Problem Set 2

Problem 1

Find σ_x and σ_p for the nth stationary state of the infinite square well. Check that the uncertainty principle is satisfied. Which state comes closest to the uncertainty limit?

Problem 2

Consider a particle in the infinite square well with an initial wave function:

$$\Psi(x,0) = A[\psi_1(x) + \psi_2(x)]$$

1. Normalize $\Psi(x,0)$.

2. Determine $\Psi(x,t)$ and $\Psi^2(x,t)$.

3. Compute $\langle x \rangle$. Notice that it oscillates in time.

4. Compute $\langle p \rangle$.

5. If you measure the energy of this particle, what are the possible values?

6. What is the probability of obtaining each of the possible energies?

7. Compute the Hamiltonian operator H and compare it to E_1 and E_2.

Problem 3

Show that E must exceed the minimum value of $V(x)$ for every normalizable solution to the time-independent Schrödinger equation. Hint: rewrite the time-independent Schrödinger equation as:

$$\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}\left(V(x) - E\right)\psi$$

and consider what happens to the shape of ψ if $E < V_{\text{min}}$. In this case, can ψ be normalized?

Problem 4

Consider a free particle with an initial normalized wave function:

$$\Psi(x,0) = (\frac{2a}{\pi})^{\frac{1}{4}} e^{-ax^2} \quad \text{“Gaussian Wave Packet”}$$

where a is a real positive constant.

1. Determine $\Psi(x,t)$.

2. Determine $|\Psi(x,t)|^2$.

3. Sketch $|\Psi(x,t)|^2$ versus x at $t = 0$ and at a later t. Describe qualitatively what happens to $|\Psi(x,t)|^2$ as a function of time.

4. Find σ_x and σ_p.

5. Is the uncertainty principle satisfied?

6. At what time does the system come closest to the uncertainty limit?

Problem 5

Prove the following two theorems:

1. For normalized wave functions $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$, the separation constant E must be real. Hint: write E as $E_0 + i\Gamma$ where E_0 and Γ are real and then use the normalization condition.

2. $\psi(x)$ can always be taken to be real [unlike $\Psi(x,t)$ which is necessarily complex]. This does not mean that every solution $\psi(x)$ to the time-independent Schrödinger equation is real. But if it is not, we can always express it as a linear combination of solutions (with the same E) that are real. Hint: if $\psi(x)$ satisfies the time-independent Schrödinger equation for a given E, show that $\psi^*(x)$ does also. Thus, real linear combinations $\psi(x) + \psi^*(x)$ and $i(\psi(x) - \psi^*(x))$ are also solutions.