Problem Set 10

Problem 1

The density of copper is $8.96\,\mathrm{g/cm^3}$, and its atomic weight is $63.5\,\mathrm{g/mole}$.

1. Calculate the Fermi energy for copper. Assume $q = 1$, and give your answers in electron volts.

2. What is the corresponding electron velocity? Is it safe to assume that electrons in copper are nonrelativistic?

3. At what temperature would the characteristic thermal energy $k_B T$ (where k_B is the Boltzmann constant and T is the temperature in Kelvin) be equal to the Fermi energy for copper? This is called the **Fermi temperature**. As long as the actual temperature of the solid is well below the Fermi temperature, the material can be regarded as "cold" with most of its electrons in the lowest accessible state. Considering that the melting point of copper is 1356 K, is copper always "cold"?

Problem 2

Calculate the Fermi energy for non-interacting electrons in a two-dimensional infinite square well. Let σ be the number of free electrons per unit area of the well.

Problem 3

In lecture, we considered a Dirac comb as a model for a one-dimensional solid of N atoms. The allowed energies for the electron were found from the equation

$$\cos(Ka) = \cos(ka) + \beta\frac{\sin(ka)}{ka}$$

where K is a real constant, a is the periodicity of the potential, $\beta = \frac{m\alpha a}{\hbar^2}$, m is the particle mass, and α is the strength of the Dirac comb. In other words, the potential is of the form

$$V(x) = \alpha\sum_{j=0}^{N-1}\delta(x - ja).$$

Find the energy at the bottom of the first allowed electronic band for the case $\beta = 10$ to three significant digits. You can assume that α/a is equal to 1 eV. Hint: use a numerical solver.

Problem 4

In lecture, we proved Bloch's theorem as $\psi(x+a) = e^{iKa}\psi(x)$ with K a real constant and a the spacing between the atoms (i.e., the periodicity of the atomic lattice). Prove that this is equivalent to $\psi(x) = e^{iKx}u(x)$ where $u(x)$ is a function that is periodic with the lattice spacing. This alternative form of Bloch's theorem states that the electronic wavefunction can be written as the product of a plane wave with a function that is the same for every atom in the lattice.

Problem 5

1. For intrinsic (i.e., undoped) silicon, calculate the approximate temperature required to thermally excite free electrons at a concentration of 2×10^{17} per cm^3. Silicon is a semiconductor with a band gap of 1.12 eV and an effective intrinsic carrier concentration, N_i, of 1.71×10^{19} per cm^3.

2. Repeat the above calculation for silicon that is doped with phosphorous at a concentration of 10^{18} per cm^3. The binding energy of the donor electron on P is 0.045 eV.

3. Explain (in words) the temperature dependence of the electron density shown in the plot below for silicon that is doped with donors at a concentration of 10^{15} per cm^3.

Appendix

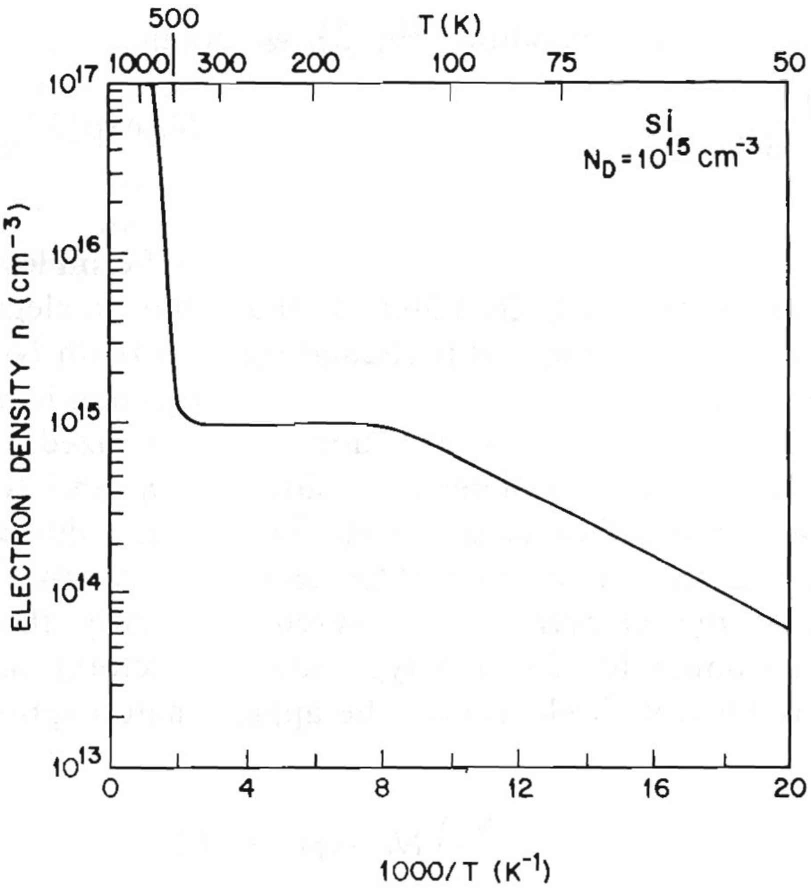

Figure 1: Temperature dependence of the electron density for doped silicon.