Defence School of
Aeronautical Engineering

# No.2 School of Technical Training

# Academic Principles Organisation

# TG5 MATHEMATICS

# 2208A, 2209A, 2211A

# Book 2

# Algebra

## WARNING

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# KEY LEARNING POINTS

| KLP | Description |
|---|---|
| 1.2.2.1 | Explain the use of algebraic symbols to represent numbers |
| 1.2.2.2 | Factorise expressions and expand brackets |
| 1.2.2.3 | Construct algebraic expressions and equations |
| 1.2.2.4 | Manipulate algebraic formulas |

# THE BASIC NOTATION AND RULES OF ALGEBRA

1. Algebra is the investigation into the relationships and properties of numbers by the use of symbols. In general, these symbols are letters, English or Greek that represent a set of values suitable for a practical situation. For example, the fruit in a fruit bowl can be given as an algebraic expression.

5 apples and 8 apples = 13 apples

where substituting the symbol 'a' to represent the apples,

$5a + 8a = 13a$

2. In this example the letter 'a' has been used to represent the apples, the number in front of the letter 'a' is known as the coefficient and it states the number of those items. The letter and the number together make up an algebraic term, the 13a is the sum of the two terms.

3. Algebraic expressions are made up from a group of terms, so when four bananas and three pears are added to the bowl:

5 apples and 8 apples and 4 bananas and 3 pears

the expression given algebraically is:

$5a + 8a + 4b + 3p$

4. The expression is a statement about different quantities, which in this case tells how many different fruits are in the bowl, an expression can be simplified by adding or subtracting like terms together.

$5a + 8a + 4b + 3p$

$= 13a + 4b + 3p$

5. The letters and symbols used in algebra always represent different sets of numbers, the letter 'a' for apples is not the same as the letter 'b' for bananas which is not the same as the letter 'p' for pears. Only like letters or symbols can be added or subtracted together and only like letters or symbols raised to the same index can be added or subtracted from each other.

$6x + 9y - 2x^2 + 3z + 7x^2$

$= 6x + 9y + 3z + 5x^2$

6. Simplified expressions are usually tidied up by writing the terms in highest power order and then alphabetic order.

$= 5x^2 + 6x + 9y + 3z$

## EXERCISE 1 - ALGEBRA – LIKE AND UNLIKE TERMS

1. $a - 3a + 4a + 2a$

2. $3x^2 - 2x^2 + x^2$

3. $5x + 2y + x - y$

Terms are only 'Like' Terms if the bases are the same and the powers are the same.

4. $7p - 4q + 3p - 5p$

5. $5m - 3m + 3n - 2m - 3n$

6. $6x^2 - 2y^2 - 4y^2 - y^2 - 2x^2$

If the bases are the same and the powers are the same, simply add and subtract the coefficients.

7. $a^3 - 5ab + 2a^2 - a^3 - a^2 + 3ab$

8. $p^2 + q^2 - 2pq + 2q^2 - 2pq - p^2 + pq$

9. $27abc - 2xyz - 26abc + xyz$

10. $a - b - c - d - 3c - b - 2a - 2d - a$

11. $2a + 3b - c - 3a + b - 2c + 4a - 2b + c$

12. $8p - 2q + 5r - 6p + 7q + 4r - p - 4q - 8r$

13. $6 - 2a - a^3 + 5a^2 + 3a^3 - 2a^2 + 2a + 2 - 3a + 4 - a^2$

14. $x^2 - 2xy + 3y^2 - x^2 - y^2 + xy - xy - 2y^2 + 3x^2$

## ALGEBRA - MULTIPLICATION: AND DIVISION

7. Algebraic terms can be multiplied and divided without any concern for whether they are like terms or not.

$$5 \times x^2 = 5x^2$$

So

$$5x^2 \times 3y^3 = 15x^2y^3$$

8. When the terms are different the coefficients are multiplied, remember that in multiplication and division, LIKE signs give POSITIVE answers, UNLIKE signs give NEGATIVE answers,

**Multiplying**

$(+ a)(+ b) = ab$

If the signs are the same the outcome is positive

$(- a)(- b) = ab$

$(+ a)(- b) = -ab$

If the signs are different the outcome is negative

$(- a)(+ b) = -ab$

$(+ 5p)(+ 6q) = 5 \times 6 \times p \times q = 30pq$

$(+ 5p)(- 2q) = -10pq$

**Dividing**

$(+a) \div (+b) = \frac{+a}{+b} = +\frac{a}{b} = \frac{a}{b}$

If the signs are the same the outcome is positive

$(-a) \div (-b) = \frac{-a}{-b} = +\frac{a}{b} = \frac{a}{b}$

$(+a) \div (-b) = \frac{+a}{-b} = -\frac{a}{b}$

$(-a) \div (+b) = \frac{-a}{+b} = -\frac{a}{b}$

If the signs are different the outcome is negative

$-12a \div 3b = \frac{-12a}{+3b} = -\frac{4a}{b}$

9. The letters and symbols are multiplied and divided using the rules of indices which are the same for letters and numbers.

## RULES OF INDICES

10. There are six basic rules of indices that are used to simplify algebraic expressions with the same base. These are:

Multiplication $a^m \times a^n = a^{m+n}$

If you are multiplying the bases, just add the indices. $2^3 \times 2^4 = 2^{3+4} = 2^7$

Division $a^m \div a^n = a^{m-n}$

If you are dividing the bases, just subtract the indices. $3^5 \div 3^3 = 3^{5-3} = 3^2$

Power of Zero $a^0 = 1$

Anything to power zero is 1

Power of Power $(a^m)^n = a^{mn}$

For a power of a power just multiply the indices. $(2^2)^3 = 2^{2\times3} = 2^6$

Negative Index $a^{-n} = \frac{1}{a^n}$

$\frac{1}{3^2}$ Can be written as $3^{-2}$

Fractional Index $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

Whatever is on the outside of the radical, goes on the bottom of the fraction

**Multiplication of terms with Indices**

$$5m^2 \times m^3 \times (-n^2) \times p^4 = -5m^5 n^2 p^4$$

**Division of terms with Indices**

$$-12a^2bc^3 \div 3ab^3c = \frac{-12a^2bc^3}{3ab^3c} = -\frac{4ac^2}{b^2}$$

## EXERCISE 2 - MULTIPLICATION AND DIVISION

**Simplify:**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | $a^3 \times a^2$ | 2. | $e^4 \times e$ | 3. | $f^6 \div f^3$ | 4. | $\frac{g^6}{g^2}$ |
| 5. | $d^4 \times d^4$ | 6. | $2g^5 \times g^2$ | 7. | $6a^3 \times 2a^4$ | 8. | $6x^3 \times 2x^4$ |
| 9. | $5n \times 2n^2$ | 10. | $3r^2 \times 2p^4$ | 11. | $4t^2 \times 2t^3$ | 12. | $(6x^2)(3x^2y^2)$ |
| 13. | $2x^3 \div x^3$ | 14. | $\frac{6p^2q^2}{2p^3q}$ | 15. | $12r^4s \div 3r^3$ | 16. | $\frac{xy}{yx}$ |
| 17. | $\frac{2ab}{2ba}$ | 18. | $\frac{12x}{12xy}$ | 19. | $\frac{h^2}{h^3k}$ | 20. | $\frac{4a^2}{6a^3k}$ |
| 21. | $(-5)(-2d)$ | 22. | $(-4n^2)(2n)$ | 23. | $(-6a^2) \div (2a)$ | | |
| 24. | $\frac{-12b^2}{-3b}$ | 25. | $\frac{-12mn}{-4}$ | 26. | $(3a^2)^2$ | 27. | $(2x^3)^2$ |
| 28. | $(2k^3m)^3$ | 29. | $(-5r^4)^2$ | 30. | $(-3p^4)^3$ | | |

## HIGHEST COMMON FACTOR.

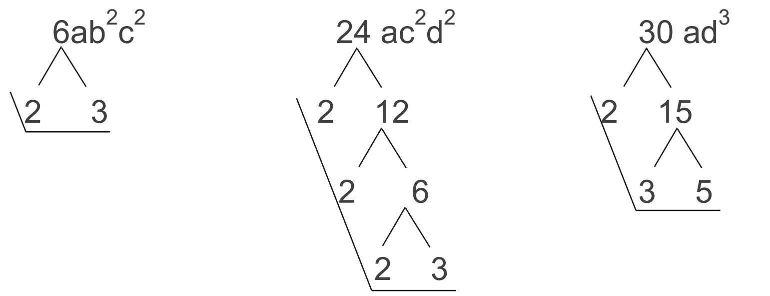
11. The Highest Common Factor (HCF) of an expression is the largest numerical and algebraic term that is common to all of the terms in the expression. For example:

> The three is common to both of the terms in the expression: $3x + 3y$, so the HCF is 3
>
> Similarly, the ‘2’ and the ‘a’ are common factors in both of the terms in the expression: $6ab + 2ac$, so the HCF is $2a$.

12. Finding the HCF is often used as a step in simplifying and solving mathematical problems.

## FINDING THE HCF USING A FACTOR TREE.

13. Break down each of the numbers into their prime factors, keeping the lowest prime factor on the left-hand side of the tree. Follow the left side of each tree, down and along the bottom, to identify the factors:




This step is important in identifying the HCF and LCM

**$2 \times 3$** **$2^3 \times 3$** **$2 \times 3 \times 5$**

14. The HCF is found by selecting the lowest power of each of the terms, but only include the terms that are common: The lowest power of the 2s is $2^1$, the lowest power of the 3s is $3^1$. Notice that the 5 is not common to all of the three expressions, so it cannot be included in the HCF. Similarly, select the lowest power of each of the letters. Notice that b, c and d are not common to all of the three expressions, so they cannot be included in the HCF.

HCF = $2^1 \times 3^1 \times a^1$

HCF = $6a$

HCF is found by selecting the **lowest** power of each of the terms, but only include the terms that are common

So, $6a$ is the highest factor which will multiply into all of the terms exactly.

## EXERCISE 3 - COMMON FACTORS

1. Which of the following numbers have six as a common factor?

   12, 32, 48, 66, 86, 90

2. List all the factors of 48.

3. a. Which of the following numbers are factors of 108 and 66?

   2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 18, 20, 22, 26, 27, 33, 36, 54

   b. From the list of common factors for 108 and 66, what is the highest common factor of both numbers?

4 Find the Highest Common Factor of the following expressions:

   a. $6p + 18q$

   b. $3x - 9xy$

   c. $5r^2 + 25r$

   d. $8a - 12ac$

   e. $16m^2n^3 - 22mn^2$

   f. $at - 6a^4$

## FACTORISING.

15. Factorising is a method of simplifying or tidying up expressions and equations. For example:

> $2a + 2b$ can be written as $2(a + b)$ by bringing the common factor to the front outside of a bracket.
>
> Similarly: $6xy + 2xz$ can be written as $2x(3y + z)$.

16. In many cases, the HCF is not immediately apparent. For example:

> $36abc – 48bc + 18ac$ could be factorised to $6c(6ab – 8b + 3a)$

17. In some cases, not all of the terms are common, so an expression can be partially factorised.

> For example: $3ab^2c – 12a^2b + d$
>
> Notice that 3 and b are common to the first two terms, but they are not in the third term.
>
> So $3ab^2c – 12a^2b + d$ would factorise to $3b(abc – 4a^2) + d$

18. Expressions can often be factorised more than once. For example:

> $ax – ay + 2x – 2y$
>
> Notice that 'a' is common to the first two terms and '2' is common to the second two terms. Factorising for a and for 2 respectively gives:
>
> $a(x – y) + 2(x – y)$
>
> It is now apparent that $(x – y)$ is common to both of the terms, so it is possible to factorise a second time:
>
> $(a + 2)(x – y)$

## EXERCISE 4 – FACTORISATION

Factorise the following expressions:

| | | | |
|---|---|---|---|
| 1. | $5x + 5y$ | 2. | $2p^2 + 8pq$ |
| 3. | $ax - ay$ | 4. | $24x^2 + 9x$ |
| 5. | $21xy^2 - 14y$ | 6. | $a(b - c) + 2(b - c)$ |
| 7. | $x(y + z) - p(y + z)$ | 8. | $2xy^2 + 6x^2y + 8x^3y$ |
| 9 | $ay + by + a + b$ | 10. | $ax + ay + bx + by$ |
| 11. | $2ab - 4ac + db - 2dc$ | 12. | $2ax + 3ay - 4bx - 6by$ |

# BIDMAS (THE ORDER OF PRECEDENCE)

19. The Order of Precedence must be used when simplifying and solving expressions and equations to ensure that the correct answer is reached.

Brackets
Indices
$\left.\begin{array}{l}\text{Divide}\\ \text{Multiply}\end{array}\right\}$ Same time
$\left.\begin{array}{l}\text{Add}\\ \text{Subtract}\end{array}\right\}$ Solve L to R

For example: $7 - 2 + 1 =$

$7 - 2 + 1 = 6$
$-2 + 1 + 7 = 6$
$1 - 2 + 7 = 6$
$7 + 1 - 2 = 6$

The equation must be solved from Left to Right
And notice that the minus belongs only to the 2

Similarly: $5 \times 4 \div 2$

$5 \times 4 \div 2 = 10$
$5 \div 2 \times 4 = 10$
$4 \div 2 \times 5 = 10$
$4 \times 5 \div 2 = 10$

All of the equations can be written in this form:

$$\frac{5 \times 4}{2} = 10$$

The divide sign only belongs only to the 2

20. Expressions and equations are simplified and solved using the rules of BIDMAS, but equations are transposed (rearranged) by using BIDMAS **in reverse**.

| Transpose<br>Rearrange ↑ | B<br>I<br>D<br>M<br>A<br>S | ↓ Simplify<br>Solve |
|---|---|---|

## BRACKETS

21. Where brackets are constructed using factorisation, they can also be expanded by multiplying the numbers and symbols out. When multiple sets of brackets are constructed, there has to be an order for multiplying them out. There are three types of bracket used in algebra; round ( ), square [ ], and curly brackets { }, and a sequence of events may produce an expression like:

$$2x - \{x + [(3 - 2x) - x(6 + 3x)] - 2x(1 - x)\}$$

22. This needs to be expanded before the like terms can be gathered together. There are three priorities when it comes to expanding brackets:

Expand round brackets first, then square brackets, and finally curly brackets.

23. Brackets are expanded by multiplying everything inside the bracket by whatever is multiplying the bracket from the outside.

$$3(a + b) = 3a + 3b$$

Similarly:

$$x(x + y) = x^2 + xy$$

$$-2b(b + c) = -2b^2 - 2bc$$

24. When expanding a series of brackets, there will be occasions when the terms can be simplified before the whole expression has been expanded:

$$2x - \{x + [(3 - 2x) - x(6 + 3x)] - 2x(1 - x)\}$$

Expand the curved brackets:

$$= 2x - \{x + [3 - 2x - 6x - 3x^2] - 2x + x^2\}$$

Simplify the terms inside the square brackets:

$$= 2x - \{x + [3 - 8x - 3x^2] - 2x + 2x^2\}$$

Expand the square brackets:

$$= 2x - \{x + 3 - 8x - 3x^2 - 2x + 2x^2\}$$

Simplify the terms inside the curly brackets:

$$= 2x - \{- 9x - x^2 + 3\}$$

Be careful: the minus sign outside of the bracket changes all of the signs inside.

Expand the curly brackets:

$= 2x + 9x + x^2 - 3$

Gather the like terms:

$= x^2 + 11x - 3$

25. Brackets can be made up of any number of terms; when there are only two terms in a bracket it is known as a binomial and when two or more binomial brackets are expanded everything in the second bracket is multiplied by everything in the first bracket. For example:

$(a - b)(a + b)$

$= (a - b)(a + b) = a^2 + ab - ba - b^2$

$= a^2 - b^2$

26. When a bracket is raised to an index, the bracket multiplies itself that number of times. For example:

$(2x - 7)^2$

$= (2x - 7)(2x - 7)$

$= (2x - 7)(2x - 7) = 4x^2 - 14x - 14x + 49$

$= 4x^2 - 28x + 49$

## EXERCISE 5 - SIMPLIFYING BRACKETS

1 $x + 2(3x - 2y) - 2(2y - 3x)$

2. $-3(b - c) - 2(c - a) + 5(a - b)$

3. $m + n - 3(mn - m) + 4(mn - n)$

4. $3x^2 - 2(y^2 + x^2) + 6(x^2 + y^2)$

5. $3x + 7[2(y - 1) + 5(x - 1)]$

6. $x(x + y) + 3[x^2 - 2x(1 + y)]$

7. $x^2 - 4\{y^2 - 2[x^2 - 8(x^2 - y^2)]\}$

8. $5(2x - 3) - 2(x - 8)$

9. $4(y + 3) - 2(7 + y) + 2$

10. $(x + y)(x + 2y)$

11. $(a + 1)(a + 3)$

12. $(p + q)(3p - 2q)$

13. $(5a - 2)(2a + 4)$

14. $(x + 3)\ (x - 6)$

15. $(a - b)^3$

## EQUATIONS AND FORMULAS

27. When an expression has been expanded and simplified it can then be used to find a solution, for a solution to be found the expression must be written as an equation; that is, the arithmetic symbols connecting the terms, which are known as operations ($+ - \times \div \sqrt{a}\ and\ x^a$ are all examples of mathematical operations), must be equal to a single solution when a set of values are substituted in. The terms can now be called variables because they can be substituted by any of the numbers from the set to which they belong and if a problem has one unknown value this can be set on one side of the equals sign, this is called the subject of the equation, while the other values are worked out on the other side.

28. Substitution is the method of replacing the known variables with their given values and transposition is the method used to move the terms about to leave the unknown variable on its own. Transposition can isolate any one of the variables either before or after substitution has been carried out by looking at each operation in turn and if necessary, using its opposite to remove the term from that side of the equation placing it the other.

## CONSTRUCTING EXPRESSIONS EQUATIONS

29. Every structure, machine, or component in the 'built environment' will have been produced following some sort of mathematical modelling. For example, the compressive strength of a building brick, the diameter of the spokes in a wheel, the circuitry in a computer etc., will all have been modelled mathematically before being produced. Simple calculations are also used in daily life.

30. Here are some examples of real-life situations that can be represented by linear functions:

- When you go on holiday abroad, the exchange rate may be $1.2 for every pound.

- Planning to run a 10-mile race, but wanting to know how many kilometres it would be. Simply divide by 5 and multiply by 8.

- The instructions for roasting a turkey state that takes 20 minutes plus 35 minutes for each kilogram.

- The cost of hiring a taxi is a fixed cost of £4 plus £1.60 per mile.

- The length of a bar, in centimetres. used to fashion a horseshoe is twice the width of horse's hoof plus 5 centimetres.

- The conversion from a temperature in degrees Celsius to its equivalent in degrees Fahrenheit can made by multiplying by 1.8 and then adding 32.

31. Models in maths are not always the groundbreaking formulas like those developed by Newton and Pythagoras, but they can be formed to help give structure to a puzzle that needs solving or help to answer the questions that are asked daily like those given above.

32. A model is developed by analysing a situation with respect to choosing the right letters or symbols for the situation then deciding how best to represent the relationship between the variables using the arithmetic operations that are available. The expressions and equations that result from the analysis are built up from previous arithmetic knowledge and leaming how to 'talk' the way through a situation mathematically. For example. 'What must I add to twenty-four to make thirty-two?'

33. This statement is completed by saying 'Twenty-four plus eight equals thirty-two', which is written using symbols: $24 + 8 = 32$ The algebraic model is then developed where:

$24 + 8 = 32$ (numerical statement)

$24 + \theta = 32$ (using a symbol to represent the unknown)

$24 + n = 32$ (using a letter to stand for the unknown).

34. Algebraic equations are formed by thinking out loud each step one at a time and jotting those thoughts onto a sheet of paper.

Try these examples:

a. Paul is given £15. He spends £7.47 - How much change does he get?

b. Concert for four adults cost £268.80. What is the cost for one adult?

c. Add 4 to my number and then multiply the result by 6. The answer is 84 What is my number?

35. The need to model problems is made clearer when the problems are more complicated. For example:

a. Joseph is four years younger than Tom. In three years time the sum of their ages will be 28. How old is Joseph now?

b. A rectangle is three times as long as it is wide. Its perimeter is 56cm. What is the length of the rectangle?

## EXERCISE 6 - CONSTRUCTING EQUATIONS

1. A used car is x years old; how much younger was it 2 years ago?

2. Find an expression for the cost of three calculators priced at p pence each and seven pens priced at q pence each.

3. Find an expression for the total mass of a carton containing m articles if the carton has a mass of 15kg and each article has a mass of 3.2kg.

4. If the cost of a jumper is reduced in a sale from x pounds to y pounds find the expression which gives the number of extra jumpers that can be bought for £130.00.

5. If x washers can be bought for £1.99, what is the cost of y washers?

6. A formula for the perimeter of the rectangle is to be constructed. Write down four different ways of calculating the perimeter in words and then rewrite the formulas, by firstly substituting in the values for the length and breadth and then writing each one as an algebraic equation.

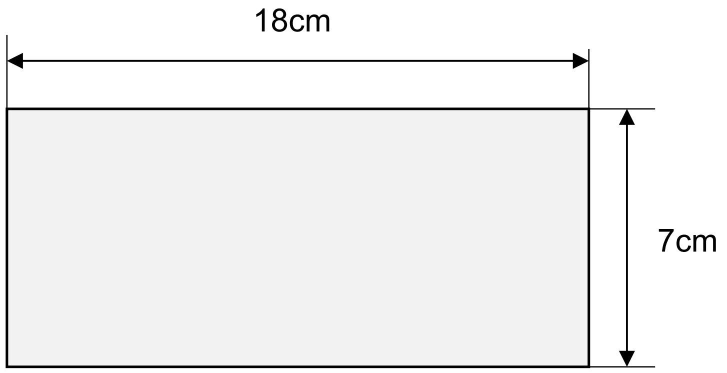



**Figure 1**

7. A rectangle has a perimeter of 56cm. The longer side of the rectangle is equal to the shorter side plus four; what are the rectangles dimensions?

8. If a car costs four times as much as a motorbike; and two cars and three motorbikes cost £91 850. Find the cost of each machine.

9. The perimeter of a triangle labelled ABC is 26m, knowing that side BC is two thirds the length of side AB and is also 2m longer than side AC. Find the lengths of each side.

10. A water tank with a capacity of 1200 litres is filled using two different water supplies. The second delivers twice the rate of the first and takes 16 minutes to fill the tank from empty. How many litres per minute does each tap deliver?

## SOLVING LINEAR EQUATIONS

36. Finding an unknown value in an equation often needs the equation to be rearranged so the unknown value is on one side of the equals sign.

**Example 1.**

$$10 = a + 7$$

To isolate the '*a'* in the equation above, the *+7* must be subtracted from the right-hand side (RHS) of the equation. However, to keep the balance of the equation, as all equations are balanced around the equals sign, 7 must also be subtracted from the left-hand side (LHS) of the equation. The process is shown below:

$$10 - 7 = a + 7 - 7$$

$$10 - 7 = a$$

$$3 = a$$

$$a = 3$$

The value can be substituted back into the original equation and the equation should balance, i.e., the LHS must equal the RHS.

There are many different forms of equation that may arise, and differing techniques may be required to solve them. Consider the equation below:

$$10 = 5b$$

To isolate the *b,* both sides of the equation must be divided by 5. Keeping the balance and dividing both sides of the equation by 5, the process is as follows:

$$\frac{10}{5} = \frac{5b}{5}$$

$$\frac{10}{5} = b$$

$$2 = b$$

$$b = 2$$

Notice that dividing through by 5 gives a value of 1 when cancelled with the 5 next to the b, and anything multiplied by 1 is itself. The LHS is also divided by 5 in order to maintain the balance.

The equation below shows how both of these techniques can be combined:

$$2x + 3 = 6$$

Using BIDMAS in reverse, the 3 can be subtracted from both sides of the equation as shown below:

$$2x + 3 - 3 = 6 - 3$$

$$2x = 3$$

The next step is to now divide both sides of the equation by 2, giving:

$$\frac{2x}{2} = \frac{3}{2}$$

$$x = \frac{3}{2}$$

$$x = 1.5$$

**Example 2.**

The following technique shows how equations with differing fractional amounts can be solved. Consider the equation:

$$\frac{x - 4}{3} = \frac{x - 2}{4}$$

Notice that there is more than one instance of x, and the equation is in fractional form.

**Note**

**When adding or subtracting values, ensure that the same operation is carried out on both sides of the equation to keep the balance.**

**When multiplying or dividing values, ensure that the same operation is carried out on each term in the equation to keep the balance.**

Using prior knowledge of fractions, multiplying both sides of the equation by the LCM will allow the denominators to cancel with the LCM.

$$\frac{x - 4}{3} = \frac{x - 2}{4}$$

Multiply both sides of the equation by the Lowest Common Multiple: LCM = 12

$$12\,\frac{(x-4)}{3} = 12\,\frac{(x-2)}{4}$$

Cancel down the 12 and the 3 and cancel down the 12 with the 4.

$$\overset{4}{\cancel{12}}\,\frac{(x-4)}{\underset{1}{\cancel{3}}} = \overset{3}{\cancel{12}}\,\frac{(x-2)}{\underset{1}{\cancel{4}}}$$

$$4(x-4) = (3x-2)$$

Expand the brackets

$$4x - 16 = 3x - 6$$

Gather the x terms on one side of the equation and the numbers on the other side. This can be done by subtracting 3x from both sides of the equation and adding 16 to both sides of the equation giving:

$$4x - 3x = 16 - 6$$

$$x = 10$$

# EXERCISE 7 - SOLVING EQUATIONS

1. Solve the following equations for x

   a. $3(2x - 3) = 33$

   b. $3(x - 4) = 2(x + 3) - 16$

2. Counting the change from the till it is found that there are an equal number of ten pence pieces and two pence pieces. Their total value comes to £1.44, how many of each coin are there in the till?

3. When three successive whole numbers: $x_1$, $x_2$ and $x_3$, are added together they total 216. What are the values of the numbers?

4. When a number; x is doubled and thirteen is added to it, the result is 39. What is the value of the number?

5. When a certain number is added to the numerator and to the denominator of the fraction $\frac{1}{8}$ the new fraction $\frac{1}{2}$ is found. What is the value of the number?

6. Solve the following equations for x:

   a. $\frac{x}{2} + \frac{x}{3} = -10$

   b. $\frac{x+3}{3} = \frac{x-3}{2}$

   c. $\frac{3x-5}{4} - \frac{9-2x}{3} = \frac{x-3}{2}$

7. If a = 2, b = 3 and c = 5; find the values of the following expressions:

   a. $2b + 4$

   b. $3b + 5c$

   c. $8 - a$

   b. $a + 2b + 5c$

   e. $\frac{5a + 9b + 8c}{ab + c}$

   f. $\frac{4b + 3c}{b(c - a)}$

8. Given that: $p = \frac{9}{16}$ $q = 6$ and $r = 3$, evaluate the following:

   a. $\sqrt{pr^2}$

   b. $q^2r^3$

   c. $\frac{p^2}{qr}$

9. The formula for kinetic energy is $KE = \frac{1}{2}mv^2$, where KE is the kinetic energy in Joules(J), m is the mass in kilograms (kg) and v is the velocity in metres per second (m/s). Find the velocity if the kinetic energy is 96J and the mass is 3kg.

## EXERCISE 8 – SOLVING APPLIED PROBLEMS

1. The electrical resistance of a cable can be found using the formula, $R = \frac{\rho L}{A}$. Given that R = 2.56 Ω, L = 0.64 m and A = $2.5 \times 10^{-4}$ m, find the value of ρ.

2. The equation used to find the size of a force is F = ma. Find the acceleration (a) when a force (F) of 5.6 kN is applied to a mass (m) Of 700kg (Note that acceleration has units' metres per second squared ($m/s^2$)).

3. Ohm's Law is the equation V = IR that defines the relationship between the voltage (V), current (I) and resistance (R) in an electric circuit. An electrical appliance takes a current of 0.6A from a 240V Supply, using the formula V=IR find the resistance of the appliance. (Note that resistance is measured in Ohms (Ω)).

4. Boyle's Law: $P_1V_1 = P_2V_2$, is a formula that relates the pressures and volumes at two different stages of the same experiment. Find the value of $P_2$ when: $P_1 = 100 \times 10^3$ Pa, $V_1 = 0.24$ l, and $V_2 = 0.32$ l. (Note that pressure is measured in Pascals (Pa) and volume is measured in litres).

5. When electrical resistors are connected in parallel to each other, their total resistance in Ohms can be found using the equation:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

a. Find the total resistance when $R_1$ = 4Ω, $R_2$ = 6Ω, $R_3$ = 3Ω,

b. Find the resistance of $R_2$, when $R_T$ = 3Ω, $R_1$ = 5Ω, $R_3$ = 10Ω

6. The conversion of Fahrenheit temperatures and Celsius temperatures can be found using the formula: $F = \frac{9}{5}℃ + 32$. Knowing this, express 140°F in degrees Celsius.

7. When the voltage and resistance in a direct current circuit are known; the power measured in Watts (W). can be found using the formula: $P = \frac{V^2}{R}$. When a dc circuit dissipates 180W of power across its 12V supply what is the resistance in the circuit?

8. The velocity of an object is its speed in a certain direction, the velocity of an object can be found using the formula: $v^2 = u^2 + 2as$. Where 'v' is its final velocity in m/s, 'u' is its initial velocity in m/s, 'a' is its acceleration in $m/s^2$ and 's' is its distance travelled in m. Find the acceleration of the object when it speeds up from rest to 8m/s over a distance of 80m.

## EXERCISE 9 – SOLVING LINEAR EQUATIONS

Solve the following equations:

| | | | | | |
|---|---|---|---|---|---|
| 1 | $n+8=17$ | 2. | $n-5=11$ | 3. | $3y=20$ |
| 4. | $\frac{x}{2}=9$ | 5. | $8x=0$ | 6. | $x+2x=18$ |
| 7. | $3n-2=10$ | 8. | $a+4=2$ | 9. | $4y-y=21$ |
| 10. | $\frac{x+2}{4}=5$ | 11. | $3c=c+5$ | 12. | $2p-8=p-3$ |
| 13. | $t+7=17-4t$ | 14. | $2a+4=19-a$ | 15. | $7m-9-m=3m$ |

## EXERCISE 10 – FURTHER LINEAR EQUATIONS

1.

| | | | | | |
|---|---|---|---|---|---|
| a. | $3(n-7)=12$ | b. | $4(2k-1)=20$ | c. | $5(2r+3)=15$ |
| d. | $4(t-5)=0$ | e. | $\frac{x}{4}+2=5$ | f. | $m+\frac{m}{5}=12$ |
| g. | $\frac{a}{3}=1+\frac{a}{4}$ | h. | $\frac{x}{5}-\frac{2}{7}=0$ | i. | $\frac{2k}{3}-\frac{k}{2}=1$ |
| j. | $\frac{a}{3}+\frac{a}{5}=1$ | k. | $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}=1$ | l. | $\frac{5w}{2}+\frac{1}{3}=\frac{3}{4}$ |
| m. | $\frac{4}{x}=\frac{2}{7}$ | n. | $\frac{3}{p}=\frac{9}{10}$ | o. | $\frac{2}{v}+\frac{3}{2}=\frac{5}{3}$ |
| p. | $\frac{3}{t}+\frac{4}{3}=\frac{17}{6}$ | | | | |

2.

| | | | |
|---|---|---|---|
| a. | $4a-(5-a)=17$ | b. | $8b-(3b+4)=11$ |
| c. | $7-(3-4m)=23$ | d. | $(8n-7)-(5n+13)=0$ |
| e. | $12t+1-7t=31$ | f. | $0=5-2x-17$ |
| g. | $9-4y=13+5-7y$ | h. | $d=12-(11+4d)$ |
| i. | $5-(a+6)=4+(7-4a)$ | j. | $6b-(7+2b)=10$ |
| k. | $7=m-(7m-25)$ | l. | $6t-23-5-t=7$ |

m. $8x+(4-5x)=24-(5+3x)$ n. $4y=(16-y)-(5+6y)$

o. $(8z-9)-(5z+12)=(2z-21)$ p. $11f+(5-2f)=21-(2-f)$

q. $18g-(7g+6)+(3-4g)=0$

## EXERCISE 11 - SUBSTITUTION

1. Find the value of x in the following equations

   a. $2ax + b = bcx + a$, where $a = 3$, $b = -1$, $c = -2$

   b. $p(mx+c)=q(-mx+c)$ Where $c=-5$, $m=2$, $p=3$ and $q=-2$

   c. $2a^2x + b^2y = c$, where $a = \frac{1}{2}$ $b = -2$, $c = -3$ and $y = -2$

   d. $y=(p-q)(x-r)$ Where $p=4.5$, $q=-2.5$, $r=3.2$ and $y=5.6$

   e. $vx + w^2 = gA$, where $A = 1.6$, $g = 3.5$, $v = 0.4$ and $w = -2$

2. Find m when $F=\frac{mv^2}{r}$ where, $F=1280$, $r=5$ and $v=20$

3. Find r when $F=\frac{mv^2}{r}$ where, $F=3600$, $m=8$ and $v=30$

4. Find $R_1$ when $\frac{1}{R_T}=\frac{1}{R_1}+\frac{1}{R_2}$ where, $R_T=3$ and $R_2=12$

5. Find a when $C=\frac{e(n-1)a}{d}$ where, $C=36$, $d=10$, $e=3$ and $n=-5$

6. If $Q^2R=\frac{L}{C}$ find C when, $L=396$, $R=5.5$ and $Q=6$

7. If $T=\frac{L}{R}$ find T when, $L=100\times10^{-3}$ and $R=10\times10^{6}$

8. If $T = CR$, find T when $C = 100 \times 10^{-6}$ and $R = 10 \times 10^3$

9. If $V = IR$, find V when $I = 1 \times 10^{-3}$ and $R = 10 \times 10^3$

10. If $X_L=2\pi fL$ find $X_L$ when, $\pi=\frac{25}{8}$, $f=400$ and $L=10\times10^{-3}$

# TRANSPOSITION OF FORMULA

37. Transposition is best explained as BIDMAS in reverse. Not all of the steps have to be carried out to isolate the unknown variable, and in some cases the order may have to be repeated more than once because of the way that the unknown variable is connected to the other variables.

| Transpose<br>Rearrange | ↑ | B<br>I<br>D<br>M<br>A<br>S | ↓ | Simplify<br>Solve |
|---|---|---|---|---|

**Example 1.**

Transpose the equation: **y** = **mx** + **c** to make **m** the subject:

There are three terms: y, mx, and c.

The first step is to look at how the terms are connected. The m and x are held together by multiplication, and the c is added to them.

$y = mx + c$

Using BIDMAS in reverse, deal with terms that can be added or subtracted first

Subtract c from both sides:

$y - c = mx$

Again, look at how the terms are connected. The m and x are multiplied together, so divide both sides by x:

$$\frac{y - c}{x} = m$$

Notice that the x divides both of the terms on the LHS

Place the subject on the LHS

$$m = \frac{y - c}{x}$$

**Example 2.**

Transpose the equation: $\mathbf{V} = \mathbf{IR}$ to make **R** the subject:

Transpose
Rearrange

B
I
D
M
A
S

There are two terms: V, and IR.

Look at how the terms are connected. The I is multiplying the R.

$V = IR$

None of the terms are added or subtracted, so deal with multiplication and division first.

Divide both sides by I:

$$\frac{V}{I} = R$$

**Example 3.**

Transpose the equation: $\mathbf{v} = \sqrt{\mathbf{b^2 - 4ac}}$ to make '**a**' the subject:

Transpose
Rearrange

B
I
D
M
A
S

There are two terms; v, and $\sqrt{b^2 - 4ac}$.

Look at how the terms are connected. All of the terms underneath the radical are bound together by the radical, like an invisible bracket.

$v = \sqrt{b^2 - 4ac}$

It is not possible to release any of the terms from underneath the radical, so the process must start with I (Indices)

Square both sides:

$v^2 = b^2 - 4ac$

Look at how the terms are connected: $(-4ac + b^2)$. Now the terms have been released it is possible to subtract $b^2$ from both sides.

$v^2 - b^2 = -4ac$

Look at how the terms on the RHS are connected (multiplication). Finally, divide both sides by – 4c to make 'a' the subject

$\dfrac{v^2 - b^2}{-4c} = a$ This could also be written as: $\dfrac{b^2 - v^2}{4c} = a$

**Example 4.**

Transpose the equation: $T = 2\pi\sqrt{\frac{l}{g}}$ to make $l$ the subject:

Transpose Rearrange

B
I
D
M
A
S

Look at how the terms are connected. The terms $l$ and $g$ are bound together inside the radical, which is being multiplied by $2\pi$.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

> It is not possible to release any of the terms from inside the radical, so the process must start with D (Division)

Divide both sides by $2\pi$:

$$\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$$

Look at how the terms are connected. Now the terms can be released from the radical by squaring both sides.

$$\left[\frac{T}{2\pi}\right]^2 = \frac{l}{g}$$

Look at how the terms on the RHS are connected (Division). Finally, multiply both sides by $g$ to make $l$ the subject.

$g\left[\frac{T}{2\pi}\right]^2 = l$ This could also be written as: $\frac{gT^2}{4\pi^2} = l$

**Example 5.**

Transpose the equation: $\mathbf{R = R_o(1 + \alpha t)}$ to make **t** the subject:

There are two terms; $R$, and $R_o(1 + \alpha t)$.

Look at how the terms are connected. The $R_o$ and the bracket are connected by multiplication.

$$R = R_o(1 + \alpha t)$$

> In this example it is better to multiply the terms in the bracket by $R_o$

$$R = R_o(1 + \alpha t)$$

$R = R_o + R_o\alpha t$

Transpose
Rearrange

B
I
D
M
A
S

Look at how the terms are connected. The $R_o$ is added to the $R_o\alpha t$.

Using BIDMAS in reverse, subtract $R_o$ from both sides:

$R - R_o = R_o\alpha t$

Again, look at how the terms are connected. The $R_o$ and $\alpha$ and t are multiplied together, so simply divide both sides by $R_o\alpha$:

$$\frac{R - R_o}{R_o\alpha} = t \qquad \text{so} \qquad t = \frac{R - R_o}{R_o\alpha}$$

**Example 6.**

Transpose the equation: $\mathbf{E = IR_1 + IR_2}$ to make **I** the subject:

There are three terms: E, $IR_1$ and $IR_2$.

Notice that the subject of the transposition ‘I’ appears twice.

Look at how the terms are connected. The $IR_1$ and the $IR_2$ are added together. As the subject of the transposition ‘I’ appears twice it is necessary to factorise the equation for I.

$E = IR_1 + IR_2$

Factorising is the opposite of multiplying out a bracket

Transpose
Rearrange

B
I
D
M
A
S

$E = I(R_1 + R_2)$

Finally, look at how the terms are connected. The bracket is multiplying the I, so divide both sides by the bracket.

$$\frac{E}{R_1 + R_2} = I \qquad \text{so} \qquad I = \frac{E}{R_1 + R_2}$$

**Example 6.**

Transpose the equation: $Z = \sqrt{R^2 + X_c^2}$ to make R the subject:

Transpose
Rearrange

B
I
D
M
A
S

Look at how the terms are connected. The terms $R^2$ and $X_c^2$ are bound together inside the radical.

$$Z = \sqrt{R^2 + X_c^2}$$

It is not possible to release any of the terms from inside the radical until both sides of the equation are squared.

Square both sides:

$$Z^2 = R^2 + X_c^2$$

Look at how the terms are connected. The $R^2$ and the $X_c^2$ are connected by addition, so by subtract $X_c^2$ from both sides.

$$Z^2 - X_c^2 = R^2$$

The question asks for R and not $R^2$, so take the square root of both sides of the equation.

$$\sqrt{Z^2 - X_c^2} = R$$

Finally, place the subject on the LHS of the equation.

$$R = \sqrt{Z^2 - X_c^2}$$

## EXERCISE 12 – TRANSPOSITION

Transpose the following equations making the letter in brackets the subject:

| | | | | | |
|---|---|---|---|---|---|
| 1. | $y = x + z$ | (x) | 2. | $a = b - c$ | (b) |
| 3. | $p = q + s$ | (s) | 4. | $l = m - n$ | (n) |
| 5. | $y = xz$ | (x) | 6. | $y = mx$ | (m) |
| 7. | $y = \frac{z}{x}$ | (z) | 8. | $a = \frac{b}{c}$ | (c) |
| 9. | $v = u + at$ | (u) | 10. | $y = mx + c$ | (c) |
| 11 | $v = e - IR$ | (I) | 12. | $v = u + at$ | (t) |
| 13 | $P = \frac{RT}{V}$ | (T) | 14. | $s = ut + \frac{1}{2}at^2$ | (u) |
| 15. | $\frac{1}{R} = \frac{1}{P}$ | (P) | | | |

## EXERCISE 13 – TRANSPOSITION (POWERS AND ROOTS)

Transpose the following equations making the letter in brackets the subject:

| | | | | | |
|---|---|---|---|---|---|
| 1. | $d = \sqrt{A}$ | (A) | 2. | $\pi^2 = q$ | ($\pi$) |
| 3. | $a = b^2$ | (b) | 4. | $A = \pi r^2$ | (r) |
| 5. | $z = \sqrt{R^2 + x^2}$ | (x) | 6. | $a = \sqrt{2bc}$ | (c) |
| 7. | $x^2 = yz^2$ | (z) | | | |

## EXERCISE 14 – FURTHER POWERS AND ROOTS

Transpose the following equations making the letter in brackets the subject:

| | | | | | |
|---|---|---|---|---|---|
| 1. | $c = \pi d$ | (d) | 2. | $v^2 = 2gh$ | (v) |
| 3 | $I = \frac{E}{R + r}$ | (R) | 4. | $v = \sqrt{2gh}$ | (h) |
| 5. | $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$ | (C) | 6. | $\frac{1}{j} = \frac{1}{u} + \frac{1}{v}$ | (u) |
| 7. | $s = ut + \frac{1}{2}at^2$ | (a) | 8. | $I = \frac{PTR}{100}$ | (R) |

## EXERCISE 15 – TRANSPOSITION CONSOLIDATION

Transpose the following equations making the letter in brackets the subject:

| | | | | | |
|---|---|---|---|---|---|
| 1 | $N = 4y$ | (y) | 2 | $x = \frac{y}{3}$ | (y) |
| 3 | $E = IR$ | (R) | 4 | $P = V – q$ | (V) |
| 5 | $P = p + 14.7$ | (p) | 6. | $x = 3y + 5$ | (y) |
| 7. | $A = \frac{1}{2}bh$ | (b) | 8. | $Q = \frac{1}{n}$ | (n) |
| 9. | $R = \frac{100I}{PT}$ | (T) | 10. | $y = a + \frac{x}{b}$ | (x) |
| 11. | $H = S + xI$ | (S) | 12. | $a - \frac{b+c}{d}$ | (c) |
| 13. | $C = \frac{N-n}{2p}$ | (N) | 14. | $v = u + at$ | (a) |
| 15. | $F = \frac{9}{5}°\mathrm{C} + 32$ | (C) | 16. | $TE = \frac{1}{2}mv^2 + mgh$ | (h) |
| 17. | $D = B – 1.2d$ | (d) | 18. | $T = g(H + t)$ | (t) |
| 19. | $s = \frac{12(D-d)}{1}$ | (d) | 20. | $m = 600(r + 1)$ | (r) |
| 21. | $V = 333(m + 2)$ | (m) | 22. | $A = \pi r^2$ | (r) |
| 23 | $KE = \frac{1}{2}mv^2$ | (v) | 24. | $y = \sqrt{2s}$ | (s) |
| 25. | $T = \sqrt{\frac{w}{ga}}$ | (a) | | | |

## EXERCISE 16 – FURTHER TRANSPOSITION

Transpose the following equations making the letter in brackets the subject:

| | | | | | |
|---|---|---|---|---|---|
| 1 | $X_L = 2\pi fL$ | (L) | 2. | $X_C = \frac{1}{2\pi fC}$ | (C) |
| 3 | $V = E - I_aR_a$ | (E) | 4. | $T^2 = \frac{Z_S}{Z_P}$ | (T) |
| 5. | $\frac{V_S}{V_P} = \frac{N_S}{N_P}$ | ($N_P$) | 6. | $V_pI_p = V_sI_s$ | ($V_s$) |
| 7. | $f_o = \frac{1}{2\pi\sqrt{LC}}$ | (L) | 8. | $T = \frac{L}{R}$ | (R) |
| 9. | T = RC | (R) | 10 | $V = IX_L$ | ($X_L$) |
| 11. | $V = IX_C$ | (I) | 12. | $P = \frac{V^2}{R}$ | (V) |
| 13. | $P = I^2R$ | (I) | 14. | P = VI | (V) |
| 15. | $Z = \sqrt{R^2 + (X_L - X_C)^2}$ | ($X_L$) | 16. | $\frac{N_1}{N_2} = \frac{I_2}{I_1}$ | ($I_2$) |
| 17 | RP = VI SinØ | (SinØ), | 18. | P = VI CosØ | (CosØ) |
| 19 | $\cos\emptyset = \frac{R}{Z}$ | (R) | 20. | Q = VC | (V) |
| 21. | V = E – IR | (I) | 22. | BP = 2πNT | (T) |
| 23 | $\frac{N_1}{N_2} = \frac{I_2}{I_1}$ | ($I_1$) | 24. | E = V + IR | (V) |
| 25. | V = IR | (I) | 26. | V = IR | (R) |
| 27. | E = I(R + r) | (R) | 28. | E = I(R + r) | (I) |
| 29. | $V_L = \sqrt{3}V_{PH}$ | ($V_{PH}$) | 30. | $V_{ma} = V_{pk} \times 0.707$ | ($V_{pk}$) |

# ANSWERS

All numerical answers given to three significant figures or 2 decimal places unless stated otherwise.

## EXERCISE 1 – LIKE AND UNLIKE TERMS

| | | | | | |
|---|---|---|---|---|---|
| 1 | $4a$ | 2. | $2x^2$ | 3. | $6x + y$ |
| 4. | $5p - 4q$ | 5. | $0$ | 6. | $4x^2 - 7y^2$ |
| 7. | $a^2 - 2ab$ | 8. | $3q^2 - 3pq$ | 9. | $abc - xyz$ |
| 10. | $-2a - 2b - 4c - 3d$ | 11. | $3a + 2b - 2c$ | 12. | $p + q + r$ |
| 13 | $2a^3 + 2a^2 - 3a + 12$ | 14. | $3x^2 - 2xy$ | | |

## EXERCISE 2 – MULTIPLICATION AND DIVISION

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | $a^5$ | 2. | $e^5$ | 3. | $f^3$ | 4. | $g^4$ |
| 5. | $d^8$ | 6. | $2g^7$ | 7. | $12a^7$ | 8. | $12x^7$ |
| 9. | $10n^3$ | 10. | $6r^2p^4$ | 11. | $8t^5$ | 12. | $18x^4y^2$ |
| 13. | $2$ | 14. | $3p^{-1}q$ | 15. | $4rs$ | 16. | $1$ |
| 17. | $1$ | 18. | $\frac{1}{y}$ | 19. | $\frac{1}{hk}$ | 20. | $\frac{2}{3ak}$ |
| 21. | $10d$ | 22. | $-8n^3$ | 23. | $-3a$ | 24. | $4b$ |
| 25. | $3mn$ | 26. | $9a^4$ | 27. | $4x^6$ | 28. | $8k^9m^3$ |
| 29. | $25r^8$ | 30. | $-27p^{12}$ | | | | |

## EXERCISE 3 – COMMON FACTORS

1. 12, 48, 66, 90
2. 2, 3, 4, 6, 8, 12, 16, 24
3. a. For 108: 2, 3, 6, 9, 12, 18, 27, 36, 54

   For 66: 2, 3, 6, 11, 22, 33

   b. HCF = 6

4.

| | | | | | |
|---|---|---|---|---|---|
| a. | 6 | b. | $3x$ | c. | $5r$ |
| d. | $4a$ | e. | $2mn^2$ | f. | A |

### EXERCISE 4 – FACTORISATION

| | | | | | |
|---|---|---|---|---|---|
| 1 | $5(x + y)$ | 2. | $2p(p + 4q)$ | 3. | $a(x - y)$ |
| 4. | $3x(8x + 3)$ | 5. | $7y(3xy - 2)$ | 6. | $(a + 2)(b - c)$ |
| 7. | $(x - p)(y + z)$ | 8. | $2xy[y + x(3 + 4x)]$ | 9. | $(a + b)(y + 1)$ |
| 10. | $(a + b)(x + y)$ | 11. | $(2a + d)(b - 2c)$ | 12 | $(2x + 3y)(a - 2b)$ |

### EXERCISE 5 – SIMPLIFYING BRACKETS

| | | | | | |
|---|---|---|---|---|---|
| 1 | $13x - 8y$ | 2 | $7a - 8b + c$ | 3 | $4m - 3n + mn$ |
| 4 | $7x^2 + 4y^2$ | 5. | $38x + 14y - 49$ | 6. | $4x^2 - 5xy - 6x$ |
| 7 | $60y^2 - 55x^2$ | 8 | $8x + 1$ | 9. | $2y$ |
| 10 | $x^2 + 3xy + 2y^2$ | 11 | $a^2 + 4a + 3$ | 12. | $3p^2 + pq - 2q^2$ |
| 13. | $10a^2 + 16a - 8$ | 14 | $x^2 - 3x - 18$ | 15. | $a^3 - 3a^2b + 3ab^2 - b^3$ |

## EXERCISE 6 – CONSTRUCTING EQUATIONS

1. $x - 2$  2. $3p + 7q$  3. $3.2m + 15$  4. $\frac{130}{y} - \frac{130}{x}$  5. $\frac{1.99y}{x}$

6. The perimeter can be found by:

   a. walking round the shape: $P = 12 + 7 + 12 + 7$, so $P = l + w + l + w$

   b. pairing up lengths and widths:

      $P = 12 + 12 + 7 + 7$, so $P = l + l + w + w$

   c. length repeated plus width repeated:

      $P = 12 \times 2 + 7 \times 2$, so $P = l \times 2 + w \times 2$

   d. two lots of length plus two lots of width:

      $P = 2 \times 12 + 2 \times 7$, so $P = 2 \times l + 2 \times w$

   e. length plus width … repeated: $P = (12 + 7) \times 2$, so $P = (l + b) \times 2$

   f. two lots of … length plus width: $P = 2 \times (12 + 7)$, so $P = 2 \times (l + b)$

7. Making the shorter side equal b then b = 12 cm and l = 16 cm.

8. A bike costs £8 350 and a car costs £33 400

9. AB = 12m, BC = 8m, AC = 6m

10. 25 litres/min and 50 litres/min

## EXERCISE 7 – SOLVING EQUATIONS

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. | a. | $x = 7$ | b. | $x = 2$ | | |
| 2. | $x = 12$ of each type of coin. | | | | | |
| 3. | $x_1 = 71$, | $x_2 = 72$, | $x_3 = 73$ | | | |
| 4. | $x = 13$ | | | | | |
| 5. | $x = 6$ (7/14) | | | | | |
| 6. | a. | $x = -12$ | b. | $x = 15$ | c. | $x = 3$ |
| 7. | a. | 10 | b. | 34 | c. | 6 |
| | d. | 33 | e. | 7 | f. | 3 |
| 8. | a. | $2\frac{1}{4}$ | b. | 972 | c. | $\frac{9}{512}$ |
| 9. | $v = 8\text{m/s}$ | | | | | |

## EXERCISE 8 – SOLVING APPLIED PROBLEMS

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | $p = 1 \times 10^{-3}$ | 2. | $8\,\text{m/s}^2$ | 3. | $400\Omega$ | 4. | $P_2 = 75\,\text{kPa}$ |
| 5. | a. $1.33\Omega$ | b. | $R_3 = 30\,\Omega$ | | | | |
| 6. | 60°C | | | | | | |
| 7. | $0.8\,\Omega$ | | | | | | |
| 8. | $0.4\,\text{m/s}^2$ | | | | | | |

## EXERCISE 9 – SOLVING LINEAR EQUATIONS

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | 9 | 2. | 16 | 3. | $6\frac{2}{3}$ | 4. | 18 |
| 5. | 0 | 6. | 6 | 7. | 4 | 8. | – 2 |
| 9. | 7 | 10. | 18 | 11. | $2\frac{1}{2}$ | 12. | 5 |
| 13. | 2 | 14. | 5 | 15. | 3 | | |

## EXERCISE 10 – FURTHER LINEAR EQUATIONS

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1. | a. | 11 | b. | 3 | c. | 0 | d. | 5 |
| | e. | 12 | f. | 10 | g. | 12 | h. | $\frac{10}{7}$ |
| | i. | 6 | j. | $1\frac{7}{8}$ | k. | $\frac{12}{13}$ | l. | $\frac{1}{6}$ |
| | m. | 14 | n. | $3\frac{1}{3}$ | o. | 12 | p. | 2 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2. | a. | $a = 4.4$ | b. | $b = 3$ | c. | $m = 4\frac{3}{4}$ | d. | $n = 6\frac{2}{3}$ |
| | e. | $t = 6$ | f. | $x = -6$ | g. | $y = 3$ | h. | $d = \frac{1}{5}$ |
| | i. | $a = 4$ | j. | $b = 4\frac{1}{4}$ | k. | $m = 3$ | l. | $t = 7$ |
| | m. | $x = 2\frac{1}{2}$ | n. | $y = 1$ | o. | $z = 0$ | p. | $f = 1\frac{3}{4}$ |
| | q. | $g = \frac{3}{7}$ | | | | | | |

## EXERCISE 11 – SUBSTITUTION

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1. | a. | $x = 1$ | b. | $x = 12\frac{1}{2}$ | c. | $x = 10$ | d. | $x = 4$ |
| | e. | $x = 4$ | | | | | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2. | 16 | 3. | 2 | 4. | 4 | 5. | – 20 |
| 6. | 2 | 7. | $1 \times 10^{-8}$ | 8. | 1 | 9. | $1 \times 10^{1}$ |
| 10. | 25 | | | | | | |

## EXERCISE 12 – TRANSPOSITION

1. $x = y - z$
2. $b = a + c$
3. $s = p - q$
4. $n = m - l$
5. $x = \frac{y}{z}$
6. $m = \frac{y}{x}$
7. $z = xy$
8. $c = \frac{b}{a}$
9. $u = v - at$
10. $c = y - mx$
11. $I = \frac{E - V}{R}$
12. $t = \frac{v - u}{a}$
13. $T = \frac{PV}{R}$
14. $u = \frac{2s - at^2}{2t}$
15. $P = R$

## EXERCISE 13 – TRANSPOSITION (POWERS AND ROOTS)

1. $a = d^2$
2. $\pi = \sqrt{q}$
3. $b = \sqrt{a}$
4. $r = \sqrt{\frac{A}{\pi}}$
5. $x = \sqrt{z^2 - r^2}$
6. $c = \frac{a^2}{2b}$
7. $c = \sqrt{\frac{x^2}{y}}$

## EXERCISE 14 – FURTHER POWERS AND ROOTS

1. $d = \frac{c}{\pi}$
2. $v = \sqrt{2gh}$
3. $R = \frac{E}{I} - r$
4. $h = \frac{v^2}{2g}$
5. $C = \frac{L}{Q^2R^2}$
6. $u = -\frac{vj}{j - v}$
7. $a = \frac{2(s - ut)}{t^2}$
8. $R = \frac{100I}{PT}$

## EXERCISE 15 – TRANSPOSITION CONSOLIDATION

| | | |
|---|---|---|
| 1. $y = \frac{N}{4}$ | 2. $y = 3x$ | 3. $R = \frac{E}{I}$ |
| 4. $V = P + q$ | 5. $p = P - 14.7$ | 6. $y = \frac{x-5}{3}$ |
| 7. $b = \frac{2A}{H}$ | 8. $n = \frac{1}{Q}$ | 9. $R = \frac{100I}{PR}$ |
| 10. $x = b(y - a)$ | 11. $S = H - xl$ | 12. $C = ad - b$ |
| 13. $N = 2pC + n$ | 14. $a = \frac{v-u}{t}$ | 15. $c = \frac{5}{9}(F - 32)$ |
| 16. $h = \frac{TE - 0.5mv^2}{mg}$ | 17. $d = \frac{B-D}{1.2}$ | 18. $t = \frac{T - gH}{g}$ |
| 19. $d = \frac{12D - s}{12}$ | 20. $r = \frac{M - 600}{600}$ | 21. $m = \frac{V - 666}{333}$ |
| 22. $r = \sqrt{\frac{A}{\pi}}$ | 23. $v = \sqrt{\frac{2KE}{m}}$ | 24. $s = \frac{y^2}{2}$ |
| 25. $a = \frac{w}{T^2 g}$ | | |

## EXERCISE 16 – FURTHER TRANSPOSITION

| | | |
|---|---|---|
| 1. $L = \frac{X_L}{2\pi f}$ | 2. $C = \frac{1}{2\pi f X_C}$ | 3. $E = V + I_a R_a$ |
| 4. $T = \sqrt{\frac{Z_s}{Z_P}}$ | 5. $N_P = \frac{N_S V_P}{V_S}$ | 6. $V_S = \frac{V_P I_P}{I_S}$ |
| 7. $L = C\sqrt{\frac{1}{2\pi f_o}}$ | 8. $R = \frac{L}{T}$ | 9. $R = \frac{T}{C}$ |
| 10. $X_L = \frac{V}{I}$ | 11. $I = \frac{V}{X_C}$ | 12. $V = \sqrt{PR}$ |
| 13. $I = \sqrt{\frac{P}{R}}$ | 14. $V = \frac{P}{I}$ | 15. $X_L = \sqrt{Z^2 - R^2} + X_c$ |
| 16. $I_2 = \frac{N_1 I_1}{N_2}$ | 17. $\sin\phi = \frac{RP}{VI}$ | 18. $\cos\phi = \frac{P}{VI}$ |
| 19. $R = Z\cos\varnothing$ | 20. $V = \frac{Q}{C}$ | 21. $I = \frac{E - V}{R}$ |
| 22. $T = \frac{BP}{2\pi N}$ | 23. $I_1 = \frac{I_2 N_2}{N_1}$ | 24. $V = E - IR$ |
| 25. $I = \frac{V}{R}$ | 26. $R = \frac{V}{I}$ | 27. $R = \frac{E - IR}{I}$ |
| 28. $I = \frac{E}{R + r}$ | 29. $V_{Ph} = \frac{V_L}{\sqrt{3}}$ | 30. $V_{Pk} = \frac{V_{rms}}{0.707}$ |

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