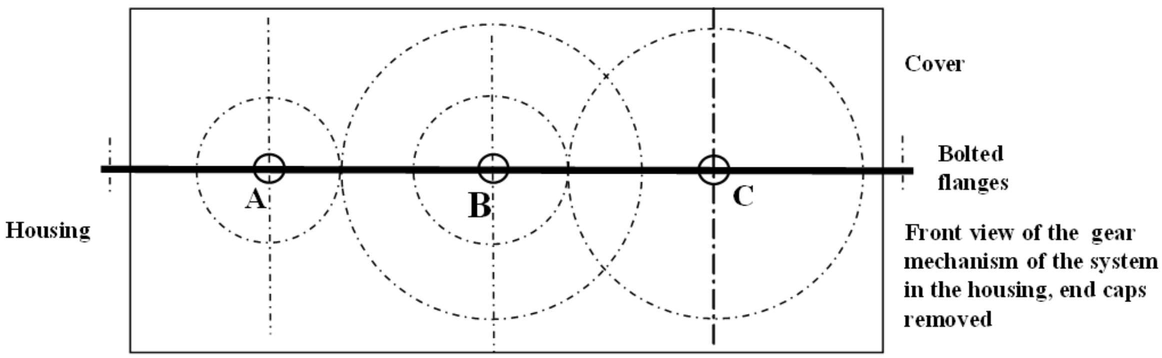
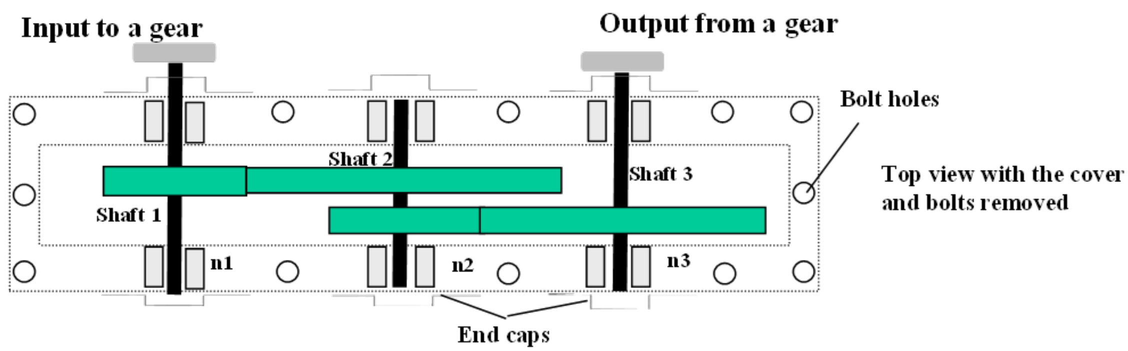
ME 315
THEORY OF MACHINES – DESIGN OF ELEMENTS
Fall 2024

Design Project
Group submission





Group Members

Mark Chauhan	Shaft System 1
Trevor Abbott	Shaft System 2
Zachary Warrin	Shaft System 3

Grade

On-time step dues	/5
Structural design, drawings	/40
Design analysis	/35
Report writing	/20

Total _______/95

Introduction

In Design of Machine Elements Class, the group designed a two-stage gear transmission. The goal of the transmission is to transmit power from input shaft A to output shaft C with a speed reduction. The motor speed feeding into input shaft A through a gear is 2700 rpm and the power input to shaft A is 60kW. The speed of shaft C will be $n_3 = 400$ rpm, with a 100% efficiency for each pair. The group's design included fatigue stress concentration factors, through bending and torsional stresses. The group's design incorporated a life of 5 years(7 days a week, 52 weeks a year, and 8 hours a day), with a reliability of 99%.

Gear Train

	Gear Train Table			
	Gear 1	**Gear 2**	**Gear 3**	**Gear 4**
N (Teeth)	19	58	23	51
Power (KW)	60	60	60	60
Speed (RPM)	2700	884.48	884.48	398.88

Summary:

Gear Train Ratio = 6.769

Output Speed(RPM) = 398.88

Error(%) = 0.28%

Gear Geometry

	Gear Geometry			
	Gear 1	**Gear 2**	**Gear 3**	**Gear 4**
Module (mm)	5	5	5	5
Pressure Angle	20	20	20	20
Face Width (mm)	65	60	65	60
Pitch Diameter (mm)	95	290	115	255
Base Circe Diameter (mm)	89.27	272.51	108.07	239.62
Outside Diameter (mm)	105	300	125	265
Root Circle Diameter	82.5	277.5	102.5	242.5
Centerline Distance (mm)		192.5		185
Total Distance AC (mm)				377.5

Gear Parameters

$n_1 = 2700$ rpm, $H = 60kW$, $n_{3_{goal}} = 400 \pm 4$,

Ideal Transmission Ratio: $n_1 / n_{3_{goal}} = 6.75$

Gear Teeth:

$N_1 = 19$

$N_2 = 58$

$N_3 = 23$

$N_4 = 51$

Module: $m = 5$ mm

Addendum: $a = 5$ mm

Dedendum: $b = 2.5$ mm

Clearance: $c = 1.25$ mm

Tooth Depth: $h = 11.25$ mm

Phi: $\phi = 20^{\circ}$

Gear Forces

	Gear Forces			
	Gear 1	**Gear 2**	**Gear 3**	**Gear 4**
Material	AISI 1045 Steel	AISI 1045 Steel	AISI 1045 Steel	AISI 1045 Steel
Grade	2	1	2	1
Hardness (HB)	500	400	500	400
Torque (NM)	212.21	647.84	647.84	1436.52
Wt (N)	4467.51	4467.51	11265.8	11265.8
Wr (N)	1626.04	1626.04	4100.45	4100.45
Bending F.S.	4.79	3.13	2.43	1.53
Contact F.S.	1.64	2.29	1.27	1.46

Shaft

	Shaft Table		
	Shaft 1	**Shaft 2**	**Shaft 3**
Materials	AISI 1045 Steel	AISI 1045 Steel	AISI 1045 Steel
Speed (RPM)	2700	884.48	398.88
Max Torque (Nm)	212.21	647.84	1436.52
Max Bending (Nm)	285.53	487.77	407.0
Critical Cross Section	Gear 1, facing output shaft (C+)	Gear 3, facing gear 2 (B+)	Bearing D, side of output shaft (D+)
Minimum diameter (mm)	20	28	37
Diameter at critical cross section (mm)	40	55	70
Goodman Factor of Safety	1.68	2.26	4.04
Yield Factor of Safety	3.93	4.58	5.93
Lifetime	Infinite life (1+ million cycles)		

Bearings

	Bearings Table		
	Shaft 1	**Shaft 2**	**Shaft 3**
Max Combined Radial Load (N)	2604.14	9755.51	11951.98
Thrust Load (N)	0	0	0
Equivalent Load Used in Calculation (N)	2604.14	9755.51	11951.98
C (N)	34231.5307	89519.43052	84105.88984
Bearing SKF Numbers	6308	6411	6314
Bore (mm)	40	55	70
Outer Diameter (mm)	90	140	150
Width (mm)	23	33	35
C, Bearing (N)	42300	99500	111000
C0, bearing (N)	24000	62000	68000
Expected Life (Mil Cycles)	4285.76	1061.01	801.03
Expected Life (Years)	9.43	6.87	11.49
Bearing Changing Services Needed in 5 year period	0	0	0

Material & Bearing Selection

We selected AISI 1045 steel for our gear reduction transmission because of its excellent hardening capability, allowing it to achieve a hardened surface through heat treatment or induction hardening while maintaining a tough, ductile core—ideal for minimizing wear and extending service life. Its good machinability makes it economical to manufacture and finish, while its toughness and moderate fatigue resistance ensure reliable performance under cyclic loading conditions typical in gear shafts. Additionally, AISI 1045 offers a cost-effective solution, providing an optimal balance between performance and affordability for medium-duty applications.

Deep groove ball bearings are an excellent choice for this shaft design due to their versatility, high efficiency, and ability to handle both radial and axial loads. Their low friction design ensures smooth operation and minimal energy loss, which is essential for maintaining performance and reducing wear over time. Additionally, deep groove ball bearings are highly durable and capable of operating at high speeds, making them ideal for applications where precision and reliability are critical. These bearings were chosen because their compact structure simplifies the design while providing excellent load-carrying capacity, ensuring the shaft system runs efficiently and reliably under varying conditions.

Cost

For cost, the shafts and gears were designed to be thicker and weigh more for a lowest factor of safety of 1.27. This would allow for a longer-term use of the shafts and greater stiffness. Although, this increases cost, this 'heavy duty' system should be extremely reliable.

Stiffness

We chose diameters for our shaft that were much larger than the minimum diameter in order to have a high stiffness. This does increase cost, but it also means that this design is heavy duty.

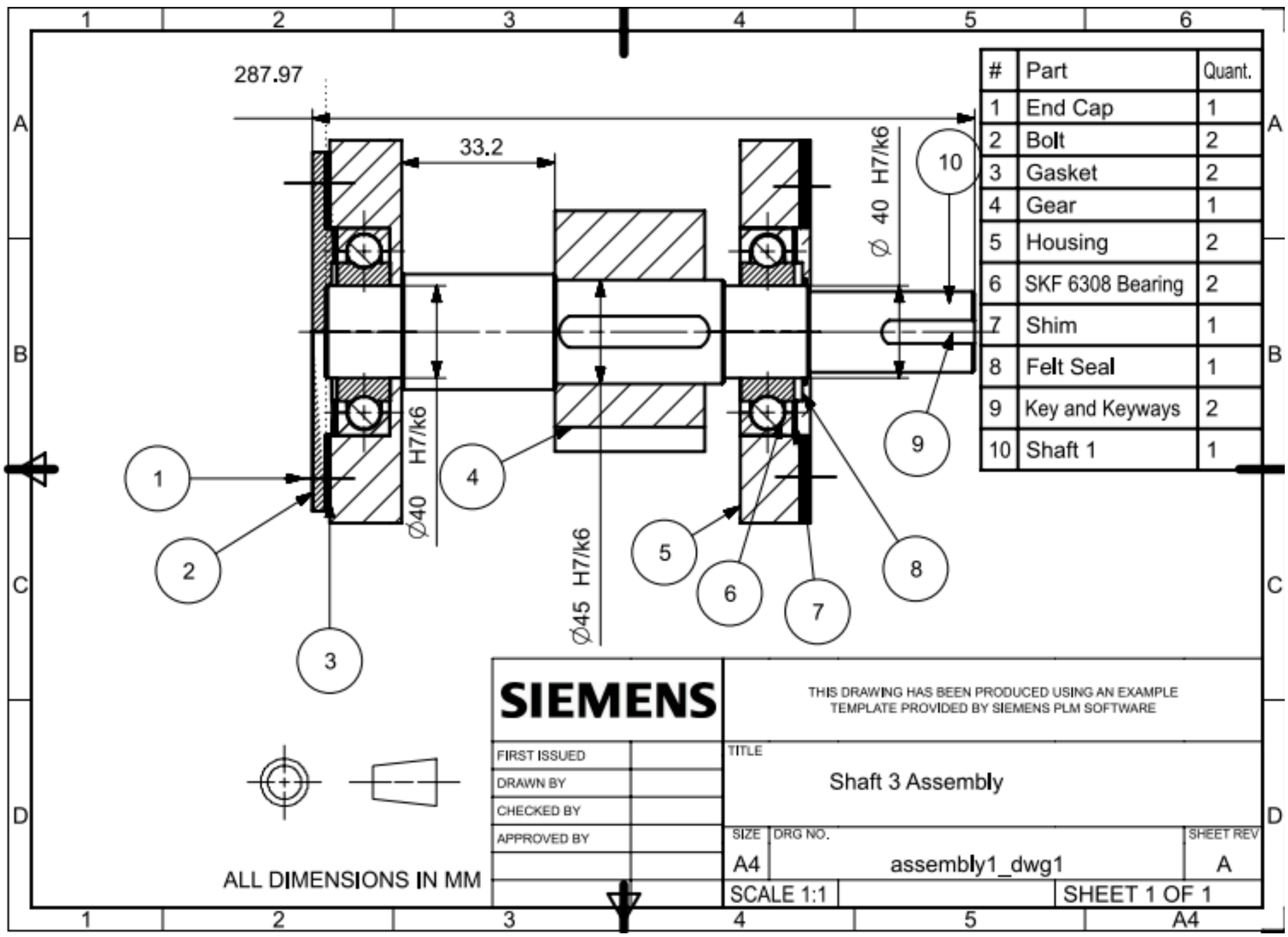
Compactness

Our design is very compact. We did this for two reasons. First was to reduce the overall footprint of the transmission, which is useful if it is needed in small locations. Second was to reduce the distance between gears and bearings, minimizing the effect of bending.

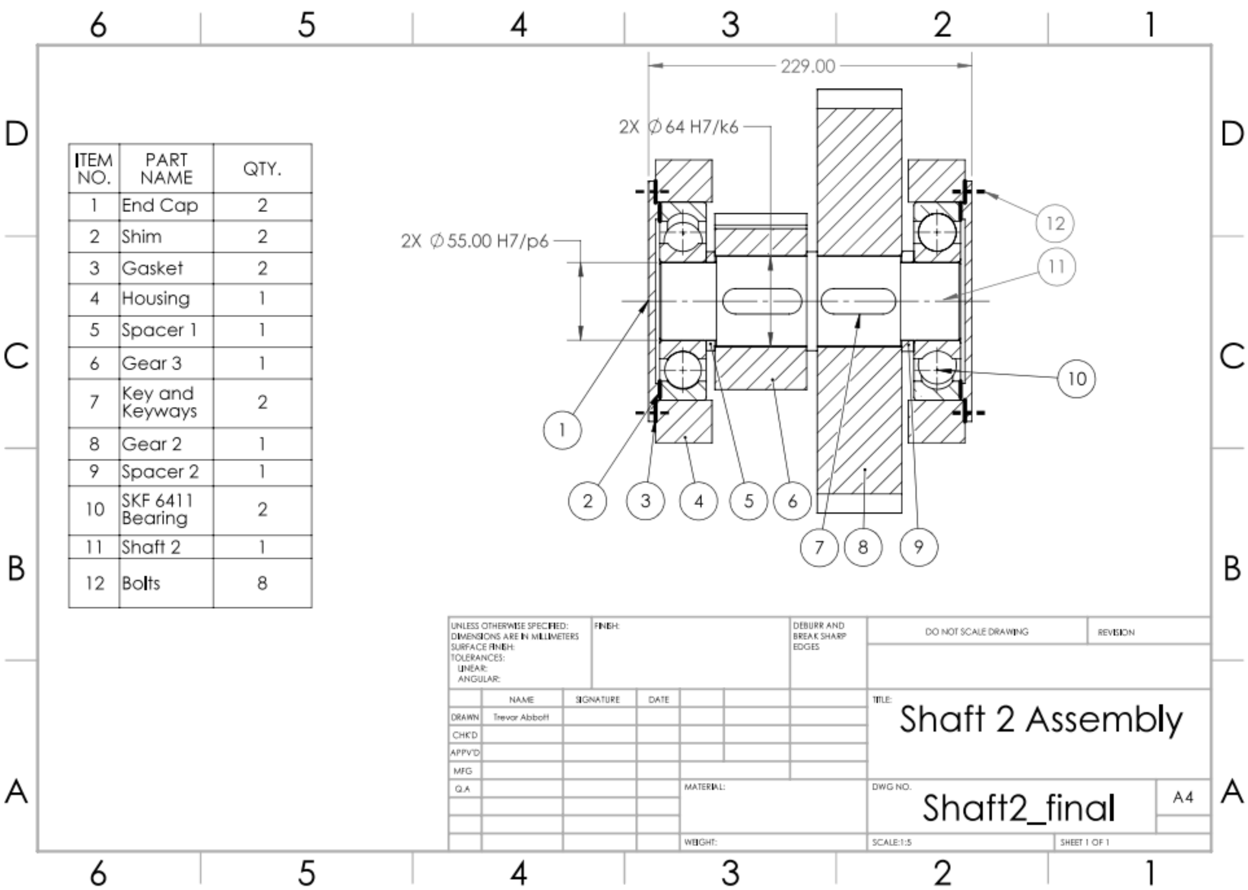
Summary Table	
Power (KW)	60
Input Speed (RPM)	2700
Output Speed (RPM)	398.88
Overall Factor of Safety	1.27
Maintenance Needs	For the 5 year life span of this system, no major maintenance is needed. However, [gear 3] has the lowest factor of safety, and should therefore be checked every so often, as this therorteically will be the first component to fail. Oil should be changed regularly to avoid breakdown in lubrication. Additionally,

Assembly Drawings

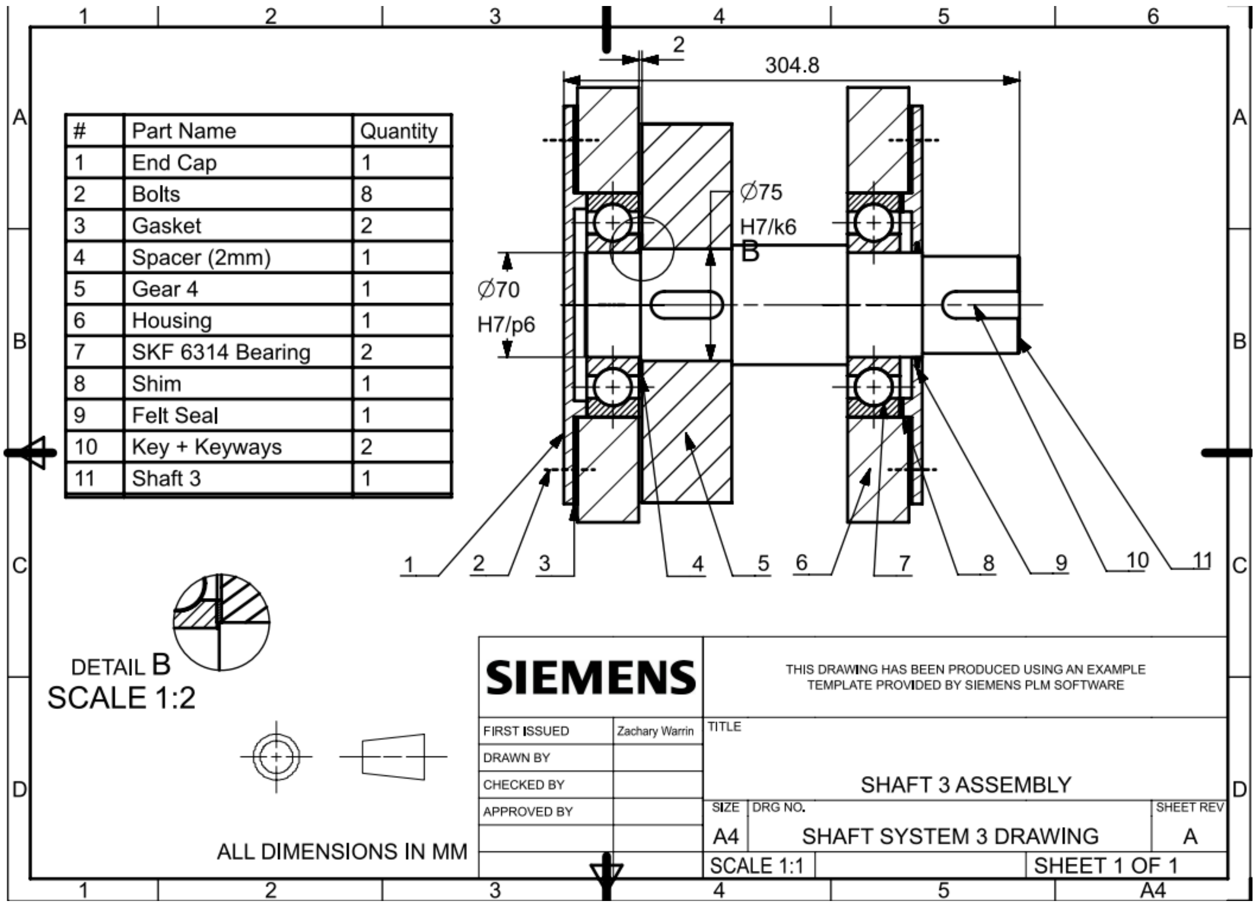
Shaft A



Shaft B


ITEM NO. | PART NAME | QTY.
1 | End Cap | 2
2 | Shim | 2
3 | Gasket | 2
4 | Housing | 1
5 | Spacer 1 | 1
6 | Gear 3 | 1
7 | Key and Keyways | 2
8 | Gear 2 | 1
9 | Spacer 2 | 1
10 | SKF 6411 Bearing | 2
11 | Shaft 2 | 1
12 | Bolts | 8
229.00
2X Ø64 H7/k6
2X Ø55.00 H7/p6
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN MILLIMETERS
SURFACE FINISH:
TOLERANCES:
LINEAR:
ANGULAR:
FINISH:
DEBURR AND BREAK SHARP EDGES
DO NOT SCALE DRAWING
REVISION
NAME
SIGNATURE
DATE
DRAWN
Trevor Abbott
CHK'D
APPV'D
MFG
Q.A
MATERIAL:
WEIGHT:
TITLE:
Shaft 2 Assembly
DWG NO.
Shaft2_final
A4
SCALE:1:5
SHEET 1 OF 1

Shaft C

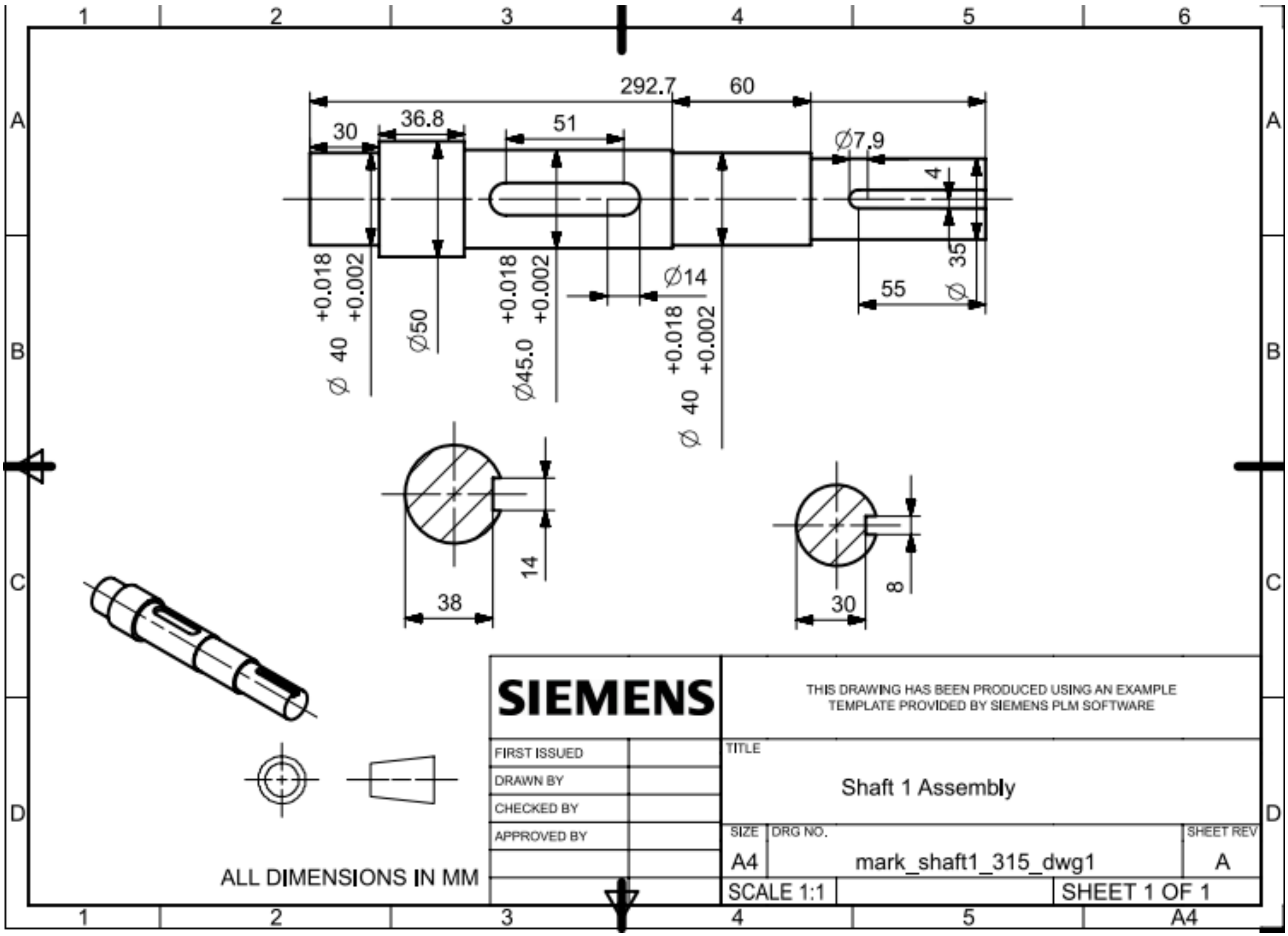


#	Part Name	Quantity
1	End Cap	1
2	Bolts	8
3	Gasket	2
4	Spacer (2mm)	1
5	Gear 4	1
6	Housing	1
7	SKF 6314 Bearing	2
8	Shim	1
9	Felt Seal	1
10	Key + Keyways	2
11	Shaft 3	1

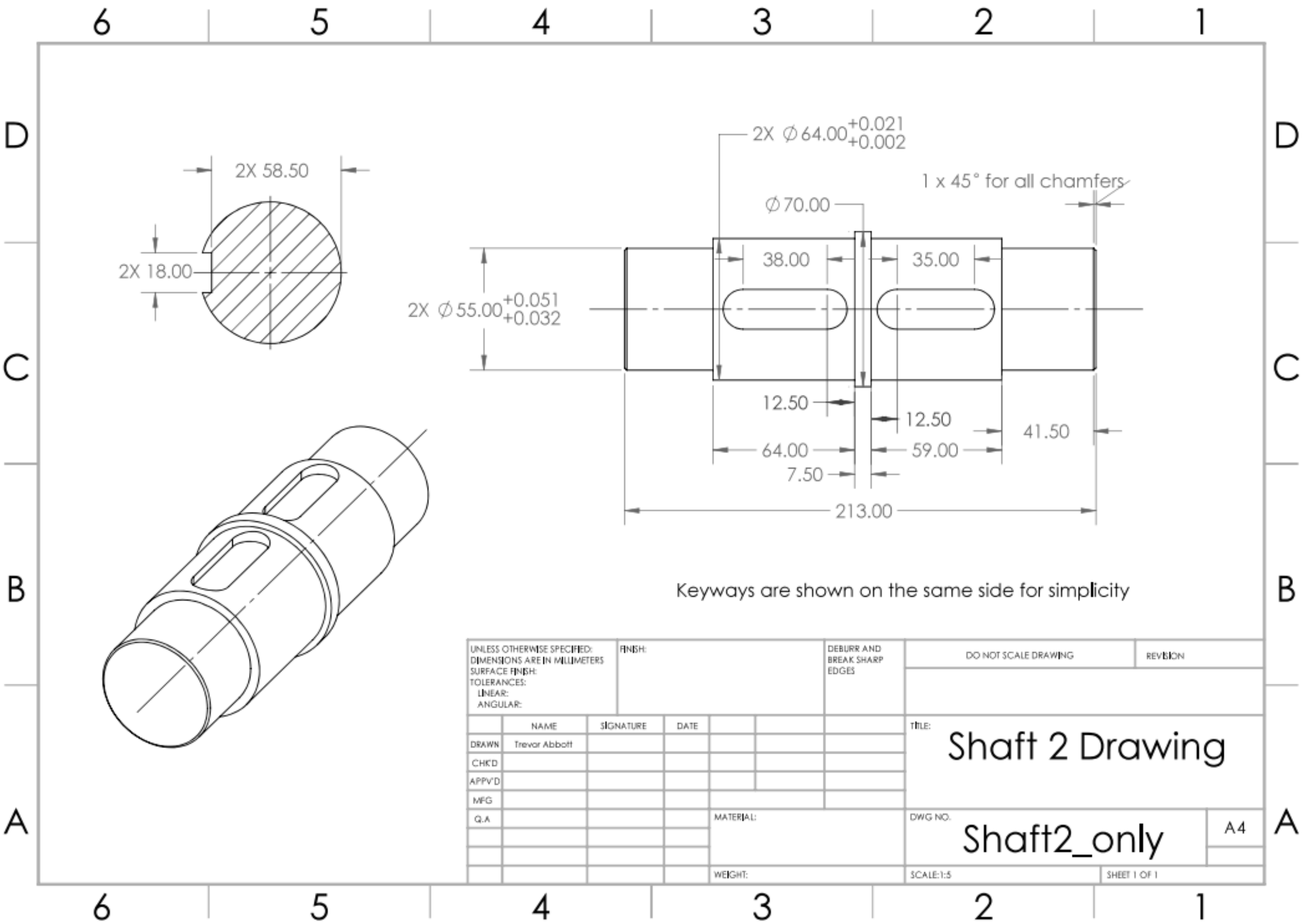
2
304.8
Ø75
H7/k6
B
Ø70
H7/p6
DETAIL B
SCALE 1:2
ALL DIMENSIONS IN MM
SIEMENS
THIS DRAWING HAS BEEN PRODUCED USING AN EXAMPLE TEMPLATE PROVIDED BY SIEMENS PLM SOFTWARE
FIRST ISSUED Zachary Warrin
DRAWN BY
CHECKED BY
APPROVED BY
TITLE
SHAFT 3 ASSEMBLY
SIZE A4
DRG NO. SHAFT SYSTEM 3 DRAWING
SHEET REV A
SCALE 1:1
SHEET 1 OF 1

Shaft Drawings

Shaft A

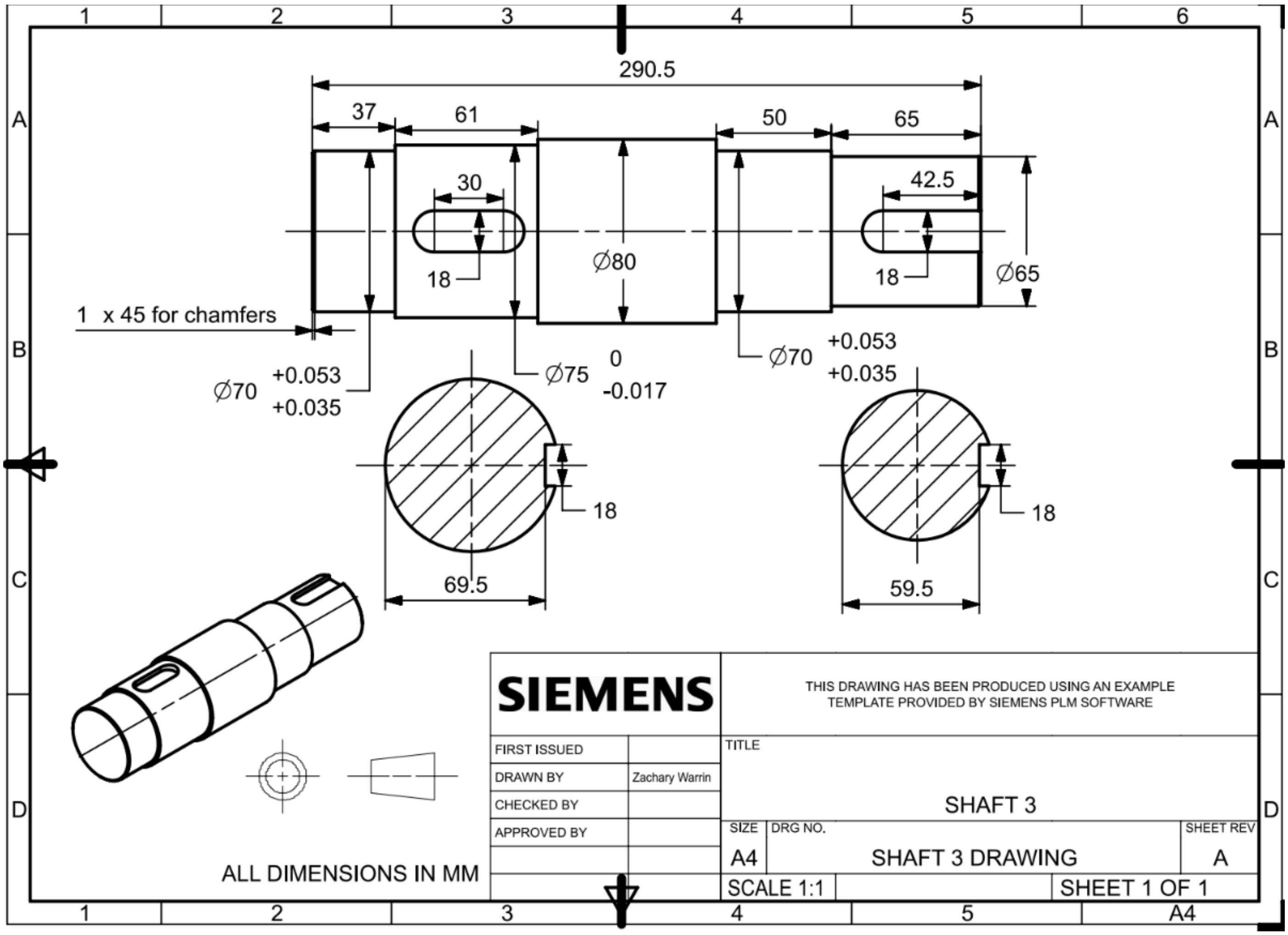


Shaft B



2X 58.50
2X 18.00
2X Ø 55.00 +0.051 +0.032
2X Ø 64.00 +0.021 +0.002
Ø 70.00
1 x 45° for all chamfers
38.00
35.00
12.50
12.50
41.50
64.00
59.00
7.50
213.00
Keyways are shown on the same side for simplicity
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN MILLIMETERS
SURFACE FINISH:
TOLERANCES:
LINEAR:
ANGULAR:
FINISH:
DEBURR AND BREAK SHARP EDGES
DO NOT SCALE DRAWING
REVISION
NAME
SIGNATURE
DATE
DRAWN
Trevor Abbott
CHK'D
APPV'D
MFG
Q.A
MATERIAL:
WEIGHT:
TITLE:
Shaft 2 Drawing
DWG NO.
Shaft2_only
A4
SCALE:1:5
SHEET 1 OF 1

Shaft C



290.5
37
61
50
65
30
42.5
18
18
∅80
∅65
1 x 45 for chamfers
∅70 +0.053 +0.035
∅75 0 -0.017
∅70 +0.053 +0.035
18
18
69.5
59.5
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Zachary Warrin
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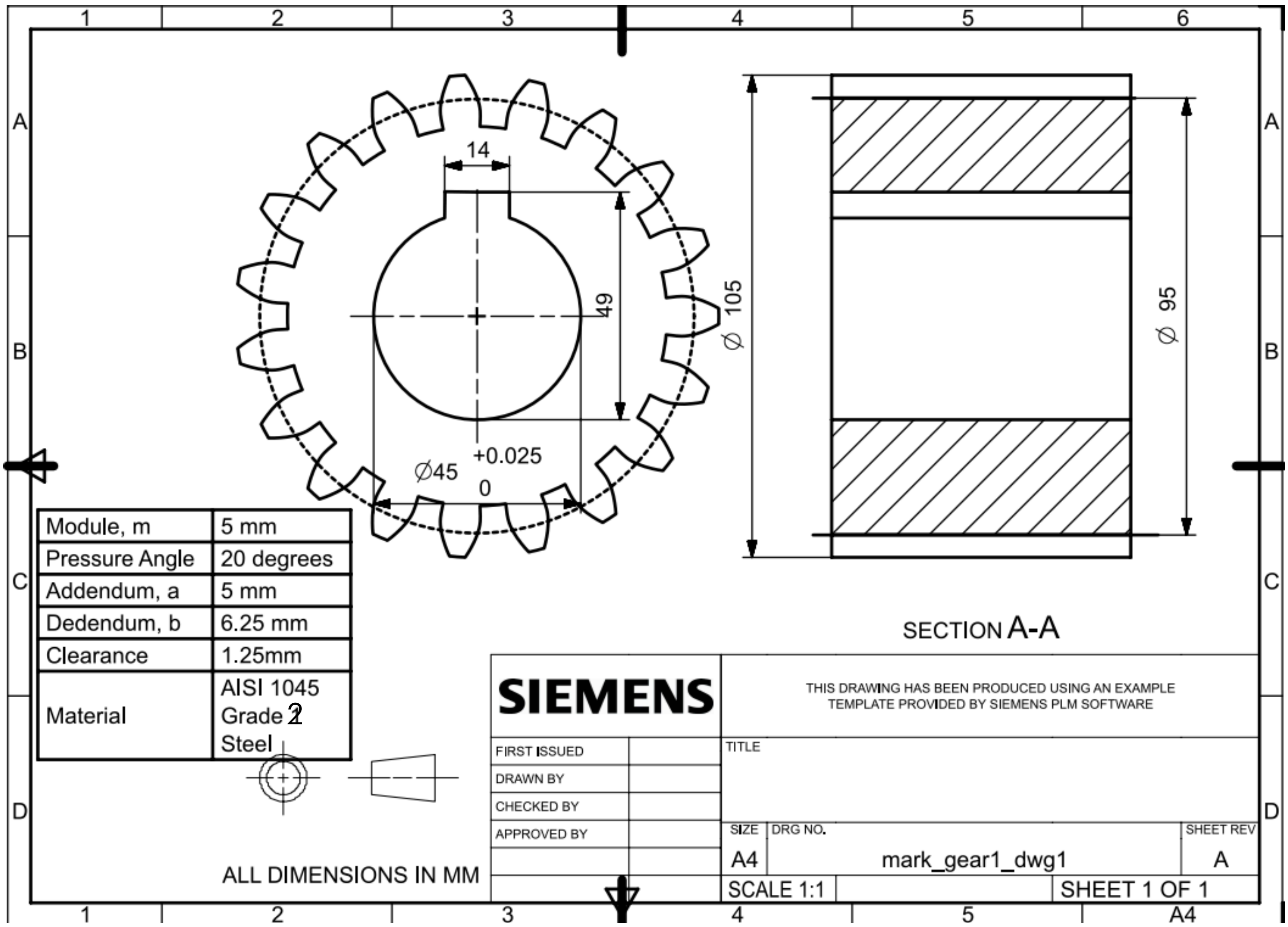
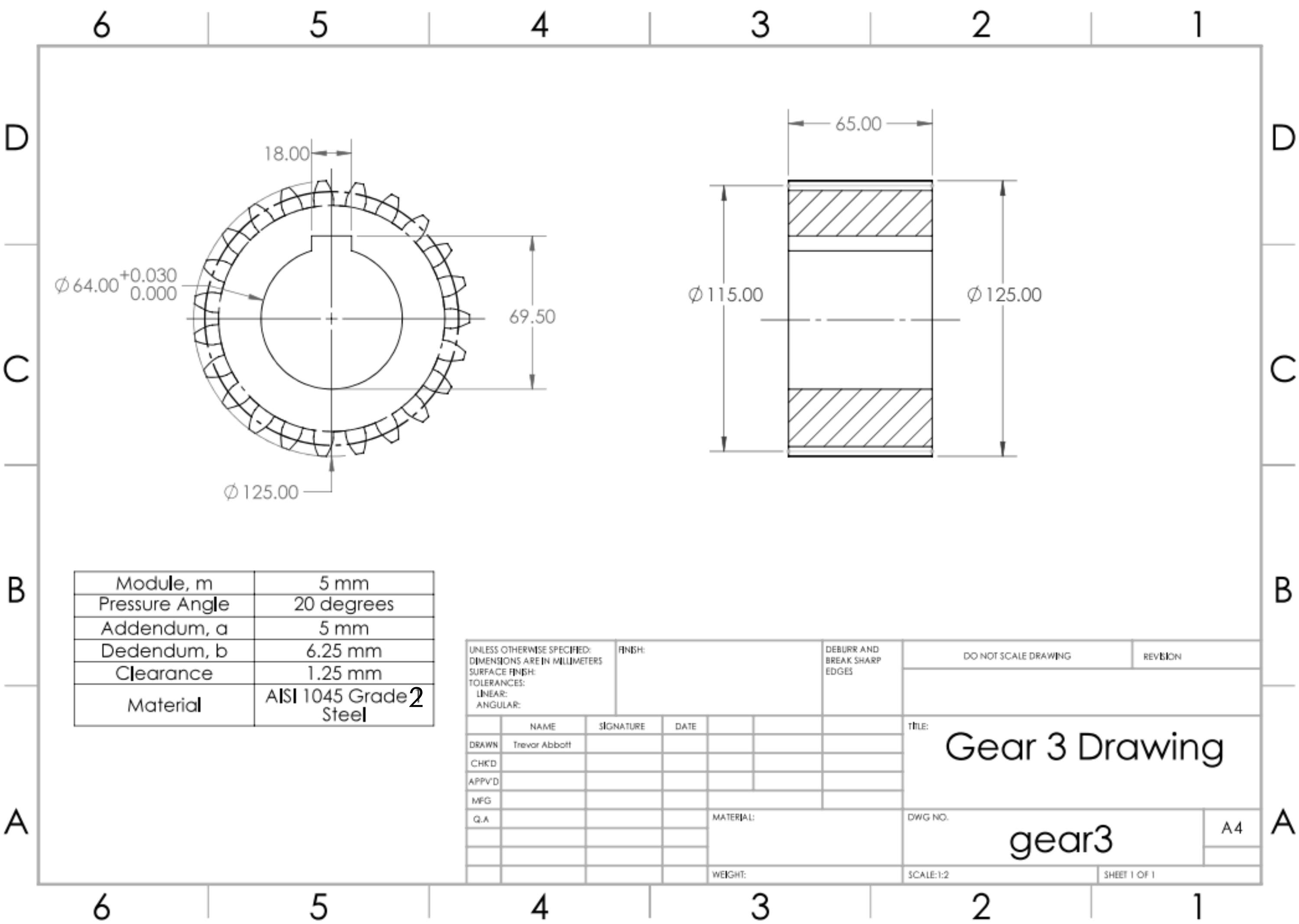
TITLE
SHAFT 3
SIZE
A4
DRG NO.
SHAFT 3 DRAWING
SHEET REV
A
SCALE 1:1
SHEET 1 OF 1
ALL DIMENSIONS IN MM

Gear Drawings

Gear 1

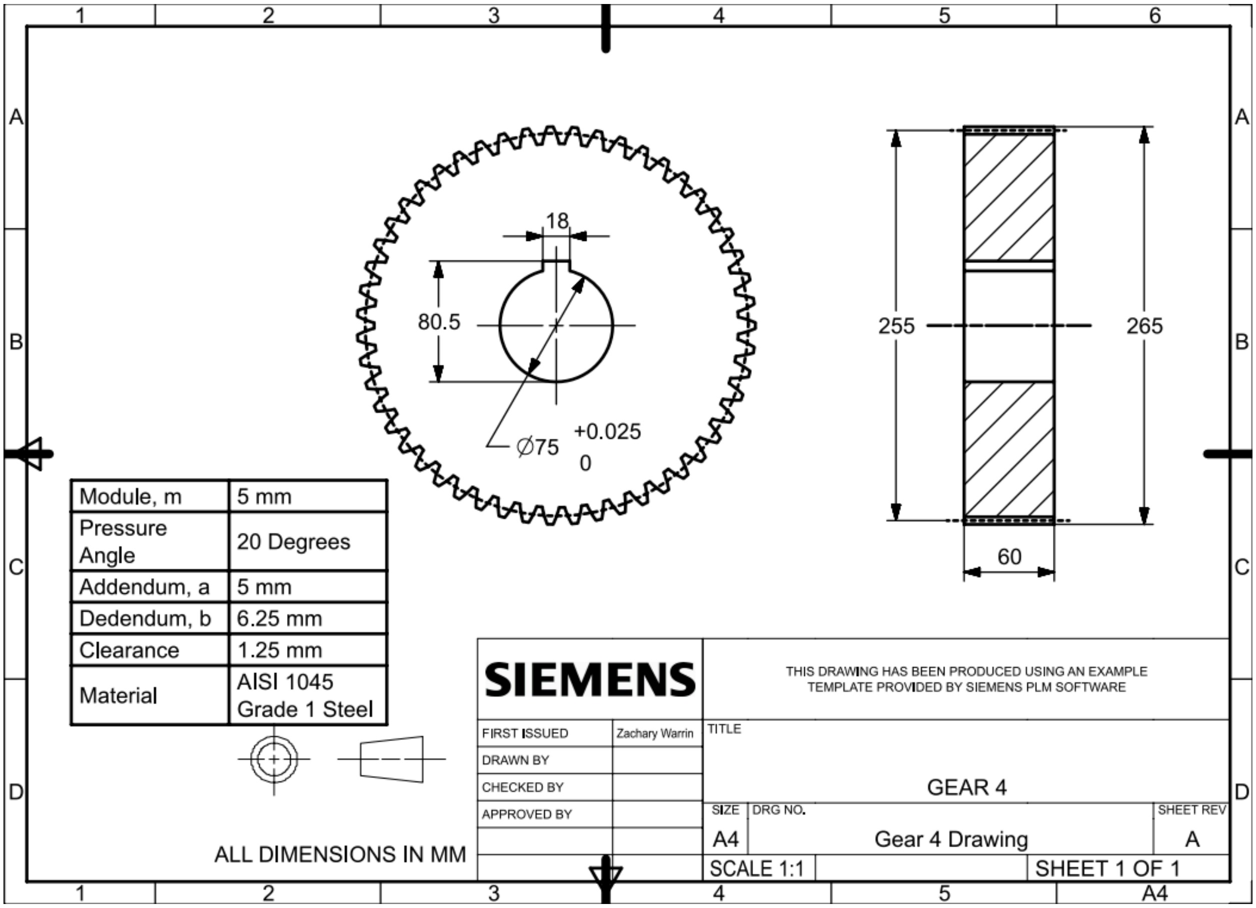


Gear 3



*According to outline Gear 2 is not required in report

Gear 4


18
80.5
∅75
+0.025
0
255
265
60
Module, m
5 mm
Pressure Angle
20 Degrees
Addendum, a
5 mm
Dedendum, b
6.25 mm
Clearance
1.25 mm
Material
AISI 1045 Grade 1 Steel
SIEMENS
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TITLE
GEAR 4
SIZE
DRG NO.
SHEET REV
A4
Gear 4 Drawing
A
ALL DIMENSIONS IN MM
SCALE 1:1
SHEET 1 OF 1
A4

Shaft A Analysis

Gearing

Geometry:

$N_1 = 19$

Pitch Diameter: $d_p = N_1 m = 95$ mm

Face Width: $b_w = 12m = 65$ mm

Choose a input gear diameter of 150 mm.

Forces:

$$T_1 = \frac{60H}{2\pi n_1} = 396.491 \text{Nm}$$

Gear Forces from the Input Gear:

$$Wr_i = W_t tan(\phi) = 1029.84 \text{ N}$$

$$W_t i = \frac{T}{\frac{d}{2}} = 2829.47 \text{ N}$$

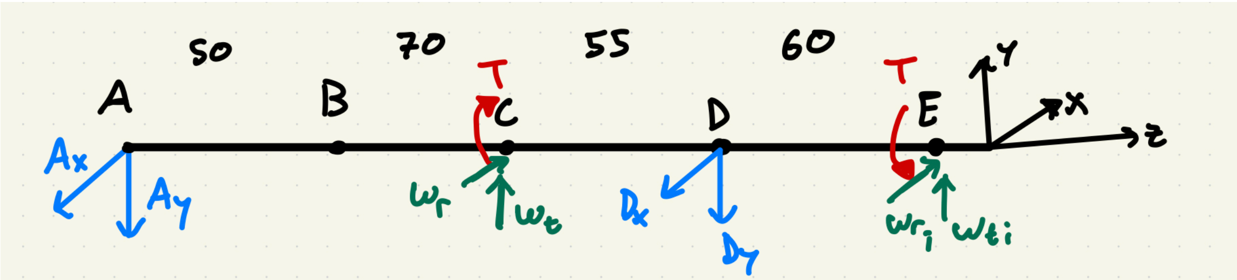
Gear Forces from Gear One onto Gear Two:

$$W_{T12} = \frac{T}{r_p} = 4467.51 \text{ N}$$

$$W_{12} = \frac{W_{T12}}{cos(\phi)} = 4754.222 \text{ N}$$

$$W_{r12} = W_{12} sin(\phi) = 1626.040 \text{ N}$$

Force Analysis



System of Equations:

$$\Sigma F_x = 0 = -A_x + W_{r1} - D_x + W_{r2}$$

$$\Sigma F_y = 0 = -A_y + D_y + W_{t1} - W_{t_2}$$

$$\Sigma M_{Ay} = 0 = W_{r1} * (50 + 70) - D_x(50 + 70 + 65) + W_{r2}(50 + 70 + 55 + 60)$$

$$\Sigma M_{Ax} = 0 = W_{t1} * (50 + 70) + D_y(50 + 70 + 65) - W_{t2}(50 + 70 + 55 + 60)$$

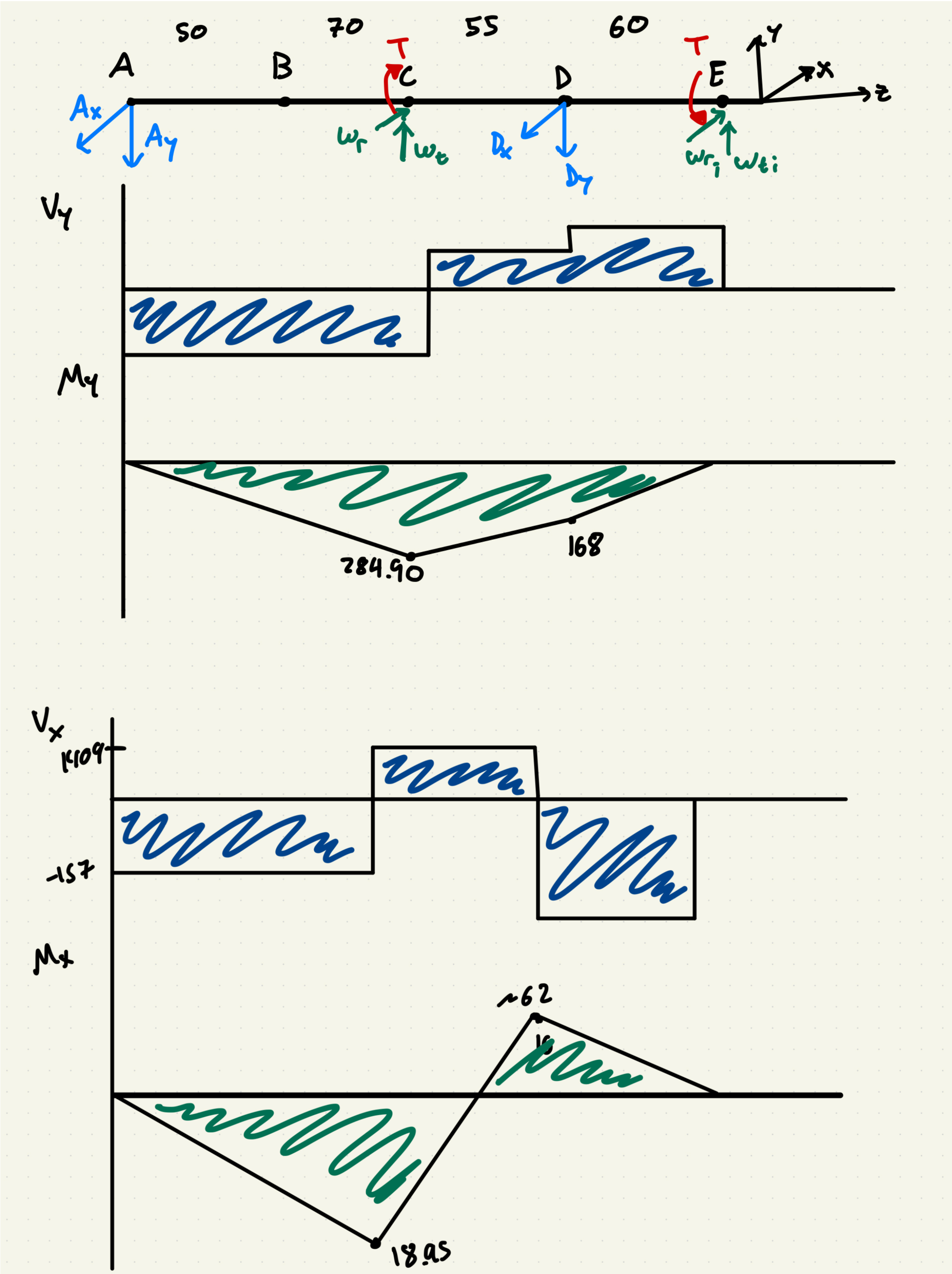
Forces:

$$A_x = 157.953N$$

$$D_x = 2497.93N$$

$$A_y = 2374.18N$$

$$D_y = 736.14N$$


50
70
55
60
T
T
A
B
C
D
E
Y
X
Z
Ax
Ay
Wr
Wt
Dx
Dy
Wri
Wti
Vy
My
284.90
168
Vx
1109
-157
Mx
~62
18.95

Stress Analysis

Factor of Safety:

$$n_s = 1.1$$

Most critical point occurs at C^+

Material:

Material	E(Pa)	UTS(MPa)	Ys(MPa)	Poisson Ratio
AISI 1045 Steel	200000	394.72	294.74	0.29

Minimum Diameter Calculation:

$$\tau_{det} = \frac{0.557Y_s}{n_s} = 154.6$$

$$d_{min} \geq \sqrt[3]{\frac{16T}{\pi \tau_{det}}} = 19.12mm$$

Keyway Effect(5% Increase):

$$d_{min} = d_{min}(1.05) = 20.076 \text{ mm}$$

Total Moment and Shear:

$$V_{total} = \sqrt{2093.33^2 + 1468.09^2} = 2556.82N$$

$$M_{total} = \sqrt{(284.9^2) + (18.95)^2} = 285.53Nm$$

Factor of Safety

Stress Analysis and Cyclic Loading with a chosen $d_{min} = 40mm$:

$$\sigma_a = \sigma_{bs} = \frac{32M}{\pi d^3} = 45.44MPa$$

$$\sigma_m = 0MPa$$

$$\tau_v = \frac{4V}{3A} = 2.71MPa$$

$$\tau_T = \frac{16T}{\pi d^3} = 16.88MPa$$

$$k_f = 1.58 * UTS^{-0.085} = 0.951$$

$$k_s = 1.189 * d^{-0.112} = 0.787$$

$$\frac{1}{n_s} = \frac{\sqrt{(K_f\sigma_{xa})^2 + 3(K_{fs}\tau_{xya})^2}}{(k_f k_s k_r k_t k_m)(0.5S_{UT})} + \frac{\sqrt{(\sigma_{xm})^2 + 3(\tau_{xym})^2}}{S_{UT}}$$

$$\frac{1}{n_s} = \frac{136.95}{263.05} + \frac{29.4}{394} = 0.52 + 0.074$$

$n_s = 1.68$ > 1.1 The shaft diameter of 40 mm is usable

Yielding:

$$\frac{1}{n_s} = \frac{\sigma_{VM-A}}{S_y} + \frac{\sigma_{VM-M}}{S_y}$$

$$\frac{1}{n_s} = \frac{\sqrt{(45.44)^2 + 3(2.71)^2}}{294.74} + \frac{\sqrt{3(16.88)^2}}{294.74}$$

$n_s = 3.93$ > 1.1 The shaft diameter doesn't pass yield line

Gear Stress and Life Analysis

Material	E(Pa)	Hardness	Poisson Ratio	Grade
AISI 1045 Steel	200000	500	0.285	2

Shaft Cycles

$$t = 5\ year * 52\frac{week}{year} * 7\frac{day}{week} * 8\frac{hr}{day} * 60\frac{min}{hr} = 873600\ mins$$

$N = t * n_1 =$ 2358.72 Million Cycles

Allowable Bending Stress

$Y_N = 1.6831 * (N * 10^6)^{-0.0323} = 0.838$

$S_b = 0.703(HB) + 113 = 464.5$ MPa

$\sigma_{b_{theoretical}} = \dfrac{S_b * Y_n}{k_t * k_r} = 389.366MPa$ $\qquad k_t = k_r = 1$

Actual Bending Stress

$Y_j = 0.34, k_a = 1.00, k_s = 1.00, k_m = 1.10, k_i = 1.00, k_b = 1.00$

$$V = \frac{\pi n d}{60} = 13.430\frac{m}{s}$$

$B = \dfrac{(12 - Q_v)^{\frac{2}{3}}}{4} = 0.825$ (assuming $Q_v = 6$)

$A = 50 + 56(1 - B) = 59.773$

$$K_v = \left(\frac{A + \sqrt{200V}}{A}\right)^B = 1.674$$

$$\sigma_b = \frac{W^t}{m b_w Y_j} k_a k_s k_m k_v k_i k_b = 81.2310MPa$$

$n_b = \dfrac{\sigma_{b_{theoretical}}}{\sigma_b} = 4.793 > 1.1$ This design will work under bending stress.

Allowable Contact Stress

$$HB_{pinion} = 500HB \qquad \frac{HB_{pinion}}{HB_{gear}} = 1.25$$

$$HB_{gear} = 400HB$$

$$Z_N = 2.466N^{-0.056} = 0.736$$

$$C_H = 1 + A'(\frac{N_g}{N_p} - 1.0) = 1.0 \text{ since } A' = 0$$

$$K_t = 1, \quad K_r = 1$$

$$S_c = 2.4 * HB_g + 237 = 1437\ MPa$$

$$\sigma_{c_{theoretical}} = \frac{S_c Z_N C_{H_g}}{K_T K_r} = 105.8230\ MPa$$

Actual Contact Stress

$$K_a = 1,\ K_s = 1,\ K_e = \sqrt{\frac{2}{\frac{1-v_p^2}{E_p} + \frac{1-v_g^2}{E_g}}} = 466563.13,\ K_m = 1.2,\ K_v = 1.674$$

$$\phi = 20°,\ d_p = 0.095m,\ d_g = 0.235m$$

$$I = \frac{\pi cos(\phi)sin(\phi)}{1 + \frac{d_p}{d_g}} = 0.761$$

$$\sigma_c = K_e\sqrt{\frac{W^t}{b_w d_p}} I K_a K_s K_m K_v = 64.5018MPa$$

$$n_c = \frac{\sigma_{c_{theoretical}}}{\sigma_c} = 1.641 > 1.1$$ This design will work under contact stress.

Bearing Selection

$$h = 5\ year * 364\frac{day}{year} * 8\frac{hr}{day} = 14560\ hrs$$

$L_{10} = 60hn_3 10^{-6} = 2271.36$ Million Revolutions

$A_p = 1$ (no shock), $m = 3$ (ball bearing), $X = 1$ (no axial force)

$$P_r = Max(A, B) = 2604.14N$$

$$C = PL_{10}^{\frac{1}{m}} = 34231.531N$$

Using information from: https://www.skf.com/us/products/rolling-bearings/ball-bearings/deep-groove-ball-bearings

Bearing Spec	C(N)	C0 (N)	d (m)	D (m)	W (m)
6308	42300	24000	0.04	0.09	0.023

Lifetime of Bearing:

$$L = \left(\frac{C}{P}\right)^m = 4285.763\ million\ revolutions$$

$$Life = L(10^6)(n_1)^-1(60\frac{min}{hr})^-1(8\frac{mhr}{day})^-1(7\frac{day}{week})^-1(52\frac{week}{yr})^-1 = 9.435\ years$$

Bearing doesn't require a replacement.

Keyways and Tolerances

- Keyway selected for:
  - 30 < d < 38, 10 X 8
  - 44 < d < 50, 14 X 9

- Tolerances:

RESULTS		
HOLE		
Parameter	**Value**	**Unit**
Designation	40 H7	---
Hole Upper Deviation	25	µm (0.001mm)
Hole Lower Deviation	0	µm (0.001mm)
Maximum Hole Size	40.025	mm
Minimum Hole Size	40	mm
SHAFT		
Parameter	**Value**	**Unit**
Designation	40 k6	---
Shaft Upper Deviation	18	µm (0.001mm)
Shaft Lower Deviation	2	µm (0.001mm)
Maximum Shaft Size	40.018	mm
Minimum Shaft Size	40.002	mm
FIT		
Parameter	**Value**	**Unit**
Designation	40 H7/k6	---
Fit Type	Transition fit	---
Maximum Interference	18	µm (0.001mm)
Maximum Clearance	23	µm (0.001mm)

*Keyway and Tolerance Reference at the end

Shaft B Analysis

Gearing

Geometry:

$N_1 = 19, N_2 = 58, N_3 = 23, N_4 = 51$

Pitch Diameter Gear 2: $d_p = N_2 m = 290$ mm

Face Width Gear 2: $b_w = 12m = 60$ mm

Pitch Diameter Gear 3: $d_p = N_3 m = 115$ mm

Face Width Gear 3: $b_w = 65$ mm

Forces:

$n_2 = \frac{N_1}{N_2} * n_1 = 884.483 \frac{m}{s}$

$T_2 = \frac{60H}{2\pi n_2} = 647.789$Nm

Gear Forces from Gear 2 translates from Gear 1:

$Wr_2 = 1626.040N$

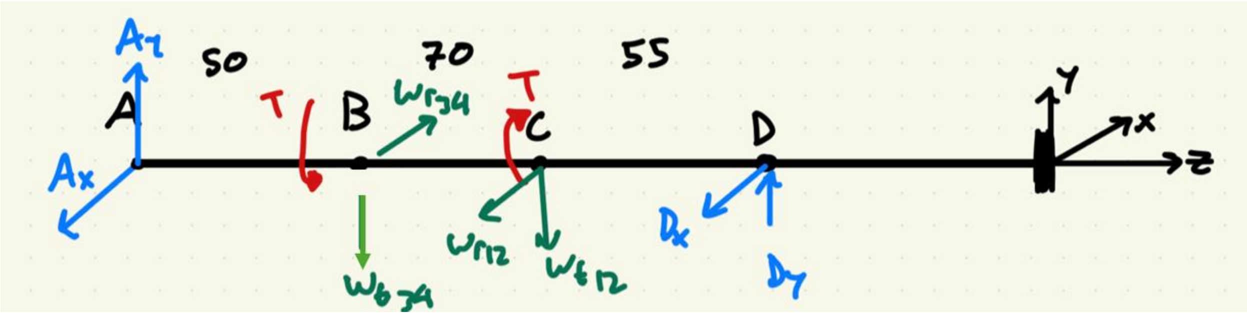
$W_{t2} = 4467.507N$

Gear Forces from Gear Three:

$W_{T34} = \frac{60*60000}{\pi * \frac{n_1}{trainratio} * \frac{d_P}{1000}} = 11265.888N$

$W_{r34} = W_{T_{34}} tan(\phi) = 4100.448N$

Force Analysis



System of Equations:

$$\Sigma F_y = 0 = A_y - W_{t12} + D_y - W_{t34}$$

$$\Sigma F_x = 0 = -A_x - W_{r12} - D_x + W_{r34}$$

$$\Sigma M_{Ay} = 0 = W_{r34} * 50 - W_{r12}(50 + 70) - D_x(50 + 70 + 55) = 0$$

$$\Sigma M_{Ax} = 0 = -W_{t34}(50) - W_{t12}(50 + 70) + D_y(50 + 70 + 55)$$

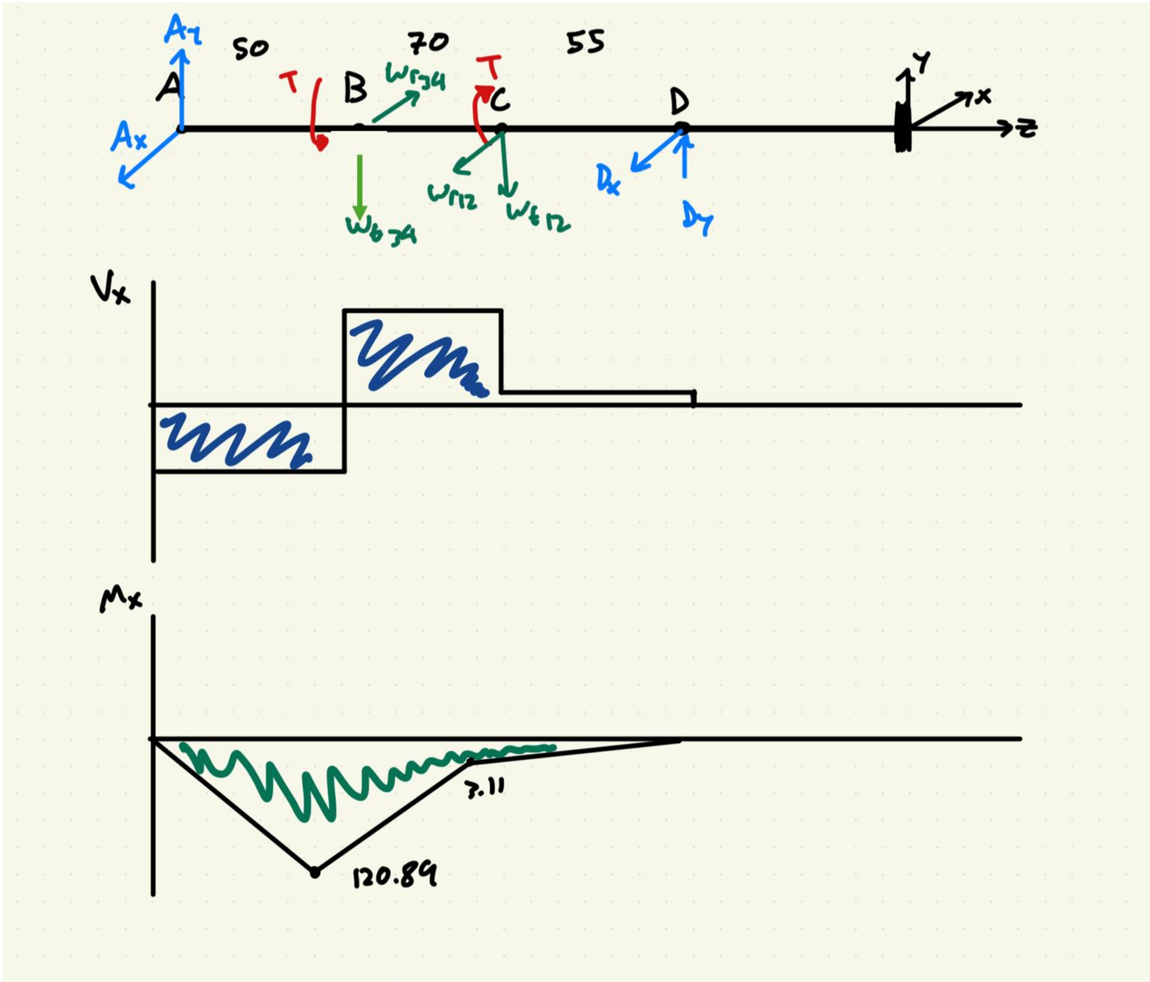
Forces:

$A_x = 2417.85N$

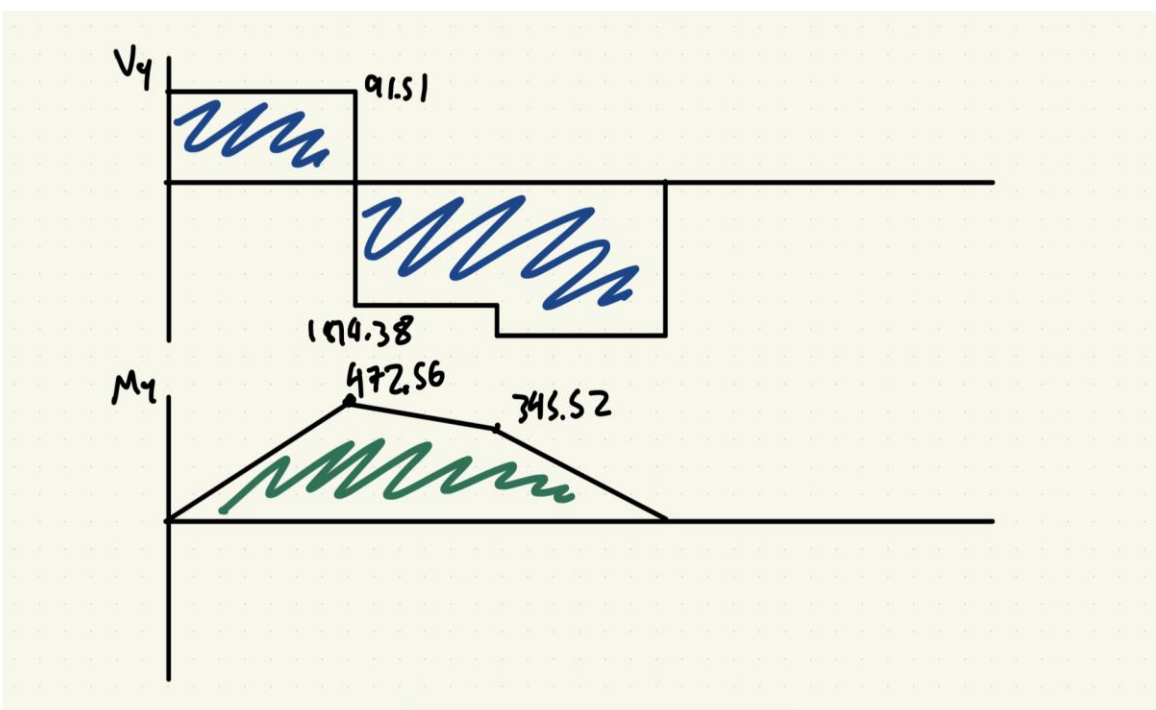
$D_x = 56.56N$

$A_y = 9451.14N$

$D_y = 6282.26N$



Ay
50
70
55
T
B
Wr34
T
C
A
D
Y
X
Ax
z
Dx
Wt34
Wr12
Wt12
Dy
Vx
Mx
7.11
120.89



Vy
91.51
184.38
My
472.56
345.52

Stress Analysis

Factor of Safety:

$$n_s = 1.1$$

Most critical cross section occurs at B^+

Material:

Material	E(Pa)	UTS(MPa)	Ys(MPa)	Poisson Ratio
AISI 1045 Steel	200000	394.72	294.74	0.29

Minimum Diameter Calculation:

$$\tau_{det} = \frac{0.557Y_s}{n_s} = 154.374$$

$$d_{min} \geq \sqrt[3]{\frac{16T}{\pi\tau_{det}}} = 27.74mm$$

Keyway Effect(5% Increase):

$d_{min} = d_{min}(1.05)$ = 29.127 mm

$$d_{min} = 30\ mm$$

Total Moment and Shear (most critical point in space- this point moves on the shaft):

$$V_{total} = \sqrt{1814.36^2 + 1682.6^2} = 2474.48N$$

$$M_{total} = \sqrt{(472.56^2) + (120.84)^2} = 487.77Nm$$

Stress and Factor of Safety

Stress Analysis and Cyclic Loading with $d_{min} = 55mm$ for stiffness:

$$\sigma_a = \sigma_{bs} = \frac{32M}{\pi d^3} = 29.86MPa$$

$$\sigma_m = 0MPa$$

$$\tau_v = \tau_a = \frac{4V}{3A} = 1.39MPa$$

$$\tau_T = \tau_m = \frac{16T}{\pi d^3} = 19.83MPa$$

$$\frac{1}{n_s} = \frac{\sqrt{(K_f\sigma_{xa})^2 + 3(K_{fs}\tau_{xya})^2}}{(k\ Values)(0.5S_{UT})} + \frac{\sqrt{(\sigma_{xm})^2 + 3(\tau_{xym})^2}}{S_{UT}}$$

$k\ values: kf = 0.95; ks = 1.645; kr = 0.82; kt = 1; km = 1$

$$\frac{1}{n_s} == 0.36 + 0.09 = 0.45$$

$n_s = 2.26$ > 1.1 The shaft diameter of 40 mm is usable

Yielding:

$$\frac{1}{n_s} = \frac{\sigma_{VM-A}}{S_y} + \frac{\sigma_{VM-M}}{S_y}$$

$$\frac{1}{n_s} = \frac{\sqrt{(29.86)^2 + 3(1.39)^2}}{294.74} + \frac{\sqrt{3(19.83)^2}}{294.74}$$

$n_s = 4.58$ > 1.1 The shaft diameter doesn't pass yield line (good to go)

Gear Stress and Life Analysis

Gear 2:

Material	E(Pa)	Hardness	Poisson Ratio	Grade
AISI 1045 Steel	200000	400	0.285	1

Shaft Cycles

$$t = 5year * 52\frac{week}{year} * 7\frac{day}{week} * 8\frac{hr}{day} * 60\frac{min}{hr} = 873{,}600\ minutes$$

$N = t * n_2 =$ 772.684 Million Revolutions

Allowable Bending Stress

$$Y_N = 1.6831 * (N * 10^6)^{-0.0323} = 0.869$$

$$S_b = 0.533(HB) + 88.3 = 301.5$$

$$\sigma_{b_{theoretical}} = \frac{S_b * Y_n}{k_t * k_r} = 262.008MPa \qquad k_t = k_r = 1$$

Actual Bending Stress

$$Y_j = 0.34,\ k_a = 1.00,\ k_s = 1.00,\ k_m = 1.14,\ k_i = 1.00,\ k_b = 1.00$$

$$V = \frac{\pi n d}{60} = 13.430\frac{m}{s}$$

$B = \frac{(12-Q_v)^{\frac{2}{3}}}{4} = 0.825$ (assuming $Q_v = 6$)

$$A = 50 + 56(1 - B) = 59.773$$

$$K_v = (\frac{A+\sqrt{200V}}{A})^B = 1.674$$

$$\sigma_b = \frac{W^t}{mb_wY_j}k_ak_sk_mk_vk_ik_b = 83600247.47\ Pa$$

$n_b = \frac{\sigma_{b_{theoretical}}}{\sigma_b} = 3.134 > 1.1$ This design will work under bending stress.

Allowable Contact Stress

$HB_{pinion} = 500HB$ $\frac{HB_{pinion}}{HB_{gear}} = 1.25$

$HB_{gear} = 400HB$

$Z_N = 2.466N^{-0.056} = 0.784$

$C_H = 1 + A^{'}\left(\frac{N_g}{N_p} - 1.0\right) = 1.005793553, A^{'} = 0.0028$

$$K_t = 1, K_r = 1$$

$S_c = 2.22 * HB_g + 200 = 1088MPa$

$\sigma_{c_{theoretical}} = \frac{S_c Z_N C_{H_g}}{K_T K_r} = 857.833MPa$

Actual Contact Stress

$K_a = 1, K_s = 1, K_e = \sqrt{\frac{2}{\frac{1-v_p^2}{E_p} + \frac{1-v_g^2}{E_g}}} = 466563.13$, $K_m = 1.14$, $K_v = 1.674$

$\phi = 20°, d_p = 0.095m, d_g = 0.235m$

$$I = \frac{\pi cos(\phi) sin(\phi)}{1 + \frac{d_p}{d_g}} = 0.761$$

$\sigma_c = K_e \sqrt{\frac{W^t K_a K_s K_m K_v}{b_w d_p I}} = 374.522MPa$

$n_c = \frac{\sigma_{c_{theoretical}}}{\sigma_c} = 2.290 > 1.1$ This design will work under contact stress.

Gear Stress and Life Analysis

Gear 3:

Material	E(Pa)	Hardness	Poisson Ratio	Grade
AISI 1045 Steel	200000	500	0.285	2

Shaft Cycles

$$t = 5year * 52\frac{week}{year} * 7\frac{day}{week} * 8\frac{hr}{day} * 60\frac{min}{hr} = 873600hours$$

$N = t * n_1 =$ 772.68 Million Cycles

Allowable Bending Stress

$$Y_N = 1.6831 * (N * 10^6)^{-0.0323} = 0.869$$

$$S_b = 0.703(HB) + 113 = 464.5$$

$$\sigma_{b_{theoretical}} = \frac{S_b * Y_n}{k_t * k_r} = 403.658MPa \qquad k_t = k_r = 1$$

Actual Bending Stress

$$Y_j = 0.35,\ k_a = 1.00,\ k_s = 1.00,\ k_m = 1.17,\ k_i = 1.00,\ k_b = 1.00$$

$$V = \frac{\pi n d}{60} = 5.326\frac{m}{s}$$

$B = \frac{(12-Q_v)^{\frac{2}{3}}}{4} = 0.825$ (assuming $Q_v = 6$)

$$A = 50 + 56(1 - B) = 59.773$$

$$K_v = (\frac{A+\sqrt{200V}}{A})^B = 1.433$$

$$\sigma_b = \frac{W^t}{m b_w Y_j} k_a k_s k_m k_v k_i k_b = 116032002.1\ Pa$$

$n_b = \frac{\sigma_{b_{theoretical}}}{\sigma_b} = 2.431 > 1.1$ This design will work under bending stress.

Allowable Contact Stress

$HB_{pinion} = 500HB$ $\frac{HB_{pinion}}{HB_{gear}} = 1.25$

$HB_{gear} = 400HB$

$Z_N = 2.466N^{-0.056} = 0.784$

$C_H = 1 + A^{'}\left(\frac{N_g}{N_p} - 1.0\right) = 1, A^{'} = 0$ $K_t = 1, K_r = 1$

$S_c = 2.4 * HB_g + 237 = 1437MPa$

$\sigma_{c_{theoretical}} = \frac{S_c Z_N C_{H_g}}{K_T K_r} = 1126.475MPa$

Actual Contact Stress

$K_a = 1, K_s = 1, K_e = \sqrt{\frac{2}{\frac{1-v_p^2}{E_p} + \frac{1-v_g^2}{E_g}}} = 466563.13$, $K_m = 1.17$, $K_v = 1.433$

$\phi = 20°, d_p = 0.095m, d_g = 0.235m$

$$I = \frac{\pi cos(\phi) sin(\phi)}{1 + \frac{d_p}{d_g}} = 0.696$$

$\sigma_c = K_e \sqrt{\frac{W^t}{b_w d_p} I K_a K_s K_m K_v} = 889.024MPa$

$n_c = \frac{\sigma_{c_{theoretical}}}{\sigma_c} = 1.267 > 1.1$ This design will work under contact stress.

Bearing Selection

$$h = 5year * 364\frac{day}{year} * 8\frac{hr}{day} = 14560hrs$$

$L_{10} = 60hn_3 10^{-6} = 772.684$ Million Revolutions

$A_p = 1$ (no shock), $m = 3$ (ball bearing), $X = 1$ (no axial force)

$$P_r = Max(A, B) = \sqrt{9451.14^2 + 2417.85^2} = 9755.51N$$

$$C = PL_{10}^{\frac{1}{m}} = 89519.431N$$

Using information from: https://www.skf.com/us/products/rolling-bearings/ball-bearings/deep-groove-ball-bearings

Bearing Spec	C(N)	C0 (N)	d (m)	D (m)	W (m)
6411	99500	62000	0.055	0.14	0.033

Lifetime of Bearing:

$L = (\frac{C}{P})^m = 1061.010$ million revolutions

$$Life = L(10^6)(n_2)^-1(60\frac{min}{hr})^-1(8\frac{mhr}{day})^-1(7\frac{day}{week})^-1(52\frac{week}{yr})^-1$$

$$Life \ = 6.866years$$

Bearing doesn't require a replacement within 5 year period.

Keyways and Tolerances

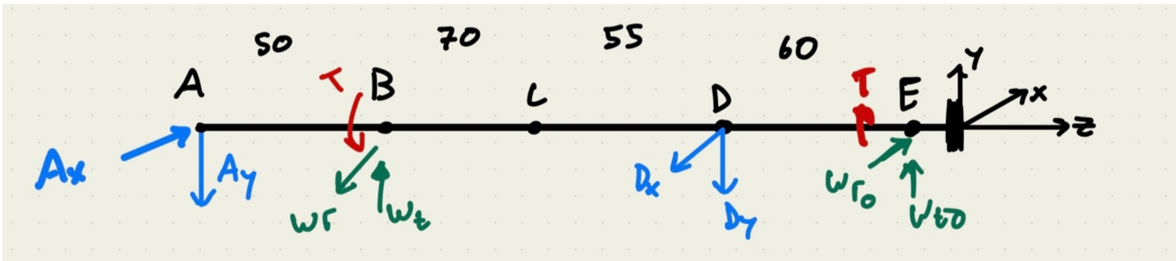
- Keyway selected for:
  - 50 < d < 58, 16 (W) X 10 (H)
  - 65 < d < 68, 18 (W) X 11 (H)

Tolerances:

RESULTS		
HOLE		
Parameter	Value	Unit
Designation	55 H7	---
Hole Upper Deviation	30	µm (0.001mm)
Hole Lower Deviation	0	µm (0.001mm)
Maximum Hole Size	55.03	mm
Minimum Hole Size	55	mm
SHAFT		
Parameter	Value	Unit
Designation	55 k6	---
Shaft Upper Deviation	21	µm (0.001mm)
Shaft Lower Deviation	2	µm (0.001mm)
Maximum Shaft Size	55.021	mm
Minimum Shaft Size	55.002	mm
FIT		
Parameter	Value	Unit
Designation	55 H7/k6	---
Fit Type	Transition fit	---
Maximum Interference	21	µm (0.001mm)
Maximum Clearance	28	µm (0.001mm)

- *Keyway and Tolerance Reference at the end

Shaft 3 Analysis



Force Analysis

System of Equations:

*Calculation for gear forces and torque calculation under gear analysis

$$\Sigma F_x = 0 = A_x - W_{r34} - D_x + W_{ro}$$

$$\Sigma F_y = 0 = -A_y - D_y + W_{t34} - W_{to}$$

$$\Sigma M_{Ax} = 0 = -W_{t34} * 50 + D_y(50 + 70 + 55) - W_{to}(50 + 70 + 5 + 60)$$

$$\Sigma M_{Ay} = 0 = -W_{r34} * (50) - D_x(50 + 70 + 45) + W_{ro}(50 + 70 + 55 + 60)$$

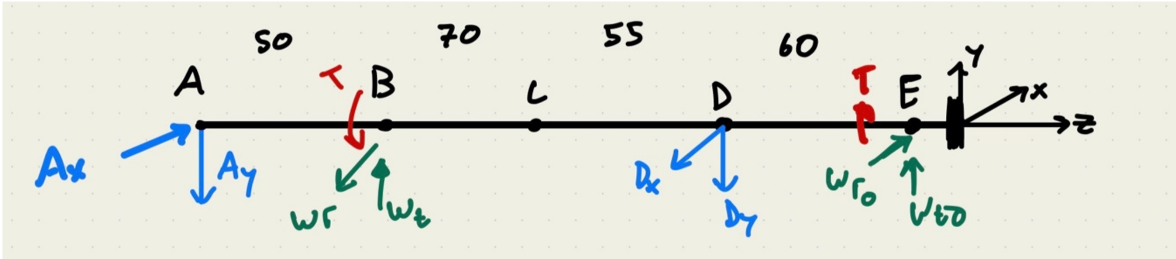
Forces:

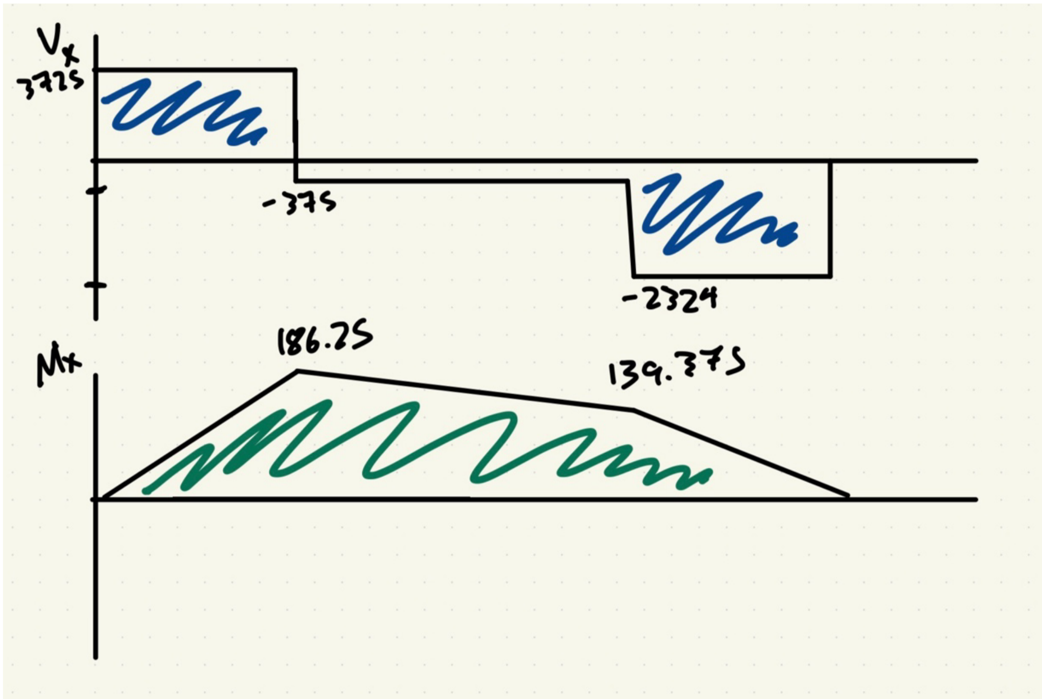
$A_x = 3725\ N$

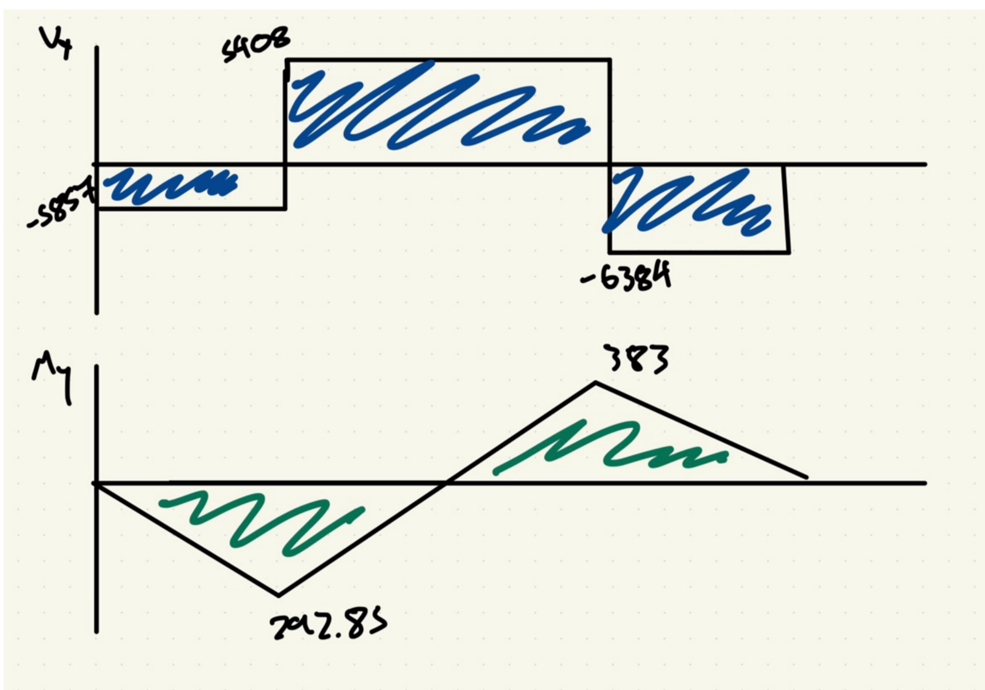
$D_x = 1949\ N$

$A_y = 5857.8\ N$

$D_y = 11792\ N$


50
70
55
60
A
B
C
D
E
T
T
Y
X
Z
Ax
Ay
Wr
Wt
Dx
Dy
Wr0
Wt0


Vx
3725
-375
-2324
Mx
186.25
139.375


Vy
-5857
-6384
My
383
292.85

Stress Analysis

Most critical cross section occurs at D^+

Material:

Material	E(Pa)	UTS(MPa)	Ys (MPa)	Poisson Ratio
AISI 1045 Steel	200000	394.72	294.74	0.29

Minimum Diameter Calculation:

$$\tau_{det} = \frac{0.557Y_s}{n_s} = 154.584$$

$$d_{min} \geq \sqrt[3]{\frac{16T}{\pi\tau_{det}}} = 36.17mm$$

Keyway Effect (5% Increase):

$d_{min} = d_{min}(1.05)$ = 37.979 mm

Rounded, **38 mm**

Total Moment and Shear (most critical point in space, points on shaft cycle through):

$$V_{total} = \sqrt{6384^2 + 2324^2} = 6793.0\ N$$

$$M_{total} = \sqrt{(383.0^2) + (139.38)^2} = 407.0\ Nm$$

Stress and Factor of Safety Calculation

Stress Analysis and Cyclic Loading with a chosen

$d_{min} = 70mm$:

$$\sigma_a = \sigma_{bs} = \frac{32M}{\pi d^3} = 12.08\ MPa$$

$$\sigma_m = 0MPa$$

$$\tau_v = \frac{4V}{3A} = 2.35\ MPa$$

$$\tau_T = \frac{16T}{\pi d^3} = 21.33\ MPa$$

Kf = 0.95, Kr = 0.82, Kt = 1, Km = 1, Ks = 1.189*d^-0.112 = 1.60

KF = 3, KFS = 2.8

$$\frac{1}{n_s} = \frac{\sqrt{(K_f\sigma_{xa})^2+3(K_{fs}\tau_{xya})^2}}{(Kf*Ks*Kr*Kt*Km)(0.5S_{UT})} + \frac{\sqrt{(\sigma_{xm})^2+3(\tau_{xym})^2}}{S_{UT}}$$

$$\frac{1}{n_s} = 0.15 + 0.0936$$

$n_s = 4.04$ > 1.1 The shaft diameter of 70 mm is usable

Yielding:

$$\frac{1}{n_s} = \frac{\sigma_{VM-A}}{S_y} + \frac{\sigma_{VM-M}}{S_y}$$

$$\frac{1}{n_s} = \frac{\sqrt{(36.32)^2+3(27.06)^2}}{K_s'+.5*S_{ut}} + \frac{\sqrt{(3*21.33^2)}}{S_{UT}}$$

$n_s = 5.93$ > 1.1 The shaft diameter doesn't pass yield line

Gear Stress and Life Analysis

Geometry:

$N_1 = 19, N_2 = 58, N_3 = 23, N_4 = 51$

Pitch Diameter: $d_p = N_4 m = 255$ mm

Face Width: $b_w = 12m = 65$ mm

Forces:

$$n_3 = \frac{n_1}{trainratio} = 398.884$$

$$\mathrm{T} = \frac{60H}{2\pi n_3} = 1436.52 \text{ Nm}$$

Gear Forces from Gear Three to Four:

$$W_{t34} = \frac{60*60000}{\pi*n3*\frac{d_p}{1000}} = 11265.888N$$

$$W_{r34} = W_{T_{34}} tan(\phi) = 4100.448N$$

Gear Forces from Output Gear:

Assumed outside gear pitch diameter of 450 mm

$$W_{to} = \frac{T}{\frac{d}{2}} = 6384.5\ N$$

$$W_{ro} = W_{to} tan(\phi) = 2323.78\ N$$

Material	E(Pa)	Hardness	Poisson Ratio	Grade
AISI 1045 Steel	200000	500	0.285	2

Shaft Cycles

$$t = 5year * 52\frac{week}{year} * 7\frac{day}{week} * 8\frac{hr}{day} * 60\frac{min}{hr} = 873600\ min$$

$N = t * n_3 =$ 348.47 Million Cycles

Allowable Bending Stress

$$Y_N = 1.6831 * (N * 10^6)^{-0.0323} = 0.892$$

$S_b = 0.533(HB) + 88.3 = 301.5$ MPa

$$\sigma_{b_{theoretical}} = \frac{S_b * Y_n}{k_T * k_R} = 268.835\ MPa \qquad k_T = k_R = 1$$

Actual Bending Stress

$Y_j = 0.35$, $k_a = 1.00$, $k_s = 1.00$, $k_m = 1.14$, $k_i = 1.00$, $k_b = 1.00$

$$V = \frac{\pi n d}{60} = 5.33\,\frac{m}{s}$$

$B = \frac{(12 - Q_v)^{\frac{2}{3}}}{4} = 0.825$ (assuming $Q_v = 6$)

$$A = 50 + 56(1 - B) = 59.773$$

$$K_v = \left(\frac{A + \sqrt{200V}}{A}\right)^B = 1.433$$

$$\sigma_b = \frac{W^t}{m b_w Y_j} k_a k_s k_m k_v k_i k_b = 175.26\ MPa$$

$n_b = \frac{\sigma_{b_{theoretical}}}{\sigma_b} = 1.534 > 1.1$ This design will work under bending stress.

Allowable Contact Stress

$$HB_{pinion} = 500HB \qquad \frac{HB_{pinion}}{HB_{gear}} = 1.25$$

$$HB_{gear} = 400HB$$

$$Z_N = 2.466N^{-0.056} = 0.820$$

$$C_H = 1 + A'\left(\frac{N_g}{N_p} - 1.0\right) = 1.003436087 \qquad K_t = 1, K_r = 1$$

$$S_c = 2.22 * HB_g + 200 = 1088MPa$$

$$\sigma_{c_{theoretical}} = \frac{S_c Z_N C_{H_g}}{K_T K_r} = 894.851\ MPa$$

Actual Contact Stress

$$K_a = 1, K_s = 1, K_e = \sqrt{\frac{2}{\frac{1-v_p^2}{E_p}+\frac{1-v_g^2}{E_g}}} = 466563.13,\ K_m = 1.14,\ K_v = 1.433$$

$$\phi = 20°, d_p = 255mm, d_g = 265mm$$

$$I = \frac{\pi cos(\phi)sin(\phi)}{1+\frac{d_p}{d_g}} = 0.696$$

$$\sigma_c = K_e\sqrt{\frac{W^t}{b_w d_p \mathbf{I}}K_a K_s K_m K_v} = 613.385\ MPa$$

$n_c = \frac{\sigma_{c_{theoretical}}}{\sigma_c} = 1.459 > 1.1$ This design will work under contact stress.

Bearing Selection

$$h = 5year * 364\frac{day}{year} * 8\frac{hr}{day} = 14560hrs$$

$L_{10} = 60hn_3 10^{-6} = 348.47$ Million Revolutions

$A_p = 1$ (no shock), $m = 3$ (ball bearing), $X = 1$ (no axial force)

Bearing D experiences the highest forces.

$$P_r = \sqrt{1949^2 + 11792^2} = 11951.98$$

$$C = PL_{10}^{\frac{1}{m}} = 84105.89N$$

Using information from: https://www.skf.com/us/products/rolling-bearings/ball-bearings/deep-groove-ball-bearings

Bearing Spec	C(N)	C0 (N)	ID (m)	OD (m)	Bw (m)
6314	111000	68000	0.07	0.15	0.035

Lifetime of Bearing:

$$L = (\frac{C}{P})^m = 801.03\ million\ revolutions$$

$$Life = L(10^6)(n_1)^-1(60\frac{min}{hr})^-1(8\frac{mhr}{day})^-1(7\frac{day}{week})^-1(52\frac{week}{yr})^-1$$

$$= 11.49\ years$$

Bearing doesn't require a replacement.

Keyways and Tolerances

Keyway selected from table for:

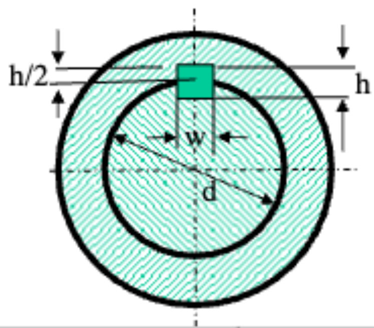
58 < d < 65, 18 X 11

RESULTS		
HOLE		
Parameter	**Value**	**Unit**
Designation	70 H7	---
Hole Upper Deviation	30	μm (0.001mm)
Hole Lower Deviation	0	μm (0.001mm)
Maximum Hole Size	70.03	mm
Minimum Hole Size	70	mm
SHAFT		
Parameter	**Value**	**Unit**
Designation	70 k6	---
Shaft Upper Deviation	21	μm (0.001mm)
Shaft Lower Deviation	2	μm (0.001mm)
Maximum Shaft Size	70.021	mm
Minimum Shaft Size	70.002	mm
FIT		
Parameter	**Value**	**Unit**
Designation	70 H7/k6	---
Fit Type	Transition fit	---
Maximum Interference	21	μm (0.001mm)
Maximum Clearance	28	μm (0.001mm)

*Keyway and Tolerance Reference at the end

References

- Keyways



Shaft diameter d (mm)	Key width W x height h
12<d≤17	5 x 5
17<d≤22	6 x 6
22<d≤30	8 x 7
30<d≤38	10 x 8
38<d≤44	12 x 8
44<d≤50	14 x 9
50<d≤58	16 x 10
58<d≤65	18 x 11

Reference

Norton, R., L., 2000, *Machine Design*, Prentice-Hall Inc.

- Tolerances:
  - https://amesweb.info/fits-tolerances/tolerance-calculator.aspx
- Bearings:
  - https://www.skf.com/us/products/rolling-bearings/ball-bearings/deep-groove-ball-bearings