Introduction to Game Theory
Prof. Roger A. McCain (Drexel)

ACME Seminar

Mark L. Chang

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History

- **Interdisciplinary approach to the study of human behavior**
- **Founded in the 1920s by John von Neumann**
- **1994 Nobel prize in Economics awarded to three researchers**
- **"Games" are a metaphor for wide range of human interactions**

The Prisoner's Dilemma

- **Two burglars, Bob and Al, are captured and separated by the police**

- **Each has to choose whether or not to confess and implicate the other**

- **If neither confesses, they both serve one year for carrying a concealed weapon**

- **If each confesses and implicates the other, they both get 10 years**

- **If one confesses and the other does not, the confessor goes free, and the other gets 20 years**

The Prisoners' Dilemma

		AI	
		Confess	Don't Confess
Bob	Confess	10 / 10	0 / 20
	Don't Confess	20 / 0	1 /1

Dominant Strategies

- **The prisoners have fallen into a "dominant strategy equilibrium"**

- **DEFINITION: Dominant Strategy**
  - **Evaluate the strategies.**
  - **For each combination, choose the one that gives the best payoff.**
  - **If the same strategy is chosen for each different combination, that strategy is called a "dominant strategy" for that player in that game**

- **DEFINITION: Dominant Strategy Equilibrium**
  - **If, in a game, each player has a dominant strategy, and each player plays the dominant strategy, then that combination of (dominant) strategies and the corresponding payoffs constitute the dominant strategy equilibrium for that game.**

Issues in the Prisoner's Dilemma

- **Individually rational decisions results in both persons being made worse off**
  - **This result and subsequent realization by von Neumann made a wide impact in modern social science.**
  - **Parallels can be drawn to: arms races, road congestion, pollution, depletion of fisheries, etc.**

- **"Unrealistic"**
  - **Only a two-person game**
  - **Assumed no communication between prisoners**
  - **Perhaps the "rational" choices are not rational at all**

Information Technology Example

- **Players**
  - **Company considering a new internal computer system**
  - **A supplier who is considering producing it**
- **Choices**
  - **To install an advanced system with more features**
  - **To install a proven system with less functionality**
- **Payoffs**
  - **Net payment of the user to the supplier**
- **Assumptions**
  - **A more advanced system really does supply more functionality**

Payoff Table

		User	
		Advanced	Proven
Supplier	Advanced	**20 / 20**	**0 / 0**
	Proven	**0 / 0**	**5 /5**

Complications

- **There are no dominant strategies.**
- **The best strategy depends on what the other player chooses!**
- **Need a new concept of game-equilibrium**
- **<u>Nash Equilibrium</u>**
  - **Occurs when each participant chooses the best strategy given the strategy chosen by the other participant**
  - **Advanced/Advanced**
  - **Proven/Proven**
  - **Can be more than one Nash equilibrium**
- **This is considered a cooperative game**

Zero-Sum Games

- **Players**
  - **Even**
  - **Odd**
- **Players show pennies simultaneously**
- **Each may show either head or tail**
- **If both show the same side, *even* wins a penny from *odd***
- **If both show different sides, *odd* wins a penny from *even***

Pennies Game

		Odd	
		Head	Tail
Even	Head	1,-1	-1,1
	Tail	-1,1	1,-1

Bottled Water Game

- **Players: Evian, Perrier**
- **Each company has a fixed cost of $5000 per period, regardless of sales**
  - **They are competing for the same market, and each must chose a high price ($2/bottle), and a low price ($1/bottle)**
  - **At $2, 5000 bottles can be sold for $10,000**
  - **At $1, 10000 bottles can be sold for $10,000**
  - **If both companies charge the same price, they split the sales evenly between them**
  - **If one company charges a higher price, the company with the lower price sells the whole amount and the higher price sells nothing**
  - **Payoffs are profits – revenue minus the $5000 fixed cost**

Price Competition

		Perrier	
		$1	$2
Evian	$1	$0, $0	$5000, -$5000
	$2	-$5000, $5000	$0, $0

The Maximin Criterion

- **There is a clear concept of a solution for the zero-sum game**
- **Each player chooses the strategy that will maximize the minimum payoff**
- **Both choose $1 prices**

What about non-constant sum?

- **Let's sell widgets**
- **Set pricing to $1, $2, or $3 per widget**
- **Payoffs are profits, allowing for costs**
- **General idea is that company with lower price gets more customers, and more profits, within limits**

Widgets

		ACME Widgets		
		$1	$2	$3
Widgeon Widgets	$1	0, 0	50, -10	40, -20
	$2	-10, 50	20, 20	90, 10
	$3	-20, 40	10, 90	50, 50

Nash Equilibrium

- **If there is a set of strategies with the property that no player can benefit by changing their strategy while the other players keep their strategies unchanged, then that set of strategies and the corresponding payoffs constitute the Nash Equilibrium**

The Queuing Game

- **Two person games don't get us very far**
- **Suppose six people are waiting at an airline boarding gate**
- **They are waiting to check in, and one of them stands up and steps to the counter to be the first in the queue**
- **As a result, others sitting feel the need to stand and wait in the queue**

Queuing Game

Order served	Gross payoff	Net payoff
First	20	18
Second	17	15
Third	14	12
Fourth	11	9
Fifth	8	6
Sixth	5	3

Application to CAD

- "A Low Power Scheduler Using Game Theory", N. Ranganathan and Ashok K. Murugavel, CSE, Nanomaterials and Nanomanufacturing Research Center, University of South Florida

- Modeled the problems of low-power scheduling and binding as auction-based game theoretic problems and solve them using the Nash equilibrium function

- Achieved 11.8% better power results than an ILP-method (best known)