Quantum Mechanics

Week 1

Mark Benazet Castells
mbenazet@ethz.ch

FS2024

Contents

Pre-Reading Note

Dear Students,

Welcome to the course on Quantum Mechanics. As part of your learning resources, I will prepare a series of educational materials and sheets designed to complement the lectures.

Please note that these materials are **abridged versions** of the content from the textbook ***"Introduction to Quantum Mechanics By David J. Griffiths"***. They have been tailored to align with the class schedule and topics, providing you with concise summaries and key points for each topic covered.

It's important to understand that these sheets are **not standalone resources**. They are intended to be used in conjunction with the class material. For a deeper understanding and a more comprehensive view of each topic, I strongly encourage you to refer to the mentioned textbook.

The book provides detailed explanations, examples, and insights that go beyond the scope of our summaries. It will be an invaluable resource for you to solidify your understanding of Quantum Mechanics.

I cannot guarantee neither correctness nor completeness of the script. Please report any mistake directly to me.

Have fun with Quantum Mechanics!

Best regards,

Mark Benazet Castells

1 Introduction to Quantum Mechanics

Quantum Mechanics (QM) offers a revolutionary perspective on the nature and behavior of matter and energy at the atomic and subatomic levels.

2 The Limitation of Classical Physics

Classical physics, which was the prevailing theory in the early 20th century, stumbled upon phenomena it couldn't explain, notably in the study of black body radiation. This led to the realization that a new framework was needed to understand the behavior of energy and matter at microscopic scales.

Remark: There are other phenomena where classical physics fails, such as the photoelectric effect, emission spectra of the hydrogen atom, and others.

2.1 Black Body Radiation

A black body, an idealized emitter and absorber of electromagnetic radiation, was a concept classical physics couldn't accurately describe. The Rayleigh-Jeans Law, which predicts the radiance of a black body, is given by:

$$B(\nu, T) = \frac{2\nu^2 k_B T}{c^2} \tag{1}$$

This law suggested an unrealistic increase in energy emission at higher frequencies, leading to the infamous 'ultraviolet catastrophe'.

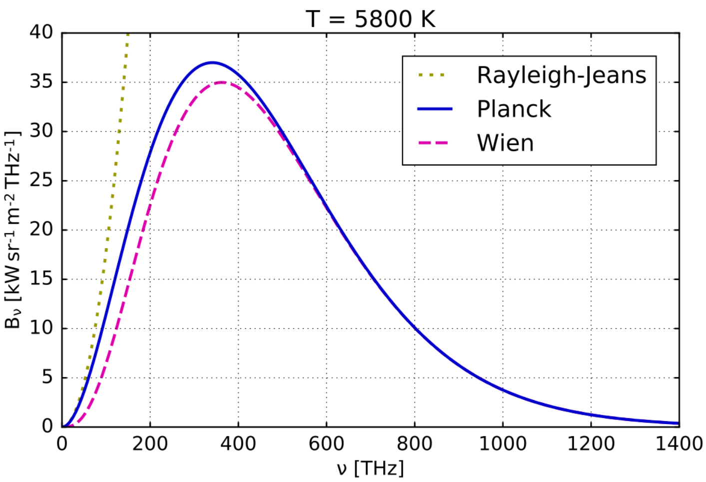

Figure 1: Comparison of black body radiation laws at 5800 K: Rayleigh-Jeans Law (yellow dotted line), Planck's Law (solid blue line), and Wien's approximation (magenta dash-dot line).

2.2 Ultraviolet Catastrophe and Planck's Solution

The ultraviolet catastrophe exposed the inadequacy of classical physics. It wrongly predicted that a black body would radiate infinite energy at high frequencies. Max Planck resolved this paradox by introducing the concept of energy quantization, positing that energy is emitted in discrete packets, or quanta. This is expressed as $E = h\nu$, where h is Planck's constant and ν is the frequency, marking the birth of Quantum Mechanics.

3 Wave-Particle Duality

One of the most fascinating concepts in quantum mechanics (QM) is wave-particle duality. This principle reveals that all matter, including not just electrons and photons but also larger entities like neutrons and protons, exhibits both wave-like and particle-like characteristics.

The double-slit experiment is a classic demonstration of this duality. When particles such as electrons pass through two closely spaced slits, they create an interference pattern characteristic of waves. This pattern emerges even when particles are sent through one at a time, suggesting that each particle interferes with itself as a wave.

Wave-particle duality is quantitatively described by the De Broglie relation, which links a particle's wave-like property (wavelength, λ) to its particle-like property (momentum, p):

$$\lambda = \frac{h}{p} \tag{2}$$

where h is Planck's constant.

Conversely, the photoelectric effect exemplifies the particle-like behavior of light. In this phenomenon, light incident on a material surface can eject electrons, demonstrating that light, typically thought of as a wave, also behaves as a stream of particles, or photons.

4 Quantum Mechanics: A Paradigm Shift

The development of QM marked a significant shift in our understanding of the universe. It revealed that classical physics is only a special case in a more comprehensive quantum framework. The Schrödinger Equation, a fundamental equation in QM, allows for the determination of a system's wave function, unlocking a deeper understanding of its quantum properties.