	236.2	1.527×10^{-5}
9	307.42	0.466	229.7	1.493×10^{-5}
10	264.36	0.413	223.2	1.458×10^{-5}
11	226.32	0.364	216.7	1.422×10^{-5}
12	193.30	0.311	216.7	1.422×10^{-5}
13	165.10	0.265	216.7	1.422×10^{-5}
14	141.02	0.227	216.7	1.422×10^{-5}
15	120.45	0.194	216.7	1.422×10^{-5}
16	102.87	0.165	216.7	1.422×10^{-5}
17	87.87	0.141	216.7	1.422×10^{-5}
18	75.05	0.121	216.7	1.422×10^{-5}
19	64.10	0.103	216.7	1.422×10^{-5}
20	54.75	0.088	216.7	1.422×10^{-5}