Defence School of
Aeronautical Engineering

Aerosystems Engineer & Management Training School

Academic Principles Organisation

SCIENCE

BOOK 3 - Dynamics

WARNING

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ENABLING OBJECTIVE

SC3 Explain the basic principles of motion and how this relates to aircraft

KEY LEARNING POINTS

SC3.1 Define speed, velocity and acceleration

SC3.2 State Newton's laws of motion

SC3.3 Explain the relationships $F = ma$ and $W = mg$

SC3.4 Define the equations of linear motion for constant acceleration

SC3.5 Solve problems related to an aircraft in flight using the equations of motion

SC3.6 Define basic terms for angular motion

SC3.7 Define terms for oscillating motion

SC3.8 Explain simple harmonic motion in terms of mass-spring and simple pendulum systems

SC3.9 Calculate the natural frequency of small oscillations in a pendulum

SC3.10 Define basic terms for simple machines

SC3.11 Solve problems involving levers, pulleys, screw-jacks and gears

SC3.12 Define basic terms used in simple gyroscopes

SC3.13 Define work, energy and power

SC3.14 Explain the conservation of energy principle

SC3.15 Solve problems involving potential and kinetic energy

SC3.16 Define basic terms used with friction

SC3.17 Explain viscosity

SC3.18 Solve simple problems involving friction on a horizontal surface

Exam

1 Question from SC3.1 to SC3.3
1 Question from SC3.4 to SC3.5
1 Question from SC3.6 or SC3.12
1 Question from SC3.7 to SC3.9
1 Question from SC3.10 to SC3.11
1 Question from SC3.13 to SC3.15
1 Question from SC3.16 to SC3.18

SC3.1 - DEFINE SPEED, VELOCITY AND ACCELERATION

Definition of quantities used

1. Displacement is a vector quantity measuring the change of position of a body in a given direction. In other words, it is the straight-line measurement between two points, including both the length of the line and its angle to some prescribed datum.

2. Distance is the change of position of a body independent of direction. When a body moves, the distance it has moved is measured by the length of its path of motion. Since this may occur in any direction, distance is a scalar quantity.

For example:

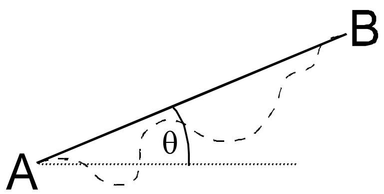


Figure 1

3. The dotted line represents distance travelled between two points A and B. The solid line represents the displacement between the two points. If the motion of a body is only in a straight line then the magnitude of the distance travelled, and the displacement will be the same.

4. The symbol for distance is ‘d’ and the symbol for displacement is ‘s’. When considering motion in a straight line we use the symbol ’s’ but, because the magnitude of distance and displacement are the same, it is common practice to use the term ‘distance’ when describing the length of the motion.

5. Speed is defined as the rate of change of distance. It is a scalar quantity.

$$\text{speed} = \frac{d}{t}$$

where:
d = distance
t = time

The units of speed are metres per second (m/s).

6. Velocity is defined as the rate of change of displacement. It is a vector quantity.

$$\text{velocity} = \frac{s}{t}$$

where:
s = displacement
t = time

The units of velocity are metres per second (m/s).

7. Acceleration is defined as the rate of change of velocity. Acceleration is a vector quantity. When a body experiences a change in the magnitude or direction of its velocity, it must have been subject to acceleration.

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{change in time}} = \frac{v - u}{t}$$

where: v = final velocity
u = initial velocity
t = time

8. The units of acceleration are the units of velocity divided by time therefore:

metres per second per second (m/s^2).

9. Gravity is the force that attracts objects together and is dependent upon the mass of the objects and the distance between them. If the objects are small then the forces are small but if the objects are large, such as planets, then the force become very noticeable. The direction of the force is always towards the centre of the object.

10. On Earth, every object is attracted towards the centre of the Earth and if placed on scales or a weighing balance it experiences a force, towards the Earth's centre. This force is called the object's weight. If an object is dropped from a height it will accelerate towards the centre of the Earth. The force is known as 'g' force.

11. The value of 'g' varies from place to place on the earth. It is largest at the poles and smallest at the equator. For this reason, most space rocket launches take place near to the equator to minimise the fuel needed.

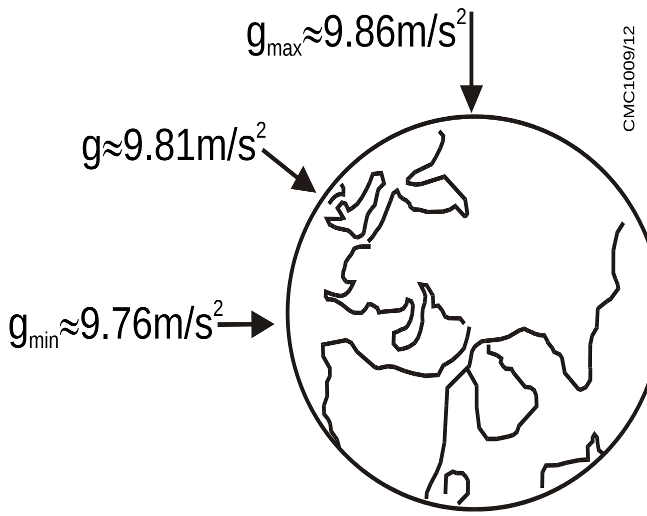


Figure 2 - Gravity differences

At our latitude, the value of 'g' is approximated to a value of 9.81 m/s2.

12. In practice, the speed and velocity of a body are seldom constant. When a car travels 40 km in one hour, then it is unlikely that its speed will remain constant. It is probable that for part of the journey the car is travelling at more than 40 km/h and for some of the time the car's speed is less than 40 km/h. Therefore, we often refer to the average speed or average velocity of a body.

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$\text{Average velocity} = \frac{\text{displacement (total distance travelled in a given direction)}}{\text{total time taken}}$$

Or $\bar{v} = \frac{s}{t}$

Where:

$\bar{v}$ = average speed/velocity

s = distance/displacement

t = time

SC3.2 - STATE NEWTON'S LAWS OF MOTION
SC3.3 - EXPLAIN THE RELATIONSHIPS F = MA & W = MG

13. We will now consider the effect that forces have on the motion of bodies. Elementary dynamics is based on Newton's three laws of motion. These laws cannot be proved in a strict sense, but their truth has been confirmed by many experiments and they remain the basis for most practical work. It is only when very high velocities are encountered, comparable to the speed of light that Newton's laws fail to give accurate results.

Newton's laws of motion

14. Newton's first law of motion states that:

"Every-body continues in its state of rest or of uniform motion in a straight line unless compelled by some external force to act otherwise"

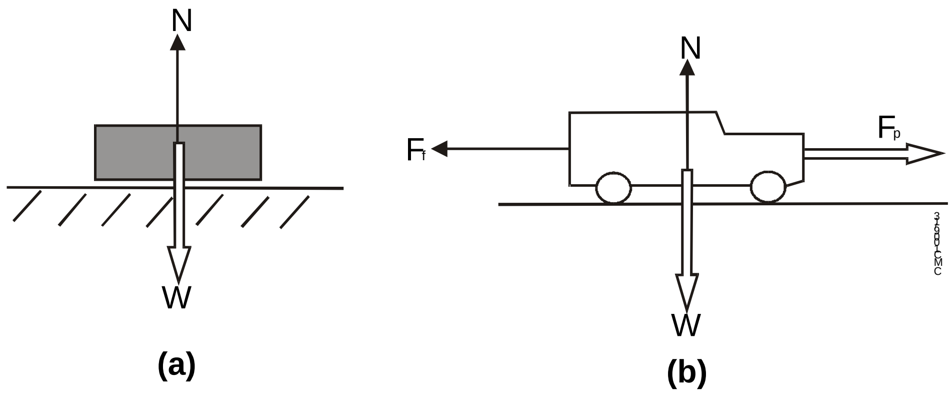


Figure 3 – Newton's 1st law

15. In figure 3(a), a block is placed on a flat surface. Application of Newton's first law tells us that for the block to remain where it is, there can be no external force acting. Here the weight of the block (W) must be equal in size and opposite in direction to the "reaction force" (N) provided by the supporting surface and so the object remains at rest.

16. Remember from previous work that forces equal in magnitude and opposite in direction are effectively cancelled out.

17. In figure 3(b), the vehicle is travelling with uniform motion (steady speed or velocity). Applying Newton's first law we have the weight (mg) cancelled by a reaction force (N) from the road surface. In addition, we can say that the propulsive force produced by the engine Fp must be cancelled by the friction force Ff. Hence with no overall external force acting the vehicle is subject to Newton's first law and has uniform motion.

18. When a body is at rest or is moving uniformly, it is said to be in equilibrium. Newton's second law of motion states that if an external force acts on a body:

“The rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts”

19. The momentum of a body is defined as the product of its mass and its velocity.

Momentum (p) = mv (kg m/s)

20. Momentum is the product of a scalar (mass) and a vector (velocity) – therefore it is a vector quantity. The momentum of a body is changed if we change either the mass or the velocity of the body.

21. Examples of changes of momentum due to changes in mass are:

a. The mass of a rocket in flight is decreasing as the fuel burns away.

b. The mass of a leaking water cart being pulled along a road is decreasing as the water leaks away.

22. Examples of changes of momentum due to changes in velocity are:

a. The momentum of a truck increases as it accelerates away from traffic lights.

b. The momentum of a water jet changes as it strikes a wall. In this case the direction of the velocity is changing as it strikes the wall.

23. Whenever a change in momentum takes place, there must always be a force either causing the change or being caused by the change.

Example

24. The vehicle below has mass 1 500 kg and its velocity is measured as shown in figure 4.

Figure 4

Note that when a body is accelerating its initial velocity is given the symbol ‘u’ and its final velocity the symbol ’v’.

25. According to Newton's second law, the rate of change of momentum is proportional to the force applied and takes place in the direction of that force. The vehicle has increased its velocity and therefore has changed its momentum.

Initial momentum = m.u = 1 500 x 10 = 15 000 kg m/s

Final momentum = m.v = 1 500 x 25 = 37 500 kg m/s

Force = rate of change of momentum $= \dfrac{\text{change of momentum}}{\text{change of time}}$

$$\text{Force} = \frac{37\,500 - 15\,000}{12} = \frac{22\,500}{12} = 1\,875 \text{ N}$$

26. So, the net force acting to cause the change of velocity is 1.875 kN and it acts in the direction of the motion.

27. If the mass or velocity of the object changes (e.g. a rocket burning up fuel) then we have to use the form of Newton's second law as written above.

28. If, however the mass of the object does not change then Newton's second law may be simplified, as follows:

Change in momentum = final momentum – initial momentum

$= m_2 v_2 - m_1 v_1$ (if mass constant, then $m_2 = m_1$)

$= m(v_2 - v_1)$

$$\text{Force} = \frac{\text{change of momentum}}{\text{change of time}} = \frac{m(v_2 - v_1)}{t_2 - t_1}$$

The rate of change of velocity $\dfrac{v_2 - v_1}{t_2 - t_1}$ is known as acceleration (a)

Force = mass x acceleration

$\therefore$ F = m a

29. Force is measured in newtons (N). A force of 1 N acting on a mass of 1 kg will provide an acceleration of 1 m/s^2.

Let's check that this formula gives us the same answer for the example in figure 4 above.

Acceleration = rate of change of velocity. $\dfrac{v-u}{t} = \dfrac{25-10}{12} = \dfrac{15}{12} = 1.25 \text{ m/s}^2$

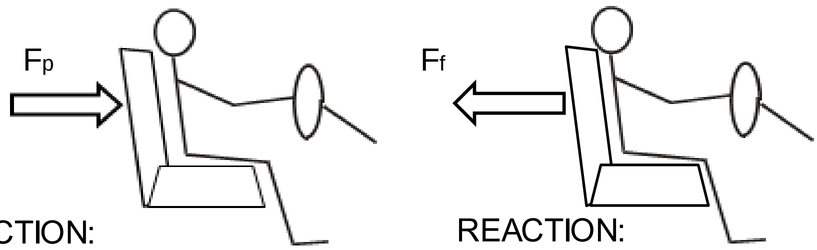
Now F = m.a = 1 500 x 1.25 = 1.875 kN (Same as before.)

30. Newton's third law of motion states:

"To every action there is an equal and opposite reaction"

31. This law is referring to a concept called inertia. This is the resistance of a body to any change in its motion (i.e. acceleration, motion along a curved path, change in rotation etc).

32. Suppose within the vehicle, accelerating in example 1 above, the driver has a mass of 90 kg. Part of the engines propulsive force is used to accelerate the driver. But as far as the driver is concerned he experiences a force pushing him back into his seat. This is the reaction referred to in the third law. Some textbooks call this the inertial force.



ACTION:

$F = ma$

$F = 90\ kg \times 1.25\ m/s^2$
$F = 112.5\ N$

REACTION:

Driver experiences a force of 112.5 N pushing him back into his seat

Figure 5 - Inertial reaction

33. In reality, the action is the only real force acting. There is no force trying to push the driver back in his seat. The seat is trying to accelerate through the driver and he is simply in the way.

34. Using the simplified version of the equation produced from Newton's second law where the mass of the object remains constant (see paragraph 27)

Force = mass x acceleration

35. As stated previously, the weight of a body is defined as the force with which it is attracted to the earth's surface by gravity and should be expressed in the SI unit of force, the newton (N).

36. To establish the mathematical relationship between mass and weight, consider a body of mass m, suspended above the earth. It will be attracted downwards with a force F, which is its weight W.

37. If the body is released it will fall towards the earth, accelerating as it falls due to the force of gravity.

By Newton's second law

$F = ma$

But $a = g$, the acceleration due to gravity

So, $F = mg$

But the force F = weight W

So, $W = mg$

38. It follows that to calculate the weight of a body we multiply its mass by the acceleration due to gravity. If the mass is in kg and g is in m/s^2, then its weight will be in newtons. We will use the accepted Northern European value of $g = 9.81\ m/s^2$.

Example:

A block of wood has a mass of 25 kg. Calculate its weight given that the acceleration due to gravity is $9.81\ m/s^2$.

$W = m \times g \qquad = 25\ kg \times 9.81\ m/s^2$

$= 245.25\ N$

Exercise 1

Note: In all the following questions assume that the value of Earth's gravity is 9.81 m/s^2.

1. Define the term acceleration.

2. Explain the difference between mass and weight.

3. Explain why the weight of a body varies according to its location (e.g., on earth, on the moon, in deep space).

4. A car engine has a mass of 300 kg. Calculate the weight of the engine.

5. An aircraft is weighed using weighing cells calibrated in newtons and its weight is 395 kN. Calculate the mass of the aircraft.

6. An astronaut has a mass of 90 kg. Calculate his weight on earth and on the moon. Assume that gravity on the moon is $1.6\ m/s^2$.

7. A beam is weighed using a spring balance calibrated in newtons. The balance reads 4.5 N. Calculate the mass of the beam.

8. State Newton's First Law of Motion.

9. State Newton's Second Law of Motion.

SC3.4 - DEFINE THE EQUATIONS OF LINEAR MOTION FOR CONSTANT ACCELERATION

39. Using the earlier definitions of velocity and acceleration, we will now look deeper at the case of a body moving with respect to time. One way to visually represent this is to use a velocity / time graph as shown in figure 6.

40. Figure 6 shows a velocity time graph for a cyclist travelling at 15 m/s for 30 s. We can calculate the distance travelled by using the previous definition of velocity. Rearranging the formula, gives:

displacement = velocity x time

= 15 x 30 = 450 m

This is also the area under the “velocity – time” graph.

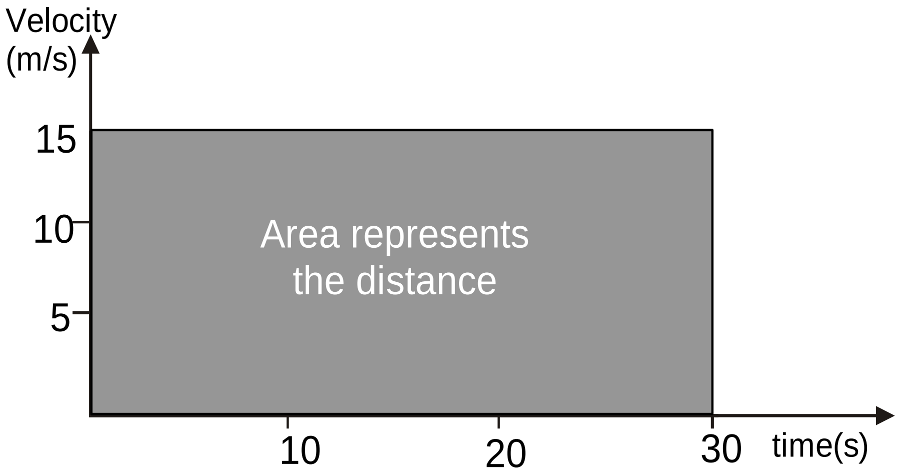


Figure 6 - Linear velocity

41. We can now apply the same concept to the generalised case of a body starting with an initial velocity ‘u’ (m/s) and accelerating with an acceleration of ‘a’ (m/s^2) to a final velocity ‘v’ (m/s) in a time of ‘t’ (seconds). The distance travelled will be ‘s’ (m).

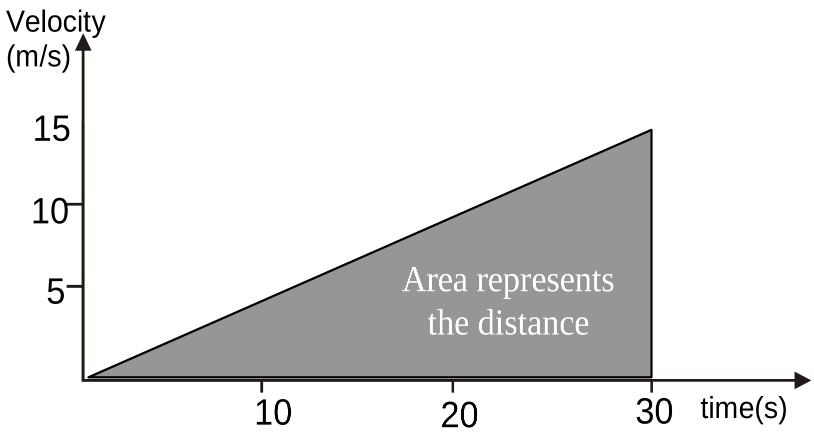
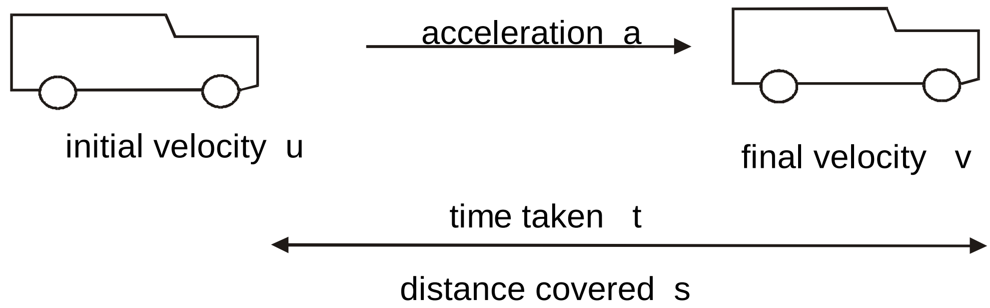


Figure 7 - Changing velocity

Note that a period of constant acceleration will produce a straight-line graph as shown.

Summary of quantities



Symbol	Physical Quantity	SI Units	unit symbol
u	initial velocity	metres/second	m/s
v	final velocity	metre/second	m/s
a	acceleration	metre/second^2	m/s^2
s	displacement/distance	metres	m
t	time	second	s

Equations of motion

42. From the previous definition of acceleration, it is possible to derive the first of the equations of motion:

We already know how to calculate the acceleration $a = \dfrac{v - u}{t}$

We usually transpose this equation to get $at = v - u$ (…Equation A)

And finally transposed for v gives $v = u + at$

This is the first of four linear motion equations: $\boxed{v = u + at}$

43. To help derive the remaining equations of motion we start by drawing the “velocity – time” graph for a generalised acceleration:

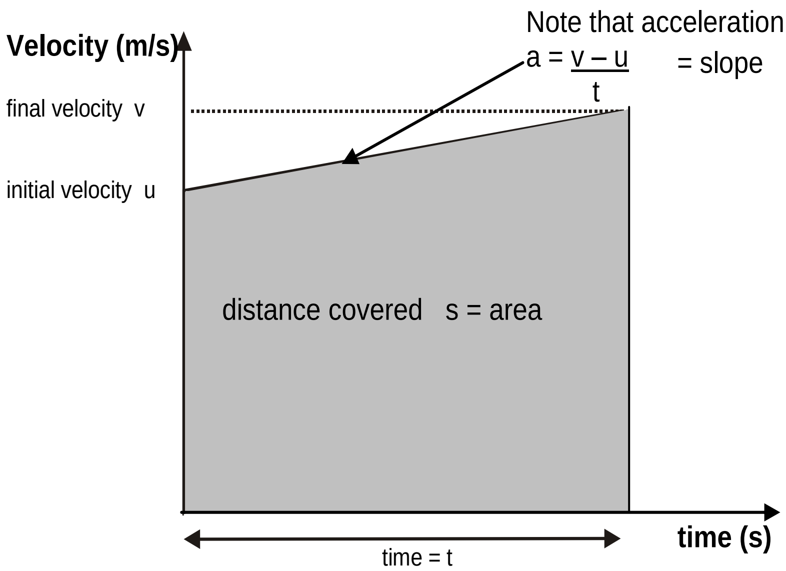


Figure 8

44. To find the distance covered (area under the graph), one way is to form a rectangle from the original graph trapezium by chopping off the top half of the triangle (shown shaded in figure 9 below) and gluing it back on top of the lower half.

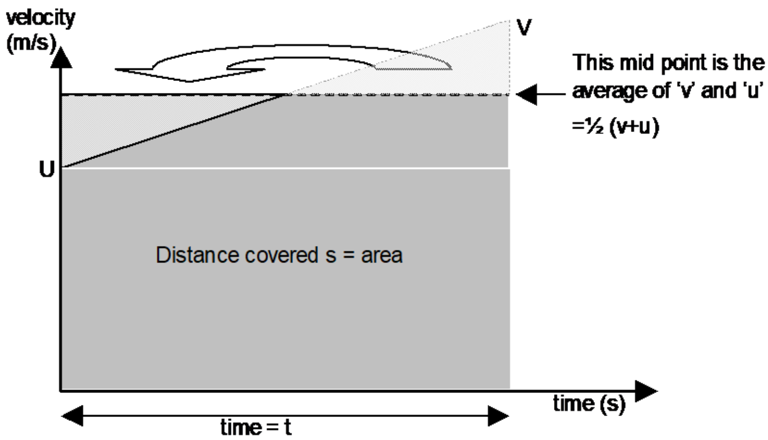


Figure 9

45. Written as an equation, the velocity at the mid-point between the initial velocity u and the final velocity v

velocity at mid-point = average velocity = ½ (v + u)

The area of this new formed rectangle will equal base x height

area = t x ½ (v + u)

or area = ½ (v + u) x t

So, the second linear motion equation is: $\boxed{s = \frac{1}{2}(v + u)\,t}$

46. Another way to find the area is to split it into a rectangle and a triangle:

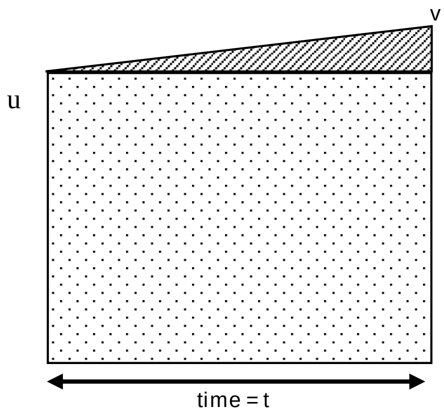
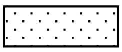
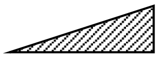


Figure 10

47. The total distance can be found by adding the area of the rectangle and the triangle together:

s =  + 

so, $s = ut + \frac{1}{2}(t(v-u))$

48. This equation has s, u, t and v in it. But the second equation, quoted above, already has these letters in it so is of no more use, but if we use a relationship from para 52 (Equation A) which gave $(v - u) = at$

and now substitute at in place of $(v - u)$

giving $s = ut + \frac{1}{2} tat$

So, the third linear motion equation is: $\boxed{s = ut + \frac{1}{2} a t^2}$

49. The fourth equation involves mathematical processes. (We have to take our existing three equations and substitute them into each other in order to get a new equation involving v, u, a and s)

Equation A (para 52) above gave: $(v - u) = at$

If linear motion equation 2 is re-arranged, we get: $(v + u) = \frac{2s}{t}$

Multiplying these two together we get

$$(v - u)(v + u) = \frac{2sat}{t}$$

which expands to: $v^2 - u^2 = 2as$

so $v^2 = u^2 + 2as$

So, the fourth linear motion equation is: $\boxed{v^2 = u^2 + 2as}$

Note that the use of the equations of linear motion is more important here, more than how they are derived.

Summary of Equations of Motion

Equation	Physical interpretation
$v = u + at$	finish speed = start speed + timed amount of acceleration
$s = \frac{1}{2}(v + u)t$	distance = average speed over a period of time
$s = ut + \frac{1}{2}at^2$	distance = distance covered at steady velocity + extra covered during acceleration
$v^2 = u^2 + 2as$	No obvious interpretation

To use these equations to solve problems we must be given at least three of the values and at least one must be a velocity.

SC3.4 - SOLVE PROBLEMS RELATED TO AN AIRCRAFT IN FLIGHT USING THE EQUATIONS OF MOTION

Free falling bodies

50. The equations of motion will apply if a body is falling freely. In this instance, the acceleration with which the body falls will be the acceleration due to gravity, 9.81 m/s^2. The equations of motion will only truly apply when a body falls in a vacuum. They do not apply when bodies fall through the air because the air will slow them down by an amount dependent on their shape. However, if bodies fall through air over short distances, then air resistance can be considered to be negligible and 9.81 m/s^2 can be assumed to be a good approximation for the acceleration due to gravity.

51. If a body is falling freely then the acceleration due to gravity is positive. If the body was thrown vertically upwards, as in the case of a ball, then the acceleration due to gravity is in the opposite direction to the motion and is therefore considered to be a negative (-9.81 m/s^2) e.g. it will slow it down.

52. A general method to solve problems which involve the equations of motion is given below:

- Tabulate the question
- Identify both known and unknown quantities
- Select the equation
- Transpose the equation (if required)
- Substitute known values into the equation and calculate the answer

Examples:

53. A brick falls from a 12 m high scaffold.

a. How fast does it strike the ground (assuming no air resistance)?

b. How long does it take to fall?

Solution:

a. We must know three things including at least one speed!

We know that distance travelled will be s = 12 m

We know that objects in free fall accelerate at 9.81 m/s^2

So, a = 9.81 m/s^2

As the brick begins its descent we know that its initial downwards velocity is zero (i.e. before it starts to accelerate) so u = 0 m/s

Now we look for an equation that has our three known factors in and the final velocity v. The fourth equation fits the bill.

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + 2 \times 9.81 \times 12 = 235.44$$

$$v = \sqrt{235.44} \quad = 15.34 \text{ m/s}$$

b. We try and find another equation that has the original three pieces of information and the time in it.

Unfortunately, the equation that meets this is: $s = ut + ½at^2$
This would generally mean solving a quadratic equation for t.

To make life easier we take one of the other (simpler) equations and solve for t using our answer from a.

$v = u + at$ therefore $t = \frac{v - u}{a}$

Now put in the values at work to find t:

So, $t = \frac{15.34}{9.81} = 1.56$ s

54. A stone takes 3 seconds to fall down a well and strike the water. Ignoring the effect of air resistance, calculate:

a. How deep is the well?

b. With what velocity will the stone strike the water at the bottom?

Solution:

What three things are known?

Initial velocity, $u = 0$m/s

Acceleration, $a = 9.81$ m/s^2

Time of travel, $t = 3$ s

The equation containing these three quantities and the distance, s is:

$s = ut + ½at^2$

$s = 0 \times 3 + ½ \times 9.81 \times 32$

$s = 0 + 44.15$ $= 44.15$ m deep

b. Now an equation with the original three 'knowns' and the final velocity in is:

$$v = u + at$$

$$\text{so,} \quad v = 0 + 9.81 \times 3 = 29.43 \text{ m/s}$$

55. A bullet is fired vertically into the air at a muzzle velocity of 200 m/s. Ignoring the effect of air resistance, calculate:

a. How high does the bullet go?
b. How long does it take to reach its maximum height?
c. How long does it take for the bullet to return to the ground from leaving the barrel?

Solution:

a. What three things are known?

i. Initial velocity u = 200 m/s
ii. Final velocity (at top of travel) v = 0 m/s
iii. Acceleration acting on bullet, a = - 9.81 m/s^2 (negative because it's slowing down the bullet!)

Find an equation with three 'knowns' and distance (s).

$$v^2 = u^2 + 2as$$

So, $0^2 = 200^2 + 2 \times -9.81 \times s$

$0 = 40\,000 - 19.62 \times s$

So, $19.62 \times s = 40\,000$

$$s = \frac{40\,000}{19.62} = 2\,039 \text{ m}$$

b. Now find an equation with u, v, a, and t in it:

$v = u + a.t$

$0 = 200 - 9.81 \times t$

So, $9.81 \times t = 200$

$$t = \frac{200}{9.81} = 20.4 \text{ s}$$

Time taken to fall will be the same as time taken to rise (assuming no friction), so total time to rise and fall back to the ground will be 2 x 20.4 = 40.8 s

Exercise 2

1. A car initially travelling at 36 km/h accelerates at 2.5 m/s^2 for 7 s.

   a. What is its final velocity in SI units?
   b. What is this velocity in km/h?

2. A cyclist starts at 8 m/s and accelerates to 24 m/s down a slope. If the acceleration rate is measured at 2 m/s^2, how long does the acceleration last?

3. A stone is dropped down a mineshaft 120 m deep.

   a. At what velocity does it strike the bottom?
   b. How long does it take to fall?
   c. How fast is it going after falling 70 m?

4. A 1 200 kg car brakes from 80 km/h to 44 km/h over 100 m

   a. What is the deceleration rate?
   b. How long does the braking last for?
   c. What is the change in momentum?
   d. What is the size of the braking force?

5. A ball kicked vertically just touches a telephone wire 13 m off the ground.

   a. How fast was the ball travelling when kicked?
   b. How long for the ball to rise and fall back down?

6. An arrow is fired vertically into the air at a velocity of 25 m/s

   a. How high does the arrow fly?
   b. How long after firing does it take the arrow to return to the ground?

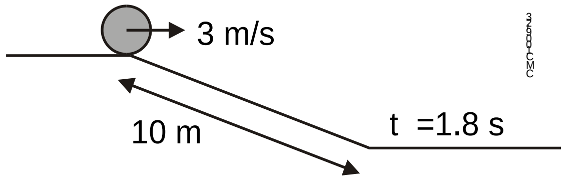
7. An ice hockey puck slides in a straight line over a horizontal sheet of ice. It passes point A with a velocity of 12 m/s, and 2.8 s later passes point B. The distance between A and B is 25 m and assuming the retardation is constant, determine:

   a. the retardation
   b. its velocity at point B
   c. how long after passing point A the puck stops moving

8. An arrow is fired at 25 m/s down a 90 m deep mineshaft.

   a. How fast will it be going when it strikes the bottom?
   b. How long does it take to reach the bottom?

9. A beach ball initially travelling at 3 m/s, starts to roll down a hill. The distance down the incline is 10 m and it travels this in 1.8 s.



a. What is the acceleration of the ball as it rolls down the hill?
b. What will be the velocity of the ball at the bottom of the incline?

10. An aircraft lands on a horizontal runway at a touchdown speed of 180 km/h. If the aircraft takes 35 s to come to rest, find the distance travelled and the deceleration.

Multiple Accelerations

56. Sometimes we have to deal with problems that involve several acceleration rates. An example of this is described below.

57. A vicar sets off, from rest, on his bike. After 12 s, of constant acceleration he reaches a steady speed of 6 m/s. He continues at this speed for 2 minutes and then he comes to rest in 3 s.

If we sketch a velocity–time graph for this motion, we get the following:

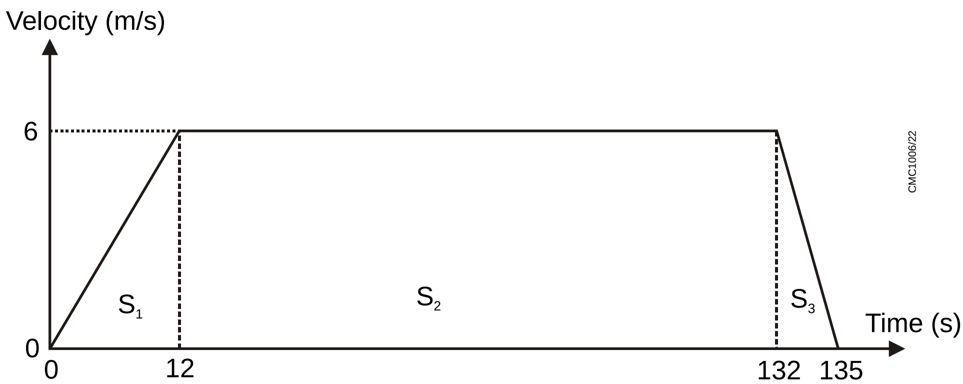


Figure 11 - Velocity time graph

58. The graph shows three distinct regions. In the first, he will have a positive acceleration and cover distance S_1 as indicated. In the second region he travels at steady velocity (acceleration = 0). In this region he covers distance S_2. In the final region he is decelerating (slowing down or retarding) and he covers distance S_3 while doing this.

59. We can apply our equations of motion to each region but not to the motion as a whole. So, for example, when trying to find the first acceleration rate we look at the first region only (the triangle in the graph.)

60. We can also find distance S_2 from the constant velocity x length of time.

61. And we can find the deceleration and S_3 from the last triangle ◺ on the graph.

Example

62. Using the vicar on his bike example in paragraph 57 above:

a. What is the acceleration rate?
b. What is the distance travelled during the acceleration?
c. What is the distance travelled while at steady speed?
d. What is the deceleration rate? (deceleration = negative acceleration)
e. What is the distance travelled during the deceleration?
f. What is the average speed?

Solution:

a. $a = \frac{v-u}{t} = \frac{6-0}{12} = 0.5 \text{ m/s}^2$

b. $S_1 = ½ (v + u).t \quad = 0.5 \times (6 + 0) \times 12 = 36 \text{ m}$

c. S_2 = velocity x time = 6 m/s x 120 s = 720 m

d. $a = \frac{v-u}{t} = \frac{0-6}{3} = -2 \text{ m/s}^2$ (negative acceleration)

So, deceleration is positive 2 m/s^2

e. $S_3 = ½ (v + u).t \quad = 0.5 \times (0 + 6) \times 3 = 9 \text{ m}$

f. Average speed = $\frac{\text{total distance}}{\text{total time}}$ =

$$\frac{36+720+9}{12+120+3} = \frac{765 \text{ m}}{135 \text{ s}} = 5.67 \text{ m/s}$$

Exercise 3

1. A cyclist accelerates from rest to 10 m/s in 5 s. He then carries on at this speed for 45 s before coming to rest in 2 s.
Draw a velocity time graph for this whole motion.

   a. Calculate the distance covered during the acceleration
   b. Calculate the distance covered during the steady speed
   c. Calculate the distance covered during the deceleration
   d. What is the total distance covered?
   e. What is the average speed of the cyclist?
   f. What is the acceleration rate?
   g. What is the deceleration rate?

2. A body starts from rest and accelerates with a constant acceleration of 1.5 m/s^2 up to a speed of 6 m/s. It then travels at 6 m/s for 12 s after which time it is retarded to a speed of 2 m/s. If the complete motion takes 18 s, find the:

   a. time taken to reach 6 m/s
   b. rate of retardation
   c. total distance travelled

3. A milk float accelerates, from rest, with a constant acceleration of 0.22 m/s^2 in a time of 10 s. It continues at this new speed for 45 s before coming to rest with a constant deceleration in a time of 15 s. Sketch the velocity-time graph to illustrate this journey and determine the:

   a. time taken to travel the first 50 m
   b. total distance travelled

SC3.6 - DEFINE BASIC TERMS FOR ANGULAR MOTION

Angular motion

63. Until now all the motion described has been in one straight direction – known as linear motion. Many machines however are based on motion in a circular direction, known as angular motion. The simplest example is the wheel, but motors, generators, and jet engines also employ angular motion.

64. Before we examine angular motion in more detail we need to be able to put rotational measurements into SI units. The SI unit of angular displacement is the radian.

Definition of a radian

65. If we draw a circle of any radius r and travel around its circumference by a distance r, then we will have moved by an angle of 1 radian.

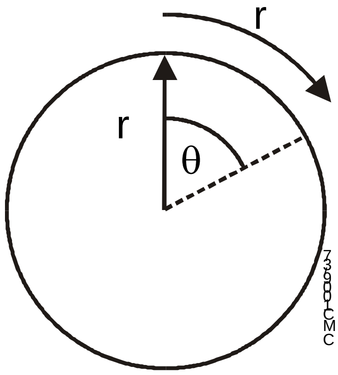


Figure 12. Radian

66. How big is a radian in the more familiar units of degrees? We first need to ask how many times the radius will fit around the circumference.

$$\frac{\text{Circumference}}{\text{radius}} = \frac{2\pi r}{r} = 2\pi \text{ radians in one complete circle}$$

So 2π radians $= 360^0$ $\Rightarrow$ 1 rad $= \frac{360^\circ}{2\pi}$ $\approx 57.296^0$

Note that calculators do not need to be in rad mode to deal with angular motion problem.

Example:

67. Convert the following:

a. 180^0 into radians $\frac{180^\circ}{360^\circ} \times 2\pi =$ π rad ≈ 3.142 rad

Fraction of a whole circle. A Whole circle in radians.

b. 4.5 revolutions into radians: $4.5 \times 2\pi$ $= 9\pi$ $\approx$ 28.27 rad

c. 2.4 rad into degrees:

$$\frac{2.4}{2\pi} \times 360^0 \approx 137.5^0$$

Whole circle in degrees

Fraction of whole circle

Angular velocity and angular acceleration

68. Because the SI unit of angular displacement is the radian, it follows that angular velocities are measured in radian/sec (rad/s). The symbol used for angular velocity is ω (omega); ω_1 is used for the initial angular velocity and ω_2 for the final angular velocity.

69. Also, angular acceleration (how quickly something increases its rate of rotation) is measured in radian/sec^2 (rad/s^2). The symbol used for angular acceleration is α (alpha).

70. The angular velocity of most rotating machines is usually measured in revolutions per minute and they often have RPM meters. Values of RPM will therefore need to be converted into SI units.

The overall conversion is: RPM x $\frac{2\pi}{60}$ = rad/s

Example:

71. An electric motor has an operating speed of 36 000 RPM, what is this in rad/s?

First convert to revolutions per second (RPS)

$$\frac{36\ 000\ \text{RPM}}{60} = 600 \text{ revolutions per sec}$$

Each revolution is one complete turn of the system = 2π rad

So now convert RPS to rad/s: $600 \times 2\pi = 1\,200\pi \approx 3\,770$ rad/s

Or directly using the conversion given in paragraph 70:

$$\text{RPM} \times \frac{2\pi}{60} = \text{rad/s} \qquad 36\,000 \times \frac{2\pi}{60} = 3\,769.9 \text{ rad/s}$$

Exercise 4

1. Convert these angles to rads

   a. 200^0
   b. 720^0
   c. 36^0
   d. 3.5 revs
   e. 244 revs

2. Convert the following to degrees

   a. 3π rad
   b. 2 rad
   c. 45 rad
   d. 3.5 revs
   e. 100 revs

3. Convert the following to revs

   a. 720^0
   b. 45 rad
   c. 100 rad

4. Convert the following to rad/s

   a. 6 000 RPM
   b. 15 000 RPM
   c. 100 RPS
   d. 25 RPS

5. A gear change in a car makes the engine speed change from 2 000 RPM to 3 000 RPM in 2 seconds.

   a. What is the start speed in rad/s?
   b. What is the finish speed in rad/s?
   c. What is the angular acceleration in rad/s^2?

6. Explain the term "radian". Draw a diagram to help the explanation.

Centripetal acceleration and centripetal force

72. Consider a body, of constant mass, moving in a circular path from point A to point B with a constant angular velocity ω, as shown in figure 13 below.

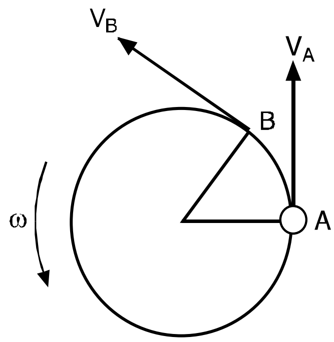


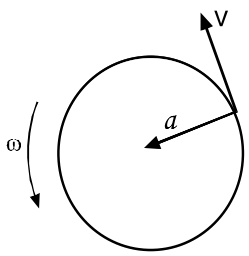
Figure 13 - Angular velocity

73. The body is moving with a constant angular velocity ω, but the instantaneous velocity at VA (tangential to the circle) is different to that at VB because the direction is changing.

74. According to Newton's first law, a body will continue in its state of rest or of uniform motion in a straight line unless compelled by some external force to act otherwise. Therefore, for this body to be moving in a circular path (non-linear motion) there must be an external force acting on it.

75. From Newton's Second Law, F = ma, when a force is present there must be an acceleration.

76. For any circular motion this acceleration is known as centripetal acceleration and acts radially inwards towards the centre of the circle.



Centripetal acceleration acts towards the centre of the circle and is given by the formula:

$a = \frac{v^2}{r}$ or $a = \omega^2 r$

where a = centripetal acceleration
v = linear velocity
ω = angular velocity
r = radius of circular motion

Example:

77. A body is rotating in a horizontal circle of radius 2 metres at a linear velocity of 3 m/s. Determine the centripetal acceleration acting towards the centre of rotation.

Centripetal acceleration $a = \frac{v^2}{r} = \frac{3^2}{2} = 4.5 \text{ m/s}^2$

78. From Newton's second law, when a body is subjected to acceleration, then a force must have been applied to the body to cause it (F = ma). This is known as centripetal force and is the force tending to pull a body in towards the centre of rotation.

79. Centripetal force is calculated using the formula below:

$$F = \frac{mv^2}{r}$$

or

$$F = m\omega^2 r$$

where F = centripetal force
m = mass
v = linear velocity
ω = angular velocity
r = radius of circular motion

Example:

80. A body of mass 2.5 kg moves with a constant angular velocity of 4 rad/s in a horizontal circle of radius 3.5 m. Find the centripetal force that must act on the body towards the centre of the circle.

Force = mass × acceleration

$F = m\omega^2 r = 2.5 \times 4^2 \times 3.5 = 140$ N

The centripetal force acting towards the centre of the circle = 140 N

81. This force is directed radially inwards towards the centre of the circle and is the force that makes a body move in a circular path. The force increases as the speed of the body increases and vice-versa if the speed decreases.

82. An example of centripetal force is the rotating of an object, such as a conker, which is tied to a piece of string. The centripetal force is represented by the tension in the string, providing a pulling force towards the centre of the circle. If the string breaks, the conker will continue in a straight line in the direction it had at the moment of breakage. Its direction of motion will now be a tangent to the circle.

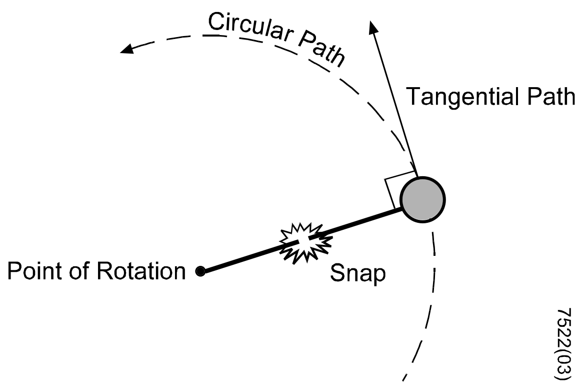


Figure 14 - Centripetal force

83. The centripetal force acting on a rotating body is the force that pulls the object into a circular motion rather than allowing it to continue in a straight line. By Newton's third law there must be opposing inertia force of equal magnitude to the centripetal force.

84. This force acts radially outwards and is commonly known as centrifugal force.

85. It must be appreciated that the centrifugal force is not applied to the body, for example, in the case of a train on a curved track, the rails exert a force on the train (centripetal) to enable it to travel in the curve, and the train pushes outwards on the rails (inertia).

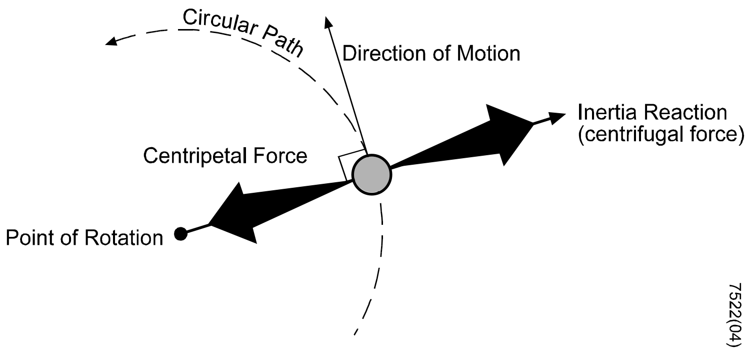


Figure 15 - Centrifugal force

Exercise 5

1. In what direction does a centripetal acceleration act?

2. A body of mass 3 kg is rotating in a horizontal circle of diameter 2.5 m and with a linear velocity of 2 m/s. Calculate the centripetal force acting on the body.

3. An athlete is swinging a 1 kg shot-putt in a horizontal circle at a radius of 1.5 m. If the shot-putt angular velocity is 5 rad/s what is the centrifugal force on the shot-putt?

4. Calculate the angular velocity of a 1 kg mass being swung at a horizontal radius of 1.5 m by a force measuring 16 N.

SC3.12 - DEFINE BASIC TERMS USED IN SIMPLE GYROSCOPES

Momentum

86. Before developing the study of the motion of a body into the principles of how a gyroscope works, a short recap.

87. In the first dynamics unit, velocity was defined as the rate of change of displacement with respect to time. It is measured in metres per second (m/s) and is a vector quantity.

88. In the fifth dynamics unit, angular velocity was defined as the rate of change of angular displacement with respect to time. It is usually measured in radians per second (rad/s) but could be quoted in revolutions per minute (rpm).

89. In the work about Newton's laws of motion, the term momentum was discussed briefly because Newton had used the word in his second law. The linear momentum of a body was defined as the product of its mass and its velocity.

$$\text{Linear momentum } (p) = mv \text{ (kg m/s)}$$

90. Momentum can only be changed by the action of a force e.g., a change in momentum will require a change in velocity, which requires an acceleration (from $F = ma$).

91. When a collision occurs between two bodies A and B, the force exerted on B by A is equal and opposite to the force exerted on A by B. This is an example of Newton's Third Law. If no external force acts on the bodies, then the momentum remains constant.

92. This is the principle of conservation of linear momentum, which states that the total linear momentum of a body or a system of bodies in any direction remains constant unless acted upon by a resultant force in that direction.

93. Consider the firing of a gun. The equal and opposite forces exerted by the gun on the bullet, and the bullet on the gun, will cause motion of each. However, as there are no external forces involved it follows that the change in momentum of the bullet due to firing must be equal and opposite to the change in momentum of the gun. The forward momentum of the bullet is evident and the backward momentum of the gun itself is shown by the kick or recoil. The difference in the resulting velocities is due entirely to their difference in mass.

$$\text{mass of bullet} \times \text{muzzle velocity} = \text{mass of gun} \times \text{recoil velocity}$$

Inertia

94. All matter resists any change in motion. The quantity resisting a change of momentum e.g., resisting acceleration, is called inertia. The inertia of a body causes it to be reluctant to change its linear velocity and is dependent upon its mass - the greater the mass, the greater the inertia.

95. Being an innate property, which only comes into effect when acceleration occurs, inertia can be used in the formulation of equations, or modelling, to help solve problems in dynamics. When an external force is applied to a body, which causes acceleration, then from Newton's third law the equal and opposite reaction to this force is the inertial force (often called the 'inertial reaction').

96. Similarly, the term used to show the reluctance of a body to change its angular velocity is called its moment of inertia. Again, the moment of inertia of a body is dependent upon its mass, but it is also dependent upon the distribution of the mass in the body.

97. The moment of inertia of an object about a given axis describes how difficult it is to induce an angular rotation of the object about that axis. For example, consider two wheels of the same mass, one large and one small in radius. The smaller wheel is easier to accelerate into spinning fast, because its mass is concentrated close to the axis of rotation. Conversely, the larger wheel takes more effort to accelerate into spinning fast, because its mass is spread out further from the axis of rotation. Quantitatively, the smaller wheel has a smaller moment of inertia, whereas the larger wheel has a larger moment of inertia, even if they have the same mass.

98. Angular momentum is the product of a body's moment of inertia and its angular velocity.

99. A gyroscope is a device for measuring or maintaining orientation in space based on the principle of conservation of angular momentum. The essence of the device is a heavy fast spinning metal wheel (the rotor) mounted on an axle. The mass of a gyroscope is concentrated towards the rim to maximise its moment of inertia – see figure 16.

Figure 16. Gyroscope

100. The axle of the rotor is mounted on a frame known as a gimbal. Most gyroscopes have more than one gimbal to allow complete freedom of movement in all axes – see figure 16.

101. The rotor, once spinning, tends to resist changes to its orientation due to the angular momentum of the wheel. In physics this ability to remain in a fixed attitude in space is also known as gyroscopic inertia or gyroscopic rigidity.

102. A gyroscope in operation - The rotor will maintain its spin axis direction regardless of the orientation of the outer frame.

103. Figure 17 shows a typical instruction sheet for a “toy” gyroscope.

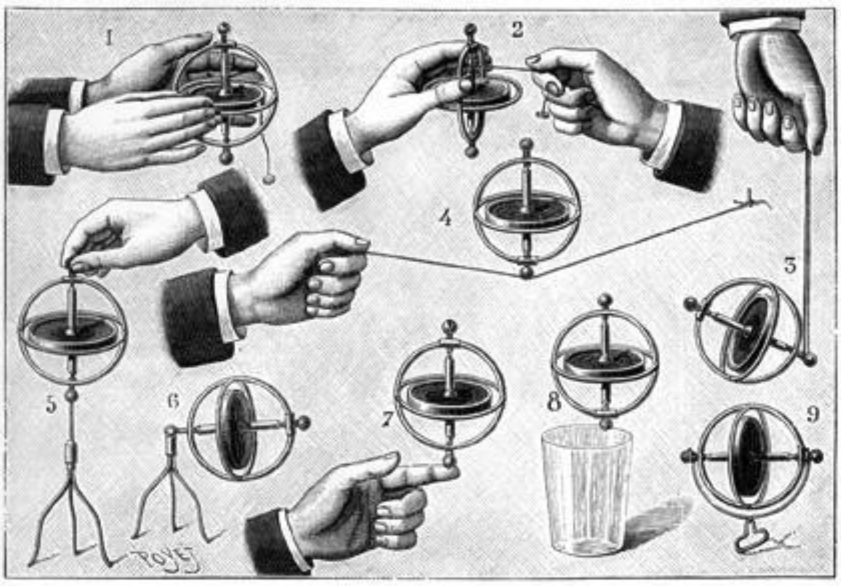


Figure 17 - Toy gyroscope

104. Once spinning, the rotor of a gyroscope can be subject to gyroscopic precession, which is when a torque is applied to the gyroscope it causes the axis to move at 90o to the applied torque. The phenomenon, known as “torque induced precession”, is commonly seen in a toy spinning top, but all rotating objects can undergo precession. If the speed of the rotation and the magnitude of the torque are constant the axis will describe a cone, its movement at any instant being at right angles to the direction of the torque.

105. This concept is easier to understand by examining the effects of inertia, which is often stated by the phrase "A body in motion tends to stay in motion". In this case the "motion" of a rotating body is in its rotation. If an external force pushes upon the rotating body, the body will resist the force by pushing back against it, but the reaction is applied at a right angle to the external force.

106. A disadvantage of precession is that it can cause fastenings under large torque loads to unscrew themselves. Bicycle pedals are left-threaded on the left-hand crank so that precession tightens the pedal rather than causing it to come loose. Before the advent of taper lug nuts, which are immune to precession, some cars also used left-threaded nuts for the left side road wheels. Precession or gyroscopic considerations have an effect on bicycle performance at high speed.

107. Precession is also the mechanism behind gyrocompasses. Gyroscopes are used extensively in-flight control systems and aircraft instrumentation, such as artificial horizons and inertial navigation systems.

108. Certain safety handling precautions have to be observed when working with gyroscopes such as allowing time for the gyroscopes to run down fully before moving them. Also due to their delicate bearings, gyroscopes must be transported using special to type transport containers. They must be “Handled like Eggs” and are often labelled as such.

SC3.7 - DEFINE TERMS FOR OSCILLATING MOTION

Simple harmonic motion

109. If a force is applied to a body it will generally produce a movement in the direction of the applied force (Newton's 2nd law). If the body is fixed in some way then the force applied may cause the body to distort, bend or twist. For example, a ruler held over the edge of a table is held firmly to the table and a force applied to the overhanging end.

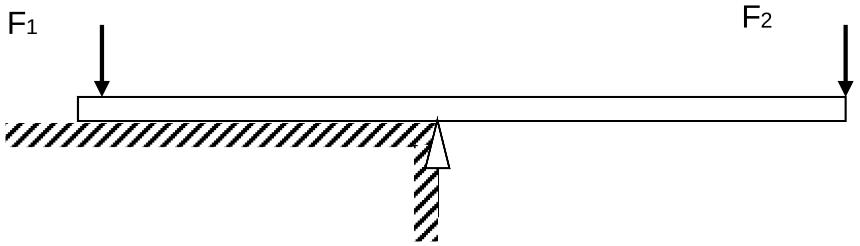


Figure 18 - Beam

110. This is similar to an example of a beam or see-saw with one very large restraining force F_1 and a smaller force F_2 applied to the free end of the beam. In previous examples the beam was assumed to be solid, rigid and unbending and would therefore pivot, in this case on the edge of the table.

111. However, in real life the beam would bend slightly under the application of force F_2.

112. The force applied in the above example will cause the ruler to bend or sag downwards by an amount dependent upon the force applied and the stiffness of the ruler material.

113. If the applied force F_2 is then removed from the ruler, or beam, it will tend to return to its original straight position.

114. However, if the movement of the end of the ruler was observed very closely it does not return immediately to the original straight position. Between the removal of the force and the final steady position, the end of the ruler will move back up passing the original neutral position before moving down and up, down and up until it finally stops moving.

115. This swinging movement about the zero point, shown in figure 19, is known as an oscillation or vibration.

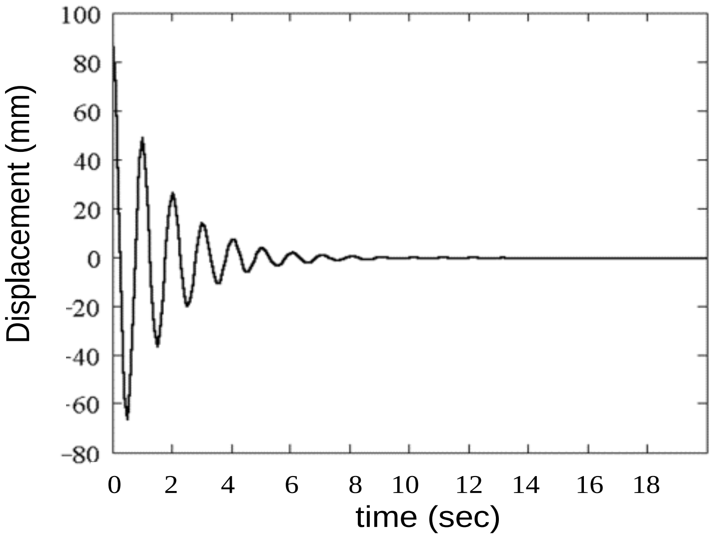


Figure 19 - Vibration

Vibration

116. By definition, a free vibration is a vibration that occurs at the natural frequency of a body when it has been displaced from the rest position and allowed to vibrate freely without the application of any periodic force.

117. The motion described, is known as simple harmonic motion. Simple harmonic motion is defined as any motion where a restoring force is applied that is proportional to the displacement and in the opposite direction to the displacement. Under simple harmonic motion an object executes an oscillatory motion about a fixed point.

118. Similar examples of simple harmonic motion occur with a mass on a spring or a simple pendulum.

119. A forced vibration is one that results from the application of a periodic force to a body capable of vibrating. This periodic force maintains the vibration rather than it just dying away to zero. An example of this driven or forced vibration is a loudspeaker emitting a continual steady tone.

120. The opposite of a forced vibration is where the free vibration is deliberately reduced to zero as quickly as possible. In this case the oscillation or vibration is described as being damped. The ruler used in figure 18 is naturally damped due to its stiffness and the oscillations shown in figure 19 quickly die away to zero.

121. The mechanical motion of vibration is occasionally desirable. For example, the motion of a tuning fork, the reed in a woodwind instrument or harmonica, or the cone of a loudspeaker is desirable vibration, necessary for the correct functioning of the device. The vibrating alert in a cell phone or the much larger vibrations of a vibration compactor are additional examples where the vibration is desired.

122. More often, vibration is undesirable, wasting energy and creating unwanted sound. For example, the motion of engines, electric motors, or any mechanical

device in operation usually suffers from unwanted vibrations. Such vibrations can be caused by imbalances in the rotating parts, uneven friction, the meshing of gear teeth, parts that are dragging together, etc. Careful design can usually minimize unwanted vibrations or 'dampers' can be added to suppress their transmission outside of the point of generation.

123. Vibrations can produce sound or even music. Sound produces vibrations in materials because sound consists of pressure waves in air (or other materials). The pressure in the waves "pushes" on objects (as you would expect pressure to do), but being a wave, it is changing rapidly, pushing harder and softer repeatedly many times a second. This pushing causes the material subjected to the sound to vibrate.

124. All objects have a natural (or free) vibration rate. When a vibrating system (mechanical or acoustic) is set into forced vibration by the application of a periodic driving force with the applied rate at or near the natural vibration rate then vibrations of maximum amplitude result. This is a condition known as resonance.

125. Figure 20 shows a graph of displacement against time for part of a system being oscillated or vibrated:

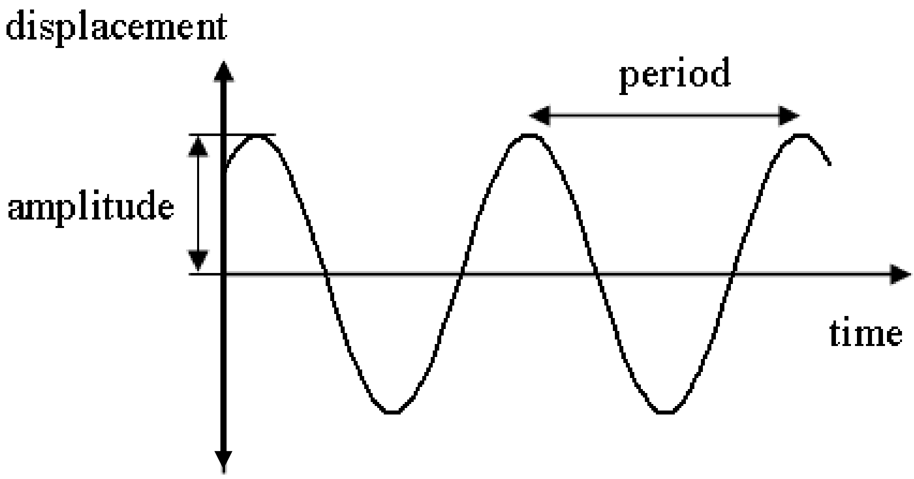


Figure 20 - Steady oscillation

126. A cycle is the name for one complete vibration or oscillation. Figure 20 shows two complete cycles of this oscillation or vibration.

127. Amplitude is the name for the maximum amount of displacement from the neutral or zero point.

128. The time period, or periodic time is the time for one complete oscillation or vibration. In figure 20, this is the time taken from one maximum positive displacement (peak) to the next corresponding maximum positive displacement. However, any point on the graph to the next corresponding point will also give the same value. e.g. the periodic time taken for one complete cycle.

129. If a waveform completes one cycle in one second it is said to have a frequency of 1 Hertz (Hz). Hertz is the SI unit of frequency.

130. Frequency is defined as the number of oscillations per second, vibrations per second or cycles per second. Therefore frequency (f) is inversely related to the periodic time (T).

$f = \frac{1}{T}\ (Hz)$ and therefore $T = \frac{1}{f}\ (sec)$

SC3.8 - EXPLAIN SIMPLE HARMONIC MOTION IN TERMS OF MASS-SPRING AND SIMPLE PENDULUM SYSTEMS

Simple harmonic motion

131. Simple harmonic motion is a term describing the periodic motion that occurs when a vibrating structure is displaced and experiences a restoring force proportional to its displacement. It can be exhibited in a variety of simple physical systems and below are two examples:

Figure 21 - Simple harmonic oscillators

Mass on a spring

132. The first example is that of a mass suspended on a spring. Here the mass will extend the spring by an amount dependent upon the mass and the stiffness of the spring used, in accordance with Hooke’s Law.

133. Application of a force vertically downwards, will pull the mass down from the initial neutral position and will further extend the spring. If the mass is now released from the maximum positive displacement, it will tend to return to the neutral position. It does however exhibit simple harmonic motion between the release and neutral position it eventually takes up.

134. In the absence of any external forces and damping, the variation of displacement with time is sinusoidal.

135. Upon release, the spring will contract, raising the mass vertically upward. The inertia gained by the mass will take it beyond the neutral position. However, gravity acting on the mass will soon overcome this inertia to firstly slow the mass before eventually stopping its upward motion. At this point of maximum negative displacement, gravity will then cause the mass to begin falling again. The mass will fall past the neutral point as gravity and its inertia both act vertically downwards to extend the spring again. The extension of the spring now slows the mass to a standstill close to the position of maximum positive displacement.

136. The process will then repeat, but in each cycle the maximum displacement reducing due to damping losses, due to air resistance and losses within the spring itself, until the mass comes to rest at the original neutral position.

137. The graph of displacement of the mass with time will be similar to that of the end of the ruler used as an example in the previous section – see figure 22.

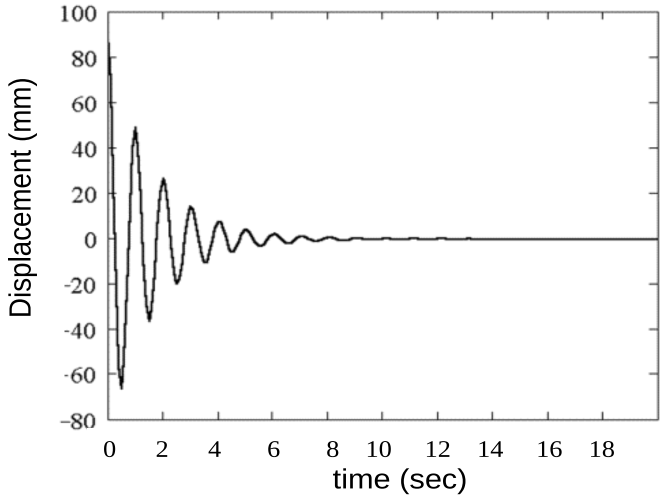


Figure 22 - Ruler oscillation

Pendulum

138. The second example is that of a gravity pendulum or bob pendulum. This is effectively a mass (bob) suspended at the end of a light rigid rod or rope which, when given an initial push, will swing back and forth under the influence of gravity.

139. The pendulum was first studied in the 10th century for its oscillatory movement but was first used in clocks in the 17th century following observations by Galileo who used his pulse to time the swinging of the lamps in the cathedral at Pisa. He concluded that the time for a lamp to swing does not depend on the angle through which it swings.

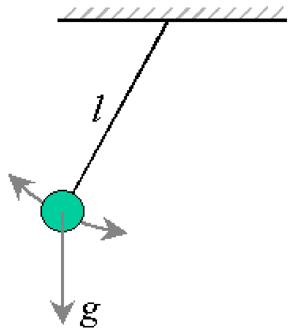


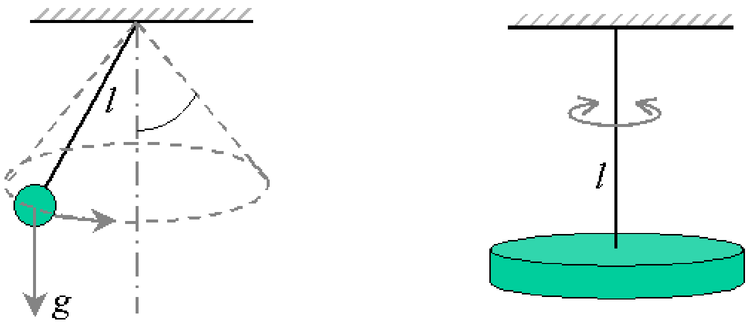
Figure 23 – Pendulum

140. If the pendulum swings through a small angle (less than 15 degrees) the motion may be approximated as simple harmonic motion. The period of a pendulum swing is significantly affected only by its length and the acceleration of gravity. The period of motion is independent of the mass of the bob and the angle through which it swings.

141. There are a number of different forms of pendulum mainly used in clock mechanisms. The main characteristic of the simple bob pendulum being that when the mass is displaced from its position of rest, it will oscillate at a fixed frequency.

142. The pendulum clock uses an escapement system that passes energy to the pendulum to keep it swinging and also releases the gear train in a step-by-step manner. The gear train also slows the rapid rotation of the escapement down to a suitable speed to match the characteristics of the motor. An indicating system shows how often the escapement has rotated and therefore how much time has passed.

143. Other forms of clock pendulum, shown below in figure 24, include the conical pendulum and the torsional pendulum. In the conical, the circular rotation of the mass produces a cone shape rather than just moving back and forth and in the torsional where the mass hangs vertically on a strip of sprung steel and spins producing a restoring torque in the suspension wire.



(a) conical (b) torsional

Figure 24 - Clock pendulum

SC3.9 - CALCULATE THE NATURAL FREQUENCY OF SMALL OSCILLATIONS IN A SIMPLE PENDULUM

Pendulum motion

144. In the previous section the pendulum was used as one example of simple harmonic motion. It was also stated that the period of a pendulum swing is dependent only on its length and the acceleration of gravity and is independent of the mass of the bob and the angle of swing (for small angles).

145. The period of the pendulum is the time taken for the pendulum to swing left to right and back again – which is one complete cycle. The formula for the periodic time, T, is given using the following formula:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where:
T = periodic time [s]
l = length [m]
g = gravitational acceleration [m/s2]

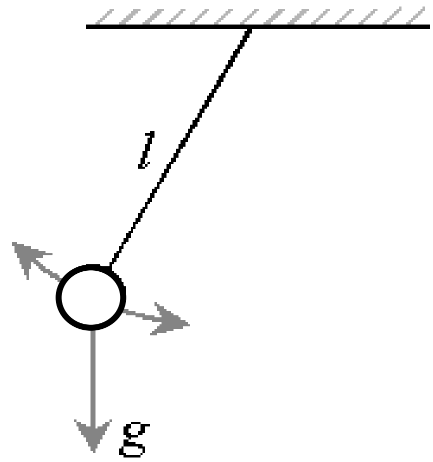


Figure 25 - Pendulum

146. It was also noted in a previous section that the periodic time was inversely related to the frequency of oscillations and was stated as follows:

$$T = \frac{1}{f}\ (\text{sec}) \qquad \text{and therefore} \qquad f = \frac{1}{T}\ (\text{Hz})$$

147. Therefore, a formula can be produced for the frequency of oscillation for a pendulum, as follows:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}\ (\text{Hz})$$

Example:

148. A simple pendulum with a bob length of 200 mm is set in motion. Assuming that the local gravity is measured at 9.81 m/s^2 calculate the frequency of oscillation of the pendulum.

Given that: l = 200 mm

g = 9.81 m/s^2

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \quad \text{so} \quad f = \frac{1}{2\pi}\sqrt{\frac{9.81}{0.2}}$$

f = 1.115 Hz

Exercise 6

1. If a clock, which holds perfect time in UK, is used on the equator without any adjustments, what will be the effect on its ability to keep time?

2. A pendulum clock has a length of 185 mm with a value for g of 9.81 m/s^2. Calculate the frequency.

3. A grandfather clock has a pendulum of length 450 mm. Given that g = 9.81 m/s^2, calculate its periodic time.

4. Describe a free vibration.

5. Describe a forced vibration.

6. Explain the term “resonance”.

7. Explain the term “simple harmonic motion” and give 3 practical examples of simple harmonic motion taking place.

PRACTICAL EXERCISE - FREQUENCY OF A PENDULUM

Aim

To investigate the effect of mass and length on the motion of a simple pendulum.

Apparatus

Peg board, stopwatch, x2 lengths of string, steel rule, various masses

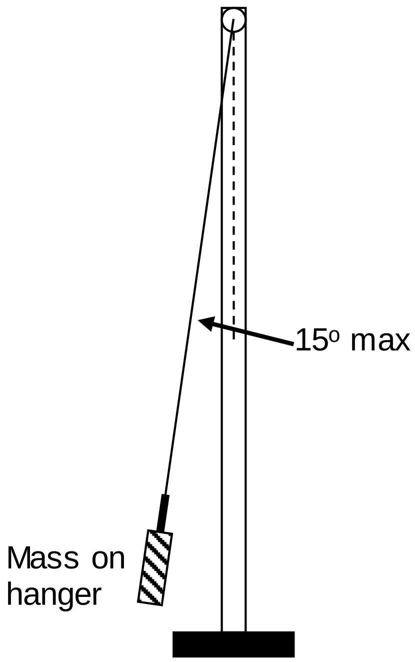


Figure 1 - The pendulum

Procedure

1. Set up the pendulum using a 100 g mass hanger and the longest piece of string provided. Ensure the clip is attached to the peg to prevent the pendulum falling off.

2. Displace the pendulum from the equilibrium by a small angle (i.e. less than 15°)

3. Using the stopwatch provided, measure and record the time for 30 complete oscillations of the pendulum. Record this time in the table below in column 2.

4. Repeat paragraph 3 to record 3 times for the 100g mass.

5. Now measure the actual pendulum length and record in column 6.

6. Add a further 100 g to the hanger and repeat 3 times for 30 swings of the 200 g, 300 g and 400 g pendulums – measuring and recording times in column 2 and the actual lengths of each pendulum in column 6.

Analysis

7. Complete the results table as follows:

Column 3 is the time for one oscillation, hence divide values in column 2 by 30.
Column 4 is the average of each of the three readings in column 3.
Column 5 uses the average value of periodic time (T) from column 4 to calculate the pendulum length using the following formulae:

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{where } g = 9.81 \text{ m/s}^2$$

$$\text{So…}T = 2.006\sqrt{\text{length}}$$

$$l = \frac{T^2}{4.024}$$

Pendulum Mass g	Time for 30 oscillations s	Periodic time s	Average periodic time T s	Calculated pendulum length m	Actual measured pendulum length m
100 g					
100 g					
100 g					
200 g					
200 g					
200 g					
300 g					
300 g					
300 g					
400 g					
400 g					
400 g					

Questions

8. Does the mass effect the periodic time?

9. Compare the calculated pendulum length to the measured lengths. Comment on any differences:

10. Comment on the accuracy of the results and suggest any factors that would cause errors in the readings/ calculations:

Procedure

11. Set up the pendulum using a 200 g mass hanger and the shorter piece of string provided. Ensure the clip is attached to the peg to prevent the pendulum falling off.

12. Displace the pendulum from equilibrium by a small angle (i.e. less than 150).

13. Using the stopwatch provided, measure and record the time for 30 complete oscillations of the pendulum. Record this time in the table below in column 2.

14. Repeat paragraph 13 to record 3 times for the 200 g mass.

15. Measure the actual pendulum length and record in column 6.

16. Complete the table to calculate the pendulum length as before.

Pendulum Mass g	Time for 30 oscillations s	Periodic time s	Average periodic time T s	Calculated pendulum length m	Actual measured pendulum length m
200 g					
200 g					
200 g					

Questions

17. Does the length of the pendulum effect the periodic time?

18. Compare the calculated pendulum length to the measured lengths. Comment on any differences:

SC3.10 - DEFINE BASIC TERMS FOR SIMPLE MACHINES

Simple machines

149. There is a limit to the maximum force that man can exert unaided and since the earliest times he has tried to devise methods of overcoming this limitation. On occasion, also, he may have wished to apply a force at some point remote from his own location or, perhaps, to increase the range of movement available.

150. Devices to help achieve these aims are known as machines. A machine is defined as a device, which changes the magnitude, direction or line of action of a force. A simple machine usually involves a small input force, known as the effort, to give a larger output force, called the load.

151. Some typical examples of simple machines to be considered later include levers, pulleys, screw-jacks and gear systems.

152. First though, it is essential to understand the definitions of the terms to be used later.

Velocity Ratio

153. The ratio of the distance moved by the effort and the distance moved by the load is known as the velocity ratio (VR).

$$\text{velocity ratio} = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

As the name implies we can also use:

$$\text{VR} = \frac{\text{velocity of effort}}{\text{velocity of load}}$$

Note: The units of distance used for both the movement of the effort and that of the load are both the metre; therefore, the velocity ratio is a ratio e.g. A number without units.

154. The velocity ratio is sometimes called the movement ratio.

Mechanical Advantage

155. The ratio of the load force to the effort force of a machine or system is known as the mechanical advantage (MA).

$$\text{mechanical advantage} = \frac{\text{load}}{\text{effort}}$$

Note: The units of force used for both the value of the load and the effort are both the newton; therefore, the mechanical advantage is also a ratio.

156. The mechanical advantage is sometimes called the force ratio.

Efficiency

157. For any system the ratio of the output to the input is known as the efficiency of the system and the symbol used for efficiency is η (eta).

$$\text{Efficiency } (\eta) = \frac{\text{work output}}{\text{work input}}$$

Note: Work is explained in detail in SC3.13.

$$\eta = \frac{\text{work out}}{\text{work in}} = \frac{\text{load} \times \text{distance travelled by load}}{\text{effort} \times \text{distance travelled by effort}}$$

$$\eta = \frac{\text{load}}{\text{effort}} \times \frac{\text{load distance}}{\text{effort distance}} = \text{MA} \times \frac{1}{\text{VR}}$$

$$\text{so, efficiency } (\eta) = \frac{\text{MA}}{\text{VR}}$$

Note: We can also use $\frac{\text{power output}}{\text{power input}}$ to give efficiency

158. Without losses, a system is deemed to be 100% efficient. Taking losses into account the efficiency of a system falls below 100%.

159. In an ideal situation where there is no friction or losses, where MA = VR, the efficiency will be 1.

$$\eta = 1$$

160. In a real system which incurs friction or other losses MA < VR and the efficiency is less than 1.

$$\eta < 1$$

Therefore, η is always ≤ 1.

Note: Efficiency is often expressed as percentage efficiency (η%). This is done by adding the % symbol to the left-hand side of the equation and multiplying the right-hand side by 100.

SC3.11 - SOLVE PROBLEMS INVOLVING LEVERS, PULLEYS, SCREW JACKS AND GEARS

Levers

161. The lever was probably the earliest device used by man to move a large load by means of the limited physical effort available. At Stonehenge, for instance, it is almost certain that wooden poles were used as levers to erect the heaviest stones, some of which have a mass of up to 45 t. The lever is a simple but very effective machine and can be used to illustrate certain fundamental characteristics that are common to all machines.

162. Consider the lever shown in figure 26. It pivots about the point O known as the fulcrum and is being used to lift an object of weight *W* on one end of the lever by the application of a force F on the other.

163. The force to be overcome, the weight of the object, W, is known as the load and the applied force F is known as the effort. The distance from the load to the fulcrum is a and the distance from the effort to the fulcrum is b.

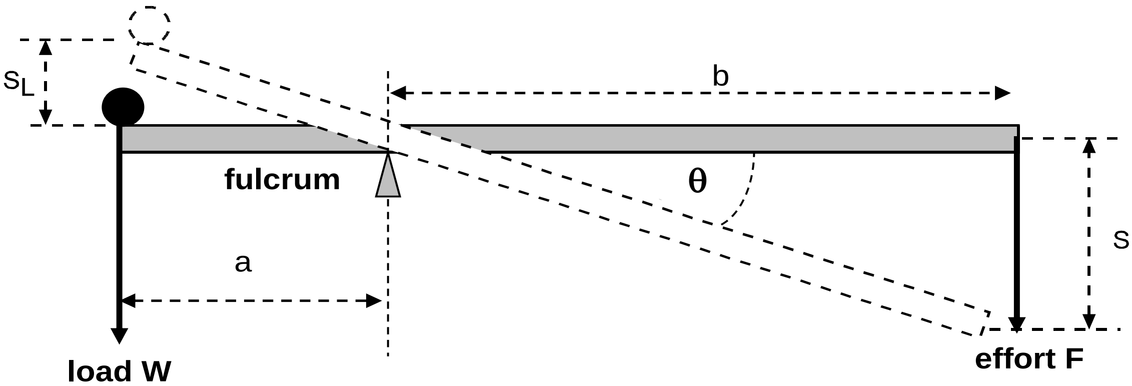


Figure 26 - Lever

164. If we now displace the lever through some small angle θ to the position shown by the dotted lines, then the effort will have moved through a distance S_E and the load through a distance S_L.

165. From the previous work on definitions, the velocity ratio is given as:

$$\text{velocity ratio} = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

$$\text{In this case} \quad VR = \frac{S_E}{S_L}$$

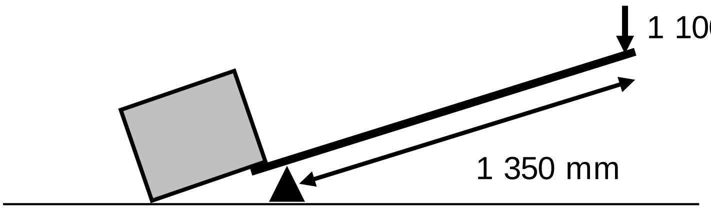
166. However, for the small angle of movement of this lever, shown as θ .

$$VR = \frac{b \sin \theta}{a \sin \theta} = \frac{b}{a}$$

$$VR = \frac{\text{distance of effort from fulcrum}}{\text{distance of load from fulcrum}}$$

Examples:

167. A 1 500 mm crowbar is used to lift a 1 tonne packing case. The crowbar pivots 150 mm from the end under the packing case and it takes an effort of 1 100 N to just lift the case from the ground. (note: gravity = 9.81 m/s^2)



a. Calculate the velocity ratio

We know that VR = $\frac{\text{distance of effort from fulcrum}}{\text{distance of load from fulcrum}}$

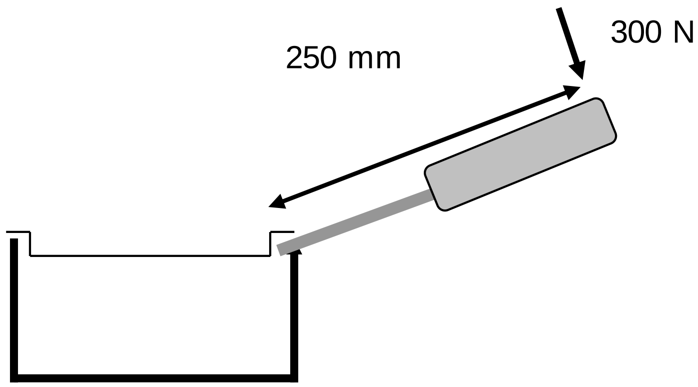
$\frac{1\ 350\text{ mm}}{150\text{ mm}}$ = 9

b. Calculate the mechanical advantage

We know that MA = $\frac{\text{load}}{\text{effort}}$

$\frac{1\ 000\text{ kg} \times 9.81}{1\ 100\text{ N}}$ = 8.92

168. A 250 mm long screwdriver is used to open the lid of a paint tin using a force of 300 N exerted on the end of the screwdriver. The screwdriver point is inserted 5 mm under the rim of the lid and it pivots on the edge of the tin.



a. Calculate the velocity ratio

b. If the efficiency is 85% calculate the friction force holding down the lid

a. We know that $VR = \dfrac{\text{distance of effort from fulcrum}}{\text{distance of load from fulcrum}}$

$$\frac{24.5 \text{ cm}}{0.5 \text{ cm}} = 49$$

b. We know that $\text{efficiency} = \dfrac{MA}{VR}$

Therefore, efficiency x VR = MA = 0.85 x 49 = 41.65

MA x effort = load

Therefore load (friction force) = 41.65 x 300 N = 12.49 kN

Exercise 7

1. What is the purpose of a machine?

2. In a real machine that has losses due to friction, what would the value of the mechanical advantage be compared with the velocity ratio.

3. A simple machine raises a load of 900 N by a distance of 0.3 m using an effort of 250 N, which moves through a distance of 1.3 m. Calculate the MA, VR and efficiency of the machine.

4. A lever system has an efficiency of 45% when used to lift a 1 tonne load. Calculate the effort needed to raise the load when the effort moves 6 m to raise the load by 0.5 m.

Pulley systems

169. Pulley systems are widely used in cranes, lifts, hoists, etc. The simplest pulley system is the single pulley, which merely changes the direction of the force not its magnitude. In the one pulley system, shown in figure 27, the effort equals the load. Thus, the mechanical advantage is equal to one.

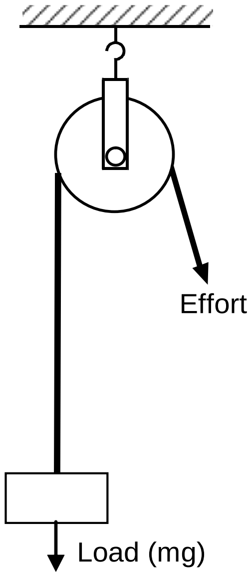


Figure 27 - Simple pulley

170. The distance moved by the effort and the distance moved by the load will also be the same, therefore the velocity ratio will also be equal to one.

171. However, most pulley systems contain several pulleys and consequently will have a larger velocity ratio.

Two Pulley Systems

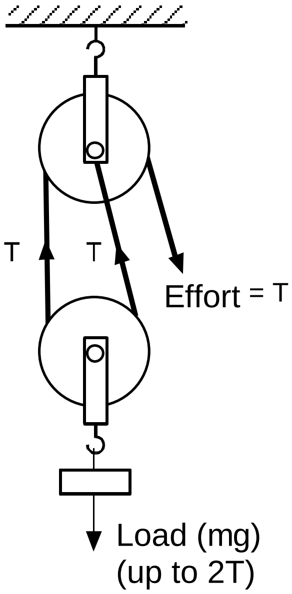


Figure 28 - Two pulley system

172. In a two-pulley system, the pulleys are arranged in separate frames and the lower one will move up or down in relation to the upper one, which is fixed to a suitable support. One continuous string is attached to the same support as the upper pulley and is passed through the system and feeds out to the effort as indicated in figure 28.

173. In the 2-pulley system, 2 strings are equally supporting the load (mg). The effort, pulling from the right side of the top pulley wheel, which is used to lift the load, would be equal to the tension (T) in the string.

174. For an ideal pulley it is assumed there is no friction in the bearings and that the rope has no weight, also the tension in all the string will be equal throughout.

$$\text{Tension T} = \frac{\text{load}}{\text{number of supporting strings}}$$

$$\text{ideal effort} = \frac{\text{load}}{\text{number of supporting strings}} \quad \ldots\ldots (1)$$

$$\text{ideal MA} = \frac{\text{load}}{\text{ideal effort}} \quad \ldots\ldots (2)$$

Substitute (1) into (2):

$$\text{Hence the ideal MA} = \frac{\text{load}}{\text{load}} \times \text{number of supporting strings}$$

$$\therefore \text{ideal MA} = \text{number of supporting strings}$$

175. And for an ideal machine where VR = MA, then the VR is also the number of strings.

Example:

176. Suppose the load in figure 28 is raised by 10 mm. Then each supporting string must shorten by 10 mm, so that a total length of 20 mm must pass over the upper pulley in the direction of the effort:

$$\text{VR} = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

$$= \frac{20}{10}$$

$$\text{VR} = 2 = \text{ideal MA}$$

$$= \text{number of strings}$$

Three Pulley Systems

177. In a three-pulley system the upper fixed pulley block carries two independently rotating pulleys, while the lower block carried one pulley as before. The arrangement of the continuous string should be clear from figure 29.

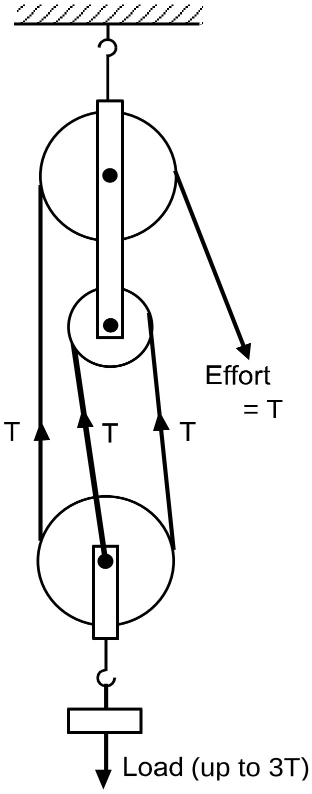


Figure 29 - Three pulley system

178. If the load is raised 10 mm then each of the three strings will each need to be shortened by 10 mm. This means that the effort must move 30 mm.

$$VR = \frac{30}{10}$$

$$= 3 = \text{ideal MA}$$

179. The three strings support the load equally so that the ideal effort is only a third of the load.

$$\text{ideal effort} = \frac{\text{load}}{\text{number of supporting strings}}$$

so, the ideal MA = number of supporting strings

= 3

Also, VR = 3

180. For any single string pulley system, the velocity ratio is equal to the number of supporting strings.

Note: The actual MA is always less than the ideal because some effort must be expended in overcoming friction and lifting the lower pulley block. The efficiency of a pulley block, as with all machines, must be less than 1 (100%).

Example:

181. In a pulley system, an effort of 20 N is required to raise a load of 200 N. If the effort moves through 150 mm to raise the load by 10 mm, find:

a. The mechanical advantage
b. The velocity ratio
c. The distance moved by the effort in raising the load 35 mm
d. The efficiency of the machine

a. Mechanical advantage $= \dfrac{\text{load}}{\text{effort}}$

$= \dfrac{200\text{ N}}{20\text{ N}} = 10$

b. Velocity ratio $= \dfrac{\text{distance moved by effort}}{\text{distance moved by load}}$

$= \dfrac{150\text{ mm}}{10\text{ mm}} = 15$

c. Distance moved by effort $=$ VR $\times$ distance moved by load

$= 15 \times 0.035\text{ m} = 0.525\text{ m}$

d. Efficiency $= \dfrac{\text{MA}}{\text{VR}} = \dfrac{10}{15}$

$= 0.666$ or 66.6%

Exercise 8

1. What would the velocity ratio of a pulley system be with 7 supporting strings between the pulleys?

2. A single string pulley system consists of 3 pulleys in the upper block and 2 pulleys in the lower block. Make a neat sketch of the arrangement. In use it was found that an effort of 100 N was required to raise a load of 430 N. Determine the velocity ratio of the system and its efficiency.

3. A mass of 80 kg is lifted by a single string three-pulley system. The applied effort is 392 N. Calculate the MA, VR and the efficiency of the pulley system.

The screw-jack

182. The screw-jack is a relatively simple machine making use of a helical screw thread to raise heavy loads by means of a small effort. A typical jack is shown in figure 30.

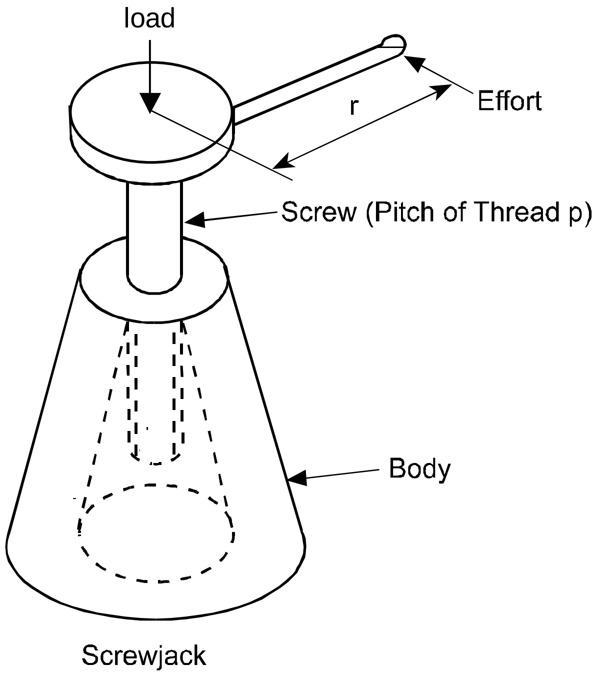


Figure 30 - Screw jack

183. If the handle of the jack is rotated through one revolution, then the effort will move through a distance S_E given by

$S_E = 2\pi r$ where r is the radius of rotation

184. The load will be raised by a distance S_L which will be equal to the pitch of the thread p.

$S_L = p$

185. The velocity ratio VR of a screw jack is therefore given by:

$$VR = \frac{S_E}{S_L} = \frac{2\pi r}{p}$$

186. As with other machines, if the jack was 100% efficient the mechanical advantage would be equal to the velocity ratio. In reality, of course, there will always be losses in the system and MA will be less than VR.

Example:

187. A screw jack has a thread with a pitch of 15 mm and the effort is applied at a radius of 0.5 m. If a load of 2 kN is raised by means of an effort of 25 N, calculate the efficiency of the jack.

Distance moved by effort in one revolution $= 2\pi r = (2\pi \times 0.5)$ m

Distance moved by load in one revolution $=$ pitch $= 15 \times 10{-}3$ m

$$\text{Therefore, VR} = \frac{2\pi \times 0.5}{15 \times 10^{-3}}$$

$$= 209.4$$

Load of 2 kN is raised by effort of 25 N

$$\text{MA} = \frac{\text{load}}{\text{effort}}$$

$$= \frac{2 \times 10^{3}}{25}$$

$$= 80$$

$$\text{efficiency \%} = \frac{MA}{VR} \times 100\%$$

$$= \frac{80 \times 100\%}{209.4}$$

$$= 38.2\%$$

188. For a screw-jack with a multi-start thread, the formula for the velocity ratio is adapted to include the number of starts that the thread has. For example:

Double start thread $\text{VR} = \frac{S_E}{S_L} = \frac{2\pi r}{2p}$

Triple start thread $\text{VR} = \frac{S_E}{S_L} = \frac{2\pi r}{3p}$ ….etc

General Formula: $\text{VR} = \frac{S_E}{S_L} = \frac{2\pi r}{(n)p}$ (where n = number of starts)

Exercise 9

1. A screw-jack has a single start thread of pitch 5 mm and the effort is applied at a radius of 0.4 m. It is found by experiment that a load of 5 kN was generated by an effort of 35 N. Calculate the:

    a. velocity ratio of the jack
    b. mechanical advantage of the jack
    c. efficiency of the jack

2. A single start screw-jack has a thread of pitch 8 mm and the effort is applied at a radius of 180 mm. If the efficiency of the jack is 35%, calculate the maximum load that can be raised by an effort of 300 N.

3. A double start screw-jack has a 10 mm pitch thread and the effort is applied at a radius of 0.14 m. If a load of 5 kN is raised by means of an effort of 290 N, determine the efficiency of the screw-jack.

PRACTICAL EXERCISE - CHARACTERISTICS OF A SIMPLE MACHINE

Aim

To investigate the properties of a simple machine, namely mechanical advantage, velocity ratio and efficiency.

Apparatus

Screw jack, force meter, steel rule, graph paper, various masses

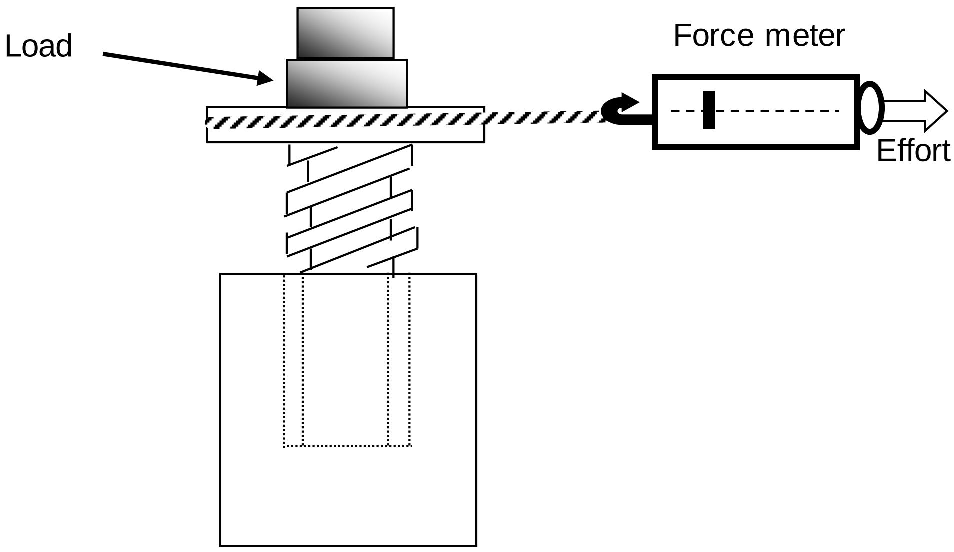


Figure 1 - The Screw Jack

Procedure

1. Take measurements from the screw jack and devise a method of determining the velocity ratio. Give details of how values were measured/calculated and record the value of the screw jack's Velocity Ratio in the table below:

Velocity Ratio (VR)	=	$\frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$
Distance moved by effort	=	
Distance moved by load	=	
Velocity Ratio	=	

2. Set the force meter to zero.

3. Wrap the string around the screw jack platform and attach the force meter to the free end of the string as per figure 1 above.

4. Ensuring the force meter is kept horizontal, measure the effort required to raise the platform with no load (mass = zero) at a slow constant speed. Record the value in column 3 of the table below against 0 kg. (Note: You may need to give the load a gentle push to overcome static friction.

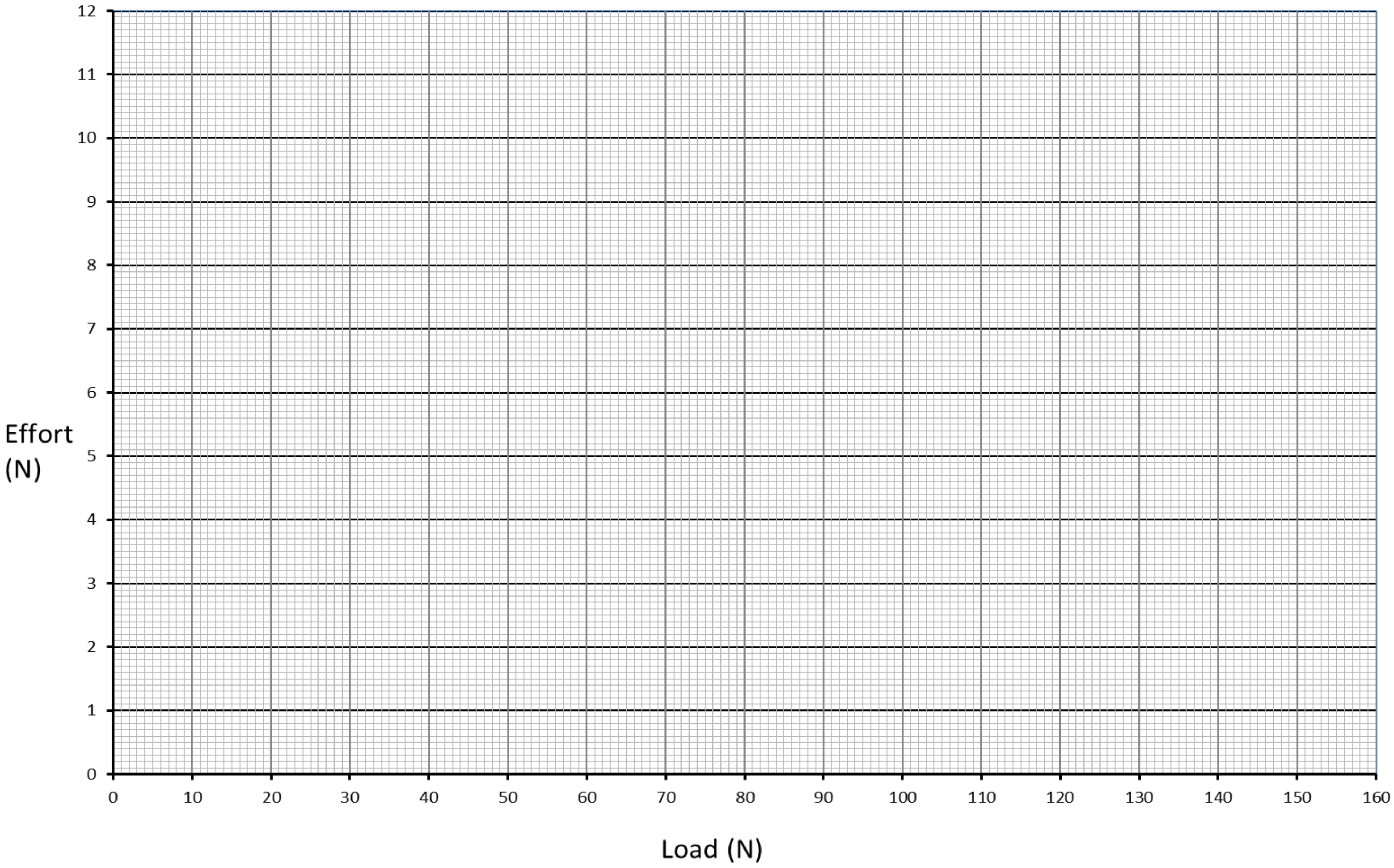
5. Place a 1.0 kg mass onto the screw jack. Wind the screw jack back to use the same portion of the thread for each added mass. Use the force meter to determine the effort force required to raise the load at a steady constant speed. Record the effort force needed in column 3 of the table.

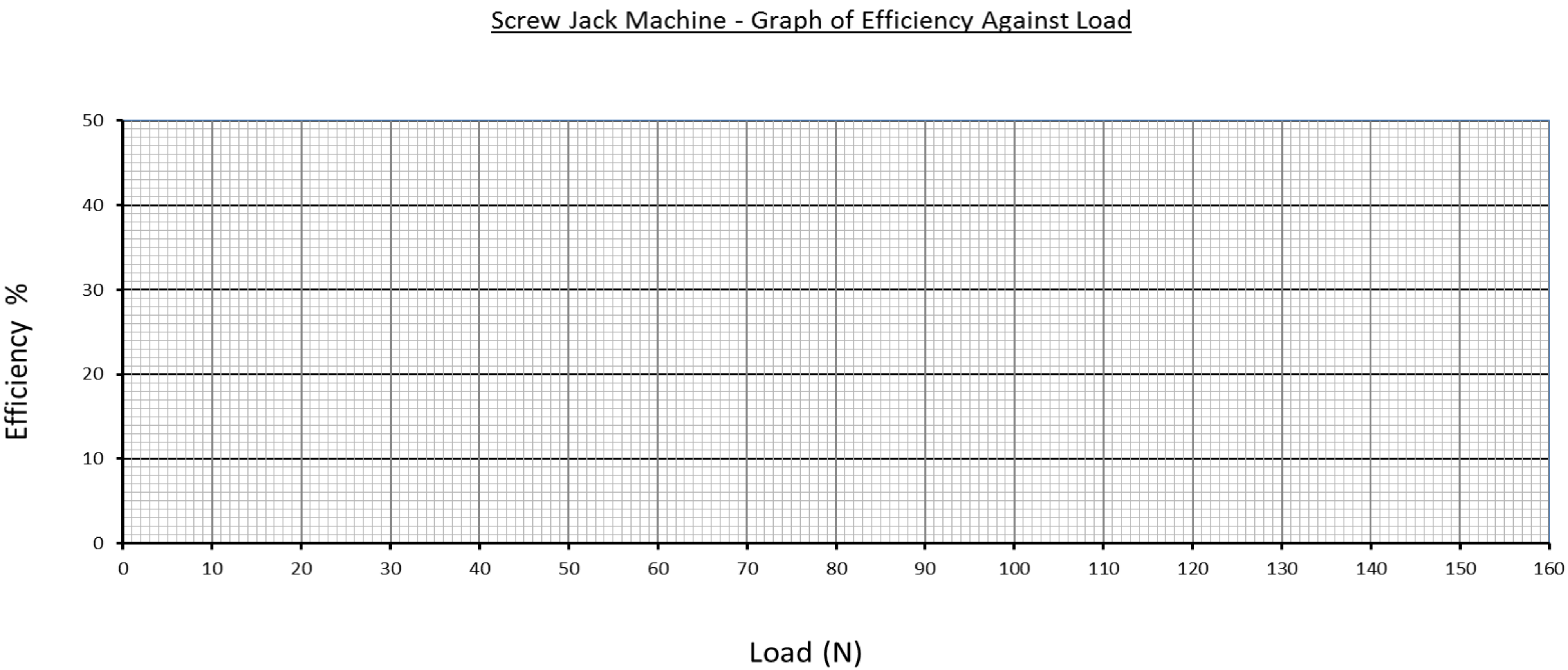
6. Repeat paragraph 5 for increasing loads of 1 kg, up to a maximum load of 16 kg.

Note: When loading the screw jack, ensure that the masses are not stacked too high. Use masses of 2, 5 and 10 kg whenever possible to ensure that the height is kept to a minimum. Take care to prevent the weights toppling from the platform.

Mass Kg	Load N	Effort N	Mechanical Advantage (MA)	Velocity Ratio (VR)	Efficiency (η) as %
0 kg	0 N				
1 kg	9.81 N				
2 kg					
3 kg					
4 kg					
5 kg					
6 kg					
7 kg					
8 kg					
9 kg					
10 kg					
11 kg					
12 kg					
13 kg					
14 kg					
15 kg					
16 kg					

Screw Jack Machine - Graph of Effort Against Load




Screw Jack Machine - Graph of Efficiency Against Load
Efficiency %
50
40
30
20
10
0
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
Load (N)

7. Calculate values of the Load, Mechanical Advantage and Efficiency to complete the table.

$$\text{Mechanical Advantage} = \frac{\text{Load}}{\text{Effort}}$$

$$\text{Efficiency}\,(\eta) = \frac{\text{Mechanical Advantage}}{\text{Velocity Ratio}}$$

8. On the graph paper draw graphs of:

   a. Effort against Load
   b. Efficiency against Load

9. Use the graph of effort against load to determine:

   the Effort to overcome Frictional Resistance = ………………N

10. Use the graph of the efficiency against load to estimate:

   the maximum efficiency possible from the screw jack = ………………. %

Conclusion

Explain briefly why screw jacks are used in some mechanical power transmission systems:

List some mechanical systems which use screw jacks to transmit power.

Gear Trains

189. Rotational motion is often transmitted by a system of meshing gear wheels or spur gears. These gear wheels can change both the magnitude and line of action of a force; hence they are another example of a simple machine.

190. A gear system in which there are only two gear wheels is known as a simple gear train. The gear that drives the system is known as the input gear or driver and the driven gear is known as the output gear driven gear or follower.

191. A velocity of rotation of a gear wheel is usually expressed in revolutions per minute (RPM) and given the symbol N.

192. In figure 31 below, the driver wheel has t_1 number of teeth and is rotating at a speed of N_1 RPM. The driven wheel or follower has t_2 number of teeth and rotates in the opposite direction to wheel 1 at a speed of N_2 RPM.

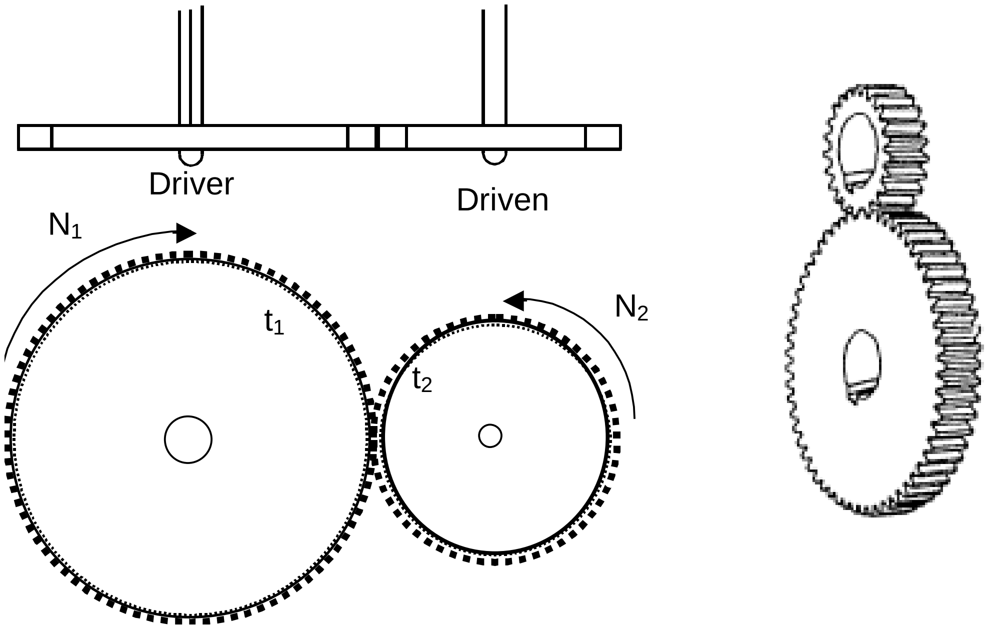


Figure 31 - Straight tooth spur gear

193. Assuming there is no slipping between the teeth on the gears and let us assume there are 40 teeth on the driver wheel ($t_1 = 40$) and 20 teeth on the follower ($t_2 = 20$). If the driver wheel makes 1 revolution its 40 teeth will engage with 40 teeth on the follower and therefore the follower will have to make 2 revolutions.

194. Written as a formula:

$$N_1 \times t_1 = N_2 \times t_2$$

$$1 \times 40 = 2 \times 20$$

$$\text{But velocity ratio} = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

$$\text{So, for a simple gear train VR} = \frac{N_1}{N_2} = \frac{t_2}{t_1}$$

195. A simple gear train is limited in its applications for the following reasons:

a. The direction of the output motion is opposite to that of the input and this may not be acceptable in certain applications.

b. The ratio between the input and output velocities depends on the difference in the number of teeth on the input and output wheels. Large ratios therefore require one of the wheels to have a large number of teeth and there may not be the space available to accommodate a wheel of sufficient size.

196. In a gear train with 3 gear wheels, the second gear is called an idler gear and its function is to rotate gear wheel 3 in the same direction as gear wheel 1.

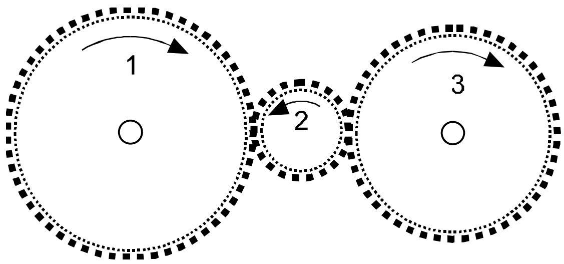


Figure 32 - Idler gear

197. The number of teeth on the idler gear has no effect as can be shown:

$$\frac{N_1}{N_2} = \frac{t_2}{t_1} \quad \text{and} \quad \frac{N_2}{N_3} = \frac{t_3}{t_2}$$

So therefore, overall VR = $\dfrac{N_1}{N_3} = \dfrac{N_2\left(\dfrac{t_2}{t_1}\right)}{N_2\left(\dfrac{t_2}{t_3}\right)} = \dfrac{\dfrac{t_2}{t_1}}{\dfrac{t_2}{t_3}} = \dfrac{t_2}{t_1} \times \dfrac{t_3}{t_2}$

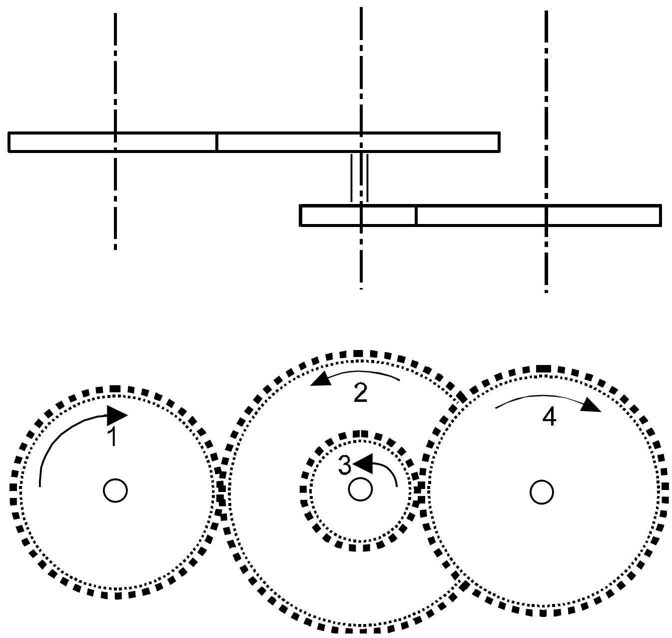
$$\therefore \frac{N_1}{N_3} = \frac{t_3}{t_1}$$

$\therefore$ for an idler gear train

$$VR = \frac{\text{number of teeth on final gear}}{\text{number of teeth on first gear}}$$

Note: the size of the idler gear is irrelevant.

198. A gear system with 2 or more gears on the same shaft is called a compound gear train. An example of a compound gear train is shown in figure 33 below.



(Gears 2 and 3 are linked by a common shaft)

Figure 33 - Compound gear train

199. Referring to figure 33, wheels 1 and 2 form a simple gear train and so do wheels 3 and 4. The speed and direction of Wheel 2 will be the same as Wheel 3 as they are on the same axle therefore $N_2 = N_3$.

$$\frac{N_1}{N_2} = \frac{t_2}{t_1} \quad \text{and} \quad \frac{N_3}{N_4} = \frac{t_4}{t_3}$$

$$VR = \frac{N_1}{N_4} = \frac{t_2}{t_1} \times \frac{t_4}{t_3}$$

In general:

$$VR = \frac{\text{input speed}}{\text{output speed}} = \frac{\text{product of number of teeth on driven wheels}}{\text{product of number of teeth on drivers}}$$

Example:

200. The input gear of a compound gear train has 36 teeth. It drives an intermediate gear with 12 teeth. The intermediate gear shaft carries a second gear with 48 teeth that drives the output gear, which has 12 teeth. Calculate the RPM of the output gear if the input is rotating at 200 RPM.

N_1 = 200 RPM
t_1 = 36
t_2 = 12
t_3 = 48
t_4 = 12
N_4 =?

For gears 1 and 2 $\frac{N_1}{N_2} = \frac{t_2}{t_1}$

so $N_2 = N_1 \times \frac{t_1}{t_2}$

For gears 2 and 3 $N_3 = N_2$ since they are on the same shaft

so $N_3 = N_1 \times \frac{t_1}{t_2}$

For gears 3 and 4 $\frac{N_3}{N_4} = \frac{t_4}{t_3}$

therefore $N_4 = N_3 \times \frac{t_3}{t_4}$

$= N_1 \times \frac{t_1}{t_2} \times \frac{t_3}{t_4}$

$= 200 \times \frac{36}{12} \times \frac{48}{12}$

= 2 400 RPM

Exercise 10

1. A compound gear train has 2 drivers 1 and 3, and 2 driven gears 2 and 4. Gears 2 and 3 are on the same shaft. The number teeth on each gear wheel are as follows:

   $t_1 = 30$, $t_2 = 60$, $t_3 = 30$, $t_4 = 80$

   If the input speed is 200 RPM, calculate the output speed.

2. A compound gear train consists of a driver gear 1 with 40 teeth meshing with 160 teeth of gear 2. Gear 3, on the same shaft as gear 2, has 50 teeth and meshes with the 100 teeth of gear 4. Calculate the velocity ratio and how many times gear 1 turns for 1 revolution of gear 4.

3. A compound gear train consists of a driver gear 1 with 30 teeth meshing with 90 teeth of gear 2. Gear 3, on the same shaft as gear 2, has 60 teeth and meshes with the 120 teeth of gear 4. Calculate the velocity ratio and efficiency if the MA is 4.32.

Worm gear

201. A worm gear consisting of a shaft with a screw thread (the worm) that meshes with a toothed wheel (the wormwheel). This combination changes the direction of axis of rotary motion by ninety degrees. Worm gears also decrease the speed of turning from worm screw to wormwheel and increase its force. An example is shown in figure 34.

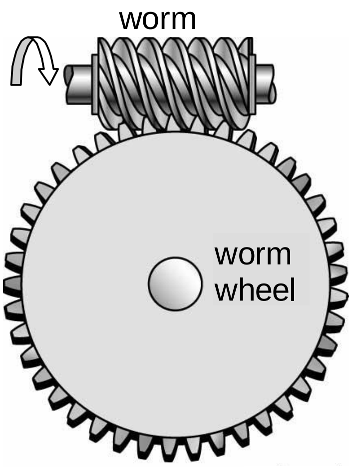


Figure 34 - Worm and wheel

202. For each complete turn of the worm the wormwheel shaft advances only one tooth of the gear. In the example in figure 34 showing a 41-tooth wormwheel gear, the speed is reduced by a factor of 41.

203. Unlike ordinary gears, the motion is not reversible, a worm can drive a gear to reduce speed, but a gear cannot drive a worm to increase it. As the speed is reduced the power to the drive increases correspondingly. Worm gears are a compact, efficient means of substantially decreasing speed and increasing drive torque. Ideal for use with small electric motors.

204. The formula for calculating the velocity ratio of a worm gear is as follows:

$$VR = \frac{N_1}{N_2} = \frac{t_2}{t_1} \qquad = \qquad \frac{\text{Number of teeth in wormwheel}}{\text{Number of teeth in worm} = 1}$$

Example:

205. The input drive of a worm gear rotates at 1200 RPM. It drives a wormwheel with 42 teeth. Calculate the velocity ratio of the worm gear and the speed of the output.

$$VR = \frac{\text{Number of teeth in wormwheel}}{1} = \frac{42}{1} = 42$$

Output speed (N_2) = Input speed (N_1) ÷ VR = 1 200 ÷ 42 = 28.57 RPM

Exercise 11

1. The output shaft from a wormwheel gear rotates at 2.75 RPM and the worm input drive shaft spins at 165 RPM. How many teeth are on the wormwheel gear?

SC3.13 - DEFINE WORK, ENERGY AND POWER

Work

206. If a force is applied to a body and it moves, then the force is said to do work on the body. Consider a block of wood of mass m sitting on a horizontal surface as shown in figure 35.

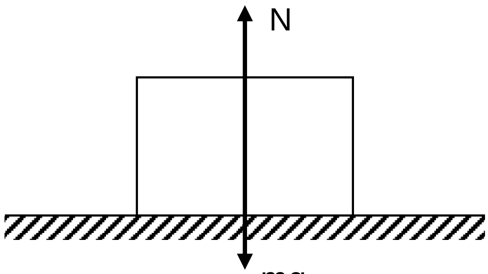


Figure 35 - Equilibrium

207. We know from previous work on forces and equilibrium that the surface will exert a reaction force N (Normal) on the block, equal in magnitude and opposite in direction to the weight of the block (mg). Although it is exerting a force on the block the surface is doing no work.

208. If, however, you take the block in your hand and raise it through some vertical distance then you are doing work - the force you are exerting is moving through a distance in the direction of the force.

209. The unit of work in the SI system is the joule (J). The joule is defined as the work done when a force of one Newton moves through a distance of one metre in the direction of the force. Hence, if a force F in Newton's is exerted through a distance s in metres the work done WD in joules is given by:

$$WD = F \times s$$

210. Note that work is a product of force and distance and the derived units of work are therefore Nm. The joule is directly equivalent to the Nm.

Energy

211. Anything that is able to do work is said to possess energy. Therefore, by definition, energy is the capacity or ability to do work.

212. When work is done, energy is converted.

213. Since energy is the capacity to do work it is measured in the same units as work. The unit of energy in the SI system is the joule (J).

Power

214. Power is defined as the rate of doing work or the rate of energy conversion. It is the amount of work done per unit of time.

$$\text{Power} = \frac{\text{Work Done}}{\text{time}}$$

215. The symbol for power is P and the unit of power is the joule per second (J/s), which is known as the watt (W).

216. Power may also be calculated from force and velocity. Consider a force F acting on a body moving with a velocity v in the direction of the force. If the body moves through a distance s, then:

$$WD = F \times s$$

$$P = \frac{WD}{t}$$

$$= F \times \frac{s}{t}$$

But $\frac{s}{t} = v$ so Power = Force x velocity

SC3.14 - EXPLAIN THE CONSERVATION OF ENERGY PRINCIPLE

The principle of conservation of energy

217. The principle of conservation of energy states that energy can neither be created nor destroyed; it can only be converted from one form to another.

218. Whilst the principle can be challenged at the nuclear level where energy can be created by the destruction of matter, as in an atomic bomb, it holds good for everyday engineering situations and can be used to solve problems involving the interchange of different categories of energy.

219. Energy can take many forms:

a. Heat or thermal energy - Molecule frictional movement

b. Electrical energy - Movement of charge within a conductor

c. Chemical energy - Batteries, matches, bullets, bombs, fuels, etc

d. Nuclear energy - Release of energy from ‘mass’ ($E = mc^2$).

e. Light energy - Electro-magnetic energy that can travel through a vacuum

f. Sound energy - Pressure waves hitting ear drum

g. Mechanical energy - Possessed by a moving body, by its position or its condition

220. In this part of the course we are primarily concerned with mechanical energy and in particular the energy possessed by a body by virtue of its position or its motion.

221. These forms of mechanical energy are dealt with in more detail in the following section.

222. According to the conservation of energy principle, the forms of energy can be converted from one form to another.

223. Here are some conversions and the device that does the conversion – known as the transducer:

- Heat energy is converted to mechanical energy by a steam engine
- Electrical energy is converted to mechanical energy by a motor
- Electrical energy is converted to heat energy by an electric fire
- Electrical energy is converted to light energy by an electric light bulb
- Electrical energy is converted to sound energy by a loudspeaker
- Chemical energy is converted to heat energy by burning fuels
- Chemical energy is converted to electrical energy by a battery
- Nuclear energy is converted to heat energy by an atomic reactor
- Light energy is converted to electrical energy by a solar cell
- Light energy is converted to chemical energy by living plants
- Sound energy is converted to electrical energy by a microphone
- Mechanical energy is converted to electrical energy by a generator
- Mechanical energy is converted to heat energy by friction

SC3.15 - SOLVE PROBLEMS INVOLVING POTENTIAL AND KINETIC ENERGY

Potential energy

224. It was stated in the previous unit that energy exists in many forms, one of which was mechanical energy.

225. The first form of mechanical energy to be considered is the energy possessed by a body by virtue of its position. This is called potential energy (PE).

226. For example, an object that is suspended at some height above the ground has the capacity to do work, namely it can fall – therefore it possesses energy. The potential energy of a body mass m at a height h is given by the formula:

$$PE = mgh$$

227. The units for PE derived from this formula are consistent with those of work as follows:

If mass is in kg, gravity in m/s^2 and height in m, then in unit terms:

$$PE = kg \; x \; \frac{m}{s^2} \; x \; m$$

But the unit of force, the newton, is derived from mass x acceleration

Newton $= kg \; x \; \frac{m}{s^2}$

Therefore, PE = N x m

= Joule (J)

228. Note that the formula for potential energy gives the energy of the body with respect to the datum from which the height is measured. This need not necessarily be ground level. For example, a block of wood suspended above the floor in a first-floor room will have a different level of PE with respect to the room floor than the PE with respect to ground level.

Example:

229. A block of concrete of mass 500 kg is suspended 5 m above a platform, which is itself 6m above ground level. Calculate the potential energy of the block:

a. with respect to the platform

b. with respect to ground level

Solution:

a. PE with respect to the platform

m = 500 kg
h = 5 m

$$\begin{aligned} PE &= mgh \\ &= 500 \times 9.81 \times 5 \\ &= 24\ 525 \text{ J} \\ &= 24.52 \text{ kJ} \end{aligned}$$

b. PE with respect to ground level

m = 500 kg
h = 11 m

$$\begin{aligned} PE &= mgh \\ &= 500 \times 9.81 \times 11 \\ &= 53\ 955 \text{ J} \\ &= 53.9 \text{ kJ} \end{aligned}$$

230. As can be seen in the above example, the block has more energy with respect to the ground than with respect to the platform.

Kinetic energy

231. The second form of mechanical energy to be considered is the energy possessed by a body by virtue of its motion. This is called kinetic energy (KE).

232. An object that is in motion has the capacity to do work - it possesses energy. The kinetic energy of a body mass m travelling with a velocity v is given by the formula:

$KE = \frac{1}{2}mv^2$

233. The units for KE derived from this formula are consistent with those of work as follows:

If mass is in kg, velocity in m/s, then in unit terms:

$$KE = \text{kg x} \left(\frac{\text{m}}{\text{s}}\right)^2 = \text{kg}\frac{\text{m}^2}{\text{s}^2} = \text{kg}\ \frac{\text{m}}{\text{s}^2}\text{ x m}$$

But the unit of force, the newton, is derived from mass x acceleration

Newton $= \text{kg x}\frac{\text{m}}{\text{s}^2}$

Therefore, KE = N x m

= Joule (J)

Example:

234. A car of mass 1.5 t is travelling at 55 km/h and accelerates to 70 km/h. Calculate:

a. The initial KE of the car
b. The final KE of the car

Solution:

a. Initial KE

m = 1.5 t = 1 500 kg

v = 55 km/h $= 55 \times \frac{10^3}{3\ 600} = 15.28$ m/s

KE $= \frac{1}{2} \times 1\ 500 \times 15.28^2$

$= 175.1$ kJ

b. Final KE

m = 1.5 t = 1 500 kg

v = 70 km/h $= 70 \times \frac{10^3}{3600} = 19.44$ m/s

KE $= \frac{1}{2} \times 1\ 500 \times 19.44^2$

$= 283.4$ kJ

235. In the previous section when discussing the principle of conservation of energy, it was stated that energy can be converted from one form to another. It can therefore be assumed that the potential energy possessed by a body can be converted into kinetic energy, and vice versa. If no energy is created or destroyed during this conversion, then the total energy remains the same. During this conversion it can be then assumed that the potential energy lost will exactly equal the kinetic energy gained or vice versa (i.e. no losses).

236. This can be explained by considering the following example.

Example:

237. A brick of mass 3 kg, sits on the ground.

a. How much potential energy does it have?

PE = mgh = 3 x 9.81 x 0 = 0 J

b. If the brick is now lifted 8 m vertically onto a scaffold platform. How much potential energy does the brick have when balanced on the platform?

PE = mgh = 3 x 9.81 x 8 = 235.44 J

c. From where does the brick get this energy?

To lift it up by 8 m, work has been done.

Work done = force x distance

But force = mass x acceleration (in this case work is done against the acceleration of gravity

Therefore, the work done = mass x gravity x distance (height)

= mgh = PE

d. If the brick is accidentally knocked from the scaffold, what happens to the brick and to this potential energy?

The brick will fall back down to the ground due to the acceleration of gravity. As it is moving it will have another form of energy, that of kinetic energy. According to the law of conservation of energy, as the brick falls the potential energy is changed into kinetic energy.

e. Assuming no energy losses due to air resistance, what will be the kinetic energy of the brick as it strikes the ground?

At the point that the brick hits the ground it will have zero potential energy again because it has no height (PE = mgh).
All of the PE will therefore become KE

KE = 235.44 J

f. With what velocity will the brick strike the ground?

Re-arrange KE equation to get velocity:

$$v = \sqrt{\frac{2 \text{ x KE}}{m}} = \sqrt{157} = 12.53 \text{ m/s}$$

Example:

238. A football of mass 1.5 kg is kicked vertically into the air. It leaves the footballers foot at a velocity of 20 m/s. Using energy conservation concepts, calculate how high it will travel.

Solution:

Kinetic energy when it leaves the footballers boot = ½ mv^2

So KE = ½ x 1.5 x 20^2 = 0.75 x 400 = 300 J

This energy is transformed into PE as the ball rises. At the top of the travel it will have no KE (velocity will be zero) so all of the KE will have transformed into PE. So, PE at top = 300 J

We know that PE = mgh so we can work out the height reached

$$h = \frac{PE}{mg} = \frac{300}{1.5x9.81} = \frac{300}{14.7} = 20.4 \text{ m}$$

Exercise 12

1. Explain the principle of conservation of energy.

2. List 4 examples of the application of the principle.

3. A trolley of mass 30 kg is travelling with a velocity of 25 m/s along a track, which is 5 m above ground level. Calculate:

    a. The KE of the trolley.
    b. The PE of the trolley.
    c. The total energy of the trolley.

4. The KE of an aircraft on take-off is 9.6 MJ. If the aircraft has a mass of 12 tonne, calculate the take-off velocity.

5. A body of mass 75 kg is supported 20 m above ground level. It is then released and falls freely to earth. Calculate its PE and KE:

    a. Before release.
    b. 5 m above ground level.
    c. Just before it strikes the ground.

6. A mass of 120 kg is dropped from a height of 10 m. Assuming no losses, calculate:

    a. The PE of the mass before release.
    b. The KE of the mass immediately before striking the ground.
    c. The velocity with which the mass strikes the ground.

SC3.16 - DEFINE BASIC TERMS USED WITH FRICTION

Friction

239. If a surface moves, or attempts to move, over another surface with which it is in contact it will experience a resistance to its motion. The value of this resistance will depend on a number of factors including the materials involved, the condition of the surfaces in contact and the forces holding the surfaces together, but it will always be present. This resistance is known as friction.

240. It is generally found that these frictional forces are greater just before the one surface starts to move over the other than when motion is actually just taking place. The point just before motion takes place is known as the point of impending motion.

241. In other words, it takes a greater force to start the surface moving than it does to keep it in motion at a uniform velocity. These two cases are known respectively as static friction (Ffs) and dynamic or sliding friction (Ffd) and the forces due to static friction are generally greater than those due to dynamic friction.

Examples and applications of friction

242. Friction is a mixed blessing. Without the friction between your shoes and the floor it would be impossible to walk across a room and without the friction between tyres and road a car would not be able to move, much less go around corners or stop! Friction can also, of course, be undesirable and great efforts are made to reduce it to a minimum in, for instance, wheel bearings or between the moving parts of an engine. In these cases, excessive friction means wasted energy, overheating and undue wear on the components concerned.

243. As stated previously, the degree of friction present in any situation depends on the type of material and condition of the surfaces involved and on the forces between them. A common method of reducing friction is by separating the surfaces involved with a viscous fluid, commonly either oil or grease - this process is known as lubrication. Alternatively, surfaces may be separated by high pressure air.

The laws of dry friction

244. Practical experiments carried out over many years have established a set of laws that govern the relationship between frictional forces and the conditions appertaining at the time. It is emphasised that, with the exception of the first, these laws of friction are not fundamental rules that can be used to predict an exact mathematical result in every circumstance but are general laws that appear to govern the majority of situations encountered.

245. The six laws of dry friction are:

a. Friction always opposes the direction of motion or impending motion

b. Frictional force is proportional to the force between the surfaces. (This is known as the normal reaction N)

c. Frictional force is independent of the area in contact.

d. Static friction is greater than dynamic friction.

e. Dynamic friction is independent of velocity.

f. Friction depends on the nature and condition of the surfaces involved.

Normal reaction and coefficient of friction

246. Consider a block resting on a surface as shown in figure 36.

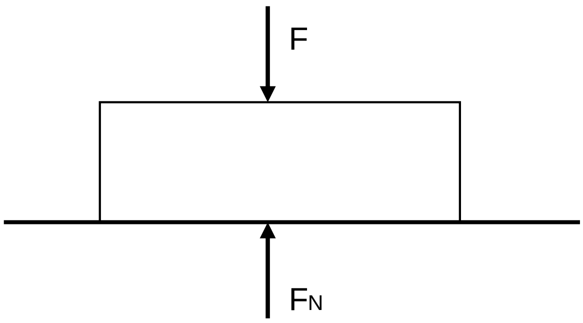


Figure 36 - Normal reaction

247. If there is a force F pressing the block on to the surface it will be balanced by a reactive force (F_N) provided by the surface. It is called the normal reaction since it always acts at right angles (perpendicular) to the surface.

248. If the block has mass m and the surface is horizontal with no other forces involved, then F will be the weight of the block (mg) and the magnitude of the normal reaction force F_N will then also equal mg. This is not always the case, for example, a block resting on a surface, which is inclined to the horizontal at an angle.

249. The value of F_N would also be affected by any external force acting on the block pressing it on to the surface. It would be increased, for example, by any magnetic attraction between the two.

Coefficient of friction

250. The law of friction which states that the frictional force F_f is directly proportional to the normal reaction F_N may be written as follows:

$$F_f \propto F_N$$

251. This may be rewritten as an equation by inserting a constant of proportionality. In the case of friction, the constant is represented by the Greek letter μ (mu), which is known as the coefficient of friction. Thus:

$$F_f = \mu F_N$$

252. This equation is often transposed to give:

$$\mu = \frac{F_f}{F_N}$$

253. If the block shown in figure 36 now has a pulling force F_P applied to it, then four forces will now be acting on the block, as shown in figure 37.

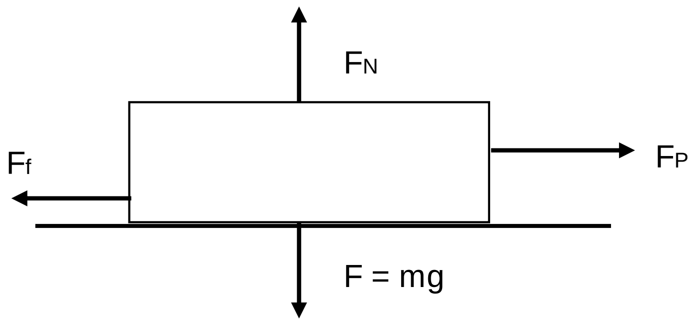


Figure 37 - Four balanced forces

254. In this case the normal reaction F_N will be equal to the weight (mg) and the pulling force, F_p will be opposed by the frictional force F_f.

255. For small values of F_p there will be sufficient friction between block and surface to prevent movement taking place. If F_p is slowly increased, then F_f will initially also increase so that the two forces remain in balance and the block will remain static.

256. Eventually, however, F_f will reach its maximum or limiting value and if we increase F_p any further, the block will start to move. This point at which the block is just on the point of moving is known as the point of impending motion.

SC3.17 - EXPLAIN VISCOSITY

Viscosity

257. As has been discussed so far, friction occurs between the surfaces of 2 objects when they move relative to each other.

258. In a moving fluid, each molecule of fluid experiences stresses exerted on it by other molecules of the fluid which surround it. The stress at each part of the surface of the molecule can be resolved into components known as pressure and shear stress. Pressures are exerted whether a fluid is moving or at rest whilst shear stresses only occur in moving fluids.

259. Viscosity is a measure of a fluid's resistance to flow and may be considered as internal friction. The higher the value of viscosity the greater the resistance to motion or flow.

260. A substance such as treacle will have a high viscosity, whilst water will have a lower viscosity. Gases such as air will have a very low value of viscosity.

261. Viscosity is a function of temperature and the viscosity of a liquid falls as the temperature increases.

262. However, gases such as air, work in the reverse sense in that the viscosity increases with a rise in temperature (due to more chaotic motion of air molecules making ordered progress more difficult).

SC3.18 - SOLVE SIMPLE PROBLEMS INVOLVING FRICTION ON A HORIZONTAL SURFACE

Solving problems involving friction

263. The concepts below can be used to solve problems involving the determination of frictional forces etc. and a number of different cases are dealt with below involving bodies on purely horizontal planes.

264. The problems may be addressed by the resolution of forces. Although the examples below involve stationary bodies on the point of impending motion, the same techniques would be used with bodies moving at constant velocity except that μ would then represent the coefficient of sliding friction.

Example:

265. A concrete block has a mass of 2 tonne and rests on a horizontal concrete floor. If the coefficient of friction between block and floor is 0.65, calculate the minimum force necessary to move the block.

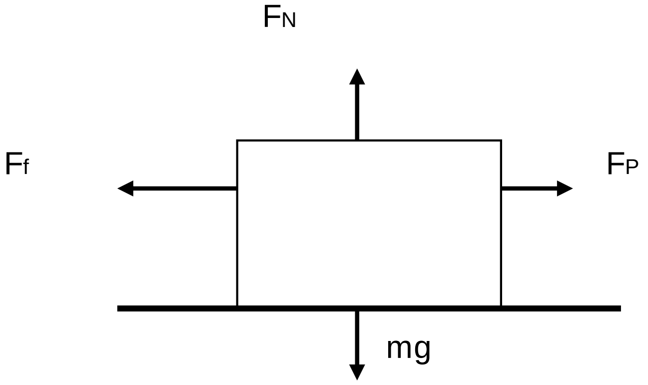


Figure 38

m = 2 t = 2x103 kg
μ = 0.65
F_p =?

Solution by Resolution of Forces:

Resolving parallel to plane

$$F_p = F_f$$

Resolving perpendicular to plane

$$N = mg$$

But F_f = μ N

$$F_f = \mu\, mg$$

And $$F_p = \mu\, mg$$

$$F_p = 0.65 \times 2 \times 103 \times 9.81 \text{ N}$$

$$F_p = 12\ 753 \text{ N} = 12.753 \text{ kN}$$

Example:

266. A wooden block, of mass 5 kg, sits on a horizontal steel surface. It is found that a horizontal force of 30 N is just sufficient to move the block. Determine the coefficient of friction between wood and steel

$m = 5$ kg
$F_p = 30$ N
$\mu = ?$

Solution by Resolution of Forces:

Resolving parallel to plane

$$F_p = F_f$$

Resolving perpendicular to plane

$$F_N = mg$$

But $F_f = \mu N$

$$F_f = \mu mg$$

$$F_p = \mu mg$$

Transposing for μ

$$\mu = \frac{F_p}{mg}$$

$$\text{so } \mu = \frac{30}{5 \times 9.81}$$

$$\mu = 0.61$$

Exercise 13

1. A steel block, mass 25 kg, rests on a horizontal wooden floor. If the coefficient of friction between steel and wood is 0.45, determine the minimum force necessary to move the block if the force is applied horizontally.

2. An aircraft lands wheels up. If the mass of the aircraft is 50 tonne and it is found that the force necessary to move it is 420 kN, determine the coefficient of friction between the aircraft and the runway.

3. An Iron block of mass 56 kg, rests on a horizontal surface. If the horizontal force required to cause impending motion is 280 N, calculate the coefficient of friction between the iron and the surface. If the coefficient is reduced by lubrication to 0.3, calculate the horizontal force necessary to cause impending motion.

4. The material used in a braking system is tested and it is found that the coefficient of friction between the material and the steel brake disk is 0.81. Calculate the normal force when the frictional force is 0.74 kN

5. List the 6 laws of dry friction.

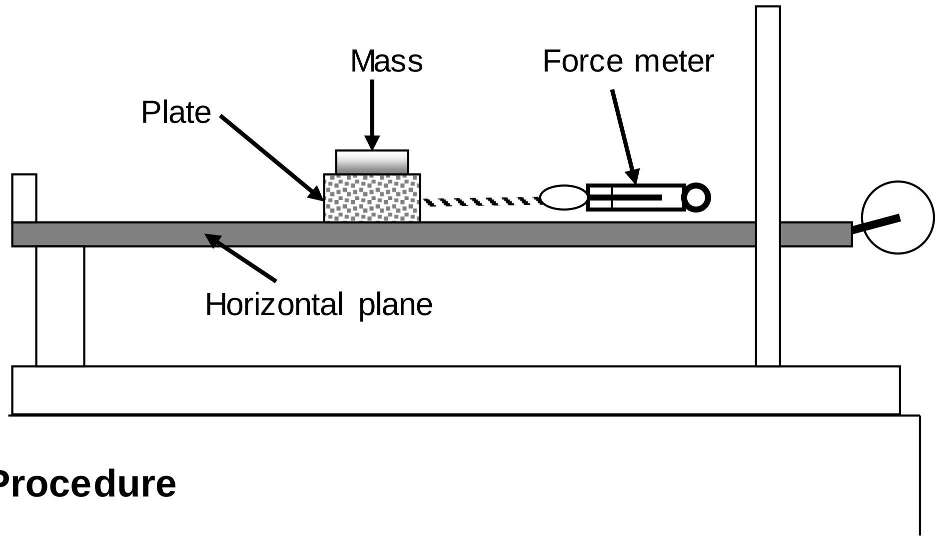
PRACTICAL EXERCISE - 2ND LAW OF FRICTION

Aim

To establish by practical means the general relationship between the frictional force and the normal reaction.

Apparatus

Horizontal plane, sliding mass plate, masses, force meter, string



Procedure

1. Ensure the force meter reads 0 – adjust as required.

2. Measure the weight of the softwood plate using the spring balance and record in table 1 column 1.

3. Add a flat plate mass of 0.5 kg on top of the softwood sliding plate.

4. Place the sliding plate at a fixed point away from the pulley end.

5. With the plane set horizontal, attach one end of the string to the softwood sliding mass plate and the other end to the force meter.

6. Apply a steadily increasing horizontal pull to the force meter until the plate just starts to move.

7. Measure the force that is required to cause the plate to move very slowly with constant velocity, and record in table 1, column 5.

8. Add a further 0.5 kg mass to the softwood sliding plate.

9. Repeat paragraphs 4 to 8, until a total of 3.5 kg is pulled on the sliding plate.

10. Calculate the weight of each mass (column 2 x gravity) and record results in column 3. Then add this to the weight of the softwood plate (column 1) to give the total weight force and record this in column 4.

Note: Total weight force is equal to normal reaction N.

Weight of softwood Plate (N)	Value of added mass (kg)	Weight of additional mass (N)	Total weight = Normal Reaction (N)	Force meter value = Friction Force Ff (N)
	0.5 kg	4.905 N		
	1.0 kg	9.81 N		
	1.5 kg			
	2.0 kg			
	2.5 kg			
	3.0 kg			
	3.5 kg			

Table 1

11. Plot a graph of F_f against N from result obtained.

Conclusions

12. Look at the results on the graph. What conclusion can you draw from the graph about the relationship between the frictional force and the normal reaction?
…………………………………………………………………………………………..
…………………………………………………………………………………………..

13. Given that the gradient of the graph is the coefficient of friction of the material (μ). Calculate the gradient of the graph.

The coefficient of friction of the softwood is …………………….

14. When the pull was applied to the spring balance what did you notice about the value of force applied before the block moved and then just as it started to move?
…………………………………………………………………………………………..
…………………………………………………………………………………………..

15. Why is this?
…………………………………………………………………………………………..
…………………………………………………………………………………………..

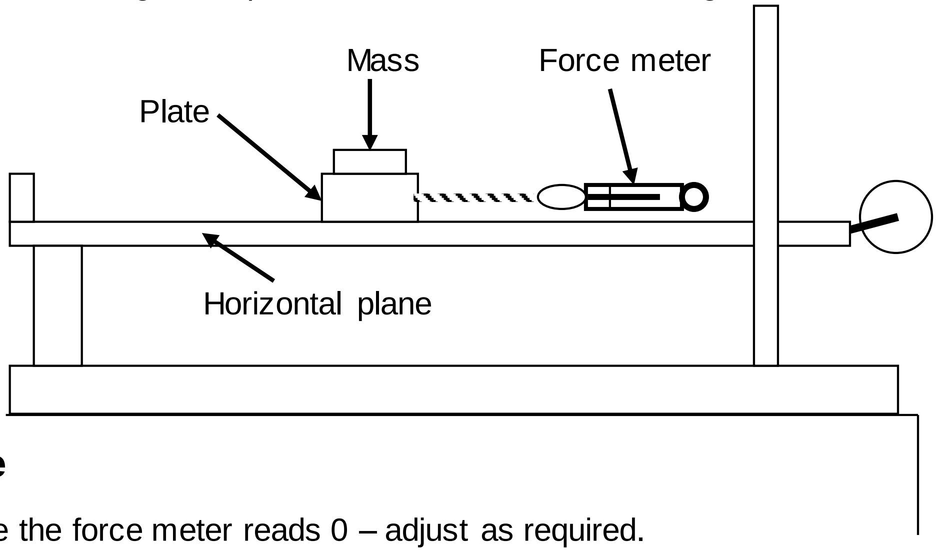
PRACTICAL EXERCISE - 3RD LAW OF FRICTION

Aim

To establish by practical means that frictional force is independent of area in contact.

Apparatus

Horizontal plane, sliding mass plate, masses, force meter, string



Procedure

1. Ensure the force meter reads 0 – adjust as required.

2. Place the fully flat surface of the wooden mass plate onto the plane.

3. Place the sliding plate at a fixed point away from the pulley end and add a flat plate mass of 2 kg on top of the sliding plate.

4. With the plane horizontal, attach one end of the string to the wooden sliding mass plate and the other end to the force meter.

5. Apply a steadily increasing horizontal pull to the force meter until the plate just starts to move.

6. Record the force that is required to cause the plate to move very slowly with constant velocity:

   Force required (full area) = ………………………………. N

7. Turn the wooden sliding plate over so that the area in contact with the plane is approximately halved.

8. Repeat paragraphs 3 to 6 and record force below.

   Force required (half area) = …………………………… …N

9. Look at the results obtained in paragraph 6 and 8. Can you draw any conclusion from the results about the relationship between the frictional force and the area in contact?……………………………………………………………………………..
……………………………………………………………………………………………..

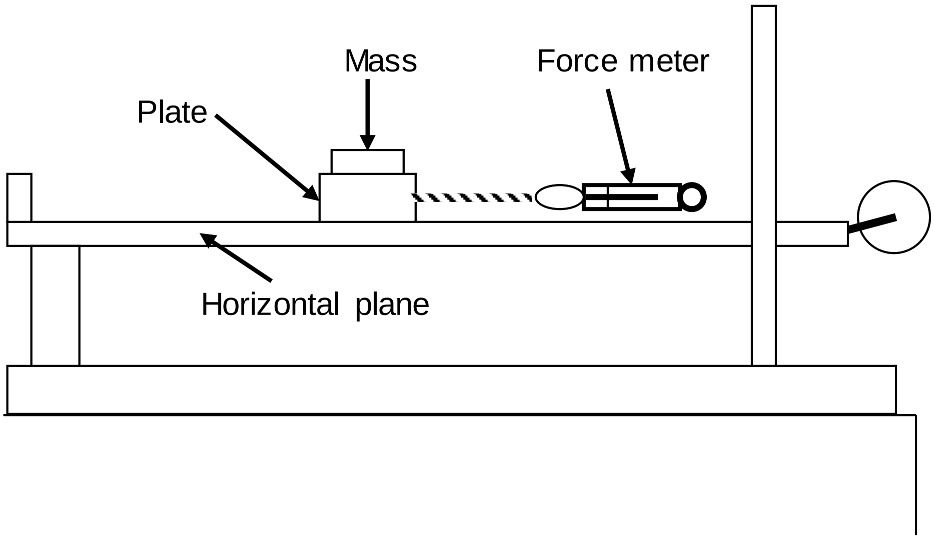
PRACTICAL EXERCISE - 4TH LAW OF FRICTION

Aim

To establish by practical means that static friction is greater than dynamic friction.

Apparatus

Horizontal plane, sliding mass plate, masses, force meter, weighing scales, string



Procedure

1. Ensure the force meter reads 0 – adjust as required.

2. Place the wooden sliding mass plate onto the plane.

3. Place the sliding plate at a fixed point away from the pulley end and add a flat plate mass of 3 kg on top of the sliding plate.

4. With the plane horizontal, attach one end of the string to the wooden sliding mass plate and the other end to the force meter.

5. Apply a steadily increasing horizontal pull to the force meter until the plate just starts to move.

6. Record the maximum force observed, just before the plate begins to move

   Maximum force observed = ………………………………..N

7. Record the force that is required to cause the plate to move very slowly with constant velocity:

   Constant force required = ………………………………..N

8. Can you draw any conclusion from the results about the relationship between the static and dynamic friction? ……………………………………………………………….. ……………………………………………………………………………………………

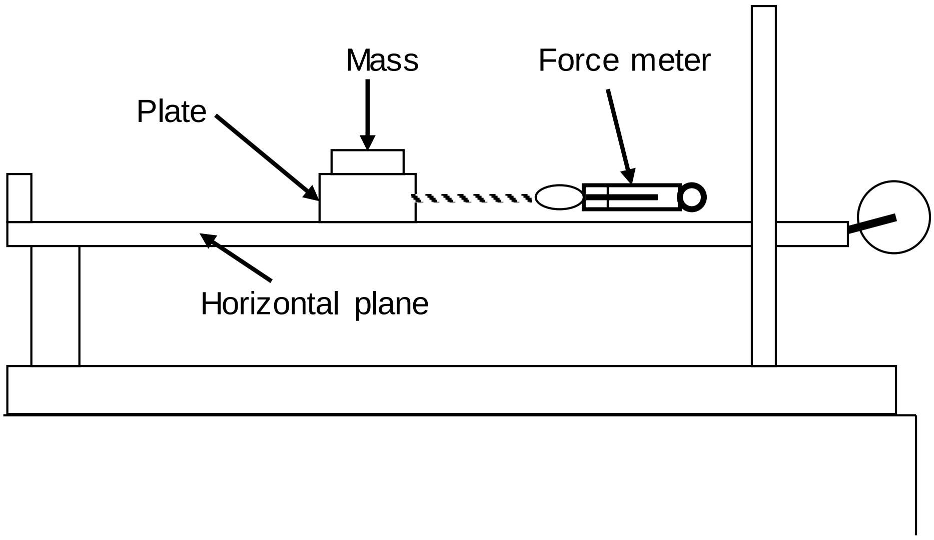
PRACTICAL EXERCISE - 6TH LAW OF FRICTION

Aim

To establish by practical means that friction depends upon the nature and condition of the surfaces in contact.

Apparatus

Horizontal plane, sliding mass plates, masses, force meter, weighing scales, string



Procedure

1. Ensure the force meter reads 0 – adjust as required.

2. Measure and record the combined weight of the perspex/leather plate, the softwood plate, hardwood plate and a 2 kg mass. Record this value in table 2, column 2.

3. Place the perspex surface of the sliding plate onto the horizontal plane and add the softwood, hardwood and a flat plate mass of 2 kg on top of the sliding plate.

4. Place the sliding plate at a fixed point away from the pulley end.

5. With the plane horizontal, attach one end of the string to the perspex sliding mass plate and attach the other end to the force meter.

6. Apply a steadily increasing horizontal pull to the force meter until the plate just starts to move.

7. Record the force that is required to cause the plate to move very slowly with constant velocity, and record in table 2, column 3.

8. Turn the perspex sliding plate over, placing the leather plate onto the horizontal plane and add the softwood, hardwood and 2 kg mass.

9. Repeat paragraphs 4 to 7 for leather, recording force required in column 3.

10. Replace the perspex/leather sliding plate with the softwood sliding plate surface now onto the plane.

11. Add the hardwood, the perspex/leather plate and the 2 kg mass, to ensure that the total mass of the slider is the same as before.

12. Repeat paragraphs 4 to 7 for softwood and record results in table 2.

13. Replace the softwood sliding plate with the hardwood sliding plate with the flat surface to the plane.

14. Again, add the softwood, the perspex/leather plate and the 2 kg mass, to ensure that the total mass of the slider is the same as before.

15. Repeat paragraphs 4 to 7 for hardwood, recording force required in column 3.

16. Calculate the co-efficient of friction μ for each of the materials used using the formula:

$$F_f = \mu N$$

17. Record the values of μ in table 2 column 4.

Material used on plane	Total weight (N)	Friction Force F_f (N)	Co-efficient of friction μ
Perspex			
Leather			
Softwood			
Hardwood			

Table 2

18. Look at the results in table 2. Can you draw any conclusion about the relationship between the frictional force and the nature and condition of the surfaces in contact material?
…………………………………………………………………………………………………...
……………………………………………………………………………………………………
……………………………………………………………………………………………………
………………………………………………………………………………………

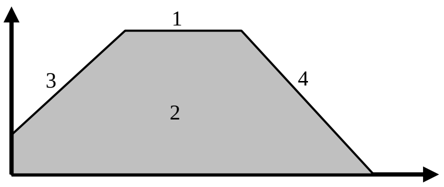
Dynamics Consolidation

1. What is Newton's first law of motion?

2. If an aircraft's landing speed and length of runway used in coming to rest are known, identify which of the equations of motion could be used to determine the deceleration rate of the aircraft.

3. A car accelerates uniformly from rest up to a velocity of 25 m/s and this takes 20 seconds. It then decelerates at a uniform rate of 2.5 m/s^2 until it stops again. Calculate the total distance travelled.

4. Identify which part of the velocity / time graph represents uniform velocity.



5. When a body is thrown vertically upwards, what does the force of gravity do?

6. A stone dropped from the roof of a building takes 4 seconds to hit the ground. Calculate the height of the building.

7. What is the definition of centripetal force?

8. When does resonance occur on a body subject to simple harmonic motion?

9. A pendulum has a bob length of 34 cm. Calculate the frequency of oscillation.

10. A 40 kg load is to be moved using a 2 m lever. Calculate the VR and MA, if the fulcrum is 25 cm from the load and the effort required to move the load is 56 N.

11. If a force of 50 N is applied to an object that moves 30 m in 15 seconds, calculate the power used.

12. Explain the theory of Conservation of energy.

13. A ball weighing 2 kg falls from a wall and strikes the ground with a Kinetic Energy (KE) of 392.4 J. Calculate the height of the wall.

14. A 3 kg wood block sits on a horizontal steel plate. If the contact area is 48 cm^2 and the force to move the block is 22 N, calculate the coefficient of friction between the wood and the steel.

15. A load of 40 kg is lifted using a single string 3 pulley system. If the applied effort is 196.2 N calculate the MA, VR and the percentage efficiency of the pulley.

16. List 4 laws of friction.

17. Define viscosity

18. As the temperature of a fluid increases what happens to its viscosity?

19. As the temperature of a gas increases what happens to its viscosity?

SCIENCE DEFINITIONS

Book 3 – Dynamics

Distance	Change of position of a body independent of direction.
Displacement	Change of position of a body in a given direction – units: metre.
Speed	The change of distance with respect to time - units are metre per second (m/s)
Velocity	The change of displacement with respect to time - units are metre per second (m/s)
Acceleration	The change of velocity with respect to time - units are metre per second squared (m/s^2)
Newton 1st law	An object continues in a state of rest or uniform motion in a straight line unless a force acts to change.
Newton 2nd law	When a force acts, the rate of change of momentum is proportional to the applied force and takes place in the direction in which the force acts. $F = ma$
Newton 3rd law	For every action there is an equal and opposite reaction.
Momentum	Product of a bodies mass and velocity - units are kilogram metre per second (kg m/s).
Inertia	Resistance of a body to change its motion.
Radian	The angle produced by the length of one radius around the circumference of a circle.
Centripetal acceleration	The acceleration of an object in a circle as it tries to take the straight path (linear) - units are radian per second squared (rad/s^2).
Centripetal force	Inwards force created as an object accelerates in a circle rather than traveling in a straight line - units are Newton (N)
Centrifugal force	Outwards force that opposes centripetal force - units Newton (N)
Angular velocity	The change of angular displacement with respect to time - units are radians per second (rad/s).
Angular acceleration	The change of angular velocity with respect to time - units are radians per second squared (rad/s^2).
Amplitude	The size of displacement.

Angular momentum	Product of a body's moment of inertia (I) and angular velocity - units are kilogram radian per second (kg.rad/s).
Simple Harmonic Motion (SHM)	The oscillatory motion caused by a restoring force acting in the opposite direction to the displacement force. Examples: weight on a spring; guitar string, ruler on a desk; pendulum.
Free vibration	After one displacement a system is allowed to vibrate at its natural frequency until it stops.
Forced vibration	A system vibrating due to a periodic force being applied to make it continue to oscillate. Examples: microphone or speaker.
Resonance	Where the frequency of a forced vibration is applied at the same rate as its natural frequency causes size of oscillations to increase.
Cycle	One complete vibration.
Periodic Time	Time taken to complete one cycle - units are seconds (s).
Frequency	Number of cycles per second - units are Hertz (Hz).
Work	The product of a force and distance - units are Joule (J)
Power	The rate of doing work - units are Watts (W).
Gyroscopic rigidity	The ability of a gyro to maintain a fixed point in space caused by angular momentum. More momentum equals more rigidity.
Gyroscopic precession	An axis of rotation will react and move at 90 degrees to an applied force on it.
Simple machine	A device that changes the magnitude, direction and/or line of action of a force.
Conservation of Energy	Energy cannot be created or destroyed only converted from one form to another. Examples: A battery takes chemical energy and converts it to electrical energy; a light bulb takes electrical energy and converts it to light and heat energy.
Static friction	The force that opposes the relative motion of two sliding surfaces - when not moving.
Dynamic friction	The force that opposes the relative motion of two sliding surfaces – when moving.
Co-efficient of friction	The relationship between the frictional force and the normal reaction force
Viscosity	A fluids resistance to flow.

Answers to Exercises

Exercise 1 p10

1. The rate of change of velocity.
2. Weight is the force produced by the action of gravity upon the mass.
3. Because the value of gravity changes due to location, so does weight.
4. 2.943 kN 5. 40.26 x 10^3 kg 6. 883 N, 144 N 7. 0.4587 kg
8. See page 6. 9. See page 7.

Exercise 2 p19-20

1. a. 27.5 m/s b. 99 km/hr
2. 71. 8 s
3. a. 48.5 m/s b. 4.95 s c. 37.1 m/s
4. a. 1.7 m/s^2 b. 5.81 s c. 12 000 kgm/s d. 2.064 kN
5. a. 15.97 m/s b. 3.26 s
6. a. 31.9 m b. 5.1 s
7. a. 2.19 m/s^2 b. 5.86 m/s c. 5.48 s
8. a. 48.9 m/s b. 2.44 s
9. a. 2.84 m/s^2 b. 8.12 m/s
10. 875 m 1.43 m/s^2

Exercise 3 p22

1. a. 25m b. 450 m c. 10 m d. 485 m e. 9.32 m/s
   f. 2 m/s^2 g. 5 m/s^2

2. a. 4 s b. 2 m/s^2 c. 92 m

3. a. 27.73 s b. 126.5 m

Exercise 4 p25

1. a. 3.49 rad b. 12.57 rad c. 0.628 rad d. 22 rad e. 1533 rad

2. a. 540^0 b. 114.6^0 c. 2 578^0 d. 1 260^0 e. 36 000^0

3. a. 2 rev b. 7.16 rev c. 15.9 rev

4. a. 628.32 rad/s b. 1 570.8 rad/s c. 628.32 rad/s d. 157.08 rad/s

5. a. 209.44 rad/s b. 314.16 rad/s c. 52.36 rad/s^2

6. The angle turned through when a point on a circle moves by distance equal to the radius.

Exercise 5 p28

1. Inwards. 2. 9.6 N 3. 37.5 N 4. 3.266 rad/s

Exercise 6 p41

1. Runs slower 2. 1.16 Hz 3. 1.35 s

4. A vibration at natural frequency after 1 displacement then allowed to vibrate freely.

5. Application of a periodic force on a body capable of vibrating.

6. Occurs when a forced vibration occurs at or near the natural vibration rate and oscillations increase rather than reduce.

7. Motion where a restoring force is applied that is proportional to the displacement and in the opposite direction to the displacement.

Exercise 7 p49

1. A device to change the magnitude and/or direction of a force.

2. MA less than VR.

3. 3.6; 4.33; 83.1% 4. 1 816.7 N

Exercise 8 p53

1. 7. 2. 5; 86% 3. 2; 3; 66.67%

Exercise 9 p56

1. a. 502.7 b. 142.9 c. 28.4%

2. 14.84 kN 3. 39.2%

Exercise 10 p65

1. 37.5 RPM 2. VR = 8; 8 revs 3. VR = 6; 72%

Exercise 11 p67

1. 60

Exercise 12 p76

1. Energy can neither be created or destroyed, just converted from one form to another.

2. See handout page 71.

3. a) 9.375 kJ b) 1.472 kJ c) 10.85 kJ

4. 40 m/s

5.	a)	14.7 kJ; 0	b)	3.68 kJ; 11.04 kJ	c)	0 ; 14.7 kJ
6.	a)	11.77 kJ	b)	11.7 kJ	c)	14 m/s

Exercise 13 p82

1. 110.4 N

2. 0.86

3. 0.51; 164.8N

4. 913.6 N

5. See handout page 78.

Answers to Dynamics Consolidation

1	See CM Dynamics page 6
2	$v^2 = u^2 + 2as$
3	375 m
4	1
5	It opposes the motion and slows the body down to a stop. Then accelerates the object back down to ground.
6	78.5 m
7	The force tending to pull the rotating body inwards to centre
8	Natural frequency adds to forced vibration making bigger
9	0.85 Hz
10	MA = 7 VR = 7
11	100 W
12	Energy cannot be created or destroyed just converted
13	20 m
14	0.75
15	MA = 2 VR = 3 efficiency = 66.6%
16	See CM Dynamics page 78
17	A measure of a fluids opposition to flow
18	Viscosity reduces
19	Viscosity increases