Causal Inference for Observational Longitudinal Survival Data

Mark Bech Knudsen

Context

Thesis written in blocks 1 & 2 autumn 2022, supervised by Torben Martinussen & Helene Rytgaard @ Biostatistics KU.

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Thesis explores 3 topics relating to causal inference for non-randomized survival data:

- Do hazard ratio estimates from the Cox model have a causal interpretation for a time-dependent treatment?
- A problem with Markov models for survival data with time-dependent covariates.
- Causal inference in continuous-time using inverse probability of treatment weights.

Defining causal effects

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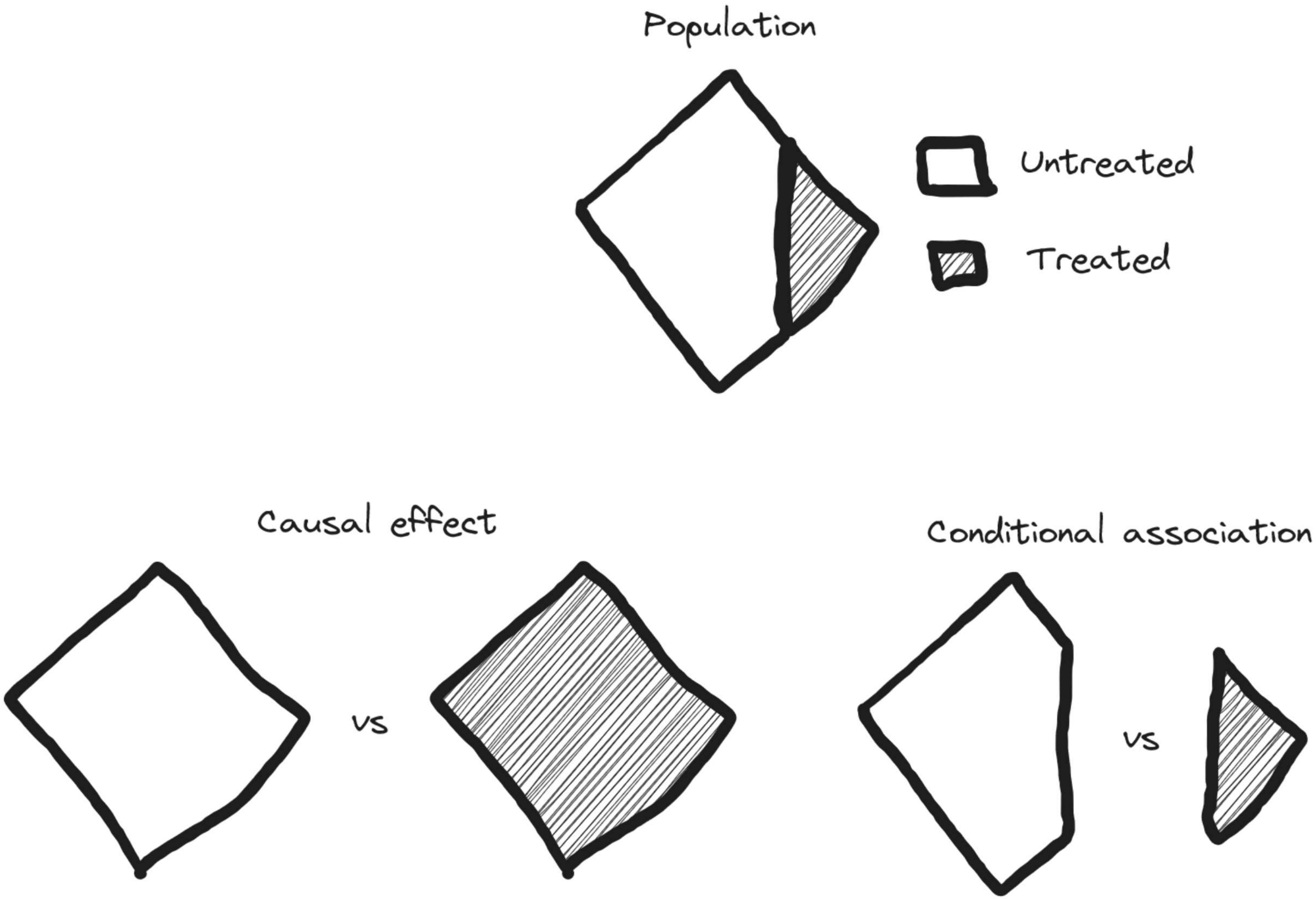
Different from the conditional association $P(T > t \mid A = 1) - P(T > t \mid A = 0)$.

Causal vs. conditional effects



*Inspired by Miguel A. Hernán: A definition of causal effect for epidemiological research (2004)

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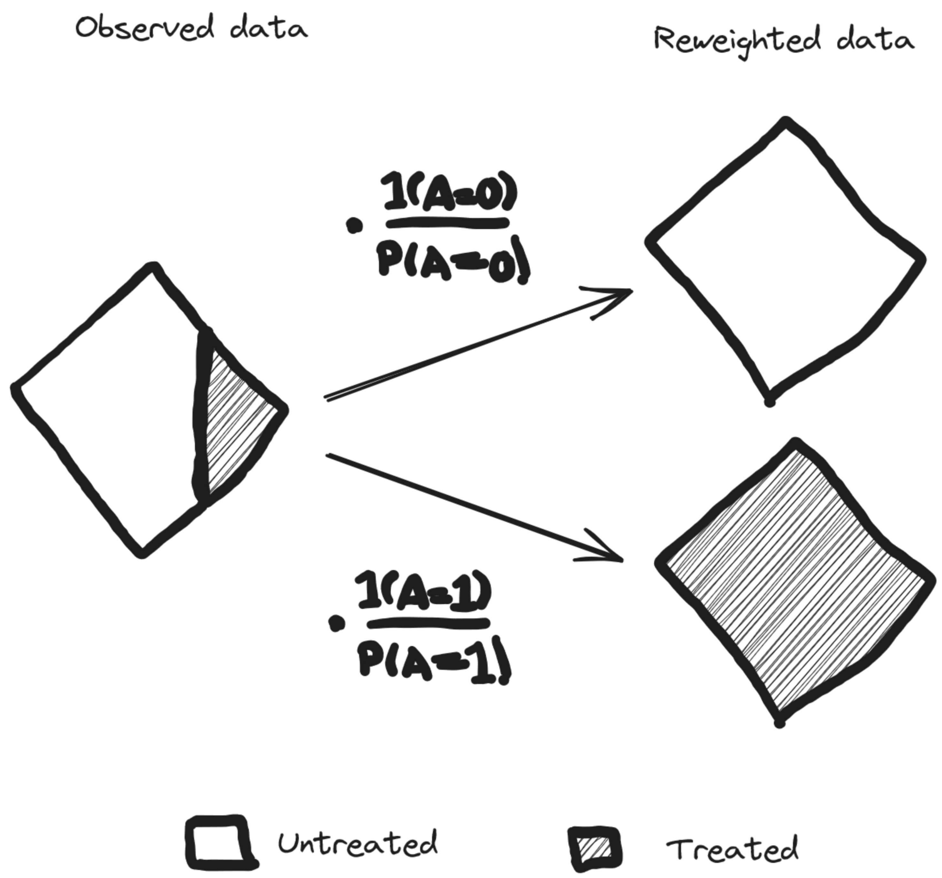

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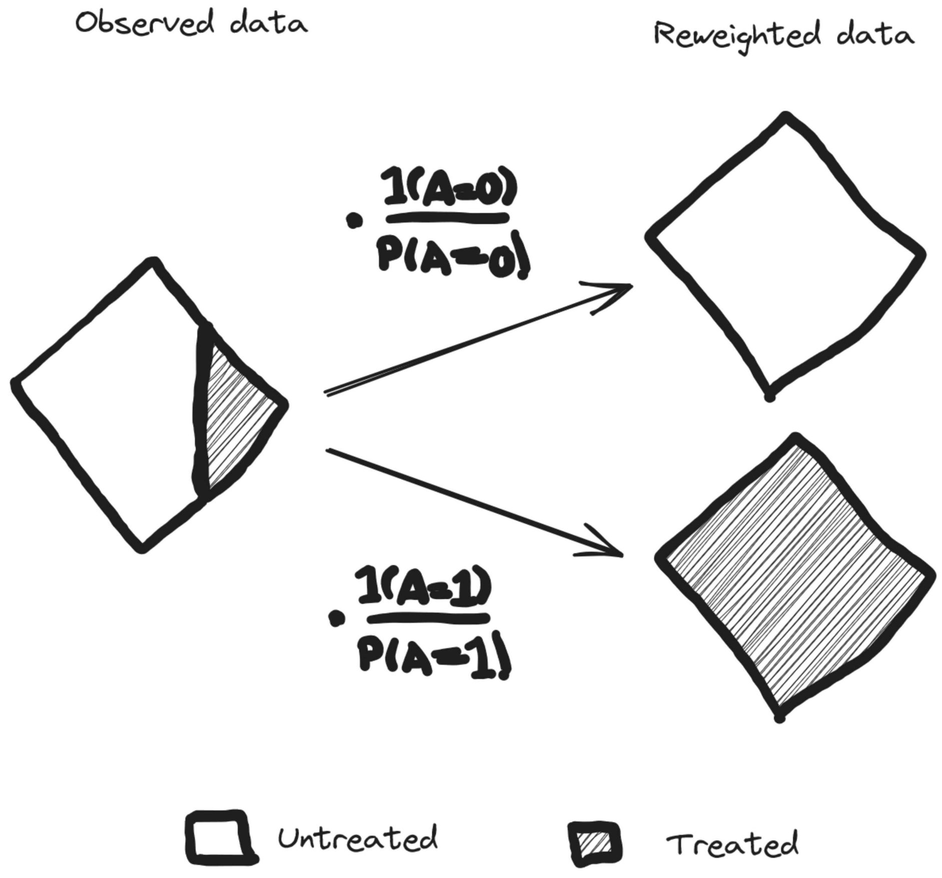

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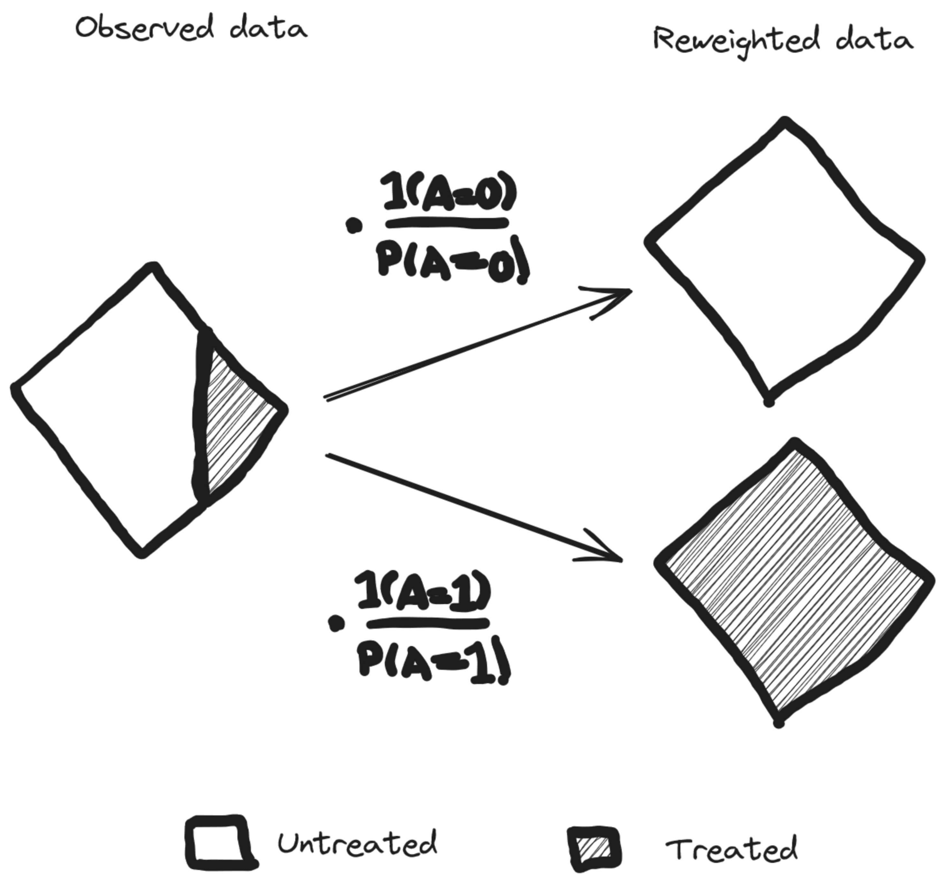
The W_i^a are referred to as *inverse probability of treatment weights (IPTW)*.



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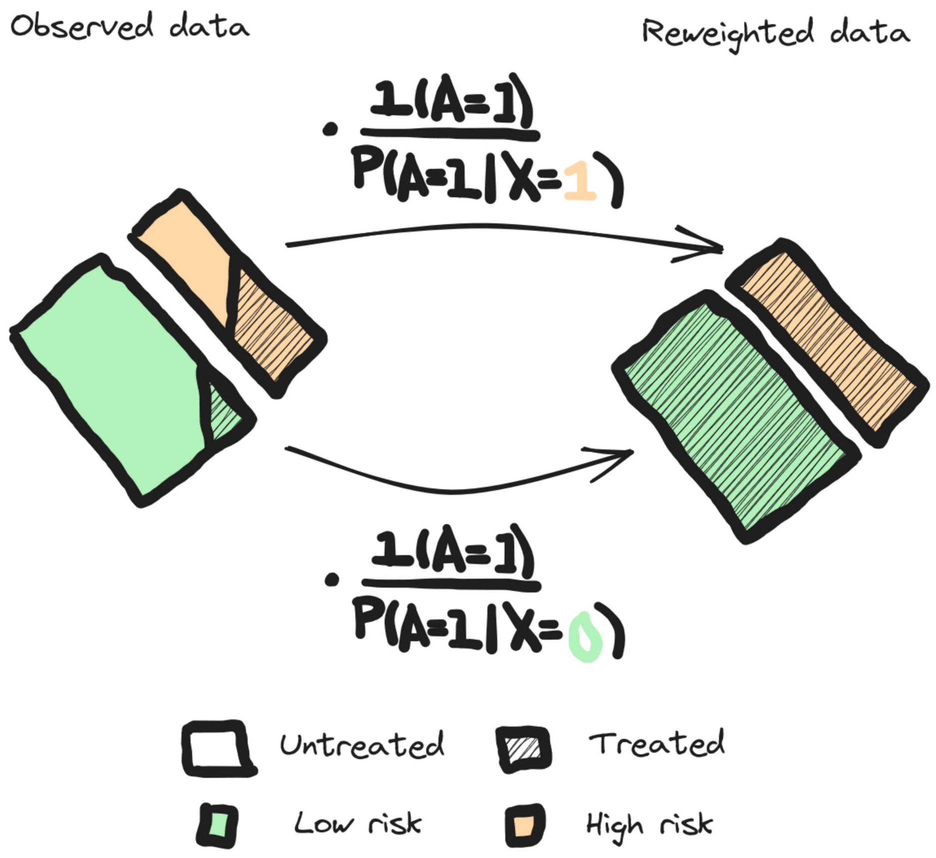

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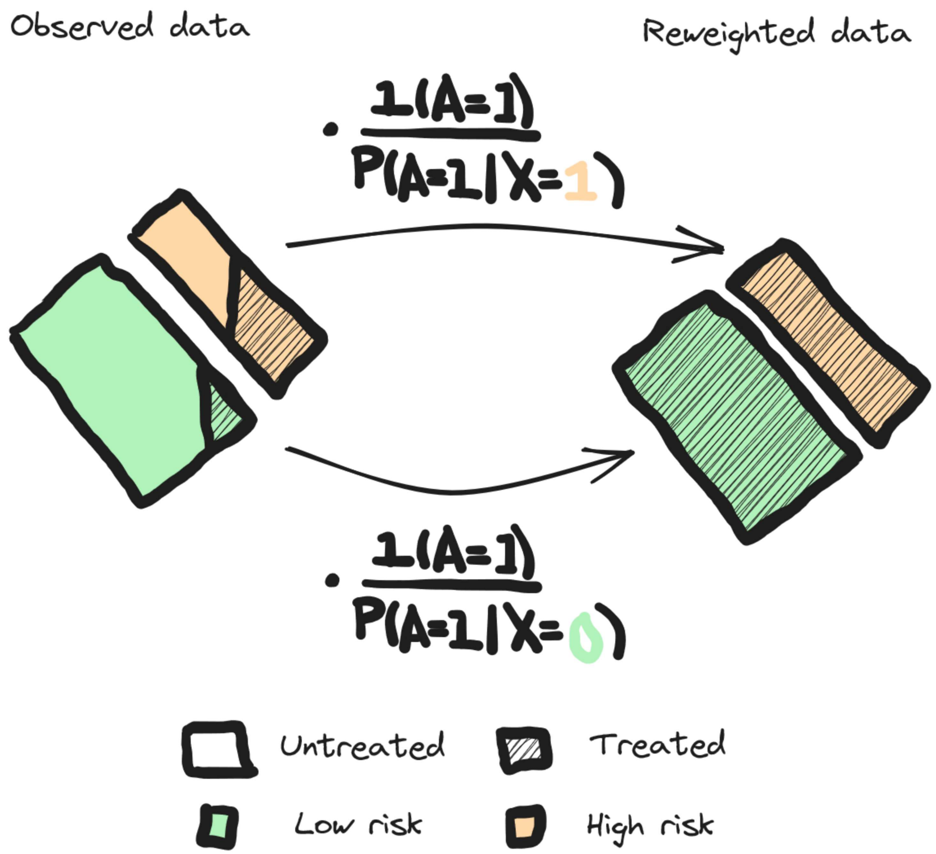

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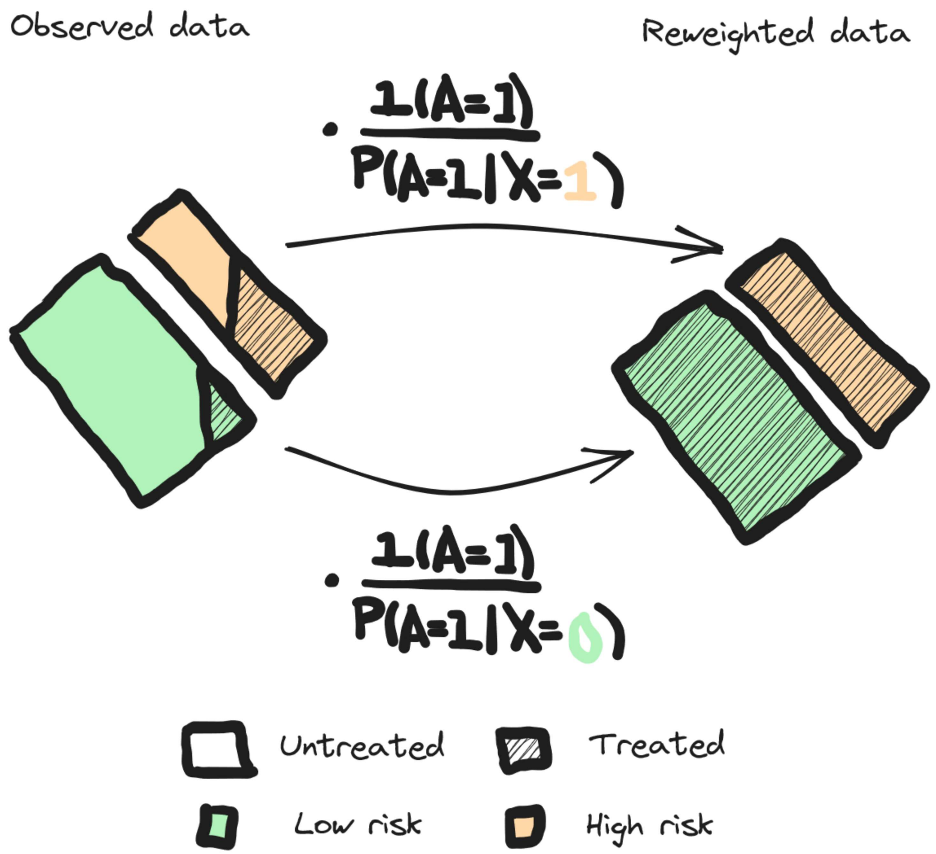
This works regardless of the dimension of X, and also for continuous covariates.



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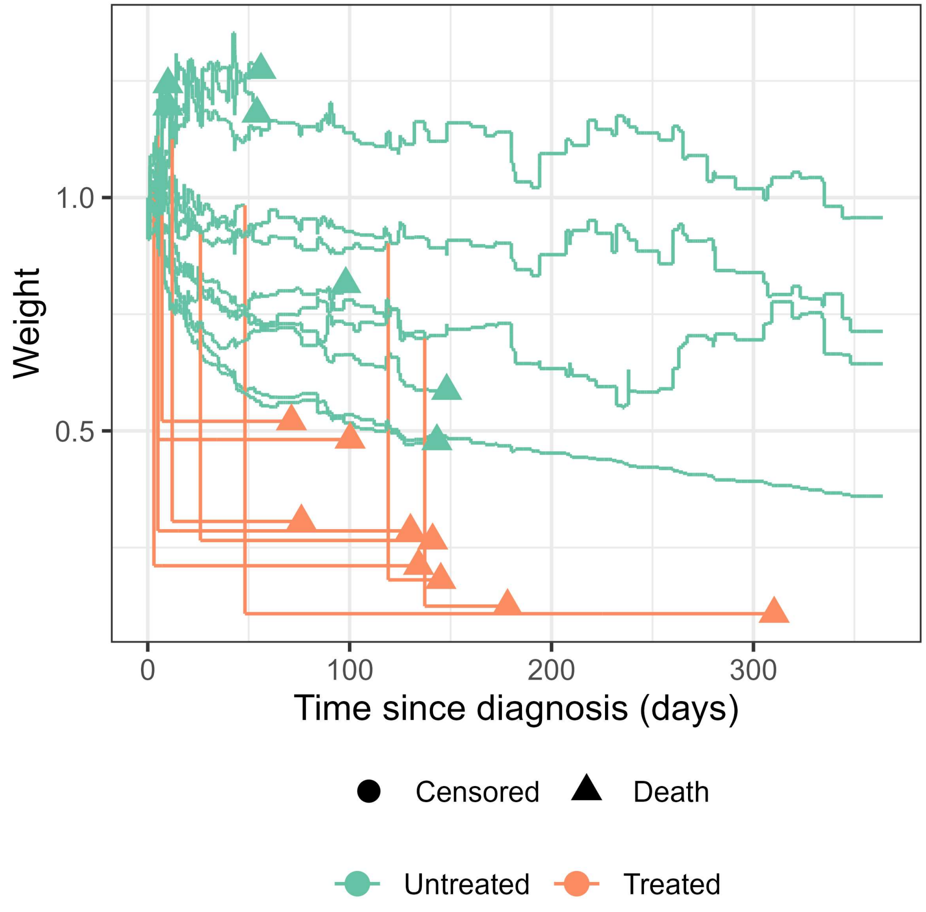

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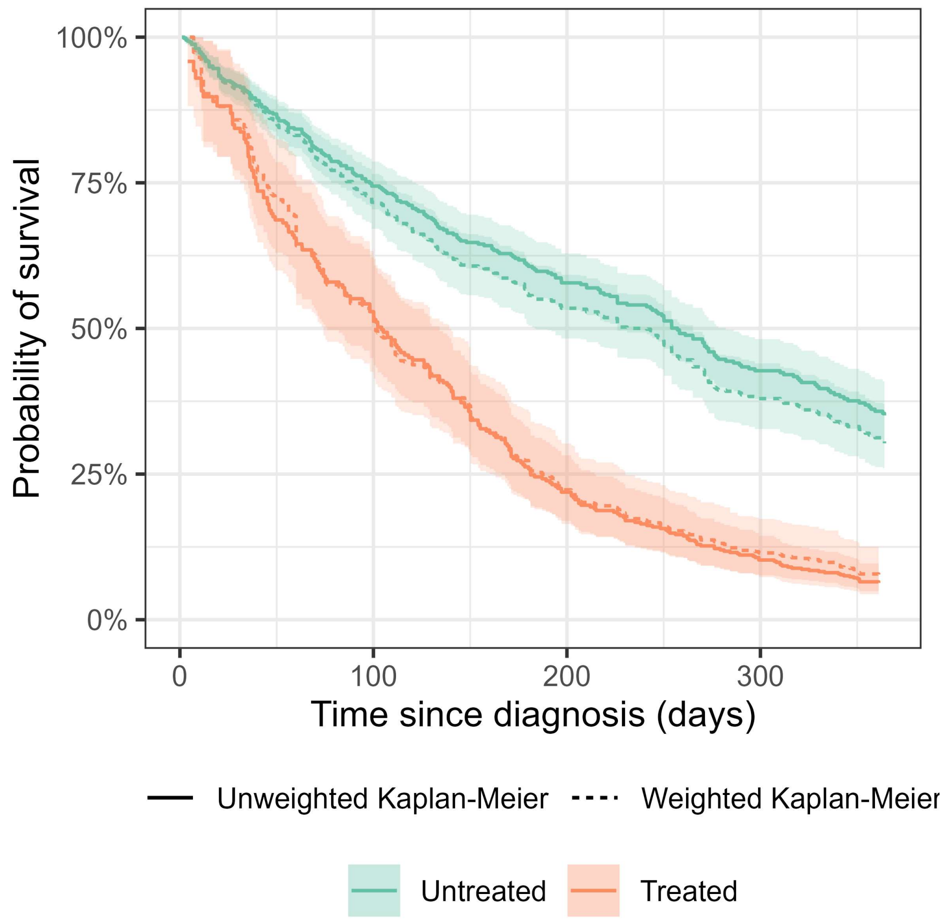

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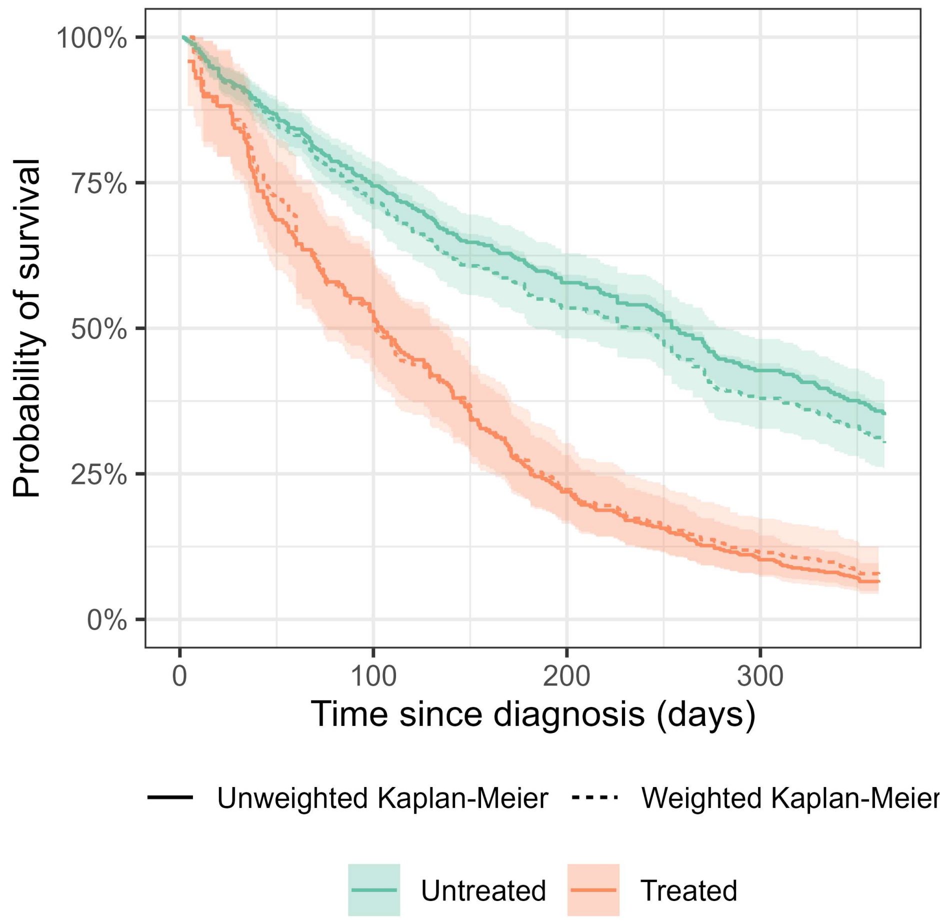

Weighting does not change the conclusion much. But we did not have enough information about *time-dependent* confounders.

Thank you!

And good luck when you write your thesis.