Problem Set 5

Problem 1

Prove the generalized uncertainty principle:

$$\sigma_{\hat{A}}^2\sigma_{\hat{B}}^2 \geq (\frac{1}{2i}\langle[\hat{A},\hat{B}]\rangle)^2$$

where $\hat{A}$ and $\hat{B}$ are linear Hermitian operators. Hint: the proof is on p. 110 of Griffiths. I just want you to go over it and understand it.

Problem 2

1. Show that $[\hat{A}\hat{B}, \hat{C}] = \hat{A}[\hat{B}, \hat{C}] + [\hat{A}, \hat{C}]\hat{B}$

2. Show that $[\hat{x}^n, \hat{p}] = i\hbar n x^{n-1}$

3. Show that $[f(\hat{x}), \hat{p}] = i\hbar \frac{df}{dx}$

Problem 3

Prove the specific uncertainty relation that is written below, which is between position ($\hat{A} = x$) and energy ($\hat{B} = \hat{H}$):

$$\sigma_{\hat{x}}\sigma_{\hat{H}} \geq \frac{\hbar}{2m}|\langle \hat{p} \rangle|$$

For a stationary state, does this relation tell you anything?

Problem 4

Given a system that is in a linear combination of infinite-square-well stationary states,

$$\Psi(x,t) = \frac{1}{2}[\psi_1(x)\exp\left(-\frac{iE_1 t}{\hbar}\right) + \psi_2(x)\exp\left(-\frac{iE_2 t}{\hbar}\right)]$$

calculate:

1. $\sigma_{\hat{H}}$

2. $\sigma_{\hat{x}}$

3. $\frac{d\langle \hat{x} \rangle}{dt}$

4. Then test the time-energy uncertainty relationship:

$$\sigma_{\hat{x}}^2 \sigma_{\hat{H}}^2 \geq \left(\frac{\hbar}{2}\right)^2 \left(\frac{d\langle \hat{x} \rangle}{dt}\right)^2$$

Note: some answers from problems 1 and 2 from Problem Set 2 will be helpful here.