Problem Set 4

Problem 1

1. For the quantum harmonic oscillator, explicitly show the symmetry (i.e., even or odd) of $\psi_1(x)$ and $\psi_3(x)$. Use the Hermite polynomial form for $\psi_n(x)$:

$$\psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \frac{1}{\sqrt{2^n n!}} H_n(\xi) \exp\left(-\frac{\xi^2}{2}\right), \text{where}$$

$$H_1(\xi) = 2\xi, H_3(\xi) = 8\xi^3 - 12\xi, \text{ and } \xi = \sqrt{\frac{m\omega}{\hbar}}x.$$

2. For the infinite square well, explicitly show that $\psi_1(x)$ and $\psi_3(x)$ are orthogonal.

Problem 2

1. Show that the position operator $\hat{x}$ is Hermitian.

2. Show that the momentum operator $\hat{p}$ is Hermitian.

3. Show that the sum of two Hermitian operators is Hermitian.

4. When is the product of two Hermitian operators also Hermitian?

Problem 3

Show that if $\langle h|\hat{Q}h\rangle = \langle \hat{Q}h|h\rangle$ for all functions h in Hilbert space, then $\langle f|\hat{Q}g\rangle = \langle \hat{Q}f|g\rangle$ for all functions f and g. Each of these expressions are used as the definition of a Hermitian operator. This problem shows that they are equivalent. Hint: First let $h = f + g$ and then let $h = f + ig$.

Problem 4

Show that the Dirac delta function, $\delta(x)$, can be written as:

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \exp(ikx)\, dk$$

Hint: Use Plancherel's theorem and take the Fourier transform of $\delta(x)$ and then take the inverse Fourier transform.

Problem 5

Is the ground state of the infinite square well an eigenfunction of momentum? If so, what is its momentum? If not, why not?