THE SYNCHROTRON MOTION SIMULATOR FOR ADIABATIC CAPTURE STUDY IN THE TLS BOOSTER

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Abstract

The synchrotron motion simulator is invented to simulate the particle motion in RF (Radio Frequency) for a ring accelerator. It is especially used to study the efficiency of adiabatic capture for a booster ring. The purpose of adiabatic capture is to optimize RF settings during the ramping of beam energy and obtain the greatest efficiency of particle capture. In this paper we study the synchrotron motion for particles in the TLS (Taiwan Light Source) booster. We compare the properties of TLS booster as a proton or electron accelerators, using the same ramping scenario of beam energy, and optimize the RF settings to have the best efficiency.

INTRODUCTION

The TLS booster is a combined function FODO lattice with twelve periods, the circumference is 72 m. Its original designed is to boost electrons energy from 50 (MeV) to 1.5 (GeV). It has been operated for more than twenty years. The new accelerator TPS (Taiwan Photon Source) is planned to replace the TLS. If the TLS storage ring is decided to decommission in future, the TLS booster can be considered to transfer to a proton accelerator for nuclear or medical researches.

The capture efficiency for a proton booster is required to be high enough to minimize the proton losses. Since the loss of protons would cause radiation contaminations and endanger the human body or environment. Given the ranges of RF operation and ramping scenario of the beam energy, we can search for the best settings for adiabatic capture by synchrotron motion simulator. In the case of TLS booster, we plan to accelerate protons with the kinetic energy from 7 (MeV) to 300 (MeV) in the time period 50 (ms). The operation range of RF voltage is from 0 to 15 (kV), and the harmonic number is 2.

EVOLUTION OF SYNCHROTRON PHASE-SPACE ELLIPSE

The RF cavity is operated in a resonance condition to provide accelerating voltage, i.e. longitudinal electric field, to particles. For simplicity, we do not consider the effects of synchro-betatron coupling. The synchrotron equations of motion can be derived from Hamiltonian. For the beam acceleration, it is suitable to choose the phase-space mapping equation in the coordinates $(\phi, \Delta E)$, where ϕ is the phase of RF voltage and ΔE is the change of a particle energy per revolution. Let n is the turn number for a particle in a ring accelerator. The evolution equations of synchrotron motion in phase-space $(\phi, \Delta E)$ are derived from [1]:

$$\begin{aligned} \Delta E_{n+1} &= \Delta E_n + eV(\sin\phi_n - \sin\phi_s) - C_\gamma \frac{E^4}{\rho}, \\ \phi_{n+1} &= \phi_n + \frac{2\pi h \eta}{\beta^2 E}\Delta E_{n+1}, \\ \text{where} & \begin{cases} eV\sin\phi_s = E(t_{n+1}) - E(t_n), \\ \gamma = \frac{E}{m_0 c^2}, \\ \beta = \sqrt{1-\frac{1}{\gamma^2}}, \\ v = \beta c, \\ \eta = \alpha_c - \frac{1}{\gamma^2}, \\ \frac{\Delta E}{\beta^2 E} = \frac{\Delta P}{P}. \end{cases} \end{aligned} \tag{1}$$

e is particle charge, V is RF voltage, E is the total energy of a particle, ρ is the local radius of curvature for a dipole magnet, h is the harmonic number for RF, ϕ_s is the phase factor for RF, m_0 is the stationary mass of a particle, v is particle velocity, α_c is the momentum compaction factor for a ring accelerator and P is the momentum of a particle. The radiation power coefficient C_γ is deduced from Larmor's theorem, which dependents on different kind of particles:

$$\begin{aligned} C_\gamma &= \frac{4\pi}{3}\frac{r_0}{(m_0c^2)^3} \\ &= \begin{cases} 8.846\times 10^{-5}\ \mathrm{m/(GeV)^3} \text{ for electrons}, \\ 7.783\times 10^{-18}\ \mathrm{m/(GeV)^3} \text{ for protons}, \end{cases} \end{aligned} \tag{2}$$

where $r_0 = e^2/4\pi\epsilon_0 m_0 c^2$ is the classical radius for a particle.

Note that V and E can be as functions of time during the ramping of beam energy. The V as a function of time in a booster ring is modeled according to [2]:

$$V(t) = \begin{cases} \left[3\left(\frac{t}{T_\nu}\right)^2 - 2\left(\frac{t}{T_\nu}\right)^3\right](V_f - V_i) + V_i \\ \qquad\qquad \text{for } 0 \le t \le T_\nu, \\ V_f \ \text{ for } t > T_\nu, \end{cases} \tag{3}$$

where T_ν the adiabatic capture time, V_i and V_f are initial and final RF voltages, respectively. The setting of T_ν would affects the efficiency for adiabatic capture. The total energy E as a function of time in a ramping cycle is modeled as:

$$\begin{aligned} E(t) &= m_0c^2 + K(t), \\ K(t) &= \left(\frac{K_f - K_i}{2}\right)\left[\left(\frac{K_f+K_i}{K_f-K_i}\right) - \cos(2\pi f t)\right], \end{aligned} \tag{4}$$

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where $K(t)$ is the kinetic energy as a function of time, K_i and K_f are initial and final kinetic energies for a particle in a ramping cycle. f is the booster ramping frequency. $K(t)$ is proportional to the bending magnetic field $B(t)$:

$$B(t) = \left(\frac{B_f - B_i}{2}\right)\left[\left(\frac{B_f + B_i}{B_f - B_i}\right) - \cos(2\pi f t)\right], \quad (5)$$

where B_i and B_f are initial and final bending magnetic fields in a ramping cycle. So in Eq. 1 the change of total energy for a particle per revolution is equal to the change of kinetic energy:

$$eV \sin\phi_s = E(t_{n+1}) - E(t_n) = K(t_{n+1}) - K(t_n), \quad (6)$$

where $t_{n+1} - t_n$ is the revolution period for a particle between n and $n+1$ turns:

$$t_{n+1} - t_n = \frac{L}{v(E(t_n))}. \quad (7)$$

L is the circumference for a ring accelerator, and the particle velocity v is the function of particle energy at the time t_n as shown in Eq. 1. The RF phase factor ϕ_s is thus calculated by

$$\phi_s = \sin^{-1}\left[\frac{K(t_{n+1}) - K(t_n)}{eV}\right]. \quad (8)$$

Note that this equation also sets the maximum rate for the ramping of beam energy with $0 \leq [K(t_{n+1}) - K(t_n)] < eV$. For a heavy particle ramping, if it is started at low kinetic energy, the particle velocity is not close to the speed of light. So the revolution period would be changed obviously at the early stage of energy ramping. Since ϕ_s is dependent on the change of kinetic energy per revolution period, it is also changed obviously, as shown in the top left and right of Fig. 2.

The synchrotron tune Q_s is calculated by

$$Q_s = \nu_s\sqrt{|\cos\phi_s|}, \quad \text{where} \quad \nu_s = \sqrt{\frac{h|\eta|eV}{2\pi\beta^2 E}}, \quad (9)$$

if $\eta \neq 0$. The adiabatic coefficient α_{ad} is then defined as

$$\alpha_{\mathrm{ad}} = \frac{1}{2\pi}\left|\frac{dT_s}{dt}\right|, \quad \text{where} \quad T_s = T_0/Q_s, \quad (10)$$

T_0 is the revolution period. The condition for adiabatic synchrotron motion is $\alpha_{\mathrm{ad}} \ll 1$. The phase-space area enclosed by the separatrix is called the bucket area, $\tilde{\mathcal{A}}_{\mathrm{B}}$, which is approximated as

$$\tilde{\mathcal{A}}_{\mathrm{B}} \approx \frac{16 Q_s}{h|\eta|\sqrt{|\cos\phi_s|}}\left(\frac{1 - \sin\phi_s}{1 + \sin\phi_s}\right). \quad (11)$$

Since bucket area is the maximum size of a beam we can make, we should avoid the zero bucket area with $\phi_s = 90°$ from Eq. 8.

OPTIMIZE THE TLS BOOSTER AS A PROTON OR ELECTRON MACHINE

For the TLS booster, the fixed parameters are $L = 72$ (m), $\rho = 5$ (m), $\alpha_c = 0.1346$, $K_i = 7$ (MeV), $K_f = 300$ (MeV) and $f = 10$ (Hz). For RF cavity, the fixed parameters are $h = 2$ and $V_f = 15$ (kV). The kinetic energy as a function of time for particles in a ramping cycle is shown in Fig. 1, which is based on Eq. 4. The RF voltage variables T_ν and V_i are crucial parameters to be optimized for the best efficiency of adiabatic capture. We assume the initial conditions for a bunch of the beam is flatly distributed in the phase $\phi = [-\pi,\ \pi]$ (rad), and the distribution of $\Delta P/P$ ($\Delta E/E$) for protons (electrons) is a Gaussian with zero mean and $\sigma = \pm 0.05\%$ or $\sigma = \pm 0.5\%$, i.e., the width of $\Delta P/P$ ($\Delta E/E$) is 0.1% or 1% for protons (electrons). Two thousand particles are generated to represent a bunch of the beam ($N = 2000$). We track these particles by the synchrotron motion simulator and obtain the capture efficiency for particles in a ramping cycle.

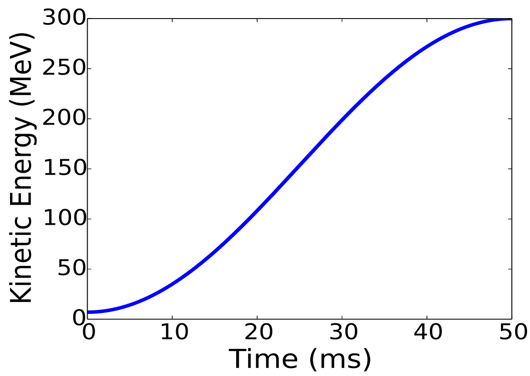

Figure 1: The kinetic energy as a function of time in a ramping cycle (based on Eq. 4), where $K_i = 7$ (MeV), $K_f = 300$ (MeV) and $f = 10$ (Hz).

For protons (electrons) acceleration, it takes about 95,500 (208,151) turns to accomplish a ramping cycle. The properties of adiabatic capture for TLS booster as a proton (electron) machine are shown in the Fig. 2 (Fig. 3). We scan the ranges of $T_\nu = [0.1,\ 4.9]$ (ms) and $V_i = [0.5,\ 15]$ (kV), obtain the capture efficiencies and find the best settings of T_ν and V_i for protons (electrons) in the TLS booster. Fig. 4 (Fig. 5) shows an example of the evolution of phase-space and capture rate for protons (electrons) in a ramping cycle. Fig. 6 (Fig. 7) shows the heat map of adiabatic capture efficiencies for protons (electrons) with respect to T_ν and V_i settings. Table 1 lists the RF voltage settings T_ν and V_i for the best efficiency of TLS booster for protons or electrons accelerations, and with different initial $\Delta P/P$ or $\Delta E/E$ distributions.

SUMMARY

We have taken the TLS booster as an example to study the adiabatic capture efficiency for beam energy ramping.

Table 1: The best capture efficiency with RF voltage settings T_ν and V_i for TLS booster

Beam type (initial Gaussian width)	T_ν (ms)	V_i (kV)	**Efficiency (%)**
protons ($\Delta P/P = 0.1\%$)	0.5	7.5	$99 \pm 2.2^*$
protons ($\Delta P/P = 1\%$)	0.1	7	$85.7 \pm 2.2^*$
electrons ($\Delta E/E = 0.1\%$)	0.1	12.5	$99.9 \pm 2.2^*$
electrons ($\Delta E/E = 1\%$)	0.1	12	$98 \pm 2.2^*$

* The standard error = $\sqrt{N}/N$, where $N = 2000$ is the total number of particles in simulation.

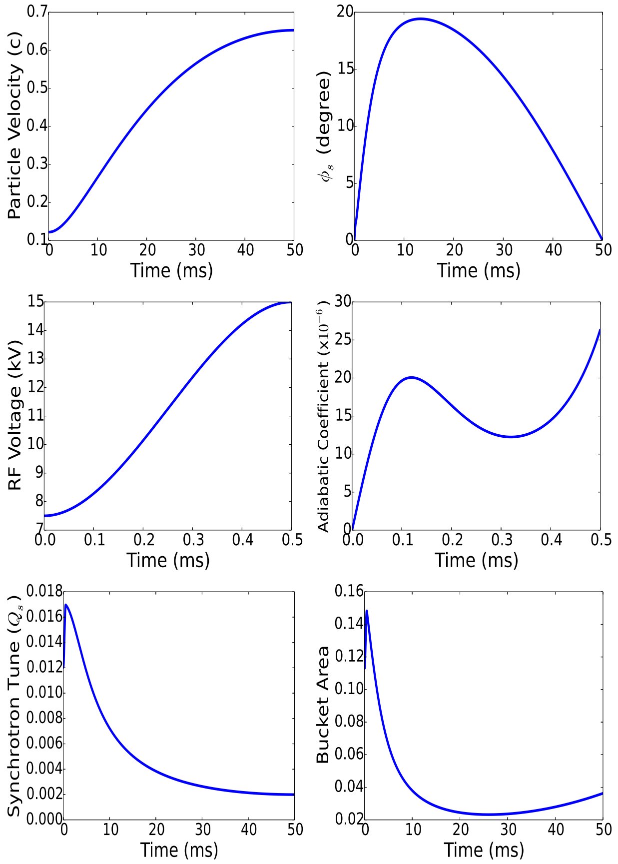

Figure 2: The proton velocity in the unit of light speed c (top left), RF phase factor ϕ_s (top right), RF voltage V (middle left), adiabatic coefficient α_{ad} (middle right), synchrotron tune Q_s (bottom left) and bucket area $\tilde{\mathcal{A}}_{\mathrm{B}}$ (bottom right) vs. time in a ramping cycle. Here we set $T_\nu = 0.5$ (ms) and $V_i = 7.5$ (kV).

For protons acceleration, it is better to have the initial beam condition with a small $\Delta P/P$ distribution and a small adiabatic capture time T_ν for RF voltage setting, in order to have the best efficiency. For electrons acceleration, the requirements for good capture efficiency is roughly same with protons. However the electrons acceleration usually has a better efficiency compared to protons or other heavy

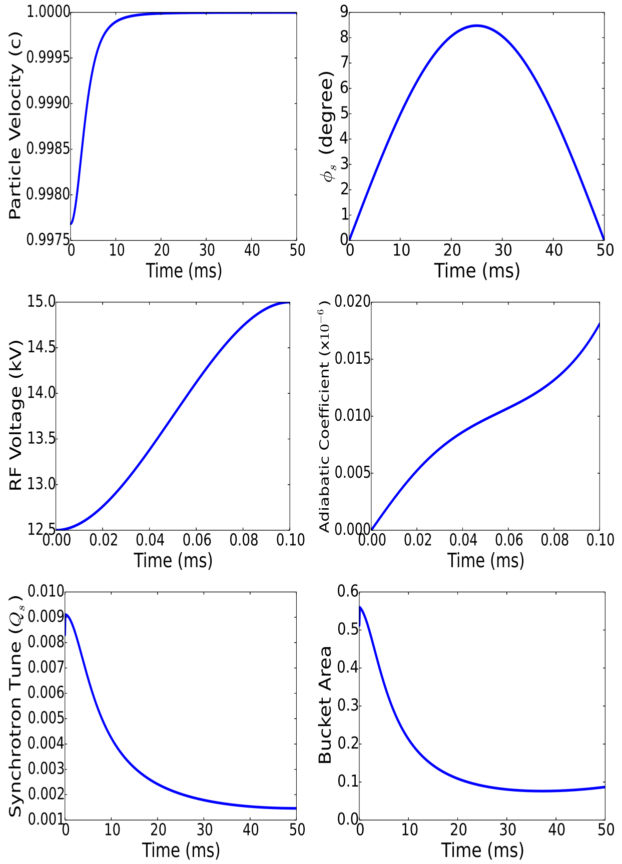

Figure 3: The electron velocity in the unit of light speed c (top left), RF phase factor ϕ_s (top right), RF voltage V (middle left), adiabatic coefficient α_{ad} (middle right), synchrotron tune Q_s (bottom left) and bucket area $\tilde{\mathcal{A}}_{\mathrm{B}}$ (bottom right) vs. time in a ramping cycle. Here we set $T_\nu = 0.1$ (ms) and $V_i = 12.5$ (kV).

particles, as shown in Table 1.

The synchrotron motion simulator is written in Python language which is easy to understand and execute on many OS platforms. It can be freely download from [3]. This program can be further developed for other kinds of particle accelerators, like carbon or heavy ion accelerators, for their design and optimization.

REFERENCES

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[3] C.C. Chiang, *Synchrotron Motion Simulator*, https://sourceforge.net/projects/synchrotronmotionsimulator/

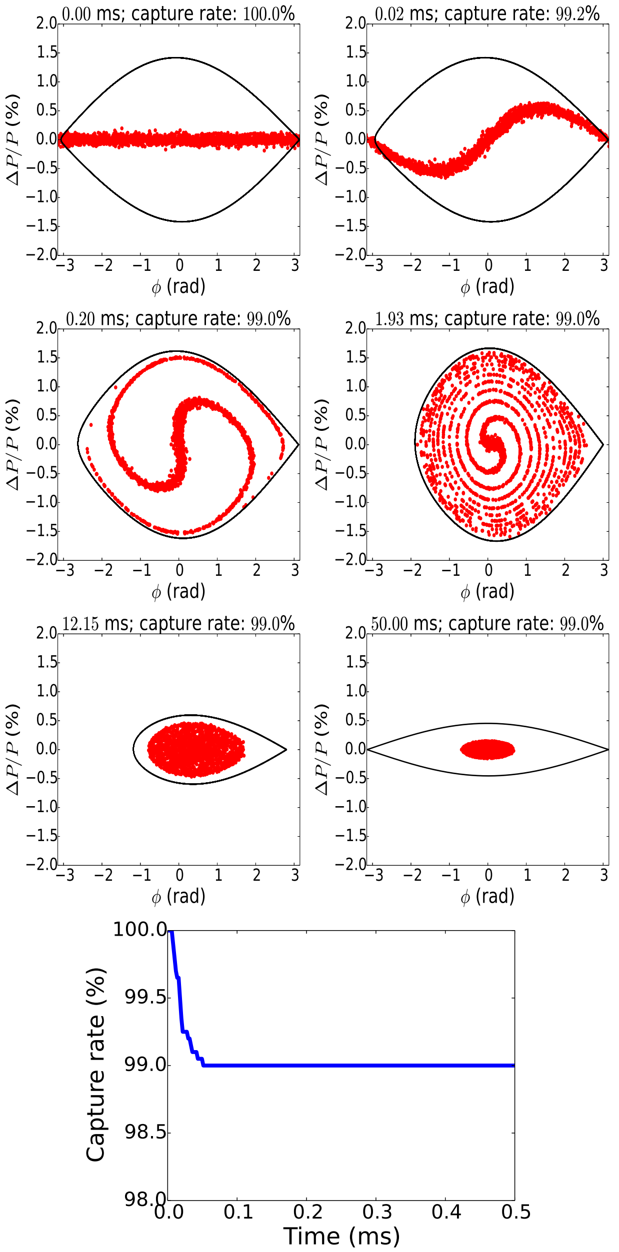

Figure 4: The evolution of phase-space $(\phi, \Delta P/P)$ for protons in a ramping cycle. We set $T_\nu = 0.5$ (ms), $V_i = 7.5$ (kV) and assume the initial conditions for a bunch of the beam is flatly distributed in the phase $\phi = [-\pi,\ \pi]$ (rad), and the $\Delta P/P$ is a Gaussian distribution with zero mean and $\sigma = \pm 0.05\%$, i.e., the width of $\Delta P/P$ is 0.1%. Bottom plot is the capture rate vs. time for protons acceleration.

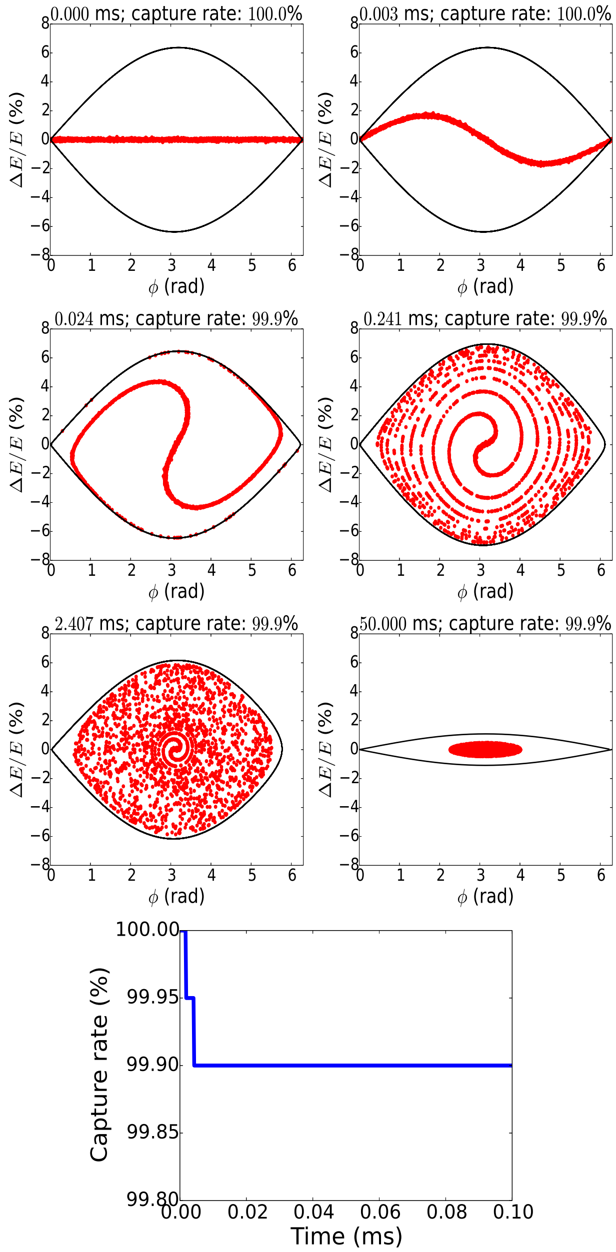

Figure 5: The evolution of phase-space $(\phi, \Delta E/E)$ for electrons in a ramping cycle. We set $T_\nu = 0.1$ (ms), $V_i = 12.5$ (kV) and assume the initial conditions for a bunch of the beam is flatly distributed in the phase $\phi = [-\pi,\ \pi]$ (rad), and the $\Delta E/E$ is a Gaussian distribution with zero mean and $\sigma = \pm 0.05\%$, i.e., the width of $\Delta E/E$ is 0.1%. Bottom plot is the capture rate vs. time for electrons acceleration.

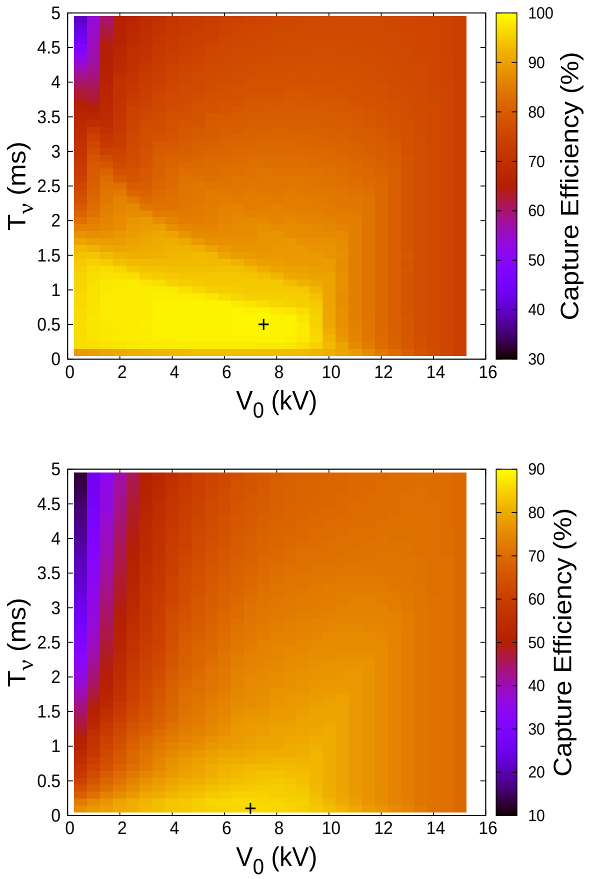

Figure 6: The adiabatic capture efficiencies with respect to RF voltage settings T_ν and V_i for proton beam with the Gaussian width $\Delta P/P = 0.1\%$ (upper) or $\Delta P/P = 1\%$ (lower). The cross marker indicates the position of greatest efficiency in the heat map.

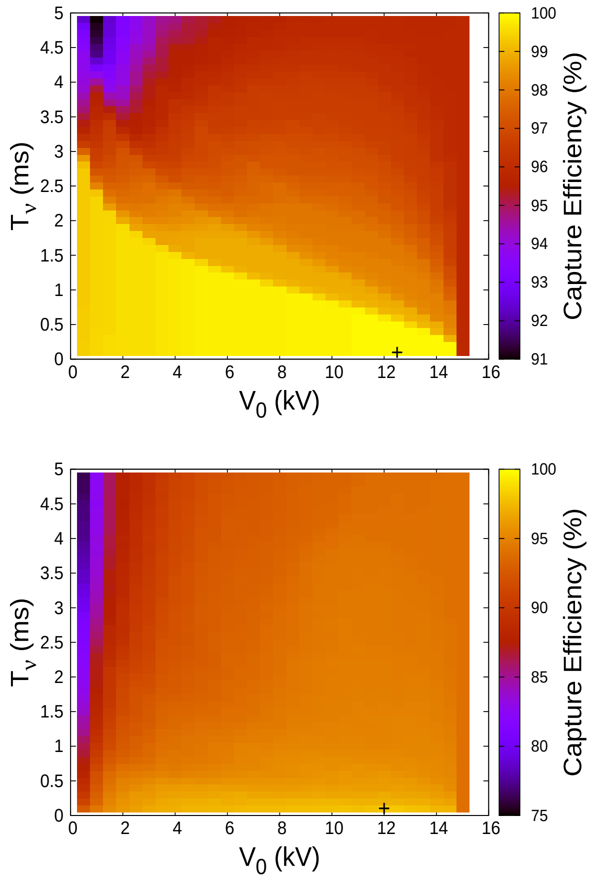

Figure 7: The adiabatic capture efficiencies with respect to RF voltage settings T_ν and V_i for electron beam with the Gaussian width $\Delta E/E = 0.1\%$ (upper) or $\Delta E/E = 1\%$ (lower). The cross marker indicates the position of greatest efficiency in the heat map.