Defence School of
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No.2 School of Technical Training

Academic Principles Organisation

TG5 MATHEMATICS

2208A, 2209A, 2211A

BOOK 1

Numeracy

WARNING

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NUMERACY

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FRACTIONS

KEY LEARNING POINTS

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1.2.1.2	Convert mixed numbers to improper fractions and vice versa.
1.2.1.3	Simplify proper and improper fractions by cancelling.
1.2.1.4	Multiply 3 fractions together.
1.2.1.5	Divide one fraction by another.
1.2.1.6	Perform mixed multiplication and division of up to 3 fractions.
1.2.1.7	Calculate the Lowest Common Denominator for up to 3 fractions.
1.2.1.8	Add together up to 3 fractions.
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FRACTIONS

1. **Proper fractions** may be defined as fractions less than 1. They are also known as Vulgar or common fractions.

For example: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{5}$, $\frac{11}{17}$, etc.

2. **Improper fractions** are fractions which are greater than 1.

For example: $\frac{7}{3}$, $\frac{5}{3}$, $\frac{17}{11}$, $\frac{8}{5}$, etc.

3. **Mixed numbers** include whole numbers and proper (vulgar) fractions.

For example: $1\frac{1}{2}$, $2\frac{3}{5}$, $6\frac{4}{11}$, $27\frac{6}{7}$, etc.

4. For all fractions the number above the bar is called the **numerator.** The number below the bar is called the **denominator.**

5. The simplest form of $\frac{30}{60}$ is $\frac{1}{2}$ Fractions can be expressed in simplest form by dividing numerator and denominator by equal numbers until they will not divide further.

6. For example: $\frac{8}{12} = \frac{2}{3}$ in simplest form (after dividing numerator and denominator by 4).

7. **Cancelling**. The process of dividing numerator and denominator by equal values is called cancelling.

For example: $\frac{27}{81} = \frac{9}{27} = \frac{3}{9} = \frac{1}{3}$

8. **Converting mixed numbers to improper fractions**. Multiply whole numbers by denominator and add to numerator.

For example: $2\frac{3}{5} = \frac{13}{5}$

9. **Converting improper fractions to mixed numbers.** Divide numerator by denominator to give whole number - remainder gives new numerator.

For example: $\frac{25}{4} = 6\frac{1}{4}$

10. Cancelling improper fractions involves exactly the same process as cancelling vulgar fractions.

For example: $\frac{28}{4} = \frac{7}{1} = 7$

and $\frac{45}{6} = \frac{15}{2} = 7\frac{1}{2}$

11. For cancelling a mixed number, first convert it to an improper fraction then cancel down as before.

For example: $7\frac{3}{6}$

$= \frac{45}{6}$

$= \frac{15}{2}$

$= 7\frac{1}{2}$

SOLVE CALCULATIONS USING STANDARD OPERATORS ON FRACTIONS

12. **Addition**

a. Express all fractions as mixed numbers in lowest terms.

b. Add the whole numbers together.

c. To add vulgar fractions, you must convert each fraction so that their denominators are all the same. This is done by finding the Lowest Common Multiple, (LCM) of the denominators.

i.

$$\frac{1}{5}+\frac{1}{6}+\frac{3}{10}$$

$$=\frac{6+5+9}{30}$$

$$=\frac{20}{30}$$

$$=\frac{2}{3}$$

Note: If your addition of fractions results in improper fractions, you must convert this to a mixed number as shown in example (ii).

ii.

$$\frac{9}{4}+\frac{5}{12}+1\frac{3}{8}$$

$$=2\frac{1}{4}+\frac{5}{12}+1\frac{3}{8}$$

$$=3+\frac{1}{4}+\frac{5}{12}+\frac{3}{8}$$

$$=3+\frac{6+10+9}{24}$$

$$=3+\frac{25}{24}=3+1\frac{1}{24}$$

$$=4\frac{1}{24}$$

13. **Subtraction**. The same basic procedure should be used for subtraction as for addition.

a. $\frac{8}{9} - \frac{2}{3}$

$= \frac{8-6}{9}$

$= \frac{2}{9}$

b. $\frac{8}{3} - 1\frac{4}{7}$

$= 2\frac{2}{3} - 1\frac{4}{7}$

$= 1 + \frac{2}{3} - \frac{4}{7}$

$= 1 + \frac{14-12}{21}$

$= 1\frac{2}{21}$

c. $4\frac{1}{3} - 1\frac{3}{4}$

$= 3 + \frac{1}{3} - \frac{3}{4}$

$= 3 + \frac{4-9}{12}$

$= 2\frac{7}{12}$

Note: As numerator (4 - 9) in example c gives a negative value, one whole unit has to be converted to $\frac{12}{12}$ before the subtraction of fractions is carried out.

14. Mixed Addition and Subtraction can be carried out exactly as above.

Example:

a. $5\frac{7}{12} - 4\frac{1}{2} + 3\frac{3}{4}$ dealing with whole numbers first gives:

$$= 4 + \frac{7 - 6 + 9}{12}$$

$$= 4 + \frac{10}{12}$$

$$= 4\frac{5}{6}$$

b. $2\frac{1}{8} - 1\frac{3}{4} + 4\frac{1}{3}$

$$= 5 + \frac{3 - 18 + 8}{24}$$

$$= 4 + \frac{24 + 3 - 18 + 8}{24}$$

$$= 4\frac{17}{24}$$

15. Remember that your final step in any calculation must be to simplify (cancel fractions).

Example:

$$3\frac{3}{5} + 1\frac{9}{10} - 2\frac{3}{4}$$

$$= 2 + \frac{12 + 18 - 15}{20}$$

$$= 2\frac{3}{4}$$

Exercise 1

1. Add the following fractions and mixed numbers:

a. $\frac{1}{3}+\frac{4}{7}$ b. $\frac{4}{5}+\frac{5}{6}$ c. $\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$

d. $\frac{1}{4}+\frac{4}{5}+\frac{7}{10}$ e. $3\frac{1}{3}+4\frac{3}{7}$ f. $2\frac{3}{4}+1\frac{1}{5}+1\frac{7}{10}$

2. Subtract the following:

a. $\frac{1}{2}-\frac{1}{3}$ b. $\frac{3}{5}-\frac{2}{7}$ c. $4\frac{5}{6}-2\frac{2}{3}$

d. $7\frac{2}{5}-2\frac{2}{3}$ e. $5\frac{1}{4}-1\frac{5}{6}$ f. $6\frac{3}{8}-5\frac{7}{12}$

3. Evaluate the following, simplifying as far as possible:

a. $\frac{2}{5}+\frac{9}{10}-\frac{3}{4}$ b. $3\frac{5}{6}-1\frac{1}{2}+\frac{3}{4}$ c. $2\frac{1}{8}-1\frac{5}{15}+2\frac{1}{2}$

d. $5\frac{5}{12}-2\frac{4}{9}+3\frac{1}{4}$ e. $4\frac{1}{4}+3\frac{1}{3}-5\frac{5}{8}$ f. $1\frac{1}{5}-3\frac{2}{3}+2\frac{1}{2}$

16. **Multiplication**

a. Express all mixed numbers as improper fractions.

b. Cancel vertically if possible.

c. Cancel across the multiplication sign if possible.

d. Multiply numerators together; multiply denominators together.

e. If the result is an improper fraction, convert to a mixed number.

f. Check that your answer is in simplest form.

Examples:

i. $\frac{2}{9}\times 4$

$= \frac{2}{9}\times\frac{4}{1}$

$= \frac{8}{9}$

ii. $1\frac{4}{5}\times 2\frac{1}{3}\times\frac{5}{14}$

$= \frac{\cancel{9}^{3}}{\cancel{5}_{1}}\times\frac{\cancel{7}^{1}}{\cancel{3}_{1}}\times\frac{\cancel{5}^{1}}{\cancel{14}_{2}}$

$= \frac{3}{2} = 1\frac{1}{2}$

17. **Division**. Convert all mixed numbers to improper fractions, invert the fractions you are dividing by and then proceed as for multiplication.

Examples:

a. $\frac{3}{4} \div 1\frac{5}{7}$

$= \frac{3}{4} \div \frac{12}{7}$

$= \frac{\cancel{3}^{1}}{4} \times \frac{7}{\cancel{12}_{4}}$

$= \frac{7}{16}$

b. $\frac{3}{4} \div 7$

$= \frac{3}{4} \times \frac{1}{7}$

$= \frac{3}{28}$

c. $\frac{3}{8} \div \frac{5}{16}$

$\frac{3}{\cancel{8}_{1}} \times \frac{\cancel{16}^{2}}{5}$

$= \frac{6}{5}$

$= 1\frac{1}{5}$

18. Mixed multiplication and division can be carried out by simply inverting all the fractions preceded by a division sign and then treating the calculation as a multiplication only.

Example:

$1\frac{3}{4} \div 4\frac{1}{2} \times 1\frac{5}{7}$

$= \frac{7}{4} \div \frac{9}{2} \times \frac{12}{7}$

$= \frac{7}{4} \times \frac{2}{9} \times \frac{12}{7}$

$= \frac{\cancel{7}^{1}}{\cancel{4}_{1}} \times \frac{2}{\cancel{9}_{3}} \times \frac{\cancel{12}^{31}}{\cancel{7}_{1}}$

$= \frac{2}{3}$

Note: You only turn upside-down the fraction you are dividing by, i.e., the fraction after the division sign.

Exercise 2

1. Convert the following mixed numbers to improper fractions:

a. $2\frac{3}{8}$ b. $6\frac{3}{4}$ c. $1\frac{7}{8}$ d. $3\frac{3}{16}$

2. Convert the following improper fractions to mixed numbers:

a. $\frac{13}{4}$ b. $\frac{7}{2}$ c. $\frac{43}{8}$ d. $\frac{39}{16}$

3. Multiply the following, simplifying as far as possible:

a. $\frac{3}{4}\times\frac{1}{2}$ b. $\frac{5}{6}\times\frac{9}{10}$ c. $\frac{1}{6}\times\frac{2}{3}$

d. $\frac{5}{8}\times\frac{4}{15}$ e. $\frac{2}{3}\times1\frac{1}{8}$ f. $1\frac{3}{5}\times1\frac{9}{16}$

g. $\frac{1}{3}\times\frac{6}{7}\times\frac{1}{2}$ h. $1\frac{1}{5}\times2\frac{1}{3}\times\frac{1}{21}$ i. $2\frac{3}{4}\times1\frac{5}{9}\times3\frac{6}{7}$

4. Divide the following, simplifying as far as possible:

a. $\frac{5}{6}\div\frac{7}{8}$ b. $\frac{7}{12}\div\frac{14}{15}$ c. $4\frac{1}{8}\div2\frac{3}{4}$

5. Evaluate the following, simplifying as far as possible.

a. $1\frac{1}{2}\times2\frac{2}{3}\div1\frac{1}{4}$ b. $2\frac{2}{15}\times3\frac{1}{8}\div2\frac{1}{7}$

c. $2\frac{1}{4}\div1\frac{1}{2}\times3\frac{1}{3}$ d. $3\frac{1}{2}\div2\frac{1}{3}\times1\frac{1}{6}$

e. $2\frac{2}{3}\times3\frac{6}{7}\div1\frac{1}{5}$

RULES OF PRECEDENCE.

KEY LEARNING POINTS

KLP	Description
1.2.1.11	Write the Letters BIDMAS in the correct order and state the meaning of each letter.
1.2.1.12	Evaluate simple arithmetic expressions involving the rules of precedence.

EVALUATING EXPRESSIONS USING BIDMAS

19. When evaluating numerical expressions, we need to know the order in which addition, subtraction, multiplication and division are carried out.

Consider evaluating	$3 + 4 \times 5$
With addition carried out first:	$3 + 4 \times 5 = 7 \times 5 = 35$
and with multiplication carried out first	$3 + 4 \times 5 = 3 + 20 = 23$

So, from the above it can be seen that the order in which numerical operations are carried out is very important.

20. Using the BIDMAS rule it tells us the order which we must carry out these numerical operations.

<table>
<tr><td>Brackets</td><td>1st priority $5(3+6) = 5 \times 9$</td></tr>
<tr><td>Indices</td><td>2nd priority
Indices are just powers and roots 2^4 $\sqrt[3]{x}$ etc.</td></tr>
<tr><td>Division ÷</td><td rowspan="2">3rd priority
Multiplication and division can be done at the same time.
$5 \times 4 \div 2 = 10$
$4 \div 2 \times 5 = 10$
$5 \div 2 \times 4 = 10$
$4 \times 5 \div 2 = 10$

This is how all of the equations really look: $\frac{5\times4}{2} = 10$
Because the ÷ belongs to the 2</td></tr>
<tr><td>Multiplication ×</td></tr>
<tr><td>Addition +</td><td rowspan="2">4th priority
Addition and Subtraction can be done at the same time.
$7 - 2 + 1 = 6$
$-2 + 7 + 1 = 6$
$7 + 1 - 2 = 6$

Always solve from left to right.
The minus belongs to the 2. (i.e. -2)
We could move the -2 anywhere in the equation and get the same result.</td></tr>
<tr><td>Subtraction -</td></tr>
</table>

21. Occasionally problems involve the term ‘of’ in these instances the ‘of’ is to be regarded as a multiplication, so ½ of 10 means ½ × 10. (Third priority)

22. Examples using BIDMAS:

a.	$2 + 3 \times 4$	**multiplication first**	
	$2 + 3 \times 4$	$= 2 + 12$	$= 14$
b.	$(2 + 3) \times 4$	**brackets first**	
	$(2 + 3) \times 4$	$= 5 \times 4$	$= 20$
c.	$4 + 4 \div 2$	**division first**	
	$4 + 4 \div 2$	$= 4 + 2$	$= 6$
d.	$20 \div (4 \times 5)$	**brackets first**	
	$20 \div (4 \times 5)$	$= 20 \div 20$	$= 1$
e.	5 + ½ of 20	**(of means ×) first**	
	5 + ½ of 20	$= 5 + 10$	$= 15$

Exercise 3

Evaluate:

1. $3 + 4 \times 2 =$
2. $8 - 4 \div 2 =$
3. $5 \times 3 + 2 =$
4. $(5 + 3) \times 4 =$
5. $(8 - 4) \div 2 =$
6. $8 + 5 \times 3 - 2 =$
7. $13 - {}^{1}\!/_{4}\ of\ 32 =$
8. $({}^{3}\!/_{4} - {}^{1}\!/_{4})of\ 22 =$
9. $3 + 8 \times 9 =$
10. $3 \times 7 - 2 =$
11. $8 \times 5 - 2 + 5 \times 6 =$
12. $5 - 15 \times (5 - 2) =$
13. $2(-7 - 7) =$
14. $9 \div 3 \times (-4) + (-6) \div (-2) =$
15. $-200 \div 10 \times (-4) + (-16) \div (-8) =$
16. $3 \times (-12) + (-44) \div (-2) =$
17. $\big((-12) \times (20) \div (-6) - (+4)\big) \times {}^{1}\!/_{2} =$
18. $15 + 6 \div 2 + 5 \times (6 \div 2) =$
19. $-21 \div (-3) \times (-4) + (-11) \times 14 \times (6 \div 2 + 4) =$
20. $108 \div 3 \times (5 + 4) \div (-6 \times 3) =$
21. $\frac{15+2}{(-2-7)} \times \frac{11\times4+3}{47} + \frac{6\times(-5)}{-2} =$
22. $\frac{3^2+4^2}{5^2} =$
23. $\sqrt{31 + 18} + 4(9 + 2) =$ (Note: Assume all roots to be positive)
24. $(2^3)^2 \div 16 + 4 =$
25. $\big((3^2 \times 9 \div 3) \div (-1)\big) \times 3 =$
26. $\frac{6^2\div\sqrt{134+10}+16-\sqrt{64}+5^2}{2^2\times3^2} =$ (Note: Assume all roots to be positive)

DECIMALS

KEY LEARNING POINTS

KLP	Description
1.2.1.13	State that decimals are fractions whose denominators are powers of 10.
1.2.1.14	Recognise and count decimal places.
1.2.1.15	Add decimals together.
1.2.1.16	Subtract decimals.
1.2.1.17	Multiply decimals.
1.2.1.18	Divide decimals.
1.2.1.19	Solve decimals involving mixed operations + − × ÷

DECIMAL NUMBERS

23. Decimals are an extension of the place values in the denary system and are used to express values less than one. They may be regarded as being the same as fractions.

24. In the same way a fraction is recognised by the use of the dividing line, a decimal is recognisable by the use of a dot separating whole numbers from values less than one, which is known as the decimal point.

25. In the number 11.111 each digit is one tenth of the digit in the place to the left. Therefore, the digit in the place after the decimal point (first decimal place) has a value of one tenth and the digit in the second decimal place one hundredth and so on.

Adding and Subtracting with Decimals

26. When setting out addition or subtraction problems with decimals it is vital that the decimal points are kept in line with each other. This ensures that all of the same place values are treated together.

Worked Example

Addition 32.15 + 8.125 + 331. 4 + 0.333 =

Is laid out as follows:

```
  32.15    +
   8.125
 331.4
   0.333
```

27. It is permissible to use zeros to even up the number of digits after the decimal point, as they will not alter the value. Then calculate the columns in the normal way.

$$\begin{array}{r}032.150 \quad + \\ 008.125 \\ 331.400 \\ \underline{000.333} \\ 372.008 \end{array}$$

Subtraction 24.32 – 8.45 =

Set out with decimal points in line.

```
 24.32
– 8.45
```

Calculate in the normal manner.

$$\begin{array}{r} 24.32 \\ \underline{-\ 8.45} \\ 15.87 \end{array}$$

Exercise 4

Calculate the following:

1. 76.435 + 113.87 + 62.322 =

2. 0.046 + 0.728 + 0.24 =

3. 27.625 + 9.25 + 14.7 + 7.066 =

4. 15.54 – 7.77 =

5. 278 – 85.36 =

6. 0.418 – 0.273 =

Multiplying Decimals

28. **Multiplying Decimals by Powers of 10**. In the section dealing with whole numbers, it was seen that multiplying a number by a power of ten shifted the digits in the number to the left so that they have a greater place value. When decimal values are involved, this shift requires the decimal point to move to the right. It will do so, at the rate of one place for every zero in the power of ten being used.

Exercise 5

Calculate the product of the following:

1. 346.874 × 100 =

2. 0.0046 × 1000 =

3. 2.625 × 1000 =

4. 13.054 × 100 =

5. 0.000278 × 10000 =

29. **Multiplying Decimals**. When multiplying decimal numbers, it is easier to treat them as whole numbers first, then place the decimal point back in the result in the correct place.

Worked Example

12.43 × 4.3 =

The first stage of this calculation is to express the numbers as whole numbers by removing the decimal points but noting how many decimal places have been removed. In this case there are 3.

Carry out the multiplication.

$$\begin{array}{rl} 1243 & \quad x \\ \underline{\quad 43} & \\ 3729 & \\ \underline{49720} & \\ 53449 & \end{array}$$

30. **Placing the Decimal Point**. Once the correct figures have been arrived at there are two means of reinstating the decimal point in the correct place.

a. Count the number of decimal places removed in the first step and put it back into the answer.

i.e., 53.449

The answer will have the same number of decimal places as the question.

b. Make a rough estimate of the answer by multiplying approximations of the original numbers. In the example above 12.5 × 4 would suffice to show that the answer is in the vicinity of 50.

Therefore 53.449 gives the closest fit.

31. Both of these methods yield the correct answer, so one may be used to check the other.

Exercise 6

Calculate the product of the following.

1. 46.74 × 1.3 =

2. 0.06 × 5.12 =

3. 2.65 × 10.5 =

4. 0.154 × 1.44 =

5. 22.2 × 2.22 × 0.222 =

Dividing Decimals

32. **Dividing a Decimal by Powers of Ten**. When a decimal number is divided by a power of ten, the decimal point for the answer will shift to the left one place for each zero in the power of ten being used.

Exercise 7

Calculate the following:

1. 0.0404 ÷ 100 =

2. 452.625 ÷ 1000 =

3. 130 ÷ 100 =

4. 492.874 ÷ 100 =

5. 30.15 ÷ 10000 =

9. **Dividing a Decimal by a Whole Number**. The division of a decimal number by a whole number is set out the same as a normal division problem.

Worked Example

4.32 ÷ 3 =

Set out thus

$$3\underline{)\ 4.3\ 2}$$

The division proceeds as normal and the decimal point is placed in the answer as it is encountered in the calculation.

Thus

$$\begin{array}{l} 3\underline{)\ 4.{}^{1}3{}^{1}2} \\ \quad 1.\ 4\ 4 \end{array}$$

33. This principle also applies when long division is called for.

Worked Example

4.32 ÷ 16 =

Set out thus

$$16\overline{)4.32}$$

4 will not divide by 16, so 0 is placed above the four and the decimal point is placed in the answer above the decimal point in the bracket.

$$\begin{array}{r} 0. \\ 16\overline{)4.32} \end{array}$$

43 will divide by 16, twice. So, 2 is written above the 3 in the bracket and 32 (2×16) is subtracted from 43 to give the remainder.

$$\begin{array}{r} 0.2 \\ 16\overline{)4.32} \\ \underline{3\,2} \\ \mathbf{1\,1} \end{array}$$

The two in the second decimal place is brought down to the remainder, giving 112, which divided by 16 will give 7.

$$\begin{array}{r} 0.27 \\ 16\overline{)4.32} \\ \underline{3\,2} \\ 1\,1\,2 \end{array}$$

Exercise 8

Calculate the following:

1. 492.872 ÷ 4 =

2. 0.406 ÷ 8 =

3. 52.625 ÷ 25 =

4. 8.64 ÷ 12 =

5. 301 ÷ 40 =

Dividing by a Decimal

34. **Recall of Relevant Information**

- A division problem can be set out as a fraction.
- Equivalent fractions express the same value in different terms.
- Multiplying decimals by powers of ten shifts the decimal point to the right.

35. To solve the problem 8.34 ÷ 0.3, there are two ways of looking at these numbers which suggest possible solutions. Given that it is easier to divide by a whole number than a decimal, this type of problem may be solved in one of the following ways:

a. The whole division may be written as a fraction.

i.e. $$\frac{8.34}{0.30}$$

Multiplying each number by the same power of ten would give an equivalent fraction.

In this case if each number is multiplied by10 the denominator will become a whole number.

Therefore:

$$8.34 \div 0.3 = 83.4 \div 3$$

Which can be set out thus

$$3\overline{)\,83.4}$$

b. Recognising 0.3 as being $\frac{3}{10}$.

$$8.34 \div 0.3 = 8.34 \div \frac{3}{10}$$

Using the division rule for fractions this will become:

$$8.34 \div 0.3 = 8.34 \times \frac{10}{3} = 83.4 \div 3$$

Multiplying across the top line and dividing by the number on the bottom line completes the solution.

Exercise 9

Calculate the following using either of the methods shown.

1. 92.872 ÷ 0.4 =
2. 0.426 ÷ 0.03 =
3. 2.625÷ 2.5 =
4. 8.64 ÷ 0.09 =
5. 301 ÷ 0.8 =

ROUNDING TO ‘SIGNIFICANT FIGURES’ AND ‘DECIMAL PLACES

Significant Figures

36. The value of any digit in a number is dependent upon its position. We call the first non-zero digit the first significant figure and the last non-zero digit the last significant figure. These three numbers all have 4 significant figures.

a.	3 0 9 7 | | 1^{st} 4^{th}	b.	3 0 . 9 7 | | 1^{st} 4^{th}	c.	0.00003097 | | 1^{st} 4^{th}

Note: Do not confuse with decimal places, sub-paragraph c has 8 decimal places.

37. Further examples:

a.	14384	(5 sig fig)	e.	1.005	(4 sig fig)
b.	0.089	(2 sig fig)	f.	72300	(3 sig fig)
c.	29.08604	(7 sig fig)	g.	10000000	(1 sig fig)
d.	72	(2 sig fig)	h.	630100	(4 sig fig)

38. To write a number to a given number of significant figures we look at one more significant figure than is required and if this digit is 5 or more, we round up the preceding figure; 4 or less we leave the preceding figure as it is.

For example:

a. 187340 is 187000 to 3 significant figures 190000 to 2 significant figures.

b. 0.001239 is 0.00124 to 3 significant figures, 0.0012 to 2 significant figures.

c. 35.603 is 35.6 to 3 significant figures, 36 to 2 significant figures.

d. 0.081778 is 0.08178 to 4 significant figures, 0.0818 to 3 significant figures.

e. 1.006 is 1 to one significant figure.

Decimal Places

39. When correcting to a certain number of decimal places (rounding off) it is only the digit immediately following the required decimal place which affects it. If the digit immediately following is 5 or more, the previous digit should be increased by 1. If it is less than 5 the previous digit is unaltered. For example:

a.	0.833	=	0.83 to 2 decimal places.
b.	0.866	=	0.87 to 2 decimal places.
c.	0.2346	=	0.23 to 2 decimal places.
d.	127.12304	=	127.1 to one decimal place
e.	0.0001449	=	0.00014 to 5 decimal places

Exercise 10

1. Write the following to 3 significant figures.

a.	6962	b.	70.406	c.	0.012392
d.	0.010991	e.	45.607	f.	2345

2. Write 24.86582 to the following number of significant figures.

a.	6 sig fig	b.	4 sig fig	c.	2 sig fig.

3. Write the following to 3 decimal places.

a.	0.19387	b.	12.30972	c.	65.4555
d.	5.9997	e.	0.004977	f.	0.00321

4. Write 12.0938157 to the following number of decimal places.

a.	1 dp	b.	4 dp	c.	6 dp

CONVERT BETWEEN FRACTIONS AND DECIMALS

40. The Imperial system of measurement makes extensive use of fractions to express partial quantities. However, there are occasions where these partial quantities are expressed as decimals. For instance, the diameter of a fastener hole may be given as a fraction of an inch and the clearance to allow the fastener to pass through the hole given as thousandths of an inch expressed as a decimal to three places.

41. Choosing the correct size of drill bit to cut a clearance hole for a fastener is an example of an occasion where it will be necessary to add a fraction to a decimal. This will involve either converting a fraction into a decimal or converting a decimal into a fraction.

CONVERTING FRACTIONS TO DECIMALS

42. In the absence of an appropriate conversion chart the decimal equivalent of a fraction may be found in the following way:

Worked Example

Write $\frac{1}{4}$ as a decimal.

Obey the instruction given by the line and divide the numerator by the denominator.

i.e., $4\overline{)1.00}$

4 will not divide into 1 and so 0 is placed above it on the answer line and the 1 is carried into the first decimal place where it has a value of 10 tenths.

10 tenths divided by 4 is 2 (which is written in the first decimal place in the answer) with 2 remaining which is carried into the second decimal place where it will be worth 20 hundredths.

20 hundredths divided by 4 is 5 exactly which is entered in the second decimal place of the answer:

$$\begin{array}{r} 0.25 \\ 4\overline{)1.00} \end{array}$$

Note: When converting a proper fraction there will be no whole number before the decimal point in the answer.

Exercise 11

Convert the following fractions into decimals.

a. $\frac{3}{4}$ b. $\frac{3}{5}$ c. $\frac{7}{20}$ d. $\frac{5}{8}$ e. $\frac{3}{16}$

43. Sometimes it is necessary to limit the number of decimal places in the answers to this type of calculation. For example, a task given to an aircraft technician may involve the accurate measurement of very small distances. The level of accuracy normally expected is no finer than a few thousandths of an inch. Therefore, it is sensible to limit these numbers to three decimal places. More or less decimal places may be specified depending on the level of accuracy required.

Worked Example

i. $\frac{5}{16}$ to 2 decimal places.

Convert the fraction into a decimal.

$\frac{5}{16}$ = 0.3125

The aim here is to express $\frac{5}{16}$ to the nearest hundredth.

0.3125 is closer to 0.31 than it is to 0.32, so the numbers in the third and fourth decimal places are discarded.

Therefore $\frac{5}{16}$ to 2 decimal places = 0.31

ii.(2) $\frac{7}{16}$ to 2 decimal places.

Convert the fraction into a decimal.

$\frac{7}{16}$ = 0.4375

0.4375 is closer to 0.44 than it is to 0.43, so the number in the second decimal place is rounded up from 3 to 4.

Therefore $\frac{7}{16}$ to 2 decimal places = 0.44

44. If the number in either of these examples were to be expressed to 3 decimal places, it can be seen that the 5 in the fourth decimal place lies exactly in the middle of the limits of the third decimal place.

45. The rule for rounding is:

When it's five or more, increase the score. i.e., 3.46 = 3.5 to 1 dp

When it's four or below, let it go. i.e., 3.43 = 3.4 to 1 dp

Exercise 12

Convert the following fractions into decimals. Answers to be written to **3 decimal places.**

a. $\frac{1}{3}$ b. $\frac{2}{3}$ c. $\frac{5}{9}$ d. $\frac{1}{6}$ e. $\frac{5}{12}$

CONVERTING DECIMALS TO FRACTIONS

46. To find the decimal equivalent of a fraction, the figures in the decimal represents the numerator and this needs to be placed over a denominator which will be a power of 10.

i.e., 0.25 will become $\frac{25}{100}$ which cancels down to $\frac{1}{4}$ in its lowest terms.

Note: The quantity of 0s after the 1 of the denominator is the same as the quantity of decimal places in the original number.

Exercise 13

Find the fractional equivalent of the following decimals:

a. 0.35 b. 0.22 c. 0.44 d. 0.375

e. 0.625

Exercise 14

Calculate the following, giving answers as fractions and decimals:

a. $0.23 + \frac{3}{5}$ b. $0.351 + \frac{2}{5}$

c. $\frac{1}{4} + 1.28$

RATIOS AND PERCENTAGES

KEY LEARNING POINTS

KLP	Description
1.2.1.20	Convert a ratio into fractional form.
1.2.1.21	Express one quantity as a ratio of another.
1.2.1.22	Divide a given quantity into a stated ratio.
1.2.1.23	Express a number as a percentage of another number.
1.2.1.24	Calculate a percentage of a quantity
1.2.1.25	Calculate the percentage increase or decrease of a quantity.

RATIOS

47. Ratios are evident in all fields of engineering. For example: In a widely used concrete mix, cement, sand and gravel are in the ratio:

1 to 2 to 4 (often written 1 : 2 : 4)

This means that for every 1 part of cement, there are 2 of sand and 4 parts of gravel.

Example 1

4 kg of cement requires 8 kg of sand and 16 kg of gravel.

20 kg of cement requires 40 kg of sand and 80 kg of gravel.

The ratio of sand to cement is always 2 to 1 (twice as much sand as cement), gravel to cement is always 4 to 1.

48. A ratio compares quantities using the same units - here kilograms. So here 1 tonne of cement would need 2 tonnes of sand and 4 tonnes of gravel.

Example 2

In a scale drawing, using the **scale** 1:50,

1 cm on the drawing stands for 50 cm on the ground.

37 mm on the drawing stands for $37 \times 50 = 1850$ mm = 1.85 metres on the ground.

3 metres on the ground, though becomes $3 \div 50 = 0.06$ m or 60 mm on the drawing.

8.5 metres on the ground becomes $8.5 \div 50 = 0.17$ m = 170 mm on the drawing.

Example 3

In a dye works, the ratio of colour A to colour B is 5:4.

So, 5 g of A requires 4 g of B

1 g of A requires $^{4}/_{5}$ g of B

And 9 g of A requires $9 \times {}^{4}/_{5}$ of B

$= {}^{36}/_{5}$

$= 7^{1}/_{5} = 7.2$ g of B

DIVIDING AN AMOUNT INTO A GIVEN RATIO

49. This involves dividing the amount into equal parts and then distributing the parts according to the given ratio.

Example

Divide £36 in the ratio 5:4

The ratio 5:4 tells us we need 5 + 4 = 9 parts

£36 divided by 9 means that each part is £4

So the distribution is: 5 × £4 : 4 × £4

Or £20 : £16

Check by adding: £20 + £16 = £36

Exercise 15

1. Divide 24 kg in the ratio 3 : 5

2. Divide 63 ml in the ratio 2 : 5

3. Divide 80 m in the ratio 1 : 3 : 4

4. Divide £48 in the ratio 2 : 3 : 7

Exercise 16

1. The length of an aircraft and the length of its model are in the ratio 200 : 1. If the aircraft is 30 m long, how long is its model?

2. A drawing is to be made 1/5 full size. If a dimension of 740 mm is to be represented on the drawing, what size will it be?

3. Divide a line 140 mm long in the ratio 4 : 3.

4. A metal suitable for high speed bearings is made from tin and lead in the ratio 8.6 : 1.4. Find the mass of each metal in a sample of metal which has mass 15 kg?

5. A bar of metal 10.5 m long is to be cut into 3 parts in the ratio ½ : 1¾ : 3. Find the length of each part?

6. A mass is composed of 3 parts copper to 2 parts zinc. Find the mass of copper and zinc in a casting which has a mass of 80 kg?

7. How much copper is required to be melted with 40 kg zinc to make a brass so that the ratio of the copper to zinc is 7 : 3?

PERCENTAGES

Percentages

50. **Definition** - A percentage is a fraction whose denominator is 100.

Example: $3\%\ means\ \frac{3}{100}$

51. **Changing a Fraction to a Percentage**. - To change a fraction to a percentage, multiply by 100.

Example: $\frac{3}{5}\ as\ a\ percentage\ = \frac{3}{5} \times 100 = 60\%$

52. **Changing a Percentage to a Fraction**. - To change a percentage to a fraction, divide by 100. For example:

a. 8% as a fraction $= \frac{8}{100} = \frac{2}{25}$

b. 12½% as a fraction $= \frac{12.5}{100} = \frac{1}{8}$

53. **Changing a Percentage to a Decimal**. - To convert a percentage to a decimal, divide by 100. (Move decimal point two places to the left). For example:

a. 65% as a decimal = 65 ÷ 100 = 0.65

b. 32.5% as a decimal = 32.5 ÷ 100 = 0.325

54. **Changing a Decimal to a Percentage**. - To convert a decimal to a percentage, multiply by 100. (Move decimal point two places to the right). For example:

a. 0.21 as a percentage = 0.21 × 100 = 21%

b. 0.037 as a percentage = 0.037 × 100 = 3.7%

Exercise 17

1. Express as a proper fraction:

a.	0.6	b.	0.35	c.	0.48
d.	0.05	e.	0.325	f.	25%
g.	13%	h.	4½%		

2. Express as a percentage:

a.	0.43	b.	0.025	c.	1.25	d.	$^{2}/_{3}$
e.	$^{3}/_{7}$	f.	$^{1}/_{12}$	g.	$^{3}/_{8}$		

PERCENTAGES

55. **Percentage of a Quantity**. - To find the value of a percentage of a quantity, multiply by the percentage value divided by 100. For example:

a. 4% of 60 = 60 × 4 ÷ 100 = 240 ÷ 100 = 2.4

b. 3.5% of 1500 = 1500 × 3.5 ÷ 100 = 15 × 3.5 = 52.5

56. **One Quantity as a Percentage of Another**. - To express one quantity as a percentage of another, make a fraction of the two quantities and multiply by 100.

For example:

a. 12 as a percentage of 50 $= \frac{12}{50} \times 100 = 24\%$

b. 4 as a percentage of 60 $= \frac{4}{60} \times 100 = 6.67\%$

Exercise 18

1. Calculate:

a. 4% of 30 b. 0.8% of 360 c. 1.5% of 60

d. 120% of 75 e. 80% of 90

2. Express:

a. 30 as a percentage of 50 b. 24 as a percentage of 16

c. 0.5 as a percentage of 12.5 d. 3.2 as a percentage of 2.4

e. 0.08 as a percentage of 0.72

DIRECTED NUMBERS

KEY LEARNING POINTS

KLP	Description
1.2.1.26	Identify positive and negative numbers (directed numbers).
1.2.1.27	Add and subtract directed numbers.
1.2.1.28	Multiply and divide directed numbers.

DIRECTED NUMBERS

56. Numbers in arithmetic and symbols in algebra, can be either **positive** or **negative**. A positive number is indicated by placing a + sign in front of it. A negative number is indicated by placing a – sign in front of it. If the number is not identified by a sign, we must assume that it is positive. For example:

a. Positive 12 is written + 12 or just 12.

b. Negative 12 is written - 12.

57. **Brackets** can be used to distinguish between, for example, the positive number ‘plus three’ and the operation ‘add three’. Similarly, between the negative number ‘minus four’ and the operation ‘subtract four’. The rules for removing brackets containing directed numbers can be seen from the following.

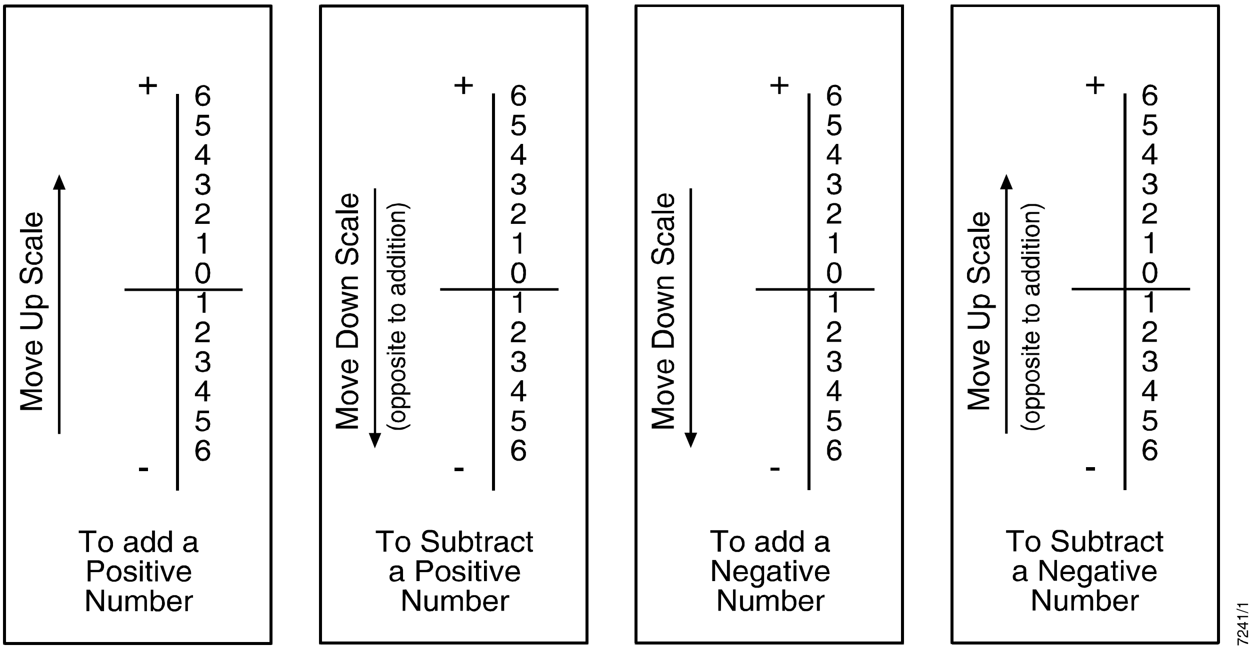
a. + (-2) = -2

b. - (+3) = -3

c. - (-4) = +4

d. + (+5) = +5

58. The logic behind the removal of the above brackets can be seen by considering the following number scales and remembering that subtraction is the opposite operation to addition.



For example:

a. $(-2) + 4 = +2$

b. $1 - 5 = -4$

c. $2 + (-4) \quad = 2 - 4 \quad = -2$

d. $(-2) - (-5) = -2 + 5 \quad = +3$

59. When evaluating expressions which contain positive and negative numbers you should combine the numbers starting from the beginning of the expression, using the above rules. For example:

a. $3 + 4 - 7 + 6 - 1$
$= 7 - 7 + 6 - 1$
$= 0 + 6 - 1$
$= 5$

b. $7 - 4 + 1 - 6 - 3$
$= 3 + 1 - 6 - 3$
$= 4 - 6 - 3$
$= -2 - 3$
$= -5$

Note however that we would not expect you to have to write down each of these lines of working.

60. When multiplying directed numbers **like** signs give **positive** answers, **unlike** signs give **negative** answers. For example:

a. $5 \times 2 = 10$

b. $5 \times (-3) = -15$

c. $(-5) \times 4 = -20$

d. $(-5) \times (-5) = 25$

61. Similarly, when dividing directed numbers, **like** signs give **positive** answers, **unlike** signs give **negative** answers. For example:

a. $25 \div (-5) = -5$

b. $(-20) \div (-5) = 4$

c. $(-15) \div 5 = -3$

d. $10 \div 2 = 5$

Exercise 19

1. Evaluate:

a. $8 - 5$

b. $7 + -4$

c. $6 - (-2)$

d. $-2 - 4$

e. $-3 + -6$

f. $-4 - (-7)$

g. $-3 + 3$

h. $0 - -4$

i. $2 + -2$

j. $8 + (-4)$

k $-7 + -3$

l. $2 - 4$

m. $-2 - -3$

n. $-6 - (-5)$

o. $-2 - 3$

p. $-(-3)$

2. Evaluate:

a. $13 - 7 - 8$

b. $14 - 15 - 3$

c. $3 - 5 - 2$

d. $14 - 52 - 3 + 7$

e. $93 - 102 - 76$

f. $19 + 7 - 6 - 4 - 7 + 3$

g. $14 - 7 + 1 - 14 - 1 + 7$

3. Evaluate:

a. $(-2) \times 3$

b. $(-3) \times (-4)$

c. $5 \times (-2)$

d. $(-5) \times 3$

e. $(-3) \times 5$

f. $(-2) \times (-1)$

g. $0 \times (-5)$

h. $6 \div (-2)$

i. $(-12) \div (-1)$

j. $(-8) \div 8$

k. $(-6) \div (-3)$

l. $\dfrac{(+12)}{(-3)}$

m. $\dfrac{(-2)}{(-4)}$

n. $0 \div (-5)$

INDICES AND STANDARD FORM

KEY LEARNING POINTS

KLP	Description
1.2.1.29	Define the terms base, index and power.
1.2.1.30	Multiply and divide powers to the same base.
1.2.1.31	Evaluate simple expressions involving powers.
1.2.1.32	Define the terms reciprocal and negative index.
1.2.1.33	Solve simple problems involving positive and negative indices.
1.2.1.34	Complete a table of powers from 10^6 to 10^{-6}.
1.2.1.35	State the meaning of a zero index.
1.2.1.36	Write numbers in standard form.
1.2.1.37	Calculate and approximate numbers in standard form.
1.2.1.38	Convert between multiples and sub-multiples of SI units using standard form (engineering form).

Index form and Indices

62. Numbers are often multiples of themselves.

For example:

100 = 10 x 10

36 = 6 x 6

125 = 5 x 5 x 5

64 = 2 x 2 x 2 x 2 x 2 x 2

63. The problem here is that it can be difficult to read long strings of numbers presented in this way. Because this situation arises quite often, a special notation has been developed, called the Index notation. For example:

10 x 10	is written as	10^2
6 x 6	is written as	6^2
5 x 5 x 5	is written as	5^3
2 x 2 x 2 x 2 x 2 x 2	is written as	2^6

Or more generally:

a x a x a … (n times)	is written as	a^n

64. The number such as “a” is called the base, and the number “n” is called the index (plural indices), which defines the number of bases that need to be multiplied together.

65. There is a big advantage in using index notation, because the indices are easy to manipulate using a few simple **Rules of Indices**

THE RULES OF INDICES

Rule 1	$a^m \times a^n$	$=$	a^{m+n}
Rule 2	$\frac{a^m}{a^n}$	$=$	a^{m-n}
Rule 3	$(a^m)^n$	$=$	$a^{m\times n}$
Rule 4	a^0	$=$	1
Rule 5	$\frac{1}{a^n}$	$=$	a^{-n}
Rule 6	$\sqrt[n]{a^m}$	$=$	$a^{\frac{m}{n}}$
Also	$\frac{\sqrt{a}}{\sqrt{b}}$	$=$	$\sqrt{\frac{a}{b}}$
And	$(ab)^x$	$=$	$a^x b^x$

66. The rules of indices **only apply if the bases are the same**:

For example: $2^2 \times 2^3 = 2^5$, but $5^2 \times 2^3$ cannot be calculated using these rules.

Rule 1

$$a^m \times a^n = a^{m+n}$$

67. If you are **multiplying** the bases, simply **add** the indices:

$$10^2 \times 10^3 = (10 \times 10) \times (10 \times 10 \times 10) = 10^5$$

$$5^2 \times 5^4 = 5^{2+4} = 5^6 = 15625$$

$$2^3 \times 2^5 = 2^{3+5} = 2^8 = 256$$

Rule 2

$$\frac{a^m}{a^n} = a^{m-n}$$

68. If you are **dividing** the bases, simply **subtract** the indices:

$$\frac{10^4}{10^2} = \frac{10 \times 10 \times \cancel{10} \times \cancel{10}}{\cancel{10} \times \cancel{10}} = 10^2 = 10^{4-2}$$

$$\frac{3^5}{3^2} = \frac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} = 3^3 = 3^{5-2}$$

Rule 3

$$(a^m)^n = a^{m \times n}$$

69. If a number in index form is raised to a power, we multiply the two indices.

$(10^2)^3 = 10^2 \times 10^2 \times 10^2$

$= (10 \times 10) \times (10 \times 10) \times (10 \times 10)$

$= 10^6$

70. If a number in index form is raised to a power, we multiply the two indices.

Rule 4

$$\mathbf{a^0 = 1}$$

71. Rule 4 implies that if any base is raised to the power 0 the answer will be 1. This can be illustrated by looking at the following.

$\frac{10^2}{10^2} = 1$ but from the second rule, $\frac{10^2}{10^2} = 10^{2-2} = 10^0$

72. So, $10^0 = 1$ and as this can be repeated for any base, the rule is that

$(\text{anything})^0 = 1$

Rule 5

$$a^{-n} = \frac{1}{a^n}$$

73. Again, to illustrate this rule consider the following:

$\frac{100}{10000} = \frac{1}{100} = \frac{1}{10^2}$ But from rule 2, $\frac{10^2}{10^4} = 10^{2-4} = 10^{-2}$

So $\frac{1}{10^{2}} = 10^{-2}$ and this can be repeated for any base or index.

74. If we consider the rule in fractional form, we notice a simple pattern occurring

$$\frac{a^{-2}}{1} = \frac{1}{a^{2}} \quad \textit{and} \quad \frac{a^{2}}{1} = \frac{1}{a^{-2}}$$

75. When the base value raised to a power transfers from being on the top as numerator to being on the bottom as a denominator (or vice versa) the power reverses its sign.

Exercise 20

Use the first 5 rules of indices to simplify the following:

1. $6^3 \times 6^5$
2. $7^6 \times 7^7$
3. $a^3 \times a^7$
4. $a^4 \times a^5 \times a^9$
5. $\frac{10^6}{10^4}$
6. $a^5 \div a^3$
7. $\frac{a^{10}}{b^7}$
8. $a^{19} \div a^{18}$
9. $(6^2)^5$
10. $(a^3)^3$
11. 1°
12. $(a^2 \times a^3)^3$
13. $\frac{1}{x^4}$
14. $\frac{1}{a^4 \times a^2}$

Check your answers on page 58

Fractional Indices.

76. The number $\sqrt{x}$ is referred to as "the positive square root of x"
It is defined as the positive number, which when multiplied by itself, equals x.

Thus, $\sqrt{9} = 3$, $\sqrt{36} = 6$, etc.

77. There are higher orders of roots, for example $\sqrt[3]{x}$ is the number when multiplied by itself 3 times, equals x, ($\sqrt[4]{x}$ 4 times etc.)

78. More generally, $\sqrt[n]{x}$ is the n^{th} root of x (i.e. the number which when multiplied by itself n times equals x)

79. The sign $\sqrt{\ }$ is called the radical and this method of presentation of the root is called the radical form of the root. As you will see there is another form of presentation, called the index form.

80. Most importantly, if there is no number (n) shown with the radical, it is implicit that it is 2. The 2 is not normally shown with the radical sign to signify a square root.

Rule 6

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

81. To illustrate this rule, consider the following:

$10 \times 10 = 100$ $\quad\therefore\ 10 = \sqrt{100}$

$\sqrt{100} \times \sqrt{100} = 100^1$

$100^x + 100^x = 1$ If you are multiplying the bases, just add the indices

$x + x = 1$ Therefore x must equal $\frac{1}{2}$ to satisfy the equation

$100^{½} + 100^{½} = 1$ Substitute this back into the original equation.

$\sqrt{100} = 100^{½}$

82. So, the reciprocal of the power of the root becomes the new index of the base.

STANDARD FORM

83. **Standard form** is a very useful method of representing both very large and very small numbers, that makes them easier to handle. The method uses powers of ten, which were used in the last section on indices.

Standard form is sometimes called **scientific notation**.

84. To express any number in standard form, it is written as a number between 1 and 10 and multiplied by a power of 10 to reflect the magnitude of the number.

Example.

The number 200 can be thought of as 2 x 100, or using index notation, 2×10^2

Likewise, $20 = 2 \times 10 = 2 \times 10^1$

85. Using this system, we can now write any decimal number quite neatly, as the following examples show:

234	$= 2.34 \times 10^2$	
23.4	$= 2.34 \times 10^1$	**Remember any base raised to the power of zero equals one**
2.34	$= 2.34 \times 10^0$	
0.234	$= 2.34 \times 10^{-1}$	
0.0234	$= 2.34 \times 10^{-2}$	

86. Numbers written in this way are said to be in **standard form**. As you can see there is only 1 digit to the left of the decimal point and that digit must only take the value of either 1 through to 9.

Examples.

1. Express 3500 in standard form.

   3500 is the same as 3.5 x 1000, or using the power of 10:

   3500 = $\mathbf{3.5 \times 10^3}$ in standard form.

2. Express 0.004 in standard form.

   0.004 is the same as $4 \times \frac{1}{1000}$, or using the power of 10:

   0.004 = $\mathbf{4 \times 10^{-3}}$ in standard form.

3. The planet Mercury is 58000000 kilometres from the Sun.

Express this distance in standard form.

58000000 is the same as 5.8 x 10 000 000, and since
10 000 000 = 10^7

58000000 = **5.8 x 10^7 i**n standard form.

87. A quick method for converting numbers to standard form is to think of any number, say n, as being n x 10^0. Then, move the decimal point until the number is a decimal between 1 and 10

88. Each move of the decimal point must be accompanied by a change in the index of the 10 to preserve the value of the number, following the rule:

Move the decimal point to the left, increase the power of 10

Move the decimal point to the right, decrease the power of 10

e.g.

623 000	= 623 000	x 100	= 6.23 x 105
0.000 002 67	= 0.000 002 67	x 100	= 2.67 x 10 -6

Exercise 21

1. Express the following in standard form:

a. 4600 b. 357 000 c. 0.00023

2. Light travels at 186 000 miles per second, express this value in standard form.

3. The top of Ben Nevis is 1343 metres above sea level, what is this height in standard form?

Check your answers on page 61

ARITHMETIC USING STANDARD FORM.

89. Expressing numbers in standard form also allows us to carry out basic arithmetic on very large and/or very small numbers, without having to deal with lots of zeros. For example, if you wanted to find the result of:

550 000 x 0.000 02

90. If we did this calculation using long multiplication, it would be difficult to know where to put the decimal point; even some calculators may have trouble with it. But if we convert each number to standard form, the calculation becomes easier to handle.

i.e., because

550 000 000 = 5.5×10^8 and 0.000 02 = 2×10^{-5}

So, we can say that:

550 000 000 x 0.000 02 = $5.5 \times 10^8 \times 2 \times 10^{-5}$

This could be written as:

$(5.5 \times 2) \times (10^8 \times 10^{-5})$

Now:

$5.5 \times 2 = 11$ and $10^8 \times 10^{-5} = 10^3$

Remember the rules of indices

So:

$5.5 \times 2 \times 10^8 \times 10^{-5} = 11 \times 10^3$

Hence:

550 000 000 x 0.000 02 = $11 \times 10^3 = 1.1 \times 10^4$

Exercise 22

Without using a calculator, simplify the following leaving your answer in standard form:

1. $\frac{68000}{340}$

2 12500×30000

3. 200000×0.0185

4. $\frac{1800}{0.036}$

5 $\frac{5\,100\,000}{0.0017}$

6 $225000 \times 400000 \div 1500000$

APPROXIMATION IN STANDARD FORM.

91. Approximation can be used when a quick estimate is required to check if a reading or value is of the correct magnitude.

Example 1:

274.6×4174

First express the numbers in standard form, then round off the numbers to the nearest whole number and simplify.

$2.746 \times 10^2 \times 4.174 \times 10^3$

$3 \times 10^2 \times 4 \times 10^3 = 3 \times 4 \times 10^{2+3} = 12 \times 10^5 = 1.2 \times 10^6$

Example 2:

568.9×0.0002883

First express the numbers in standard form, then round off the numbers to the nearest whole number and simplify.

$5.689 \times 10^2 \times 2.883 \times 10^4$

$6 \times 10^2 \times 3 \times 10^4 = 6 \times 3 \times 10^{2+4} = 18 \times 10^6 = 1.8 \times 10^7$

Example 3:

$\frac{134200}{0.0046} =$

First express the numbers in standard form, then round off the numbers to the nearest whole number and simplify.

$\frac{1.342 \times 10^5}{4.6 \times 10^{-3}} = \frac{1 \times 10^5}{5 \times 10^{-3}}$

Notice in this case it is better to manipulate the indices to make the numbers easier to manage.

$= \frac{1 \times 10^5}{5 \times 10^{-3}} = \frac{10 \times 10^4}{5 \times 10^{-3}} = 2 \times 10^{4 - -3} = 2 \times 10^{4+3} = 2 \times 10^7$

Exercise 23

Without using a calculator, approximate the following, leaving your answer in standard form:

1. $\dfrac{89\,420}{2670}$

2. $12\,530 \times 644\,329$

3. $192\,300 \times 0.0192$

4. $\dfrac{1800}{0.036}$

5 $\dfrac{5\,310\,048}{0.001\,66}$

6. $236\,220 \times 399\,070 \div 1754\,711$

SI UNITS OF MEASUREMENT

Physical Units

92. Physical science looks at the way in which ‘physical’ systems respond to various kinds of change. For example, this could be response to a force or to pressure or an increase in temperature. In order to quantify any change or response an agreed system of ‘measurement’ has to be used so that all Scientists, Engineers and Technicians understand each other’s figures. Various agreed systems exist and aircraft technicians need to be aware of two: The Imperial system and the Système Internationale d’Unités or ‘SI’ system.

93. All changes in physical systems can be reduced to changes in just seven ‘base’ units. These can be thought of as building blocks, from which any other type of measurement can be built. They are:

- Physical length
- Mass
- Time
- Electrical current
- Temperature
- Amount of substance
- Luminosity

94. In Aerospace engineering we are interested in the first five. In the SI system the units of measure for these five are:

Unit	**Measure**	**Symbol**	
Physical length	Metre	m	
Mass	Kilogram	kg	
Time	Second	s	
Electrical current	Ampere	A	
Temperature	Kelvin	K	(Note: do not use a ° symbol)

95. The SI base units can be combined as required to measure any non-base quantity. For example, an area is not a base unit but can easily be measured as length x width for a rectangle which is a length x a length. This gives us SI derived units of:

Area = length x length
Area = m x m = m^2 (or metres squared)

Volume = length x length x length
Volume = m x m x m = m^3 (or metres cubed)

96. Measuring the speed of an object is defined as the distance travelled divided by time taken. This gives compound SI derived units of:

Speed = distance travelled / time taken
Speed = m/s (or ms^{-1}) i.e. metres per second

97. Acceleration is similarly the change in speed divided by the time taken:

Acceleration = change in speed / time taken

Acceleration = (m/s)/s = $\frac{m}{s} \cdot \frac{1}{s} = m/s^2$ (metres per second squared)

Force is defined as a push or a pull, which moves or tries to move an object. Force can be calculated as mass x acceleration for simple objects.

Force = mass x acceleration (*explained in Science*)
Force = kg x m/s^2 (Kilogram metre per second squared)

98. The unit of Force is used so frequently and complex that a shorthand method is adopted to ease confusion. Force is a compound SI derived unit however a shorthand method to refer to force is the Newton and it is more commonly used in its named unit form.

Named Unit		Compound SI derived Unit
1 Newton	=	1 $kg \cdot m \cdot s^{-2}$

Further examples of compound SI derived units:

Pressure is defined as force acting per unit area:

Pressure = Force / area
Pressure = N/m^2 (Newtons per metre squared)

Torque is the turning force multiplied by perpendicular distance to the pivot.

Torque = Force × distance
Torque = Nm (Newton metres of torque)

Work is found by the force acting multiplied by the distance moved.

Work = Force x distance (NB: distance is not perpendicular this time)
Work = Nm (To distinguish from 'torque' we write J for Joules)
Work = J (Joules)

Density is defined as the mass per unit volume of a substance.

Density = mass / Volume
Density = kg/m^3 (Kilograms per metre cubed)

Note. A quantity raised to a power appears on the bottom of an expression as in kg/m^3, it can also be expressed as a negative power on the top of the expression from indices law 5; therefore, the unit of density may also be expressed as kgm^{-3}

Engineering Prefixes

99. When writing very large or very small measurements, engineering prefixes are used to prevent having to record too many decimal places or trailing zeros.

Prefix	Symbol	Multiplying factor	Prefix	Symbol	Multiplying factor
tera	T	1 000 000 000 000 $=10^{12}$	milli	m	0.001 = 10^{-3}
giga	G	1 000 000 000 = 10^{9}	micro	μ	0.000 001 = 10^{-6}
mega	M	1 000 000 = 10^{6}	nano	n	0.000 000 001 = 10^{-9}
kilo	k	1 000 = 10^{3}	pico	p	0.000 000 000 001 = 10^{-12}

Examples:

a. 210 GN/m^2, means 210 x 10^9 N/m^2.

Written in full: 210 000 000 000 N/m^2

b. 110 μm, means 110 x 10^{-6} m.

Written in full: 0.000 110 m

100. It is usual to combine these specified engineering prefixes with SI units.

Examples include:

MN (Mega Newton),

GW (Giga Watt),

μs (microsecond),

kV (kilo volt), mm (milli metre),

pF (pico Farad)

101. Further examples are listed in the table below:

Power of 10	Name	Symbol	Example of use
10^{-12} (0.000 000 000 001)	Pico	p	A capacitor has value 12 pF
10^{-9} (0.000 000 001)	Nano	n	The wavelength of this light is 400 nm
10^{-6} (0.000 001)	Micro	μ	The circuit took 20 μs to switch on.
10^{-3} (0.001)	Milli	m	The wire was 0.46 mm in diameter
10^{3} (1 000)	Kilo	k	The car had a mass of 1500 kg
10^{6} (1 000 000)	Mega	M	The fuel contains 1 MJ of energy
10^{9} (1 000 000 000)	Giga	G	A hydraulic pump produced 3 GN/m^2
10^{12} (1 000 000 000 000)	Tera	T	The power station produced 1.2 TW

102. There are exceptions, such as the kilogram, whose base unit is already kilo. It would be confusing to say a ‘kilo-kilogram’ to represent 1 000 kg, so instead the term ‘tonne’ (t) is used to represent 1 000 kg.

103. Other units technically do not fall into the SI system. These include: the gram (g), milligram (mg), microgram (μg), ‘kilometre per hour’ (kph), centimetres (cm, ‘centi’ is not a preferred prefix), revolutions per minute (rpm).

Engineering Form (Preferred Standard Form)

104. In Engineering Form the power of 10 is restricted to be a multiple of 3, which aligns with the SI prefixes (G, M, k, m, μ, n, p etc.). The number accompanying the power of 10 can now take any value between 1 and <1000. Most scientific calculators have a function to convert answers to Engineering Form. For example:

580000000 $= 5.8 \times 10^{8}$ in standard form.

$= 580 \times 10^{6}$ in engineering form.

0.000036 $= 3.6 \times 10^{-5}$ in standard form.

$= 36 \times 10^{-6}$ in engineering form.

2400 $= 2.4 \times 10^{3}$ in standard form.

$= 2.4 \times 10^{3}$ in engineering form.

Exercise 24

1. Express the following in engineering form:

a. 4 600 b. 357 000 000 c. 0.000 23

2. Light travels at 186 000 miles per second. Express this value in engineering form.

3. The top of Ben Nevis is 1343 m above sea level. What is this height in engineering form?

ANSWERS

Exercise 1

1	a.	$^{19}/_{21}$	b.	$1^{19}/_{30}$	c.	$^{47}/_{60}$
	d.	$1^{3}/_{4}$	e.	$7^{16}/_{21}$	f.	$5^{13}/_{20}$
2.	a.	$^{1}/_{6}$	b.	$^{11}/_{35}$	c.	$2^{1}/_{6}$
	d.	$4^{11}/_{15}$	e.	$3^{5}/_{12}$	f.	$^{19}/_{24}$
3.	a.	$^{11}/_{20}$	b.	$3^{1}/_{12}$	c.	$3^{7}/_{24}$
	d.	$6^{2}/_{9}$	e.	$1^{23}/_{24}$	f.	$^{1}/_{30}$

Exercise 2

1.	a.	$^{19}/_{8}$	b.	$^{27}/_{4}$	c.	$^{15}/_{8}$	d.	$^{51}/_{16}$
2.	a.	$3^{1}/_{4}$	b.	$3^{1}/_{2}$	c.	$5^{3}/_{8}$	d.	$2^{7}/_{16}$
3.	a.	$^{3}/_{8}$	b.	¾	c.	$^{1}/_{9}$	d.	$^{1}/_{6}$
	e.	¾	f.	$2^{1}/_{2}$	g.	$^{1}/_{7}$	h.	$^{2}/_{15}$
	j.	$16^{1}/_{2}$						
4.	a.	$^{20}/_{21}$	b	$^{5}/_{8}$	c.	$1^{1}/_{2}$		
5.	a.	$3^{1}/_{5}$	b.	$3^{1}/_{9}$	c.	5	d.	$1^{3}/_{4}$
	e.	$8^{4}/_{7}$						

Exercise 3

1.	11	2.	6	3.	17	4.	32	5.	2
6.	21	7.	5	8.	11	9.	75	10.	19
11.	68	12.	-40	13.	-28	14.	-9	15.	82
16.	-14	17.	18	18.	33	19.	-1106		
20.	-18	21.	11	22.	1	23.	51	24.	8
25.	-81	26.	1						

Exercise 4

1.	252.627	2.	1.014	3.	58.641
4.	7.77	5.	192.64	6.	0.145

Exercise 5

1.	34687.4	2.	4.6	3.	2625
4.	1305.4	5.	2.78		

Exercise 6

1.	60.762	2.	0.3072	3.	27.825
4.	0.22176	5.	10.941048		

Exercise 7

1.	0.000404	2.	0.452625	3.	1.3
4.	4.92874	5.	0.003015		

Exercise 8

1.	123.218	2.	0.05075	3.	2.105
4.	0.72	5.	7.525		

Exercise 9

1.	232.18	2.	14.2	3.	1.05
4.	96	5.	376.25		

Exercise 10

1.	a.	6960	b.	70.4	c.	0.0124
	d.	0.0110	e.	45.6	f.	2350
2.	a.	24.8658	b.	24.87	c.	25
3.	a.	0.194	b.	12.310	c.	65.456
	d.	6.000	e.	0.005	f.	0.003
4.	a.	12.1	b.	12.0938	c.	12.093816

Exercise 11

a.	0.75	b.	0.6	c.	0.35
d.	0.625	e.	0.1875		

Exercise 12

a.	0.333	b.	0.667	c.	0.556
d.	0.167	e.	0.417		

Exercise 13

a	$^{7}/_{20}$	b,	$^{11}/_{50}$	c.	$^{11}/_{25}$
d.	$^{3}/_{8}$	e.	$^{5}/_{8}$		

Exercise 14

a	$\frac{83}{100} = 0.83$	b.	$\frac{751}{1000} = 0.751$	c.	$\frac{153}{100} = 1.53$

Exercise 15

1. 9 kg, 15 kg
2. 18 ml, 45 ml
3. 10 m, 30 m, 40 m
4. £8, £12, £28

Exercise 16

1. 0.15 m
2. 148 mm
3. 80 mm and 60 mm
4. 12.9 kg tin and 2.1 kg lead
5. 1 m, 3½ m, 6 m
6. 48 kg copper, 32 kg zinc
7. $93^{1}/_{3}$

Exercise 17

1	a	$^{3}/_{5}$	b	$^{7}/_{20}$	c.	$^{12}/_{25}$
	d.	$^{1}/_{20}$	e.	$^{13}/_{40}$	f.	¼
	g.	$^{13}/_{100}$	h.	$^{9}/_{200}$		
2	a	43%	b.	2.5%	c.	125%
	d.	$66.^{2}/_{3}\%$	e.	$42^{6}/_{7}\%$	f.	$8^{1}/_{3}\%$
	g.	$37^{1}/_{2}\%$				

Exercise 18

1.	a.	$1^{1}/_{5}$ or 1.2	b.	2.88	c.	0.9
	d.	90	e.	72		
2.	a.	60%	b.	150%	c.	4%
	d.	133.33…% (or $133^{1}/_{3}\%$)	e.	11.11…% (or $11^{1}/_{9}\%$)		

Exercise 19

1.	a.	3	b.	3	c.	8	d.	-6
	e.	-9	f.	3	g.	0	h.	4
	i.	0	j.	4	k.	-10	l.	-2
	m.	1	n.	-1	o.	-5		
2.	a.	-2	b.	-4	c.	-4	d.	-34
	e.	-85	f.	12	g.	0		
3.	a.	-6	b.	12	c.	-10	d.	-15
	e.	-15	f.	2	g.	0	h.	-3
	i.	12	j.	-1	k.	2	l.	-4
	m.	½	n.	0	p.	3		

Exercise 20

1.	6^8	2.	7^{13}	3.	a^{10}	4.	a^{18}
5	10^2	6.	a^2	7.	$\frac{a^{10}}{b^7}$	8.	a
9.	6^{10}	10.	a^9	11.	1	12.	a^{15}
13.	x^{-4}	14.	a^{-6}				

Exercise 21

1. a. 4.6×10^3 b. 3.57×10^5 c. 2.3×10^{-4}

2. 1.86×10^5

3. 1.343×10^3

Exercise 22

1.	2×10^2	2.	3.75×10^8	3.	3.7×10^3
4.	5×10^4	5.	3×10^9	6.	6×10^4

Exercise 23

1.	3×10^1	2.	6×10^9	3.	4×10^3
4.	5×10^4	5.	2.5×10^9	6.	4×10^4

Exercise 24

1 a. 4.6×10^3 b. 357×10^6 c. 230×10^{-6}

2. 186×10^3

3. 1.343×10^3

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