Introduction to Quantum Mechanics

Useful Info Sheet

Version: September 8, 2020 Micha Bosshart

Values Of Some Physical Constants

Constants	Symbol	Value	$\underline{\text{Unit}}$
Avogadro's number	N_0	6.02205×10^{23}	mol^{-1}
Proton charge	e	1.60219×10^{-19}	C
Planck's constant	h	6.62618×10^{-34}	$J \cdot s$
	\hbar	1.05459×10^{-34}	$J \cdot s$
Speed of light (vac.)	c	2.997925×10^{8}	$m \cdot s^{-1}$
Atomic mass unit	amu	1.66056×10^{-27}	kg
Electron rest mass	m_e	9.10953×10^{-31}	kg
Proton rest mass	m_p	1.67265×10^{-27}	kg
Boltzmann constant	k_B	1.38066×10^{-23}	$J \cdot K^{-1}$
		0.69509	cm^{-1}
Molar gas constant	R	8.31441	$JK^{-1}mol^{-1}$
Permittivity of vac.	ε_0	8.854188×10^{-12}	$C^2 s^2 k g^{-1} m^{-3}$
	$4\pi\varepsilon_0$	1.112650×10^{-10}	$C^2 s^2 k g^{-1} m^{-3}$
Rydberg const.	R_{∞}	2.179914×10^{-23}	J
		1.097373	cm^{-1}
First Bohr radius	a_0	5.29177×10^{-11}	m
Bohr magneton	μ_B	9.27409×10^{-24}	JT^{-1}
Stefan-Boltzmann const.	σ	5.67032×10^{-8}	$Jm^{-2}K^{-4}s^{-1}$

Useful Integrals

$$\int_0^a \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi x}{a}\right) = \int_0^a \cos\left(\frac{n\pi x}{a}\right) \cos\left(\frac{m\pi x}{a}\right) = \frac{a}{2} \cdot \delta_{nm}$$

$$\int \cos^2(x) dx = \frac{1}{2}x + \frac{1}{4}\sin(2x) + \text{Constant}$$

$$\int \sin^2(kx)dx = \frac{1}{2}x - \frac{1}{4k}\sin(2kx) + \text{Constant}$$

$$\int_0^a x(a-x)\sin\left(\frac{n\pi}{a}x\right)dx = 2\cdot \left[\frac{a}{n\pi}\right]^3 \left[1-\cos(n\pi)\right]$$

$$\int_0^a x \cdot \sin^2\left(\frac{n\pi}{a}x\right) dx = \frac{a^2}{4}$$

$$\int_0^a x^2 \cdot \sin^2\left(\frac{n\pi}{a}x\right) dx = \left(\frac{a}{2\pi n}\right)^3 \left(\frac{4\pi^3 n^3}{3} - 2n\pi\right)$$

$$\int_0^\infty e^{-ax^2} dx = \left(\frac{\pi}{4a}\right)^{1/2}$$

$$\int_{-\infty}^{\infty} e^{-(ax^2 + bx + c)} dx = \sqrt{\frac{\pi}{a}} \cdot e^{\frac{b^2}{4a} - c}$$

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}} , \quad n \in \mathbb{N}^+$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{2^{n+1} a^n} \left(\frac{\pi}{a} \right)^{1/2} , \quad n \in \mathbb{N}^+$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}} \ , \quad n \in \mathbb{N}^+$$

Mathematical Formulas

Trigonometric Identities

$$\sin(\alpha)\sin(\beta) = \frac{1}{2}\cos(\alpha - \beta) - \frac{1}{2}\cos(\alpha + \beta)$$

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta)$$

$$\sin(\alpha)\cos(\beta) = \frac{1}{2}\sin(\alpha - \beta) + \frac{1}{2}\sin(\alpha + \beta)$$

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

$$e^{\pm ix} = \cos(x) \pm i \cdot \sin(x)$$

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$$

Taylor Series

$$f(x) = f(a) + f'(a)(x - a) + \frac{1}{2!}f''(a)(x - a)^2 + \frac{1}{3!}f'''(a)(x - a)^3 + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\frac{1}{1 - x} = 1 + x + x^2 + x^3 + x^4 + \dots, \quad x^2 < 1$$

$$(1 \pm x)^n = 1 \pm nx + \frac{n(n - 1)}{2!}x^2 \pm \frac{n(n - 1)(n - 2)}{3!}x^3 + \dots, \quad x^2 < 1$$

 $\sin(x) = \frac{e^{ix} - e^{-ix}}{2}$

Spherical Harmonics

$$Y_l^m(\theta,\varphi) = \varepsilon \sqrt{\frac{(2l+1)}{4\pi} \frac{(l-|m|)!}{(l+|m|)!}} \cdot e^{im\varphi} \cdot P_l^m(\cos(\theta))$$

where $\varepsilon = (-1)^m$ for m > 0 and $\varepsilon = 1$ for m < 0

Associated Legendre function

$$P_l^m(x) = (1 - x^2)^{\frac{|m|}{2}} \left(\frac{d}{dx}\right)^{|m|} \cdot P_l(x)$$

Legendre polynomial defined by the Rodrigues formula

$$P_l(x) = \frac{1}{2^l \cdot l!} \left(\frac{d}{dx}\right)^l (x^2 - 1)^l$$

Tables of Solutions

The First Few Spherical Harmonics

$$Y_0^0 = \left(\frac{1}{4\pi}\right)^{\frac{1}{2}} \qquad Y_2^{\pm 2} = \left(\frac{15}{32\pi}\right)^{\frac{1}{2}} \sin^2 \theta \cdot e^{\pm 2i\varphi}$$

$$Y_1^0 = \left(\frac{3}{4\pi}\right)^{\frac{1}{2}} \cos \theta \qquad Y_3^0 = \left(\frac{7}{16\pi}\right)^{\frac{1}{2}} (5\cos^3 \theta - 3\cos \theta)$$

$$Y_1^{\pm 1} = \mp \left(\frac{3}{2\pi}\right)^{\frac{1}{2}} \sin \theta \cdot e^{\pm i\varphi} \qquad Y_2^{\pm 1} = \mp \left(\frac{2\pi}{2\pi}\right)^{\frac{1}{2}} \sin \theta (5\cos^2 \theta - 1)e^{\pm i\varphi}$$

$$Y_2^0 = \left(\frac{5}{16\pi}\right)^{\frac{1}{2}} (3\cos^2\theta - 1)$$
 $Y_3^{\pm 2} = \left(\frac{105}{32\pi}\right)^{\frac{1}{2}} \sin^2\theta \cos^2\theta \cdot e^{\pm 2i\varphi}$

$$Y_2^{\pm 1} = \mp \left(\frac{15}{8\pi}\right)^{\frac{1}{2}}\sin\theta\cos\theta \cdot e^{\pm i\varphi} \qquad Y_3^{\pm 3} = \mp \left(\frac{35}{64\pi}\right)^{\frac{1}{2}}\sin^3\theta \cdot e^{\pm 3i\varphi}$$

The first few radial wave functions for hydrogen

$$R_{10} = 2a^{-3/2} \exp(-r/a)$$

$$R_{20} = \frac{1}{\sqrt{2}} a^{-3/2} \left(1 - \frac{r}{2a} \right) \exp(-r/2a)$$

$$R_{21} = \frac{1}{\sqrt{24}} a^{-3/2} \frac{r}{a} \exp(-r/2a)$$

$$R_{30} = \frac{2}{\sqrt{27}}a^{-3/2}\left(1 - \frac{2r}{3a} + \frac{2}{27}\left(\frac{r}{a}\right)^2\right)\exp(-r/3a)$$

$$R_{31} = \frac{8}{27\sqrt{6}}a^{-3/2}\left(1 - \frac{r}{6a}\right)\left(\frac{r}{a}\right)\exp(-r/3a)$$

$$R_{32} = \frac{4}{81\sqrt{30}}a^{-3/2}\left(\frac{r}{a}\right)^2 \exp(-r/3a)$$

$$R_{40} = \frac{1}{4}a^{-3/2} \left(1 - \frac{3r}{4a} + \frac{1}{8} \left(\frac{r}{a}\right)^2 - \frac{1}{192} \left(\frac{r}{a}\right)^3\right) \exp(-r/4a)$$

$$R_{41} = \frac{\sqrt{5}}{16\sqrt{3}}a^{-3/2} \left(1 - \frac{r}{4a} + \frac{1}{80} \left(\frac{r}{a}\right)^2\right) \frac{r}{a} \exp(-r/4a)$$

$$R_{42} = \frac{1}{64\sqrt{5}}a^{-3/2}\left(1 - \frac{r}{12a}\right)\left(\frac{r}{a}\right)^2\exp(-r/4a)$$

$$R_{43} = \frac{1}{768 \sqrt{2\pi}} a^{-3/2} \left(\frac{r}{a}\right)^3 \exp(-r/4a)$$

The first few Laguerre polynomials

$L_q(x)$

 $L_{q-p}^{p}(x)$

 $Y_l^m(\theta,\varphi)$

$$L_0 = 1$$

$$L_1 = -x + 1$$

$$L_2 = x^2 - 4x + 2$$

$$L_3 = -x^3 + 9x^2 - 18x + 6$$

$$L_4 = x^4 - 16x^3 + 72x^2 - 96x + 24$$

$$L_5 = -x^5 + 25x^4 - 200x^3 + 600x^2 - 600x + 120$$

$$L_5 = -x^2 + 25x^2 - 200x^2 + 600x^2 - 600x + 120$$

$$L_6 = x^6 - 36x^5 + 450x^4 - 2400x^3 + 5400x^2 - 4320x + 720$$

Some associated Laguerre polynomials

$$L_0^0 = 1$$
 $L_0^2 = 2$ $L_1^0 = -x + 1$ $L_1^2 = -6x + 18$

$$L_2^0 = x^2 - 4x + 2$$
 $L_2^2 = 12x^2 - 96x + 144$

$$L_0^1 = 1 L_0^3 = 6$$

$$L_1^1 = -2x + 4 L_1^3 = -24x + 96$$

$$L_2^1 = 3x^2 - 18x + 18$$
 $L_2^3 = 60x^2 - 600x + 1200$

hydrogen 1	200																1993	helium 2
H																		He
1.0079 lithium	beryllium	ĺ										Ī	boron	carbon	nitrogen	oxygen	fluorine	4.0026 neon
3	4												5	6	7	8	9	10
Li	Be												В	C	N	0	F	Ne
6.941	9.0122												10.811	12.011	14.007	15.999	18.998	20.180
sodium 11	magnesium 12												aluminium 13	silicon 14	phosphorus 15	sulfur 16	chlorine 17	argon 18
Na	Mg												Al	Si	P	S	CI	Ar
22.990	24.305			49 1		I		Taxani .		Colobora 1		-to-	26.982	28.086	30.974	32.065	35.453	39.948
potassium 19	calcium 20		scandium 21	titanium 22	vanadium 23	chromium 24	manganese 25	26	cobalt 27	nickel 28	copper 29	zinc 30	gallium 31	germanium 32	arsenic 33	selenium 34	bromine 35	krypton 36
K	Ca		Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
	Ou	l	7 10 110 10 10 10 10 10	100000000000000000000000000000000000000		0.										10 to		
39.098	40.078		44.956	47.867	50.942	51.996	54.938	55.845	58.933	58.693	63.546	65.39	69.723	72.61	74.922	78.96	79.904	83.80
39.098 rubidium	40.078 strontium		44.956 yttrium	47.867 zirconium	50.942 niobium	51.996 molybdenum	54.938 technetium	55.845 ruthenium	751-00115-00126-0	58.693 palladium	300 000 000 000	100			74.922 antimony	78.96 tellurium	79,904 lodine	83.80 xenon
39.098 rubidium 37	40.078 strontium 38		44.956	47.867 zirconium 40	50.942 niobium 41	51.996 molybdenum 42	54.938 technetium 43	55.845 ruthenium 44	58.933 rhodium 45	58,693 palladium 46	63,546 silver 47	65.39 cadmium 48	69,723 Indium 49	72.61 tin 50	74.922 antimony 51	78.96 tellurium 52	79.904	83.80 xenon 54
39.098 rubidium 37 Rb	40.078 strontium		44.956 yttrium 39	47.867 zirconium	50.942 niobium	51.996 molybdenum	technetium 43 TC	55.845 ruthenium	58.933 rhodium	58.693 palladium	63.546 silver	65.39 cadmium	69.723 indium	72.61 tin	74.922 antimony	78.96 tellurium	79,904 lodine	83.80 xenon
39.098 rubidium 37 Rb 85.468 caesium	strontlum 38 Sr 87.62 barium	F7.70	44.956 yttrium 39 Y 88.906 lutetium	47.867 zirconium 40 Zr 91.224 hafnium	50.942 niobium 41 Nb 92.906 tantalum	51.996 molybdenum 42 Mo 95.94 tungsten	54.938 technetium 43 TC [98] rhenium	55.845 ruthenium 44 Ru 101.07 osmium	58.933 rhodium 45 Rh 102.91 iridium	palladium 46 Pd 106.42 platinum	63.546 silver 47 Ag 107.87 gold	65.39 cadmium 48 Cd 112.41 mercury	69.723 indium 49 In 114.82 thallium	72.61 tin 50 Sn 118.71 lead	74.922 antimony 51 Sb 121.76 bismuth	78.96 tellurium 52 Te 127.60 polonium	79.904 lodine 53 1 126.90 astatine	83.80 xenon 54 Xe 131.29 radon
39.098 rubidium 37 Rb 85.468 caesium 55	40.078 strontium 38 Sr 87.62 barium 56	57-70	44.956 yttrium 39 Y 88.906 lutetium 71	47.867 zirconium 40 Zr 91.224 hafnium 72	50.942 niobium 41 Nb 92.906 tantalum 73	51,996 molybdenum 42 Mo 95,94 tungsten 74	54.938 technetium 43 TC [98] rhenium 75	55,845 ruthenium 44 Ru 101.07 osmium 76	58,933 rhodium 45 Rh 102,91 iridium 77	58.693 palladium 46 Pd 106.42 platinum 78	63.546 silver 47 Ag 107.87 gold 79	65.39 cadmium 48 Cd 112.41 mercury 80	69.723 Indium 49 In	72.61 tin 50 Sn 118.71 lead 82	74.922 antimony 51 Sb 121.76 bismuth 83	78.96 tellurium 52 Te 127.60 polonium 84	79.904 lodine 53 1 126.90 astatine 85	83.80 xenon 54 Xe 131.29 radon 86
39.098 rubidium 37 Rb 85.468 caesium	40.078 strontium 38 Sr 87.62 barium 56 Ba	57-70 X	44,956 yttrium 39 Y 88,906 lutetium 71 Lu	47.867 zirconium 40 Zr 91.224 hafnium 72 Hf	50.942 niobium 41 Nb 92.906 tantalum	51,996 molybdenum 42 Mo 95,94 tungsten 74	technetium 43 Tc [98] rhenium 75 Re	ruthenium 44 Ru 101.07 osmium 76 Os	rhodium 45 Rh 102.91 iridium 77 Ir	palladium 46 Pd 106.42 platinum	63,546 silver 47 Ag 107.87 gold 79 Au	65.39 cadmium 48 Cd 112.41 mercury	69.723 indium 49 In 114.82 thallium	72.61 tin 50 Sn 118.71 lead 82 Pb	74,922 antimony 51 Sb 121.76 bismuth 83 Bi	78.96 tellurium 52 Te 127.60 polonium 84 Po	79.904 lodine 53 l 126.90 astatine 85	83.80 xenon 54 Xe 131.29 radon 86 Rn
39.098 rubidium 37 Rb 85.468 caesium 55 Cs 132.91	40.078 strontium 38 Sr 87.62 barium 56 Ba 137.33	1 10 10 10 10 10 10 10 10 10 10 10 10 10	44,956 yttrium 39 Y 88,906 lutetium 71 Lu 174,97	47.867 zirconlum 40 Zr 91.224 hafnium 72 Hf 178.49	50.942 niobium 41 Nb 92.906 tantalum 73 Ta 180.95	51,996 molybdenum 42 Mo 95,94 tungsten 74 W	54.938 technellum 43 TC [98] thenium 75 Re 186.21	55.845 ruthenium 44 Ru 101.07 osmium 76 Os 190.23	58.933 rhodlum 45 Rh 102.91 iridlum 77 Ir	58.693 palladium 46 Pd 106.42 platinum 78 Pt 195.08	63,546 silver 47 Ag 107,87 gold 79 Au 196,97	65.39 cadmium 48 Cd 112.41 mercury 80 Hg 200.59	69.723 indium 49 In 114.82 thallium	72.61 tin 50 Sn 118.71 lead 82 Pb 207.2	74.922 antimony 51 Sb 121.76 bismuth 83	78.96 tellurium 52 Te 127.60 polonium 84	79.904 lodine 53 1 126.90 astatine 85	83.80 xenon 54 Xe 131.29 radon 86
39.098 rubidium 37 Rb 85.468 caesium 55 Cs	40.078 strontium 38 Sr 87.62 barium 56 Ba	1 10 10 10 10 10 10 10 10 10 10 10 10 10	44,956 yttrium 39 Y 88,906 lutetium 71 Lu	47.867 zirconium 40 Zr 91.224 hafnium 72 Hf	50.942 niobium 41 Nb 92.906 tantalum 73 Ta	51,996 molybdenum 42 Mo 95,94 tungsten 74	technetium 43 Tc [98] rhenium 75 Re	ruthenium 44 Ru 101.07 osmium 76 Os	rhodium 45 Rh 102.91 iridium 77 Ir	palladium 46 Pd 106.42 platinum 78 Pt	63,546 silver 47 Ag 107.87 gold 79 Au	65.39 cadmlum 48 Cd 112.41 mercury 80 Hg	69,723 Indium 49 In 114.82 thallium 81	72.61 tin 50 Sn 118.71 lead 82 Pb	74,922 antimony 51 Sb 121.76 bismuth 83 Bi	78.96 tellurium 52 Te 127.60 polonium 84 Po	79.904 lodine 53 l 126.90 astatine 85	83.80 xenon 54 Xe 131.29 radon 86 Rn
39.098 rubidium 37 Rb 85.468 caesium 55 Cs 132.91 francium 87	40.078 strontlum 38 Sr 87.62 barium 56 Ba 137.33 radium 88	89-102	44,956 yttrium 39 Y 88,906 lutetium 71 Lu 174,97 lawrencium 103	47.867 zirconlum 40 Zr 91.224 hafnium 72 Hf 178.49 rutherfordium 104	50.942 niobium 41 Nb 92.906 tantalum 73 Ta 180.95 dubnium 105	51,996 molybdenum 42 MO 95,94 tungsten 74 W 183,84 seaborgium 106	technellum 43 TC [98] rhenium 75 Re 186.21 bohrium 107	ruthenium 44 Ru 101.07 osmium 76 Os 190.23 hassium 108	rhodium 45 Rh 102,91 iridium 77 Ir 192,22 meitnerium 109	palladium 46 Pd 106.42 platinum 78 Pt 195.08 ununniilium 110	63,546 silver 47 Ag 107,87 gold 79 Au 196,97 unununium	65.39 cadmium 48 Cd 112.41 mercury 80 Hg 200.59 ununbium 112	69,723 Indium 49 In 114.82 thallium 81	72.61 tin 50 Sn 118.71 lead 82 Pb 207.2 ununquadium 114	74,922 antimony 51 Sb 121.76 bismuth 83 Bi	78.96 tellurium 52 Te 127.60 polonium 84 Po	79.904 lodine 53 l 126.90 astatine 85	83.80 xenon 54 Xe 131.29 radon 86 Rn
39,098 rubidium 37 Rb 85,468 caesium 55 Cs 132,91 francium	40.078 strontium 38 Sr 87.62 barium 56 Ba 137.33 radium	*	44.956 yttrium 39 Y 88.906 lutetium 71 Lu 174.97 lawrencium	47.867 zirconlum 40 Zr 91.224 hafinium 72 Hf 178.49 rutherfordium	50.942 niobium 41 Nb 92.906 tantalum 73 Ta 180.95 dubnium	51,996 molybdenum 42 Mo 95,94 tungsten 74 W 183,84 seaborgium	54.938 technetium 43 TC [98] rhenium 75 Re 196.21 bohrium	55,845 ruthenium 44 Ru 101,07 osmium 76 Os 190,23 hassium	58,933 rhodlum 45 Rh 102,91 iridium 77 Ir 192,22 meilnerium	palladium 46 Pd 106.42 platinum 78 Pt 195.08 ununniilium 110	63,546 silver 47 Ag 107.87 gold 79 Au 196.97 unununium	65.39 cadmium 48 Cd 112.41 mercury 80 Hg 200.59 ununbium 112	69,723 Indium 49 In 114.82 thallium 81	72.61 tin 50 Sn 118.71 lead 82 Pb 207.2 ununquadium	74,922 antimony 51 Sb 121.76 bismuth 83 Bi	78.96 tellurium 52 Te 127.60 polonium 84 Po	79.904 lodine 53 l 126.90 astatine 85	83.80 xenon 54 Xe 131.29 radon 86 Rn

*Lanthanide series

* * Actinide series

	lanthanum 57	cerium 58	praseodymium 59	neodymium 60	promethium 61	samarium 62	europium 63	gadolinium 64	terbium 65	dysprosium 66	holmium 67	erbium 68	thulium 69	ytterbium 70
	La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Но	Er	Tm	Yb
ı	138.91	140.12	140.91	144.24	[145]	150.36	151.96	157.25	158.93	162.50	164.93	167.26	168.93	173.04
-[actinium	thorium	protactinium	uranium	neptunium	plutonium	americium	curium	berkelium	californium	einsteinium	fermium	mendelevium	nobelium
- 1	89	90	91	92	93	94	95	96	97	98	99	100	101	102
	Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No
Į	[227]	232.04	231.04	238.03	[237]	[244]	[243]	[247]	[247]	[251]	[252]	[257]	[258]	[259]