Introduction to Electricity, Magnetism, and Optics, Physics 1402

Practice Midterm Quiz.

You should complete all four problems (25 points each). Some useful information is included on the last three pages.

1. Triangle of charges.

Two particles, each with charge q and mass m, are placed at corners of an equilateral triangle with side length d, as shown in Fig. 1.

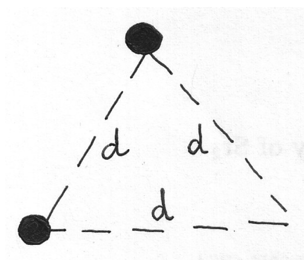


FIG. 1: Charges on an equilateral triangle.

(a) Find the electric field $\vec{E}$ at the empty corner of the triangle.

(b) Find the electric potential V at the empty corner.

(c) If the triangle is completed with a third (identical) charge, and the system is released from rest, find the speed v of each charge when they are far apart.

(d) Compute v from part (c) if $d = 1$ cm and the particles are protons.

2. Particle trajectory.

A charged particle is traversing a circular trajectory with period T in a magnetic field $\vec{B}$, as in Fig. 2.

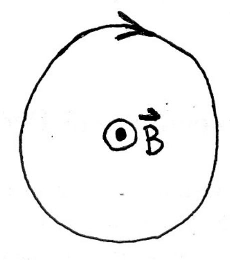


FIG. 2: Particle trajectory. Magnetic field points 'out of the page'.

(a) Is the charge of the particle positive or negative?

(b) Find the charge-to-mass ratio of the particle, q/m.

(c) An electric field $\vec{E}$ is added along the direction of $\vec{B}$. What is the trajectory now?

(d) Find the acceleration of the particle along $\vec{E}$.

3. Charged hollow sphere.

A hollow sphere has inner radius a and outer radius b, and carries a uniform charge density ρ, as shown in Fig. 3.

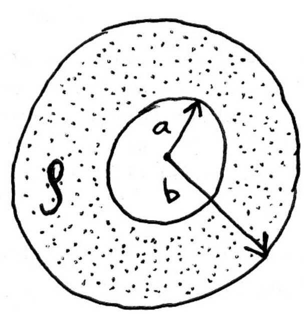


FIG. 3: Uniformly charged hollow sphere.

(a) Find the electric field $\vec{E}$ everywhere in space.

(b) Plot E from part (a).

4. Qualitative physics.

(a) A Van de Graaff generator consists of a moving rubber belt that deposits large quantities of charge on a hollow spherical conductor. When a stack of aluminum pie tins is placed on top of the sphere, they fly away from the sphere one by one. Explain why this happens.

(b) A piece of resistive material is connected in series with a battery and a capacitor in a closed loop. If the length of the resistor is doubled, does the capacitor take more or less time to charge to half the maximum charge? By what factor?

(c) Magnetic field $\vec{B}$ points upward from the ground. When a copper cube is tossed to the east, it develops a voltage V between two faces. Between which pair of faces should V be measured? Which of the two faces is at a higher potential?

Physics C1402 Formula Sheet

$$i = \frac{dq}{dt}$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} \quad \text{(Coulomb's Law)}$$

$$\vec{E} = \frac{\vec{F}}{q_0}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \quad \text{(point charge)}$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3} \quad \text{(dipole)}$$

$$|\vec{p}| = qd \quad \text{(electric dipole moment)}$$

$$\vec{F} = q\vec{E} \quad \text{(point charge)}$$

$$\vec{\tau} = \vec{p}\times\vec{E} \quad \text{(electric dipole)}$$

$$U = -\vec{p}\cdot\vec{E} \quad \text{(electric dipole)}$$

$$\Phi_E = \int \vec{E}\cdot d\vec{A} \quad \text{(electric flux)}$$

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \quad \text{(Gauss' Law, electric)}$$

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{(\textit{close} to \textit{any} conducting surface)}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \text{(line of charge)}$$

$$E = \frac{\sigma}{2\varepsilon_0} \quad \text{(sheet of charge)}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \text{(outside sph. shell of charge)}$$

$$E = 0 \quad \text{(inside spherical shell of charge)}$$

$$E = \frac{q}{4\pi\varepsilon_0 R^3} r \quad \text{(inside uniform sph. of charge)}$$

$$\Delta U = -W^{elec}$$

$$U = -W_\infty^{elec}$$

$$\Delta V = \frac{-W^{elec}}{q}$$

$$V = \frac{-W_\infty^{elec}}{q}$$

$$\Delta K = W^{appl} + W^{elec}$$

$$\Delta V = -\int \vec{E}\cdot d\vec{s}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \quad \text{(point charge)}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \quad \text{(dipole: } r >> d\text{)}$$

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

$$E_s = -\frac{\partial V}{\partial s}$$

$$U = W = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

$$q = CV$$

$$C = \frac{\varepsilon_0 A}{d} \quad \text{(parallel-plate capacitor)}$$

$$C = 2\pi\varepsilon_0\frac{L}{\ln(b/a)} \quad \text{(cylindrical capacitor)}$$

$$C = 4\pi\varepsilon_0\frac{ab}{b-a} \quad \text{(spherical capacitor)}$$

$$C = 4\pi\varepsilon_0 R \quad \text{(isolated sphere)}$$

$$C_{eq} = \sum_{i=1}^{n} C_i \quad \text{(capacitors in parallel)}$$

$$\frac{1}{C_{eq}} = \sum_{i=1}^{n}\frac{1}{C_i} \quad \text{(capacitors in series)}$$

$$U_E = \frac{q^2}{2C} = \frac{1}{2}CV^2$$

$$C = \kappa C_{air} \quad \text{(with dielectric const. } \kappa\text{)}$$

$$\varepsilon_0\oint \kappa\vec{E}\cdot d\vec{A} = q \quad \text{(Gauss' Law with dielectric)}$$

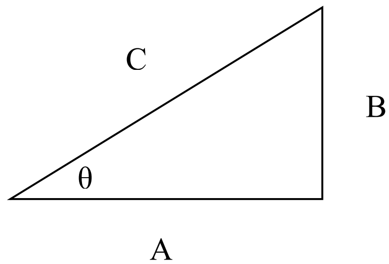
$$i = \frac{dq}{dt}$$
$$i = \int \vec{J} \cdot d\vec{A}$$
$$\vec{J} = ne\vec{v}_d$$
$$R = \frac{V}{i}$$
$$\rho = \frac{1}{\sigma} = \frac{E}{J}$$
$$\vec{E} = \rho\vec{J}$$
$$R = \rho\frac{L}{A}$$
$$\rho = \frac{m}{e^2 n\tau}$$
$$P = iV$$
$$P = i^2 R = \frac{V^2}{R} \quad \text{(for resistor)}$$

$$\varepsilon = \frac{dW}{dq}$$
$$i = \frac{\varepsilon}{R}$$
$$P_{emf} = i\varepsilon$$
$$R_{eq} = \sum_{i=1}^{n} R_i \quad \text{(resistors in series)}$$
$$\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i} \quad \text{(resistors in parallel)}$$
$$q = C\varepsilon(1 - e^{-t/RC}) \quad \text{(charging a capacitor)}$$
$$q = q_0 e^{-t/RC} \quad \text{(discharging a capacitor)}$$
$$\tau_C = RC$$

$$\vec{F}_B = q(\vec{v} \times \vec{B}) = |q| vB\sin\phi$$
$$\vec{F} = q(\vec{E} + (\vec{v} \times \vec{B})) \quad \text{(Lorentz force)}$$
$$n = \frac{Bi}{Vle} \quad \text{(Hall effect)}$$
$$|q| vB = \frac{mv^2}{r} \quad \text{(circular motion in B field)}$$
$$\vec{F}_B = i(\vec{L} \times \vec{B}) \quad \text{(force on current/wire)}$$
$$\tau = (NiA)B\sin\theta \quad \text{(torque on coil)}$$

$$|\vec{\mu}| = NiA \quad \text{(magnetic dipole moment)}$$
$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad \text{(magnetic dipole)}$$
$$U = -\vec{\mu} \cdot \vec{B} \quad \text{(magnetic dipole)}$$

$$\sin\theta = \frac{B}{C}$$
$$\cos\theta = \frac{A}{C}$$
$$\tan\theta = \frac{B}{A}$$
$$C^2 = A^2 + B^2$$



If $ax^2 + bx + c = 0$, then: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = ab\cos\phi$ (either angle between $\vec{a}$ and $\vec{b}$ may be used)

$\left|\vec{a} \times \vec{b}\right| = (a_y b_z - b_y a_z)\hat{i} + (a_z b_x - b_z a_x)\hat{j} + (a_x b_y - b_x a_y)\hat{k} = ab\sin\phi$ (use smaller angle; direction of resulting vector from Right Hand Rule)

In the following, *u* and *v* are functions of *x*:

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$
$$\frac{d}{dx}\sin x = \cos x$$
$$\frac{d}{dx}\cos x = -\sin x$$
$$\frac{d}{dx}\sin u = \cos u\frac{du}{dx}$$
$$\frac{d}{dx}\cos u = -\sin u\frac{du}{dx}$$
$$\frac{d}{dx}e^u = e^u\frac{du}{dx}$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$
$$\int \sin x\,dx = -\cos x$$
$$\int \cos x\,dx = \sin x$$
$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2})$$
$$\int \frac{x\,dx}{\left(x^2 + a^2\right)^{3/2}} = -\frac{1}{(x^2 + a^2)^{1/2}}$$
$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2(x^2 + a^2)^{1/2}}$$

Circle of radius *r*: circumference = $2\pi r$, area = πr^2

Sphere or radius *r*: area = $4\pi r^2$, volume = $\frac{4}{3}\pi r^3$

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g = 9.81 m/s^2

c = 3.00×10^8 m/s

$\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/N·m^2 = 8.85×10^{-12} F/m

k = $1/4\pi\varepsilon_0 = 8.99 \times 10^9$ N·m^2/C^2

$\mu_0 = 4\pi \times 10^{-7}$ T.m/A = $4\pi \times 10^{-7}$ H/m

e = 1.60×10^{-19} C

$m_e = 9.11 \times 10^{-31}$ kg

$m_p = 1.67 \times 10^{-27}$ kg