Quantum Mechanics
FS2024
Midterm MC & Solutions
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Questions

Lecture 1

1. A quantum-mechanical object can act as a particle and as a wave at the same time.

   ☐ T ☐ F

2. The de Broglie relation is valid for both light and matter.

   ☐ T ☐ F

3. If an object reflects all light that strikes it, it is a black body.

   ☐ T ☐ F

4. Every photon in the universe carries the same amount of energy.

   ☐ T ☐ F

5. When a single photon is sent through a double-slit experiment, quantum mechanics does not allow us to know which slit the photon traveled through.

   ☐ T ☐ F

6. In the quantum mechanical description of black body radiation, every photon mode out to infinite frequency carries $k_B T$ of energy, where k_B is Boltzmann's constant and T is the absolute temperature.

   ☐ T ☐ F

7. According to the de Broglie relation, an electron cannot act simultaneously as a particle and a wave at small lengths scales.

   ☐ T ☐ F

Lecture 2

1. All solutions to the Schrödinger equation are physically meaningful.

   ☐ T ☐ F

2. $\langle j^2 \rangle$ is always larger than $\langle j \rangle^2$.

   ☐ T ☐ F

3. If $f(x)$ is a square-integrable function with respect to x then $\frac{df}{dx}$ must go to zero as x goes to ∞.

   ☐ T ☐ F

4. $\frac{d}{dt}\int_{-\infty}^{\infty} |\Psi(x,t)|^2 \, dx = 0$ arises only for unphysical solutions to the 1D Schrödinger equation.

   ☐ T ☐ F

5. The variance of a discrete variable j is given by $\sum_{j=0}^{\infty}(j - \langle j \rangle)P(j)$.

   ☐ T ☐ F

6. According to Ehrenfest's theorem, expectation values in quantum mechanics follow classical laws.

   ☐ T ☐ F

7. $\Psi(x,t)$ has no direct physical meaning.

   ☐ T ☐ F

8. The kinetic energy of a wavefunction is related to its curvature.

   ☐ T ☐ F

9. According to the uncertainty principle, if σ_x is very large, then the momentum is well determined.

   ☐ T ☐ F

Lecture 3

1. The energy levels, E_n, of the infinite square well are equally spaced in energy.

   ☐ T ☐ F

2. At a temperature of absolute zero ($T = 0K$), a quantum-mechanical particle in an infinite square well will have finite kinetic energy.

   ☐ T ☐ F

3. If a quantum-mechanical system is in an eigenstate of $\hat{H}$, measurements of energy are deterministic.

   ☐ T ☐ F

4. The expectation value of the Hamiltonian is always a positive value.

   ☐ T ☐ F

5. Stationary states are the eigenfunctions of the Hamiltonian.

   ☐ T ☐ F

6. For a particle in a stationary state, the expectation value of the momentum is always zero.

   ☐ T ☐ F

7. A particle feels $V(x) = 0$ for $x \geq 0$ and $V(x) = \infty$ for $x \leq 0$. If we solve the time-independent Schrödinger equation (TISE) for this system, the solutions do not lie in Hilbert space.

   ☐ T ☐ F

8. If a quantum-mechanical particle is in a stationary state, its expectation values are constant in time.

   ☐ T ☐ F

9. The spectrum of the Hamiltonian for the 1D infinite square well is degenerate.

   ☐ T ☐ F

10. If Ψ_1 and Ψ_2 are solutions to the TDSE, any linear combination of Ψ_1 and Ψ_2 will also be a solution to the TDSE.

    ☐ T ☐ F

11. The stationary states of the 1D infinite square well are equally spaced in energy.

    ☐ T ☐ F

12. If a quantum-mechanical system is in a general state, Ψ_{general}, all expectation values are constant in time.

    ☐ T ☐ F

13. Separable solutions are not the only solutions to the Schrödinger equation.

☐ T ☐ F

14. For the infinite square well, the energy difference between two consecutive eigenstates, i.e., $\psi_n(x)$ and $\psi_{n+1}(x)$, decreases as the well width decreases.

☐ T ☐ F

15. The expectation value of the Hamiltonian is the sum of the energies of all of its stationary states.

☐ T ☐ F

16. Two different eigenfunctions of a Hermitian operator can have the same eigenvalue.

☐ T ☐ F

17. In quantum mechanics a free particle can have a definite energy.

☐ T ☐ F

18. Given a set of stationary states, $\psi_n(x)$, for an electron in a symmetric potential, $V(x)$, the following equation always holds: $\int_{-\infty}^{\infty} \psi_m(x)\psi_{m+1}(x)\psi_{m+2}(x)\, dx = 0$.

☐ T ☐ F

19. $\psi_n(x,t) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \exp\left[-i\left(\frac{n^2\pi^2\hbar}{2ma^2}\right) t\right]$ denotes a stationary state for a particle of mass m in a one-dimensional infinite square well of length a, with $n = 0, 1, 2 \ldots$

☐ T ☐ F

20. The energy of a quantum mechanical system can be negative but not less than the minimum in the potential energy function.

☐ T ☐ F

21. The eigenfunctions for any observable in quantum mechanics can be determined by solving the T.I.S.E.

☐ T ☐ F

22. To obtain the T.I.S.E. from the T.D.S.E., we had to assume that the potential energy function was time-independent.

☐ T ☐ F

23. Any solution of the T.I.S.E. can be normalized.

☐ T ☐ F

24. In quantum mechanics, all measurements are probabilistic.

☐ T ☐ F

25. For a free particle, $[\hat{H}, \hat{p}] = 0$.

☐ T ☐ F

26. Stationary states have a probability density that does not change with time.

☐ T ☐ F

27. If we confine an electron inside a finite volume with $V(x) = 0$, its ground state energy can never be exactly zero.

☐ T ☐ F

Lecture 4

1. The spacing between the energy levels, E_n, of the quantum-mechanical harmonic oscillator increases as the spring constant is increased ("stiffened").

   ☐ T ☐ F

2. The raising and lowering operators, $\hat{a}_+$ and $\hat{a}_-$, commute.

   ☐ T ☐ F

3. Incompatible observables share the same eigenfunctions.

   ☐ T ☐ F

4. For a one-dimensional quantum-mechanical harmonic oscillator, the solutions ψ_n will alternate between even and odd symmetry with increasing n.

   ☐ T ☐ F

5. The expectation value $\langle H \rangle$ for the standard 1D quantum-mechanical harmonic oscillator [i.e. with $V(x) = \frac{1}{2}m\omega^2 x^2$] can have any positive value.

   ☐ T ☐ F

6. Given an ensemble of particles sitting in a specific eigenstate, ψ_n, of the 1D quantum-mechanical harmonic oscillator, uncertainty exists for measurements of both x and p.

   ☐ T ☐ F

7. An operator in quantum mechanics that represents a physical observable commutes with any constant.

   ☐ T ☐ F

8. The stationary states $\psi_n(x)$ of the quantum-mechanical harmonic oscillator are eigenfunctions of $\hat{a}_+$.

   ☐ T ☐ F

9. $\hat{x}$ and $\hat{H}$ are operators that represent compatible observables.

   ☐ T ☐ F

10. For the quantum-mechanical harmonic oscillator, $\hat{a}_+\psi_0 = 0$.

    ☐ T ☐ F

11. A phonon is a quantum of vibration in a solid.

    ☐ T ☐ F

12. If a particle is described by a quantum mechanical harmonic oscillator, the probability of finding it near the center of the potential (at $x = 0$) is always greater than finding it near the edges (at x near the classical turning point).

    ☐ T ☐ F

13. For a harmonic oscillator if $\sigma_{\hat{p}}$ is very large then the energy is well determined.

    ☐ T ☐ F

14. The ψ_n's for the quantum harmonic oscillator go to zero at the classical turning points, that is $\psi_n(x) = 0$ when $V(x) = E_n$, where $V(x)$ is the harmonic potential.

    ☐ T ☐ F

Lecture 5

1. Every observable quantity in classical mechanics is represented in quantum mechanics by a linear Hermitian operator.

   ☐ T ☐ F

2. Two general ket's in Hilbert space are always orthogonal.

   ☐ T ☐ F

3. In quantum mechanics, operators that represent observable quantities are linear.

   ☐ T ☐ F

4. When an electron tunnels through a thick potential barrier, its total energy decreases.

   ☐ T ☐ F

5. For a quantum-mechanical particle in a bound state of a one-dimensional square well of finite depth for $0 < x < a$ the probability of finding the particle with $x > a$ is zero.

   ☐ T ☐ F

6. A Hermitian operator always yields real expectation values.

   ☐ T ☐ F

7. The probability of a quantum-mechanical particle to tunnel through a tall and wide 1D rectangular potential barrier, as discussed in lecture, decreases quadratically with the barrier width.

   ☐ T ☐ F

8. As a finite potential-energy barrier gets thinner in x, the probability for quantum-mechanical tunneling increases.

   ☐ T ☐ F

9. For a finite square potential well, both the scattering and bound solutions show non-classical behavior.

   ☐ T ☐ F

10. If the quantum-mechanical operator for a time-independent observable commutes with the Hamiltonian, it is a conserved quantity of the system.

    ☐ T ☐ F

11. In one dimension, $\langle f + ig | h \rangle$ in Dirac notation is the same as the integral sum $\int_{-\infty}^{\infty} f^*(x) h \, dx + \int_{-\infty}^{\infty} i(g^* h) \, dx$.

    ☐ T ☐ F

12. For any finite square well, there is always at least one bound state

    ☐ T ☐ F

13. An electron in a bound state of a finite square well will penetrate the potential barrier.

    ☐ T ☐ F

14. If an electron has an energy E that is higher than the minimum potential energy, V_{min}, of a finite well, then it must be in a bound state.

    ☐ T ☐ F

15. The number operator, $N \equiv \hat{a}_+\hat{a}_-$, which we mentioned in our discussion of the harmonic oscillator, is Hermitian.

    ☐ T ☐ F

16. For a general one-dimensional wavefunction $\psi(x)$, the wavevector $|\psi\rangle$ is infinite-dimensional.

    ☐ T ☐ F

17. If a single particle approaches a potential barrier, its wavefunction is always completely transmitted if it has kinetic energy above the height of the barrier.

    ☐ T ☐ F

Lecture 6

1. An uncertainty relation exists for every pair of observables whose quantum mechanical operators do not commute.

   ☐ T ☐ F

2. Any two eigenfunctions of momentum, $f_{p'}$ and f_p, satisfy Kronecker orthonormality.

   ☐ T ☐ F

3. In the equation $\Delta E \Delta t \geq \frac{\hbar}{2}$, Δt represents the standard deviation for measurement of time.

   ☐ T ☐ F

4. If a quantum mechanical system is in a mixture of two non-degenerate eigenstates for a given observable, measurements of this observable are probabilistic.

   ☐ T ☐ F

5. Momentum and energy eigenfunctions are always continuous.

   ☐ T ☐ F

6. A non-zero uncertainty relation exists for any two observables whose operators do not commute.

   ☐ T ☐ F

7. The eigenstate $\psi_1(x)$ of a particle in an infinite square well can be written as a linear combination of the momentum eigenfunctions.

   ☐ T ☐ F

8. The energy-time uncertainty principle comes directly from the generalized uncertainty principle.

   ☐ T ☐ F

9. A and B are incompatible quantum mechanical observables. We measure first A, then B, and then A again. The second measurement of A will always yield the same value as the first measurement of A.

   ☐ T ☐ F

10. Any two incompatible observables have an uncertainty relation.

    ☐ T ☐ F

11. When we measure a quantum mechanical observable $\hat{Q}$ that has a discrete set of eigenfunctions, we will always obtain one of the corresponding eigenvalues.

    ☐ T ☐ F

12. The uncertainty principle applies to many incompatible observables, not just $\hat{x}$ and $\hat{p}$.

    ☐ T ☐ F

13. Given a general wavefunction $\Psi(x, t)$ and two compatible observables, $\hat{A}$ and $\hat{B}$, any measurement of $\hat{B}$ yields the same result as a measurement of $\hat{A}$ and then $\hat{B}$.

    ☐ T ☐ F

Solutions

Lecture 1

1. A quantum-mechanical object can act as a particle and as a wave at the same time.

   ■ **T** ☐ F De Broglie relationship $p = \frac{h}{\lambda}$ relates wave-particle behavior

2. The de Broglie relation is valid for both light and matter.

   ■ **T** ☐ F $p = \frac{h}{\lambda} = mv$ reflects the use case for both behaviors.

3. If an object reflects all light that strikes it, it is a black body.

   ☐ T ■ **F** It absorbs all light that strikes it, not reflects it.

4. Every photon in the universe carries the same amount of energy.

   ☐ T ■ **F** Depends on their frequency according to $E = \hbar\omega$.

5. When a single photon is sent through a double-slit experiment, quantum mechanics does not allow us to know which slit the photon traveled through.

   ☐ T ■ **F** We can measure it, but then no interference pattern would result from an ensemble of such measurements

6. In the quantum mechanical description of black body radiation, every photon mode out to infinite frequency carries $k_B T$ of energy, where k_B is Boltzmann's constant and T is the absolute temperature.

   ☐ T ■ **F** Due to $E = h\nu$, when $h\nu >> k_B T$ the high frequency nodes carry no energy

7. According to the de Broglie relation, an electron cannot act simultaneously as a particle and a wave at small lengths scales.

   ☐ T ■ **F** The de Broglie relation actually says the opposite, at small length scales it can simultaneously act as a particle and wave

Lecture 2

1. All solutions to the Schrödinger equation are physically meaningful.

   ☐ T ■ **F** Only solutions which are square integrable and normalized.

2. $\langle j^2 \rangle$ is always larger than $\langle j \rangle^2$.

   ☐ T ■ **F** $\langle j^2 \rangle \geq \langle j \rangle^2$, they can be equal

3. If $f(x)$ is a square-integrable function with respect to x then $\frac{df}{dx}$ must go to zero as x goes to ∞.

   ■ **T** ☐ F Consequence of a square integrable function

4. $\frac{d}{dt}\int_{-\infty}^{\infty} |\Psi(x,t)|^2 \, dx = 0$ arises only for unphysical solutions to the 1D Schrödinger equation.

   ☐ T ■ **F** For physically meaningful solutions the probability density is time independent

5. The variance of a discrete variable j is given by $\sum_{j=0}^{\infty}(j - \langle j \rangle)P(j)$.

   ☐ T ■ **F** Should be $\sum_{j=0}^{\infty}(j - \langle j \rangle)^2 P(j)$

6. According to Ehrenfest's theorem, expectation values in quantum mechanics follow classical laws.

   ■ **T** ☐ F Definition of the Ehrenfest's theorem

7. $\Psi(x,t)$ has no direct physical meaning.

   ■ **T** ☐ F Only $|\Psi|^2$ gives us information about the particle state

8. The kinetic energy of a wavefunction is related to its curvature.

   ■ **T** ☐ F $\hat{E}_{\text{kin}} = \frac{\hat{p}^2}{2m} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$ has a second derivative, indicating curvature relationship

9. According to the uncertainty principle, if σ_x is very large, then the momentum is well determined.

   ☐ T ■ **F** σ_x being very large does not imply σ_p being zero

Lecture 3

1. The energy levels, E_n, of the infinite square well are equally spaced in energy.

   ☐ T ■ **F** They follow the formula $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$, where n is the quantum number and therefore scale with $\propto n^2$

2. At a temperature of absolute zero ($T = 0K$), a quantum-mechanical particle in an infinite square well will have finite kinetic energy.

   ■ **T** ☐ F Even at absolute zero, quantum particles have zero-point energy

3. If a quantum-mechanical system is in an eigenstate of $\hat{H}$, measurements of energy are deterministic.

   ■ **T** ☐ F If in eigenstate, $\hat{H}\psi = E\psi$ will always yield the same eigenvalue

4. The expectation value of the Hamiltonian is always a positive value.

   ☐ T ■ **F** Can be negative if the system has negative potential energy greater than its kinetic energy.

5. Stationary states are the eigenfunctions of the Hamiltonian.

   ■ **T** ☐ F Eigenvalue equation: $\hat{H}\psi = E\psi$.

6. For a particle in a stationary state, the expectation value of the momentum is always zero.

   ■ **T** ☐ F In stationary states, due to time-independence of expectation values, $\frac{d\langle x\rangle}{dt} = 0$. By Ehrenfest's theorem, $\langle p\rangle = m\frac{d\langle x\rangle}{dt}$, hence $\langle p\rangle = 0$.

7. A particle feels $V(x) = 0$ for $x \geq 0$ and $V(x) = \infty$ for $x \leq 0$. If we solve the time-independent Schrödinger equation (TISE) for this system, the solutions do not lie in Hilbert space.

   ■ **T** ☐ F The state is a scattering state, akin to that of a free particle. Its solutions are not normalizable and, as such, do not reside within the Hilbert space.

8. If a quantum-mechanical particle is in a stationary state, its expectation values are constant in time.

   ■ **T** ☐ F One of the implication of stationary states.

9. The spectrum of the Hamiltonian for the 1D infinite square well is degenerate.

   ☐ T ■ **F** $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}, n = 1, 2, 3, ...$ each level is non-degenerate.

10. If Ψ_1 and Ψ_2 are solutions to the TDSE, any linear combination of Ψ_1 and Ψ_2 will also be a solution to the TDSE.

    ■ **T** ☐ F The TDSE is linear; thus, any linear combination of solutions remains a valid solution.

11. The stationary states of the 1D infinite square well are equally spaced in energy.

☐ T ■ **F** They follow the formula $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$, where n is the quantum number and therefore scale with $\propto n^2$

12. If a quantum-mechanical system is in a general state, Ψ_{general}, all expectation values are constant in time.

☐ T ■ **F** Only stationary states have time-constant expectation values. For $\Psi_{general}$, a superposition of stationary states, expectation values fluctuate over time due to cross-term phase factors.

13. Separable solutions are not the only solutions to the Schrödinger equation.

■ **T** ☐ F Any linear combination is also a solution.

14. For the infinite square well, the energy difference between two consecutive eigenstates, i.e., $\psi_n(x)$ and $\psi_{n+1}(x)$, decreases as the well width decreases.

☐ T ■ **F** $\Delta E = \frac{(n_2^2 - n_1^2)\pi^2\hbar^2}{2ma^2}$, $n = 1, 2, 3, ...$ If $a \downarrow$ then $\Delta E \uparrow$.

15. The expectation value of the Hamiltonian is the sum of the energies of all of its stationary states.

☐ T ■ **F** $\langle H \rangle = \sum_{n=1}^{\infty} |c_n|^2 E_n$.

16. Two different eigenfunctions of a Hermitian operator can have the same eigenvalue.

■ **T** ☐ F Definition of degenerate eigenstates.

17. In quantum mechanics a free particle can have a definite energy.

☐ T ■ **F** It contains a range of k and therefore a range of E

18. Given a set of stationary states, $\psi_n(x)$, for an electron in a symmetric potential, $V(x)$, the following equation always holds: $\int_{-\infty}^{\infty} \psi_m(x)\psi_{m+1}(x)\psi_{m+2}(x)\, dx = 0$.

☐ T ■ **F** If ψ_m is even, the integral is zero. If ψ_m is odd, then different from zero.

19. $\Psi_n(x,t) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \exp\left[-i\left(\frac{n^2\pi^2\hbar}{2ma^2}\right)t\right]$ denotes a stationary state for a particle of mass m in a one-dimensional infinite square well of length a, with $n = 0, 1, 2\ldots$

☐ T ■ **F** $n = 1, 2, 3$

20. The energy of a quantum mechanical system can be negative but not less than the minimum in the potential energy function.

■ **T** ☐ F As the potential energy can be negative, the energy can also take negative values.

21. The eigenfunctions for any observable in quantum mechanics can be determined by solving the T.I.S.E.

☐ T ■ **F** Only the eigenfunctions $\hat{H}$ are determined by solving the T.I.S.E

22. To obtain the T.I.S.E. from the T.D.S.E., we had to assume that the potential energy function was time-independent.

    ■ **T** ☐ F Simplification for the T.I.S.E. needed

23. Any solution of the T.I.S.E. can be normalized.

    ☐ T ■ **F** Only T.I.S.E solutions which are square-integrable can be normalized.

24. In quantum mechanics, all measurements are probabilistic.

    ☐ T ■ **F** If the system is in an eigenfunction of the observable the measuremet is deterministic

25. For a free particle, $[\hat{H}, \hat{p}] = 0$.

    ■ **T** ☐ F For a free particle where $V(x) = 0$, the Hamiltonian simplifies to $\hat{H} = \frac{\hat{p}^2}{2m}$, solely dependent on the momentum operator $\hat{p}$. This leads to $\hat{H}$ and $\hat{p}$ commuting since they share a direct dependence.

26. Stationary states have a probability density that does not change with time.

    ■ **T** ☐ F Characteristic of stationary states

27. If we confine an electron inside a finite volume with $V(x) = 0$, its ground state energy can never be exactly zero.

    ■ **T** ☐ F Analog to the ISW, $n > 0$ in order for the existence of any state

Lecture 4

1. The spacing between the energy levels, E_n, of the quantum-mechanical harmonic oscillator increases as the spring constant is increased ("stiffened").

   ■ **T** ☐ F The energy levels of a quantum harmonic oscillator are given by $E_n = \hbar\omega(n + \frac{1}{2})$, where ω is the angular frequency related to the spring constant k by $\omega = \sqrt{\frac{k}{m}}$

2. The raising and lowering operators, $\hat{a}_+$ and $\hat{a}_-$, commute.

   ☐ T ■ **F** $[\hat{a}_+, \hat{a}_-] = 1$

3. Incompatible observables share the same eigenfunctions.

   ☐ T ■ **F** Compatible observables share the same eigenfunctions.

4. For a one-dimensional quantum-mechanical harmonic oscillator, the solutions ψ_n will alternate between even and odd symmetry with increasing n.

   ■ **T** ☐ F Characteristics of the H.O

5. The expectation value $\langle H \rangle$ for the standard 1D quantum-mechanical harmonic oscillator [i.e. with $V(x) = \frac{1}{2}m\omega^2 x^2$] can have any positive value.

   ☐ T ■ **F** The expectation value cannot be smaller than the ground state energy $E_0 = \frac{1}{2}\hbar\omega$

6. Given an ensemble of particles sitting in a specific eigenstate, ψ_n, of the 1D quantum-mechanical harmonic oscillator, uncertainty exists for measurements of both x and p.

   ■ **T** ☐ F $\sigma_x \cdot \sigma_p \geq \frac{\hbar}{2}$

7. An operator in quantum mechanics that represents a physical observable commutes with any constant.

   ■ **T** ☐ F The operator is linear

8. The stationary states $\psi_n(x)$ of the quantum-mechanical harmonic oscillator are eigenfunctions of $\hat{a}_+$.

   ☐ T ■ **F** $\hat{a}_+\psi_n = \sqrt{n+1}\psi_{n+1}$, therefore not eigenvalue equation.

9. $\hat{x}$ and $\hat{H}$ are operators that represent compatible observables.

   ☐ T ■ **F** $\hat{H}$ contains $\hat{p}$ which is incompatible with $\hat{x}$.

10. For the quantum-mechanical harmonic oscillator, $\hat{a}_+\psi_0 = 0$.

    ☐ T ■ **F** $\hat{a}_+\psi_0 = \psi_1$

11. A phonon is a quantum of vibration in a solid.

    ■ **T** ☐ F

12. If a particle is described by a quantum mechanical harmonic oscillator, the probability of finding it near the center of the potential (at $x = 0$) is always greater than finding it near the edges (at x near the classical turning point).

☐ T ■ **F** It can be zero at the center

13. For a harmonic oscillator if $\sigma_{\hat{p}}$ is very large then the energy is well determined.

☐ T ■ **F** If σ_p is very large the momentum is not well determined and therefore the energy is not well determined

14. The ψ_n's for the quantum harmonic oscillator go to zero at the classical turning points, that is $\psi_n(x) = 0$ when $V(x) = E_n$, where $V(x)$ is the harmonic potential.

☐ T ■ **F** As we have a finite potential, the wave will penetrate into the barrier

Lecture 5

1. Every observable quantity in classical mechanics is represented in quantum mechanics by a linear Hermitian operator.

   ■ **T** ☐ F Ensures that the eigenvalues (possible measurement outcomes) are real.

2. Two general ket's in Hilbert space are always orthogonal.

   ☐ T ■ **F** Orthogonality depends on the specific vectors and inner product involved.

3. In quantum mechanics, operators that represent observable quantities are linear.

   ■ **T** ☐ F Needed to preserve the properties of superposition in measurements.

4. When an electron tunnels through a thick potential barrier, its total energy decreases.

   ☐ T ■ **F** Transmission probability decreases with increasing thickness. Energy is conserved!

5. For a quantum-mechanical particle in a bound state of a one-dimensional square well of finite depth for $0 < x < a$ the probability of finding the particle with $x > a$ is zero.

   ☐ T ■ **F** With the potential being finite, there exists a non-zero probability of a particle penetrating and existing outside the potential barrier.

6. A Hermitian operator always yields real expectation values.

   ■ **T** ☐ F Definition of an Hermitian operator.

7. The probability of a quantum-mechanical particle to tunnel through a tall and wide 1D rectangular potential barrier, as discussed in lecture, decreases quadratically with the barrier width.

   ☐ T ■ **F** The transmission probability decreases exponentially with the width of the barrier, as shown in the formula: $T \approx \frac{16E(V_0-E)}{V_0^2} \exp\left[-4\frac{\sqrt{2m(V_0-E)}}{\pi}a\right]$, where a is the width of the barrier

8. As a finite potential-energy barrier gets thinner in x, the probability for quantum-mechanical tunneling increases.

   ■ **T** ☐ F $T \approx \frac{16E(V_0-E)}{V_0^2} \exp\left[-4\frac{\sqrt{2m(V_0-E)}}{\pi}a\right]$ If $a \downarrow$ then $T \uparrow$.

9. For a finite square potential well, both the scattering and bound solutions show non-classical behavior.

   ■ **T** ☐ F Bound solutions display quantum tunneling, while scattering solutions show non-classical reflection in a finite square potential well.

10. If the quantum-mechanical operator for a time-independent observable commutes with the Hamiltonian, it is a conserved quantity of the system.

■ **T** ☐ F $\frac{d\langle Q\rangle}{dt} = \frac{i}{\hbar}\langle[\hat{H},\hat{Q}]\rangle + \langle\frac{\partial\hat{Q}}{\partial t}\rangle = 0$, therefore it is a conserved quantity.

11. In one dimension, $\langle f+ig|h\rangle$ in Dirac notation is the same as the integral sum $\int_{-\infty}^{\infty} f^*(x)h\,dx + \int_{-\infty}^{\infty} i(g^*h)\,dx$.

☐ T ■ **F** Should be $\int_{-\infty}^{\infty} f^*(x)h\,dx - \int_{-\infty}^{\infty} i(g^*h)\,dx$

12. For any finite square well, there is always at least one bound state

■ **T** ☐ F Consequence of the existence of any potential $V(x) \neq 0$

13. An electron in a bound state of a finite square well will penetrate the potential barrier.

■ **T** ☐ F Consequence of the existence of any potential $V(x) \neq \infty$

14. If an electron has an energy E that is higher than the minimum potential energy, V_{min}, of a finite well, then it must be in a bound state.

☐ T ■ **F** If the energy $E > V_{min}$ the particle could be in a bound state or a scattering state

15. The number operator, $N \equiv \hat{a}_+\hat{a}_-$, which we mentioned in our discussion of the harmonic oscillator, is Hermitian.

■ **T** ☐ F The number operator is a physical observable which measures the number of quanta in the oscillator, Thus, Hermitian

16. For a general one-dimensional wavefunction $\psi(x)$, the wavevector $|\psi\rangle$ is infinite-dimensional.

■ **T** ☐ F Infinite superposition of stationary states

17. If a single particle approaches a potential barrier, its wavefunction is always completely transmitted if it has kinetic energy above the height of the barrier.

☐ T ■ **F** In quantum mechanics, particles can reflect off potential barriers even when their energy exceeds the barrier height

Lecture 6

1. An uncertainty relation exists for every pair of observables whose quantum mechanical operators do not commute.

   ■ **T** ☐ F $\sigma_{\hat{A}} \cdot \sigma_{\hat{B}} \geq \frac{1}{2}|\langle[\hat{A}, \hat{B}]\rangle|$

2. Any two eigenfunctions of momentum, $f_{p'}$ and f_p, satisfy Kronecker orthonormality.

   ☐ T ■ **F** They satisfy Dirac orthonormality: $\langle f_{p'}|f_p\rangle = \delta(p - p')$

3. In the equation $\Delta E \Delta t \geq \frac{\hbar}{2}$, Δt represents the standard deviation for measurement of time.

   ☐ T ■ **F** Δt represents time it takes the expectation value of $\hat{H}$ to change substantially (by one std.deviation). Time is the independent variable!

4. If a quantum mechanical system is in a mixture of two non-degenerate eigenstates for a given observable, measurements of this observable are probabilistic.

   ■ **T** ☐ F If $\psi = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$, the system is in a superposition state, and the probability of measuring either ψ_1 or ψ_2 eigenstate is 50%. This is because the square of the amplitude, $|\frac{1}{\sqrt{2}}|^2$, for each state equals $\frac{1}{2}$, indicating a 50% chance for observing the system in either state upon measurement.

5. A non-zero uncertainty relation exists for any two observables whose operators do not commute.

   ■ **T** ☐ F Generealizied Uncertainty Principle for non-commuting operators: $\sigma_A^2 \cdot \sigma_B^2 \geq \left(\frac{1}{2i}\langle[\hat{A}, \hat{B}]\rangle\right)^2$

6. If an operator $\hat{Q}$ represents a quantum-mechanical observable and $\frac{\partial \hat{Q}}{\partial t} = 0$, then $\frac{d\langle Q\rangle}{dt}$ must also be zero.

   ☐ T ■ **F** $\frac{d\langle Q\rangle}{dt} = \frac{i}{\hbar}\langle[\hat{H}, \hat{Q}]\rangle + \langle\frac{\partial \hat{Q}}{\partial t}\rangle$, it also depends on $\frac{i}{\hbar}\langle[\hat{H}, \hat{Q}]\rangle$.

7. The eigenstate $\psi_1(x)$ of a particle in an infinite square well can be written as a linear combination of the momentum eigenfunctions.

   ■ **T** ☐ F As the momentum eigenfunctions are complete.

8. The energy-time uncertainty principle comes directly from the generalized uncertainty principle.

   ☐ T ■ **F** Time is an independent variable, σ_t does not exist because we do not measure time.

9. A and B are incompatible quantum mechanical observables. We measure first A, then B, and then A again. The second measurement of A will always yield the same value as the first measurement of A.

☐ T ■ **F** The second measurement is probabilistic as A and B do not share the same eigenfunctions

10. Any two incompatible observables have an uncertainty relation.

■ **T** ☐ F Generealizied Uncertainty Principle for non-commuting operators: $\sigma_A^2 \cdot \sigma_B^2 \geq \left(\frac{1}{2i}\langle[\hat{A}, \hat{B}]\rangle\right)^2$

11. When we measure a quantum mechanical observable $\hat{Q}$ that has a discrete set of eigenfunctions, we will always obtain one of the corresponding eigenvalues.

■ **T** ☐ F Superposition of eigenstates in a discrete spectrum

12. The uncertainty principle applies to many incompatible observables, not just $\hat{x}$ and $\hat{p}$.

■ **T** ☐ F Generealizied Uncertainty Principle for non-commuting operators: $\sigma_A^2 \cdot \sigma_B^2 \geq \left(\frac{1}{2i}\langle[\hat{A}, \hat{B}]\rangle\right)^2$

13. Given a general wavefunction $\Psi(x, t)$ and two compatible observables, $\hat{A}$ and $\hat{B}$, any measurement of $\hat{B}$ yields the same result as a measurement of $\hat{A}$ and then $\hat{B}$.

☐ T ■ **F** The compatibility of observables $\hat{A}$ and $\hat{B}$, indicated by $[\hat{A}, \hat{B}] = 0$, means their measurements do not affect each other's outcomes due to a common set of eigenstates. However, the result of a measurement depends on the system's state, not the measurement sequence, leading to potentially different outcomes.