Defence School of
Aeronautical Engineering

No.2 School of Technical Training

Academic Principles Organisation

TG5 MATHEMATICS

2208A, 2209A, 2211A

Book 3

Graphs & Statistics

GRAPHS AND STATISTICS

KEY LEARNING POINTS

KLP	Description
1.2.3.1	Plot graphs and interpret their meaning from given data.
1.2.3.2	Sketch and interpret graphs of the linear function $y = mx + c$.
1.2.3.3	Analyse data to solve problems using statistical techniques.

GRAPHS

1.2.3.1 Plot graphs and interpret their meaning from given data.

1. Service publications and reports make extensive use of pictorial illustrations to assist the reader in gaining a greater depth of understanding of their content. The most common form of illustration that is used is the **graph**.

AXES

2. The first step in drawing a graph is to draw two lines at right angles to each other, as shown below. These lines are called the **axes of reference**. The vertical axis is usually called the **y-axis** and the horizontal one is the **x-axis**. The point where the two axes cross is called the **origin** of the graph.

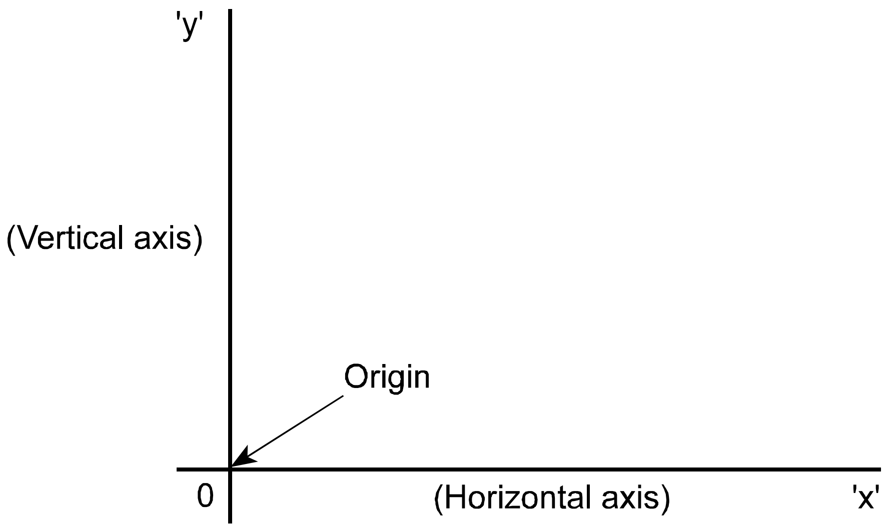

Figure 1

3. When plotting a graph, it is important to ensure that the following points are covered.

a. Draw the graph as large as the paper will conveniently allow.

b. Label each axis clearly.

c. Indicate the scale of each axis.

CO-ORDINATES (CARTESIAN/RECTANGULAR)

4. An ordered pair **of co-ordinates are used** to identify points on a graph. The system is the same as that used in map reading to locate or identify a particular point on the ground. In the graph below values of **y** are to be plotted against values of **x**. The point **P** has been plotted so that x = 4 and y = 6. The values of 4 and 6 are the **co-ordinates** of **P** and are written in brackets separated by a comma i.e. (4, 6).

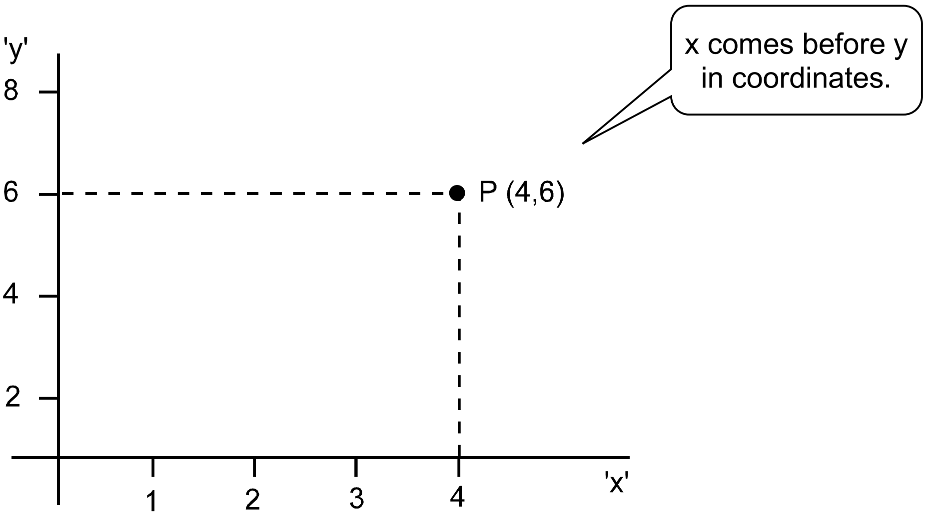

Figure 2

Example:

Give the co-ordinates of the point B and C in the graph below.

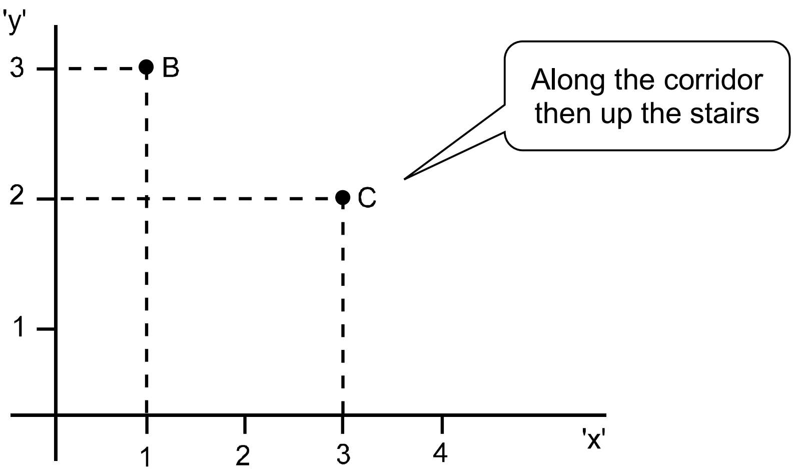

Figure 3

B is 1 along the x-axis and 3 up the y-axis and has coordinates (1, 3).

C is 3 along the x-axis and 2 up the axis and has coordinates (3,2)

GRAPHS OF EXPERIMENTAL DATA

5. Every graph shows a relationship between two sets of numbers. Graphs are commonly used to illustrate the results of an experiment or test. When drawing graphs of experimental data, the quantity that is being varied by the operator is called the **independent variable** and is usually put on the horizontal axis. The quantity that is measured is called the **response** or **dependent variable** and is usually put on the vertical axis.

Example:

The following table gives the amount of stretch measured in a control cable run under a range of increasing loads. Plot a graph to represent this information.

Load (kg)	10	20	30	40	50
Stretch (mm)	3.5	7	10.5	14	17.5

Table 1

Here the stretch that occurs in the cable is dependent upon the load that is being applied, so the load will appear on the horizontal axis as it is the independent variable.

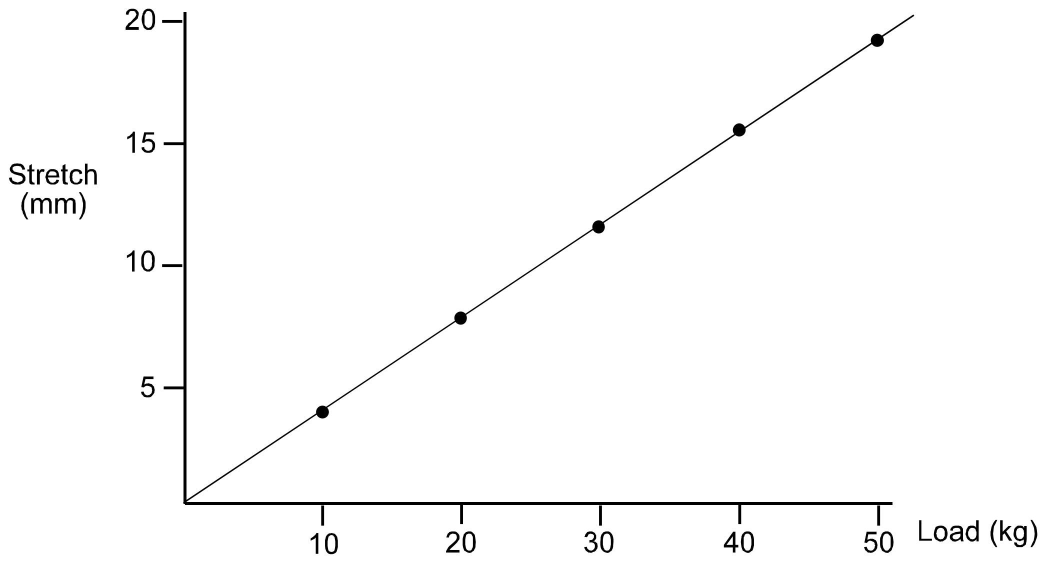

Figure 4

NEGATIVE VALUES

6. When experimental data includes negative values, the straightforward L axes arrangement can no longer be used. Other arrangements commonly include:

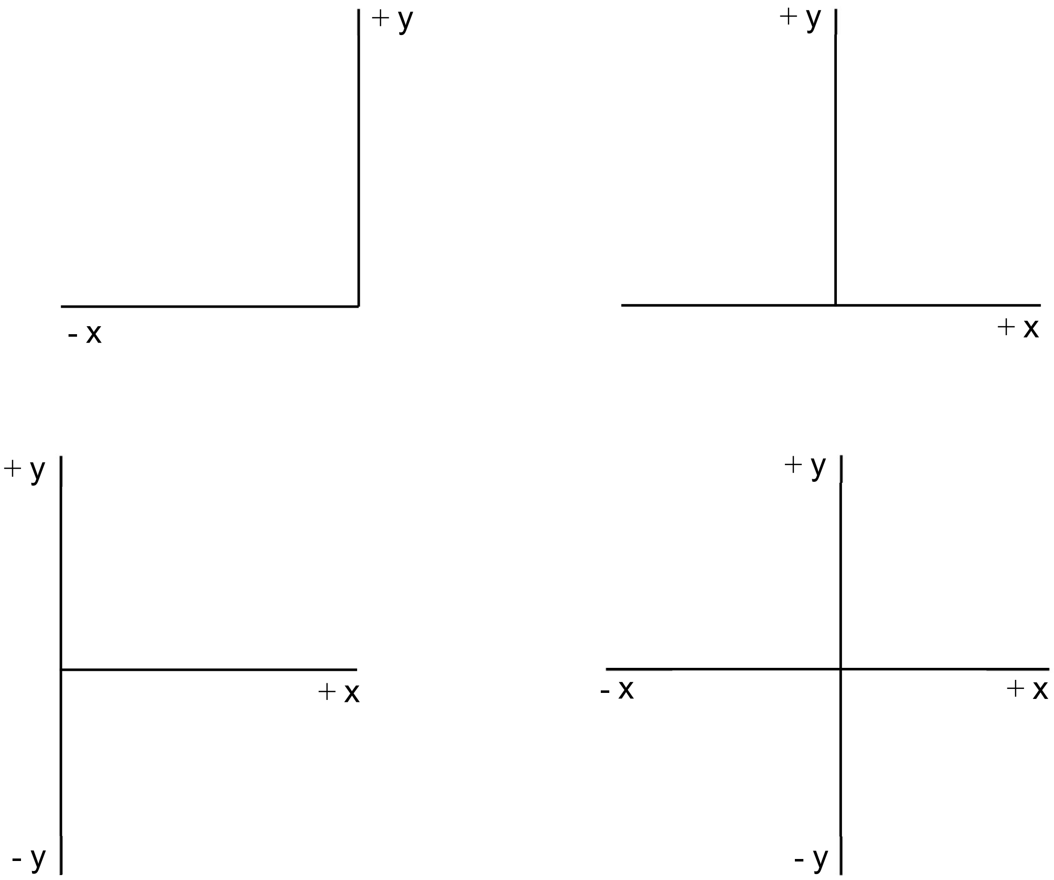

Figure 5

EXERCISE 1

The values below give the relationship between force applied and resultant acceleration, in an experiment carried out on a trolley rolling along an inclined plane.

Force in Newtons	-3	-2	-1	0	1	2	3
Acceleration in m/s^2	-5	-2	1	4	7	10	13

Table 2

Plot these results in a suitable graph.

Answer on p32

LINE OF BEST FIT

7. When the **results** obtained in a trial are plotted, the points are rarely exactly on a straight line or curve, so a **line of best fit** is drawn. The difference between an individual data point and the line is called **the residual.** The line of best fit minimises the square of the residuals and it can be found by using an application such as Excel or by eye. The line of best fit will tend to average out any errors the results.

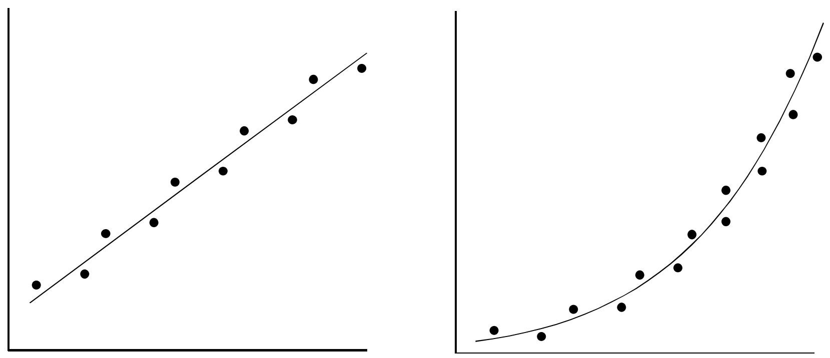

Figure 6

INTERPOLATION

8. When readings are taken from a graph from between the plotted points making use of the best line, this is known as **interpolation** of the graph. When the best line method is used, all subsequent readings must come from the line, even if they coincide with the observed values. This has the effect of averaging out errors in the observations

EXTRAPOLATION

9. When readings are estimated from points outside of the bounds of the data, this is known as **extrapolation**. Extrapolating data is generally higher risk than interpolation. If a relationship appears to be linear within the bounds of the known data, it does not mean that it will continue in that way. The diagram below shows frictional force plotted against pulling force. Notice that it only stays linear up to the point of impending movement.

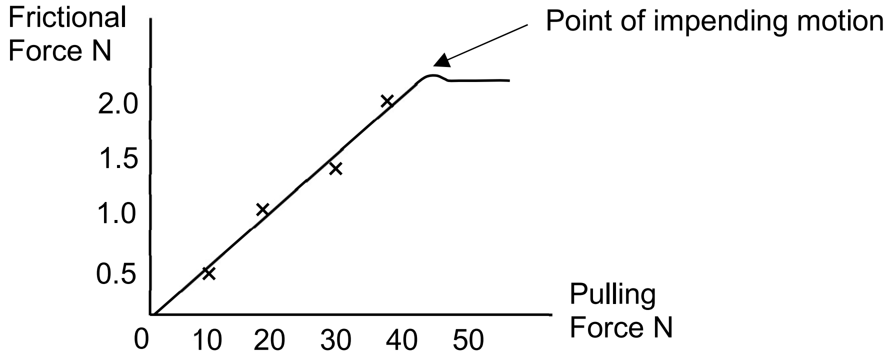

Figure 7

EXERCISE 2

1. The table below shows the comparative values of two temperature scales, Celsius (°C) and Fahrenheit (°F).

°C	0	3	9	12	15
°F	32	37	48	54	59

Table 3

Draw a line graph to represent this data and determine the following:

a. The equivalent temperature (°F) at 6°C.

b. The equivalent temperature (°F) at 18°C.

2. The following table shows how the resistance of a certain wire varies with temperature:

Temperature: T (°C)	0	10	30	50	70	90
Resistance: R (ohms)	150	156	168	180	192	204

Table 4

Plot a graph of R against T and from the graph determine:

a. The temperature when the resistance is 190 ohms.

b. The resistance when the temperature is 100°C.

3. The following table gives the results of an experiment in which the current was measured for various values of applied potential difference:

Potential Difference PD (volts)	1	2	3	5	6	7
Current 1 (mA)	6.3	8	9.3	12.6	14	16

Table 5

Plot the results and determine from the graph:

a. The current when the PD is 4 volts

b. The PD required to give a current of 8.5 mA.

Answers on p32

GRAPHS OF FUNCTIONS

10. A function is a mapping of one variable to another, following a **rule** (or **law**), for example:

$$y = 3x + 2$$

Straight Line

11. The function $y = 3x + 2$ will produce a straight line when plotted, because both x and y are of order 1 (i.e., the highest power is 1).

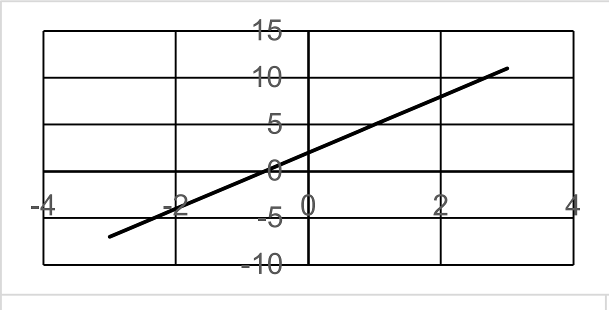

Figure 8

Quadratic

12. The function $s = 3t^2 + 1.5t – 2$ will ***not*** produce a straight line when plotted, because the variable t has order 2 (i.e., the highest power is 2).

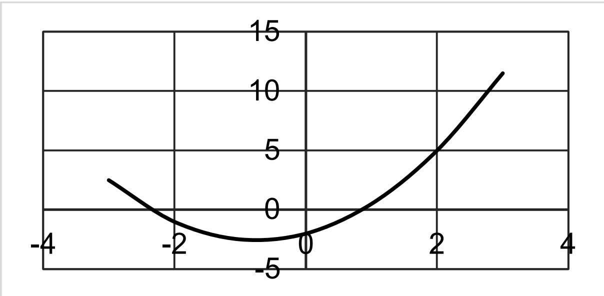

Figure 9

Inverse

13. The function $y = \frac{2}{x} + 3$ will ***not*** produce a straight line when plotted, because the variable x has order (index) = -1 (i.e., inverse function).

This could also be: $y = 2x^{-1} + 3$.

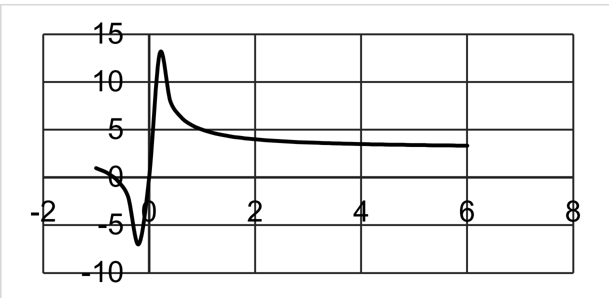

Figure 10

Straight Line Graph

14. To produce a graph of given function, for example: y = 5x + 3, start by compiling a table of values, to cover the range required, say from x = -3 to +3

x	**-3**	**0**	**3**
5x	-15	0	15
+3	+3	+3	+3
y	-12	+3	18

Table 6

15. It may be possible to get away with calculating and plotting only 2 points, as the graph will be a straight line, but it is good practice to plot at least 3 points, to confirm the arithmetic.

16. Now on the graph paper, construct suitable axes, plot the points and join up the points with a straight line. If the points do not line up, then something is wrong and the calculations need to be checked.

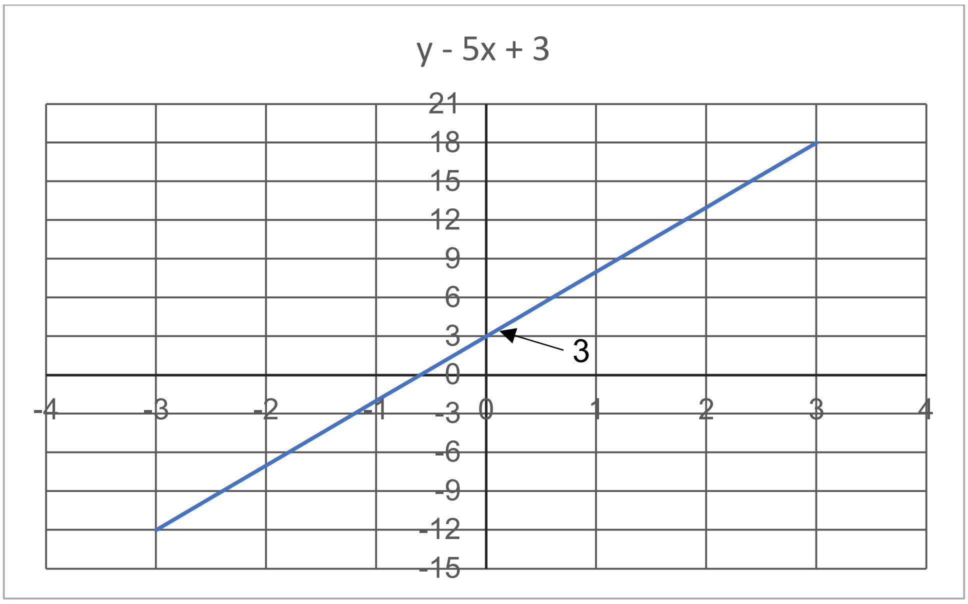

Figure 11

EXERCISE 3

1. Compile a table and draw the graph for the function y = 2x + 4, covering values of x from -3 to + 3. Clearly indicate the y intercept.

2. Compile a table and draw the graph for the function y = -x + 1, covering values of x from -3 to + 3. Clearly indicate the y intercept.

Answer on page 33

FUNCTIONS FROM GRAPHS

17. From the graph of a function, it is possible to use it to find the equation of the line.

18. Providing that the graph is a straight line, the equation will be of the form $y = mx + c$, where y and x are the two variables, m is the coefficient and c is the constant. The task is to find the coefficient m and the constant c from the graph.

19. The coefficient m is the slope, or **gradient** of the line. This is the change in the value of y divided by the corresponding change in the value of x, (i.e. **rise** over **run**) between any two points on the line.

Example:

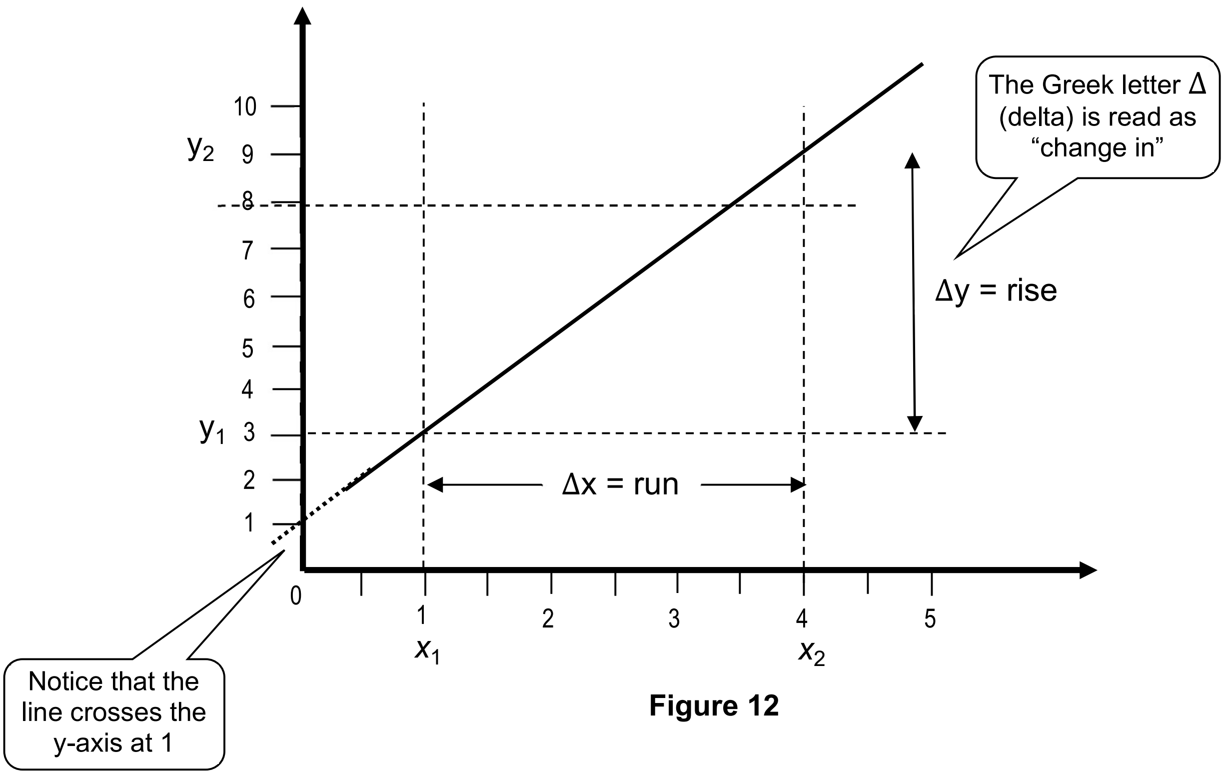

Figure 12

20. First, choose any two convenient points on the graph.

For example: (1,3) and (4,9).

Then the gradient is:

$$\text{Gradient } (m) = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-3}{4-1} = \frac{6}{3} = 2$$

So, the value of m in the formula is 2.

21. Note that the rise ($y_2 - y_1$) and run ($x_2 - x_1$) is measured in terms of the scale on the axes and not the number of squares on the graph paper.

22. Note also that if the graph slopes **upwards** (i.e., bottom left to top right, as in figure 8) then the value of m is positive, but if the slope is **downwards**, then m is negative. (If the line is horizontal, then m = 0 and if vertical, m is indeterminate).

23. The constant **c** is the value of y when x = 0. This is the **y-intercept** of the line and can simply be read from the graph. In the case of the example shown in Fig 8, the value of the y intercept is 1 and so c = 1

Thus, the full equation of the line on the graph shown in fig 8 is: $y = 2x + 1$

EXERCISE 4

For the four straight line graphs shown below, find the equations of the lines.

1.

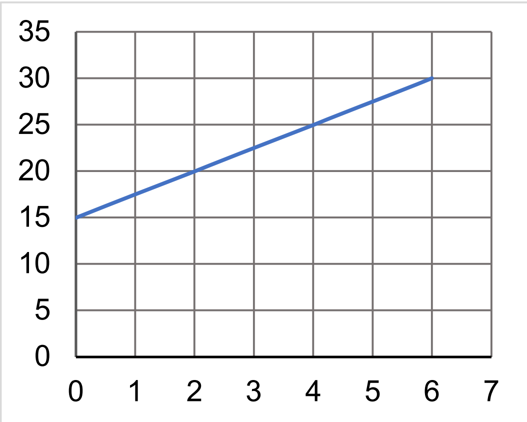

Figure 13

2.

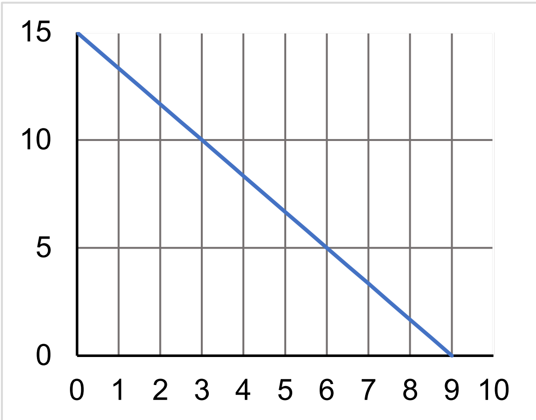

Figure 14

3.

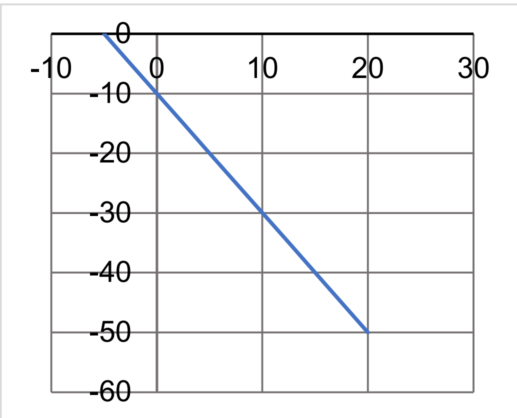

Figure 15

4.

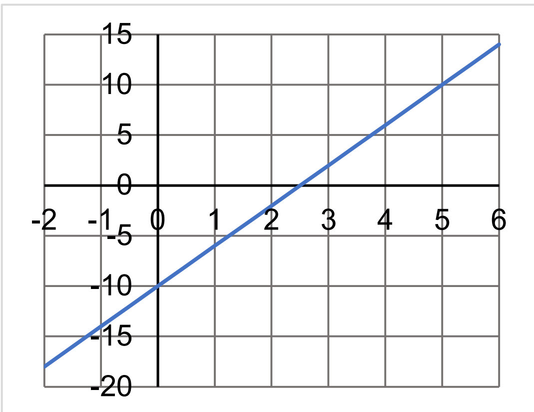

Figure 16

Answers on page 33

EQUATIONS WITHOUT GRAPHS

24. Provided the graph of a function is a straight line, it is possible to find the equation of the line, given two points on the line, without drawing the graph.

25. For example, consider the straight line which passes through the Cartesian points (1,5) and (4,14). As the line is straight, that the equation will be of the form y=mx + c

Using the formula for the gradient:

Then the gradient is:

$$\text{Gradient (m)} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - 5}{4 - 1} = \frac{9}{3} = 3$$

So far, the equation is:

$y = 3x + c$

26. To find the value of c, substitute a value for x and a value for y that is known from one of the points, say (1 , 5).

$5 = 3(1) + c$

27. This gives an equation with just one unknown. Transpose to make c the subject:

$5 = 3(1) + c$

$5 – 3(1) = c$

$c = 2$

28. The equation of the straight line which passes through points (1,5) and (4,14) is:

$y = 3x + c$

EXERCISE 5

Find the equations of the straight lines which pass through the following pairs of Cartesian points.

1. (0,4) and (2,2)

2. (3,5) and (5,3)

Answers on page 33

EXERCISE 6 CONSOLIDATION

1. Compile a table and draw a graph of the function y = -2x + 1, covering values of x from -3 to + 3. Clearly indicate the y intercept.

2. Compile a table and draw a graph of the function y = 3x – 2, covering values of x from -3 to + 3. Clearly indicate the y intercept.

3. Find the equation for the line of the following graph:

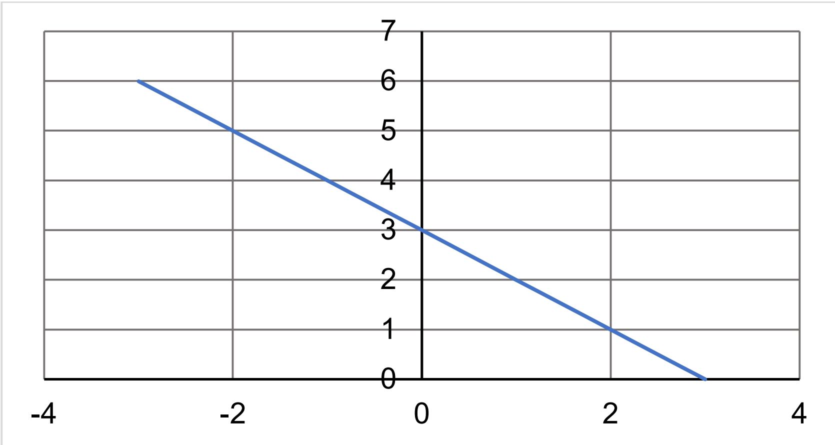

Figure 17

4. Find the equation for the line of the following graph:

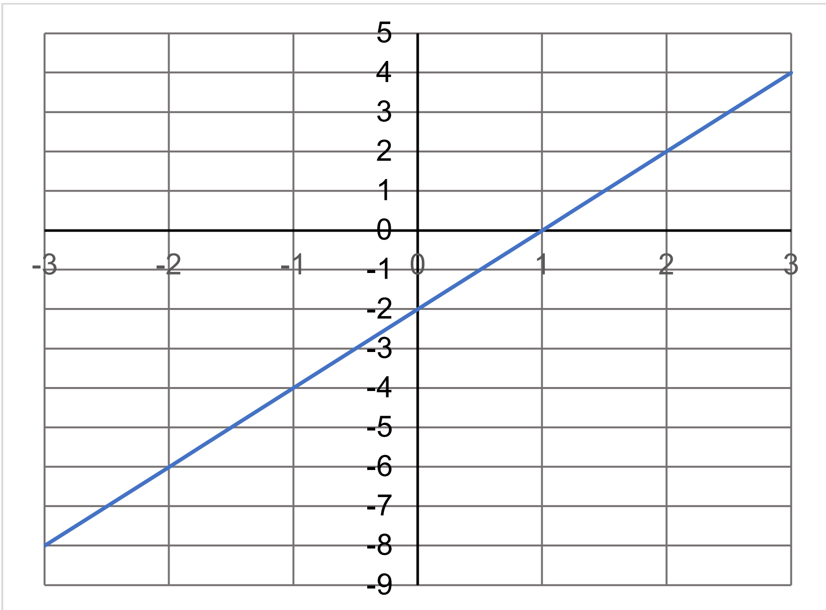

Figure 18

Answer on p34

STATISTICS

KEY LEARNING POINTS

KLP	Description
1.2.3.1	Analyse data to solve problems using statistical Techniques.

STATISTICS

1.2.3.3 Analyse data to solve problems using statistical techniques.

TERMINOLOGY

29. **Data** - data is the information which is obtained during a statistical investigation.

30. **Variables** – something that can change from one item to the next is called a variable. For example: the colour of the cars in a car park (qualitative variable – non-numerical) or the height of the trees in a park (quantitative variable – numerical). There are two types of numerical or quantitative variable:

- **Continuous**. - A continuous variable can take any value within a given range. For example: the height of children playing in a game of football. **Note: Continuous variables are rounded off to a certain degree of accuracy. E.g., 1.83 m.**

- **Discrete**. - A discrete variable increases in steps or whole numbers. For example: The number of aircraft in a hangar, or shoe sizes or the colour of cars. **Note: In the case of shoe sizes the variables can increase in half increments.**

31. **Population** – The term population refers to everything in the category under investigation. For example, the people who work in an office block, or the number of cars on the road in the UK.

32. **Surveys** – Surveys are carried out to gather information. The most accurate way to carry out a survey is to undertake a census. This involves questioning every member of a population and generally requires a great deal of time and effort. To overcome this problem, statisticians usually examine a small part of a population by taking a sample. Though a sample can provide very useful information about a population, care must be taken to ensure that the information collected is representative.

33. **Hypothesis Testing** – A statistical survey should have a purpose. It may be set to establish the quality or accuracy of a particular instrument, or to establish peoples’ opinions or to test a theory. A statement made about a population is called a hypothesis, and it is tested by statistical analysis.

34. **Classifying and Tabulating Data** – the purpose of classifying and tabulating data is to enable it to be organised and easy to read in order to carry out analysis. It is important that the tables produced are clearly laid out.

a. **Tally Charts** – One useful method for collecting data is to produce a tally chart. For example: when noting the types of drinks bought in a bar, the bar person may use a frequency table (Tally Chart).

Drink	Tally	Frequency
Bitter (pints)	𝍸 𝍸 𝍸 I	16
Lager (pints)	𝍸 𝍸 𝍸 𝍸 II	22
Red Wine (glasses)	𝍸 𝍸 II	12
White Wine (glasses)	𝍸 𝍸 III	14
Spirits (shots)	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	30
Total Drinks Sold:		94

Table 7

Notice that the lines are arranged in groups of four, with the fifth line crossing the group diagonally. It is much easier to count the data in multiples of five.

REPRESENTING DATA

35. Although data can be displayed or analysed from tables, it is often easier to interpret the information if it is displayed graphically, or pictorially. The type of representation will depend on the type of data gathered. These methods include:

- Pictograms.
- Bar Charts
- Pie Charts
- Line Graphs
- Histograms.

Pictograms

36. A pictogram represents data by the categorisation and use of symbols. For example:

37. A survey of 1000 people living at DSAE Cosford was taken to see what colour of cars they owned. The data was collected in a tally chart, but with the large numbers involved the data was difficult to read, so an alternative method of displaying the data was needed.

Colour of car	**Number of cars**
Red	60
White	100
Blue	200
Grey	50
Gold	80
Black	30

Table 8

38. There are many ways of representing the data pictorially. The following is just one option. A full car symbol represents 20 cars, half a car represents 10 cars. It is not possible to show small fractions of a symbol accurately, and the detail required should not normally be to more than half a symbol, but certain symbols may allow for a quarter.

Key:  = 20 cars

Colour of car	
Red	
White	
Blue	
Grey	
Gold	
Black	

Table 9

Bar Charts

39. A bar chart is made up of columns or bars, the heights, or lengths of which represent the frequency. For example, the children's heights in a school were recorded as follows:

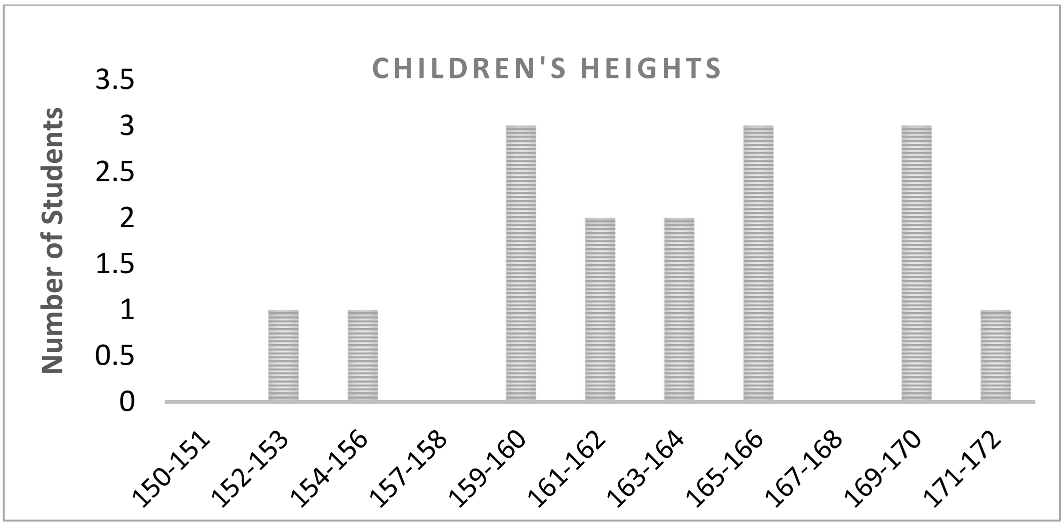

Figure 19

40. In a bar chart the length of each bar or column represents the total as shown on the scale. This can be shown vertically or horizontally.

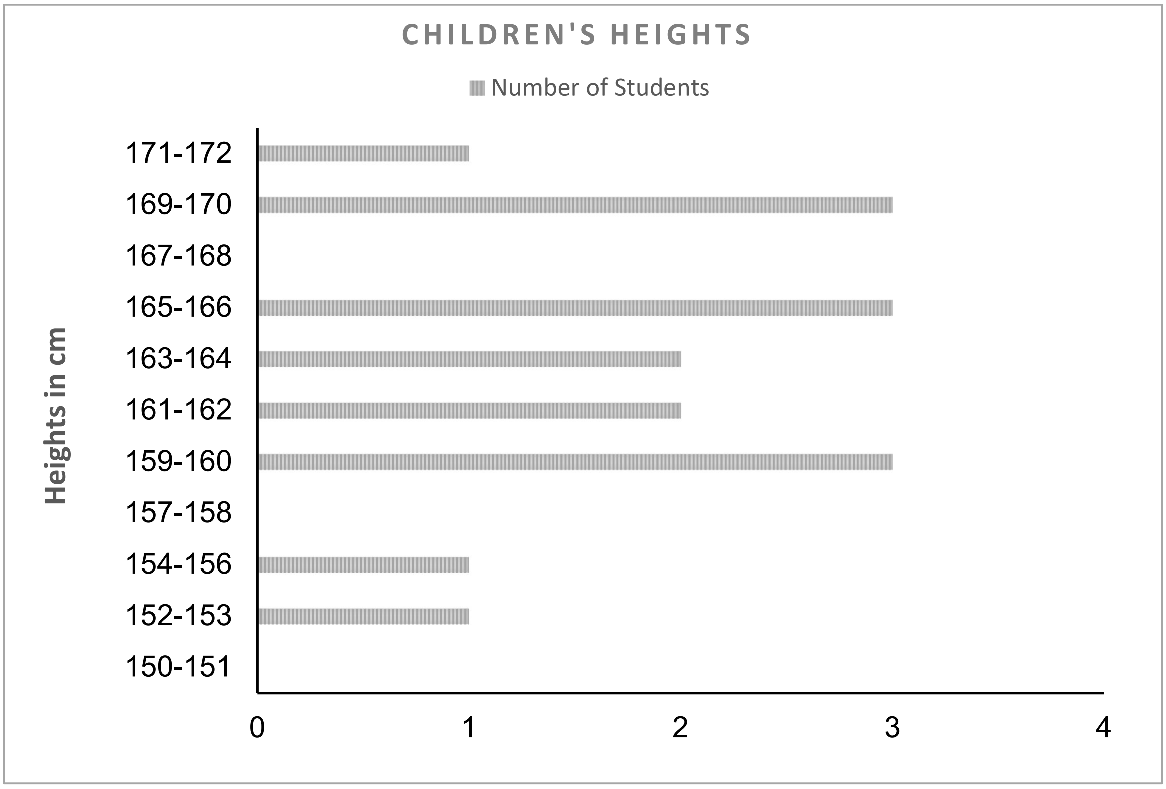

Figure 20

Dual Bar Chart

41. The number of students over 17 years old on GTM/GTE courses and the number of them holding driving licences was recorded over a period of years. These data are shown in the table below:

Year	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**
No of students over 17	32	27	29	31	33	39
No of students with a driving licence	12	17	19	11	24	28

Table 10

42. These data were represented in a dual bar chart.

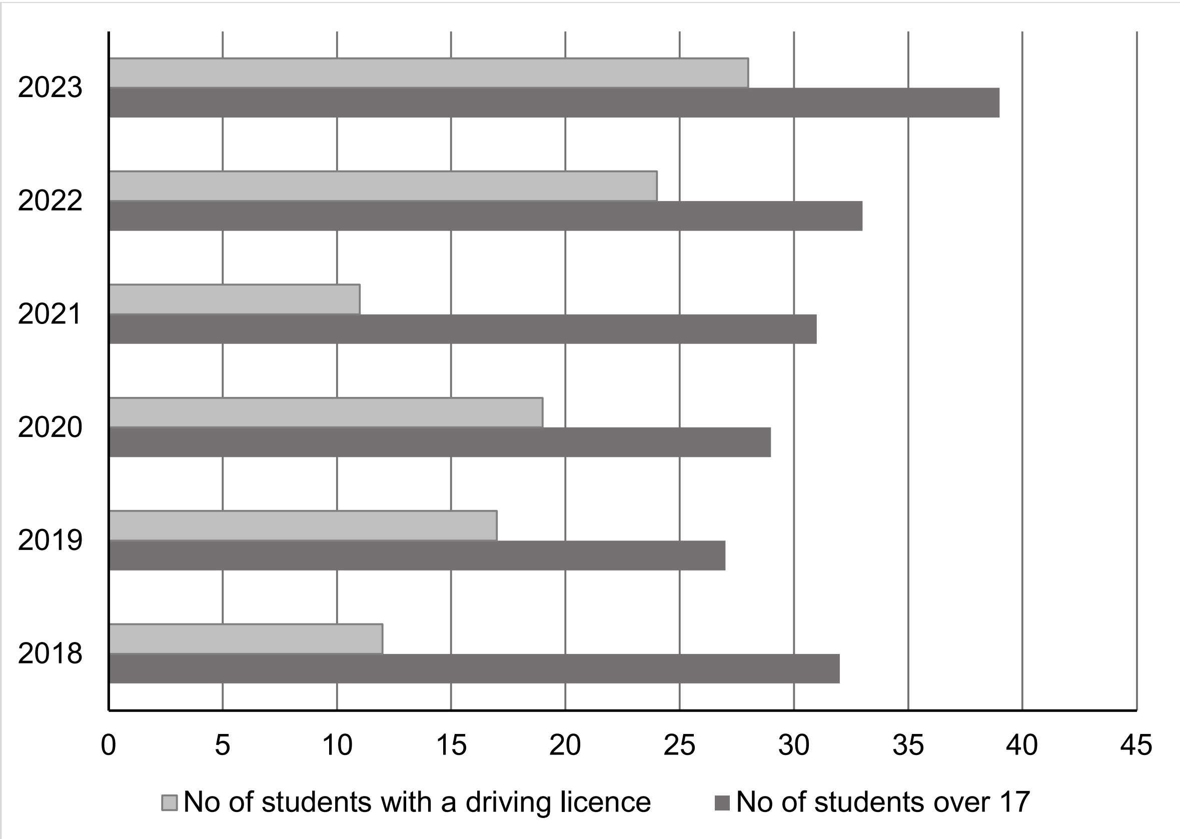

Figure 21

Stacked (Sectional) Bar Chart

43. The number of saloon and hatchback cars sold by a garage was recorded by month. These data are shown in the table below:

Month	Jan	Feb	Mar	Apr	May	June
Saloon	17	7	8	12	10	13
Hatchback	16	12	9	7	9	8

Table 11

44. These data were represented in a stacked bar chart:

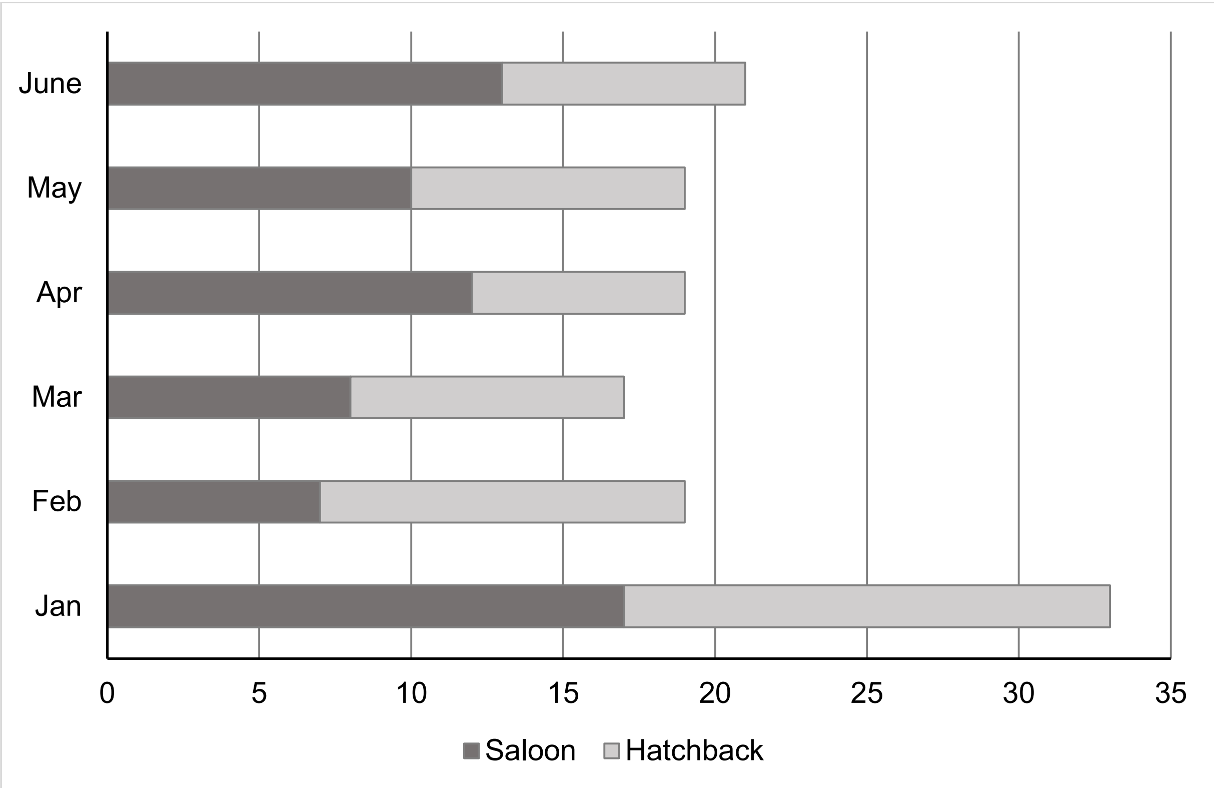

Figure 22

45. Notice that each horizontal column gives:

a. The number of saloons sold

b. The number of hatchbacks sold

c. The total number of cars sold during that month.

Pie Charts

46. A pie chart illustrates data by dividing the population into portions of a ‘pie’ or circle.

Example 1

180 people were asked which was their favourite News channel. The results were:

News Channel	Number of Viewers from group
BBC	65
ITV	20
Channel 4	63
CNN	19
Sky News	13
Total	**180**

Table 12

Add up all of the frequencies to find the total population = 180

Divide the number **into** 360 (as there are 360° in a circle): $\frac{360}{180} = 2$

So, each person’s choice represents 2° of the pie.

Multiply each frequency by the result:

BBC	$65 \times 2° = 130°$
ITV	$20 \times 2° = 40°$
Channel 4	$63 \times 2° = 126°$
CNN	$19 \times 2° = 38°$
Sky News	$13 \times 2° = 26°$
Total	**360**

Table 13

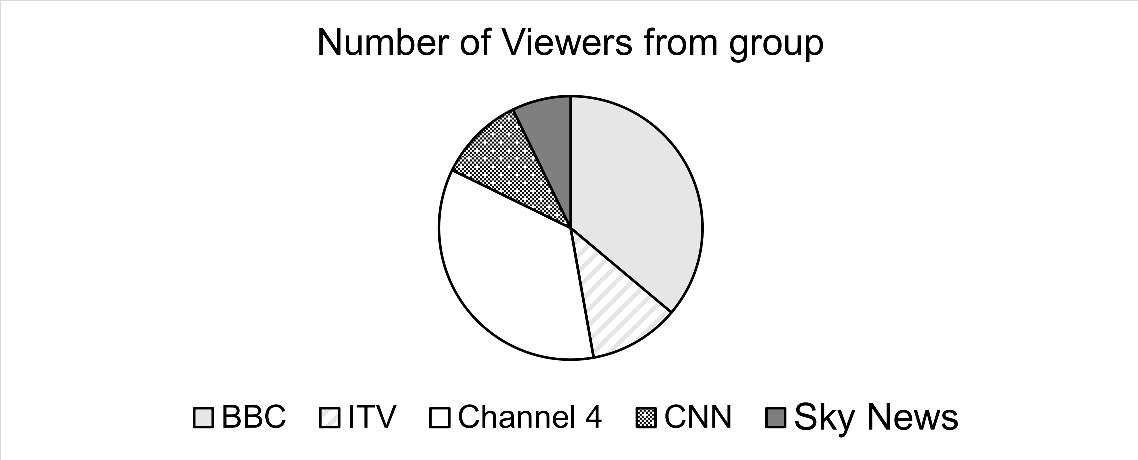

Figure 23

Example 2

The mode of transport for students into college was recorded as:

Mode of Transport	Number of Students
Walking	12
Cycling	8
Bus	26
Train	33
Car	11
Total	**90**

Table 14

Add up all of the frequencies to find the total population = 90

Divide the number **into** 360 (as there are 360° in a circle): $\frac{360}{90} = 4$

So, each person represents 4° of the pie.

Multiply each frequency by the result:

Walking	$12 \times 4° = 48°$
Cycling	$8 \times 4° = 32°$
Bus	$26 \times 4° = 104°$
Train	$33 \times 4° = 132°$
Car	$11 \times 4° = 44°$
Total	**360**

Table 15

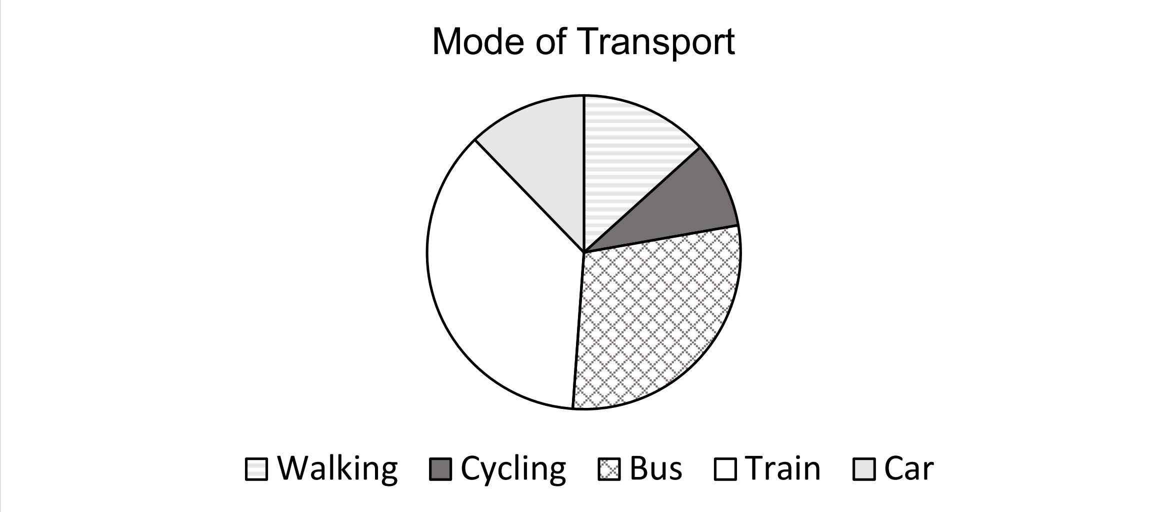

Figure 24

Interpreting Pie Charts

47. The pie chart below shows the number of students in different departments of a college.

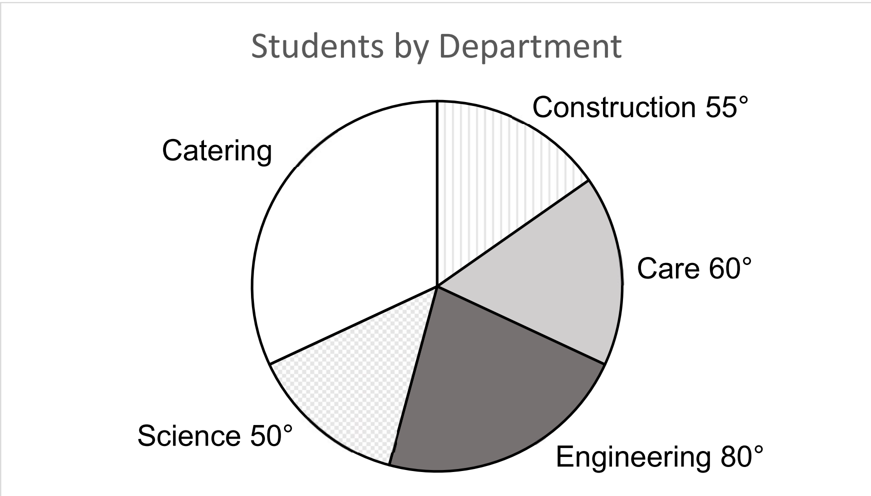

Figure 25

There are 220 students in the construction department.

1. How many students are in the college?

55° represents 220 students, so $1° = \frac{220}{55} = 4$ students

The complete circle (360°) represents $4 \times 360°$ = **1440 students in the college**.

2. How many students are there in catering?

The angle for catering is $360° - (80° + 55° + 60° + 50°) = 115°$

The **catering** department has: $4 \times 115°$ = **460 students**.

Histograms

48. A histogram is very similar in appearance to a bar chart, but it uses the area of the rectangular blocks to show the different sizes of groups of data. They are useful for showing data from groups of different sizes.

Line Graphs

49. If the quantity on the horizontal axis is a continuous variable, (temperature, age, time, height etc.) then a bar chart can be replaced by a line graph.

Example 1

The temperature of a patient in a hospital ward is recorded every 6 hours.

Day	Mon				Tue				Wed	
Time (hrs)	06	12	18	00	06	12	18	00	06	12
Temp (°F)	99.0	99.12	99.12	99.2	99.2	98.88	96.88	98.6	98.6	

Table 16

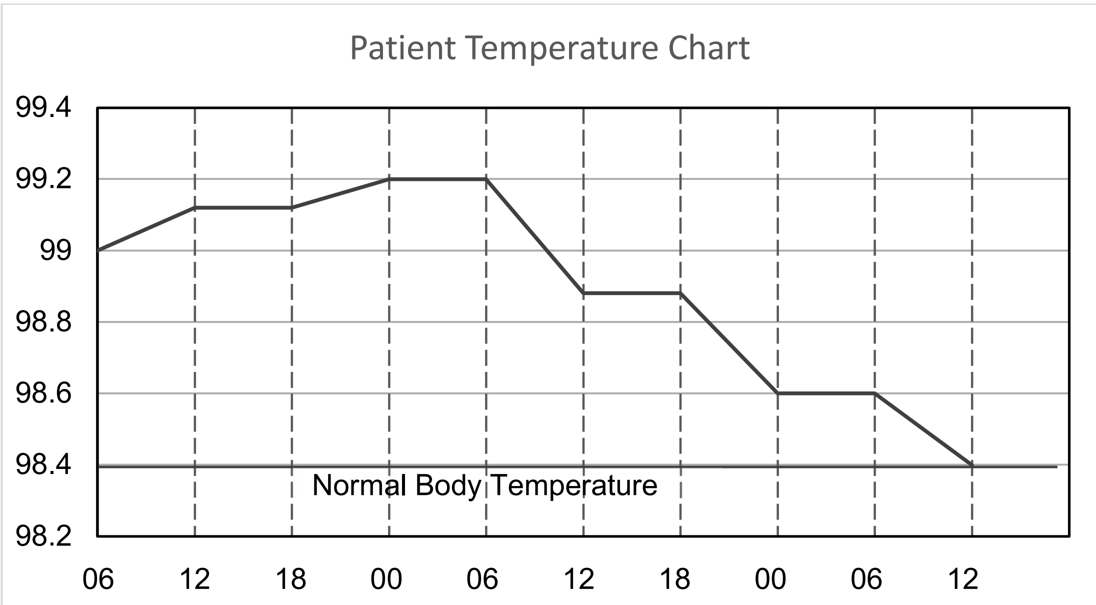

Figure 26

50. The line graph connects defined points in the data, so the points in between have no real meaning. In this example, a curve between points would be more representative, because the body temperature would change more smoothly as time increased. However, if the air temperature had been recorded, there could have been several fluctuations of temperature between readings.

AVERAGES

51. There are three different measures of the average (the centre of the data set); these are Mean, Mode and Median.

Arithmetic Mean

52. The arithmetic Mean is the average of the numbers, (a calculated "central" value of a set of numbers).

53. Simply add up all the numbers, then divide by how many numbers there are.

54. There is a formula for us to work out the arithmetic Mean: It means “**the sum of the x values, divided by the number of x values**”.

‘The sum of’

$$\text{Mean } \bar{x} = \frac{\Sigma x}{n}$$

Each x is a data value.

Total number of data values

Example 1

Find the Mean of the following data set:

2, 3, 4, 7, 10

$$\text{Mean } \bar{x} = \frac{2+3+4+7+10}{5} = \frac{26}{5} = 5.2$$

Example 2

Find the Mean of the following data set.:

1, 4, 6, 12, 14, 89

$$\text{Mean } \bar{x} = \frac{1+4+6+12+14+}{6} = \frac{126}{6} = 21$$

55. Notice that the mean in this case is larger than all of the numbers except for 89. One issue with the arithmetic mean is that it can be affected by anomalies, such as outlying values at either end of the scale.

56. The mean is a good average because all of the data is used in the calculation, but it can be heavily affected by extreme values / outliers. It can only be used with numerical data (i.e., numbers).

Median

57. The median is the middle data value, when all data values are placed in order of size, this means half of the data will be smaller than it and half of the data will be larger than it. For an odd number of data values, the median is the middle number. However, for an even number of data values, the median is the average of the middle two values:

Example 1

Find the Median of the following data set:

2, 3, 6, 2, 5, 9, 3, 8, 7, 2

Firstly, place the data in size order:

2, 2, 2, 3, 3, 5, 6, 7, 8, 9

The median is the central data value:

2, 2, 2, 3, 3, 5, 6, 7, 8, 9
⬆

For an even number of data values, the median is the average of the middle two values:

Median = $\frac{3+5}{2}$ = 4

Example 2

Find the median of the following data set:

7, 4, 6, 3, 5, 11, 4, 8, 3

Firstly, place the data in size order:

3, 3, 4, 4, 5, 6, 7, 8, 11

The median is the central data value:

3, 3, 4, 4, 5, 6, 7, 8, 11
⬆

For an odd number of data values, the median is the middle number:

Median = 5

The Median is NOT affected by extreme values, so this is a good average to use when there are outliers.

Mode

58. This is the most frequent data value. (The one that appears the most times). The mode is used to identify the most popular items. For example: A shoe shop will order more of the most common sizes. The mode can be used even with non-numerical (qualitative) data, such as colours, types or sizes. Some data sets can have more than one mode.

Example 1

Find the Mode of the following data set:

2, 3, 6, 2, 5, 9, 3, 8, 7, 2

It is much easier to find the mode if the numbers are placed in ascending order.

2, 2, 2, 3, 3, 5, 6, 7, 8, 9

By inspection, the Mode is 2

Example 2

Find the mode of the following data set:

7, 4, 6, 3, 5, 11, 4, 8, 3

Place the numbers in ascending order:

3, 3, 4, 4, 5, 6, 7, 8, 11

By inspection, the modes are 3 and 4. There are two modes (bi-modal).

The mode can be more difficult to find in large groups of numbers, so a tally chart is used as follows:

Number (x)	Tally	Frequency (f)	xf
2	IIIIII	6	12
4	IIIIIIII	8	32
6	IIIIIIIII	9	54
8	IIIIIIII	8	64
10	IIIII	5	50
12	IIII	4	48
	Total	40	260

Table 17

The mode is the number with the highest frequency. In this case **the mode is 6**, because it appears 9 times.

Frequency

59. The frequency in the table can be used to calculate the arithmetic mean.

    Instead of calculating 2 + 2 + 2 + 2 + 3 + 3 + 3 + 4 + 4

    It is easier to use (2 × **4**) + (3 × **3**) + (2 × **4**), where the numbers in bold are the frequency.

So, the arithmetic mean can easily be calculated from the table above:

1. Multiply each element (x) by its respective frequency (f) to give xf.
2. Find the sum of all of the xf values.
3. Then divide by the number of elements.

$$\frac{\text{Sum of xf}}{\text{Number of elements}} = \frac{260}{40} = 6.5$$

Example 1

Find the mean of the following scores:

Score (x)	1	2	3	4	5
Frequency (f)	3	13	11	1	5

Table 18

The score of 2 occurred 13 times, but instead of totalling 2 + 2 + 2 + 2 + 2, it is quicker to multiply 2 × 13.

Similarly, instead of totalling 3 + 3 + … 11 times, it is quicker to multiply 3 × 11.

The table can be set out as follows:

Score (x)	Frequency (f)	Score (x) × Frequency (f)
1	4	4
2	13	26
3	11	33
4	1	4
5	5	25
Totals	∑ (f) = 34	∑ (xf) = 92
	Total Frequency	Total of 34 scores

Table 19

The mean score = $\frac{\sum (xf)}{\sum (f)} = \frac{\text{Total of 34 scores}}{\text{Total Frequency}} = \frac{92}{34} = 2.705882353 = 2.71$ to 2dp

Example 2

Find the mean of the following scores:

Range	0-4	5-9	10-14	15-19	20-24
Frequency (f)	2	6	7	4	1

Table 20

With grouped data it is important to find the midpoint of each group in order to calculate the mean. This is done by taking the average of each range group.

For example, the midpoint of group (0-4) is $\frac{0+4}{2} = 2$ and (5-9) is $\frac{5+9}{2} = 7$

The table can be set out as follows:

Range	Mid-point	Frequency (f)	Midpoint × Frequency (f)
0-4	2	2	4
5-9	7	6	42
10-14	12	7	84
15-19	17	4	68
20-24	22	1	22
Totals		$\sum (f) = 20$	$\sum$ (mpf) = 220
		Total Frequency	Total mp(f)

Table 21

The mean score = $\frac{\sum (mpf)}{\sum (f)} = \frac{\text{Total Midpoint} \times \text{Frequency (f)}}{\text{Total Frequency}} = \frac{221}{20} = 11$

The mean of grouped data is not an exact value, so this is known as the **estimated mean**.

EXERCISE 7

1. Find the Mean, Mode and Median of the following data sets.

   a. 9, 6, 10, 11, 7, 5, 8, 6, 4

   b. 8.5, 5.2, 10.6, 7.7, 10.6, 5.3, 7.2, 8.1

   c. 1.6, 2.5, 3.5, 1.8, 2.2, 3.1, 2.5

   d. 8, 5, 11, 7, 10, 4, 7

Answers on page 35

2. The monthly running hours for the hydraulic rigs used on the aircraft squadrons were recorded below:

Running Hours	10 – 19	20 – 29	30 – 39	40 – 49
Frequency	2	5	6	3

Table 22

Calculate the mean running hours for the rigs.

Answers on page 35

EXERCISE 8 (GROUP EXERCISE)

1. Discuss the following sampling methods and suggest how they could be improved upon.

   a. Journalists at a local newspaper want to canvas popular opinion about plans to build a new shopping centre in town. They go to the town's high street and ask people until they have a list of 25 different opinions.

   b. For a survey into the smoking habits of teenagers, the researcher went to a tobacconist near the local college at 3:30 pm and asked everyone entering the shop how much they spent on cigarettes each week.

   c. To investigate what factors influenced the way people decided to travel to work each day, the researcher visited the local train station just before the 7:00 am train to the city arrived and asked as many people as they could.

Answers on page 35

EXERCISE 9

1. The number of people on the 87 bus was counted each time it arrived at the city centre. The data is:

    11, 25, 60, 16, 23, 2, 44, 26, 49, 58, 29, 8, 14, 24, 7, 16, 47, 5, 30, 34, 12, 33, 10, 55, 21, 32, 19, 6, 1, 21, 21, 42, 9, 35, 25, 55, 37, 46, 32, 14, 59.

Using the intervals 1-10, 11-20, 21-30, 31-40, 41-50, 51-60, draw up a tally chart and obtain the frequency distribution.

Answer on page 36

EXERCISE 10

1. The holiday destinations of 100 skiers entering an airport was as follows: Construct a sectional (stacked) bar chart to show the data:

Passengers	French Alps	Swiss Alps	Austria	Andorra	USA	Canada
Male	18	9	9	3	5	8
Female	11	1	10	4	6	2

Table 23

2. Students arriving at No4 SofTT were asked which trade they were hoping to join. Of the 240 TG 5 and 6 students, the results were as follows:

    80 Gen Tech E
    86 Gen Tech GSE
    64 MT Tech
    10 Gen Tech WS

Construct a pie chart to illustrate this data.

3. The pie chart below shows the different fuels sold in one week at a garage.

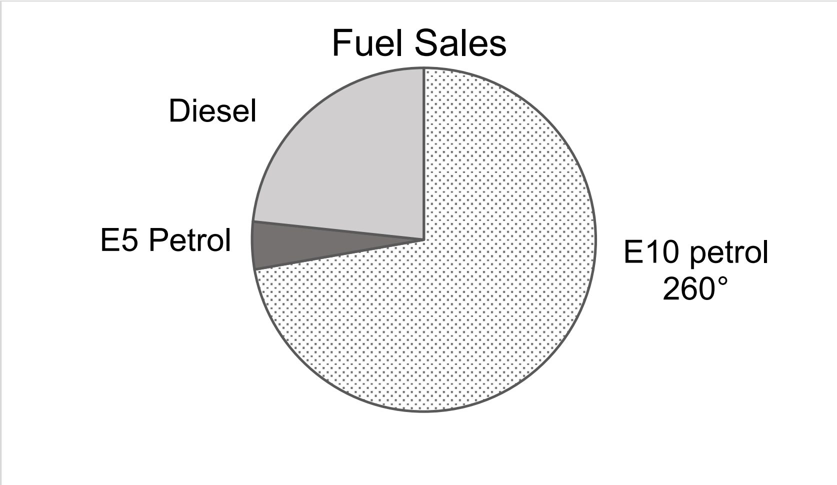

Figure 27

If 126 000 litres of diesel and 390 000 litres of E10 petrol were sold?

a. How much E5 petrol was sold?

b. What was the total quantity of fuel sold?

Answers on page 36

ANSWERS TO EXERCISES

EXERCISE 1

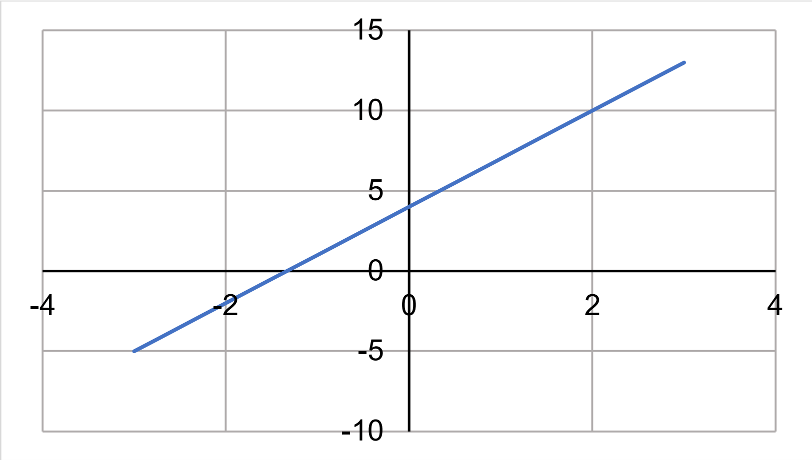

Figure 28

EXERCISE 2

1. a. 42.8 °F
   b. 64.4 °F

2. a. 67 °C
   b. 210 Ohms

3. a. 11 A
   b. 2.4 V

EXERCISE 3

1.

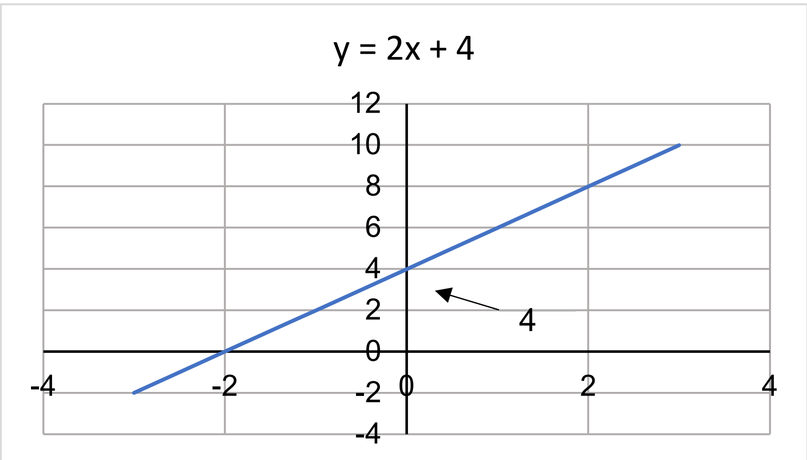

Figure 29

2.

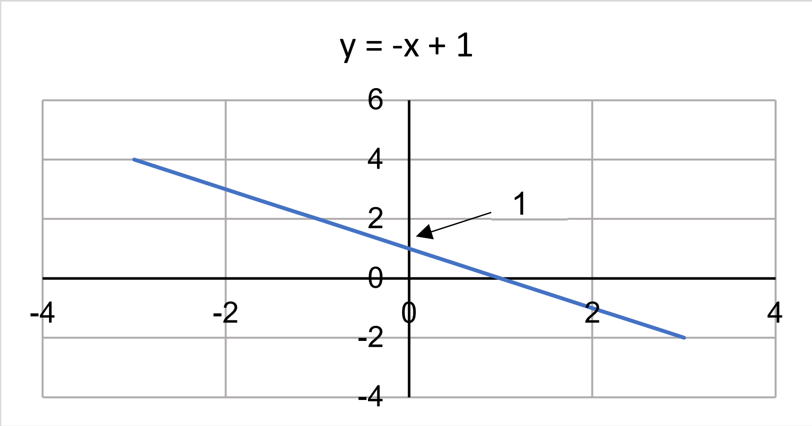

Figure 30

EXERCISE 4 ANSWERS

1. $y = \frac{5x}{2} + 15$ or $y = 2\frac{1}{2}x + 15$

2. $y = -1\frac{2}{3}x + 15$ or $y = -\frac{5x}{3} + 15$

3. $y = -2x - 10$

4. $y = 4x - 10$

EXERCISE 5 ANSWERS

1. $y = -x + 4$

2. $y = -x + 8$

EXERCISE 6 CONSOLIDATION ANSWERS

1.

x	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**
-2x	6	4	2	0	-2	-4	-6
+1	+1	+1	+1	+1	+1	+1	+1
y = -2x + 1	**7**	**5**	**3**	**1**	**-1**	**-3**	**-5**

Table 24

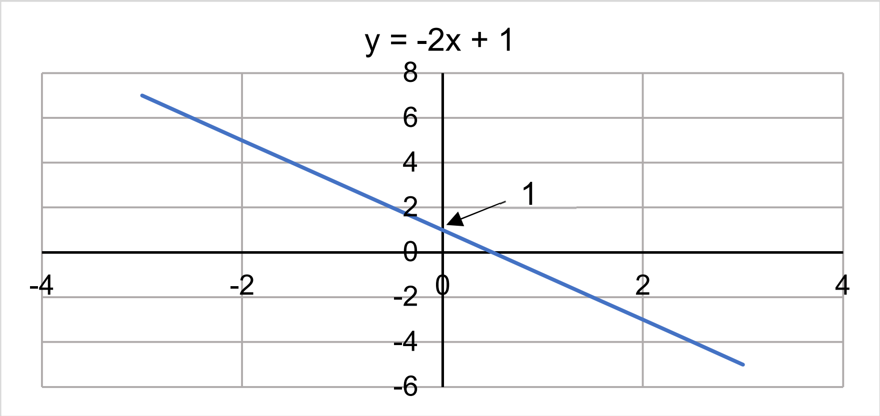

Figure 31

2.

x	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**
3x	-9	-6	-3	0	3	6	9
-2	-2	-2	-2	-2	-2	-2	-2
y = 3x - 2	**-11**	**-8**	**-5**	**-2**	**1**	**4**	**7**

Table 25

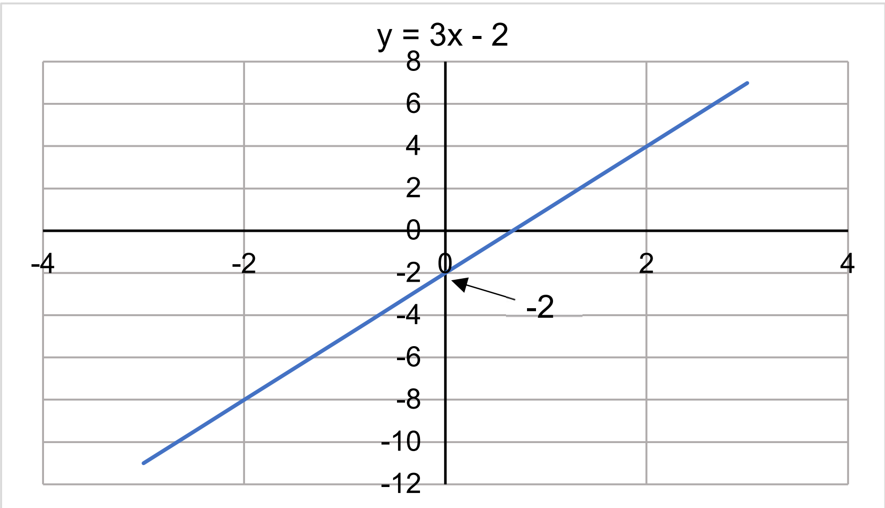

Figure 33

3. y = -x + 3

4. y = 2x -2

EXERCISE 7 ANSWERS

1.

	Mean	Mode	Median
a.	7.33	6	7
b.	7.9	10.6	7.9
c.	2.46	2.5	2.5
d	7.43	7	7

Table 26

2.

Range	Mid-point	Frequency (f)	Midpoint × Frequency (f)
10 – 19	14.5	2	29
20 – 29	24.5	5	122.5
30 – 39	34.5	6	207
40 – 49	44.5	3	133.5
Totals		$\sum$ (f) = 16	$\sum$ (mpf) = 492
		Total Frequency	Total mp(f)

Table 27

The mean score $= \dfrac{\sum (\text{mpf})}{\sum (\text{f})} = \dfrac{\text{Total Midpoint} \times \text{Frequency (f)}}{\text{Total Frequency}} = \dfrac{492}{16} = 30.75$ to 2dp

EXERCISE 8 (GROUP EXERCISE) ANSWERS

1. a. The journalist is only asking people who go to town, so the data will be skewed. A better survey would be to ask people in multiple locations including out-of-town retail parks, along with an online survey, advertised on social media for the area.

b. The researcher will have biased results if they only ask the people who smoke. The sample should include a cross section of the population including non-smokers.

c. The researcher is only asking people who already travel by train, and at a specific time of the day. The sample should include different types of transport and different times of the day to be of any value.

EXERCISE 9 ANSWERS

1.

Passengers	1-10	11-20	21-30	31-40	41-50	51-60
Tally	卌 III	卌 II	卌 卌	卌 I	卌	卌
Frequency	8	7	10	6	5	5

Table 28

EXERCISE 10 ANSWERS

1.

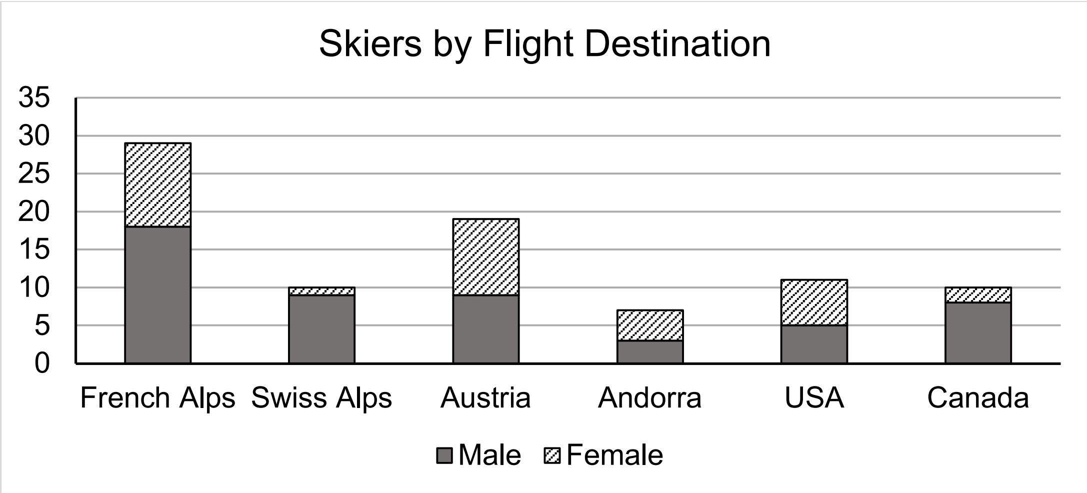

Figure 34

2.

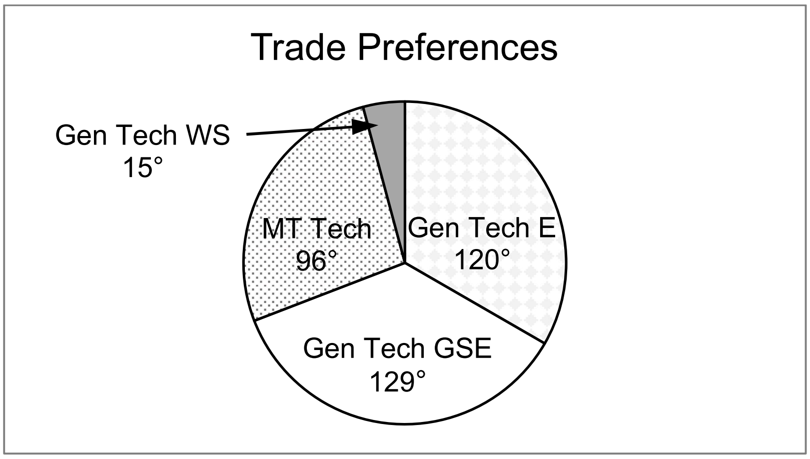

Figure 35

3 a. $390\,000 \div 260° = 1500$, so $1° = 1500$ litres

Diesel $= 126\,000 \div 1\,500 = 84°$

E5 $= 360° - (260° + 84°) = 16°$ so, $16 \times 1500 = 24\,000$ litres

b. Total $= 24\,000 + 126000 + 390000 = 540\,000$ litres