Problem Set 3

Problem 1

Evaluate the following commutators:

1. $[\hat{a}_+, C]$, where $\hat{a}_+$ is the raising operator and C is any constant.

2. $[\hat{x}, \hat{p}]$, where $\hat{x}$ is the position operator and $\hat{p}$ is the momentum operator.

3. $[\hat{a}_-, \hat{a}_+]$, where $\hat{a}_-$ is the lowering operator and $\hat{a}_+$ is the raising operator.

4. $[\hat{O}_1, \hat{x}]$, where $\hat{O}_1 = \frac{d^2}{dx^2}$.

Problem 2

For the quantum harmonic oscillator:

(i) Prove that if ψ is a solution to the Schrödinger equation with energy E, then $\hat{a}_+\psi$ is also a solution with energy $E + \hbar\omega$.

(ii) Determine the functional form of $\psi_0(x)$ by solving $\hat{a}_-\psi_0 = 0$.

Problem 3

Determine the first excited state, $\psi_1(x)$, for the harmonic oscillator. Make sure your answer is normalized.

Problem 4

For both the ground state and the first excited state of the harmonic oscillator:

(i) Compute $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, and $\langle p^2 \rangle$. Hint: introduce $\xi = \sqrt{\frac{m\omega}{\hbar}}x$ and $\alpha = (\frac{m\omega}{\pi\hbar})^{\frac{1}{4}}$ and then do only the integrations you must.

(ii) Check the uncertainty principle.

(iii) Evaluate $\langle T \rangle$, the average kinetic energy, and $\langle V \rangle$, the average potential energy. Is the sum of $\langle T \rangle$ and $\langle V \rangle$ what you would expect?