Problem Set 1

Problem 1

Consider the Gaussian distribution, $\rho(x) = Ae^{-\lambda(x-a)^2}$, where A, a, and λ are positive real constants.

1. If ρ is a normalized probability density, determine A.

2. Find $\langle x \rangle$, $\langle x^2 \rangle$, and σ_x.

(iii) Sketch the graph of $\rho(x)$.

Problem 2

Consider the wave function, $\Psi(x,t) = Ae^{-\lambda|x|}e^{-i\omega t}$, where A, λ, and ω are positive real constants.

1. Normalize $\Psi(x,t)$.

2. Determine the expectation values of x and x^2.

(iii) Find σ_x, sketch the graph of $|\Psi(x,t)|^2$ as a function of x, and mark the points $(x = +\sigma_x)$ and $(x = -\sigma_x)$. What is the probability that the particle would be found outside this range?

Problem 3

Show that $\frac{d\langle p\rangle}{dt} = \langle -\frac{\partial V}{\partial x}\rangle$, where p is the linear momentum and V is the potential energy function. Note that this is an example of Ehrenfest's theorem, which states that expectation values obey classical laws.

Problem 4

In general, quantum mechanics is relevant when the de Broglie wavelength of the particle in question ($\lambda = \frac{h}{p}$) is greater than the characteristic size of the system d. In thermal equilibrium at temperature T (in Kelvin), the average kinetic energy of a particle is given by:

$$\frac{p^2}{2m} = \frac{3}{2} k_B T,$$

where k_B is Boltzmann's constant. Thus, the typical de Broglie wavelength is $\lambda = \frac{h}{\sqrt{3mk_BT}}$.

(i) In a typical solid, the atomic spacing is around $d = 0.3$ nm. Find the temperature below which the valence electrons in a solid are quantum mechanical. Below what temperature are the nuclei in a solid quantum mechanical? (Use sodium as a typical case.) Note: your answer would be the same for most liquids.

1. Show that one would expect that an ideal gas at pressure P would show quantum mechanical behavior for $T < \frac{1}{k_B}\left(\frac{h^2}{3m}\right)^{3/5} P^{2/5}$. Thus, a gas molecule with small m at high P will require a quantum mechanical description. Calculate the temperature needed for helium at atmospheric pressure. Is hydrogen in outer space (where $d = 1$ cm and $T = 3$ K) quantum mechanical?