Defence School of
Aeronautical Engineering

No.2 School of Technical Training

Academic Principles Organisation

TG5 MATHEMATICS

2208A, 2209A, 2211A

BOOK 4

Probability

WARNING

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WARNING

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OBJECTIVES

E.O.	Description
1.2.4	Use probability to describe situations.

KEY LEARNING POINTS

KLP	Description
1.2.4.1	Define the probability of a particular outcome
1.2.4.1.1	Explain probability of outcome
1.2.4.2	Explain and show the difference between theoretical and experimental probability

PROBABILITY

1. Probability is a measure of how likely something is to happen.

2. The likelihood or probability of a given outcome is represented on a scale from the probability that the event cannot happen (zero probability) to when it is certain to (a probability of one).

Example:

A ski instructor assesses a class 15 skiers on the first morning of an expedition. The instructor summarizes the following:

a. Nine of the skiers are female.

b. Five of the skiers are wearing ‘one-piece’ ski suits.

c. One of the skiers has a beard.

d. However, luckily, they all have skis, boots and poles.

3. Since 9 out of 15 pupils are female, the probability of picking a female at random is 9 out of 15 or $\frac{9}{15}$.

4. Conversely, the probability of picking a male at random is $\frac{6}{15}$ (since the other 6 pupils must be male).

5. Since 5 of the skiers are wearing one piece ski suits; the probability of picking a ‘one piece ski suit wearer' at random must be 5 out of 15 or $\frac{5}{15}$.

6. Since everyone has remembered their skis. boots and poles, the probability of picking a pupil at random who has their skis with them must be $\frac{15}{15}$ or one.

7. Conversely the probability of picking a pupil at random who has forgotten their ski poles must be $\frac{0}{15}$ or zero.

8. To summarize:

a. If an event is bound to occur its probability is one.

b. If an event cannot occur its probability is zero.

c. The probability of an event not happening added to the probability of the event happening must be one.

9. For example, the probability of picking a female at random $\left(\frac{9}{15}\right)$ added to the probability of not picking a female at random:

$$\left(\frac{6}{15}\right) = \frac{9}{15} + \frac{6}{15} = 1$$

10. From this we can say:

a. The total probability for all possible outcomes is one.

b. The probability of an event happening is ‘1 minus the Probability of it not happening’.

11. This scale of probability can be used to make comparisons.

12. For example, the probability of picking someone with a beard $\left(\frac{1}{15}\right)$ is much less than picking someone who is female $\left(\frac{9}{15}\right)$.

Theoretical and experimental probability

13. The probability of an event occurring can be found by either theory or by experiment. The theory relies on logical thought, whilst the experimental method relies on taking the results derived from repeating an event many times.

14. Find the probability of getting a head when tossing an unbiased coin.

Theory method

a. 'Unbiased’ means both events (head. tail) are equally likely to occur. Heads and tails are the only outcomes,

So, the Probability of a head + Probability of a tail = 1 (since the total of all possible outcomes is 1).

b. Probability of a head = Probability of a tail,
So, the probability of a tail = ½

Experimental Method

a. Toss an unbiased coin 100 times and count the number of tails you obtain. You may obtain 49 tails.

The experimental probability of getting a tail is $\frac{49}{100}$.

b. If the event is repeated many times, it is usual for the experimental probability to be close to the theoretical probability, but not to be exactly the same.

Exercise 1.

1. The probability that a streetlight is lit is 0.91. What is the probability that it is not lit?

2. You toss a coin, marked with a head on both sides. What is the probability of getting a tail?

3. The probability of a plane being late is ½. What is the probability of a plane not being late?

4. A piece of fruit is selected from a bag containing 6 oranges and 4 apples at random. What is the probability that it is a pear?

5. Probabilities can be estimated either by making a subjective estimate or by making use of statistical evidence. State the method which would be used in the following cases:

    a. The probability that more females than males will listen to the news on television.

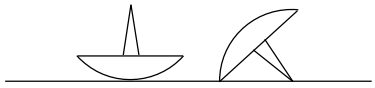
    b. The probability that humans will live on the moon in the year 2050.

    c. The probability that the next vehicle passing a college will be a motorcycle.

    d. The probability that the next book issued at a library will be a novel.

    e. The probability that there will be a cure for deafness within 10 years.

6. In an experiment, a drawing pin is thrown. The number of times it lands point up is recorded.



    a. The drawing pin is thrown 10 times and lands point up 6 times. From this data estimate the probability that it lands point up.

    b. The drawing pin is thrown 100 times and lands point up 57 times. From this data estimate the probability that it lands point up.

    c. Give a reason why the second answer is a more reliable estimate.

Simple Probabilities

15. The general formula for the probability of a given event occurring, provided the results are equally likely to happen, is:

$$\text{Number of possible events} = \frac{\text{Probability of results giving the event}}{\text{Probability of event}}$$

16. An unbiased cubical die marked 1 to 6 is thrown.

    a. What is the probability of getting a 5?

    b. There are 6 equally likely results.

    c. The probability of getting a 5 is $\frac{1}{6}$.

17. A bag contains 7 balls, identical in shape and size.

    3 balls are white and 4 are blue.

    One ball is selected from the bag at random.

    a. What is the probability that the ball is white?

    b. There are 7 balls. The probability that any one ball is selected is 1.

    c. There are 3 white balls.

    The probability of selecting a white ball = $\frac{\text{Number of white balls}}{\text{Total number of balls}} = \frac{3}{7}$

Exercise 2

1. A piece of fruit is selected at random, from a bag containing only 7 oranges and 3 apples. What is the probability that it is an apple?

2. A die, numbered 1 to 6, is thrown. What is the probability of getting a 6?

3. A die, numbered 1 to 6, is thrown. What is the probability of getting an even number?

4. One card is drawn from a pack of 52 cards. What is the probability that it is:

   a. Red.

   b. A Jack.

   c. The Queen Spades?

5. What is the probability of picking an even number from the numbers 10 to 20 (inclusive)?

6. A bag containing 20 balls, has 7 red balls, 8 blue balls, 3 yellow balls, and the rest are white. What is the probability of drawing:

   a. A red ball?

   b. A white ball?

7. Out of 20 lamp bulbs, 3 are faulty. What is the probability of selecting a bulb that works?

ANSWERS

All answers are given to 3 significant figures (sf) or 2 decimal places unless stated otherwise.

Exercise 1

1. 0.09 2. 0 3. ½ 4. 0

5. a. Statistical.

   b. Subjective.

   c. Statistical.

   d. Statistical.

   e. Subjective.

6. a. $\frac{6}{10}$

   b. $\frac{57}{100}$

   c. More reliable as an increase in tests brings the proportion of times an event occurs closer to the probability.

Exercise 2

1. $\frac{3}{10}$ 2. $\frac{1}{6}$ 3. $\frac{1}{2}$

4. a. $\frac{1}{2}$ b. $\frac{1}{13}$ c. $\frac{1}{52}$

5. $\frac{6}{11}$

6. a. $\frac{7}{20}$ b. $\frac{1}{10}$

7 $\frac{17}{20}$