SOME APPLICATIONS OF MODEL-BASED PARAMETER ESTIMATION IN ELECTROMAGNETICS

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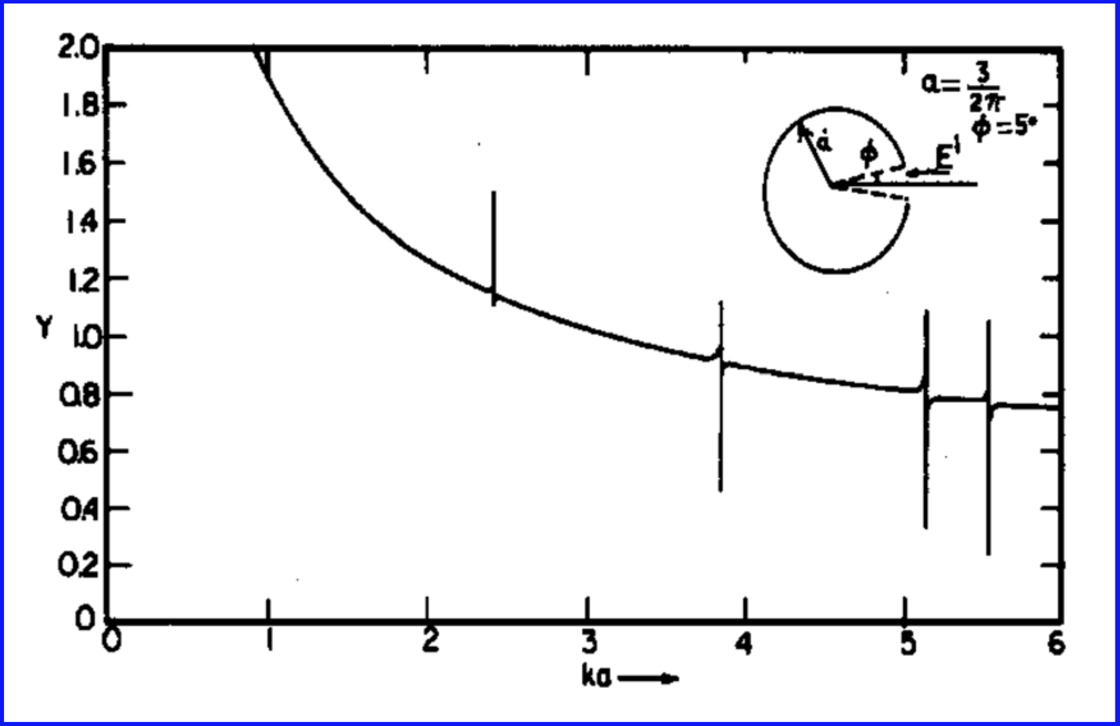
Obscured by the mathematical detail associated with most electromagnetic analysis is the possibility of representing physical observables in simpler ways by using reduced-order models, a process that is generically called "model-order reduction" (MOR). Knowledge of such models, ideally physically based, can be helpful in ways ranging from reducing the computer cost of achieving desired solutions to developing **more compact representations of observables**. One specific approach to MOR is to estimate unknown parameters of the reduced-order models from sampled data, a process called "**model-based parameter estimation**" (MBPE). This lecture will survey some applications of MBPE in electromagnetic modeling and demonstrate various benefits that result. It should be observed that the designation MBPE has evolved in EM primarily from application to integral-equation (IE) models, whereas the term MOR has primarily been associated with application to differential-equation (DE) models. While conceptually equivalent, it might be observed that MOR describes a general goal whereas MBPE is a particular approach to achieving that goal.

QUESTION: WHAT ALTERNATIVES ARE AVAILABLE FOR COMPUTING FRQUENCY RESPONSES LIKE . . .



- WHEN THE COST PER SAMPLE IS HIGH
- WHAT MODELS ARE APPROPRIATE?
- CAN NUMBER OF SAMPLES BE REDUCED?
- CAN GENERATING-MODEL OPERATION COUNT BE MINIMIZED?

MBPE IS A POWERFUL TOOL FOR COMPUTING A TRANSFER FUNCTION

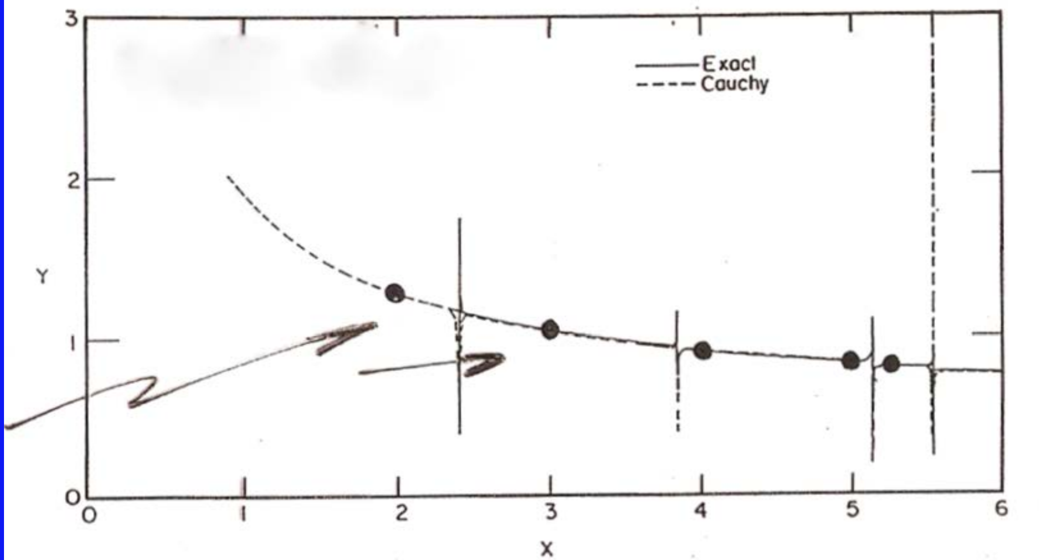


Fig. 4. Scattered far field of a conducting (slit) cylinder using Cauchy's technique (5 points). Vertical axis: V/m.

- RESPONSE AND FIRST FOUR DERIVATIVES COMPUTED AT 5 FREQUENCIES AS DESCRIBED BELOW

PRESENTATION EXPLORES SOME ISSUES IN ESTIMATING & REPRESENTING EM OBSERVABLES

1) THE SCIENTIFIC METHOD

2) MODEL-BASED PARAMETER ESTIMATION

3) FITTING MODELS FOR WAVEFORM AND SPECTRAL DATA

4) FUNCTION SAMPLING AND DERIVATIVE SAMPLING

5) ADAPTIVE SAMPLING OF FREQUENCY SPECTRA

6) ADAPTIVE SAMPLING OF RADIATION AND SCATTERING PATTERNS

7) USING MBPE TO ESTIMATE MODEL UNCERTAINTY

8) OTHER FITTING MODELS FOR EM OBSERVABLES

9) USING MBPE FOR GENERATING-MODEL COMPUTATION

PRESENTATION EXPLORES SOME ISSUES IN ESTIMATING & REPRESENTING EM OBSERVABLES

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THE SCIENTIFIC METHOD BEGAN AS AN OBSERVATIONAL ACTIVITY . . .

MAKING OBSERVATIONS
QUALITATIVE, RAW NUMBERS, PASSIVE

. . . THAT EVENTUALLY BECAME MORE SYSTEMATIC LEADING TO . . .

MAKING OBSERVATIONS

QUALITATIVE,
RAW NUMBERS,
PASSIVE

↓

TAKING DATA

ACTIVE

MORE QUANTITATIVE,

SEEKING PATTERNS

. . . WITH INCREASING AMOUNTS OF DATA RESULTING IN . . .

MAKING OBSERVATIONS
QUALITATIVE,
RAW NUMBERS,
PASSIVE

↓

TAKING DATA
ACTIVE
MORE QUANTITATIVE,
SEEKING PATTERNS

↓

ACQUIRING INFORMATION
MODELING,
PARAMETER
ESTIMATION

. . . AND FROM THIS INFORMATION DEVELOPED:

MAKING OBSERVATIONS
QUALITATIVE,
RAW NUMBERS,
PASSIVE

TAKING DATA
ACTIVE
MORE QUANTITATIVE,
SEEKING PATTERNS

ACQUIRING INFORMATION
MODELING,
PARAMETER ESTIMATION

DEVELOPING KNOWLEDGE
INTERPOLATION,
TEST THEORY,
DEDUCTION

AS THE KNOWLEDGE BASE EXPANDED THERE DEVELOPED . . .

MAKING OBSERVATIONS
QUALITATIVE,
RAW NUMBERS,
PASSIVE

TAKING DATA
ACTIVE
MORE QUANTITATIVE,
SEEKING PATTERNS

ACQUIRING INFORMATION
MODELING,
PARAMETER ESTIMATION

DEVELOPING KNOWLEDGE
INTERPOLATION,
TEST THEORY,
DEDUCTION

WISDOM
EXTRAPOLATION,
PREDICTION,
INDUCTION

. . . AN ATTEMPT TO UNDERSTAND, PREDICT AND GENERALIZE

MAKING OBSERVATIONS
QUALITATIVE,
RAW NUMBERS,
PASSIVE

TAKING DATA
ACTIVE
MORE QUANTITATIVE,
SEEKING PATTERNS

ACQUIRING INFORMATION
MODELING,
PARAMETER ESTIMATION

DEVELOPING KNOWLEDGE
INTERPOLATION,
TEST THEORY,
DEDUCTION

WISDOM
EXTRAPOLATION,
PREDICTION,
INDUCTION

VISION
SPECULATION,
HYPOTHESES,
GENERALIZATION

PASSIVE (*MERELY OBSERVING*) OR ACTIVE (*USING A PROBING FIELD*) EXPERIMENTS ARE DESIGNED . . .

- TO ACQUIRE DATA

. . . FROM WHICH . . .

- **INFORMATION IS SOUGHT**

. . . FOR THE PURPOSE OF . . .

- DEVELOPING, CONFIRMING THEORY

. . . ABOUT SOME PHYSICAL PHENOMENON OF INTEREST

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ONE APPROACH TO DEVELOPING UNDERSTANDING CAN BE DESCRIBED AS *MODEL-BASED PARAMETER ESTIMATION*

- MBPE INVOLVES A **MODEL** **IDEALLY** **BASED** **ON PHYSICAL PRINCIPLES**

  Observable(s) = Function(Variables,Parameters)

- WHOSE **PARAMETERS** ARE TO BE **ESTIMATED** FROM SAMPLED OBSERVABLES

- USING VARIOUS SIGNAL-PROCESSING TECHNIQUES

- MOST OFTEN IN THE PRESENCE OF NOISE, INCOMPLETE DATA AND/OR UNCERTAINTY

MBPE IS OBSERVED TO CONNECT DIGITAL SIGNAL PROCESSING AND CLASSICAL NUMERICAL ANALYSIS

- DIGITAL SIGNAL PROCESSING IS CONCERNED WITH THE REPRESENTATION OF SIGNALS BY SEQUENCES OF NUMBERS OR SYMBOLS AND THE PROCESSING OF THESE SEQUENCES[x]

- CLASSICAL NUMERICAL ANALYSIS FORMULAS SUCH AS INTERPOLATION, INTEGRATION AND DIFFERENTIATION "ARE CERTAINLY DIGITAL SIGNAL PROCESSING ALGORITHMS"[x]

[x]Oppenheim and Schafer, *Digital Signal Processing*, Prentice-Hall, Inc., 1975

MODEL-BASED PARAMETER ESTIMATION IS STRAIGHTFORWARD . . .

- DATA OR COMPLEX FORMULATIONS ARE REPLACED BY SIMPLER MATHEMATICAL REPRESENTATIONS

  --THE "FITTING MODEL" WHICH IDEALLY IS PHYSICALLY BASED

- MODEL PARAMETERS ARE OBTAINED FROM FITTING THE ORIGINAL DESCRIPTION, A FIRST-PRINCIPLES OR GENERATING MODEL

  --THE "PARAMETER-ESTIMATION" STEP

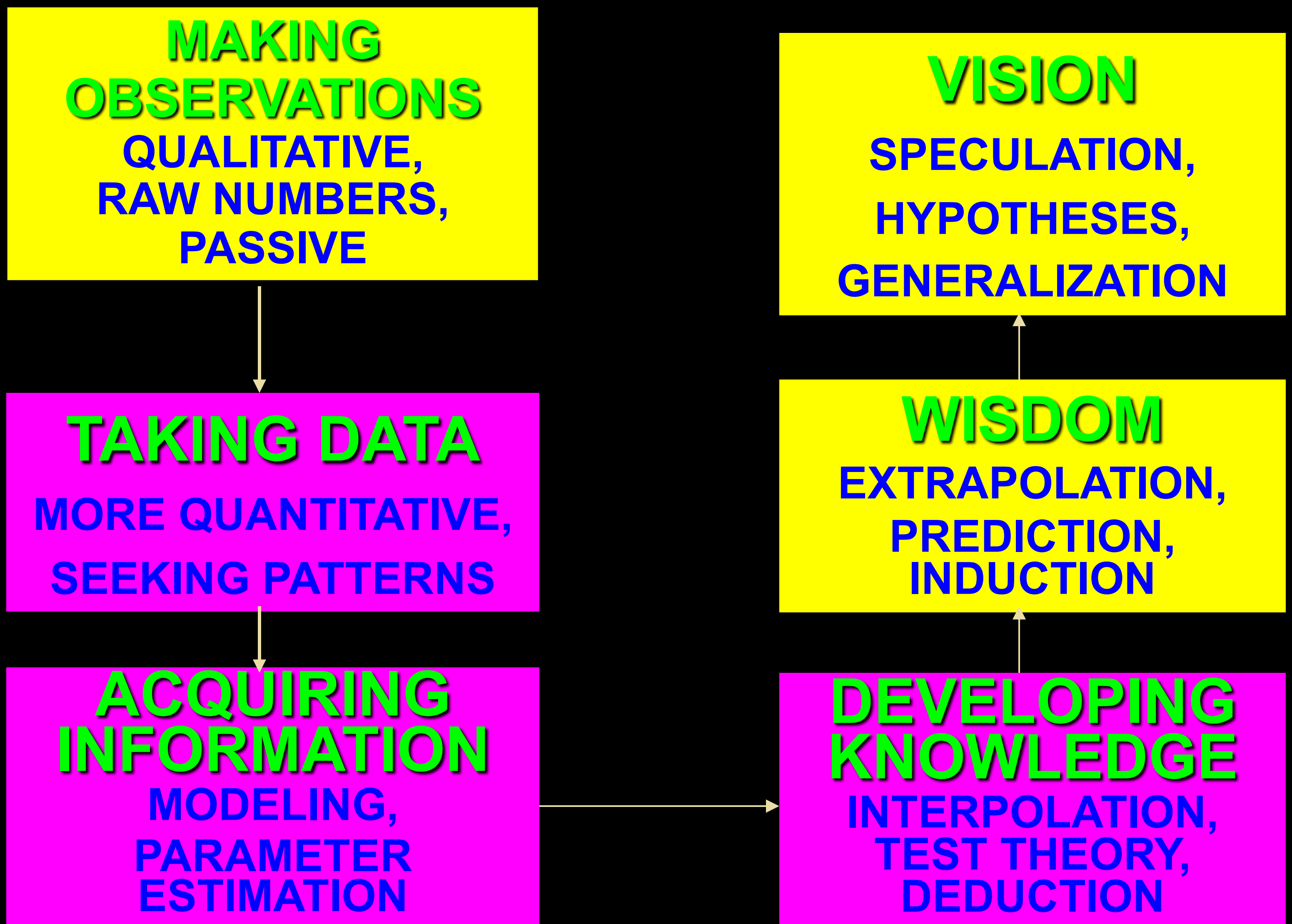
- SUCH FITTING MODELS CAN PROVIDE A MORE EFFICIENT NUMERICAL APPROACH BY:

  --REDUCING COMPLEXITY/COST OF GENERATING-FUNCTION COMPUTATION

  --MAKING MORE EFFICIENT USE OF THE RESULTS THAT ARE COMPUTED

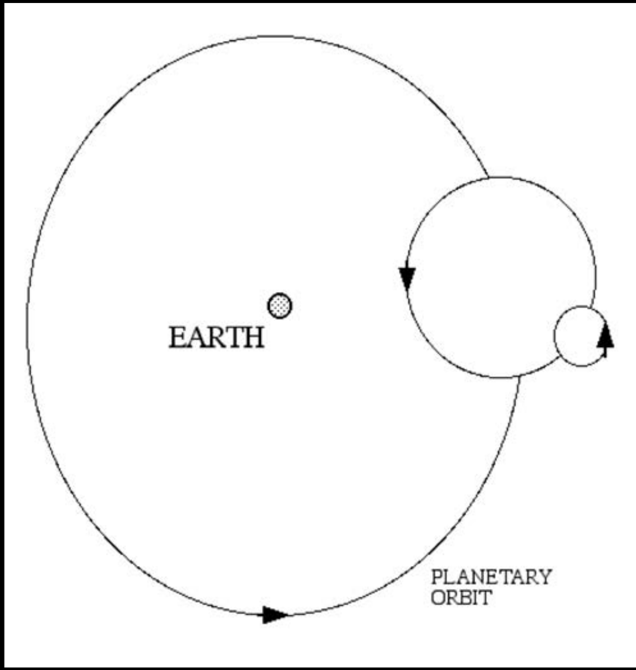
. . . OR "SMART" CURVE FITTING

MBPE CAN BE SEEN TO BE A SUBSET OF THE SCIENTIFIC METHOD

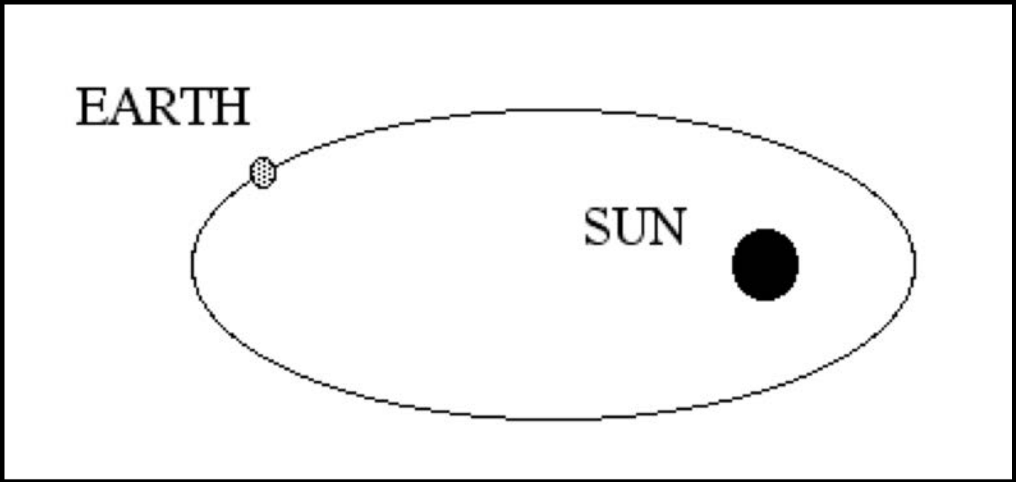


EARLY EXAMPLE OF MBPE IS PTOLEMY'S EARTH-CENTERED SOLAR-SYSTEM MODEL . . .

- INITIAL ORBITS BASED ON EARTH-CENTERED CIRCLES
- MODEL NEEDED EPICYCLE ADDED TO ACCOUNT FOR RETROGRADE MOTION
- LIMITED PREDICTIVE CAPABILITY
- MORE EPICYCLES ARE NEEDED WITH BETTER DATA
- WITH APOLOGIES TO PTOLEMY THIS IS AN EXAMPLE OF CURVE FITTING WITHOUT UNDERSTANDING THE BASIC PHYSICS OF THE PROBLEM



. . . THAT WAS SUCCEEDED BY THE CORRECT COPERNICAN, SUN-CENTERED MODEL



- CHANGED TO SUN-ORIENTED, ELIPSE-BASED PATTERN
- NEED FOR EPICYCLES ELIMINATED
- ACCURATE PREDICTIONS WITHOUT MODIFICATION
- ORIGIN OF KEPLER’S LAWS
- AN EXAMPLE OF “SMART” CURVE FITTING

MODEL-BASED PARAMETER ESTIMATION CAN REDUCE THE COST OF EM COMPUTATIONS & MEASUREMENTS BY REDUCING . . .

--THE COST OF OBTAINING THE ORIGINAL DATA

. . . AND . . .

--THE NUMBER OF DATA POINTS NEEDED TO REPRESENT THE DESIRED INFORMATION

. . . WITH THE LATTER BEING THE PRIMARY TOPIC OF THIS PRESENTATION

MBPE CAN REDUCE COST OF A SAMPLE BASED ON A FIRST-PRINCIPLES MODEL BY . . .

- e.g., SIMPLIFYING EVALUATION OF INTEGRAL-EQUATION KERNEL

VIA . . .

- EXPLOITING UNDERLYING PHYSICAL BEHAVIOR (AND SIMPLICITY THEREOF)
  - --USING A MODEL THAT EMPLOYS A REDUCED-ORDER MATHEMATICAL DESCRIPTION
- APPLICATIONS INCLUDE--
  - --SOMMERFELD INTEGRALS FOR INTERFACE PROBLEM (SPEEDUP OF 100 OR MORE)
  - --PERIODIC BOUNDARIES SUCH AS RECTANGULAR AND SPHERICAL CAVITIES

. . . OR CAN REDUCE THE NUMBER OF SAMPLES NEEDED FOR CERTAIN APPLICATIONS . . .

- e.g., TO OBTAIN WIDEBAND FREQUENCY RESPONSE OR TO OBTAIN ANGLE SCATTERING OR RADIATION PATTERNS

. . . AGAIN VIA . . .

- EXPLOITING PHYSICAL SIMPLICITY AND REDUCED-ORDER MATHEMATICAL MODEL USED TO REPRESENT THE RESPONSE

- OBTAINING MODEL PARAMETERS FROM APPROPRIATE SAMPLES OF DESIRED RESPONSE--

  --TIME OR FREQUENCY OR ANGLE OR SPACE OR . . .

  --ACTUAL RESPONSE OR DERIVATIVES THEREOF

MBPE IS USED IN MANY DIVERSE AREAS FOR WHICH A FEW EXAMPLES ARE:

Model-based parameter estimation applied on electrocardiogram signal

Model-Based Material Parameter Estimation for Terahertz Reflection Spectroscopy

THE RESULTS OF THRESHOLD SETTINGS ON MODEL BASED RECOGNITION AND PARAMETER ESTIMATION OF BUILDINGS FROM MULTI-VIEW AERIAL IMAGERY

Model Based Signal Processing for GPR Data Inversion

On Stochastic Parameter Estimation using Data Assimilation

Set-based parameter estimation for symmetric network motifs

ASAC - Analysis/Synthesis Audio Codec for Very Low Bit Rates

AN EFFICIENT ADAPTIVE FREQUENCY SAMPLING ALGORITHM FOR MODEL-BASED PARAMETER ESTIMATION AS APPLIED TO AGGRESSIVE SPACE MAPPING

VARIOUS SOURCES THAT HAVE MATERIAL COVERED IN THIS SHORT COURSE INCLUDE:

G. J. Burke, E. K. Miller, S. Chakrabarti, and K. R. Demarest (1989), "Using Model-Based Parameter Estimation to Increase the Efficiency of Computing Electromagnetic Transfer Functions", *IEEE Transactions on Magnetics*, **25** (4), pp. 2807-2809.

E. K. Miller (1991), "Model-Based Parameter-Estimation Applications in Electromagnetics", in *Electromagnetic Modeling and Measurements for Analysis and Synthesis Problems,* edited by B. de Neumann, Kluwer Academic Publishers, NATO ASI Series E: Applied Sciences Vol.. 199, pp. 205-256.

K. Kottapalli,, T. K. Sarkar, Y. Hua, E. K. Miller, and G. J. Burke (1991), "Accurate Computation of Wideband Response of Electromagnetic Systems Utilizing Narrowband Information," *IEEE Trans. Microwave Theory and Techniques*. **MTT-39**, April, pp. 682-687.

E. K. Miller and G. J. Burke (1991), "Using Model-Based Parameter Estimation to Increase the Physical Interpretability and Numerical Efficiency of Computational Electromagnetics," *Computer Physics Communications*, **68**, 43-75.

E. K. Miller (1995), “Model-Based Parameter Estimation in Electromagnetics: I--Background and Theoretical Development,” *Applied Computational Electromagnetics Society Newsletter,* **10** (3), November, pp. 40-63.

E. K. Miller and T. K. Sarkar (1999), “Model-Order Reduction in Electromagnetics Using Model-Based Parameter Estimation, in *Frontiers in EM*, D. H. Werner and R. Mittra, Ed., IEEE Press, New York, pp. 371-436.

E. K. Miller and T. K. Sarkar (1999), “An Introduction to the Use of Model-Based Parameter Estimation in Electromagnetics,” *Review of Radios Science 1996-1999*, Ed. W. Ross Stone, Oxford University Press, 139-174.

E. K. Miller (2002), “Using Adaptive Estimation to Minimize the Number of Samples Needed to Develop a Radiation or Scattering Pattern to a Specified Uncertainty,” *Applied Computational Electromagnetics Society Journal*, **17**, (3), pp. 176-186.

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RATIONAL-FUNCTION FM ARISES FROM RESONANCE BEHAVIOR OF EM PHENOMENA . . .

- DESCRIBED AS AN EXPONENTIAL SERIES IN WAVEFORM DOMAIN (WD) . . .

$$-- \; f(x) = f_p(x) + f_{np}(x) = \sum_{\alpha=1}^{P} R_\alpha e^{s_\alpha x} + f_{np}(x)$$

- . . . AND AS A POLE SERIES IN SPECTRAL DOMAIN (SD)

$$-- \; F(X) = F_p(X) + F_{np}(X) = \sum_{\alpha=1}^{P} \frac{R_\alpha}{(X - s_\alpha)} + F_{np}(X)$$

. . . WHERE "np" DENOTES A NON-POLE COMPONENT

RATIONAL-FUNCTION FM ARISES FROM RESONANCE BEHAVIOR OF EM PHENOMENA WHERE. . .

- "x" REPRESENTS THE WD INDEPENDENT VARIABLE AND "X" ITS SD, OR TRANSFORMED, COUNTERPART

- FOR TIME-FREQUENCY TRANSFORM PAIR . . .

  --x IS THE TIME VARIABLE, t

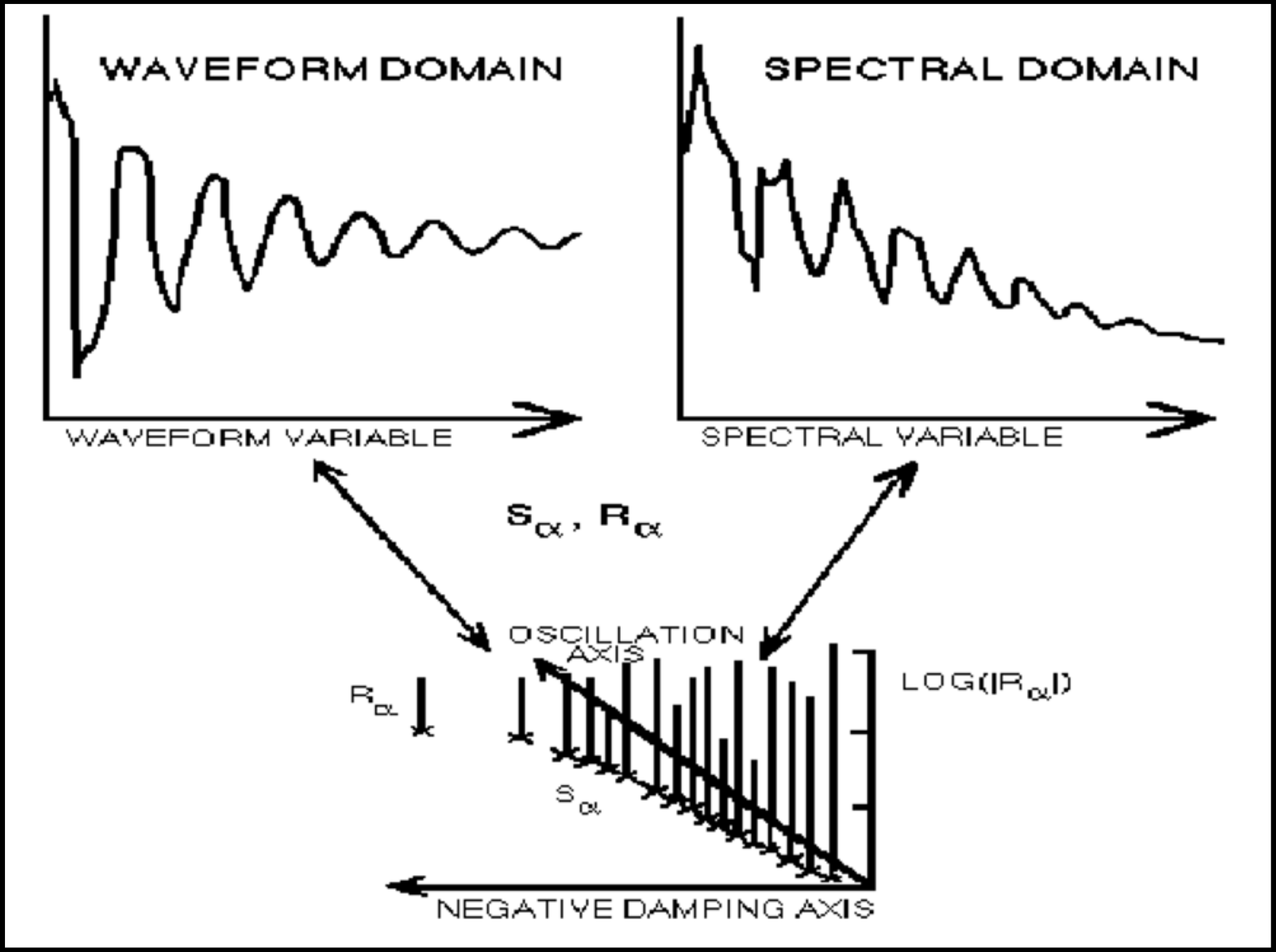
  --X IS THE COMPLEX FREQUENCY, $s = j\omega - \sigma$

- PARAMETERS FOR PAIRED FMs ARE:

  --COMPLEX RESONANCES (OR POLES) "s_α"

  --MODAL AMPLITUDES (OR RESIDUES) "R_α"

VIEWING R_α AND s_α ON A PERSPECTIVE PLOT PROVIDES CONCISE FORMAT



MODEL-BASED PARAMETER ESTIMATION THUS INVOLVES THREE THINGS . . .

1. A PROCESS OR DATA TO BE MODELED

2. A MODEL TO REPRESENT THAT PROCESS/DATA

3. PARAMETERS TO BE ESTIMATED FROM THAT PROCESS/DATA

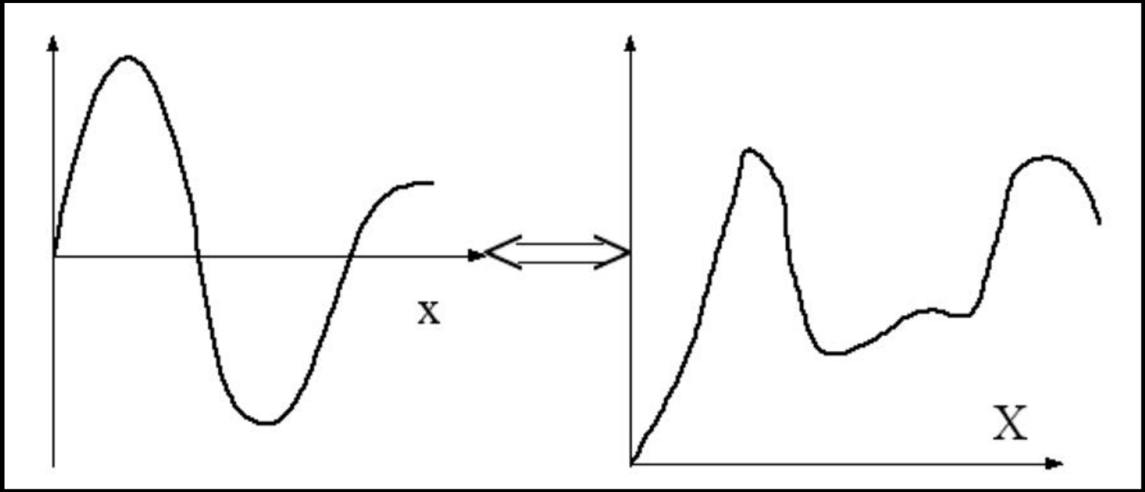
. . . WHERE THE NUMBER OF PARAMETERS IS THE MODEL ***RANK***

AMONG THE OBSERVABLES AVAILABLE IN WAVE-EQUATION PROBLEMS ARE:

- FIELD TYPE
  - --ELECTRIC OR MAGNETIC
- FIELD LOCATION
  - --NEAR ZONE AND FAR ZONE
- SOURCES
  - --CONDUCTION CURRENT, CHARGE
- SELF AND MUTUAL IMPEDANCES

POLES AND RESIDUES YIELD CONCISE PARAMETRIC DESCRIPTION . . .

- $f(x) = \sum R_\alpha \exp(s_\alpha x)$ $F(X) = \sum R_\alpha/(X - s_\alpha)$



- . . . AND PHYSICALLY USEFUL QUANTITIES
- RESONANCE FREQUENCIES
- SOURCE LOCATIONS
- TIME CONSTANTS
- DIRECTION ANGLES

ONE APPROACH USEFUL FOR SHOWING HOW SUCH DATA MIGHT BE PROCESSED IS PRONY'S METHOD . . .

- FOR WAVEFORM DATA--
  - --REQUIRES UNIFORM SAMPLING INTERVAL
  - --MATRIX OF SAMPLED DATA IS DEVELOPED
  - --MATRIX SOLUTION YIELDS COEFFICIENTS OF PREDICTOR, OR CHARACTERISTIC, EQUATION
  - --ROOTS OF CHARACTERISTIC EQUATION LEAD TO POLES

- FOR SPECTRAL DATA--
  - --PERMITS NONUNIFORM SAMPLING INTERVAL
  - --MATRIX OF SAMPLED DATA IS DEVELOPED
  - --MATRIX SOLUTION YIELDS COEFFICIENTS OF RATIONAL-FUNCTION MODEL
  - --ROOTS OF DENOMINATOR POLYNOMIAL PROVIDE POLES

. . . ALTHOUGH ITS NOISE SENSITIVITY MIGHT REQUIRE USE OF MORE SOPHISTICATED METHODS

PRONY'S METHOD YIELDS COMPLEX-EXPONENTIAL PARAMETERS FROM UNIFORMLY SPACED DATA SAMPLES:

IN $$f(t) = \sum_{\alpha=1}^{P} R_\alpha e^{s_\alpha t}$$ LET $$X_\alpha = e^{s_\alpha \delta t}$$

AND $$f_i \equiv f(t_i) = f(i\delta t), i = 0, 1, \ldots, P-1, \ldots, 2P-1, \ldots$$

SO THAT

$$\begin{aligned}
&R_1 + R_2 + \cdots + R_P = f_0 \\
&R_1 X_1 + R_2 X_2 + \cdots + R_P X_P = f_1 \\
&\vdots \\
&R_1 X_1^{P-1} + R_2 X_2^{P-1} + \cdots + R_P X_P^{P-1} = f_{P-1} \\
&\vdots
\end{aligned}$$

… TOGETHER WITH A CHARACTERISTIC EQUATION . . .

GIVEN BY $$\sum_{\alpha=0}^{P} a_\alpha X^\alpha = (X - X_1)(X - X_2)\cdots(X - X_P)$$

TO THEN FORM

$$\begin{aligned}
a_0(R_1 + R_2 + \cdots + R_P) &= a_0 f_0 \\
a_1(R_1 X_1 + R_2 X_2 + \cdots + R_P X_P) &= a_1 f_1 \\
&\vdots \\
a_P(R_1 X_1^P + R_2 X_2^P + \cdots + R_P X_P^P) &= a_P f_P
\end{aligned}$$

. . . AND ADD FIRST *P* EQUATIONS TO GET:

$$a_0 f_0 + a_1 f_1 + \cdots + a_{P-1} f_{P-1} = -f_P$$

CONTINUE WITH THE 2nd, 3rd, etc. EQUATIONS TO GET. . .

$$
\begin{aligned}
&a_0 f_0 + a_1 f_1 + \cdots + a_{P-1} f_{P-1} = - f_P \\
&a_0 f_1 + a_1 f_2 + \cdots + a_{P-1} f_P = - f_{P+1} \\
&a_0 f_2 + a_1 f_3 + \cdots + a_{P-1} f_{P+1} = - f_{P+2} \\
&\vdots \\
&a_0 f_{P-1} + a_1 f_P + \cdots + a_{P-1} f_{2P-2} = - f_{2P-1}
\end{aligned}
$$

. . . AND FIND POLES FROM ROOTS OF CHARACTERISTIC EQUATION AS:

$$
s_\alpha = \frac{1}{\delta t} Ln(X_\alpha)
$$

CHARACTERISTIC EQUATION AND POLYNOMIAL ARE NOT UNIQUE--

IN $a_0 + a_1 X + a_2 X^2 + \cdots + a_P X^P$

--AN ADDITIONAL CONSTRAINT IS NEEDED TO PRODUCE A NON-HOMOGENEOUS SYSTEM

- PRONY'S METHOD USES $a_P = 1$ WHICH LEADS TO A LINEAR-PREDICTOR FORM SHOWN ABOVE, i.e.

$$a_0 f_i + a_1 f_{i+1} + \cdots + a_{P-1} f_{i+P-1} + f_P = 0$$

CHARACTERISTIC EQUATION AND POLYNOMIAL ARE NOT UNIQUE--

IN $$a_0 + a_1 X + a_2 X^2 + \mathrm{L} + a_N X^P$$

--AN ADDITIONAL CONSTRAINT IS NEEDED TO PRODUCE A NON-HOMOGENEOUS SYSTEM

- "ENERGY" CONSTRAINT COULD USE:

$$\sum_{i=0}^{N} a_i^2 = 1$$

- NON-CAUSAL CONSTRAINT COULD HAVE FORM:

$$a_k = 1, k \neq P$$

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WAVEFORM SAMPLING USES SAMPLES BASED ON AN EXPONENTIAL MODEL . . .

$$f_i = f(x_i) = \sum_{\alpha=1}^{P} \hat{R}_\alpha (\hat{X}_\alpha)^i; i = 0,1,\ldots,2P-1,$$

$$X_\alpha = e^{\hat{s}_\alpha \delta},$$

. . . WHICH TOGETHER WITH THE CHARACTERISTIC EQUATION . . .

$$\sum_{\alpha=0}^{P} \hat{a}_\alpha X^\alpha = 0; X = X_i, i = 0,1,\ldots,P-1$$

. . . LEADS TO THE LINEAR SYSTEM . . .

$$\begin{bmatrix} f_0 & f_1 & \cdots & f_{P-1} \\ f_1 & f_2 & \cdots & f_P \\ \vdots & \vdots & \vdots & \vdots \\ f_{P-1} & f_P & \cdots & f_{2P-2} \end{bmatrix} \begin{bmatrix} \hat{a}_0 \\ \hat{a}_1 \\ \vdots \\ \hat{a}_{P-1} \end{bmatrix} = \begin{bmatrix} f_P \\ f_{P+1} \\ \vdots \\ f_{2P-1} \end{bmatrix}$$

. . . FOR THE PREDICTOR COEFFICIENTS

-- ESTIMATED QUANTITIES ARE DENOTED BY THE OVERSTRIKE

WAVEFORM-DERIVATIVE SAMPLING CAN ALSO BE USED . . .

$$f^{(i)} = \frac{d^i f(x)}{dx^i} = \sum_{\alpha=1}^{P} s_\alpha^i R_\alpha e^{s_\alpha x}$$

. . . WHICH TOGETHER WITH . . .

$$\sum_{\alpha=0}^{P} \hat{b}_\alpha s^\alpha = 0; s = s_i, i = 1, 2, \ldots, 2P$$

. . . LEADS TO THE LINEAR SYSTEM . . .

$$\begin{bmatrix} f^{(0)} & f^{(1)} & \cdots & f^{(P-1)} \\ f^{(1)} & f^{(2)} & \cdots & f^{(P)} \\ \vdots & \vdots & \vdots & \vdots \\ f^{(P-1)} & f^{(P)} & \cdots & f^{(2P-2)} \end{bmatrix} \begin{bmatrix} \hat{b}_0 \\ \hat{b}_1 \\ \vdots \\ \hat{b}_{N-1} \end{bmatrix} = - \begin{bmatrix} f^{(P)} \\ f^{(P+1)} \\ \vdots \\ f^{(2P-1)} \end{bmatrix}$$

. . . FOR THE PREDICTOR COEFFICIENTS $\hat{b}_\alpha$

SPECTRAL SAMPLES ARE MODELED USING A RATIONAL-FUNCTION

THUS $$F(X)D(X) = N(X)$$

WHERE $$N(X) = N_0 + N_1X^1 + N_2X^2 + \cdots + N_nX^n$$

$$D(X) = D_0 + D_1X^1 + D_2X^2 + \cdots + D_dX^d$$

UPON INSERTING *D* SAMPLES $F_i = F(X_i)$ **IN** $F_i\hat{D}(X_i) = \hat{N}(X_i)$

A LINEAR SYSTEM IS GENERATED FOR THE FITTING-MODEL COEFFICIENTS WITH $D_d = 1$ . . .

$$\begin{bmatrix} F_0 & X_0F_0 & \cdots & X_0^{d-1}F_0 & -1 & -X_0 & \cdots & -X_0^n \\ F_1 & X_1F_1 & \cdots & X_1^{d-1}F_1 & -1 & -X_1 & \cdots & -X_1^n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ F_{D-1} & X_{D-1}F_{D-1} & \cdots & X_{D-1}^{d-1}F_{D-1} & -1 & -X_{D-1} & \cdots & -X_{D-1}^n \end{bmatrix} \begin{bmatrix} \hat{D}_0 \\ \vdots \\ \hat{D}_{d-1} \\ \hat{N}_0 \\ \vdots \\ \hat{N}_n \end{bmatrix} = - \begin{bmatrix} X_0^dF_0 \\ X_1^dF_1 \\ \vdots \\ \vdots \\ X_{D-2}^dF_{D-1} \\ X_{D-1}^dF_{D-1} \end{bmatrix}$$

RATIONAL-FUNCTION MODEL IS LOGICAL GENERALIZATION OF A POLE-SERIES . . .

$$\sum_{\alpha=1}^{P} \frac{R_\alpha}{X - s_\alpha} = F_p(X) = \frac{N_0 + N_1 X + \cdots + N_{P-1} X^{P-1}}{1 + D_1 X + \cdots + D_P X^P} = \frac{N(X, P-1)}{D(X,P)}$$

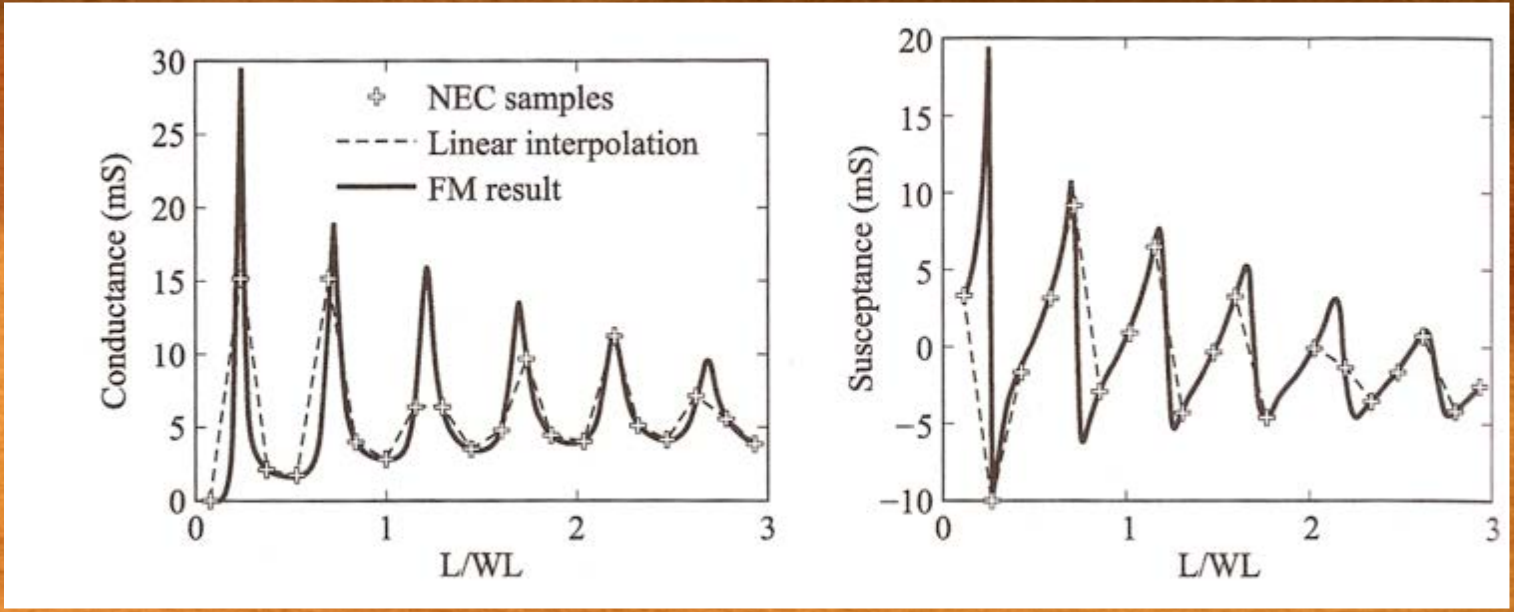
. . . IN WHICH THE NONPOLE PART CAN BE APPROXIMATED AS:

$$F_{np}(X) \cong CONSTANT \Rightarrow F(X) \approx \frac{N(X,P)}{D(X,P)}$$

OR

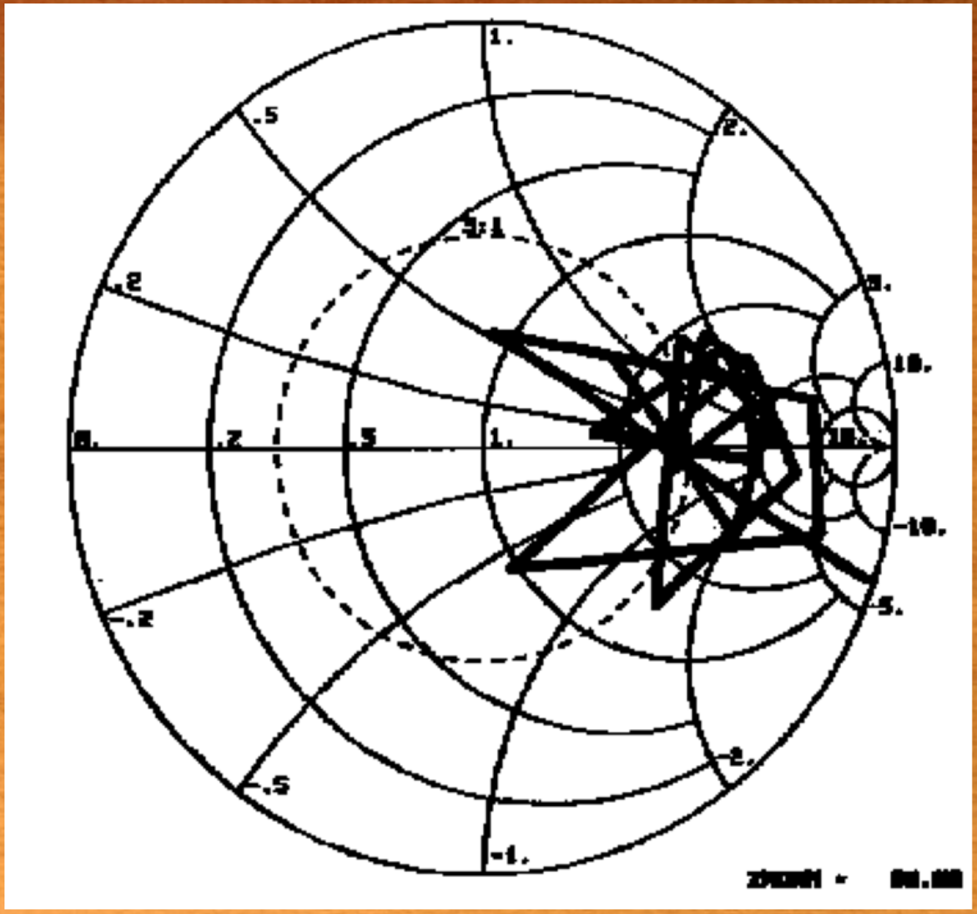
$$F_{np}(X) \cong C_1 + C_2 X \Rightarrow F(X) \approx \frac{N(X,P+1)}{D(X,P)}$$

WINDOWED RATIONAL-FUNCTION MODEL CAN BE "SLID" THROUGH SPARSELY SAMPLED DATA . . .

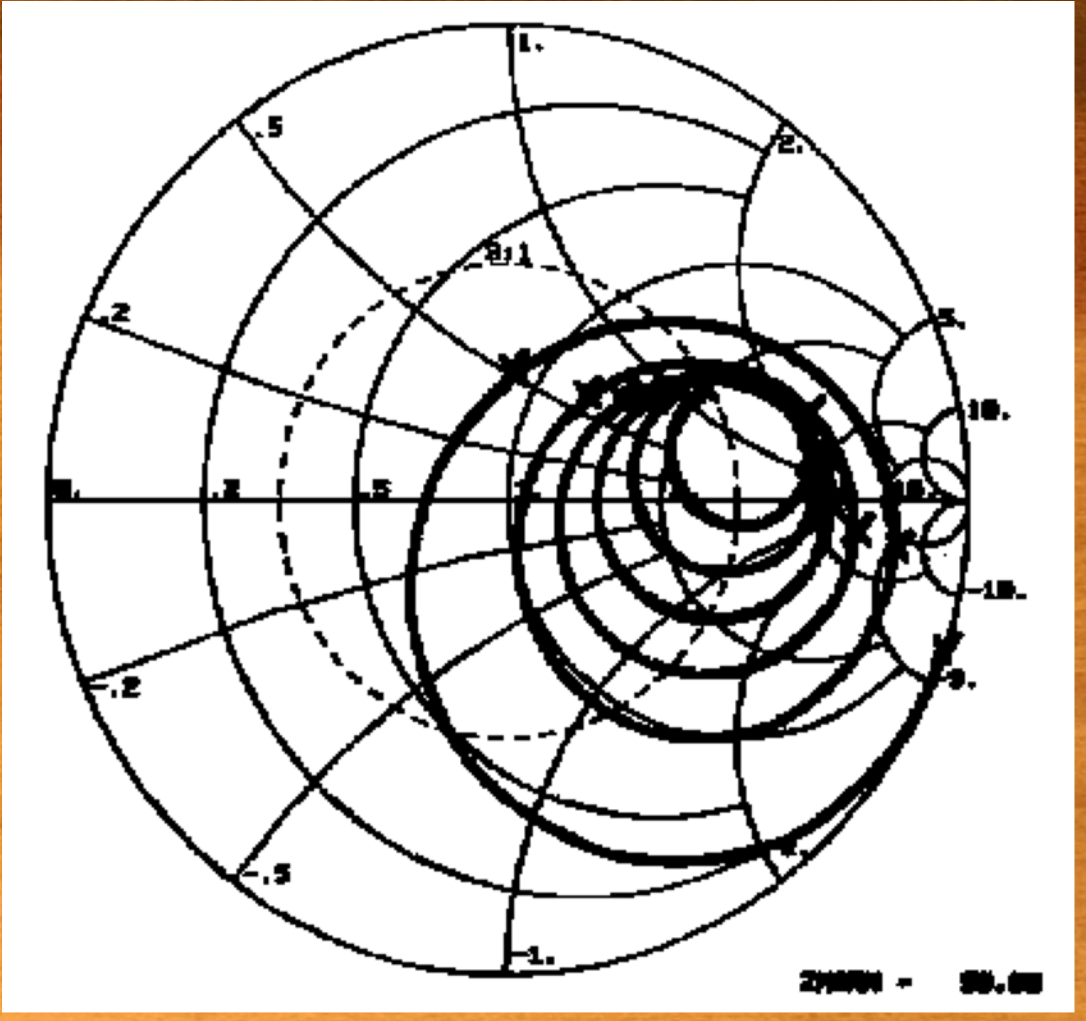


Input conductance and susceptance of monopole antenna (a/L=0.01) versus length in wavelengths as obtained from a series of overlapping rational-function fitting models using d = n = 3 (the solid line) based on 20 GM samples spaced 0.15 apart in L/wavelength which are shown as open crosses and joined by a dashed line [Burke (1992)]. A comparison of the FM values with GM samples at other frequencies reveals a numerical agreement of 1% or better.

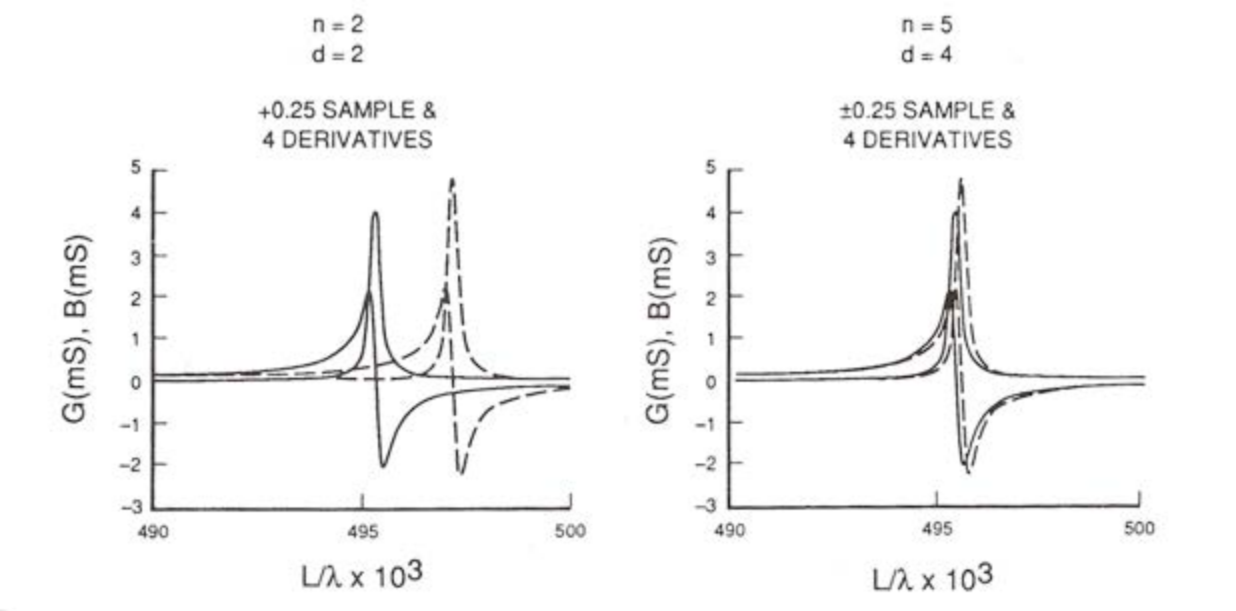
. . . WHERE STRAIGHT-LINE INTERPOLATION ON SMITH CHART EMPHASIZES INADEQUATE SAMPLING



RATIONAL-FUNCTION FITTING MODEL IS CLEARLY BETTER



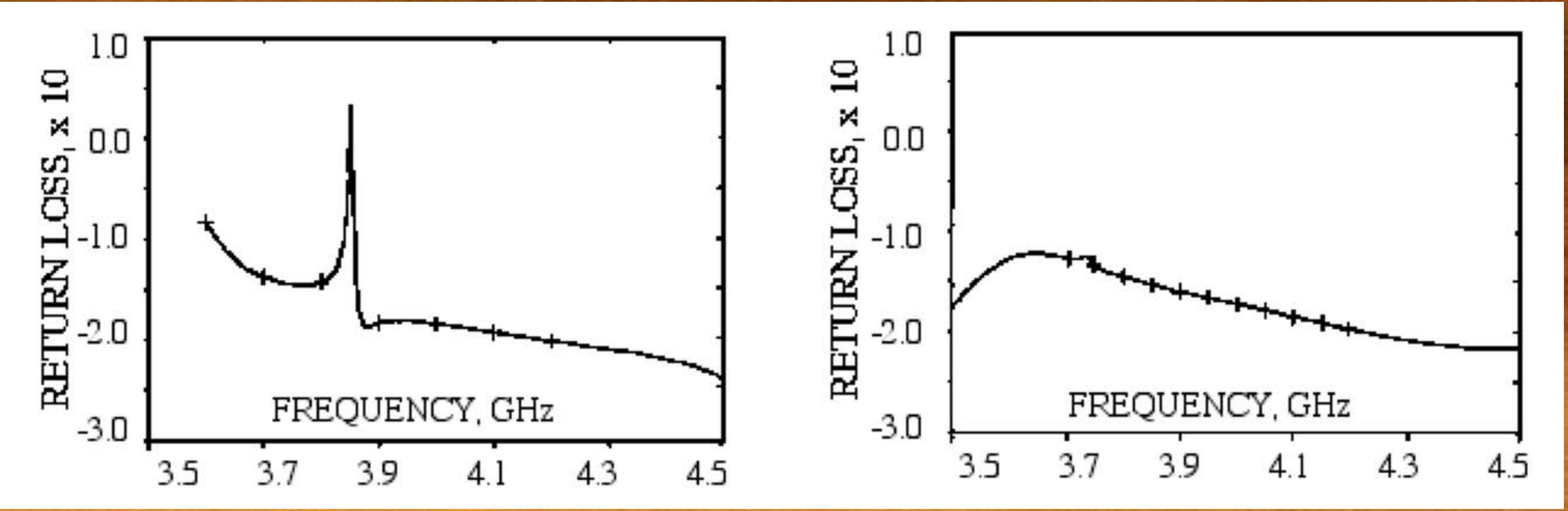
LOWER-ORDER MODEL USING 5 FREQUENCY SAMPLES MATCHES RESONANCE WITHIN 0.1% . . .



- SAMPLED AT 0.2, 0.25, 0.3, 0.35,0.4
- n = 2, d = 2
- ________ NEC COMPUTED, -------------- MBPE ESTIMATED

. . . FOR DIPOLE OVER PEC GROUND

MBPE IS ALSO USEFUL FOR SYN-THESIZING A FREQUENCY RESPONSE



Results obtained from synthesizing a corrugated-horn antenna without using MBPE, (left), and when incorporating a SD FM as part of the synthesis procedure (right) [Fermelia et al. (1993)]. The FM revealed the sharp spike in the return loss that was not apparent in the original design, permitting it to be removed in the MBPE-based design.*

*Fermelia, L. R., G. Z. Rollins and P. Ramanujam (1993), "The Use of Model-Based Parameter Estimation (MBPE) for the Design of Corrugated Horns," Program Digest of US URSI Radio Science Meeting, University of Michigan, p. 388.

SPECTRAL-DERIVATIVES ARE OBTAINED FROM THE DIFFERENTIATED RATIONAL-FUNCTION MODEL:

--WITH SAMPLES UP TO THE t'th DERIVATIVEGIVEN BY . . .

$$
\begin{aligned}
&FD = N \\
&F'D + FD' = N' \\
&F''D + 2F'D' + FD'' = N'' \\
&\vdots \\
&F^{(t)}D + tF^{(t-1)}D^{(1)} + \cdots + C_{t,t-m}F^{(m)}D^{(t-m)} + \cdots + FD^{(t)} = N^{(t)}
\end{aligned}
$$

. . . WHERE A BRACKETED SUPERSCRIPT (*m*) DENOTES AN *m*'th DERIVATIVE . . .

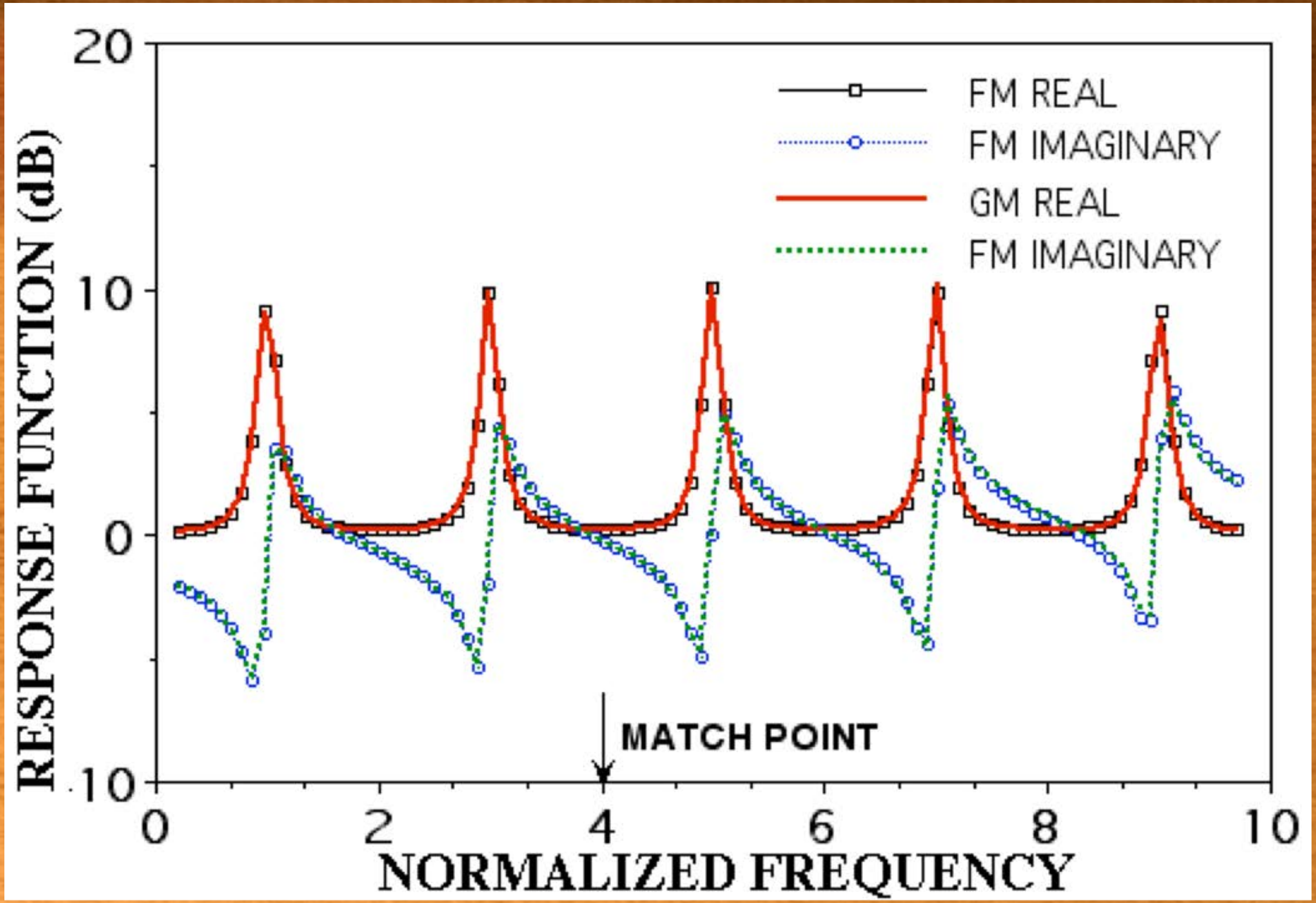
. . . LEADING TO THE LINEAR SYSTEM:

$$
\begin{bmatrix}
1 & 0 & L & 0 & 0 & 0 & L & 0 \\
0 & 1 & L & M & -F_0 & 0 & L & 0 \\
0 & 0 & L & 0 & -F_1 & -F_0 & L & 0 \\
0 & 0 & L & 1 & -F_2 & -F_1 & M & M \\
0 & 0 & L & 0 & M & -F_2 & M & M \\
M & M & M & M & M & M & M & 0 \\
0 & 0 & L & 0 & -F_{D-1} & F_{D-2} & L & -F_{D-d}
\end{bmatrix}
\begin{bmatrix}
\hat{N}_0 \\ \hat{N}_1 \\ M \\ \hat{N}_n \\ \hat{D}_1 \\ M \\ \hat{D}_d
\end{bmatrix}
=
\begin{bmatrix}
F_0 \\ F_1 \\ F_2 \\ M \\ M \\ M \\ F_D
\end{bmatrix}
$$

. . . SOLUTION OF WHICH PROVIDES THE FITTING-MODEL COEFFICIENTS

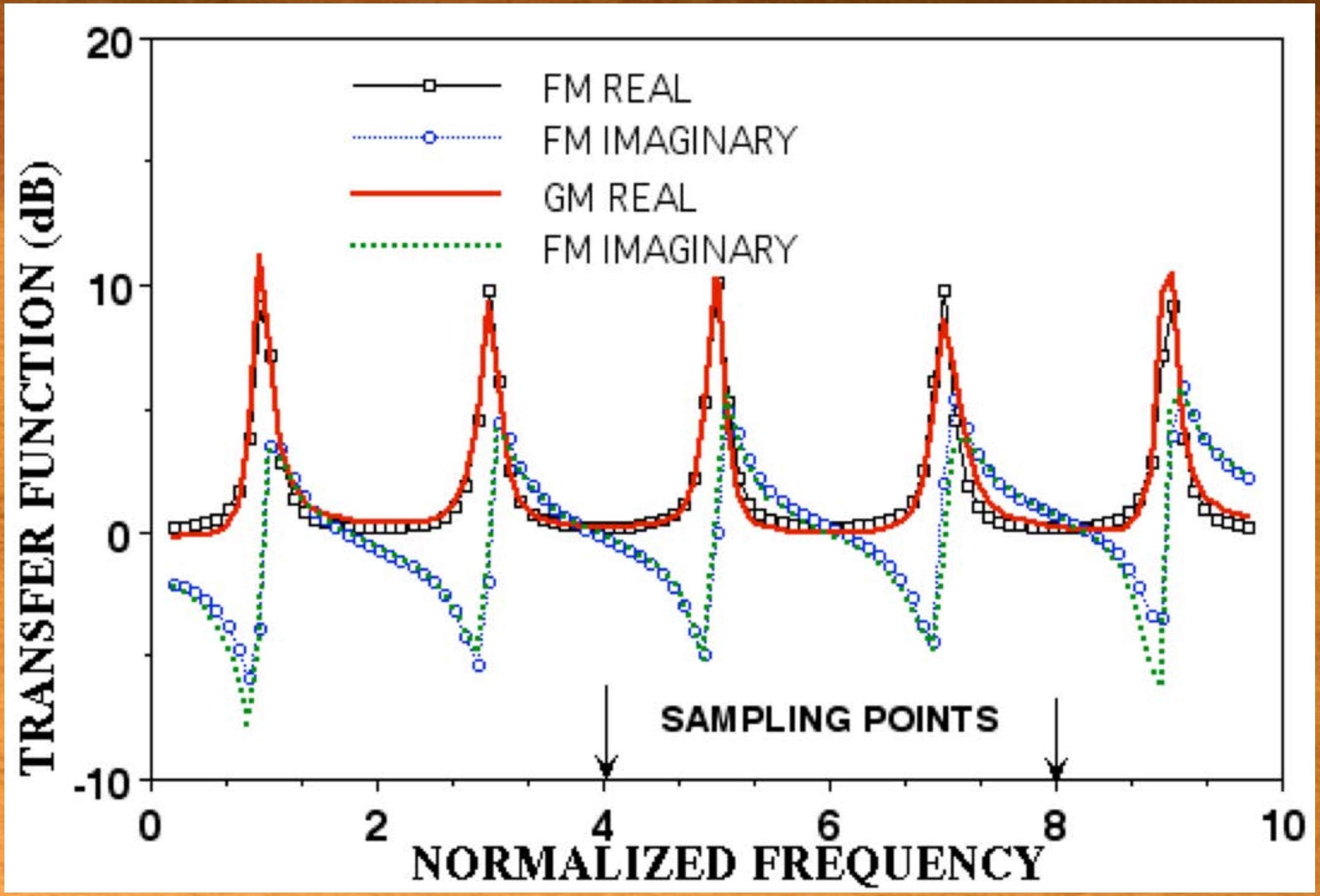
--WHERE $F_m = F^{(m)}/m!$

DERIVATIVE SAMPLING OF A 5-POLE SPECTRUM PROVIDES FEW PERCENT MATCH



ONE FREQUENCY SAMPLE & 9 DERIVATIVES

THUS FUNCTION AND DERIVATIVE SAMPLING AT TWO (OR MORE) FREQUENCIES IS ALSO FEASIBLE



TWO FREQUENCY SAMPLES AND 4 DERIVATIVES AT EACH

MBPE CAN BE USED IN TWO DIFFERENT WAYS FOR OBTAINING BROADBAND EM RESULTS . . .

FORMULATION DOMAIN	SOLUTION DOMAIN
e.g., COMPUTING IE MATRIX	REPRESENTING FREQUENCY RESPONSE

. . . USING . . .

WAVEFORM MODELS	SPECTRAL MODELS
$\sum R_i \exp(s_i x)$	$\sum R_i/(X - X_i)$

. . . BASED ON FUNCTION SAMPLES OR DERIVATIVE SAMPLES

DERIVATIVE INFORMATION IS READILY DEVELOPED FROM FDIE (& OTHER) MODELS . . .

$$\sum_{i=1}^{N} Z_{i,j}(\omega) I_j(\omega) = V_i(\omega) \Rightarrow I_i(\omega) = \sum_{j=1}^{N} Y_{i,j}(\omega) V_j(\omega)$$

--IS THE USUAL MoM SOLUTION AND--

$$\sum_{i=1}^{N} \left[Z_{i,j}(\omega) I'_j(\omega) + Z'_{i,j}(\omega) I_i(\omega) \right] = V'_i(\omega)$$

$$\Rightarrow I'_i(\omega) = \sum_{j=1}^{N} Y_{i,j}(\omega) \left(V'_j(\omega) - \sum_{k=1}^{N} Z'_{j,k}(\omega) I_k(\omega) \right)$$

--GIVES THE FIRST DERIVATIVE OF THE CURRENT [BURKE (1969)]

DERIVATIVE INFORMATION IS READILY DEVELOPED FROM FDIE (& OTHER) MODELS . . .

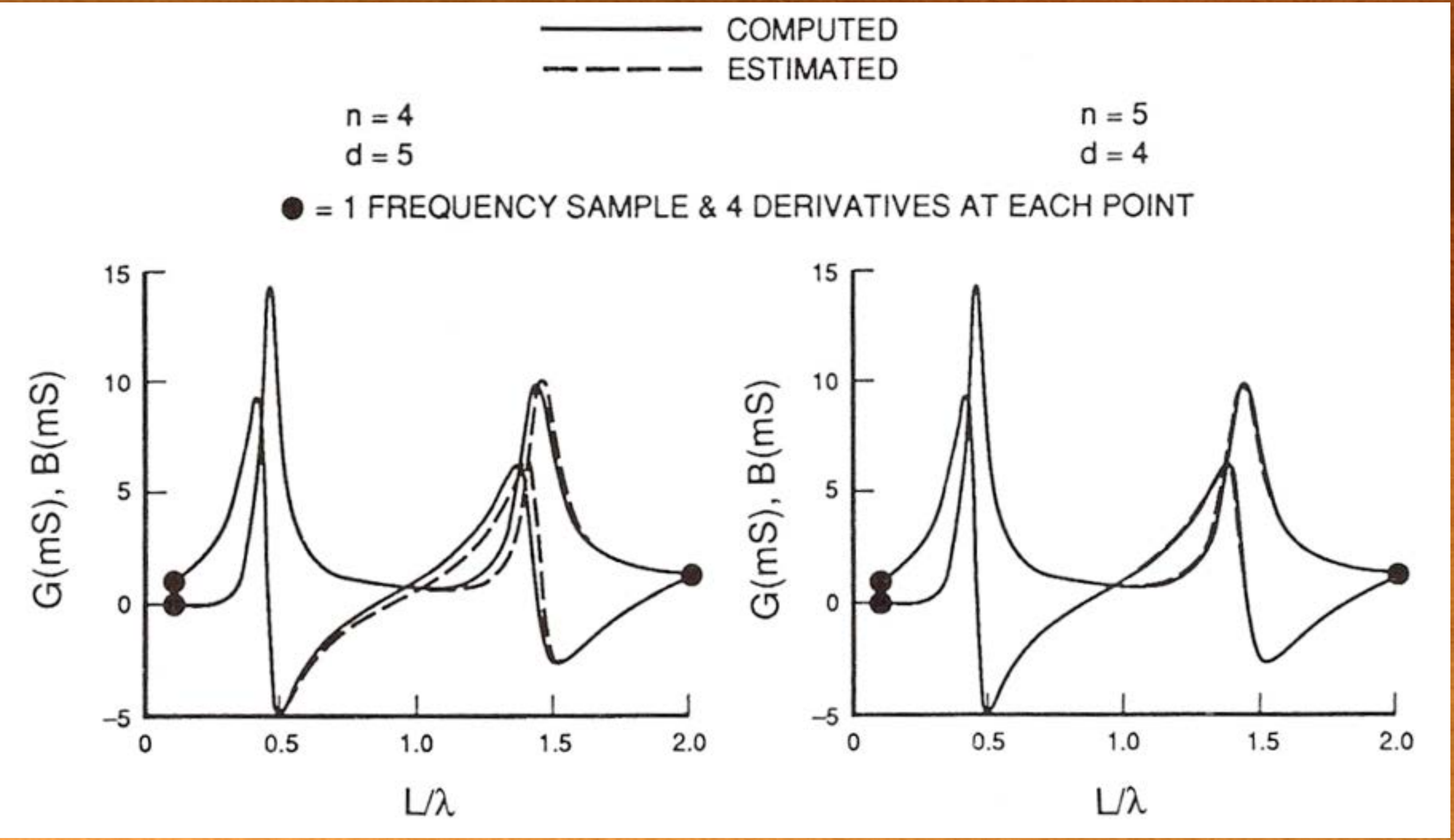
$$I_i^{(n)} = \sum_{j=1}^{N} Y_{i,j}(\omega)\left[V_j^{(n)}(\omega) - \sum_{m=0}^{n-1} C_{n,m} Z_{j,k}^{(n-m)} I_k^{(m)}\right]$$

--YIELDS THE n'th CURRENT DERIVATIVE

. . . AND WE OBSERVE THAT . . .

- OPERATION COUNT FOR EACH ADDITIONAL DERIVATIVE IS PROPORTIONAL TO N^2 AS OPPOSED TO N^3 O.C. OF Y_{ij}
  - --FOR EACH RHS AND ASSUMING $n \ll N$
- ASSUMING A DERIVATIVE PROVIDES INFORMATION EQUIVALENT TO AN ADDITIONAL FREQUENCY SAMPLE, O.C. CAN BE FURTHER REDUCED

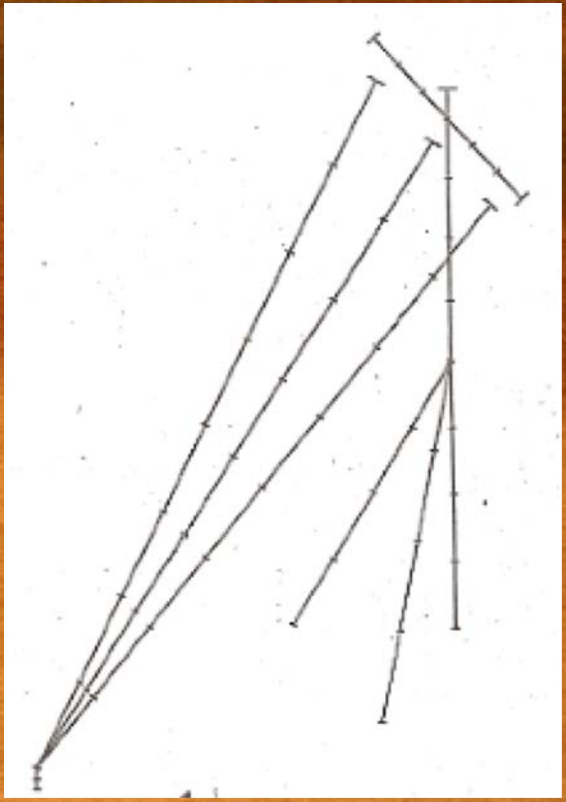
DERIVATIVE APPROACH WORKS FOR DIPOLE ADMITTANCE



HIGHER-ORDER NUMERATOR WORKS BETTER HERE

FORKED-MONOPOLE AND FAN ANTENNA* PROVIDE CHALLENGING TEST CASES

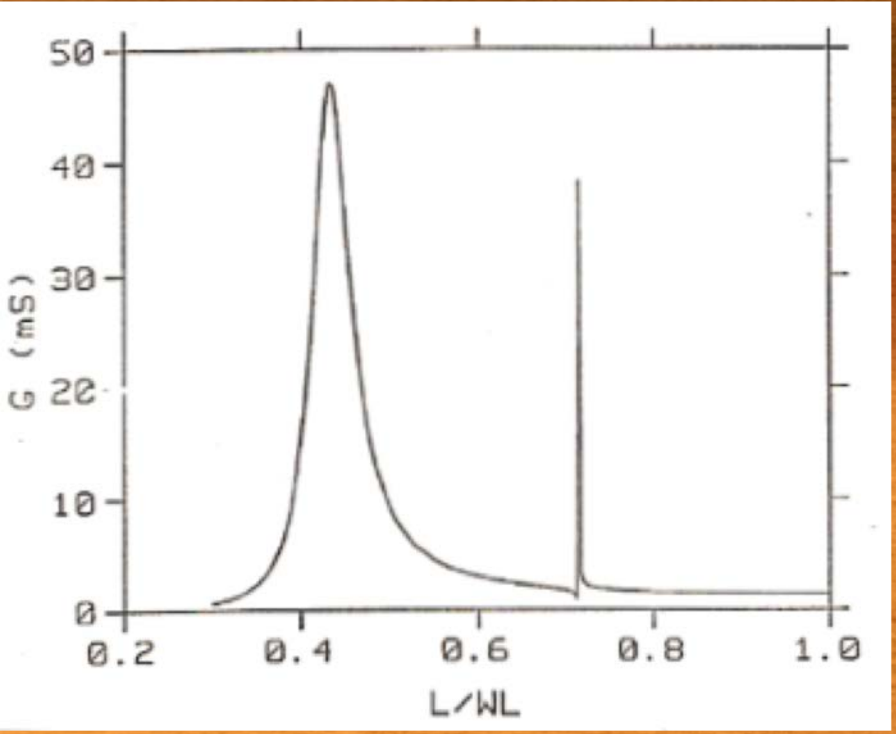
FORKED MONOPOLE

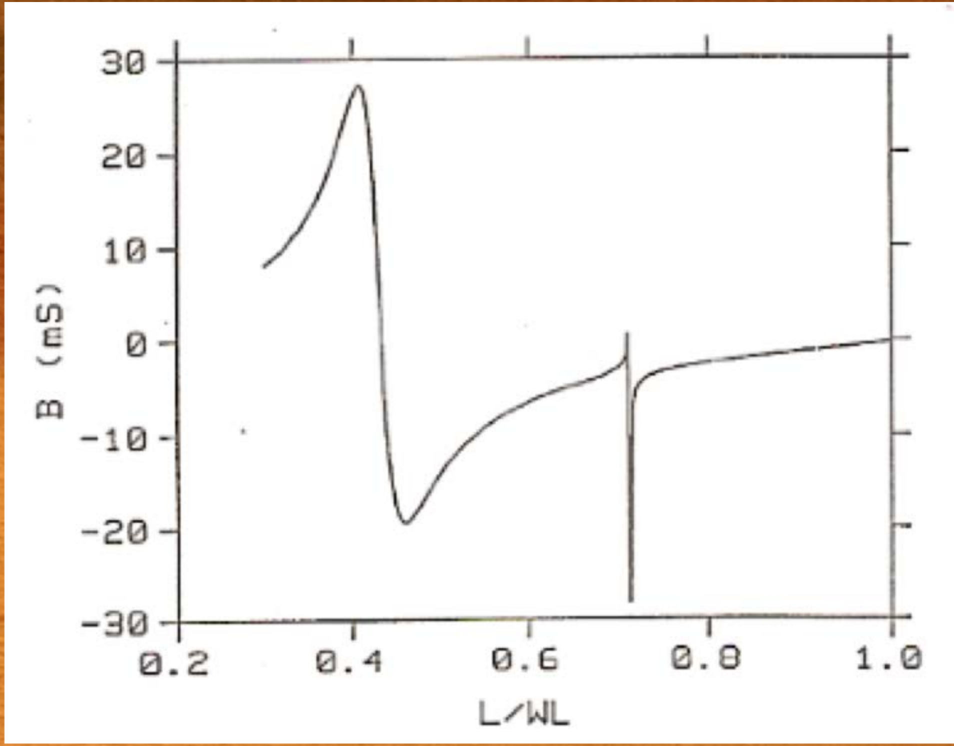


FAN ANTENNA

*BURKE (1992)

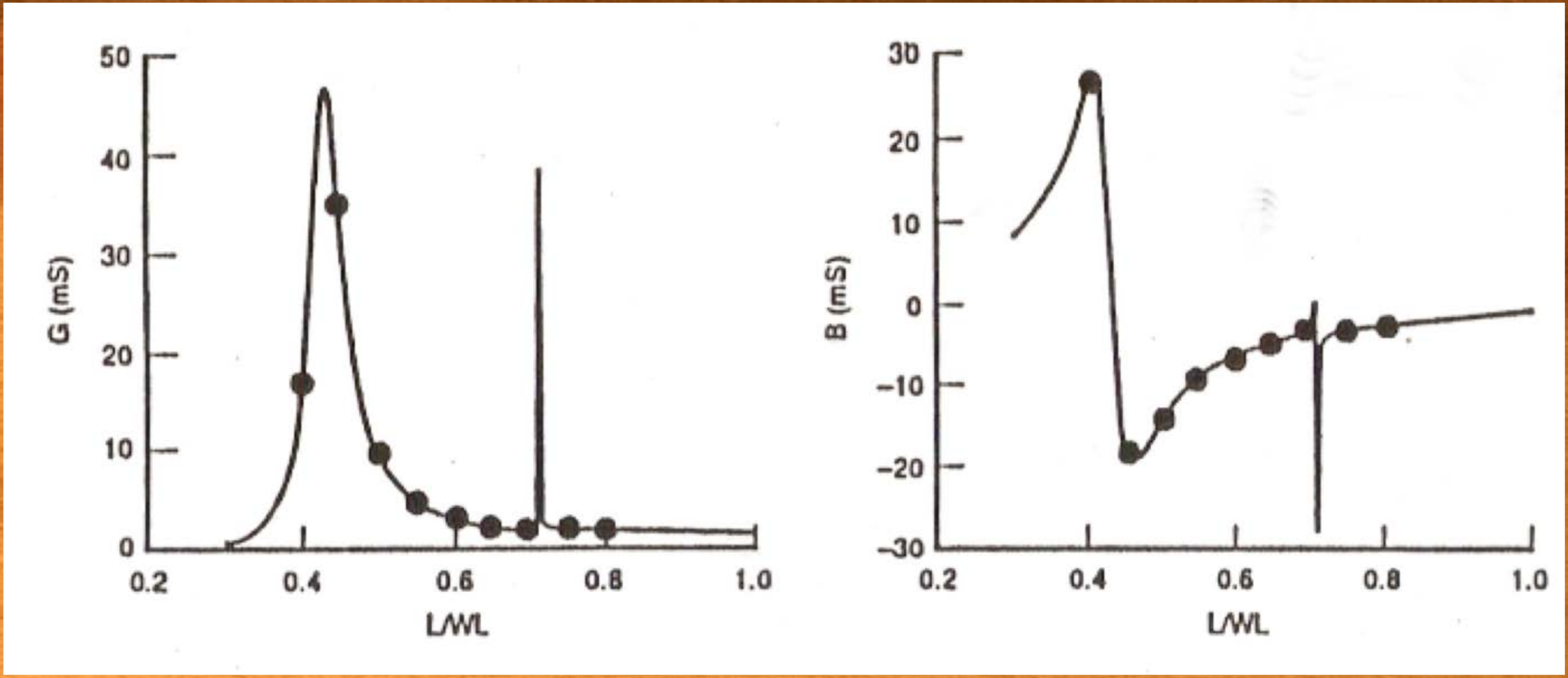
FORKED MONOPOLE HAS VERY SHARP RESONANCE STRUCTURE . . .





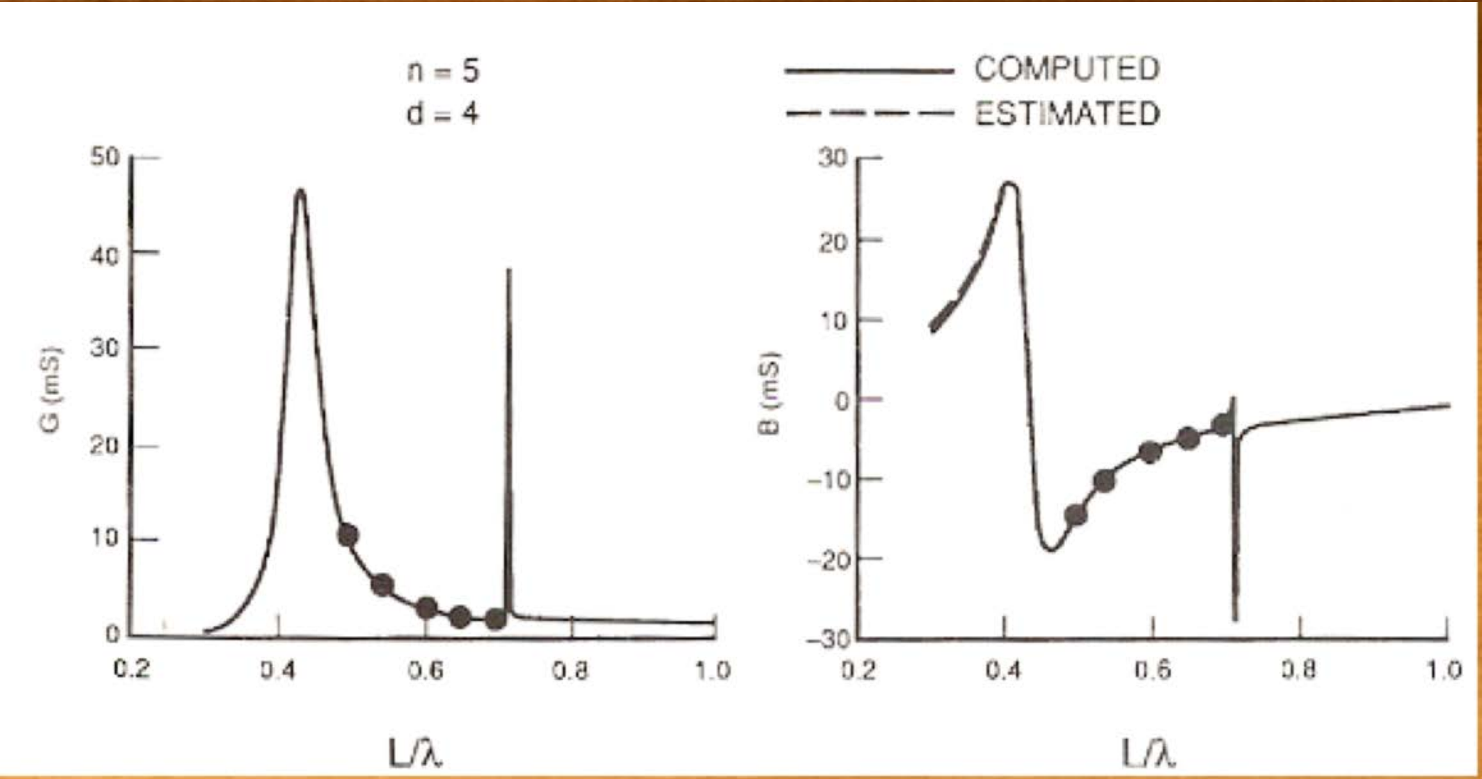
. . . AND PRESENTS A GOOD TEST CASE FOR FUNCTION & DERIVATIVE SAMPLING

FM USING ONLY FUNCTION SAMPLING YIELDS ~1% MATCH WITH GM FOR FORKED MONOPOLE



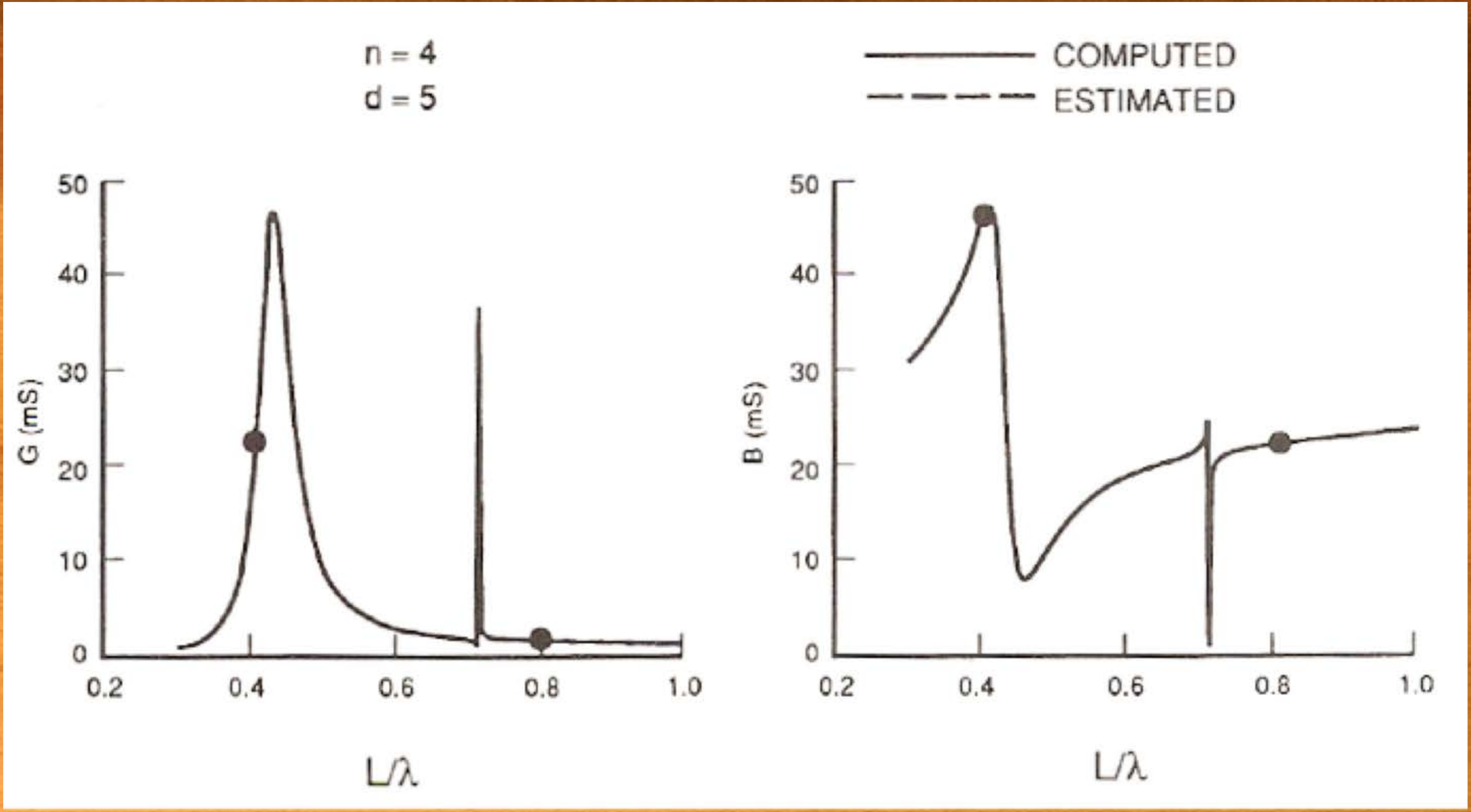
- SAMPLES AT 0.4, 0.45, . . ., 0.8 (9 GM samples)
- FM USES n = 4 and d = 4
- ________ NEC COMPUTED, -------------- MBPE ESTIMATED

USING FIVE FREQUENCIES WITH ONE DERIVATIVE AT EACH PRODUCES COMPARABLE RESULTS . . .

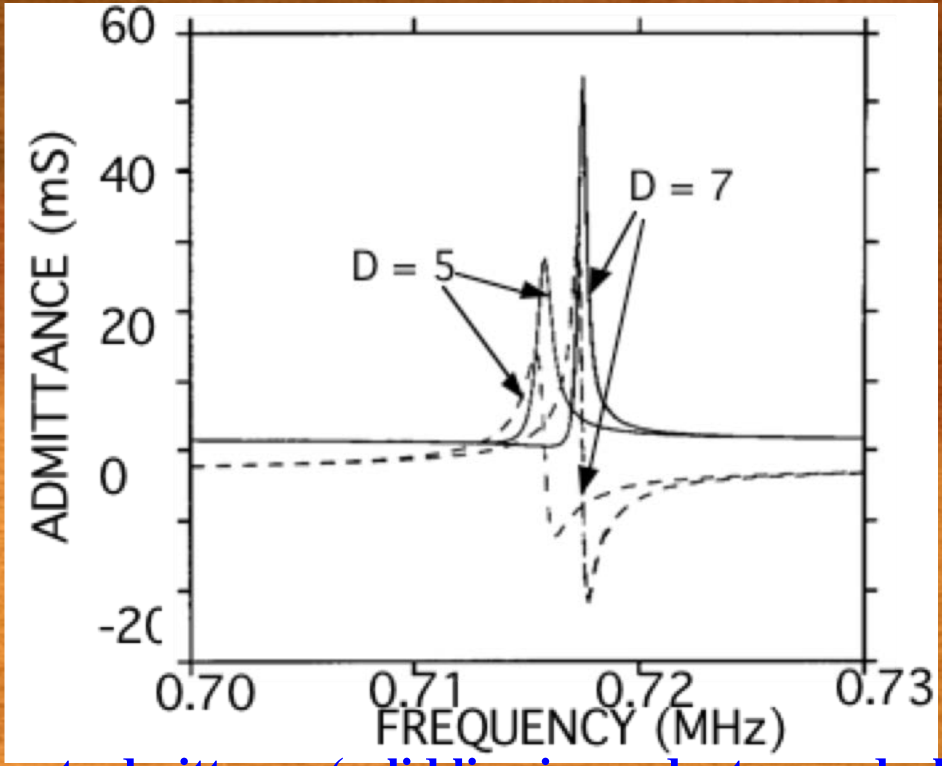


- SAMPLES AT 0.5, 0.55, 0.6, 0.65 AND 0.7 (10 SAMPLES)
- FM USES n = 5 and d = 4
- ________ NEC COMPUTED, --------------- MBPE ESTIMATED

. . . AS DOES USING 4 DERIVATIVES AT 2 FREQUENCIES

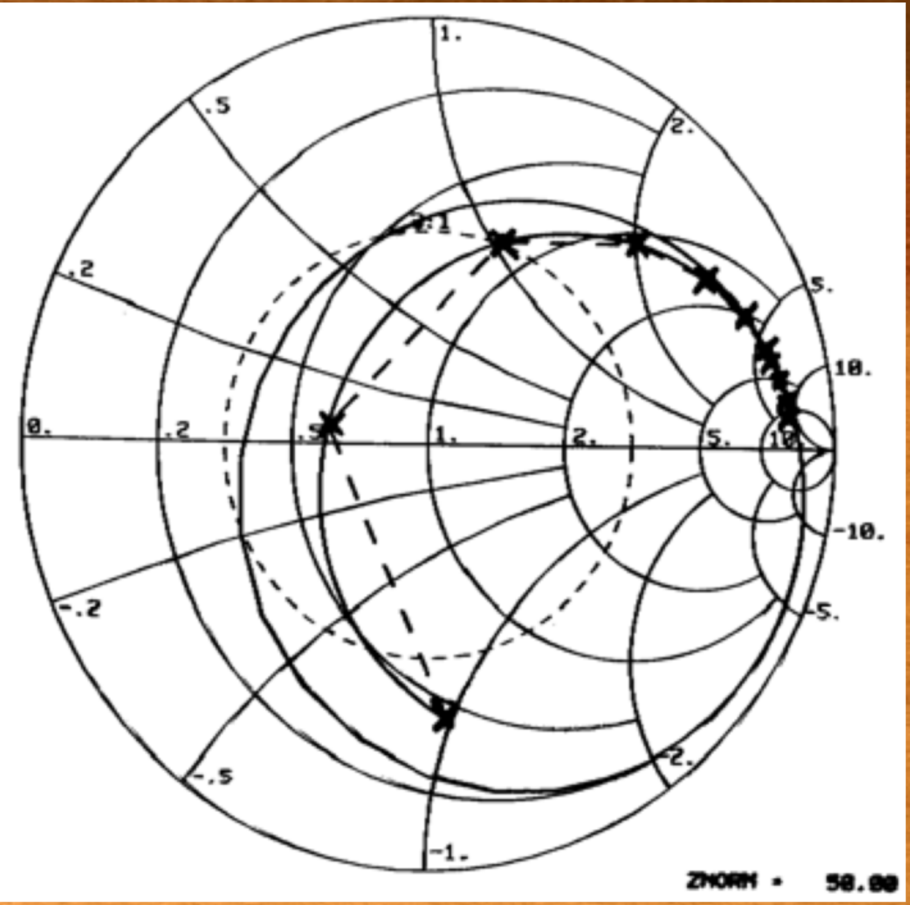


NARROW RESONANCE REVEALS ONLY SLIGHT FM DIFFERENCES



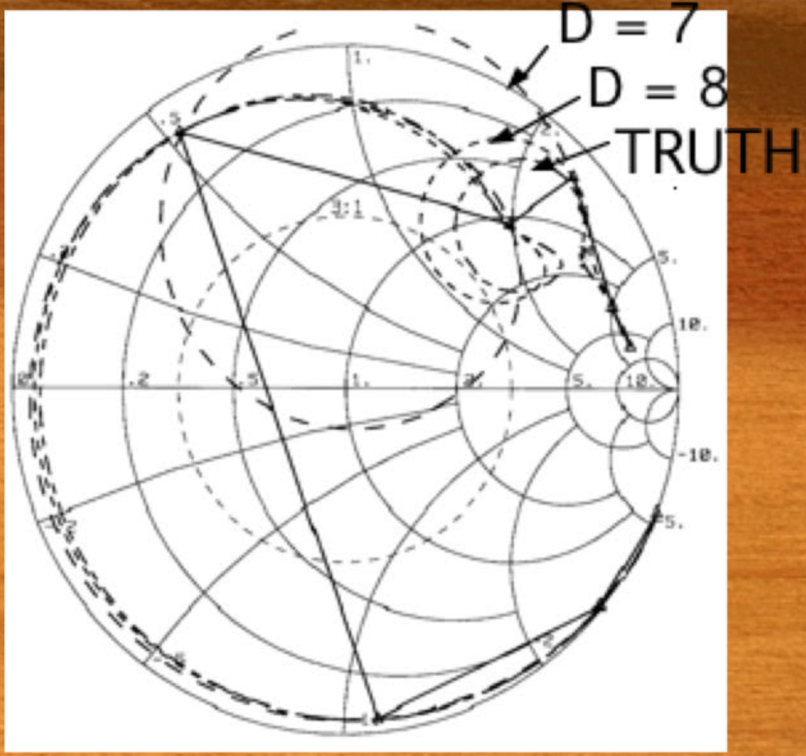
Results for the input admittance (solid line is conductance, dashed line is susceptance) of a forked-monopole antenna in the vicinity of a sharp resonance, where a differential-mode current can exist on the two unequal-length arms of the dipole [Burke (1992)]. Although the resonance is quite accurately located (to within 1% or so in frequency), there is some variation in the admittance values provided by two *FM*s, one using *D* = 5, and the other *D* = 7, function samples. The 7-sample model is the more accurate, as it is found to agree within a few percent with 21 additional *GM* samples spaced 10^{-4} L/wavelength apart beginning at 0.717 MHz.

IMPEDANCE PLOT OF FORKED MONOPOLE ON SMITH CHART EXHIBITS RAPIDITY OF VARIATION



- $\Delta 2h/\lambda = 0.05$, $N_{fit} = 7$, $n = d = 3$

NON-PHYSICAL RESULT FOR FAN ANTENNA IS ELIMINATED BY INCREASING *FM* ORDER BY 1

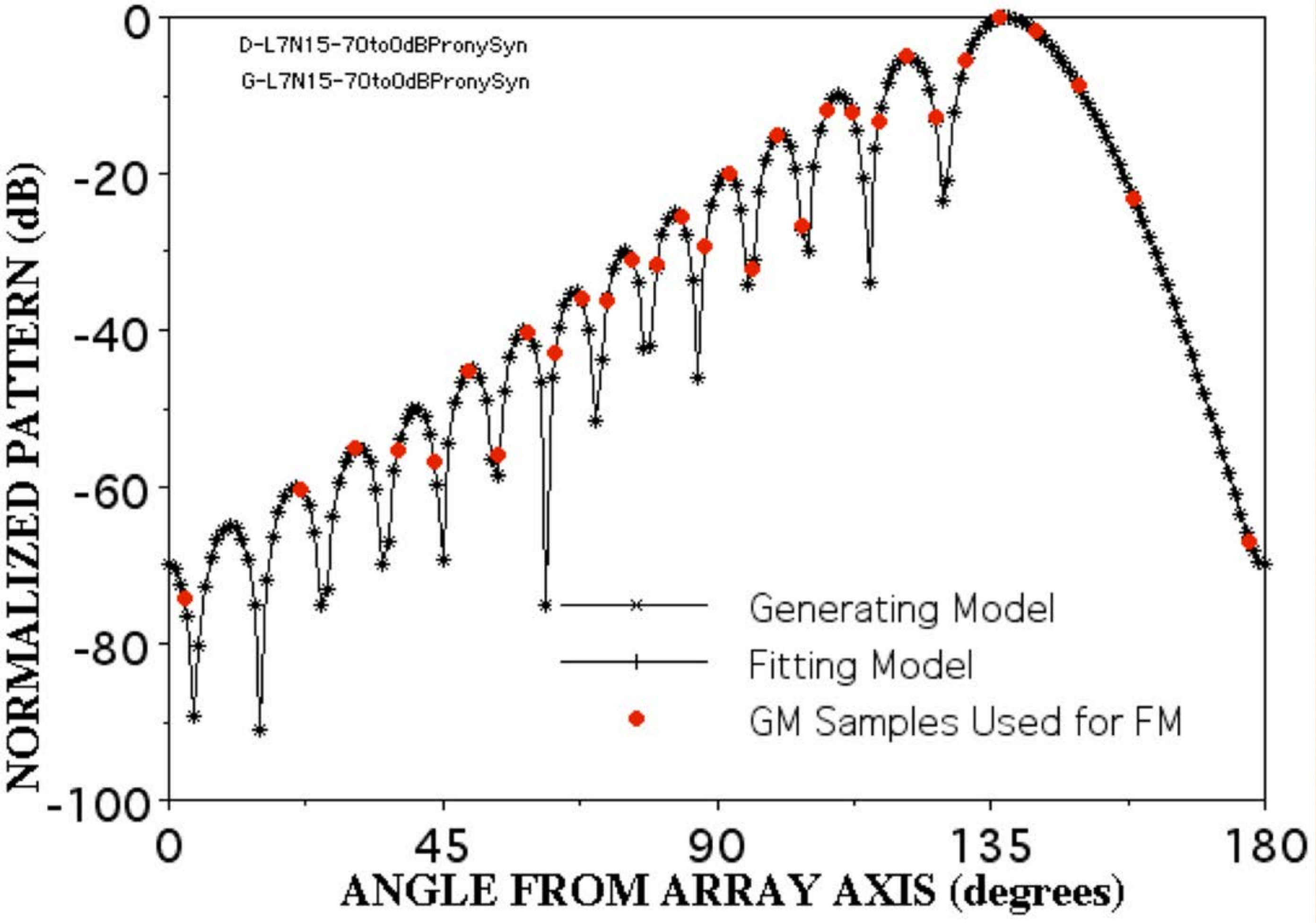


Smith-chart representation of the input impedance of the fan antenna over the frequency range 2 to 9 MHz [Burke (1992)]. The *GM* samples are shown by the triangles and are connected by a straight, solid line. The 7-sample *FM* ($d = n = 3$) produces a non-physical input resistance near the resonance "loop." Simply increasing the *FM* order by one brings it into to close agreement with the *GM* samples labeled "truth" which are computed at 0.14 MHz intervals and connected by straight lines.

WAVEFORM-DOMAIN MODELING IS USEFUL FOR A VARIETY OF EM DATA

- TRANSIENT WAVEFORMS
- FAR-FIELD PATTERNS
- DIRECTION FINDING
- FREQUENCY RESPONSES OF COLLECTIONS OF DISCRETE SCATTERERS
- REFLECTION FROM LAYERED MEDIA

WD MBPE CAN ALSO BE USED FOR PATTERN SYNTHESIS . . .

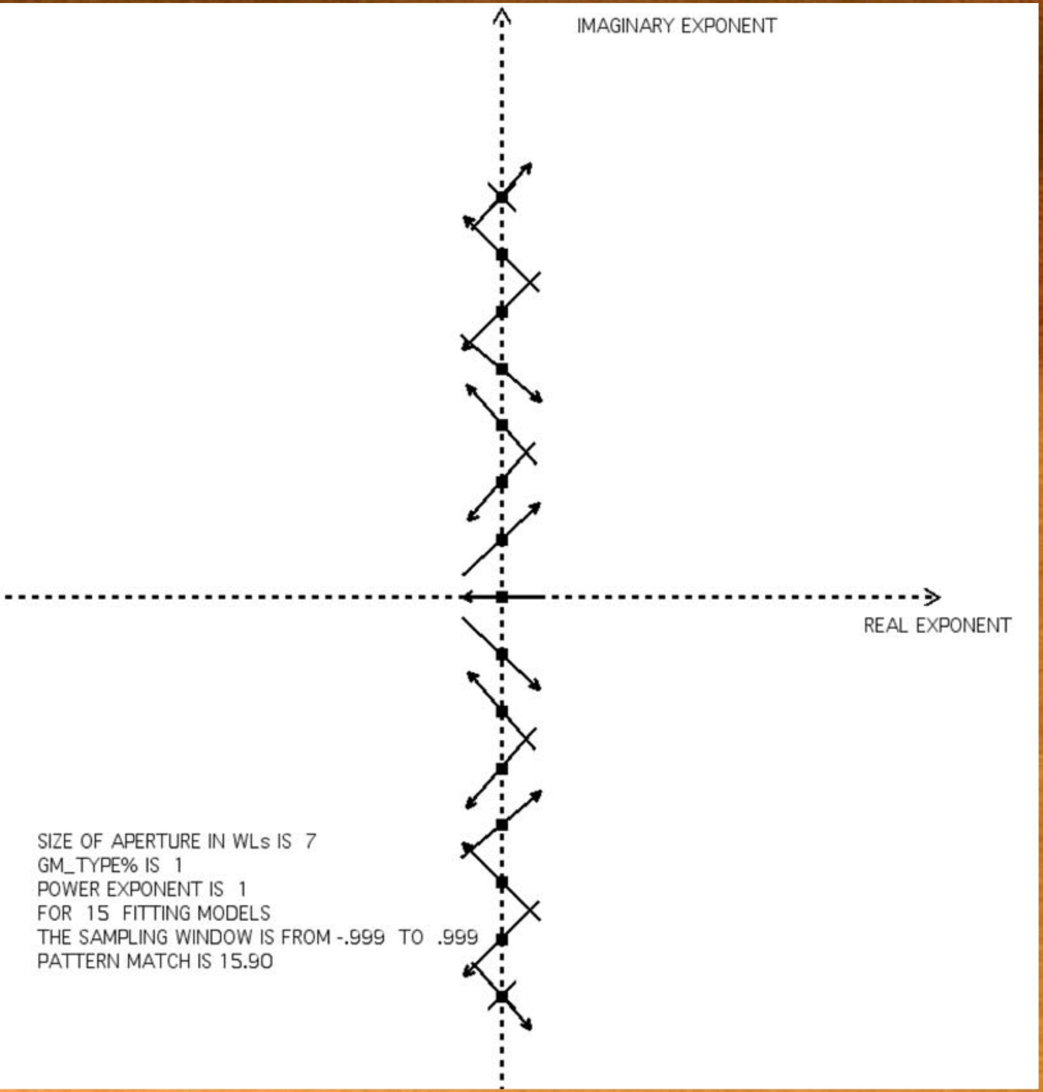


A PRONY *FM* WAS USED TO SYNTHESIZE A PATTERN VARYING FROM -70 TO 0 dB

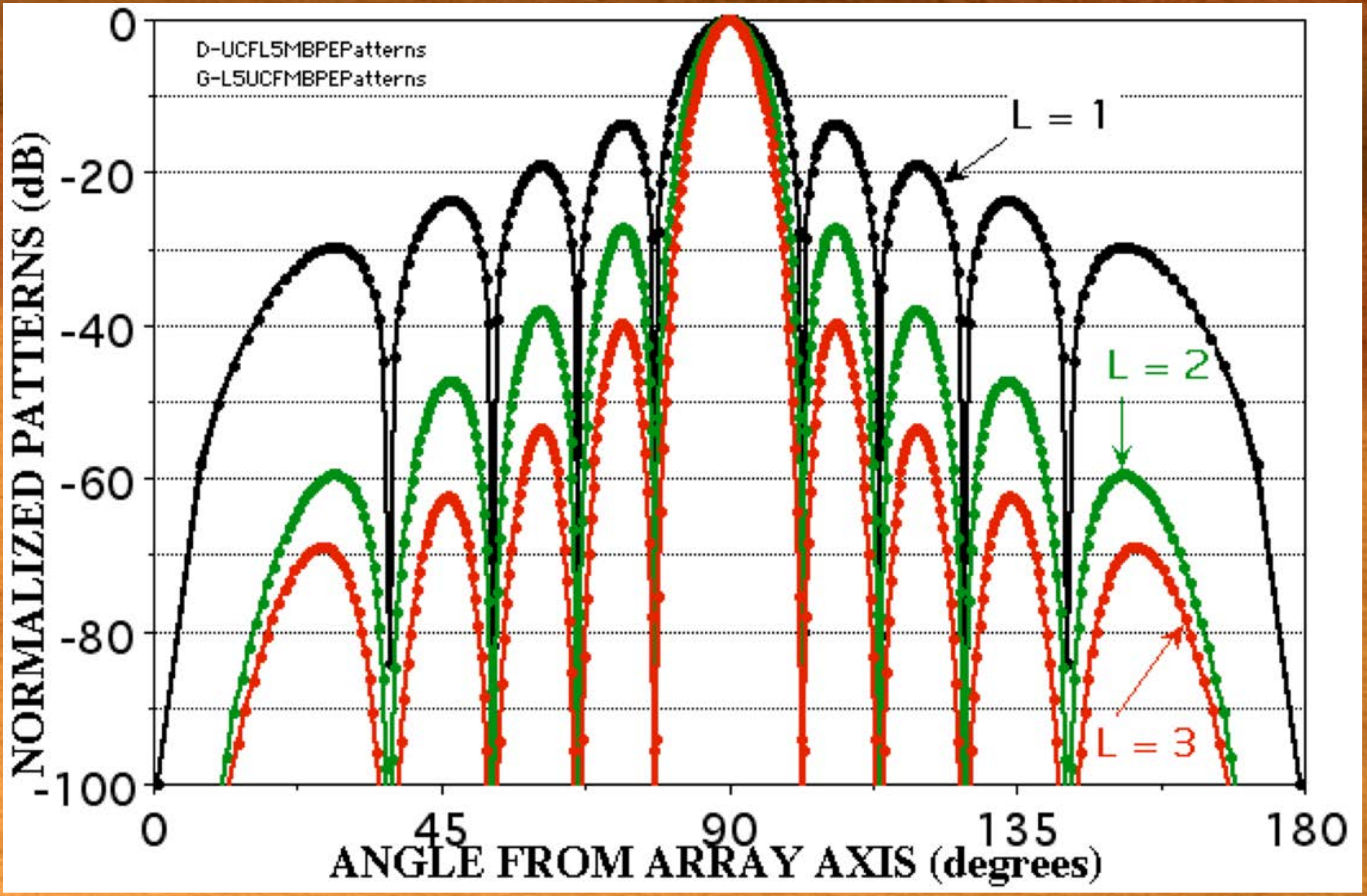
. . . PRODUCING THIS DISCRETE SOURCE DISTRIBUTION

The arrows display the complex source magnitudes on a 3-decade log scale and phase relative to the real axis with the squares showing the source locations d_α in complex space in the expression

$$P_\alpha = e^{d_\alpha \cos\theta}$$.



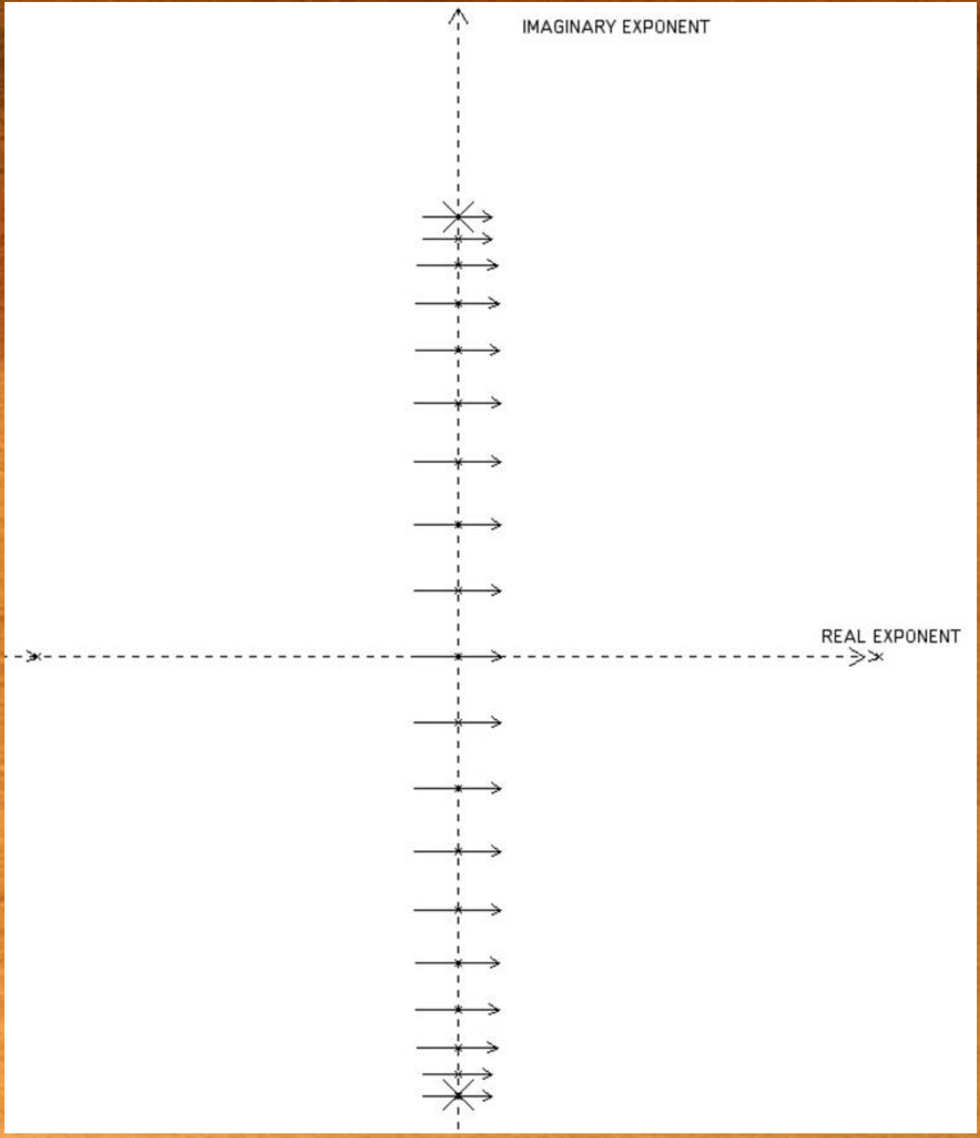
WD MBPE CAN ALSO BE USED FOR PATTERN SYNTHESIS . . .



A PRONY *FM* WAS USED TO SYNTHESIZE THE PATTERN OF A UNIFORM APERTURE RAISED TO THE POWER *L* AS $P_{UA}(\theta)^L$

. . . PRODUCING THIS DISCRETE SOURCE DISTRIBUTION

The arrows display the complex source magnitudes on a 3-decade log scale and phase relative to the real axis with the squares showing the source locations d_α in complex space in the expression $P_\alpha = e^{d_\alpha \cos\theta}$ for the $L = 1$ case. Note that the element spacing varies along the array.



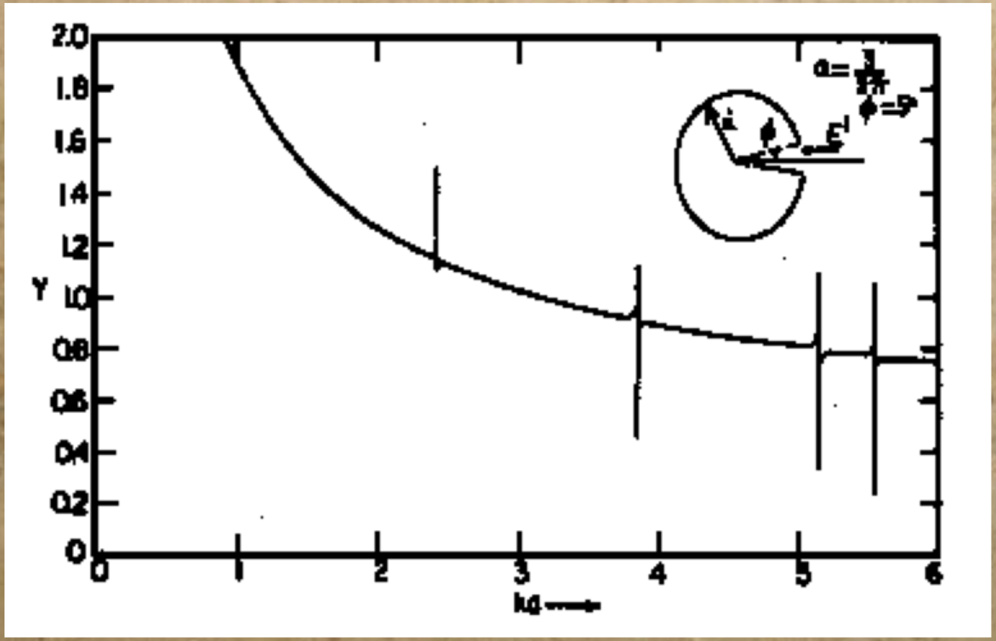
PRESENTATION EXPLORES SOME ISSUES IN ESTIMATING & REPRESENTING EM OBSERVABLES

1) THE SCIENTIFIC METHOD
2) MODEL-BASED PARAMETER ESTIMATION
3) FITTING MODELS FOR WAVEFORM AND SPECTRAL DATA
4) FUNCTION SAMPLING AND DERIVATIVE SAMPLING
5) **ADAPTIVE SAMPLING OF FREQUENCY SPECTRA**
6) ADAPTIVE SAMPLING OF RADIATION AND SCATTERING PATTERNS
7) USING MBPE TO ESTIMATE MODEL UNCERTAINTY
8) OTHER FITTING MODELS FOR EM OBSERVABLES
9) USING MBPE FOR GENERATING-MODEL COMPUTATION

USING MBPE IT IS POSSIBLE TO ADAPTIVELY SAMPLE AND ESTIMATE A TRANSFER FUNCTION TO A PRESCRIBED ACCURACY

- THE BASIC RATIONAL-FUNCTION FITTING MODEL (*FM*)
- QUANTIFYING A *FM* BY SAMPLING A FIRST PRINCIPLES OR GENERATING MODEL (*FPM* OR *GM*)
- ESTIMATING ERROR BY OVERLAPPING THE *FM*s
- CHOOSING A FREQUENCY FOR THE NEXT *GM* SAMPLE
- SOME REPRESENTATIVE RESULTS

DETERMINING A TRANSFER FUNCTION FROM DISCRETE SAMPLES CAN BE CHALLENGING . . .



- HOW DENSELY MUST THE SPECTRUM BE SAMPLED?
- CAN THE CONTINUOUS SPECTRUM BE DETERMINED?
- IS THERE ANY ASSURANCE THAT IMPORTANT FEATURES ARE NOT MISSED?
- IS AN ERROR ESTIMATE AVAILABLE?

BASIC IDEA IS STRAIGHTFORWARD . . .

- USE FIRST-PRINCIPLES (GENERATING) MODEL TO OBTAIN DESIRED SAMPLES
- USE PHYSICALLY-MOTIVATED, REDUCED-ORDER (FITTING) MODEL AS A CONTINUOUS REPRESENTATION OF THE DISCRETE SAMPLES

. . . HAVING THE TWO GOALS OF . . .

- MINIMIZING THE NUMBER OF GENERATING-MODEL SAMPLES REQUIRED
- PRODUCING A MORE ANALYTICALLY USEFUL FORM OF THE SAMPLED RESULTS

ESTIMATING A TRANSFER FUNCTION AND DATA COMPRESSION ARE COMPLEMENTARY OPERATIONS

- DATA COMPRESSION REPLACES A SAMPLED-DATA SET WITH A REDUCED-ORDER DESCRIPTION
  - --GENERALLY USING A FITTING MODEL OF THE PROCESS FROM WHICH THE DATA WAS GENERATED

- TRANSFER-FUNCTION ESTIMATION ATTEMPTS TO MINIMIZE NUMBER OF SAMPLES REQUIRED TO APPROXIMATE CONTINUOUS FUNCTION
  - --USING A FITTING MODEL WHICH REPRESENTS THE PROCESS BEING SAMPLED

EARLIER EXAMPLE OF MBPE IN EM IS THE GEOMETRICAL THEORY OF DIFFRACTION

- MODEL BASED ON RAY OPTICS AND DIFFRACTION THEORY
- PARAMETERS DERIVED FROM ANALYSIS OF CANONICAL PROBLEMS
- COULD BE BASED INSTEAD ON FITTING NUMERICAL SOLUTIONS TO GTD MODELS

OTHER KINDS OF APPLICATIONS COULD USE DIFFERENT MODELS AND PROCESSING APPROACHES

- DEVELOPING SPATIAL DESCRIPTIONS OF OBJECT GEOMETRIES

- DETERMINING MATERIAL PROPERTIES OF PROPAGATION PATHS

- OBTAINING EXCITATION FIELDS FROM INTERACTION WITH KNOWN OBJECTS/MEDIA

- . . .

GOAL IS TO REDUCE THE TOTAL COST (OPERATION COUNT) OF REPRESENTING WIDEBAND RESPONSES IN . . .

- TRANSIENT
- SPECTRAL

. . . FORMS USING FREQUENCY-SAMPLED DATA, AND WHERE THE O.C. FOR FDIE MODELS IS APPROXIMATED BY . . .

$$O.C. \propto \sum \left[A_{fill} (f_i)^4 + B_{solve} (f_i)^6 \right]$$

. . . WHEN SAMPLING AT FREQUENCIES f_i FOR 3D PROBLEMS

. . . AND IS DRIVEN IN EITHER CASE BY INFORMATION REQUIREMENTS . . .

- WHAT IS INFORMATION CONTENT OF OBSERVABLE OF INTEREST?
- WHAT IS INFORMATION PAYOFF OF MOST EXPENSIVE STEP OF MODEL COMPUTATION?

. . . SUGGESTING THAT . . .

- MODELING SHOULD ATTEMPT TO MAXIMIZE RATIO OF INFORMATION YIELD TO COMPUTATION EFFORT
- SAMPLED FREQUENCIES SHOULD BE CHOSEN FOR THEIR INFORMATION YIELD

FITTING MODEL CAN BE USED WITH GENERATING-MODEL SAMPLES IN TWO DISTINCT WAYS

1) FOR INTERPOLATING PRE-SAMPLED (OR MEASURED) GENERATING-MODEL DATA

    --USING WINDOWED, OVERLAPPING SUBSETS TO TEST ADEQUACY OF PRE-COMPUTED DATA

    --IDENTIFYING AREAS THAT MAY NEED FURTHER SAMPLING

    --ESTIMATING THE NUMERICAL ACCURACY OF THE GENERATING MODEL ITSELF

2) FOR ADAPTIVELY SELECTING FREQUENCIES WHERE GENERATING MODEL IS TO BE SAMPLED

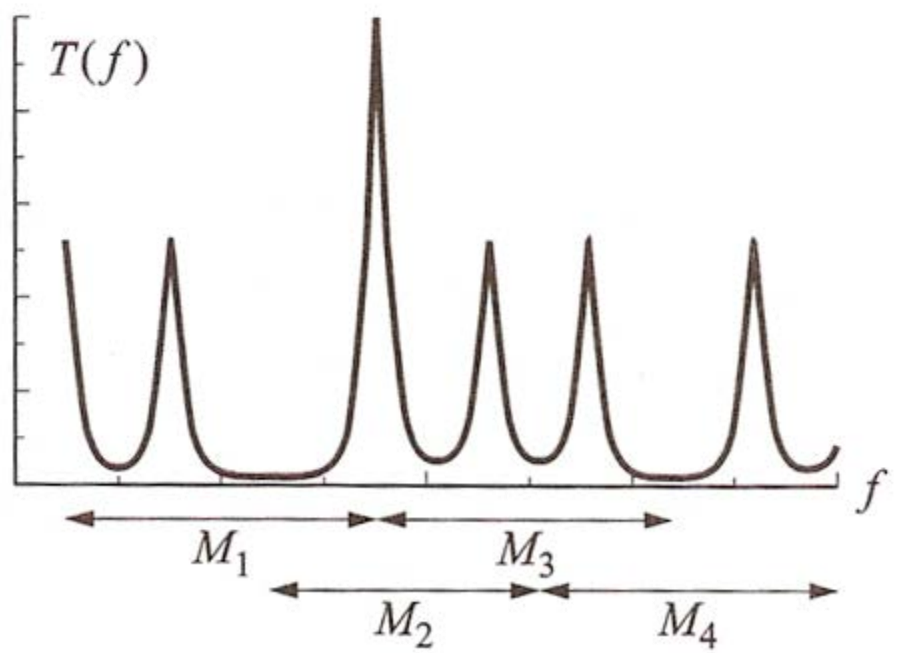
    --MAXIMIZING THE INFORMATION PROVIDED BY EACH ADDITIONAL SAMPLE

    --ENSURING THAT IMPORTANT PHYSICAL BEHAVIOR IS NOT MISSED

    --SELECTING SAMPLED FREQUENCIES ACCORDING TO ERROR/UNCERTAINTY CRITERION

THE INITIAL SEQUENCE OF LOW-ORDER FMs IS SET UP WITHOUT *A PRIORI* KNOWLEDGE OF THE GM SPECTRUM . . .

The possibility of developing a FM-representation of a transfer function over a wide frequency interval by employing a number of subinterval, overlapping, lower-order FMs, M_i, is illustrated conceptually here.

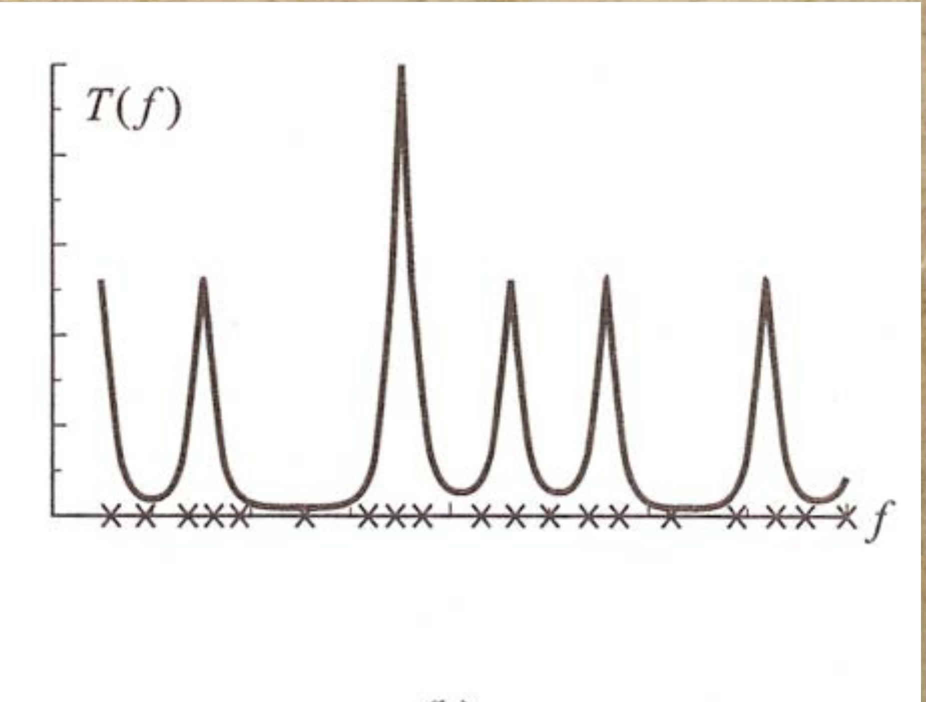


(a)

. . . BUT CAN EXPLOIT ANTICIPATED BEHAVIOR OF GIVEN OBJECT SIZE IN λs ..

. . . WITH THE OUTCOME BEING A SAMPLING DENSITY THAT VARIES WITH THE DETAILS OF THE SPECTRUM . . .

Each additional *GM* sample used for the *FM* computation [the *x*'s] is taken where the maximum *FM-FM* mismatch has been found, continuing until the overall mismatch falls below a specified estimation uncertainty. This results in a generally non-uniform placement of the *GM* samples. For presampled data the *x*'s are restricted to the available sample locations.



(b)

. . . TO ADD SAMPLES WHERE THEY ARE MOST INFORMATIVE

- i.e., WHERE THEY MAXIMIZE ADDED INFORMATION

IMPLEMENTATION OF *FM* ADAPTATION IS CONCEPTUALLY SIMPLE . . .

- BEGIN WITH A SEQUENCE OF *FM*s DESIGNATED BY $M_i(n_i,d_i)$
  - --INDEX NUMBER i; $i = 1,\ldots, M$
  - --NUMERATOR AND DENOMINATOR POLYNOMIAL ORDERS OF n_i AND d_i RESPECTIVELY

- SOME OF THE *GM* SAMPLES USED FOR M_i ARE SHARED BY *FM*s M_{i-1} and M_{i+1}, etc.

IMPLEMENTATION OF *FM* ADAPTATION IS CONCEPTUALLY SIMPLE . . .

•SMALL NUMBER OF GM SAMPLES ARE COMPUTED ACROSS BANDWIDTH OF INTEREST

--SUBSET ASSIGNED TO EACH INITIAL FM WITH A SPECIFIED OVERLAP

--MORE CLOSELY SPACED *FM* SAMPLES ARE COMPUTED AND COMPARED TO DETERMINE MINIMUM MATCH IN DIGITS, E_i, [OR MAXIMUM ERROR $10^{(-E_i)}$] FOR EACH *FM*

•IF ANY E_i < A SPECIFIED ESTIMATION ERROR, *EE*, A NEW *GM* SAMPLE IS COMPUTED . . .

-- WHERE $MM_1 = \min[E_1, E2, \ldots, E_N]_1$ OCCURS

. . . ADAPTING THE *FM*s AFFECTED BY EACH NEW GM SAMPLE . . .

- EACH *FM* CONTAINING THE NEW *GM* SAMPLE IS INCREASED BY ONE IN ORDER
  - --ALTERNATING BETWEEN INCREASING n AND d WITH EACH NEW GM SAMPLE
- NEW E_i's ARE COMPUTED FOR EACH *FM* AFFECTED
  - --AND A NEW $MM_2 = \min\,[E_1,E_2,\ldots,E_N]_2$ IS DETERMINED
- IF ANY *FM* ORDER EXCEEDS A SPECIFIED MAXIMUM, IT IS DIVIDED INTO TWO LOWER-ORDER, OVERLAPPING *FM*s
  - --TO MAINTAIN *FM* DATA MATRIX CONDITION NUMBER BELOW SOME MAXIMUM

. . . WITH A RESULTING *GM* SAMPLING DENSITY OF ABOUT 3 PER RESONANCE FOR AN *EE* OF 10^{-2}

NOTE THAT THE *FM* DATA MATRIX CAN BECOME VERY ILL-CONDITIONED

- DYNAMIC RANGE OF COEFFICIENTS IS DETERMINED BY MODEL ORDER
  --1:$(X_i)^n$ AND 1:$(X_i)^d$ FOR NUMERATOR AND DENOMINATOR CONTRIBUTIONS AND FREQUENCY X_i

- EFFECT CAN BE REDUCED BY SCALING . . .
  --DIVIDE FREQUENCIES BY COMMON FACTOR S
  --SO THAT $(X_i)^n$ ---> $(X_i/S)^n$

- . . . AND BY TRANSLATION
  --USING A REFERENCE FREQUENCY
  --SO THAT $(X_i)^n$ ---> $(X_i - X_{ref})^n$

- CONDITION-NUMBER EFFECT CAN ALSO BE CIRCUMVENTED BY USING HIGH COMPUTER PRECISION
  --RESULTS SHOWN HERE DONE IN 24-DIGIT COMPUTATIONS

OTHER ADAPTATION STRATEGIES MIGHT BE FEASIBLE

- ROMBERG-QUADRATURE TYPE PROCEDURE
  - --CHOOSE STARTING SUBINTERVAL IN FREQUENCY
  - --SAMPLE AT FIVE EQUALLY SPACED POINTS
  - --DEVELOP THREE FMs USING SAMPLES 1 & 5, 1,3 & 5, AND ALL 5
  - --COMPUTE ERROR ESTIMATES FROM FM DIFFERENCES
  - --EXPAND OR CONTRACT NEXT SUBINTERVAL DEPENDING ON THESE RESULTS*
- SUCCESSIVELY HALVE THE SAMPLE SPACING IN THE INITIAL SUBINTERVAL UNTIL SOME CONVERGENCE CRITERION IS SATISFIED
  - --DOUBLE OR HALVE THE SUCCEEDING SUBINTERVAL ACCORDINGLY
- . . .

* E. K. Miller (1970), "A Variable Interval Width Quadrature Technique Based on Romberg's Method", *Journal of Computational Physics*, 5, pp. 265-279.

ADAPTIVE SAMPLING IS A LOGICAL EXTENSION OF THE BASIC MBPE METHOD . . .

- MODEL(S) USED TO ESTIMATE ERROR VARIATION WITH FREQUENCY

- SAMPLE PLACEMENT DESIGNED TO MAXIMIZE INFORMATION

- SAMPLING CONCLUDED WHEN ERROR CRITERION IS SATISFIED

. . . AS A WAY TO MINIMIZE THE TOTAL NUMBER OF REQUIRED SAMPLES

ERROR ESTIMATES ARE ESSENTIAL TO PERFORM ADAPTIVE SAMPLING . . .

- COMPARING GENERATING WITH FITTING MODELS IS MORE ACCURATE
  - --REQUIRES MORE EVALUATIONS OF THE GENERATING MODEL
  - --PROVIDES ONLY A POINTWISE MEASURE

- COMPARING TWO (OR MORE) FITTING MODELS IS MORE EFFICIENT
  - --REQUIRES LESS COMPUTATION
  - --PROVIDES ONLY AN ESTIMATE

. . . TO DETERMINE WHERE ADDITIONAL SAMPLES ARE NEEDED THAT . . .

- SATISFY SOME ERROR CRITERION

THERE ARE TWO ESSENTIAL ADVANTAGES OF USING A FITTING MODEL . . .

- CAN BE USED FOR ADAPTIVE, IDEALLY OPTIMAL, SAMPLING OF THE *FPM* OR *GM*
  - --FOR EXAMPLE, ADD NEW SAMPLE WHERE MAXIMUM REDUCTION IN UNCERTAINTY OF THE ACTUAL RESPONSE CAN BE ACHIEVED
  - --DEFINED BY POINT OF MAXIMUM DISAGREEMENT BETWEEN *FM*s
  - --*FM* DISAGREEMENT PROVIDES HOWEVER ONLY ESTIMATE OF UNCERTAINTY

- YIELDS A SIMPLE, ANALYTICAL ESTIMATE OF THE ACTUAL RESPONSE BETWEEN THE SAMPLES $F(X_i)$
  - --ORDERS OF MAGNITUDE FASTER TO EVALUATE THAN THE *GM*

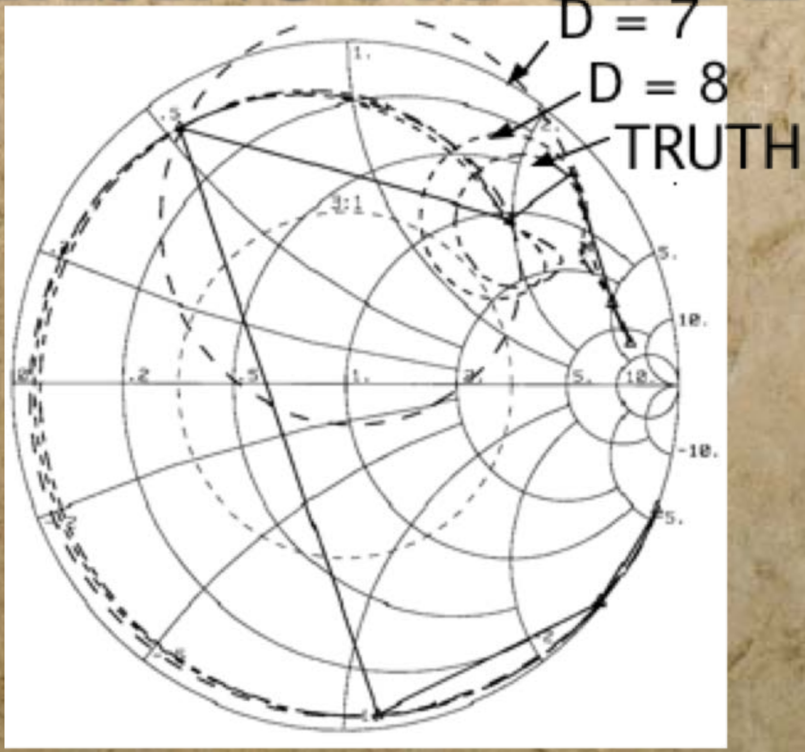
. . . BUT DETERMINING ACTUAL ERROR REQUIRES FINDING DIFFERENCE BETWEEN *GM* AND *FM*(s) . . .

- TO BE AVOIDED WHERE POSSIBLE BECAUSE REQUIRES ADDITIONAL *GM* COMPUTATION TO BE PERFORMED

. . . AND ARE OF THREE BASIC TYPES:

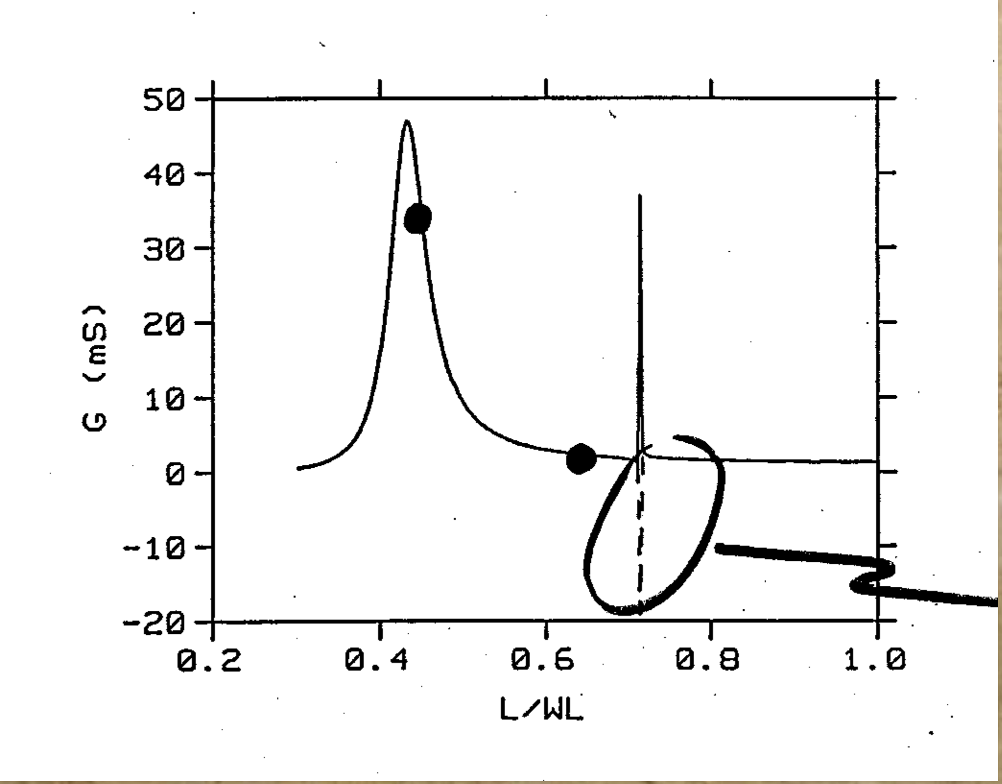
- NON-PHYSICAL RESULTS
  - --e.g., NEGATIVE CONDUCTANCE
- BASELINE SHIFTS
  - --MODEL-MODEL OR MODEL-ACTUAL
- RESONANCE SHIFTS
  - --MODEL-MODEL OR MODEL-ACTUAL

NON-PHYSICAL RESULT FOR FAN ANTENNA IS ELIMINATED BY INCREASING *FM* ORDER BY 1



Smith-chart representation of the input impedance of the fan antenna over the frequency range 2 to 9 MHz [Burke (1992)]. The *GM* samples are shown by the triangles and are connected by a straight, solid line. The 7-sample *FM* ($d = n = 3$) produces a non-physical input resistance near the resonance "loop." Simply increasing the *FM* order by one brings it into to close agreement with the *GM* samples labeled "truth" which are computed at 0.14 MHz intervals and connected by straight lines.

EVEN WHEN FM OBVIOUSLY FAILS RESULTS CAN BE USEFUL



--FORKED MONOPOLE WITH SAMPLES ATA 0.45 AMD 0.65 USING 4 DERIVATES

--FM USED $n = 5$ AND $d = 4$

DIFFERENT SAMPLING DECISIONS MIGHT BE MADE DEPENDING ON THE ERROR(S) ENCOUNTERED . . .

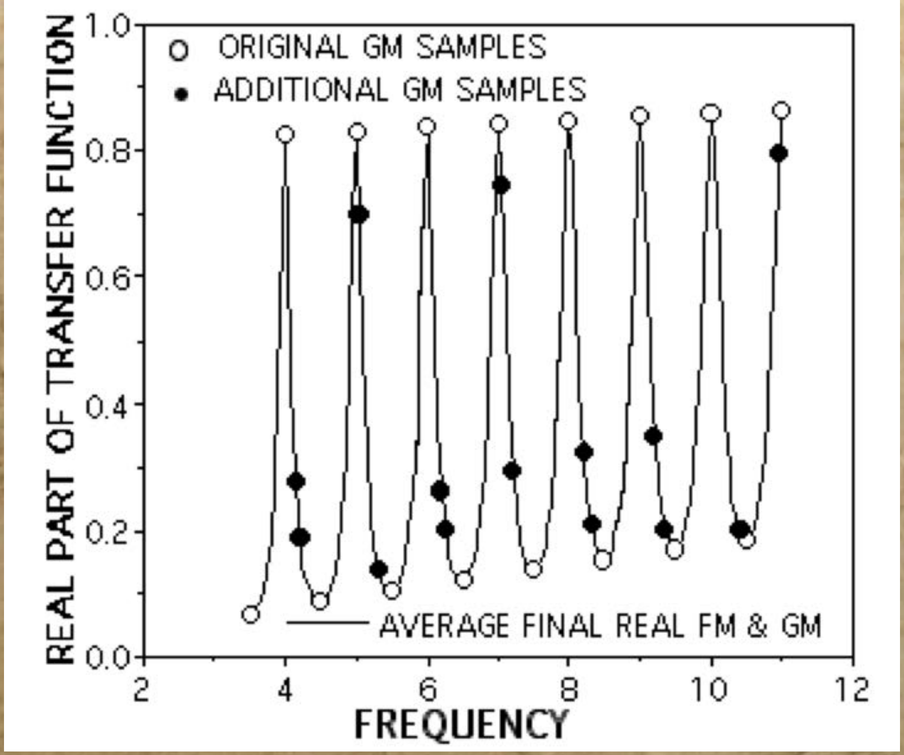
1) NON-PHYSICAL ERRORS
   --e.g., SAMPLE AT NEGATIVE PEAK IN CONDUCTANCE

2) BASELINE-SHIFT ERRORS
   --SAMPLE AT MAXIMUM DIFFERENCE IF IT EXCEEDS ERROR CRITERION

3) RESONANCE-SHIFT ERRORS
   --SAMPLE BETWEEN PEAKS IF SHIFT EXCEEDS ERROR CRITERION

. . . WHERE RESULTS THAT FOLLOW ARE LIMITED TO 2)

SIMPLE SPECTRUM WAS USED TO TEST THE CONCEPT . . .



--20-POLE SERIES HAVING $s_i = -\sqrt{i}/20 + j*i$; i = 1,2,...,20

--16 INITIAL *GM* SAMPLES PLACED AT 0.5 INTERVALS FROM s = j3.5 to j11

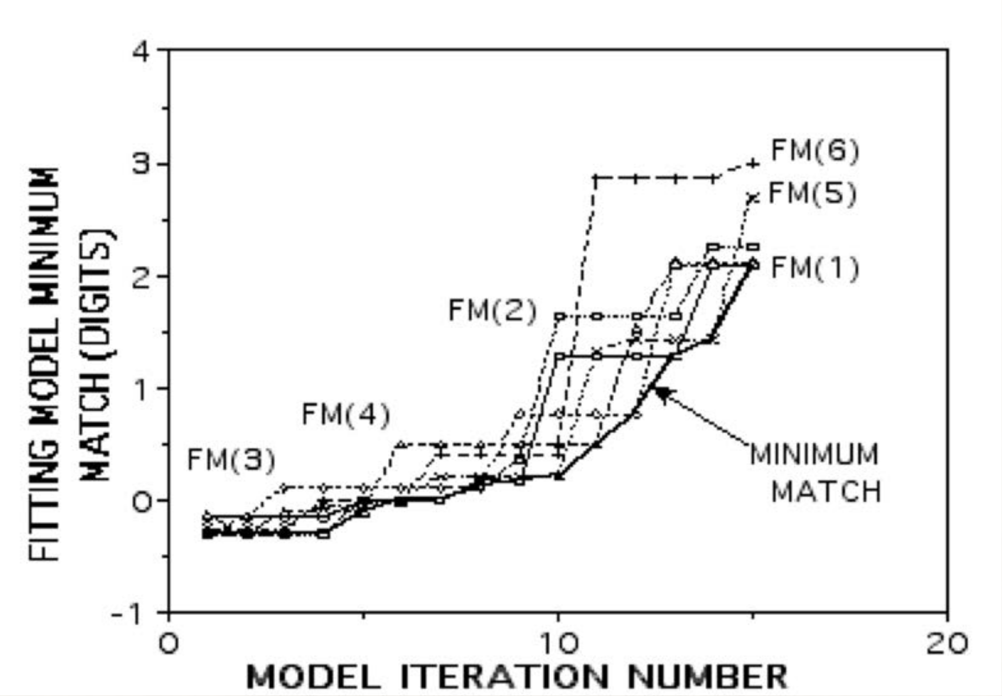
--SIX INITIAL *FM*s USING SAMPLES 1-6, 1-8, 5-10, 7-12, 9-16, 11-16

--FOUR *FM*s (1,3,4,6) OF ORDER 6 ($n = 3$, $d = 2$), TWO (2,5) OF ORDER 8 ($n = 4$, $d = 3$)

--EE SET TO 10^{-2}

--14 NEW *GM* SAMPLES ADDED IN ORDER AT 6.2, 7.2, 9.2, 5.3, 8.2, 9.35, 4.15, 6.25, 5.05, 1.05, 8.3, 7.05, 4.2, and 10.95

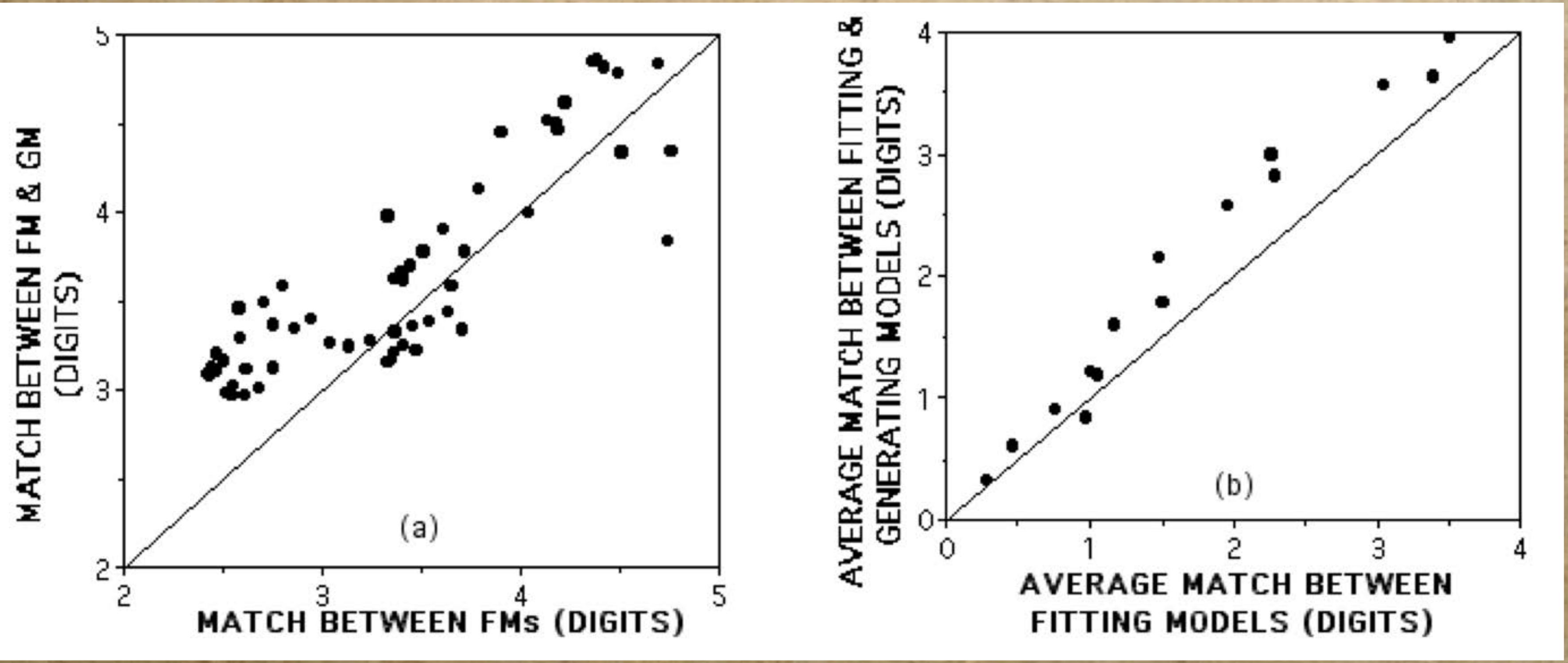
. . . RESULTING IN A MONOTONICALLY INCREASING MINIMUM MATCH $MM_i(f)$ BETWEEN THE OVERLAPPED *FMs* . .



$MM_i(f) = \mathrm{Log10}(max_j\{[|M_i(f) - M_j(f)|]/[|M_i(f)| + | M_j(f)|]\})$ digits
= MINIMUM MATCH OR MAXIMUM ERROR

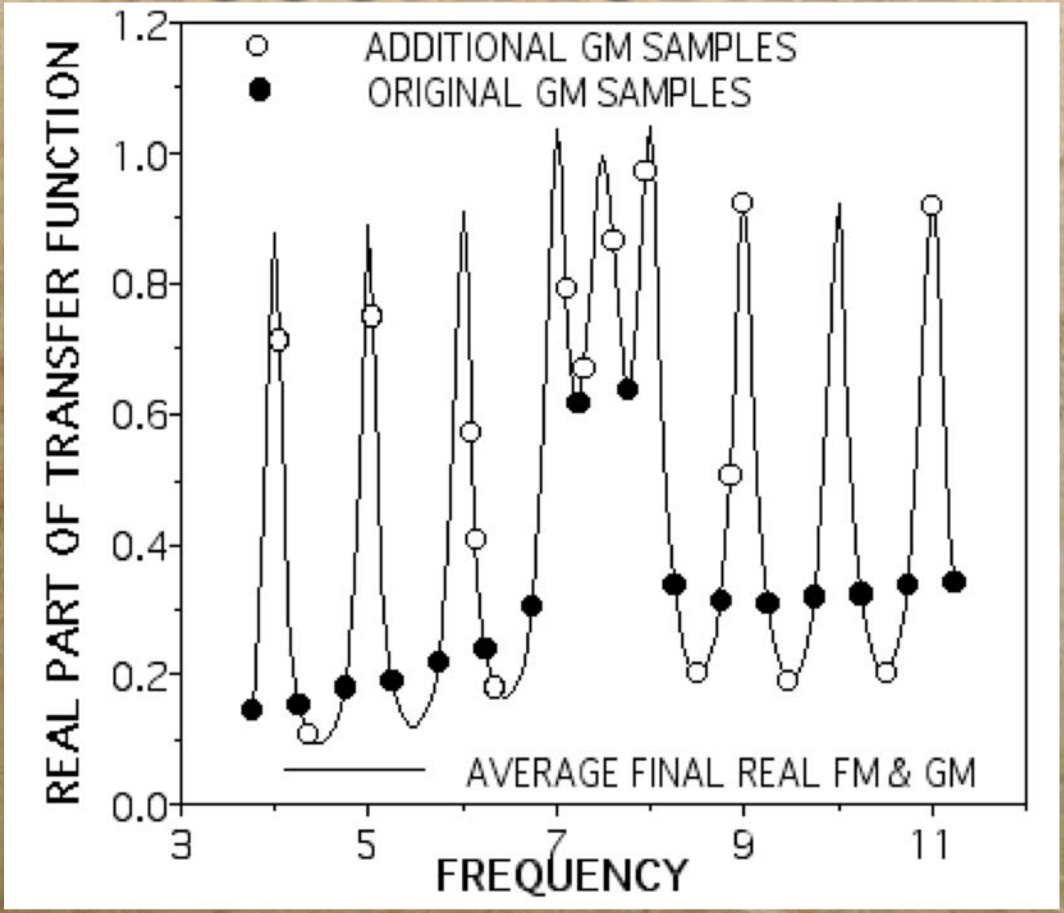
. . . WITH THE ADDITION OF EACH SUCCESSIVE GM-SAMPLE

MATCH BETWEEN *FMs* IS SIMILAR TO THE ACTUAL ERROR BETWEEN *FMs* AND *GM* . . .



. . . SHOWING THAT A *FM-FM* COMPARISON PROVIDES AN ACCURATE ERROR ESTIMATE

ADDED POLE IN *GM* IS READILY ACCOMMODATED



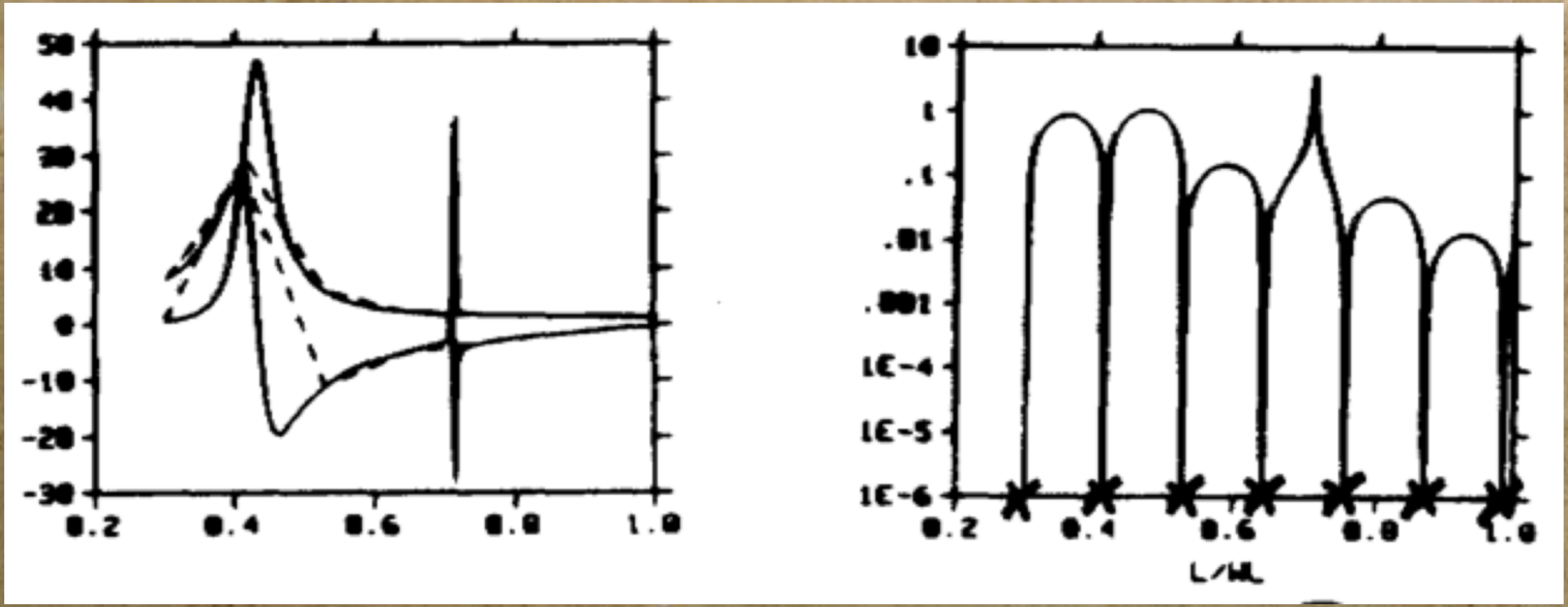
--16 MORE *GM* SAMPLES ARE NOW NEEDED COMPARED WITH 14 IN THE PREVIOUS CASE

--FINAL COMPARISON WITH ACTUAL SPECTRUM EXHIBITS SIMILAR ACCURACY

LINEAR (2-POINT) INTERPOLATION FOR FORKED MONOPOLE PRODUCES LARGE ERROR FOR 7 EQUALLY SPACED SAMPLES . . .

____________ACTUAL (NEC) -------------- MBPE ESTIMATED

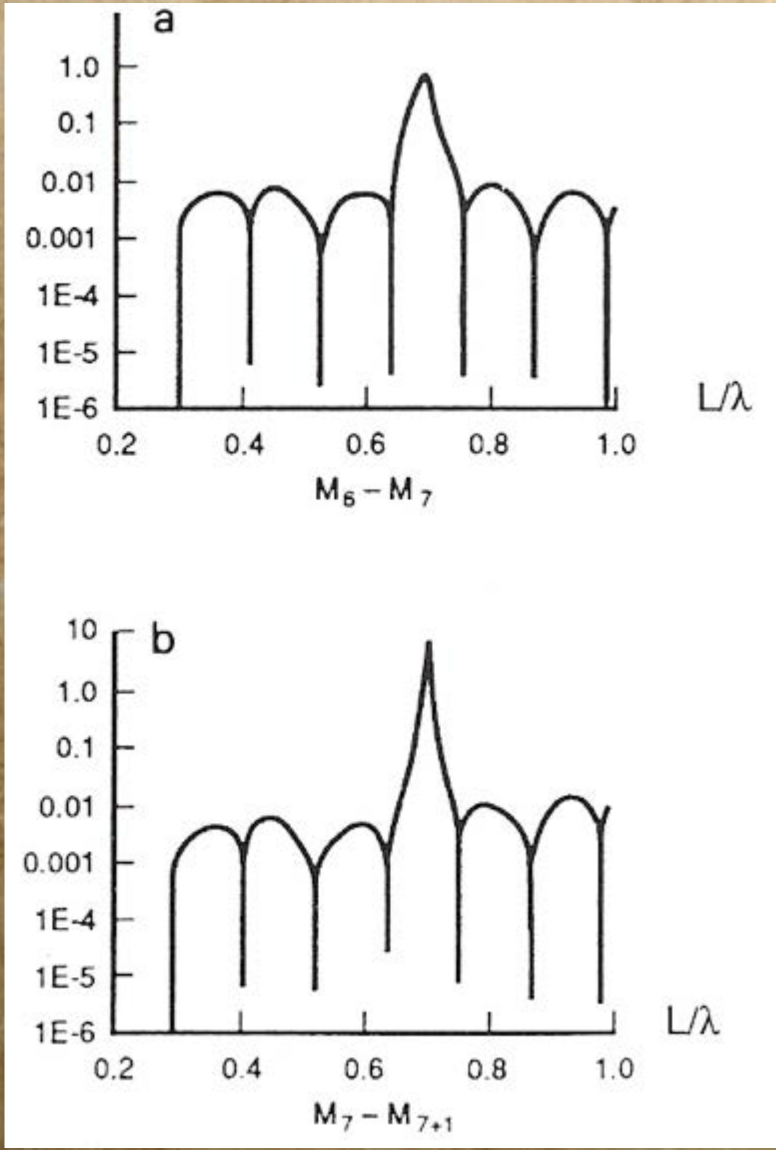
X, POINTS SAMPLED

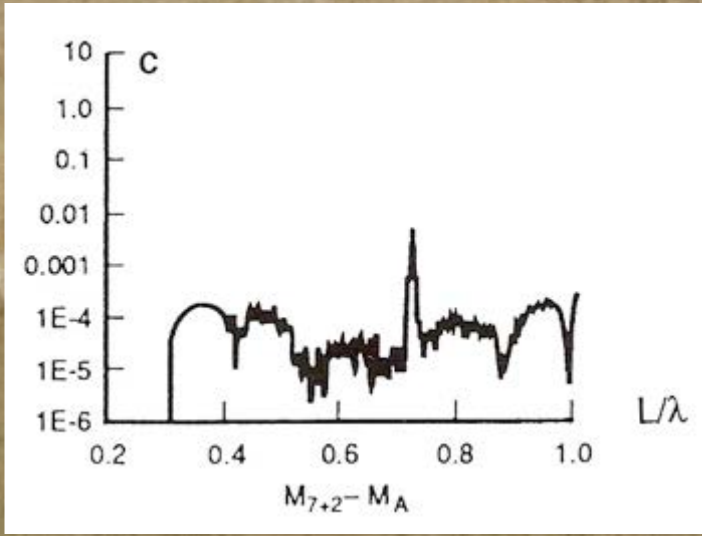


G & B |M_7 - ACTUAL|

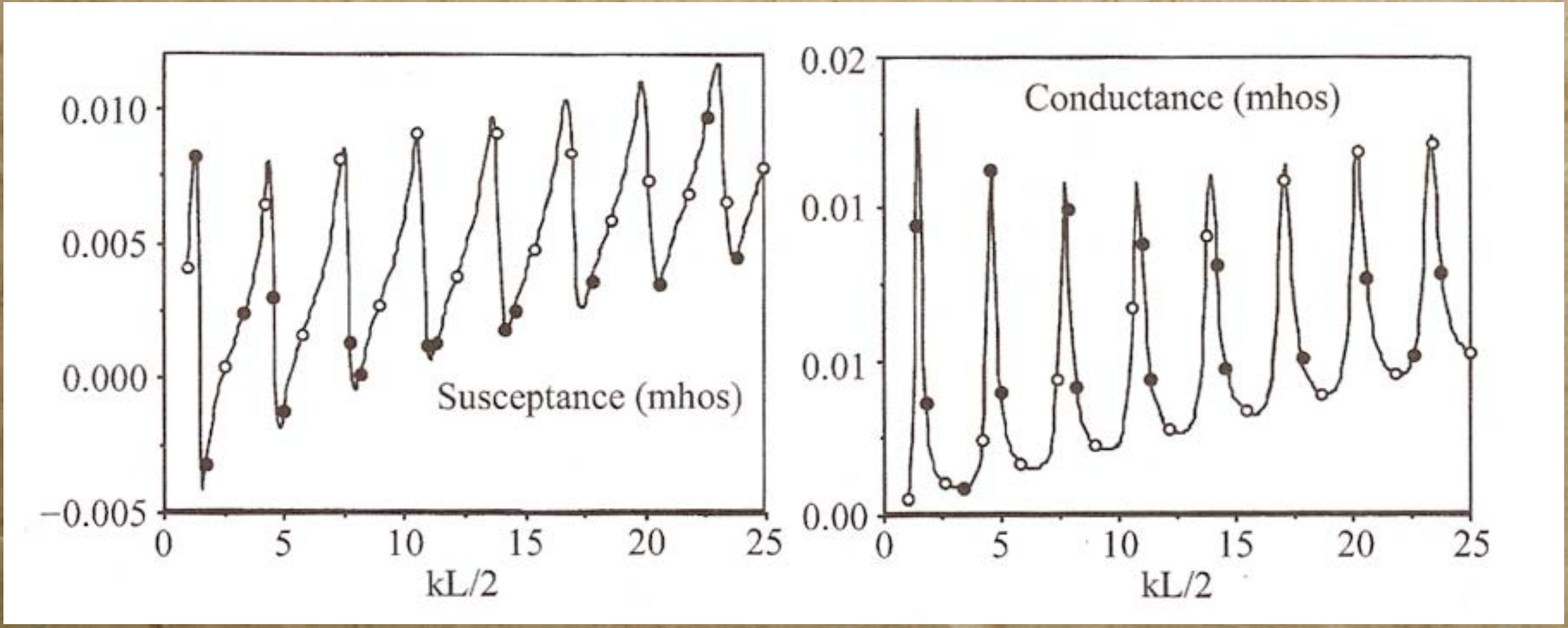
MODEL-MODEL ERROR ESTIMATES ARE CONSERVATIVE . . .

BUT DO SHOW WHERE EXTRA SAMPLES ARE NEEDED RELATIVE TO ACTUAL ERROR



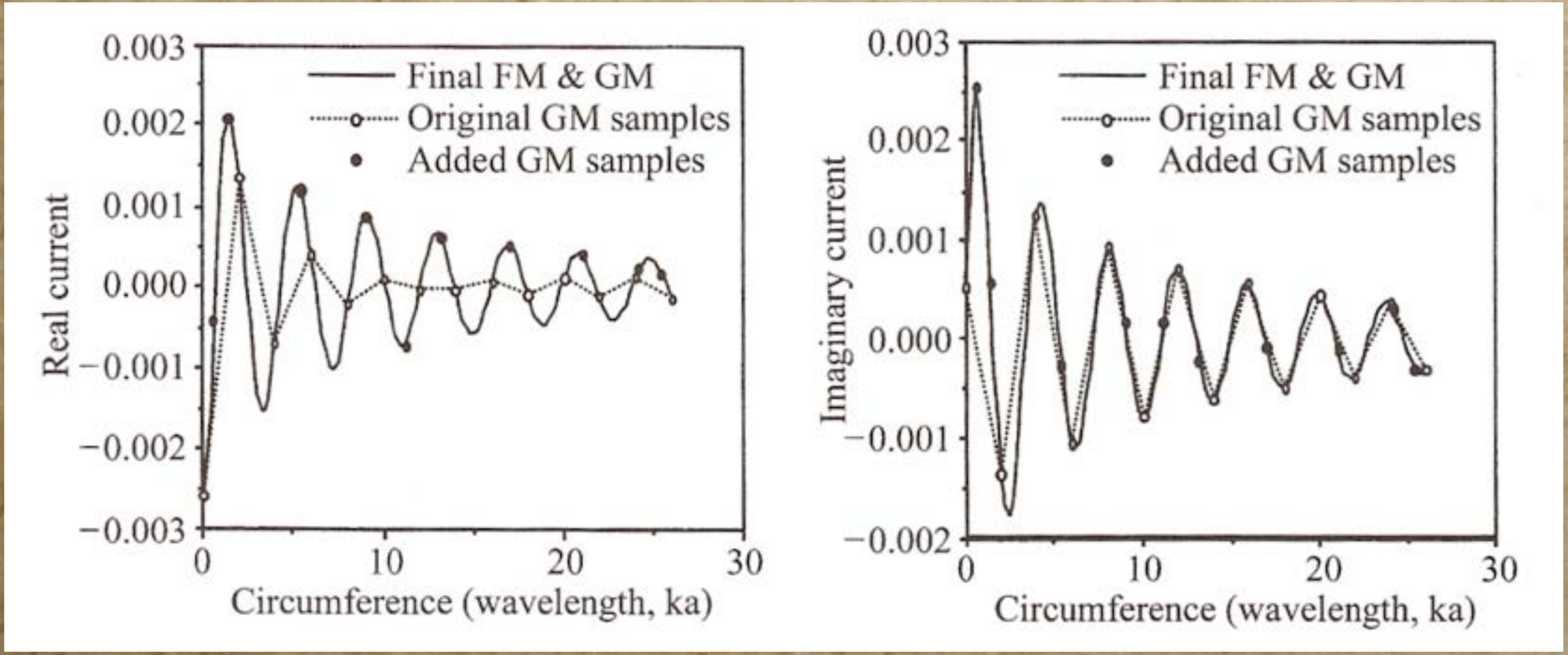


DIPOLE ADMITTANCE ADAPTIVELY SAMPLED FROM AN ANALYTICAL APPROXIMATION



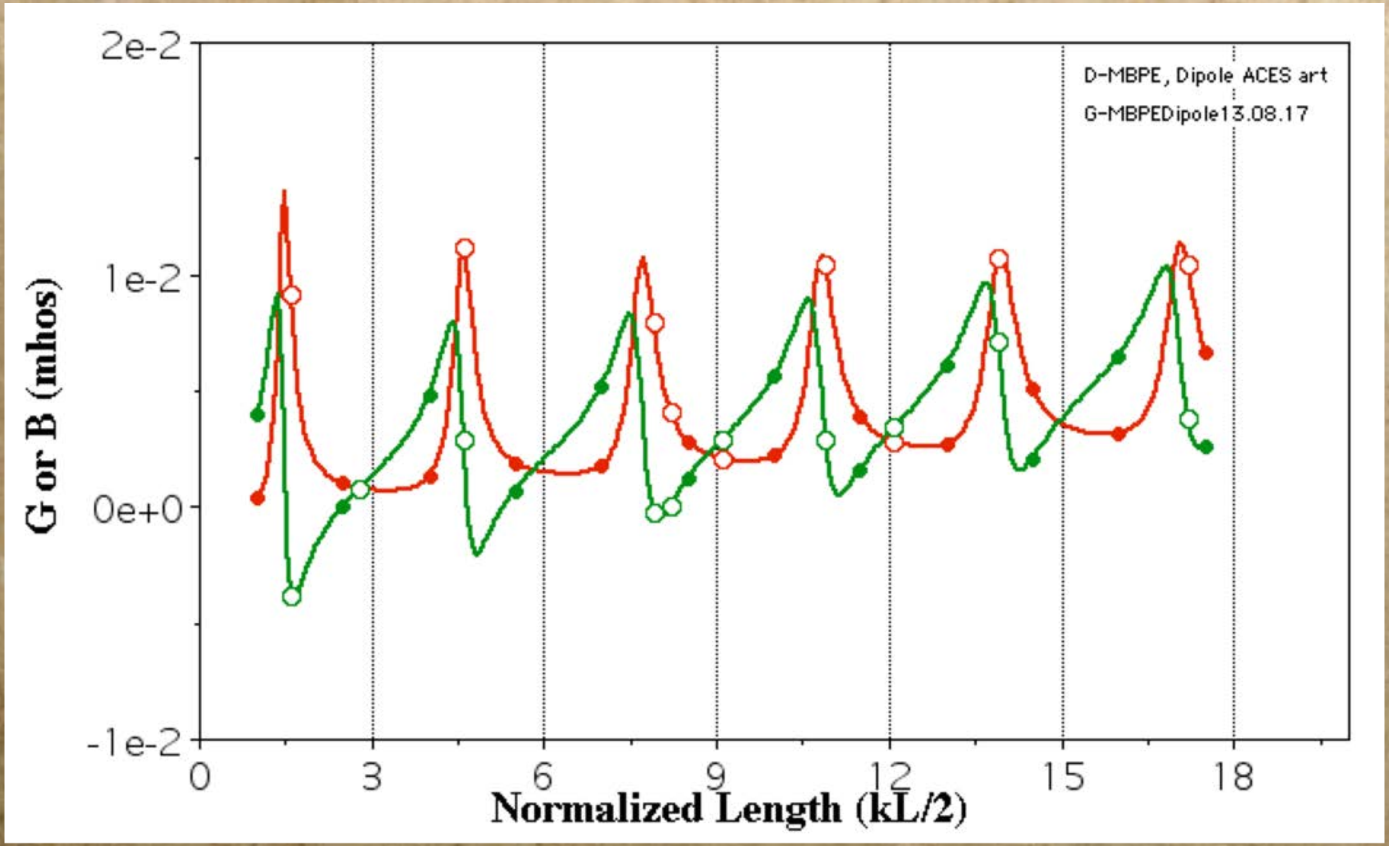
--16 INITIAL GM SAMPLES (OPEN CIRCLES) AND 31 TOTAL, ABOUT 4 SAMPLES/RESONANCE

0.1% ERROR IS ACHIEVED FOR ADAPTIVE SAMPLING OF TE CURRENT ON FRONT OF INFINITE CYLINDER



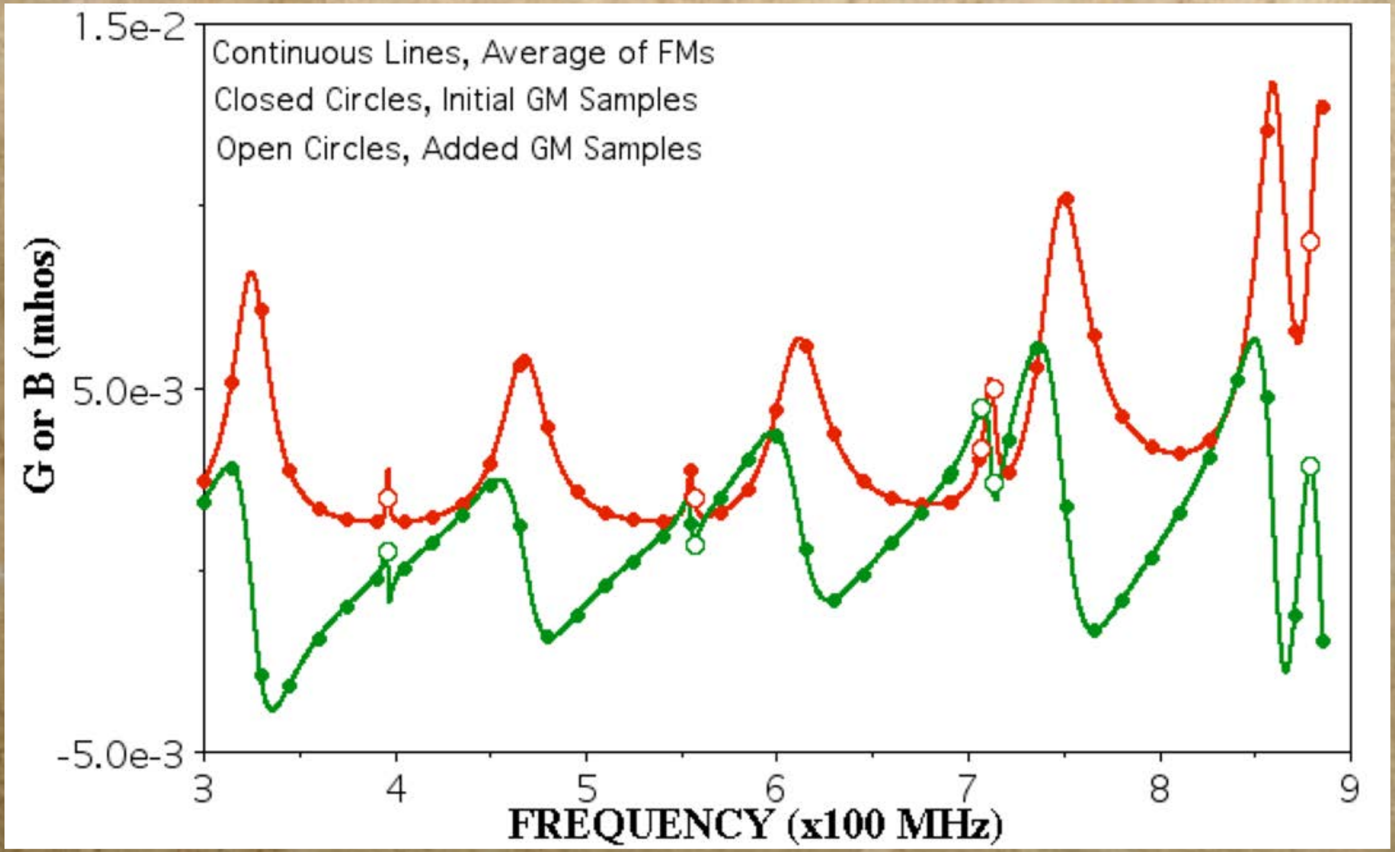
Results for adaptive sampling of the frequency-dependent current on the front side of an infinite, circular cylinder excited by a normally incident TE plane wave. Four overlapping *FM*s are used with 14 initial *GM* samples and 10 additional *GM* samples were needed to achieve a mismatch error less than 3 digits.

ADAPTIVE SAMPLING OF A DIPOLE ANTENNA MODELED USING NEC



Adaptively developed response of a dipole antenna with conductance and susceptance shown by red and green lines respectively where the *GM* and *FM* are graphically indistinguishable. The *GM* samples used for the original *FM* computation are shown by solid circles while the samples added during adaptation are indicated by open circles.

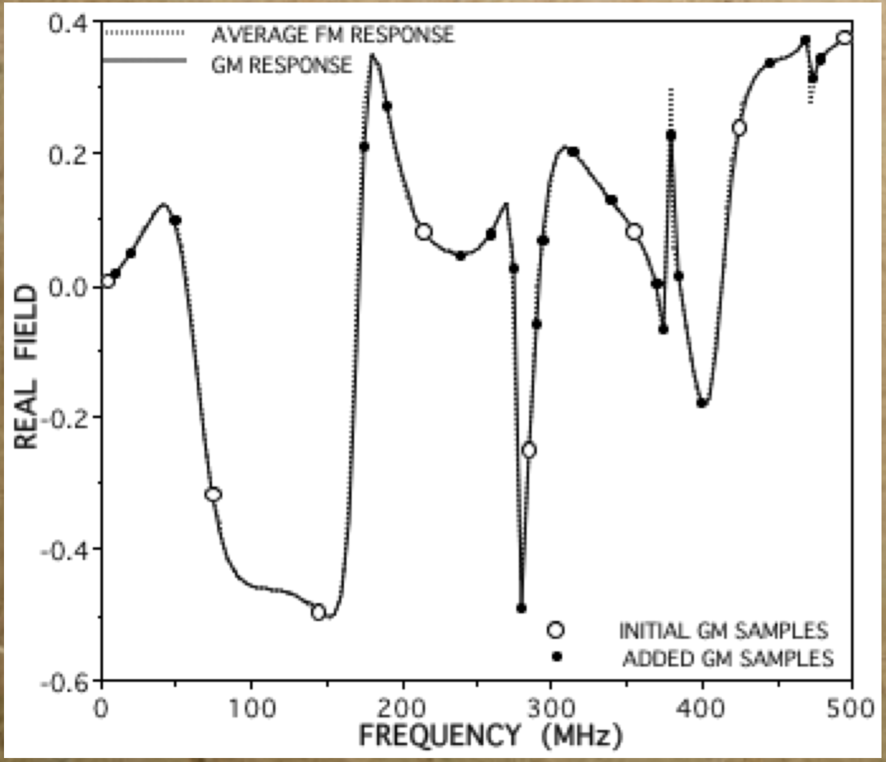
ADAPTIVE SAMPLING OF A GOAL-POST ANTENNA MODELED USING NEC

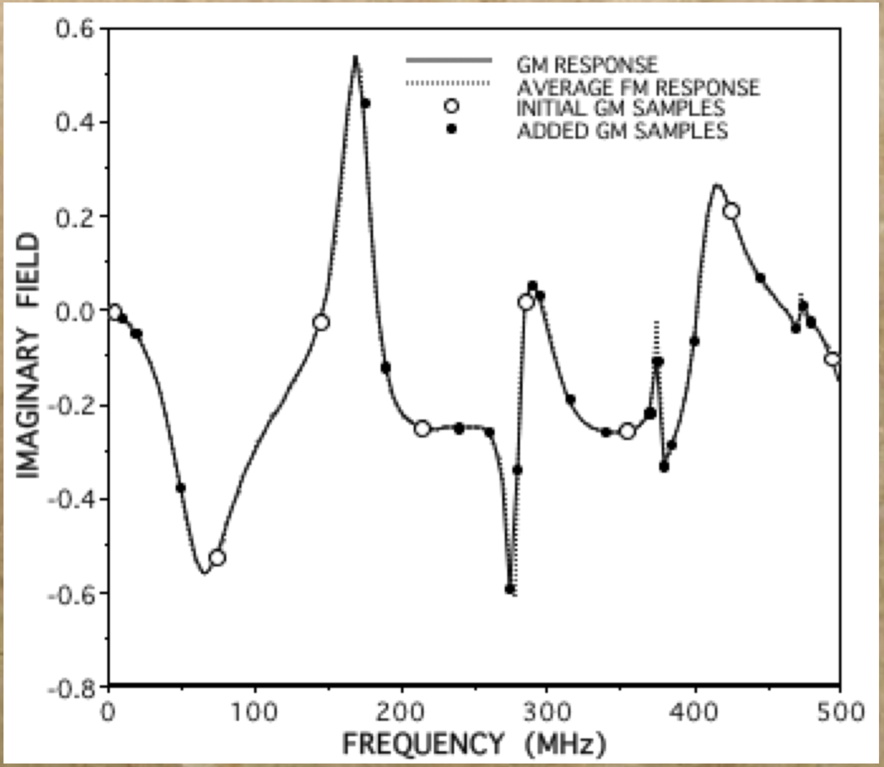


Adaptively developed response of a goal-post antenna with conductance and susceptance shown by red and green lines respectively where the *GM* and *FM* are graphically indistinguishable. The *GM* samples used for the original *FM* computation are shown by solid circles while the samples added during adaptation are indicated by open circles.

PRE-COMPUTED DATA CAN ALSO BE ADAPTIVELY MODELED . . .

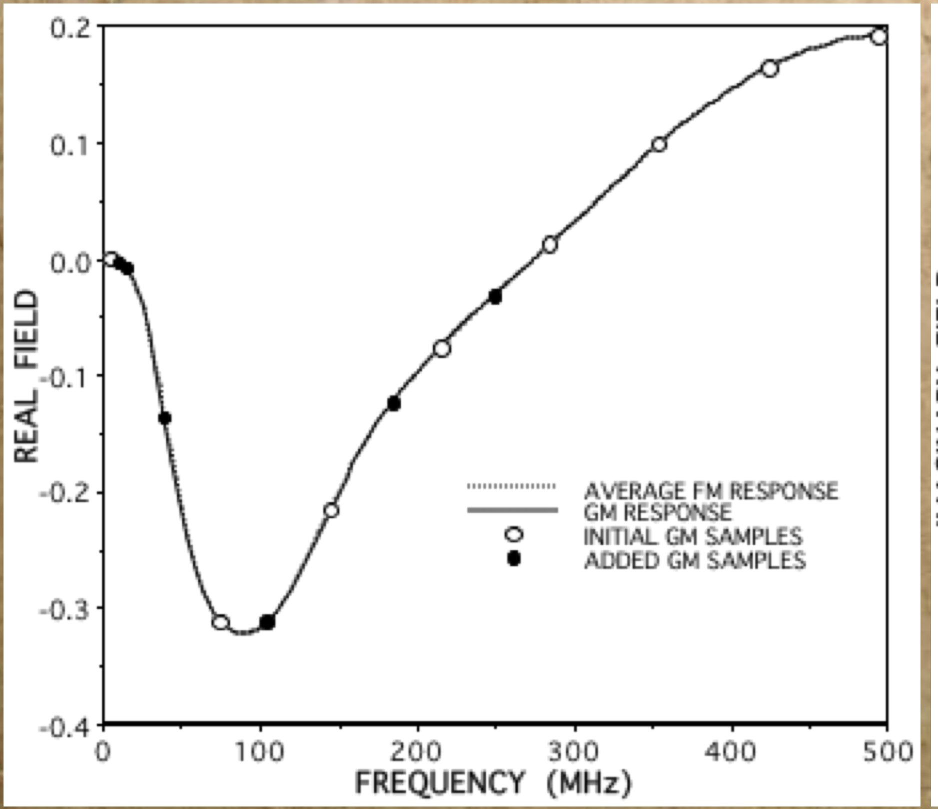
SCATTERED FIELD OF PENETRABLE, INFINITE, CIRCULAR CYLINDER . . .

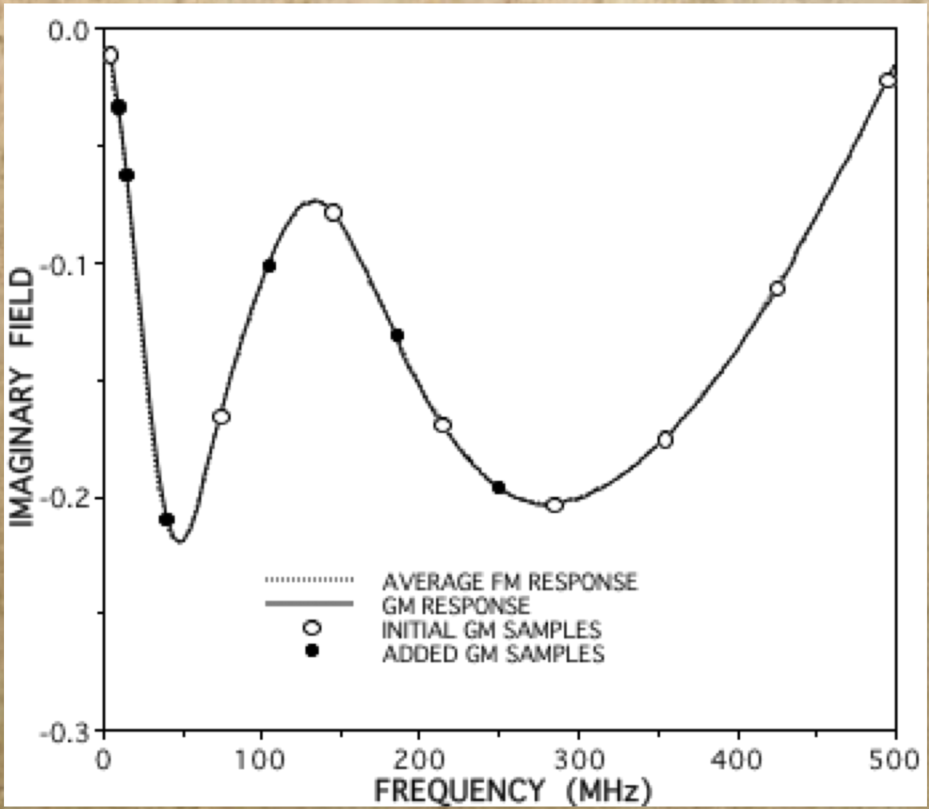




Adaptive modeling using precomputed data (from Kluskens) for the scattered field from an infinite, circular cylinder of relative permittivity 16. The precomputed *GM* samples, spaced at 5-MHz intervals, are joined by a solid line and the average FM results are shown by the dotted line. The *GM* samples used for the original *FM* computation are shown by open circles while the samples added during adaptation are indicated by solid circles.

. . . WHICH WORKS COMPARABLY WELL WHEN CYLINDER IS LOSSY





Adaptive sampling using precomputed data (from Kluskens) for the scattered field from an infinite, circular cylinder whose relative permittivity is 16 - j16. The precomputed *GM* samples, spaced at 5-MHz intervals, are joined by a solid line while the average *FM* results are shown by the dotted line. The *GM* samples used for the original *FM* computation are shown by open circles while the samples added during the adaptation process are indicated by solid circles.

MODEL COEFFICIENTS CAN BE COMPUTED FROM VARIOUS DATA COMBINATIONS

- FOR n'th-ORDER NUMERATOR, d'th-ORDER DENOMINATOR $n + d + 1$ SAMPLES ARE NEEDED

- COULD RANGE FROM:
  - --$n + d + 1$ DIFFERENT FREQUENCIES TO
  - --ONE FREQUENCY AND FIRST $n + d$ DERIVATIVES

. . . WITH "BEST" APPROACH

- MAXIMIZING BANDWIDTH

- MINIMIZING O.C. AND ERROR MEASURE

ADAPTIVE SAMPLING IS A VIABLE PROCEDURE FOR ESTIMATING EM TRANSFER FUNCTIONS TO PRESCRIBED ACCURACY

- USING A RATIONAL-FUNCTION FITTING MODEL (*FM*)
- QUANTIFYING THE FM BY SAMPLING A FIRST PRINCIPLES OR GENERATING MODEL (*FPM* or *GM*)
- ESTIMATING ERROR BY OVERLAPPING THE *FM*s
- CHOOSING A FREQUENCY FOR THE NEXT *GM* SAMPLE BASED ON MAXIMUM *FM* DIFFERENCE
- REPEATING PROCESS UNTIL SPECIFIED OVERALL ESTIMATION ERROR IS SATISFIED

PRESENTATION EXPLORES SOME ISSUES IN ESTIMATING & REPRESENTING EM OBSERVABLES

1) THE SCIENTIFIC METHOD
2) MODEL-BASED PARAMETER ESTIMATION
3) FITTING MODELS FOR WAVEFORM AND SPECTRAL DATA
4) FUNCTION SAMPLING AND DERIVATIVE SAMPLING
5) ADAPTIVE SAMPLING OF FREQUENCY SPECTRA
6) **ADAPTIVE SAMPLING OF RADIATION AND SCATTERING PATTERNS**
7) USING MBPE TO ESTIMATE MODEL UNCERTAINTY
8) OTHER FITTING MODELS FOR EM OBSERVABLES
9) USING MBPE FOR GENERATING-MODEL COMPUTATION

ADAPTIVE PATTERN SAMPLING IS OUTLINED AND DEMONSTRATED

- WHY MINIMIZE NUMBER OF FAR-FIELD SAMPLES?
- MODELING THE SOURCE DISTRIBUTION
- MODELING THE PATTERN
- USING OVERLAPPING FITTING MODELS
- STRATEGY FOR SELECTING NEW SAMPLING ANGLES
- SOME REPRESENTATIVE RESULTS

ALTHOUGH GENERALLY LESS EXPENSIVE THAN COMPUTING SOURCE DISTRIBUTIONS . . .

- FIRST-PRINCIPLES SOURCE COMPUTATION VARIES FROM ~ $N \log N$ TO $N^3/3$ FOR SINGLE EXCITATION
- FOR R EXCITATIONS, ADDITIONAL SOURCE COMPUTATIONS VARY FROM ~ $RN \log N$ TO RN^2
- SINGLE FAR-FIELD EVALUATION COSTS ~ N
- SO TOTAL PATTERN COMPUTATION COST CAN VARY FROM BEING ~N FOR ANTENNA TO ~ RN^2 FOR SCATTERER

. . . PATTERN EVALUATION CAN DRIVE THE TOTAL COMPUTATION COST FOR SOME PROBLEMS

MBPE PROCEDURE CAN REDUCE COST AND IMPROVE OUTCOME OF PATTERN COMPUTATION. . .

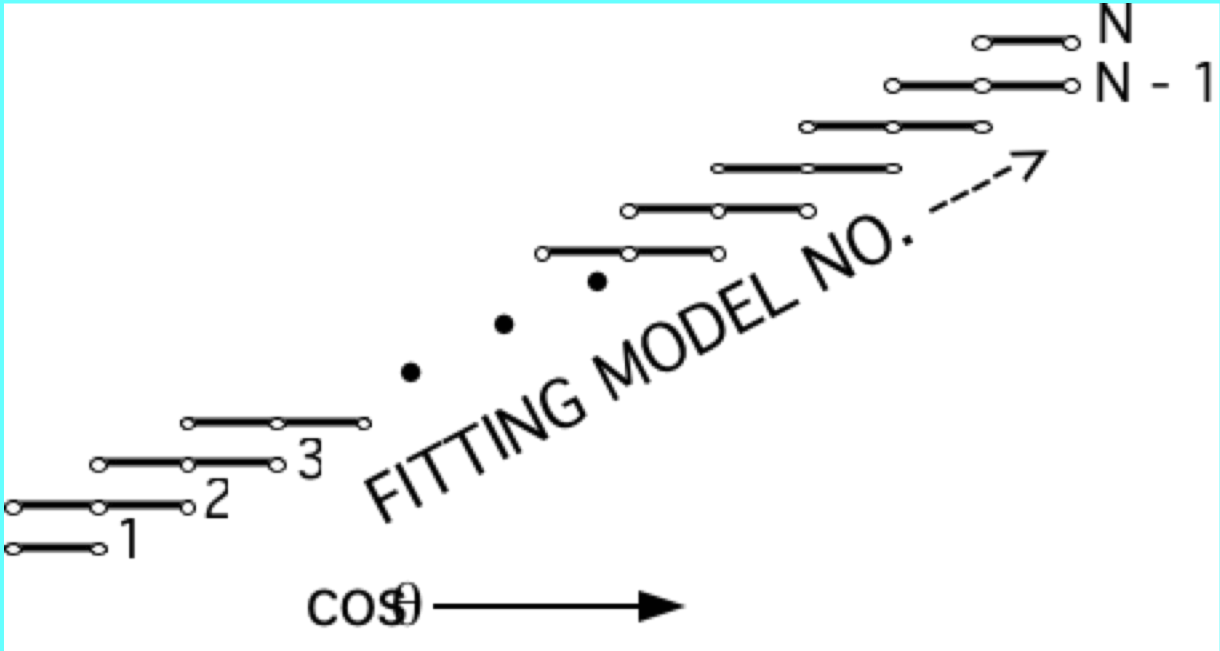
- IF NUMBER OF FAR-FIELD SAMPLES CAN BE REDUCED TO ~ *r* FROM ~ *R* THEN COST IS RDUCED TO ~ *r*/*R*

- THE FITTING-MODEL PROVIDES CONTINUOUS ANALYTICAL REPRESENTATION RATHER THAN A SET OF NUMERS

. . . WHILE ALSO ENSURING PATTERN IS OBTAINED TO SOME PRESCRIBED ESTIMATION UNCERTAINTY . . .

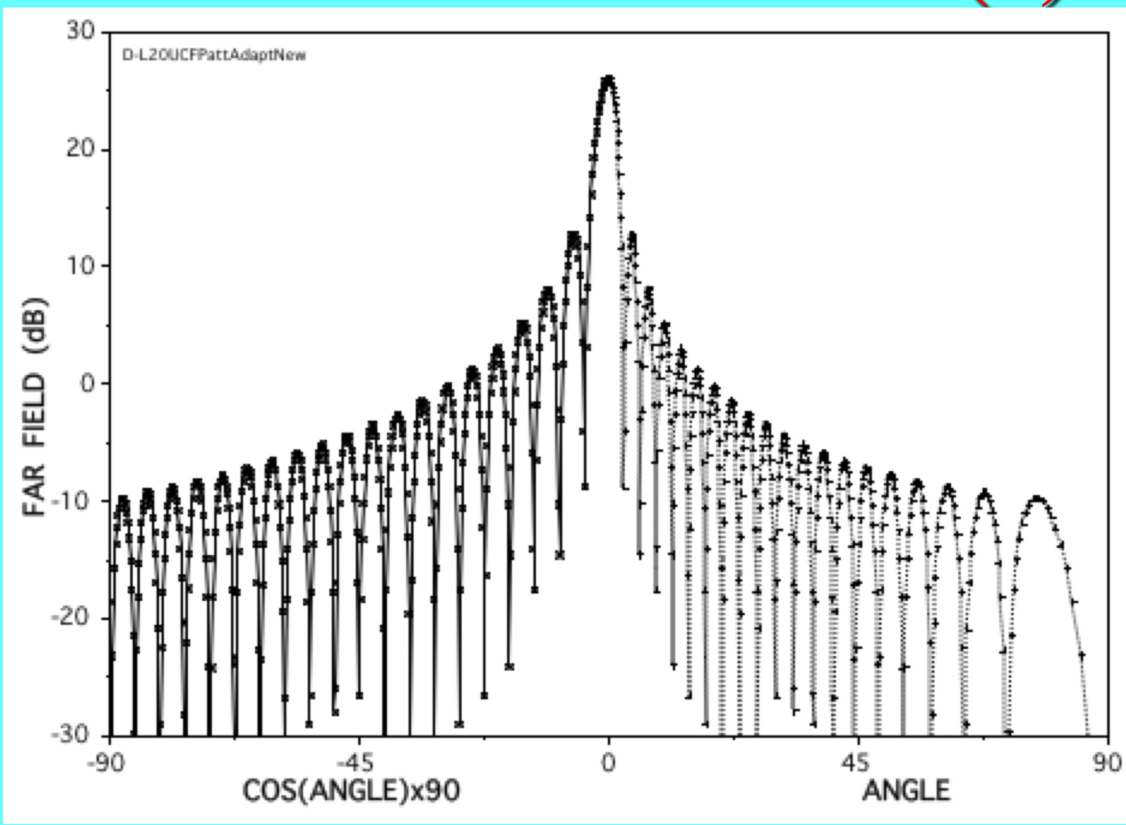
- BY USING OVERLAPPING FITTING MODELS WHOSE MAXIMUM MISMATCH IS REQUIRED TO BE LESS THAN A SPECIFIED VALUE

. . . AGAIN USING A SLIDING FM WINDOW FOR ADAPTIVE MODELING:



- HORIZONTAL LINES INDICATE THE ANGULAR EXTENT OF EACH FITTING MODEL (*FM*)
- THE OPEN CIRCLES SHOW WHERE THE PATTERN [OR GENERATING MODEL (*GM*)] IS INITIALLY SAMPLED
- FOR THIS REPRESENTATIVE CASE, ALL INTERIOR *FM*s BEGIN WITH THREE *GM* SAMPLES, WHILE THOSE AT EITHER END USE ONLY TWO FOR A TOTAL OF $N + 1$
- ADDITIONAL SAMPLES ARE SYSTEMATICALLY ADDED WHERE THE DIFFERENCE BETWEEN *FM*s IS MAXIMUM

LOBE DISTRIBUTION FOR FAR-FIELD OF LINEAR APERTURE IS DISTRIBUTED UNIFORMLY IN cos(θ) . .



. . . SO SAMPLING IS DONE IN TERMS OF cos(θ) RATHER THAN θ

PATTERN ESTIMATION CAN PROCEED IN AT LEAST TWO WAYS, BY MODELING EITHER . . .

1) THE FAR FIELD USING WINDOWED FOURIER SERIES AS FITTING MODELS

. . . OR . . .

2) APPROXIMATING THE SOURCE DISTRIBUTION USING A SERIES OF DISCRETE POINT SOURCES AS FITTING MODELS [THE DISCRETE-SOURCE APPROXIMATION (DSA)]

. . . WHERE THE LATTER CAN INVOLVE:

1) DETERMINING BOTH THE LOCATIONS AND AMPLITUDES OF THE POINT SOURCES

OR

2) SPECIFYING THE SOURCE LOCATIONS AND DETERMINING ONLY THEIR AMPLITUDES

EXPONENTIAL FITTING MODELS ARE APPROPRIATE FOR RADIATION AND SCATTERING PATTERNS . . .

- MODELING THE PATTERN* CAN BE APPROACHED USING THE FITTING MODEL

$$f_m(\theta) = \sum_{n=S}^{N} R_{mn} e^{[ikn\cos\theta]} + \sum_{n'=-S'}^{N'} R'_{mn'} e^{[-ikn'\cos\theta]}$$

- WHERE--

  --θ IS THE OBSERVATION ANGLE

  --$N = \mathrm{int}(A + 1)$, WITH THE APERTURE A IN WAVELENGTHS

  --$2N - S - S' = F$, WITH F THE TOTAL NUMBER OF *FM* TERMS

  --$|S - S'| \leq 1$

  --m IS THE *FM* NUMBER, THERE BEING A TOTAL OF M

  --R_{mn} ARE THE *FM* PARAMETERS (UNKNOWNS) OBTAINED FROM POINT MATCHING TO $f(\theta)$

- THE PROCESS IS "INITIALIZED" BY CHOOSING A STARTING VALUE FOR F, USUALLY 3 OR 4, AND THE TOTAL NUMBER OF *FM*s OR WINDOWS, M µ A

EXPONENTIAL FITTING MODELS ARE APPROPRIATE FOR RADIATION AND SCATTERING PATTERNS . . .

- MODELING THE SOURCE DISTRIBUTION CAN BE ACHIEVED USING PRONY'S METHOD AND A FITTING MODEL GIVEN BY

$$f_m(\theta) = \sum_{\alpha=1}^{W} R_{m\alpha} e^{[ikd_{m\alpha} \cos\theta]}$$

- WHERE--
  - --NUMBER OF POINT SOURCES IS W
  - --UNKNOWN SOURCE STRENGTHS ARE GIVEN BY $R_{m\alpha}$
  - --UNKNOWN SOURCE LOCATIONS ARE GIVEN BY d_{α}
  - --OBSERVATION ANGLE MEASURED FROM SOURCE AXIS IS θ AND $f(\theta)$ IS SAMPLED IN UNIFORM STEPS IN $\cos\theta$
- BOTH THE SOURCE STRENGTHS $R_{m\alpha}$ AND LOCATIONS $d_{m\alpha}$ ARE ESTIMATED
- CAN LEAD TO ILL-CONDITIONED DATA MATRIX UNLESS SIZE OF OBSERVATION WINDOWS IS LIMITED
- SOURCE DISTRIBUTION IS GENERALLY NON-UNIQUE

EXPONENTIAL FITTING MODELS ARE APPROPRIATE FOR RADIATION AND SCATTERING PATTERNS . . .

- AN ALTERNATE MODEL FOR THE SOURCE DISTRIBUTION IS TO SPECIFY THE POINT-SOURCE LOCATIONS AND TO DETERMINE THEIR AMPLITUDES, i.e., USING

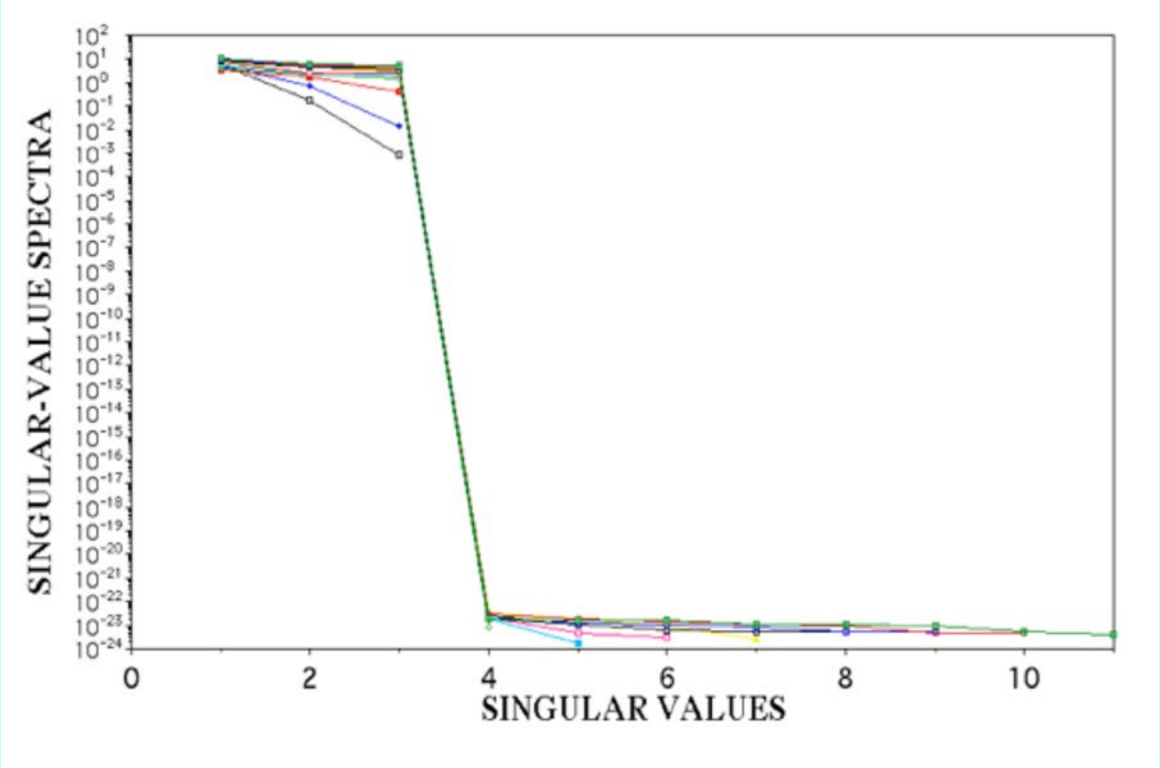
$$f_m(\theta) = \sum_{\alpha=1}^{W} R_{m\alpha} e^{[ikd_{m\alpha}\cos\theta]}$$

- WHERE--
  - --NUMBER OF POINT SOURCES IS W
  - --UNKNOWN SOURCE STRENGTHS ARE GIVEN BY $R_{m\alpha}$
  - --UNKNOWN SOURCE LOCATIONS ARE GIVEN BY $d_{m\alpha}$
- ALSO CAN LEAD TO ILL-CONDITIONED DATA MATRIX UNLESS SIZE OF OBSERVATION WINDOWS IS LIMITED
- IS MORE EFFICIENT THAN PRONY'S METHOD
- IS BETTER SUITED FOR ADAPTIVE SAMPLING SINCE FAR-FIELD SAMPLES CAN BE ARBITRARILY LOCATED IN ANGLE
- AGAIN, SOURCE DISTRIBUTION IS NON-UNIQUE

IT'S USEFUL TO COMPARE SOURCE AND FAR-FIELD MODELING FOR PATTERN ESTIMATION

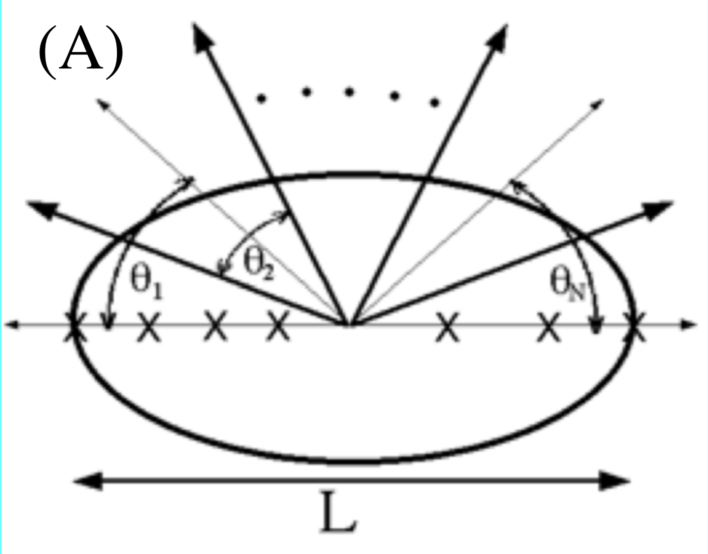
- BOTH SHARE A COMMON *RANK* OR NUMBER OF DEGREES OF FREEDOM (*DoF*)
  - --DETERMINED BY NUMBER OF RESOLVABLE SOURCES OBSERVABLE IN THE FAR FIELD
  - --APPROXIMATE UPPER LIMIT IS 2*A*
- RANK OF FAR FIELD CAN APPEAR TO BE GREATER THAN ACTUAL SOURCE RANK
  - --CONSIDER TWO POINT SOURCES MOVED FARTHER & FARTHER APART
  - --FIELD EXHIBITS MORE AND MORE LOBES
  - --YET THE FIELD'S RANK DOES NOT EXCEED TWO WHEN THE SOURCE DISTRIBUTION IS MODELED
- MODELING THE SOURCE DISTRIBUTION MAY BE BETTER FOR LARGE, SPARSELY FILLED APERTURES
- MAXIMUM FM RANK IS DETERMINED BY WINDOW SIZE RELATIVE TO SOURCE SIZE, *A*

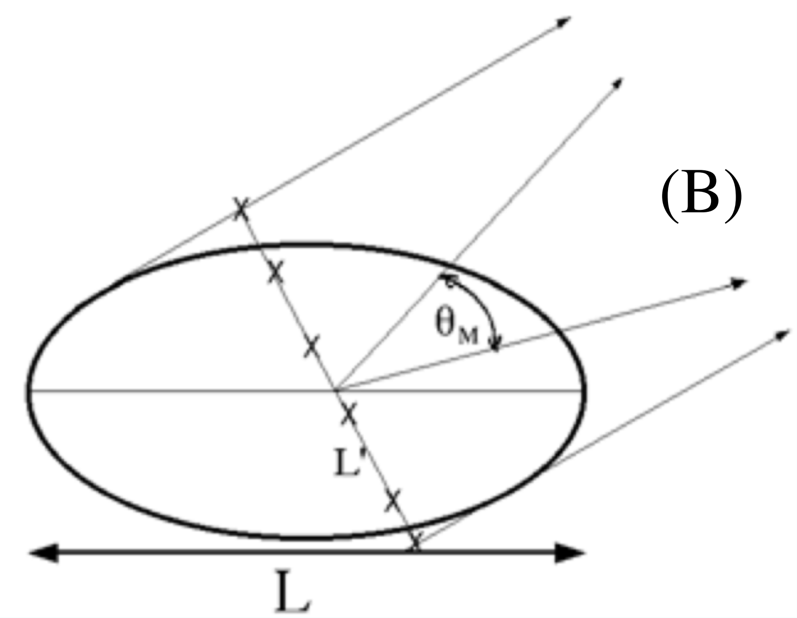
SINGULAR-VALUE SPECTRUM OF DATA MATIX REVEALS SOURCE AND INFORMATION CONTENT



- FOR 3 POINT SOURCES USING EXPONENTIAL FM HAVING FROM 3 TO 11 TERMS
- AS WIDTH OF OBSERVATION WINDOW IS INCREASED MORE FM TERMS ARE NEED FOR CONVERGENCE

THE ANGLE-SAMPLING WINDOWS AND DSA ARRAYS CAN BE ARRANGED IN DIFFERENT WAYS:





- IN (A) THE SAMPLING WINDOWS ROTATE ABOUT THE FIXED RADIATOR/SCATTERER & DSA AXES, BOTH DENOTED BY *L*, THE APPROACH USED HERE
- IN (B) THE DSA AXIS, *L*', ROTATES WITH RESPECT TO *L* SO AS TO BE PERPENDICULAR TO A LINE BISECTING THE SAMPLING WINDOW
- DSA SOURCES ARE UNIFORMLY SPACED ALONG *L* OR *L*'

ADAPTIVE SAMPLING REQUIRES AN ERROR CRITERION TO BE SPECIFIED . .

- FITTING-MODEL ERROR [$FME(\theta)$] USED HERE IS

$$FME(\theta) = A_1 + A_2\left[\left| GM(\theta_{\max}) - \frac{1}{M}\sum_{\alpha=1}^{M} FM_\alpha(\theta) \right| \right]$$

WHERE:

--$GM(\theta_{max})$ IS THE MAXIMUM VALUE FOUND IN THE PATTERN BEING MODELED

--$FM(\theta_\alpha)$ IS THE VALUE OF THE α'th FM AT OBSERVATION ANGLE θ WHERE A TOTAL OF M FITTING MODELS OVERLAP

--THE PARAMETERS A_1 AND A_2 ARE CHOSEN TO FIT THE NEEDS OF A PARTICULAR PROBLEM

--A_1 ESTABLISHES THE MAXIMUM ACCEPTABLE ESTIMATION ERROR IN THE VICINITY OF PEAK(S) OF THE *GM*

--A_2 CAUSES THE PERMITTED ESTIMATION ERROR TO INCREASE IN PROPORTION TO THE DECREASE IN THE FM VALUES RELATIVE TO $GM(\theta_{max})$

- FOR THE RESULTS PRESENTED THE *GM* AND *FM* VALUES ARE EXPRESSED IN dBs

. . . AND ALSO AN ERROR ESTIMATE:

1) ONE POSSIBILITY IS TO COMPUTE THE MAXIMUM DIFFERENCE BETWEEN OVERLAPPING FMs

$$d_{i,j}(\theta_{\max}) = \max|\left[FM_i(\theta) - FM_j(\theta)\right]|, \theta_{j,\min} \le \theta \le \theta_{i,\max}, i, j = 1, \ldots, M$$

WHERE:

--$\theta_{i,min}$ & $\theta_{i,max}$ DEFINE THE ANGLE WINDOW OF FM_i AND θ_{max} IS THE ANGLE WHERE THE MAXIMUM DIFFERENCE OCCURS

--ON THE FIRST PASS DO ALL PAIRS OF *FMs* BUT SUBSEQUENTLY ONLY COMPUTE THE DIFFERENCE FOR THOSE *FMs* THAT ARE AFFECTED BY A NEW PATTERN SAMPLE

2) IF $d_{i,j}(\theta_{max}) > FME(\theta_{max})$, SET $D_{i,j}(\theta_{max}) = d_{i,j}(\theta_{max})$; OTHERWISE SET $D_{i,j}(\theta_{max}) = 0$

3) DETERMINE THE ANGLE θ_{MAX} FOR WHICH THE MAXIMUM VALUE OF ALL THE $D_{i,j}(\theta_{max}) = D_{MAX}$ OCCURS

4) ADD A NEW PATTERN SAMPLE AT ANGLE θ_{MAX}, & UPDATE THE COEFFICIENTS OF EACH OF THE AFFECTED *FMs*

5) RETURN TO 1) AND CONTINUE UNTIL $D_{MAX} = 0$

THE FINAL PATTERN ESTIMATE IS OBTAINED BY AVERAGING THE OVERLAPPING FITTING MODELS . . .

$$F_{ave,M}(\theta) = \frac{1}{M}\left[F_i(\theta) + F_{i+1}(\theta) + \cdots + F_{i+M-1}(\theta)\right]$$

- SAMPLED AS FINELY IN ANGLE AS DESIRED
- THE *FM* COMPUTATION WILL TYPICALLY REQUIRE MUCH LESS COMPUTER TIME THAN DOES EVALUATING THE *GM*

. . . WHILE UPPER- AND LOWER-BOUND ESTIMATES TO ESTABLISH THE UNCERTAINTY IN $F_{ave,M}(\theta)$ ARE GIVEN BY

$$F_{ave,M\pm}(\theta) = F_{ave,M}(\theta) \pm \frac{1}{2} FME(\theta)$$

RADIATION PATTERN OF A UNIFORM CURRENT FILAMENT PROVIDES A USEFUL TEST CASE

- FAR FIELD IS GIVEN BY

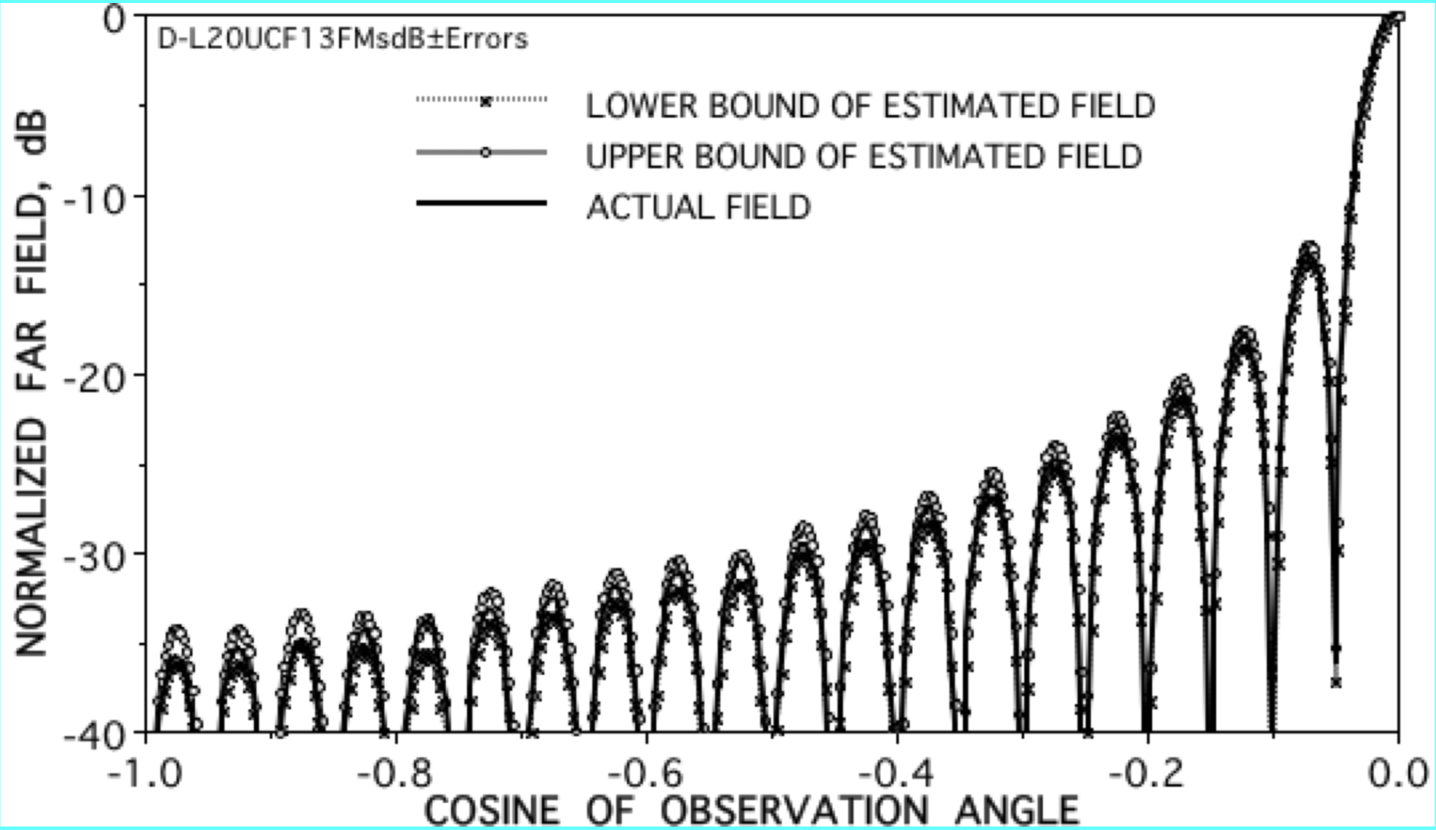
$$F_{UCF}(\theta) = L\frac{\sin[\pi L \sin(\theta)]}{\pi L \sin(\theta)}$$

--WITH L THE FILAMENT LENGTH IN WAVELENGTHS
--OBSERVATION ANGLE MEASURED FROM THE FILAMENT AXIS

- EXAMPLE USES:

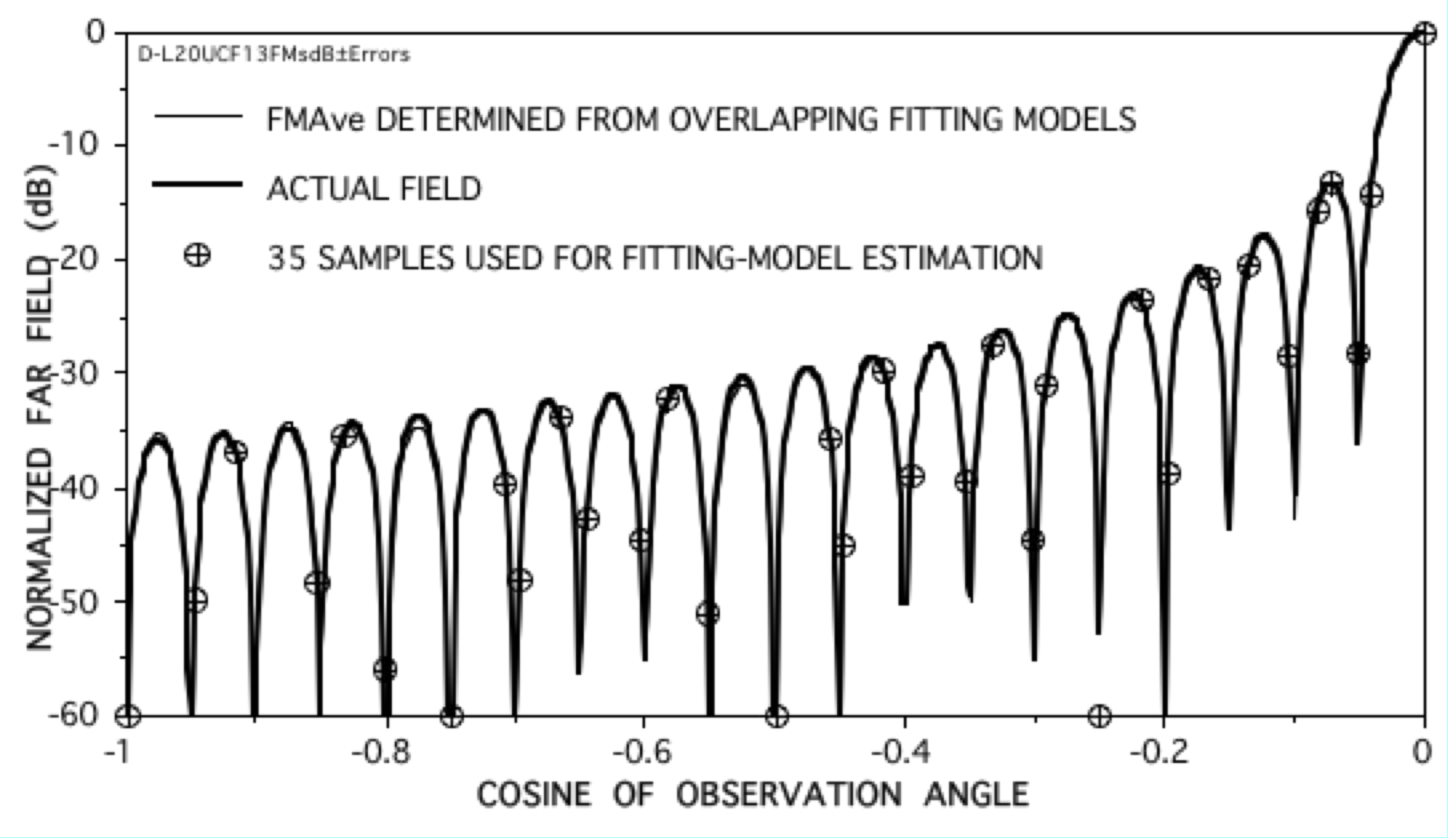
--$L = 20$
--13 FITTING MODELS
--TOTAL OBSERVATION ANGLE FROM π TO $\pi/2$
--TWO END *FMs* INITIALLY USE 2 GM SAMPLES
--REMAINING 11 USE 3
--EACH FM SHARES 2 SAMPLES WITH ITS NEIGHBOR(S)
--INITIAL ERROR SPECIFICATION USES $A_1 = 0.1$ and $A_2 = 0.05$ dB

ACTUAL FIELD IS CLOSELY BRACKETED BY THE UPPER- AND LOWER-BOUND ESTIMATES



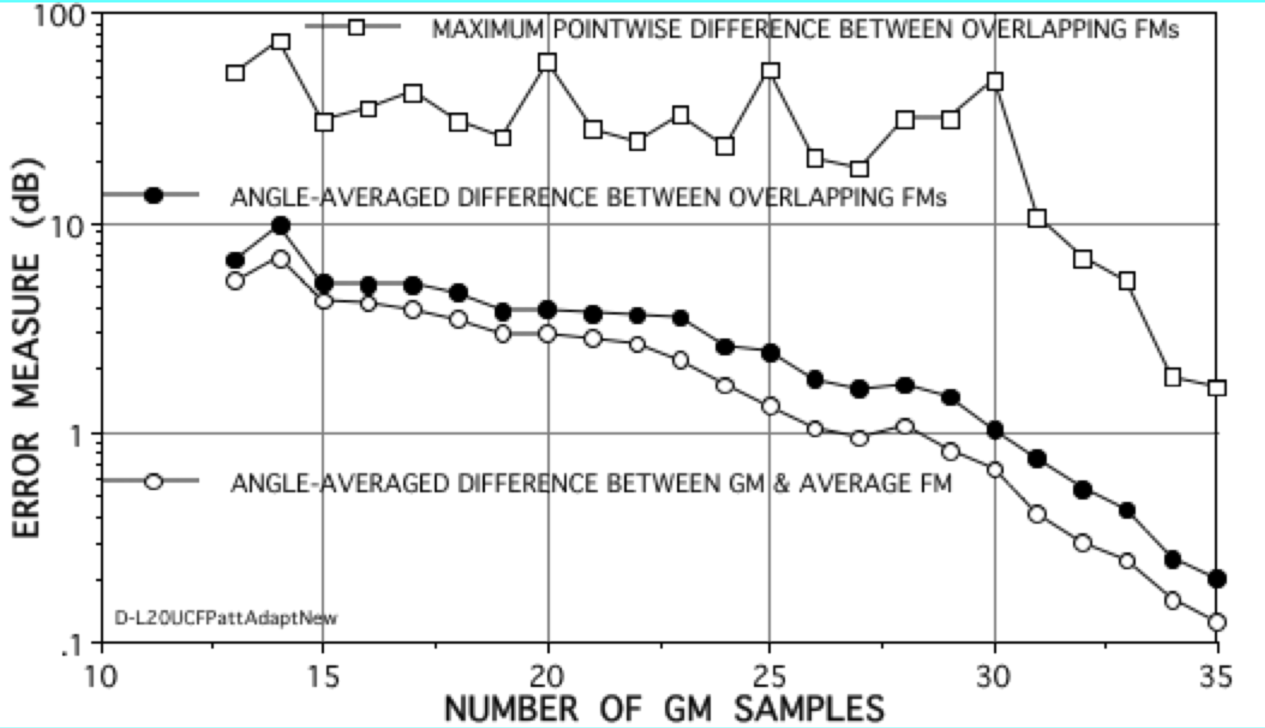
- PLOT CUTOFF AT -40 dB SHOW RESULTS MORE CLEARLY

COMPARISON OF THE ACTUAL AND ESTIMATED FIELDS SHOWS THEM TO BE NEARLY INDISTINGUISHABLE . . .



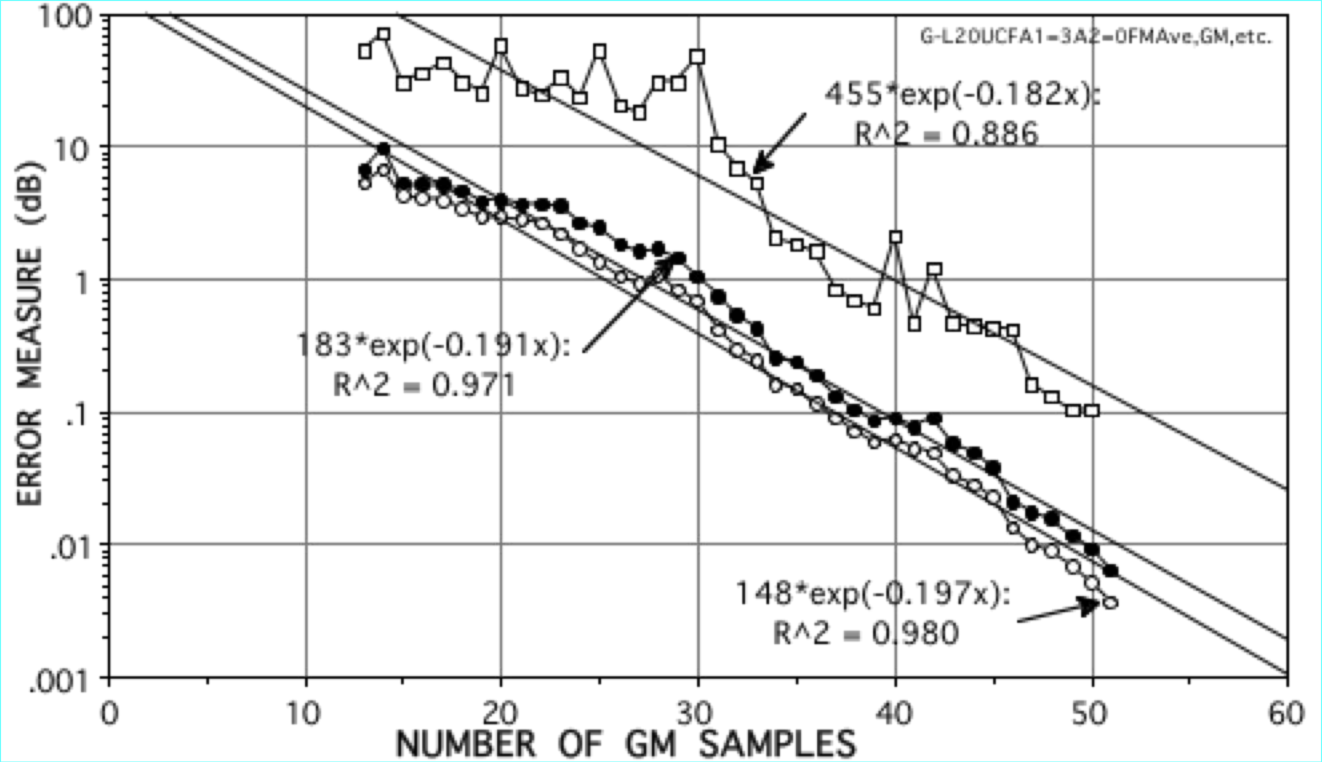
. . . UNTIL LEVEL FALLS BELOW ~ -30 dB

THE DIFFERENCE BETWEEN THE OVERLAPPING FMs IS FOUND TO BE A CONSERVATIVE ERROR MEASURE



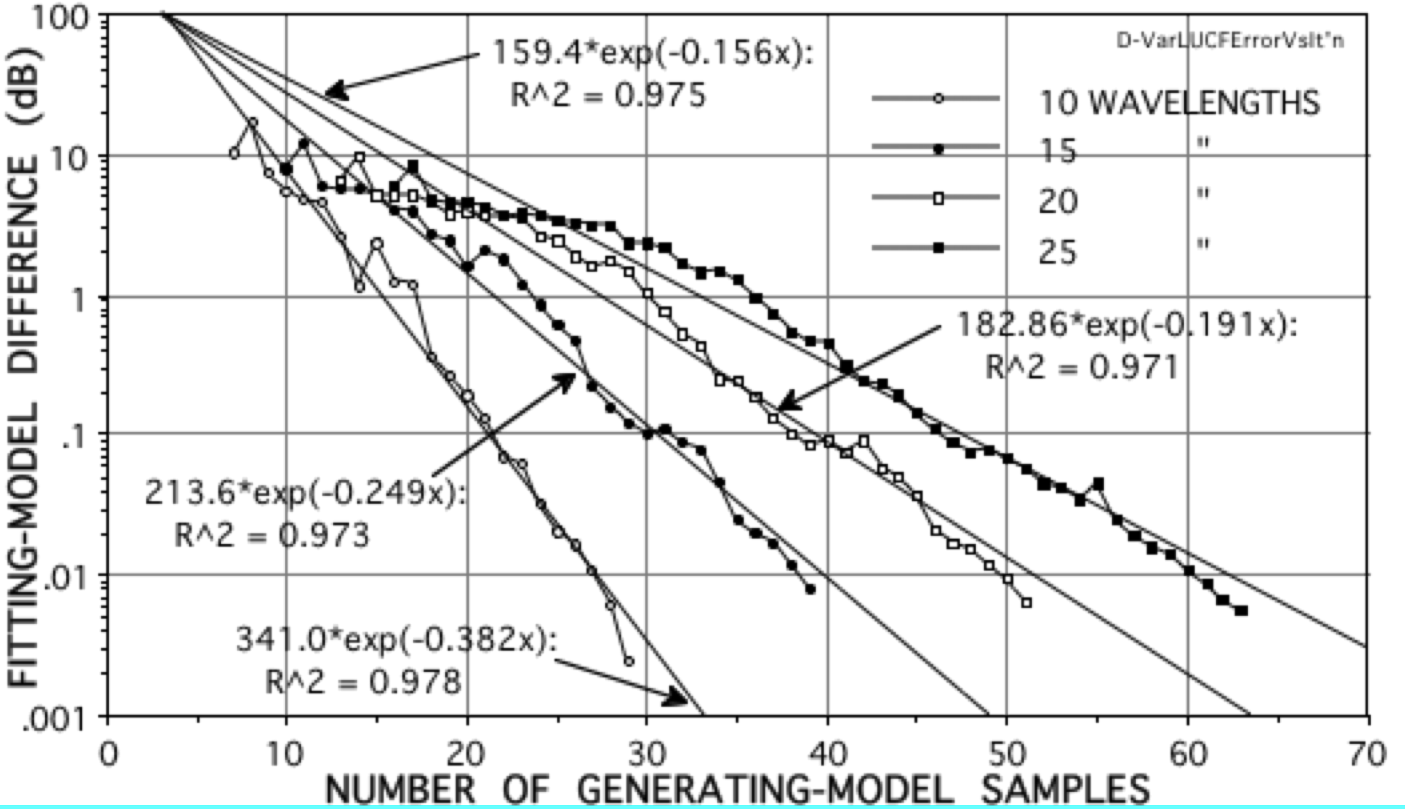
- D_{MAX} IS SHOWN IN THE UPPER CURVE
- THE ANGLE-AVERAGED DIFFERENCE OF THE OVERLAPPING *FMs* ALWAYS EXCEEDS THE DIFFERENCE BETWEEN THE FM_{AVE} AND THE *GM*

USING $A_2 = 0$ SHOWS THAT ERROR MEASURES FALL EXPONENTIALLY WITH NUMBER OF GM SAMPLES



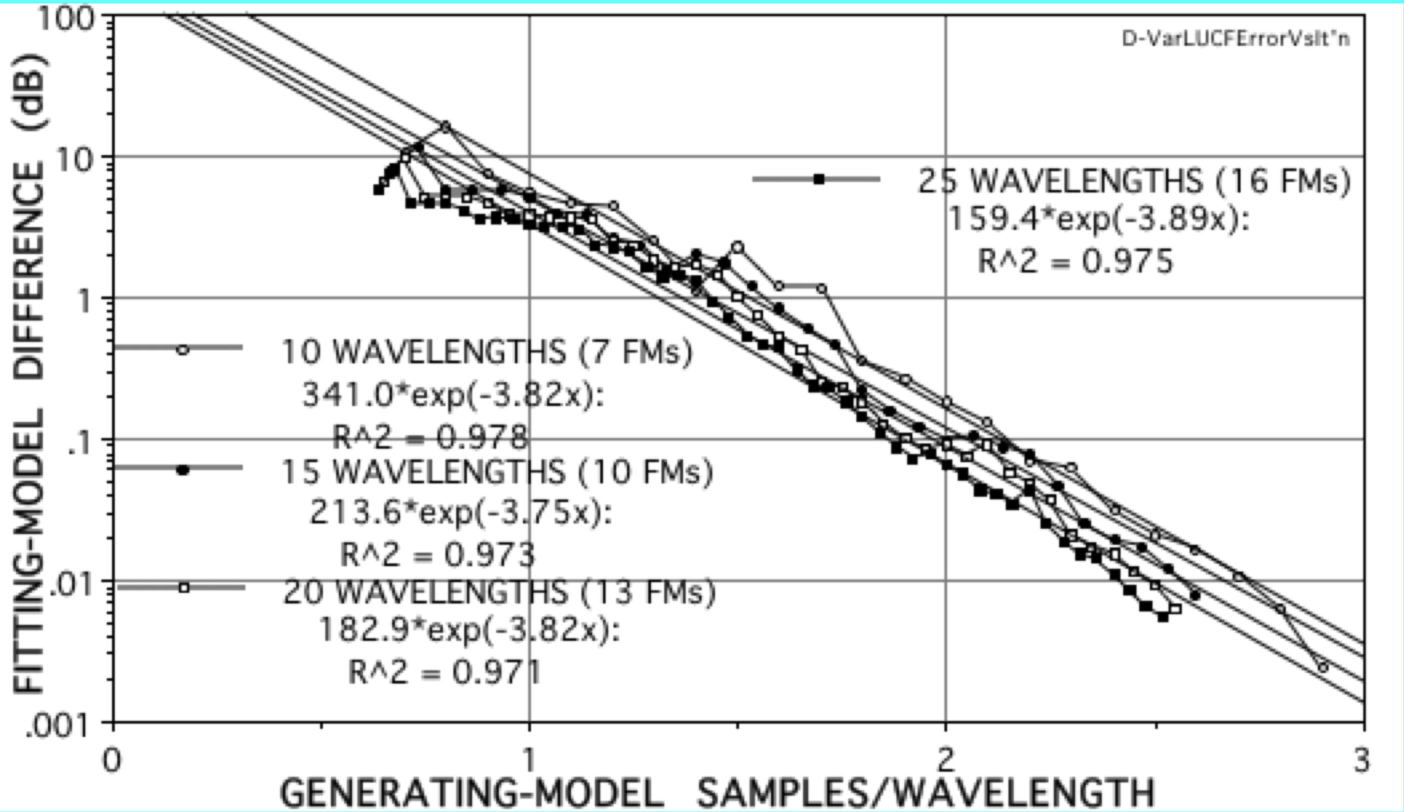
- $A_1 = 0.05$
- BEST-FIT STRAIGHT LINES ARE SHOWN WHERE R^2 IS THE SQUARE OF THE CORRELATION COEFFICIENT
- ANGLE-AVERAGED ERROR REDUCED BY MORE THAN THREE ORDERS OF MAGNITUDE

ANGLE-AVERAGED FM DIFFERENCE FOR 4 APERTURES EXHIBIT DIFFERENT EXPONENTIAL SLOPES . . .



- AGAIN USES $A_1 = 0.05$
- SLOPE DECREASES AS APERTURE SIZE INCREASES
- IMPLIES NUMBER OF FAR-FIELD SAMPLES µ APERTURE

. . . WHICH ARE NEARLY IDENTICAL WHEN PLOTTED IN TERMS OF THE NUMBER OF *GM* SAMPLES/λ



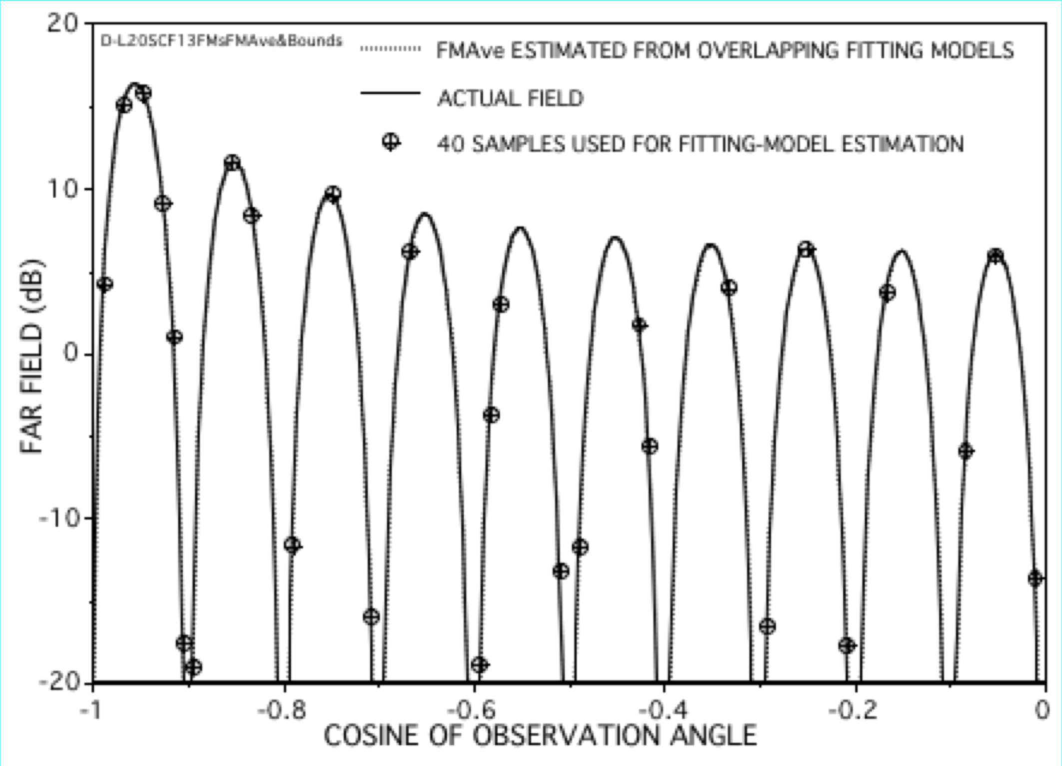
- ESSENTIALLY SAME SLOPE DEPENDENCE
- BUT MULTIPLIER DECREASES WITH INCREASING APERTURE SIZE, *A*
- SEEMS TO IMPLY THAT PATTERN RANK IS NOT LINEARLY PROPORTIONAL TO *A*

ESTIMATING THE PATTERN OF AN L = 20 SINUSOIDAL CURRENT FILAMENT . .

$$F_{SCF}(\theta) = \frac{\cos(\pi L \cos\theta) - \cos(\pi L)}{\sin\theta}$$

. . . REQUIRES MORE SAMPLES THAN THE UCF

- EVEN THOUGH FEWER LOBES ARE INVOLVED
- USING A_1 = 0.1 AND A_2 = 0.05 dB

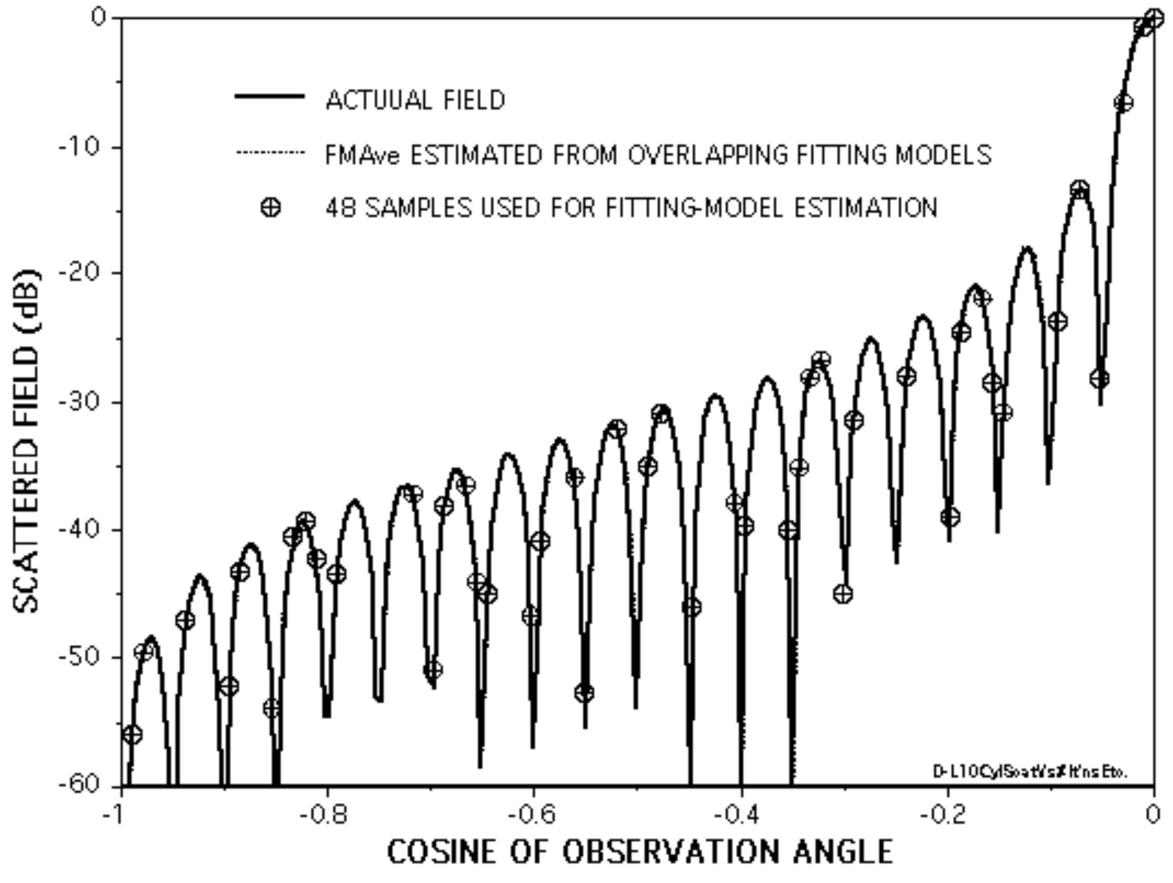


FIELD SCATTERED BY AN L = 10 THIN, CIRCULAR CYLINDER . . .

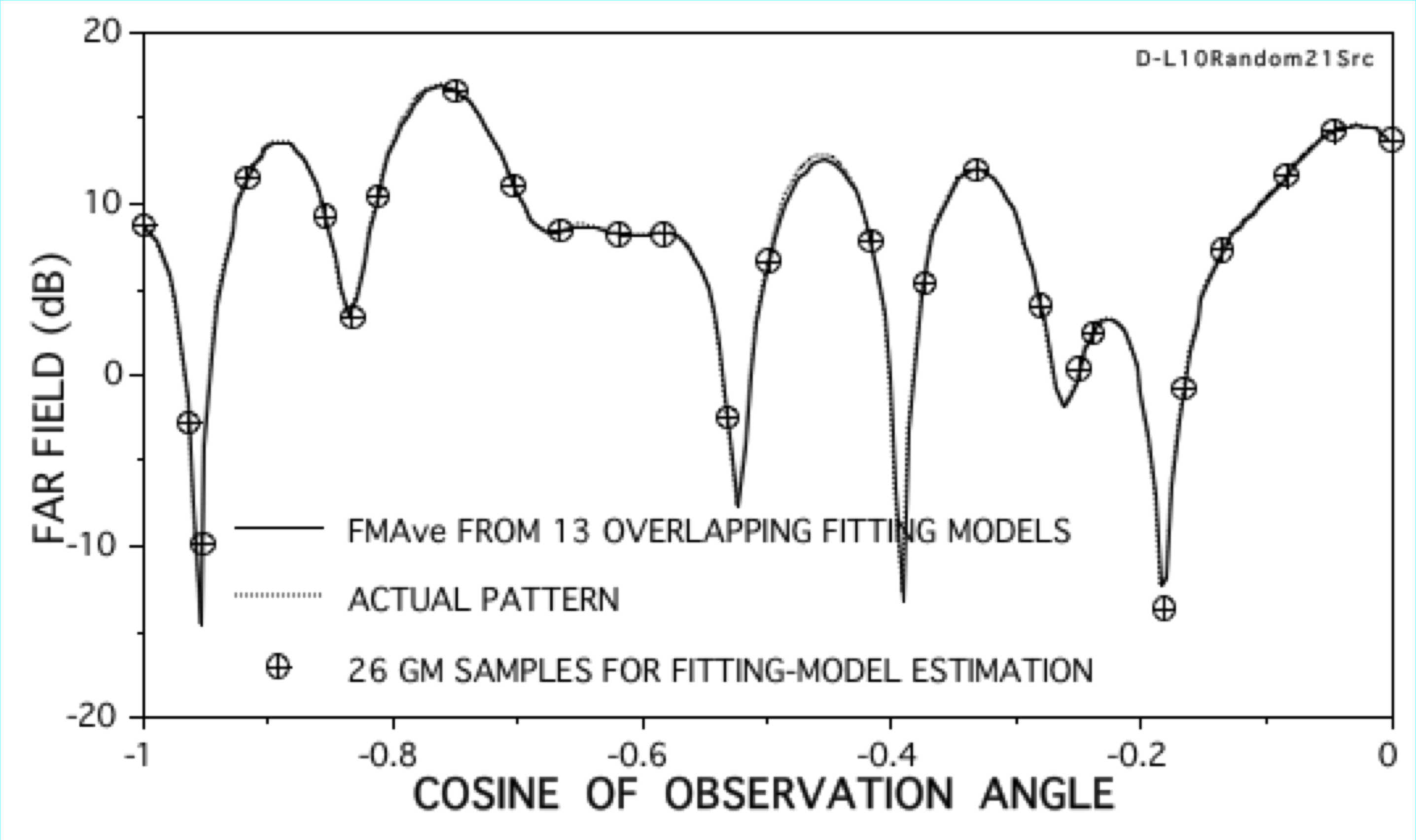
$$RCS \propto F_{CYL}^2(\theta) = \left[\cos\theta_i \frac{\sin(2\pi L \sin\theta_i)}{2\pi L \sin\theta_i}\right]^2$$

. . . REQUIRES STILL MORE SAMPLES

- USING $A_1 = 0.1$ AND $A_2 = 0.05$ dB



PATTERN FROM L = 10 ARRAY OF 21 RANDOMIZED POINT SOURCES IS ESTIMATED COMPARABLY WELL



- USING $A_1 = 0.1$ AND $A_2 = 0.05$ dB
- LARGEST DIFFERENCE BETWEEN GM AND FMAVE IS ABOUT 0.35 dB NEAR COS(ANGLE) = -0.45

SOME ADDITIONAL ASPECTS OF PATTERN ESTIMATION ARE WORTH CONSIDERING

- REQUIRED ESTIMATION UNCERTAINTY MUST BE COMMENSURATE WITH THE ACCURACY OF THE PATTERN DATA BEING MODELED
  - --OTHERWISE THE ADAPTATION PROCESS MAY STAGNATE
- A HYBRID APPROACH THAT COMBINES FAR-FIELD AND DISCRETE-SOURCE APPROXIMATION FITTING MODELS MIGHT BE ATTRACTIVE
- FURTHER EXPLORATION IS NEEDED FOR MORE COMPLEX SOURCE DISTRIBUTIONS AND MORE COMPLETE CUTS OF THE FAR FIELD
- TO AUGMENT OTHER METHODS AND/OR DATA
  - --FOR PRE-SAMPLED DATA TO ASSESS ITS ACCURACY
  - --WITH "FAST" MODELS TO AVOID EXCESSIVE OVERSAMPLING

ALTHOUGH GENERALLY LESS EXPENSIVE THAN COMPUTING SOURCE DISTRIBUTIONS . . .

- FIRST-PRINCIPLES SOURCE COMPUTATION VARIES FROM BEING ~ $NlogN$ TO ~ $N^3/3$ FOR SINGLE EXCITATION
- FOR R EXCITATIONS, ADDITIONAL SOURCE COMPUTATIONS VARY FROM ~ $RNlogN$ TO ~ RN^2
- SINGLE FAR-FIELD EVALUATION COSTS ~ N
- SO TOTAL PATTERN COMPUTATION COST CAN VARY FROM BEING ~ N FOR ANTENNA TO ~ RN^2 FOR SCATTERER

. . . PATTERN EVALUATION CAN DRIVE THE TOTAL COMPUTATION COST FOR SOME PROBLEMS

ANGLE AND FREQUENCY MODELING HAVE BEEN COMBINED IN MBPE . . .

IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, VOL. 48, NO. 3, MARCH 2000 383
The Simultaneous Interpolation of Antenna Radiation Patterns in Both the Spatial and Frequency Domains Using Model-Based Parameter Estimation
Douglas H. Werner and Rene J. Allard

IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, VOL. 51, NO. 8, AUGUST 2003 1891
The Model-Based Parameter Estimation of Antenna Radiation Patterns Using Windowed Interpolation and Spherical Harmonics
Rene J. Allard and Douglas H. Werner

. . . AND IN OTHER WAYS AS WELL

IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, VOL. 54, NO. 3, MARCH 2006
A Model-Based Parameter Estimation Technique for Wide-Band Interpolation of Periodic Moment Method Impedance Matrices With Application to Genetic Algorithm Optimization of Frequency Selective Surfaces
Ling Li, Douglas H. Werner, Jeremy A. Bossard, and Theresa S. Mayer

IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, VOL. 58, NO. 1, JANUARY 2010
Improved Model-Based Parameter Estimation Approach for Accelerated Periodic Method of Moments Solutions With Application to the Analysis of Convoluted Frequency Selected Surfaces and Metamaterials
Xiande Wang and Douglas H. Werner.

THEY USED A RATIONAL FUNCTION FOR THE FREQUENCY VARIATION . . .

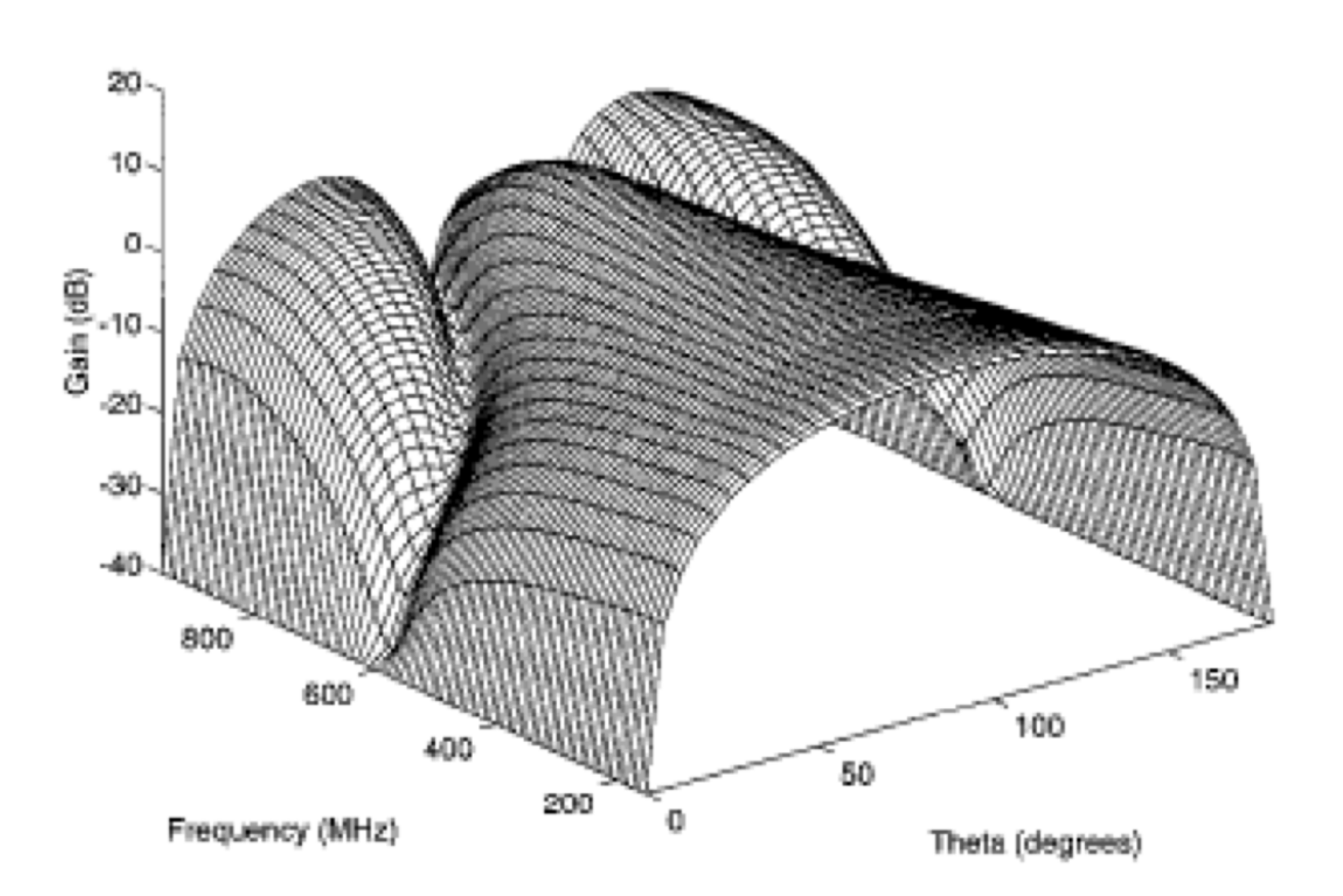
$$\begin{aligned} F(\theta, s) &= \frac{N(\theta, s)}{D(\theta, s)} \\ &= \frac{N_0(\theta)+N_1(\theta)s+N_2(\theta)s^2+\cdots+N_n(\theta)s^n}{D_0(\theta)+D_1(\theta)s+D_2(\theta)s^2+\cdots+D_{d-1}(\theta)s^{d-1}+s^d} \end{aligned}$$

AND POLYNOMIALS FOR THE ANGLE DEPENDENCE:

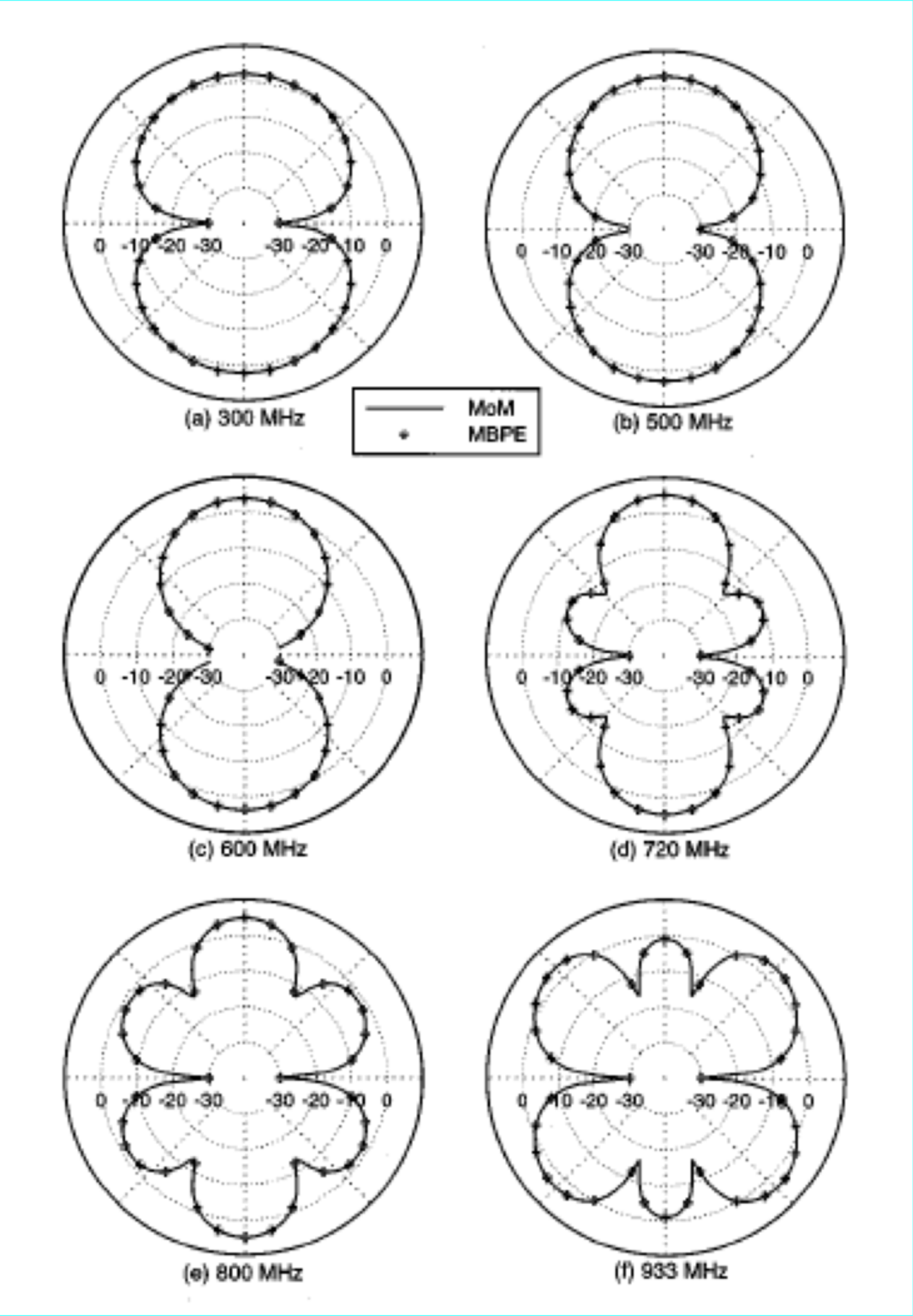
$$\begin{aligned} N_0(\theta) &= N_0^0 + N_0^1\theta + N_0^2\theta^2 + \cdots + N_0^k\theta^k \\ N_1(\theta) &= N_1^0 + N_1^1\theta + N_1^2\theta^2 + \cdots + N_1^k\theta^k \\ &\vdots \\ N_n(\theta) &= N_n^0 + N_n^1\theta + N_n^2\theta^2 + \cdots + N_n^k\theta^k \end{aligned}$$

$$\begin{aligned} D_0(\theta) &= D_0^0 + D_0^1\theta + D_0^2\theta^2 + \cdots + D_0^k\theta^k \\ D_1(\theta) &= D_1^0 + D_1^1\theta + D_1^2\theta^2 + \cdots + D_1^k\theta^k \\ &\vdots \\ D_{d-1}(\theta) &= D_{d-1}^0 + D_{d-1}^1\theta + D_{d-1}^2\theta^2 + \cdots + D_{d-1}^k\theta^k \end{aligned}$$

THE PATTERN CAN BE SUBSEQUENTLY DISPLAYED AS A FUNCTION OF BOTH FREQUENCY AND ANGLE



SOME OF THE PATTERN CUTS SHOWN HERE ARE FOR FREQUENCIES OTHER THAN USED FOR THE FM



<--- THOSE IN THE RIGHT-HAND COLUMN

PRESENTATION EXPLORES SOME ISSUES IN ESTIMATING & REPRESENTING EM OBSERVABLES

1) THE SCIENTIFIC METHOD
2) MODEL-BASED PARAMETER ESTIMATION
3) FITTING MODELS FOR WAVEFORM AND SPECTRAL DATA
4) FUNCTION SAMPLING AND DERIVATIVE SAMPLING
5) ADAPTIVE SAMPLING OF FREQUENCY SPECTRA
6) ADAPTIVE SAMPLING OF RADIATION AND SCATTERING PATTERNS
7) **USING MBPE TO ESTIMATE MODEL UNCERTAINTY**
8) OTHER FITTING MODELS FOR EM OBSERVABLES
9) USING MBPE FOR GENERATING-MODEL COMPUTATION

MODEL-BASED PARAMETER ESTIMATION CAN BE USED TO ASSESS ACCURACY (UNDERTAINTY) OF CEM RESULTS

- THE PROBLEM OF DETERMING CEM MODEL ACCURACY
- A DIFFERENT APPROACH USING MBPE
- IMPLEMENTATION
- SOME TEST APPLICATIONS

DETERMINING THE UNCERTAINTY OF CEM RESULTS REMAINS A CHALLENGING PROBLEM

- THE MOST CONVINCING APPROACH IS TO USE EXPERIMENTAL MEASUREMENTS
  - --SUCH A COMPARISON INTERMINGLES THE NUMERICAL (NME) AND PHYSICAL MODELING (PME) ERRORS
  - --PHYSICAL MODELING ERROR REQUIRES INDEPENDENT CONSIDERATION
  - --SIZE OF NUMERICAL MODELING ERROR IS NEEDED IN ANY CASE TO GUIDE MODEL DEVELOPMENT AND APPLICATION

- ASSESSING THE NME IS GENERALLY EXPENSIVE AND NOT RELIABLE . . .
  - --CONVERGENCE TESTS
  - --BOUNDARY-CONDITION MISMATCH

- . . . OR IS OTHERWISE PROBLEMATIC
  - --ENERGY CONSERVATION & RECIPROCITY
  - --COMPARISON WITH INDEPENDENT NUMERICAL RESULTS
  - --. . .

MORE USEFUL TEST WOULD AVOID THESE PROBLEMS, YIELD A MEASURE RELEVANT TO APPLICATION AND . . .

- MINIMIZE ADDITIONAL CEM MODEL COMPUTATIONS
- BE DIRECTLY APPLICABLE TO OBSERVABLES OF INTEREST
- PROVIDE EASILY INTERPRETABLE QUANTITATIVE RESULTS
- BE IMPLEMENTABLE WITHIN THE MODEL TO BE TESTED

. . . WITH ONE POSSIBILITY BEING MODEL-BASED PARAMETER ESTIMATION

DETERMINING THE "NOISE" LEVEL IN COMPUTED OBSERVABLES IS POSSIBLE USING MBPE . . .

- USES COMPUTED FREQUENCY SAMPLES FROM CEM MODEL TO BE TESTED
- LOW-ORDER, PHYSICALLY MOTIVATED FM IS DEVELOPED FROM SAMPLED DATA
- ADDTIONAL GM SAMPLES WITHIN BANDWIDTH OF FM ARE COMPARED WITH THE FM PREDICTIONS
- VARIATION BETWEEN GM AND FM RESULTS INDICATES INTERNAL GM NOISE LEVEL (OR INCONSISTENCY)

. . . THUS PERMITTING GM-DATA UNCERTAINTY TO BE ESTIMATED

DETERMINING THE “NOISE” LEVEL IN COMPUTED OBSERVABLES IS POSSIBLE USING *MBPE* . . .

One approach is to vary the parameters [order of the numerator polynomial (n) and denominator polynomial (d), and window width (W)] of a single *FM*, to seek a best-fit between that *FM* and the unused *GM* data in that window.

A second approach computes *FM*s using fixed n, d and W values from *GM* data that has additive random noise of systematically reducing amplitude until no further change is found in the mismatch error between the *FM*s and unused *GM* data.

A third approach solves an over-determined data matrix using a pseudo-inverse solution from which a mismatch error between all of the *GM* data spanned by that *FM* representation is obtained.

•VARIATION BETWEEN *GM* AND *FM* RESULTS INDICATES INTERNAL *GM* NOISE LEVEL (OR INCONSISTENCY)

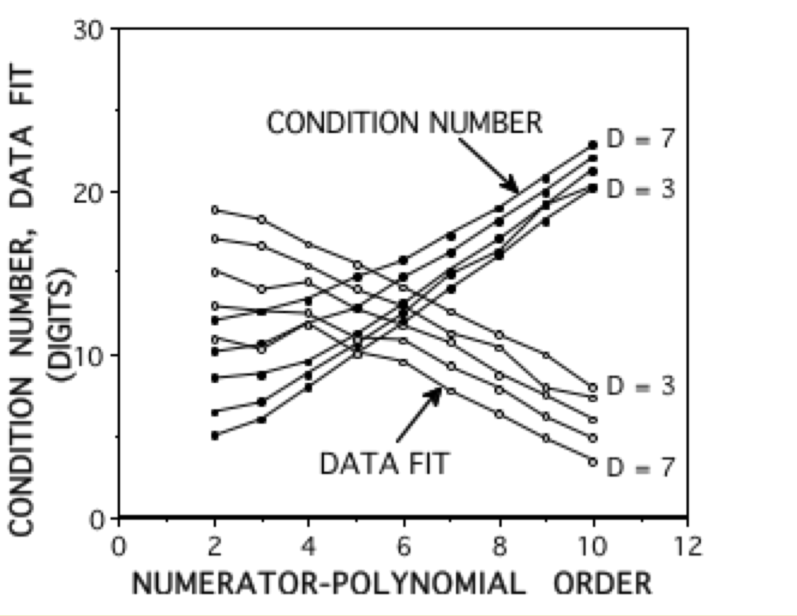
. . . THUS PERMITTING *GM*-DATA UNCERTAINTY TO BE ESTIMATED

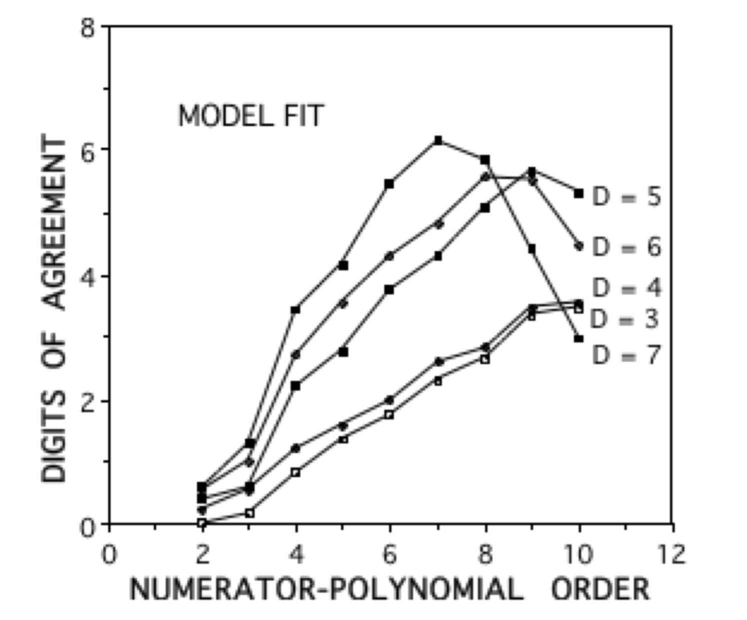
CONDITION NUMBER AND *FM* MATCH TO *GM* BEGIN THE PROCESS

- FOR FIXED FREQUENCY INTERVAL, CONDITION NUMBER (*CN*) GROWS WITH *FM* ORDER *M*
  - --$CN \sim N + D = M$ IN DIGITS
- *FM* MATCH TO ORIGINAL *GM* SAMPLES (DATA) DECREASES MONOTONICALLY WITH INCREASING *FM* ORDER
- *FM* MATCH TO ADDITIONAL *GM* DATA SAMPLES (MODEL FIT) AT FIRST INCREASES WITH MODEL ORDER . . .
  - --UNTIL EFFECT OF INCREASING CN BEGINS TO OFFSET
  - --SO MODEL FIT REACHES A MAXIMUM AT $M = M'$ AND THEN DECREASES
- SO THERE IS SOME OPTIMUM *FM* ORDER FOR TESTING *GM* UNCERTAINTY ACROSS A FIXED BANDWIDTH

. . . PROVIDING A WAY FOR *GM*-DATA UNCERTAINTY TO BE ESTIMATED

SIMPLE POLE SPECTRUM DEMONSTRATES POSSIBILITY





--COMPUTE PRECISION (CP) IS 24 DIGITS

--*GM* IS A 15-POLE SPECTRUM HAVE $s_i = \sqrt{i}/20 + j*i$; $i = 1,2,...,15$

--$M = N + D + 1$ EVENLY SPACED *GM* SAMPLES BETWEEN 5.5 AND 8.5

--RESULTS OBTAINED FOR $N = 2$ TO 10 AND $D = 3$ TO 7

--RESULTS DISPLAYED IN DIGITS, i.e., $CN = 10$ ===> CONDITION NUMBER = 10^{10}

--OBSERVE THAT DATA FIT $DF + CN \sim CP$

--MODEL FIT MEASURES *FM* MATCH TO ADDITIONAL *GM* SAMPLES

--INDICATES THAT ONLY ACCURACY LESS THAN ~ 6 DIGITS CAN BE DETECTED FOR THIS PARTICULAR CASE AND PARAMETERS

DETERMING GM ACCURACY COULD BE REALIZED IF EXACT ANSWER WERE AVAILABLE . . .

- ACCURACY OF *GM* SAMPLES S_i CAN BE EXPRESSED RELATIVE TO EXACT ANSWER E_i

$E_i = E(f_i)$ AS $A_i = [|(S_i - E_i)/E_i|]$

- OR IN DIGITS OF AGREEMENT D_i AS

$$D_i = \log_{10}[|(S_i - E_i)/E_i|]$$

. . . BUT SINCE E_i IS NOT KNOWN NEITHER ARE A_i OR D_i

- GOAL OF *FM* APPROACH IS TO DEVELOP A MEANS FOR ESTIMATING B_i WITHOUT KNOWING E_i

ONE WAY TO ESTIMATE *GM* UNCERTAINTY IS TO SEEK THE "NOISE" FLOOR OF ITS DATA SAMPLES

- ANY RESULT CAN BE EXPRESSED AS $C.CCCNNNNx10^E$
  - --"*C*" REPRESENTS THE CORRECT DIGITS, HERE SHOWN AS 4
  - --"*N*" REPRESENTS ADDITIONAL NOISE DIGITS

- THE PLACE WHERE "*C*" CHANGES TO "*N*" DETERMINES THE NOISE FLOOR

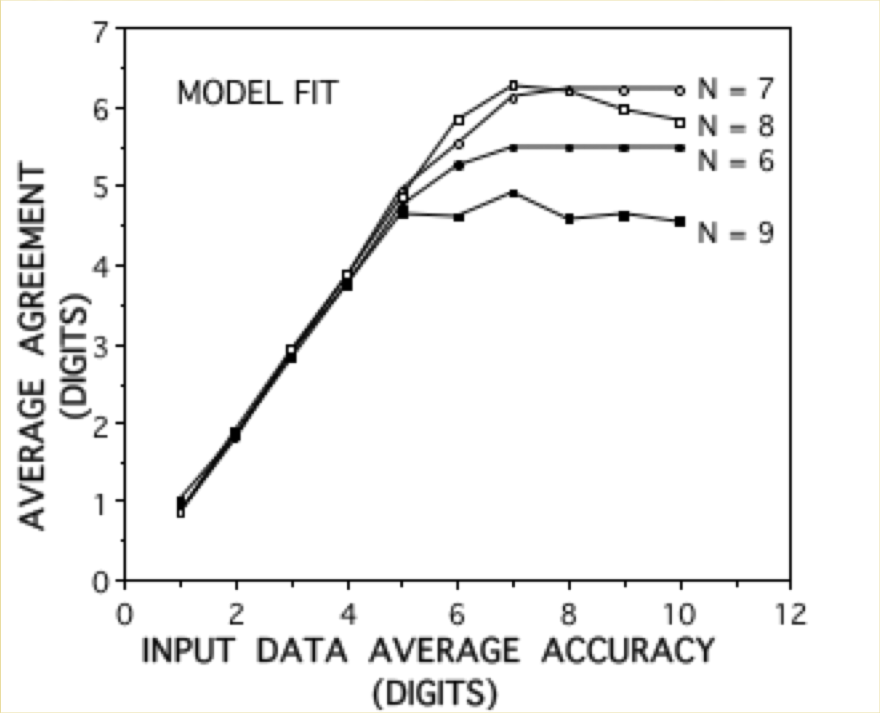
- IF THE NUMERICAL NOISE IS RANDOM, THE S_i SAMPLES SHOULD BE RANDOMLY DISTRIBUTED AROUND THE EXACT ANSWERS

- PROVIDES THE BASIS FOR ESTIMATING "*C*"
  - --USING THE MODEL FIT TOADDITIONAL *GM* SAMPLES

PROCEDURE IS STRAIGHTFORWARD:

- ACQUIRE *S* SAMPLES OF *GM* ACROSS A BANDWIDTH *F*
- USE SUBSET OF THESE SAMPLES TO DEVELOP A *FM* THAT SPANS *F'* <= *F*
- COMPUTE AVERAGE DATA FIT IN <u>DIGITS</u>, D_{DF}, BETWEEN THIS *FM* AND THE SUBSET DATA
  - --AS LIMITED BY CONDITION NUMBER (*CN*) AND COMPUTE PRECISION (*CP*)
  - --COULD EXCEED AVERAGE *GM* ACCURACY D_{GM}
- COMPUTE AVERAGE MODEL FIT, D_{MF}, BETWEEN THE *FM* & THE UNUSED *GM* SAMPLES
  - --LIMITED IN VALUE TO THE LESSER OF D_{DF} AND D_{GM}, SINCE *FM* CAN MATCH THESE SAMPLES NO BETTER THAN THOSE ON WHICH IT IS BASED
- THEREFORE PROVIDES LOWER-BOUND ESTIMATE FOR D_{GM}
- SIMILAR TO USING LINEAR REGRESSION TO FIT A STRAIGHT LINE AND ESTIMATE DATA UNCERTAINTY

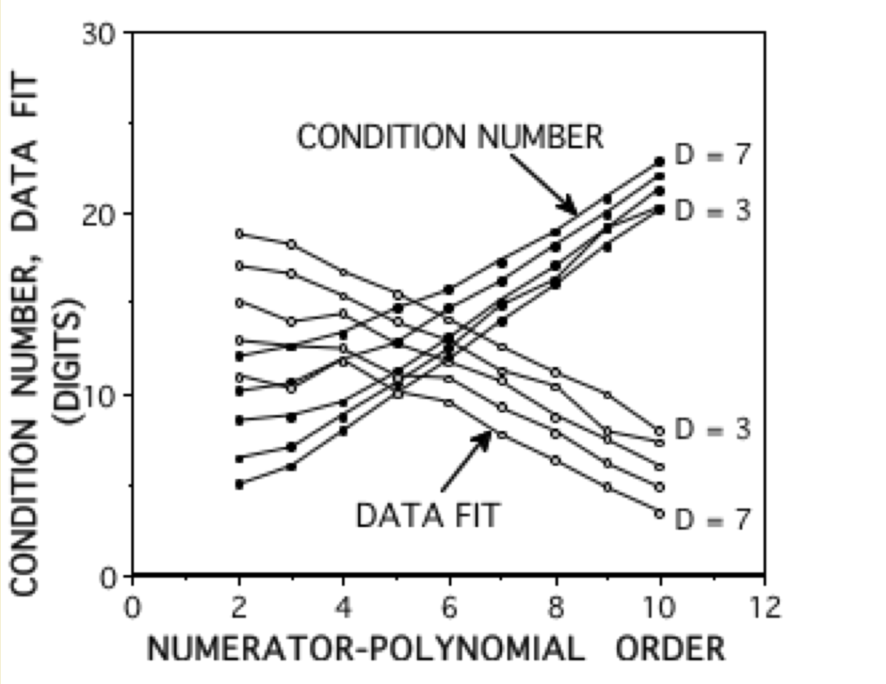
PREVIOUS POLE SPECTRUM DEMONSTRATES FEASIBILITY . . .

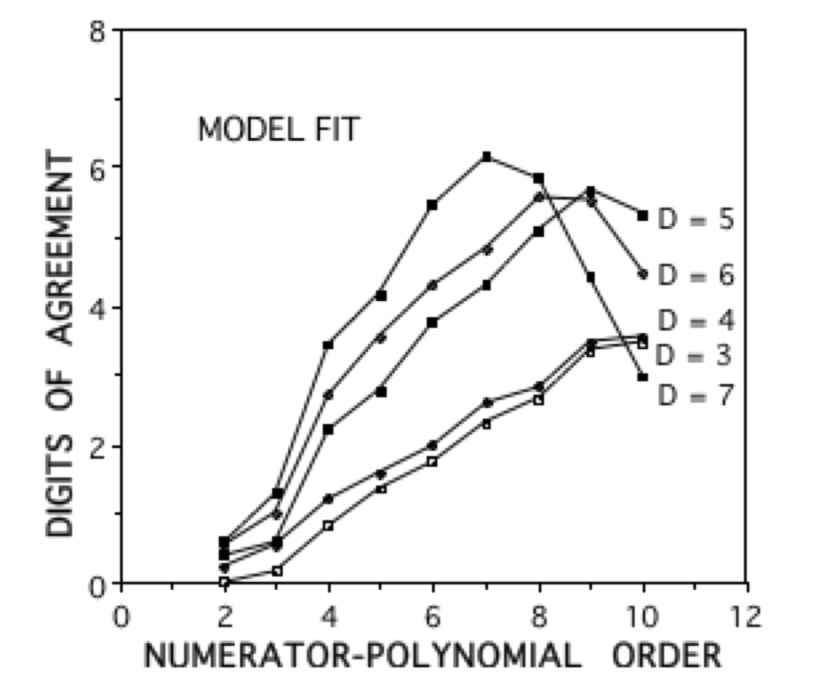


- *FM* HAVING *D* = 7 AND *N* VARYING FROM 6 TO 9
- UNIFORMLY DISTRIBUTED NOISE ADDED TO *GM* SAMPLES
- OPTIMUM *N* FOR THIS CASE IS 7

. . . PRODUCING D_{MF} (VERTICAL AXIS) WITHIN A FEW PERCENT OF THE INPUT-DATA NOISE LEVEL

CURRENT ON INFINITE CIRCULAR CYLINDER PROVIDES EM TEST . . .





--COMPUTE PRECISION (*CP*) IS 24 DIGITS

--*GM* IS CURRENT ON FRONT SIDE OF CYLINDER

--*ka* RANGE OF 3 TO 6 USED FOR $M = N + D + 1$ *GM* SAMPLES

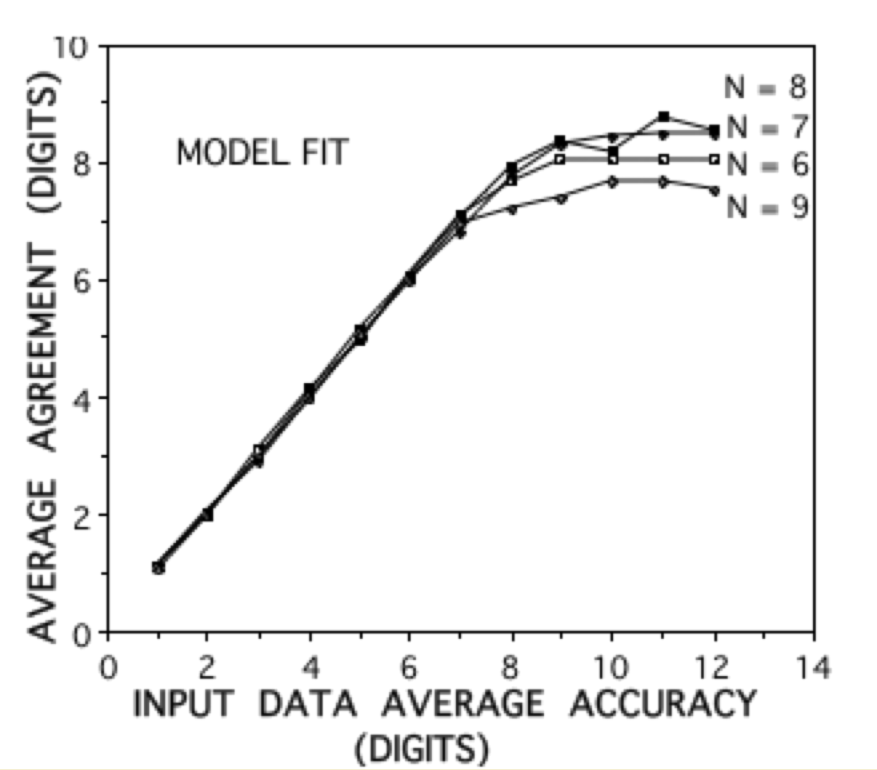
--RESULTS OBTAINED FOR $N = 2$ TO 10 AND $D = 3$ TO 7

--AGAIN OBSERVE THAT $DF + CN \sim CP$

--BEST DETECTABLE UNCERTAINTY IN *GM* IS ABOUT 6.5 DIGITS

. . . COMPARABLE TO SIMULATED SPECRUM

VARYING CONVERGENCE *L* IS ALSO "DETECTED" BY THE *FM* TEST . .



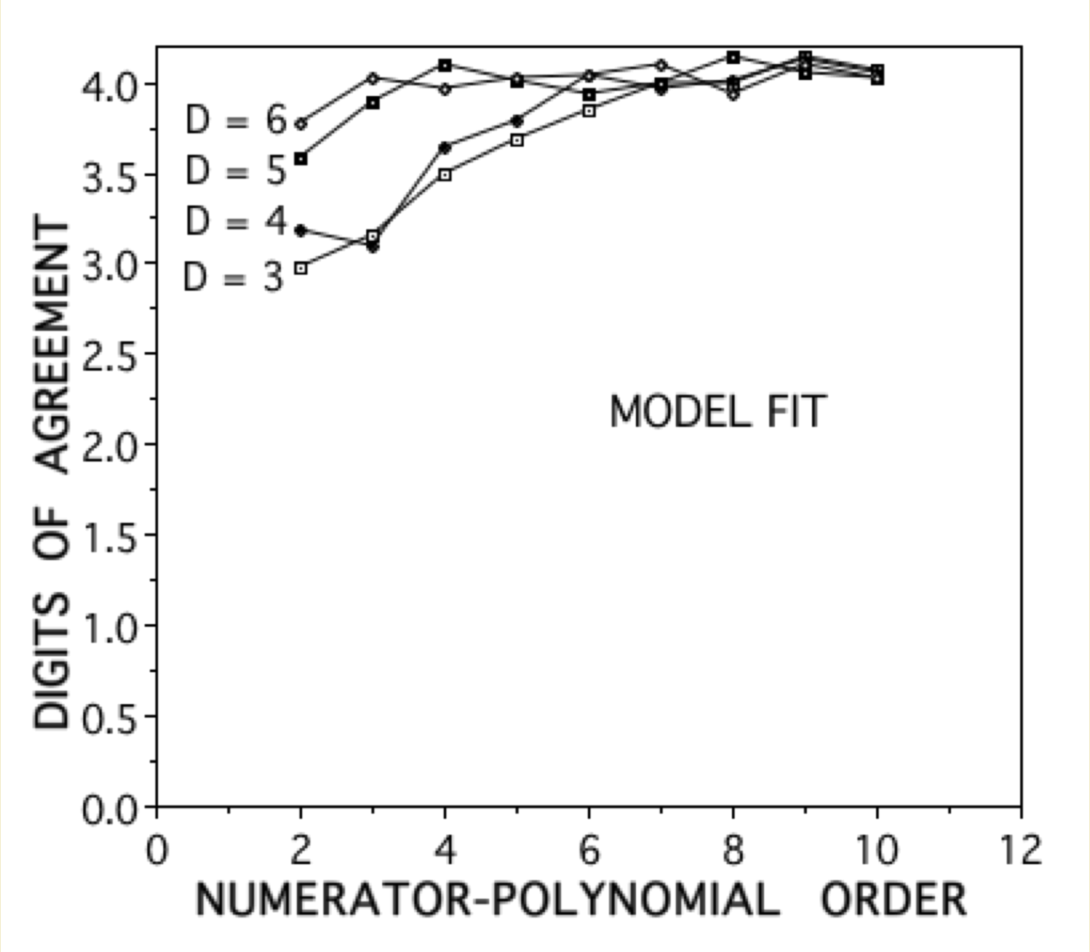
--*FM* HAVING $D = 7$ AND *N* VARYING FROM 6 TO 9

--OPTIMUM *N* FOR THIS CASE IS 7

--D_{MF} (VERTICAL AXIS) AGREES WITH AVERAGE ACCURACY OF INPUT DATA, D_{GM}

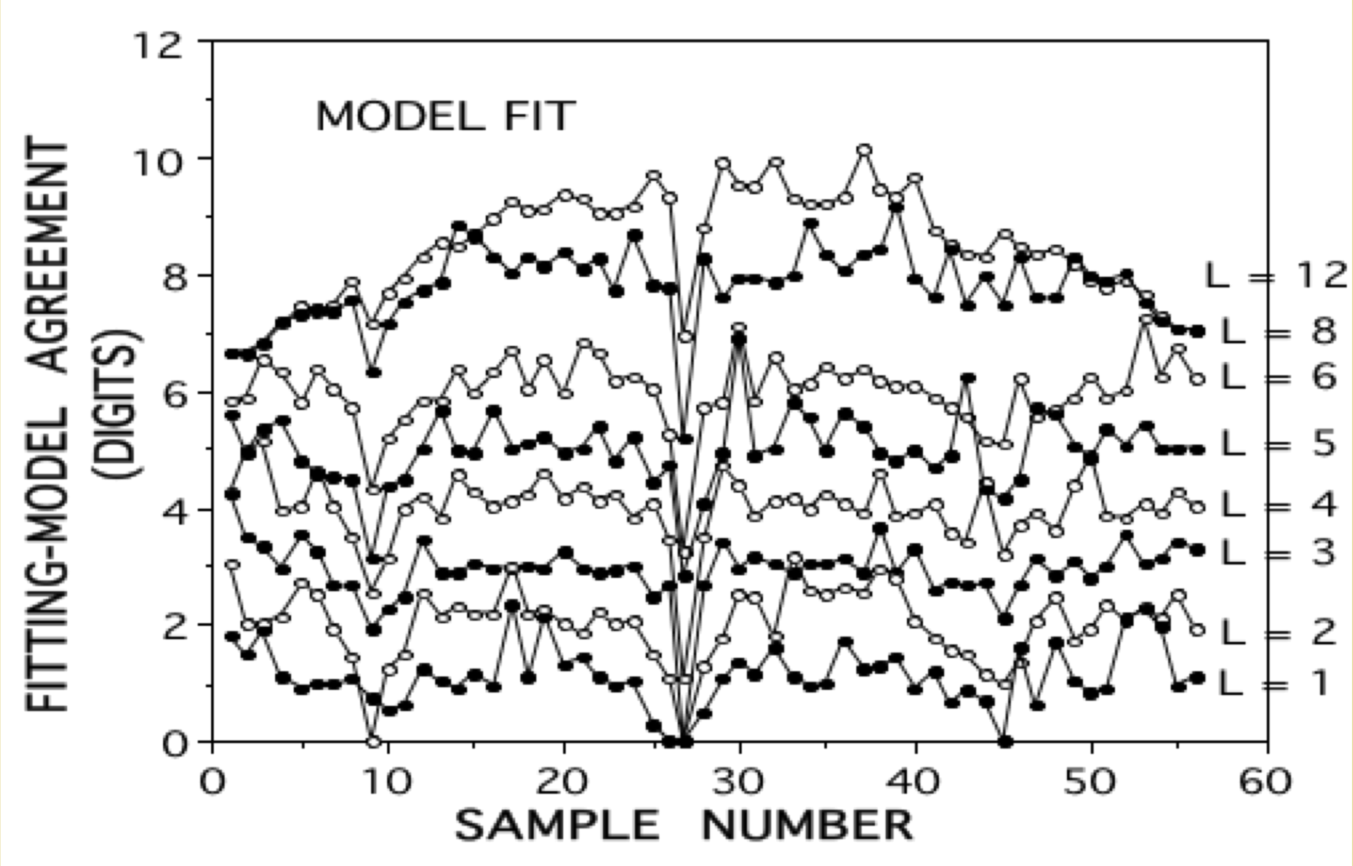
--SERIES SUMMATION STOPPED WHEN TERM i < 10^{-L} OF SUM TO THAT POINT

4-DIGIT *GM* ACCURACY IS CLEARLY INDICATED BY MAXIMUM D_{MF} . . .



. . . WHEN *FM* PARAMETERS ARE SYSTEM-ATICALLY VARIED FOR CYLINDER CURRENT

FREQUENCY DEPENDENCE OF *FM* WITH *L* PROVIDES FINER DETAIL

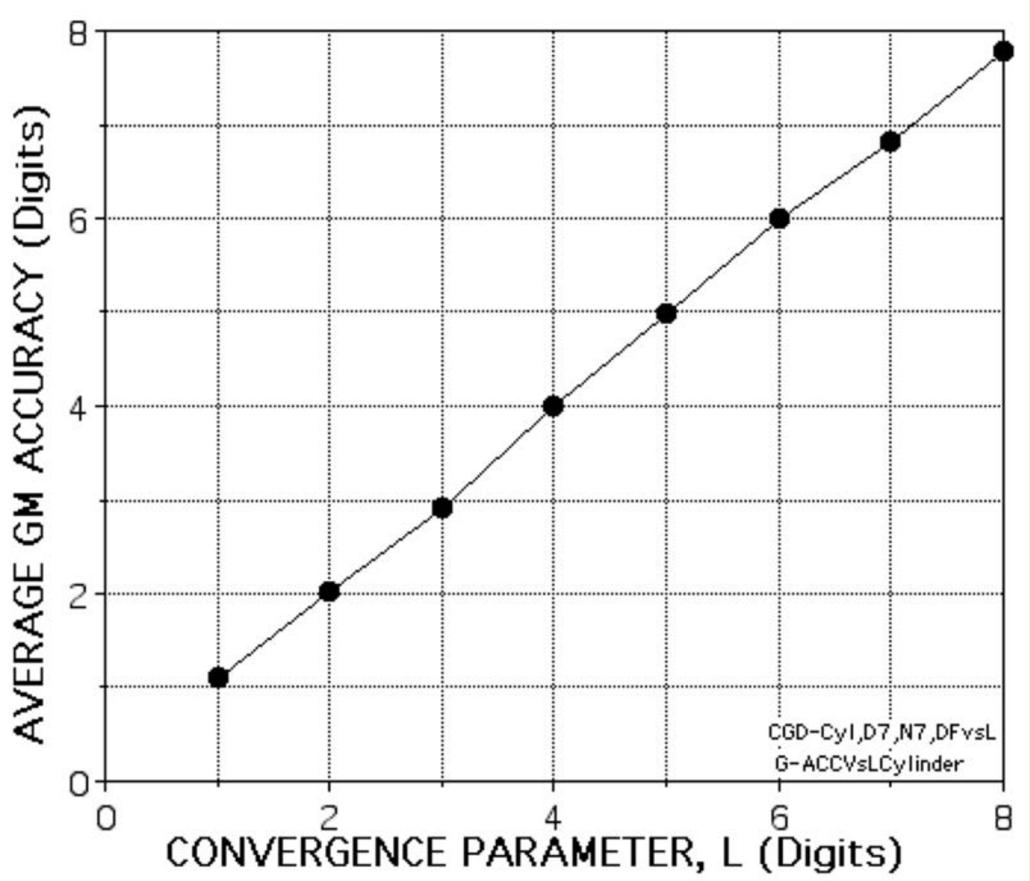


--CURRENT SERIES TERMINATED WHEN $|TERM_i/SUM(i)| < 10^{-L}$

--D_{MF} COMES FROM AVERAGING THESE INDIVIDUAL D_{iMF} VALUES

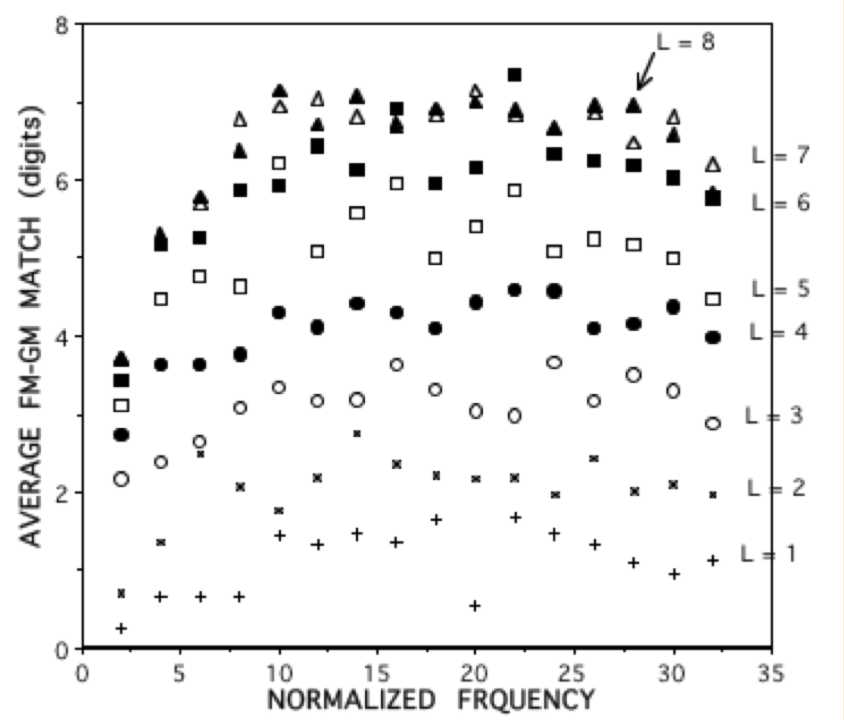
--MAXIMUM DETECTABLE UNCERTAINTY IS SEEN TO BE ABOUT 9 DIGITS

GM ACCURACY FROM *FM* AGREES CLOSELY WITH THE CURRENT-CONVERGENCE TEST



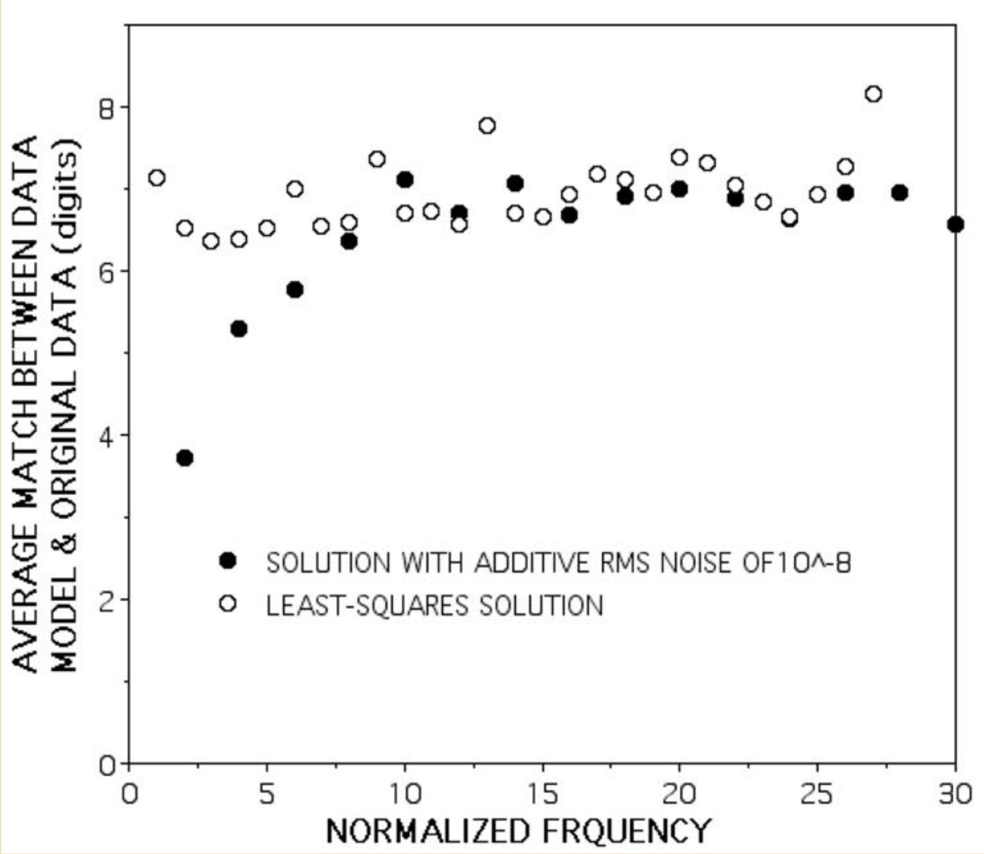
D_{GM} ESSENTIALLY MATCHES CONVERGENCE PARAMETER L

INCREASING LEVEL OF NOISE ADDED TO *GM* ALSO PROVIDES NOISE ESTIMATE



Data fit between *FM* and *GM* samples as a function of normalized frequency for the Kluskens data with additive random noise of maximum value 10^{-L}. The data fit is seen to saturate at about 6.5 digits for $L > 6$ implying the data accuracy exceeds 1 part in 10^{-6}.

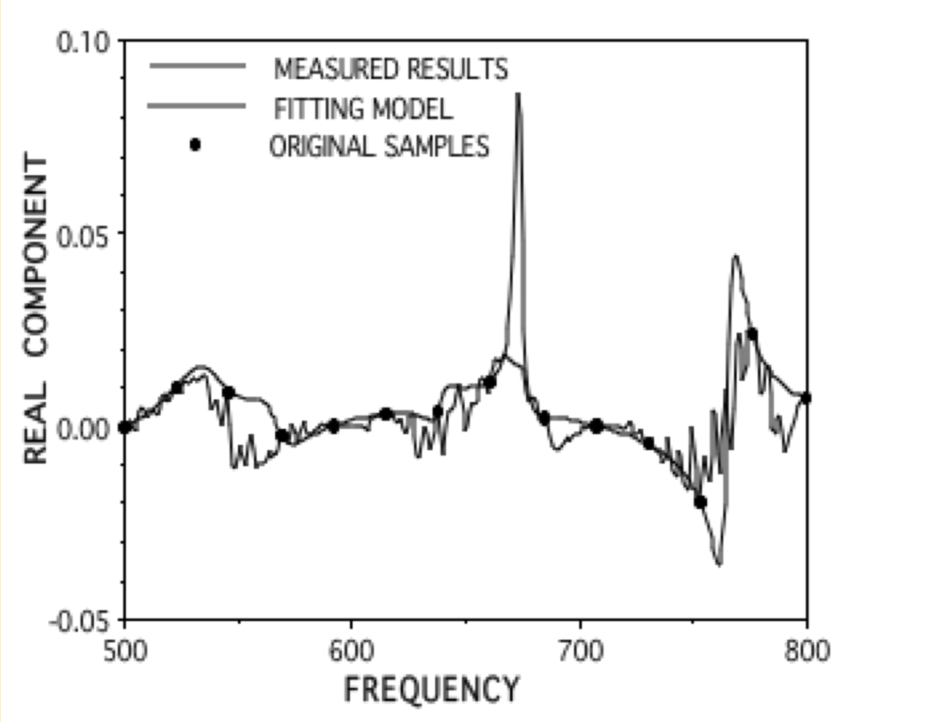
OVER-DETERMINED FM COMPUTATION OFFERS USEFUL ALTERNATIVE . .

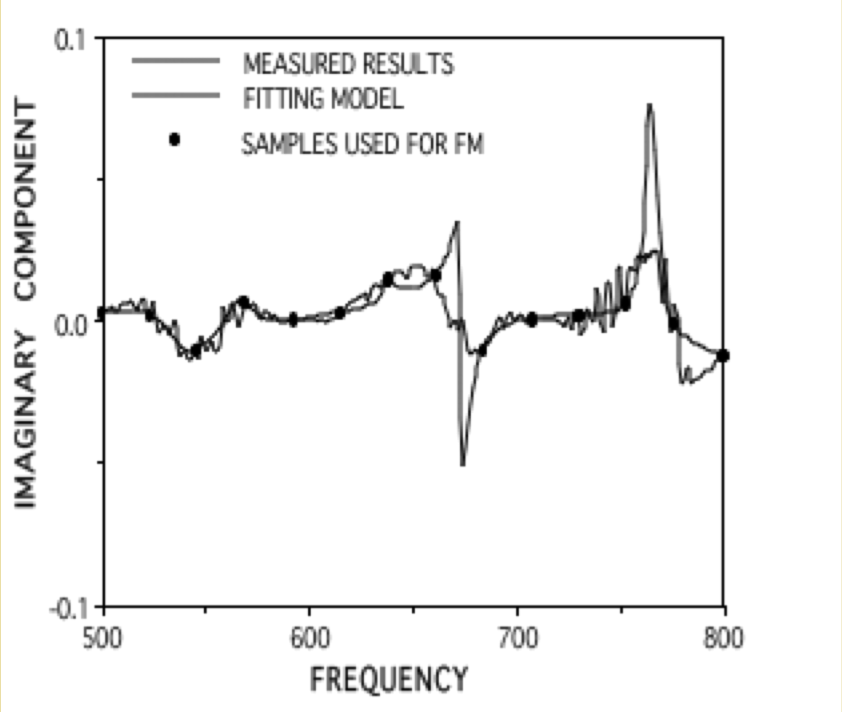


Data fit between the average FM and GM samples as a function of normalized frequency for the Kluskens data. Each computation employed four FMs and 10 extra GM samples with the over-determined data matrix solved using a pseudo inverse.

. . . COMPARABLE TO ADDED NOISE

UNCERTAINTY ESTIMATE OF MEAS-URED DATA DOES NOT LOOK USEFUL:

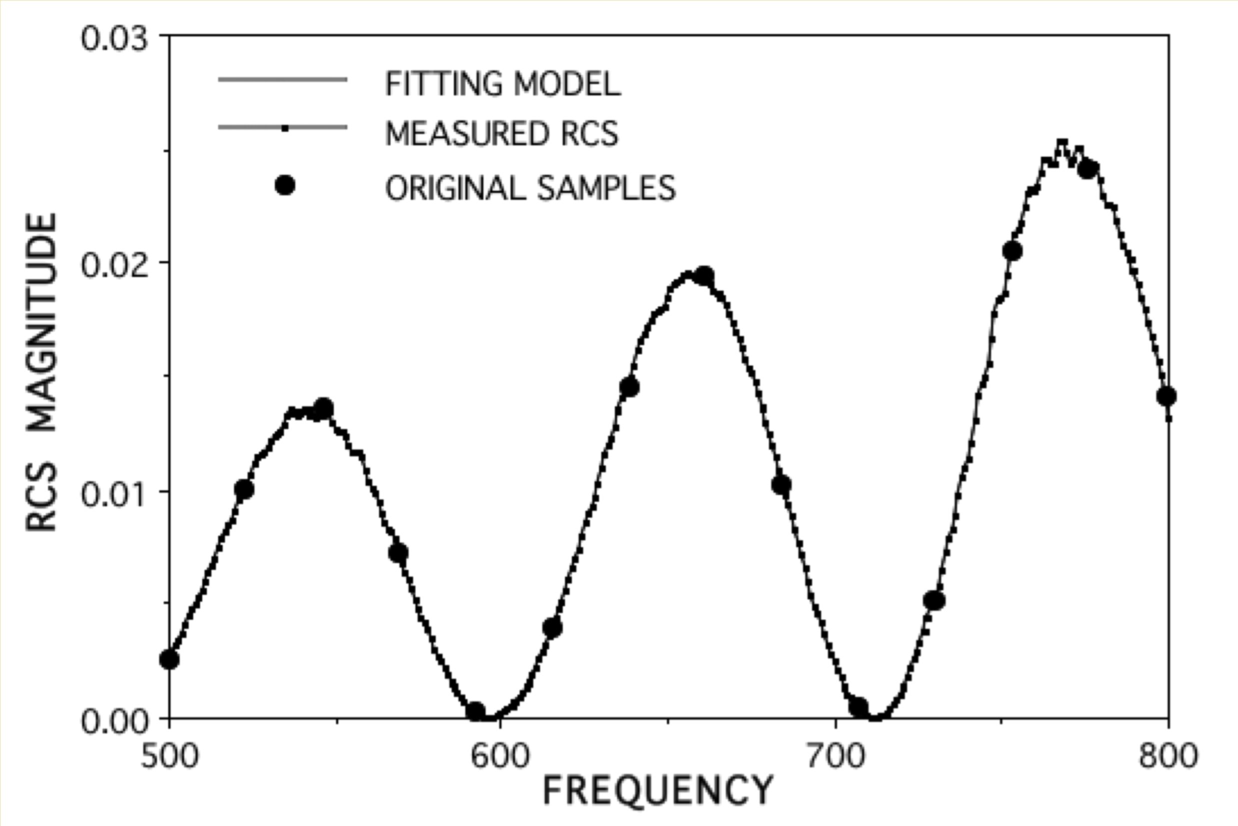




Results obtained from a 14-coefficient complex *FM* with $n = 7$ and $d = 6$ (the dark solid line) using data samples from a PEC cube (shown by the solid circles), with the narrow line showing the original data (from Mishra). Although the FM seems to average the oscillations seen in the experimental data, the overall agreement is poor.

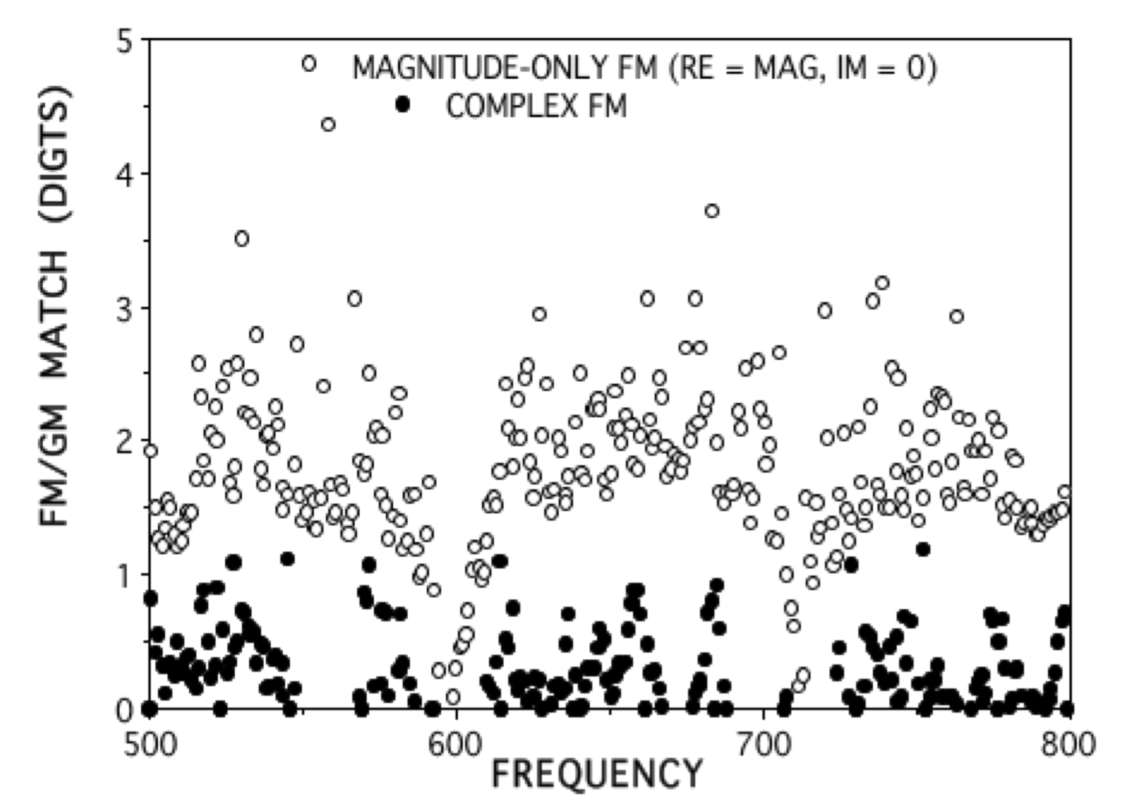
- HERE THE REAL AND IMAGINARY COMPONENTS ARE COMPUTED FROM THE MEASURED MAGNITUDE AND PHASE

. . . WHICH APPEARS MUCH IMPROVED BY USING MAGNITUDE DATA ONLY . . .



Repeat of of the cube RCS but using instead the magnitude of the measured data (from Mishra) as the input. The data fit is much better here than when using complex input data.

. . . A RESULT CONFIRMED BY COMPARING THEIR *FM-GM* MATCHES

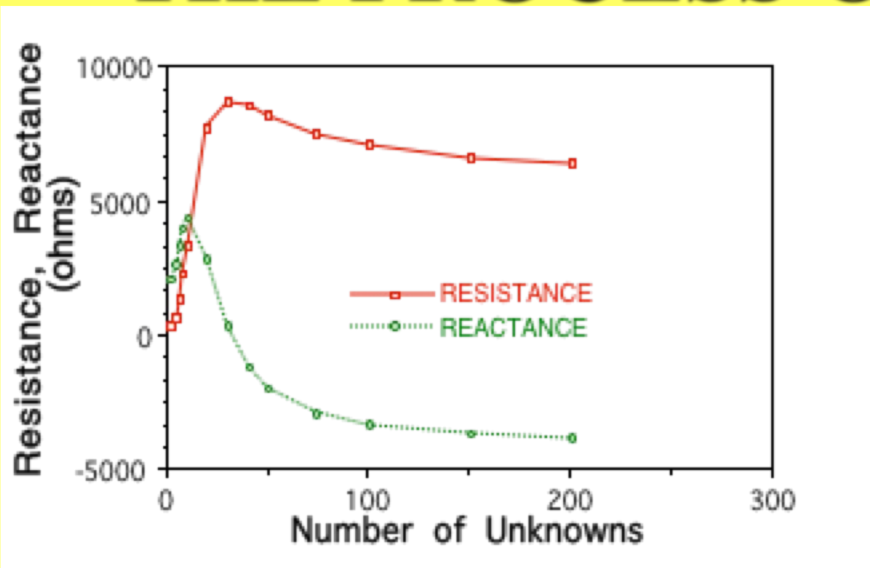


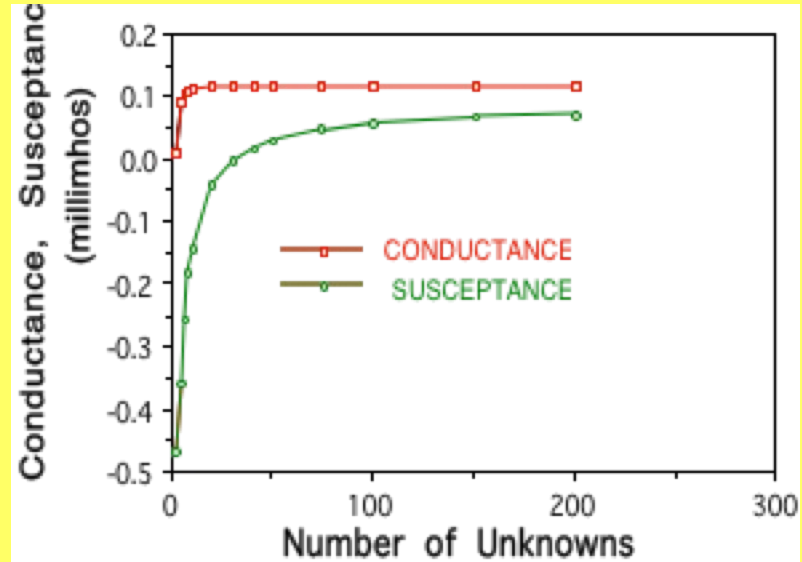
Match in digits obtained using the complex *FM* (solid symbols) and magnitude-only *FM* (open symbols) for the Mishra cube *RCS*. Average agreement for the magnitude-only *FM* in is about 2 digits, with minima occurring in the vicinity of minima in the *RCS*.

PRESENTATION EXPLORES SOME ISSUES IN ESTIMATING & REPRESENTING EM OBSERVABLES

1) THE SCIENTIFIC METHOD
2) MODEL-BASED PARAMETER ESTIMATION
3) FITTING MODELS FOR WAVEFORM AND SPECTRAL DATA
4) FUNCTION SAMPLING AND DERIVATIVE SAMPLING
5) ADAPTIVE SAMPLING OF FREQUENCY SPECTRA
6) ADAPTIVE SAMPLING OF RADIATION AND SCATTERING PATTERNS
7) USING MBPE TO ESTIMATE MODEL UNCERTAINTY
8) **OTHER FITTING MODELS FOR EM OBSERVABLES**
9) USING MBPE FOR GENERATING-MODEL COMPUTATION

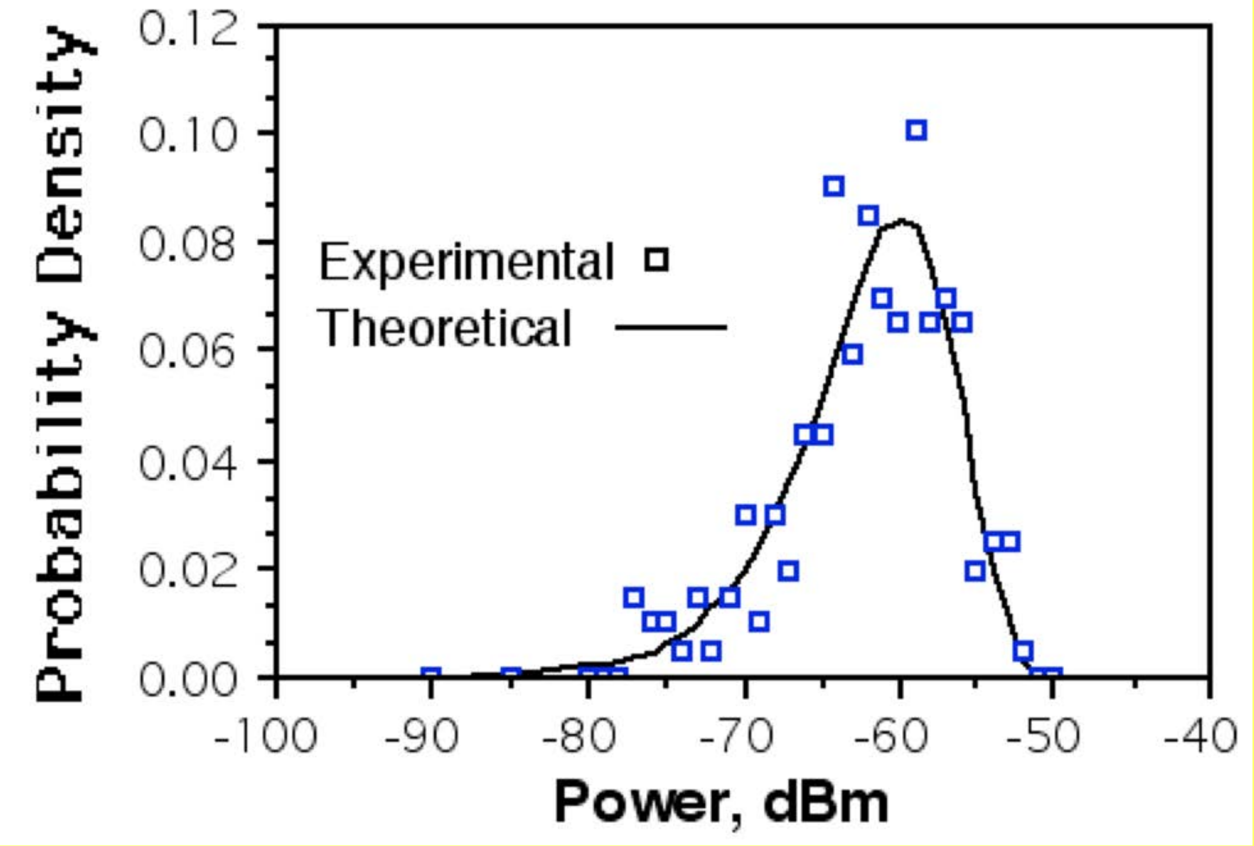
MBPE CAN EMPLOY ANY MODEL THAT REASONABLY REPRESENTS THE PROCESS OF INTEREST . . .





Results for integral-equation model (NEC) of the input impedance (left plot) and input admittance (right plot) as a function of the number of unknowns, X_s, used in the model for a center-fed, two-wavelength dipole. The impedance has not satisfactorily converged over the range shown which the admittance results demonstrate is due to a susceptance variation. In this case, the exciting source was a tangential field applied to the center segment of the antenna, whose decreasing length simulates a changing antenna source gap given by $B(X_s) \approx A_1 + A_2/[Ln(L/X_s) + A_3]$. The susceptance curve exhibits the kind of variation expected from changing the gap size. This problem can be avoided by increasing the number of source segments in proportion to X_s to maintain a fixed gap width.

STATISTICAL DATA CAN ALSO BE USED IN MBPE



Probability densities and cumulative distributions for the power coupled to a shielded cable in a mode-stirred chamber measured by Kaman Sciences [Smith (1990)] compared with Lehman's theory. This result is typical of the comparisons that Lehman's theory has produced with experimental data obtained in such experiments. Note that the only "floating" parameter needed to complete this comparison is knowledge of the experimental power density. [After Lehman (1993)].*

PRESENTATION EXPLORES SOME ISSUES IN ESTIMATING & REPRESENTING EM OBSERVABLES

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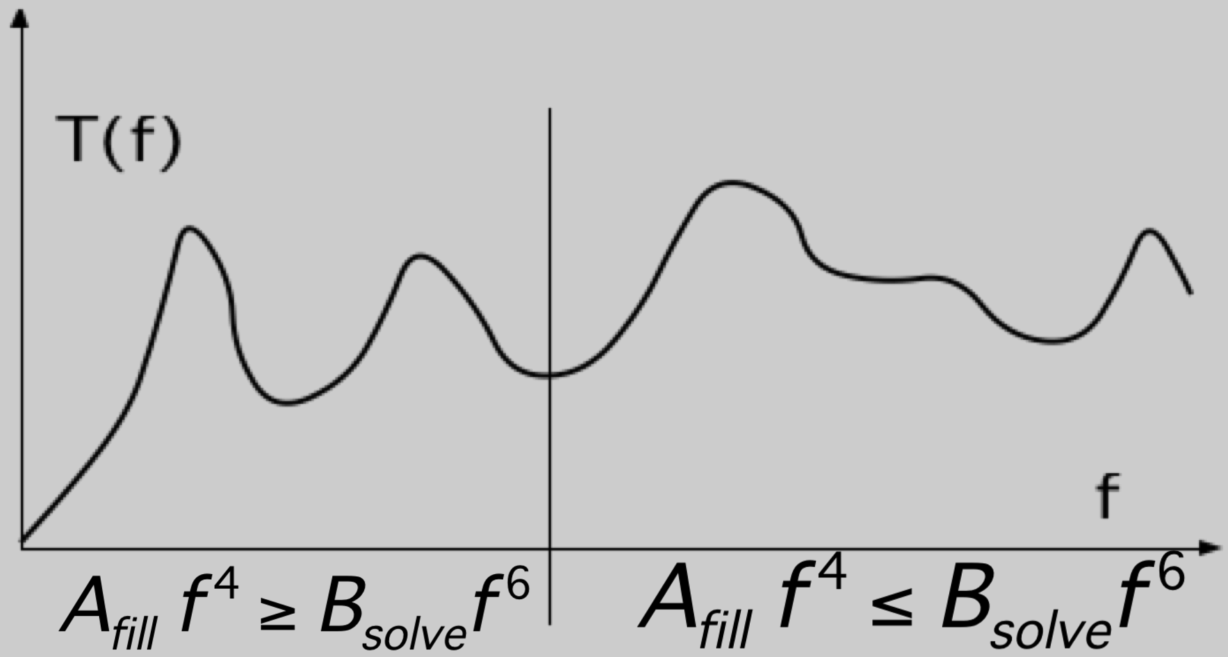
MBPE CAN IMPROVE FIRST-PRINCIPLES MODELS EFFICIENCY . . .



Formulation-domain modeling: reduce complexity of coefficient computation and minimize number of matrix-fill frequencies.

Solution-domain modeling: reduce cost of matrix solution and minimize number of matrix-solution frequencies.

. . . BY REDUCING MATRIX-FILL AND SOLUTION TIMES

MBPE CAN BE DONE IN EITHER THE FORMULATION DOMAIN . . .

$$Z_{m,n}(\omega) = \int_{\Delta_n} S_n(\omega) K_{R,m,n}(\omega) K_{\Delta,m,n}(\omega) d\Delta_n$$

. . . MODELING THE IMPEDANCE MATRIX OR THE SOLUTION DOMAIN . .

$$Y_{m,n}(\omega) = \left[Z_{m,n}(\omega) \right]^{-1}$$

. . . WHERE THE ADMITTANCE MATRIX IS MODELED, BOTH AS A FUNCTION OF:

- SPACE
- FREQUENCY
- OTHER PROBLEM PARAMETERS

SPATIAL MODELING OF $Z_{m,n}$ REQUIRES DECOMPOSITION INTO TWO PARTS . . .

- $K_{R,m,n}$ IS THE PATCH-TO-PATCH (OR P-P) PART OF THE IE KERNEL, OF THE FORM

$$K_{R,m,n}(\omega) = \frac{e^{jkr_{m,n}}}{r_{m,n}}$$

- $K_{\Delta,m,n}$ IS THE IN-PATCH (OR I-P) PART OF THE KERNEL

. . . SO THAT $Z_{m,n}$ CAN BE WRITTEN AS--

$$Z_{m,n}(\omega) = e^{jkR_{m,n}} \int_{\Delta_n} S_n(\omega) K'_{\Delta,m,n}(\omega)\, d\Delta_n$$

- WHERE $R_{m,n} + \Delta r_{m,n} = r_{m,n}$ and $K'_{\Delta,m,n}$ IS A MODIFIED SLOW-VARIATION KERNEL

THIS FORM OF $Z_{m,n}$ SHOWS THAT AN APPROPRIATE MODEL FOR IT MIGHT BE:

$$Z_{m,n}(\omega_2) \approx Z_{m,n}(\omega_1) e^{jk(\omega_2-\omega_1)R_{mn}} M_{m,n}(\omega_2-\omega_1)$$

- WHERE $M_{m,n}(\omega_2\text{-}\omega_1)$ IS AN INTERPOLATION MODEL THAT ACCOUNTS FOR THE SLOWLY VARYING PART OF THE KERNEL

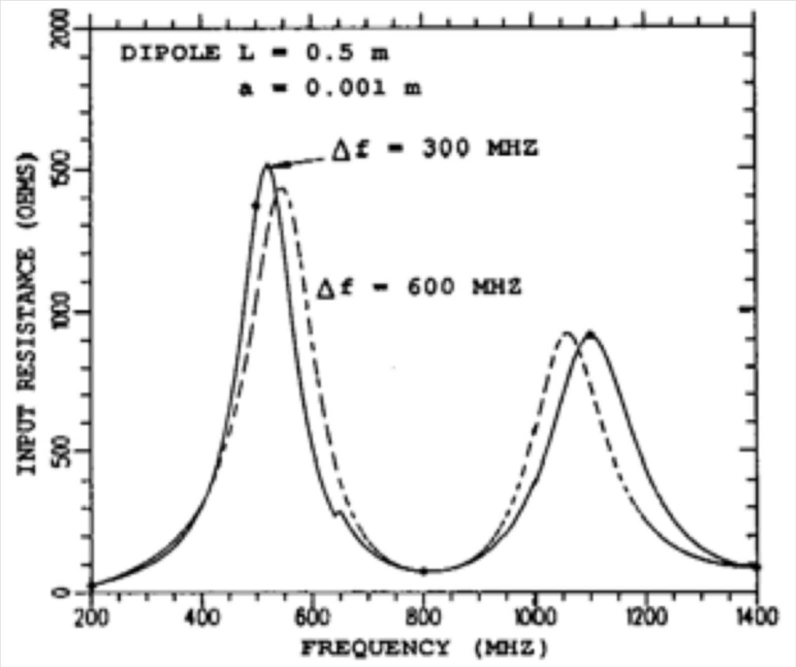
- NEWMAN (1988) USED

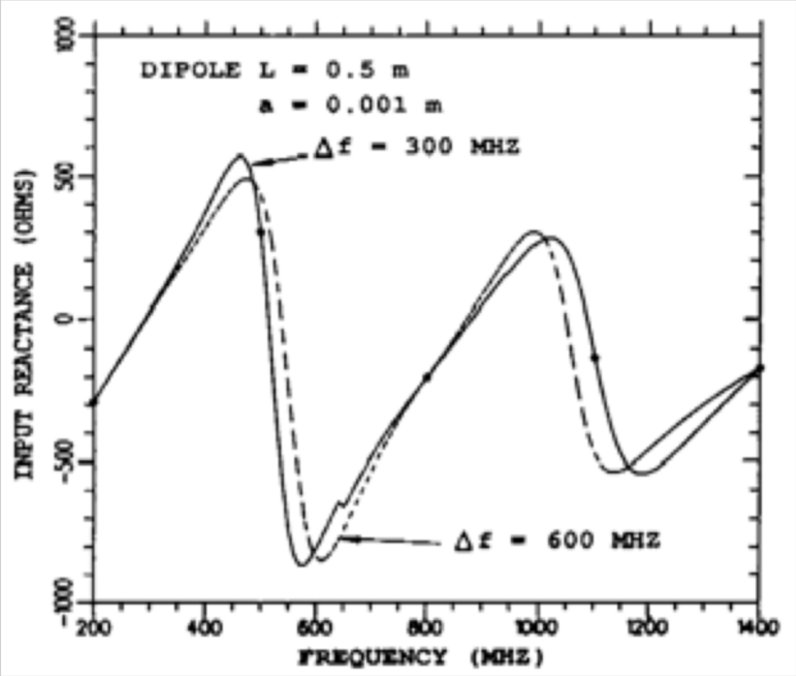
$$M_{m,n}(\omega_2-\omega_1)_{imag} = A_i + B_i \ln(\omega_2-\omega_1) + C_i(\omega_2-\omega_1)$$

$$M_{m,n}(\omega_2-\omega_1)_{real} = A_r + B_r(\omega_2-\omega_1) + Cr(\omega_2-\omega_1)^2$$

Newman, E. H. (1988), "Generation of Wide band Data From the Method of Moments by Interpolating the Impedance Matrix," *IEEE Trans. Antennas Propagat.*, Vol. 36, No. 12, pp. 1820-1824.

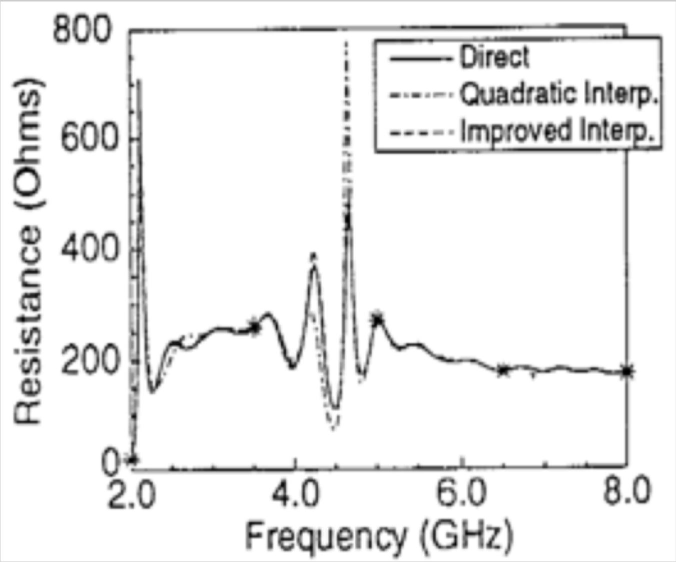
FREQUENCIES AT WHICH $Z_{m,n}$ MUST BE COMPUTED FROM FIRST PRINCIPLES CAN BE MUCH REDUCED USING MBPE

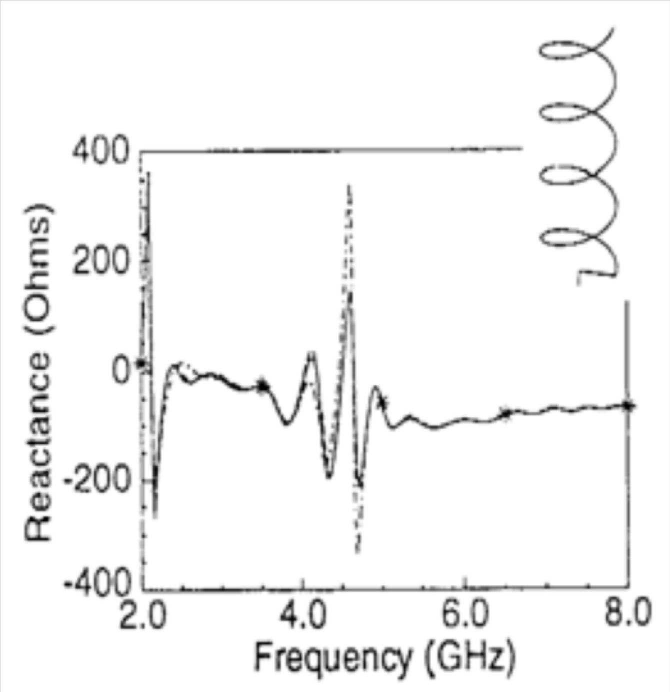




Results from using MBPE and two different *FM*s to represent the interaction coefficients of the $\underline{Z}$ matrix of a center-fed, half-wave dipole antenna [Newman (1988)]. The *FM*s employ the previous equations and are based on *GM* samples spaced 300 MHz apart (the solid line) and 600 MHz apart (the dashed line). The discontinuity in the impedance curves occurs at the point where a *FM* replaces a *GM* sample at one end of its span with a new one at the other as successive *GM* samples are employed.

A SIMILAR RESULT IS OBTAINED FOR A HELICAL ANTENNA

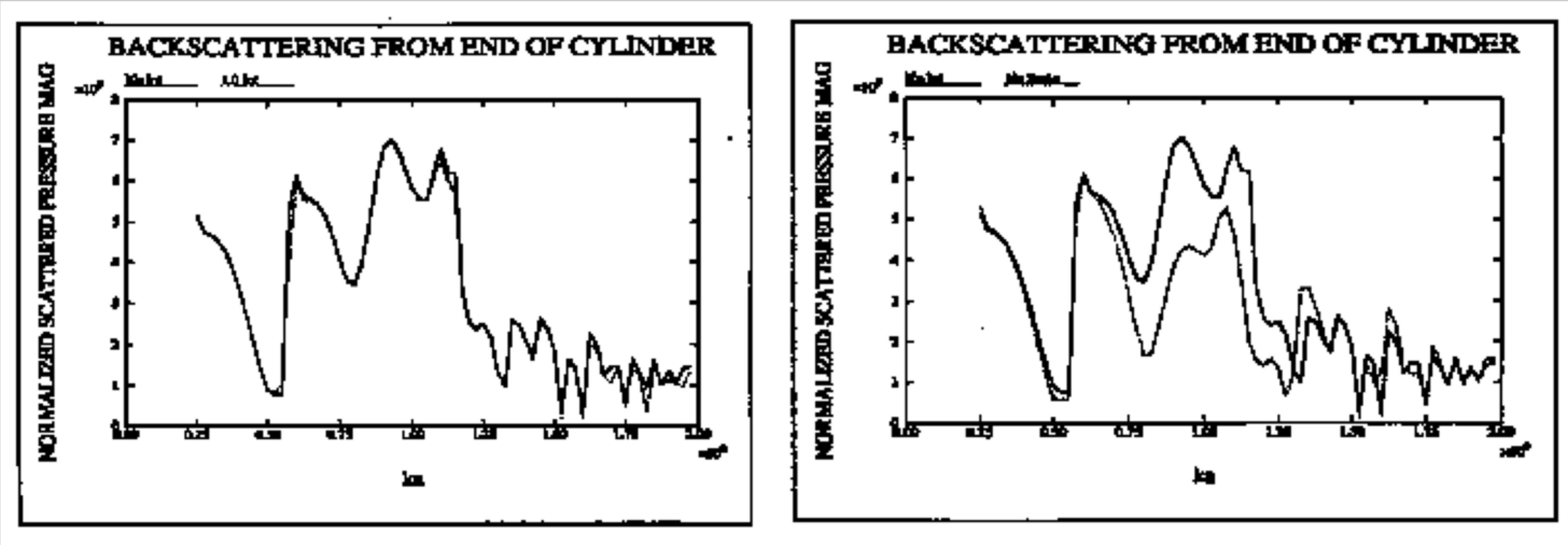




Results from using MBPE and two different *FM*s of the interaction coefficients of the $\underline{Z}$ matrix for a helical antenna. Values obtained from straight-line interpolation of 301 GM samples (solid line) are compared with a quadratic *FM* (dot-dash line) and a FM defined by previous equations (dashed line) [Virga and Rahmat-Samii (1995)]. The GM samples used for the *FM* results are indicated by the starred points.

Virga, K. and Y. Rahmat-Samii (1995), "Wide-Band Evaluation of Communications Antennas Using [Z] Matrix Interpolation with the Method of Moments," in *IEEE Antennas and Propagation Society International Symposium*, Newport Beach, CA, pp. 1262-1265.

ELASTODYNAMIC SCATTERING PROVIDES ANOTHER EXAMPLE OF MBPE

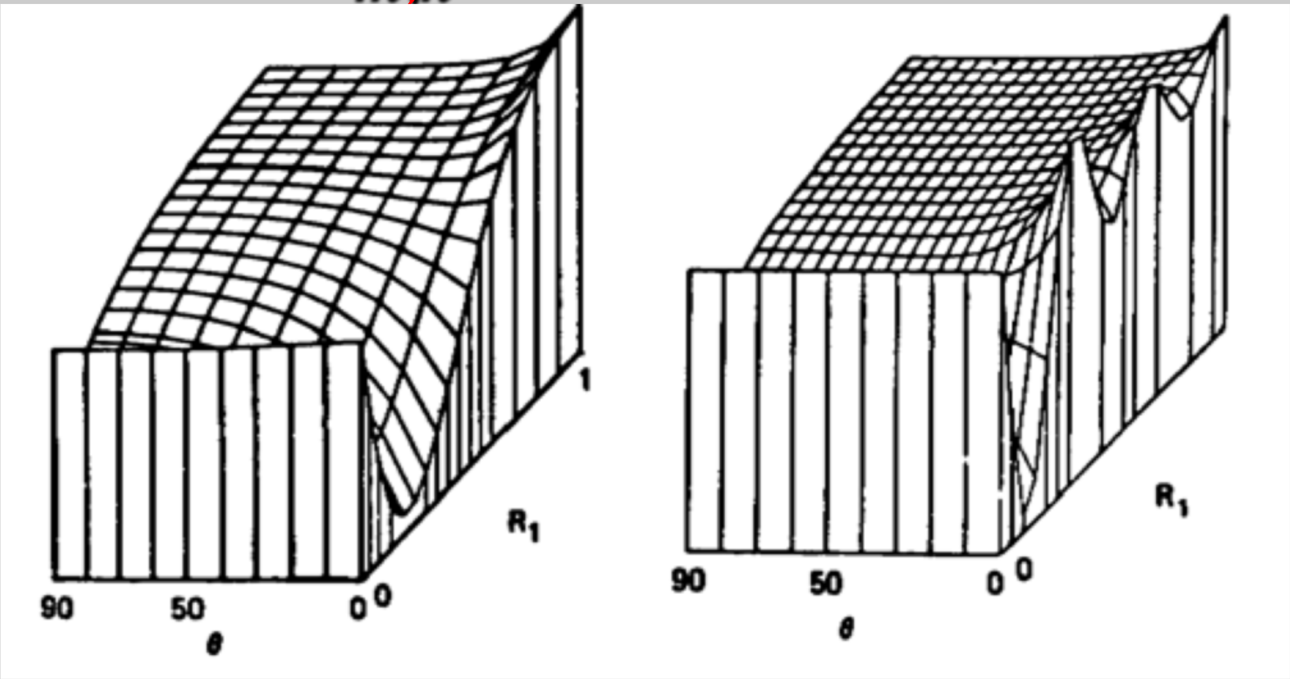


(a) (b)

Results for acoustic backscattering from the end of a circular cylinder as a function of frequency obtained from the basic model without interpolation (solid line) and using MBPE on samples spaced 1.0 unit apart in *ka* (dotted line) [Benthien and Schenck (1991)]. A linear interpolation model is used for $M_{m,n}(\omega_2 - \omega_1)$ to obtain the estimated interaction coefficients with $\exp(ikR_{m,n})/c$ explicitly factored out in (a) and implicitly included in $M_{m,n}(\omega_2 - \omega_1)$ in (b)

Benthien, G. W. and H. A. Schenck (1991), “Structural-Acoustic Coupling,” in *Boundary Element Methods in Acoustics*, ed. R. D. Ciskowski and C. A. Brebbia, Computational Mechanics Publications.

SPATIAL MBPE CAN ALSO BE USED TO REDUCE $Z_{m,n}$ COMPUTATION TIME



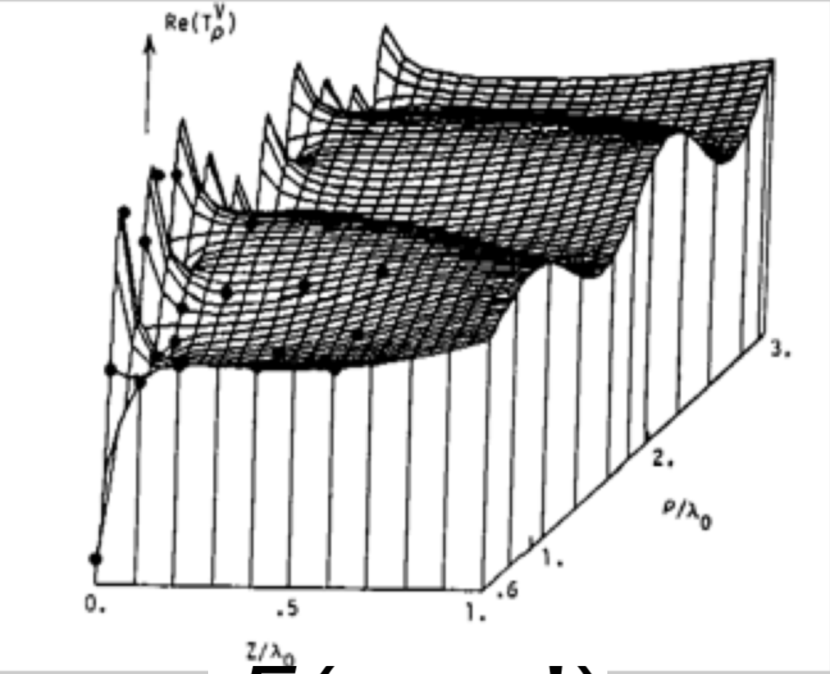
. . . AS SHOWN BY THESE PLOTS OF THE SOMMERFELD FIELDS FOR THE ONE-SIDED INTERFACE PROBLEM

- IT IS ESSENTIALLY THE GREEN'S FUNCTION THAT IS BEING MODELED

MBPE IS EFFECTIVE FOR THE CROSS-INTERFACE PROBLEM . . .

$$E(\rho,z,z') \approx \sum_{n=1}^{N} A_n f_n(\rho,z,z')$$

- THE $f_n(\rho,z,z')$ ARE BASED ON ASYMPTOTIC APPROXIMATIONS TO THE RIGOROUS INTEGRALS SHOWN BY THE POINTS

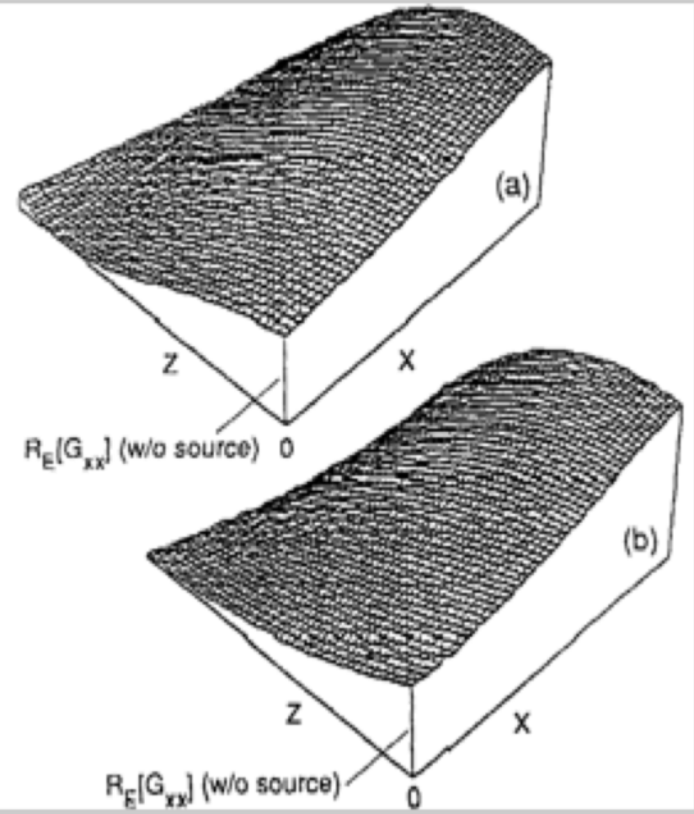


$$E(\rho,z,z')$$

. . . WHERE THE SOMMERFELD FIELDS ARE SAMPLED TO QUANTIFY A PHYSICALLY BASED *FM*

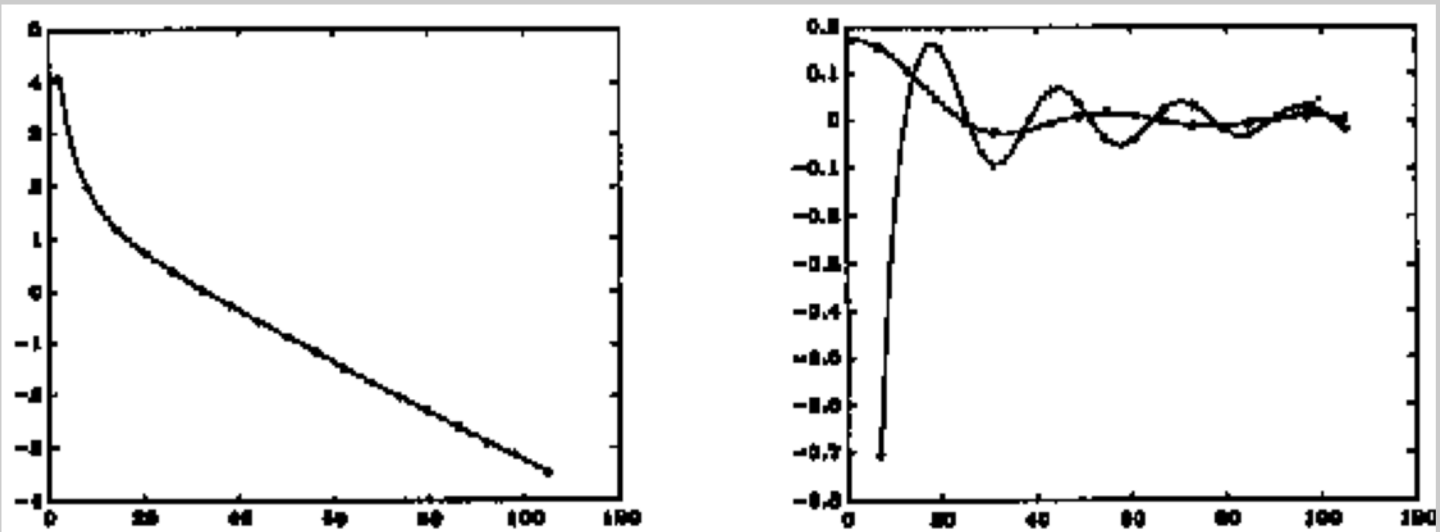
GEOMETRY-DEPENDENT GREEN'S FUNCTIONS ARE ALSO CANDIDATES FOR SPATIAL MBPE

Results for the G_{xx} component of the dyadic Green's function (x-directed source perpendicular to waveguide walls, and x-directed field) as obtained from direct evaluation of the defining equation, (a), and as evaluated using a *FM* consisting of two multiplied 3rd-order polynomials in *x* and *z*, (b) [Demarest et al. (1989)]. The current on a dipole antenna located midway between the waveguide walls obtained from using (b) are within 5% of those resulting from (a).



Demarest, K. R., E. K. Miller, K. Kalbasi, and L-K Wu (1989), "A Computationally Efficient Method of Evaluating Green's Functions for 1-, 2-, and 3-D Enclosures", *IEEE Transactions on Magnetics*, **25**(4), pp. 2878-2880.

SPATIAL VARIATIONS OF A MICRO-STRIP GREEN'S FUNCTION CAN BE VIEWED AS A GENERALIZED SIGNAL . .



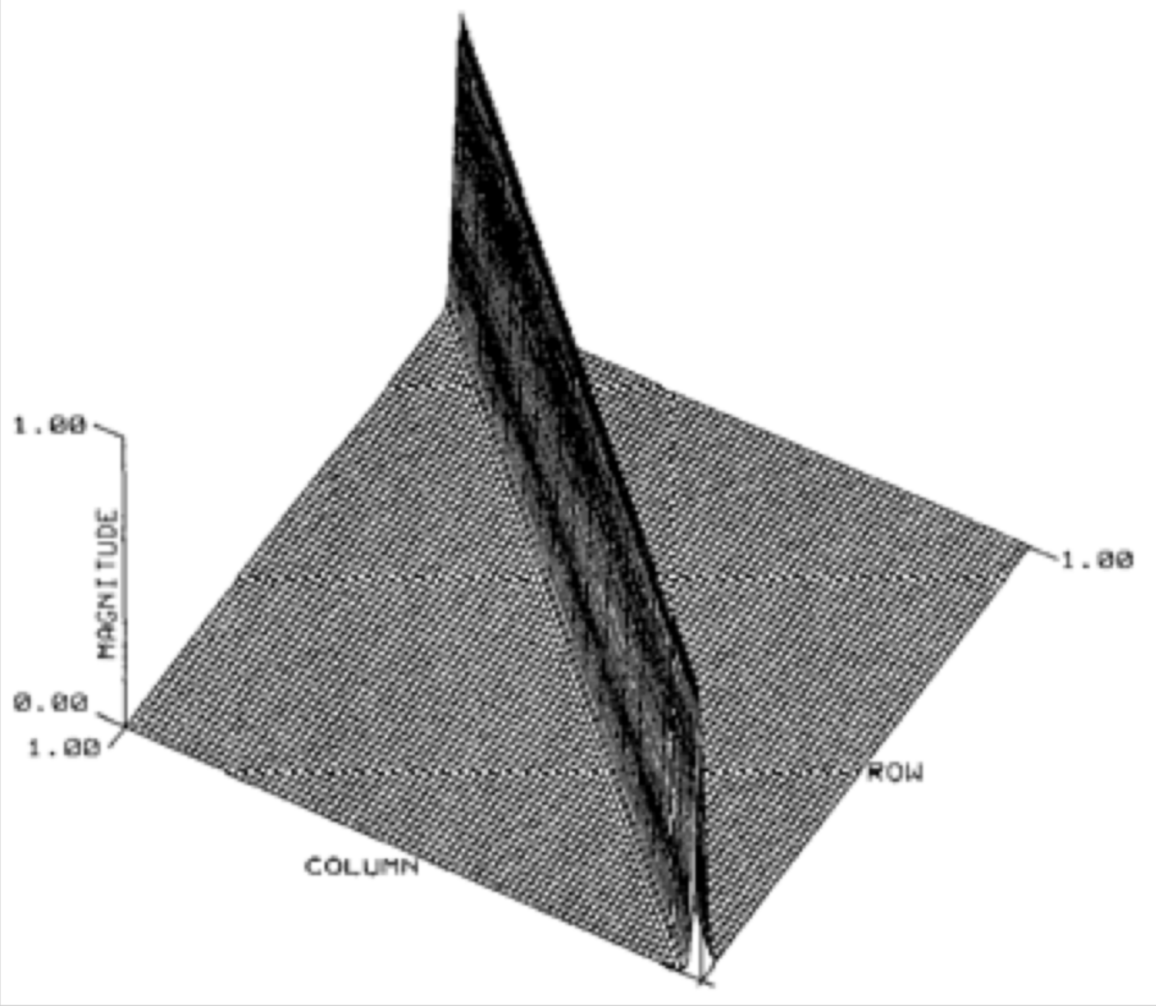
Example of modeling the spatial variation of an interaction coefficients for an integral-equation system matrix of a microstrip line with a coupled dipole using a Galerkin subdomain [Vecchi et al. (1994)]. Result on left is log10 of the static, singular part of the self-impedance matrix for the line as a function of the difference between the source and observation subdomain indices. The +'s show the points where the interactions are sampled for the FM, the solid line is the exact result and the dashed line is the FM result. The FM in this case is a polynomial applied to Z_s[log(q)]. The results on right are obtained for the frequency-dependent part of the interaction coefficient, where the solid line is the exact result, the dashed line is *FM* approximation, again using a polynomial, and the o's and +'s indicate the real and imaginary samples used for evaluating the *FM*s.

. . . AND MODELED ACCORDINGLY

Vecchi, G., P. Pirinoli, L. Matekovits and M. Orefice (1993), "Singular Value Decomposition in Resonant Printed Antenna Analysis," in *Proceedings of Joint 3rd International Conference of Electromagnetics in Aerospace Applications*, pp. 343-346.

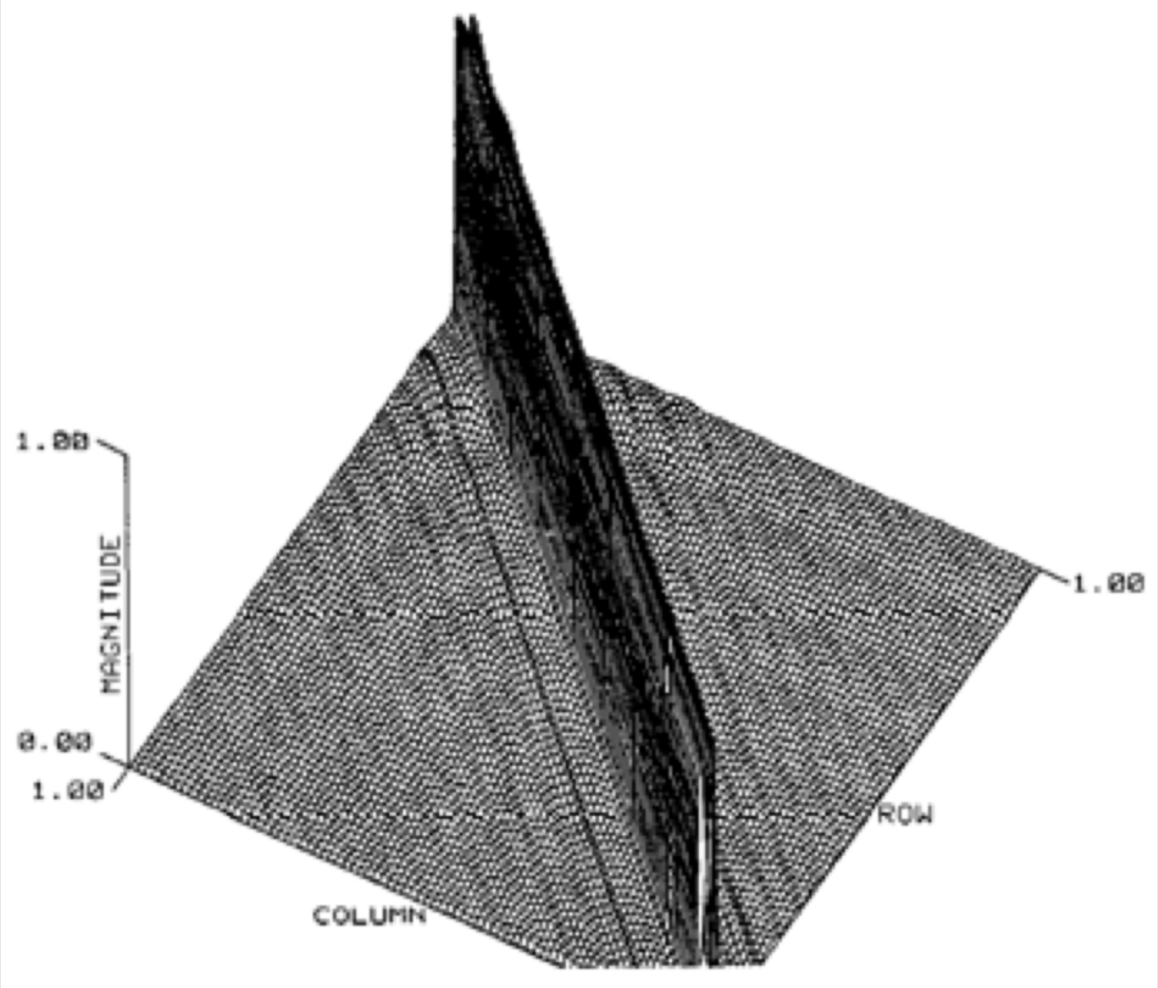
POSSIBLE MODELING OF $Z_{m,n}$ FOLLOWS FROM KNOWN EXPONENTIAL FORM OF GREEN'S FUNCTION . . .

Surface plot of the magnitude of the impedance-matrix coefficients of a straight wire 2 wavelengths long. . .



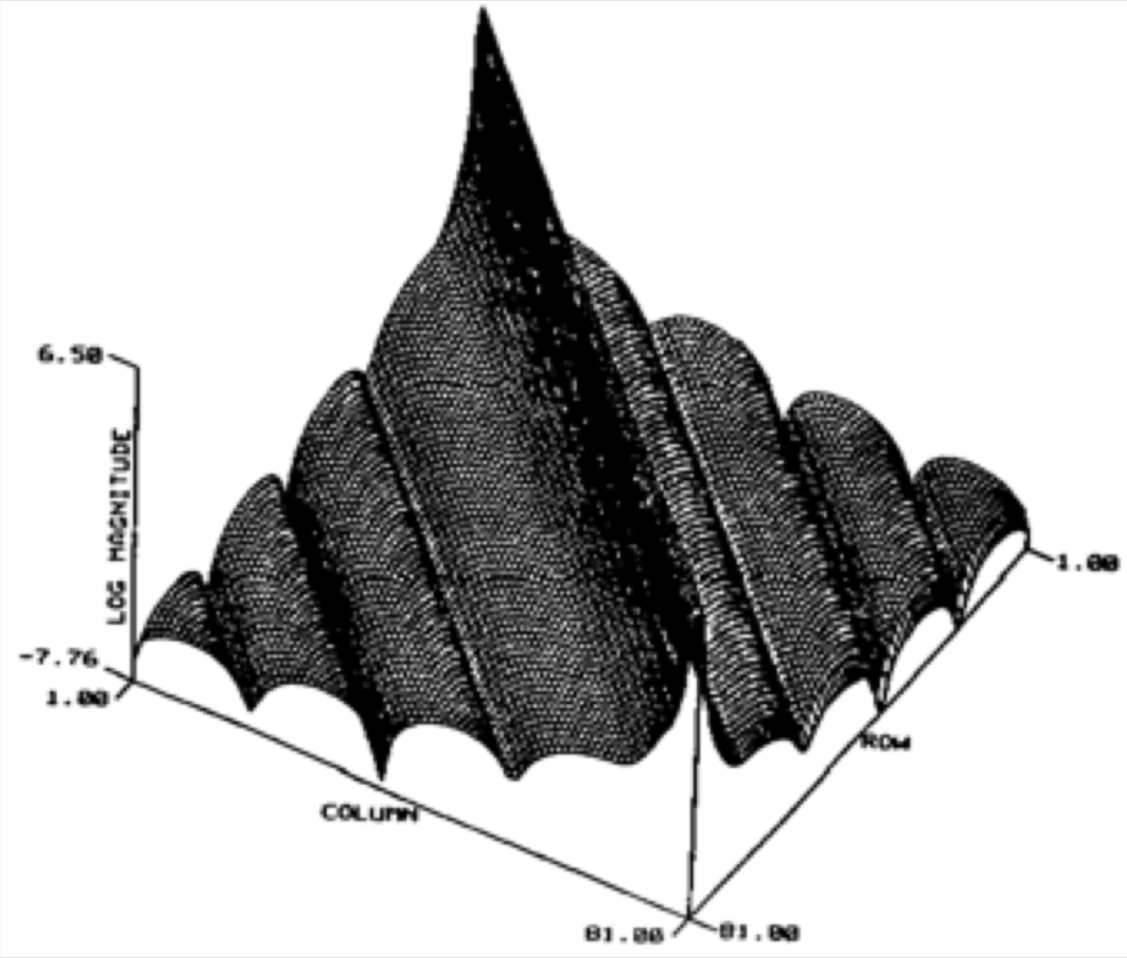
POSSIBLE MODELING OF $Z_{m,n}$ FOLLOWS FROM KNOWN EXPONENTIAL FORM OF GREEN'S FUNCTION . . .

Surface plot of the magnitude of the impedance-matrix coefficients of an 8-turn helical spiral 16 wavelengths long.



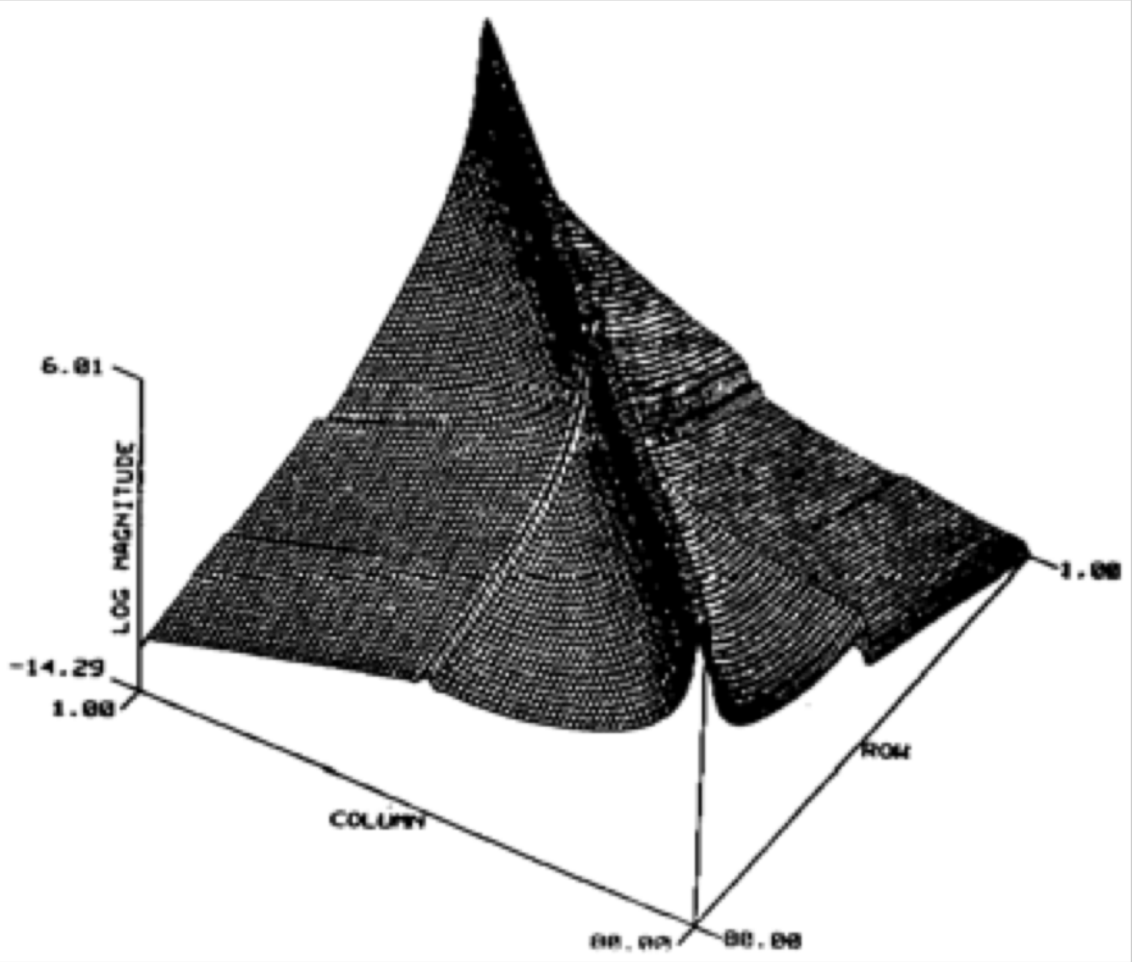
LOG OF $|Z_{m,n}|$ CONVEYS MORE INFORMATION ABOUT DISTANT INTERACTIONS . . .

Surface plot of $|Z_{m,n}|$ coefficients of a straight wire 2 wavelengths long in free space when parallel to and 10^{-4} wavelengths above an air-ground ($\varepsilon_r = 10$) interface . . .



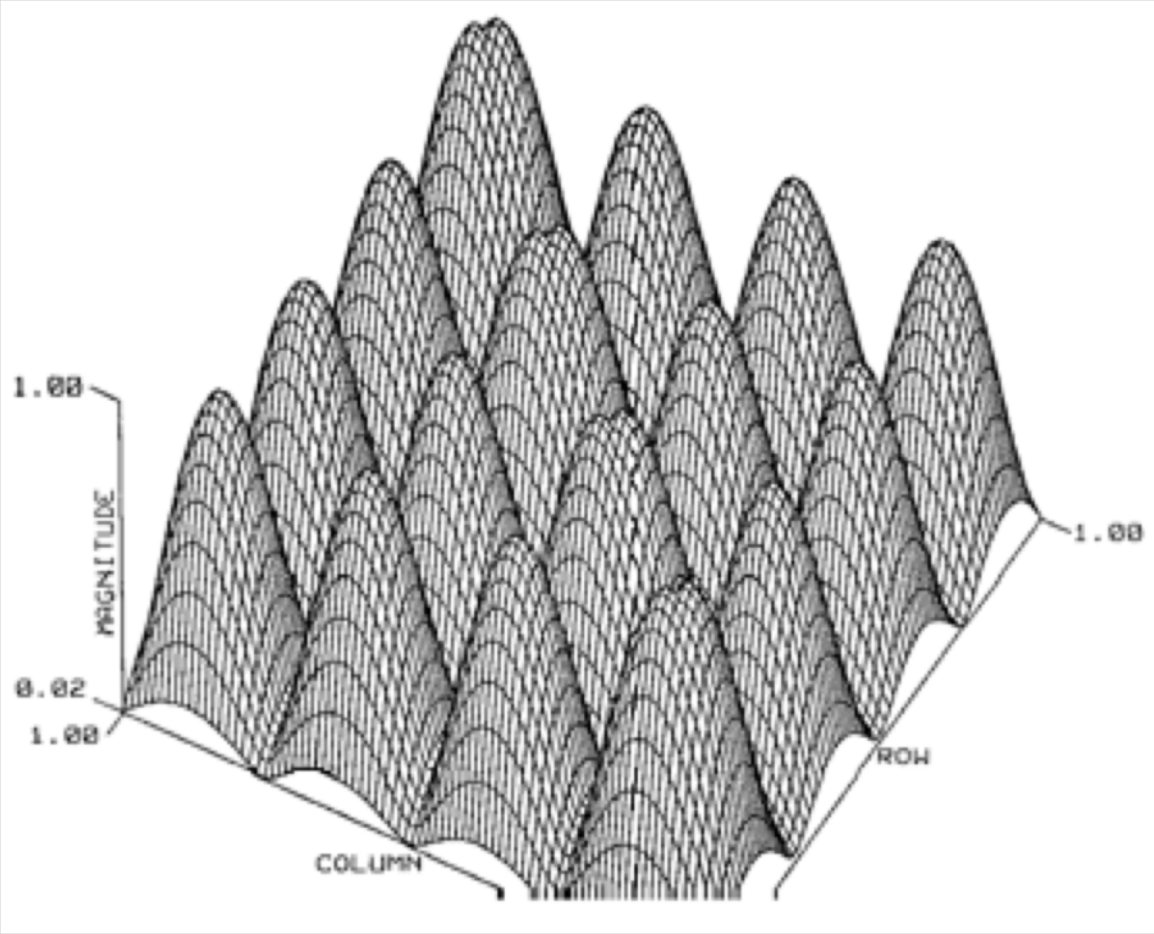
LOG OF $|Z_{m,n}|$ CONVEYS MORE INFORMATION ABOUT DISTANT INTERACTIONS . . .

Surface plot of $|Z_{m,n}|$ coefficients of a straight wire 2 wavelengths long oriented normally to the interface with half its length in each half space.



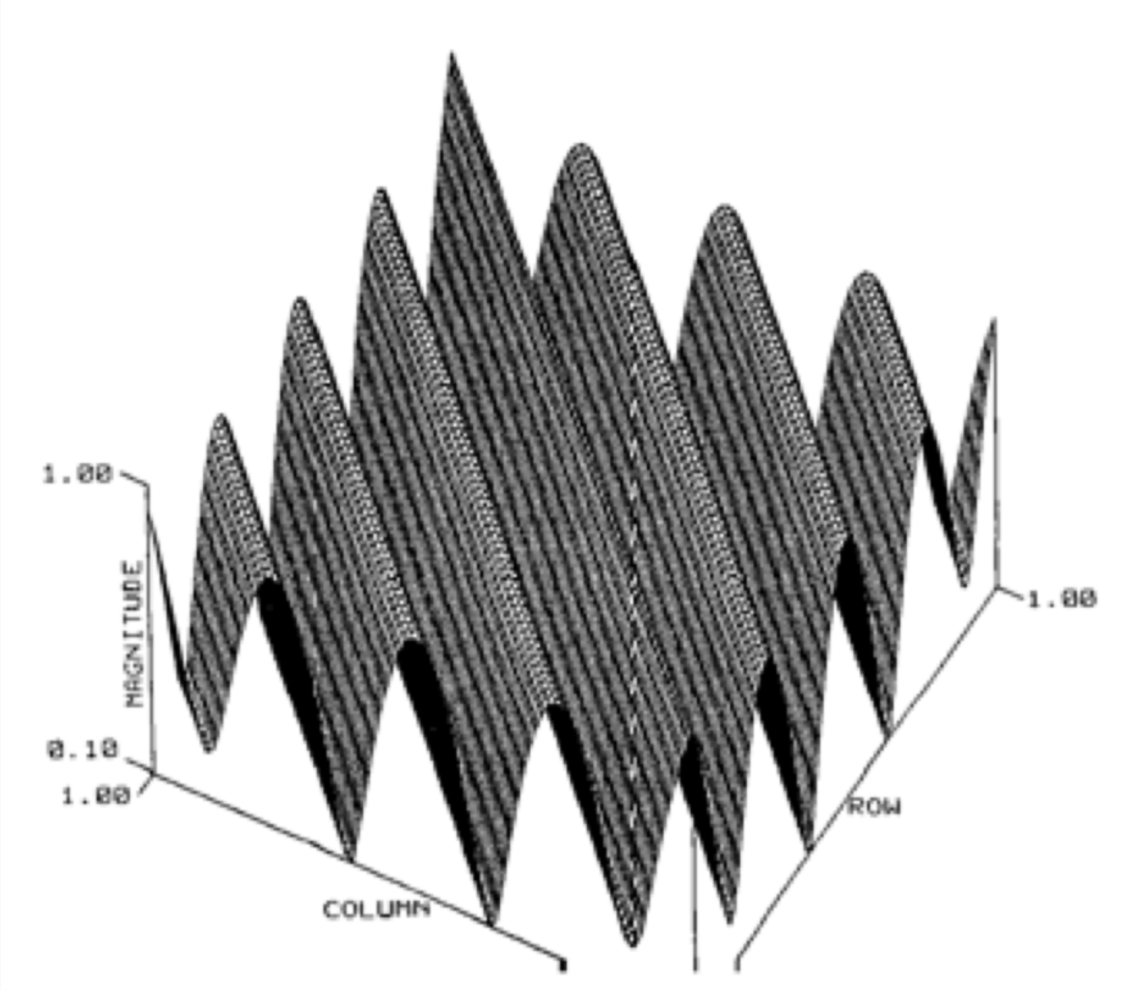
THE $|Y_{m,n}|$ COEFFICIENTS ARE ALSO CANDIDATES FOR WD MBPE . . .

Surface plots of $|Y_{m,n}|$ for a two-wavelength-long straight wire . . .



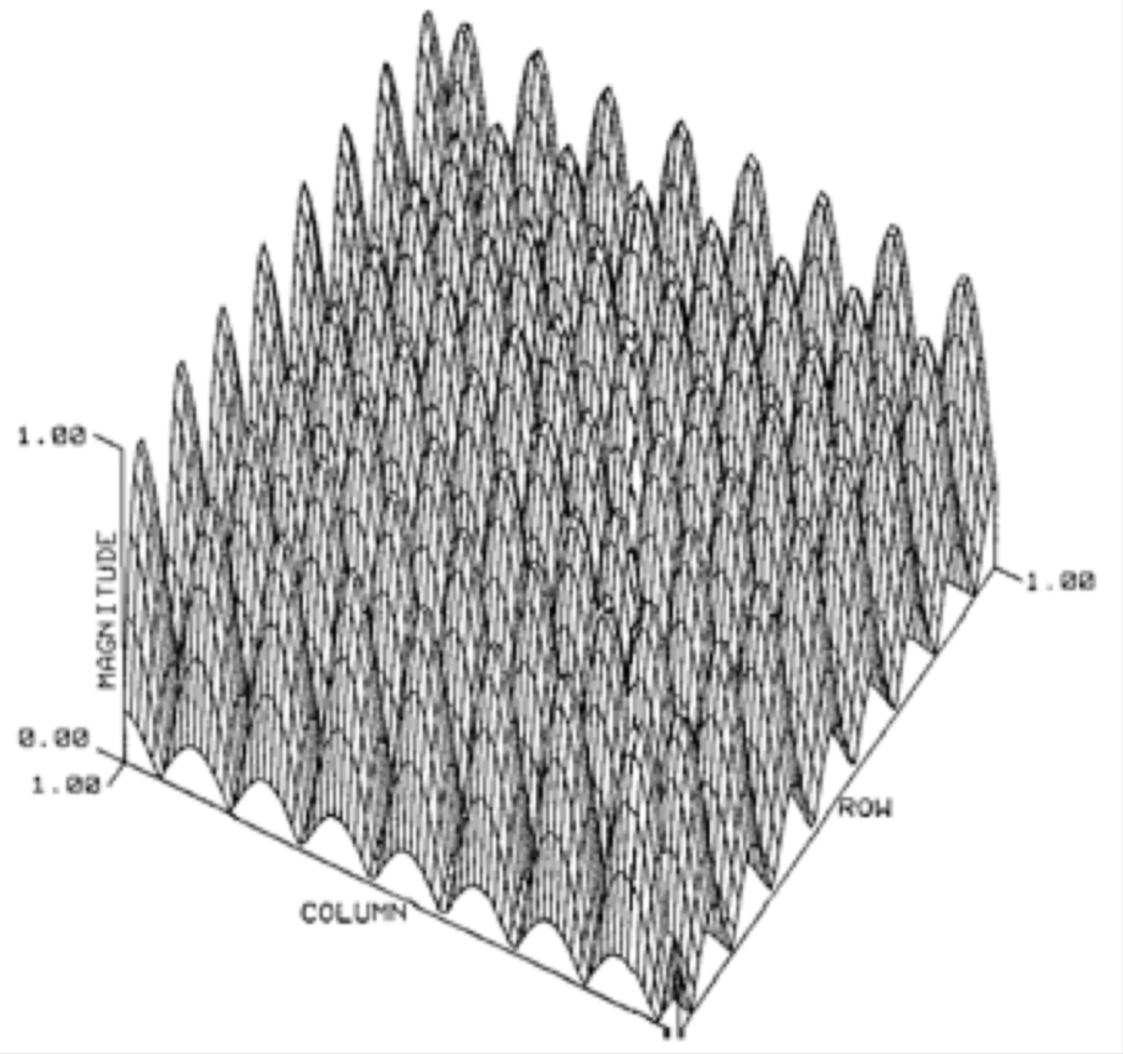
THE $|Y_{m,n}|$ COEFFICIENTS ARE ALSO CANDIDATES FOR WD MBPE . .

Surface plots of $|Y_{m,n}|$ for a two-wavelength-long a circular loop.



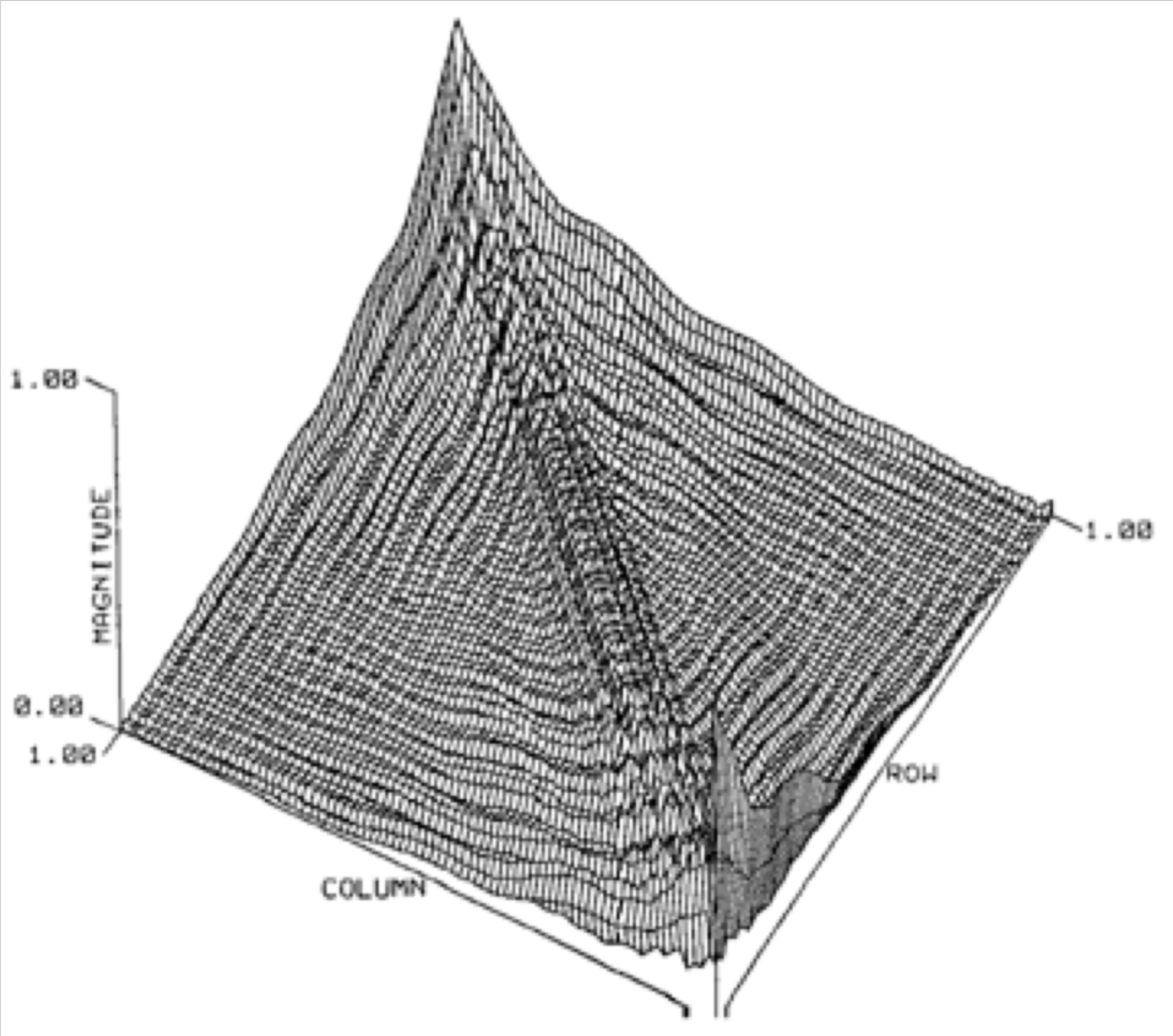
$|Y_{m,n}|$ REPRESENT CURRENTS THAT ARE PROPAGATING OR EVANESCENT

Surface plot of the magnitude of the admittance-matrix coefficients for an 8-turn helix of total wire length 4 wavelengths.The structure is below cutoff since the helix circumference, C, is less than λ and the current is a standing wave.



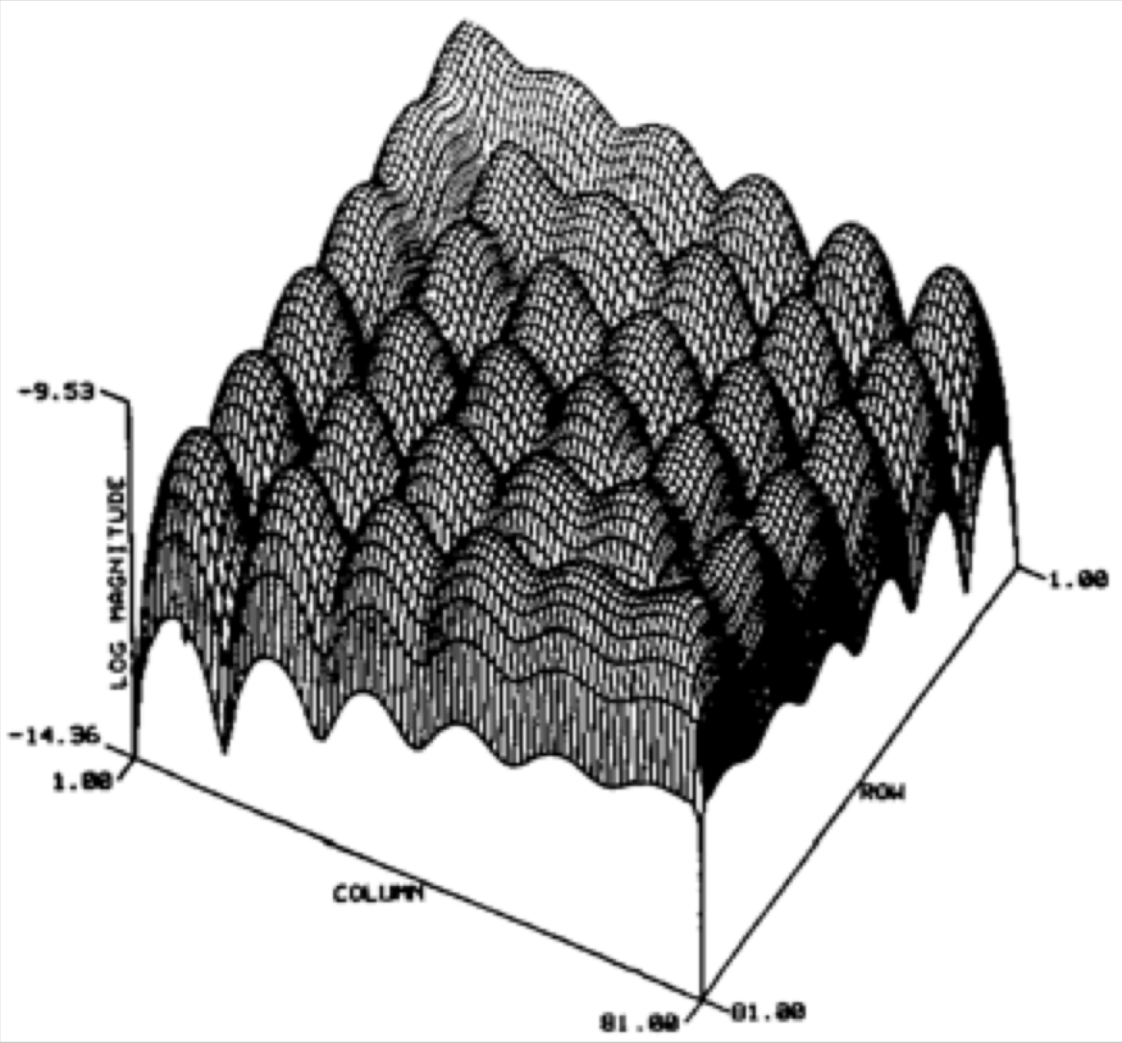
$|Y_{m,n}|$ REPRESENT CURRENTS THAT ARE PROPAGATING OR EVANESCENT

Surface plot of the magnitude of the admittance-matrix coefficients for an 8-turn helix of total wire length 16 wavelengths.The structure is now above cutoff and the current is a damped traveling wave.



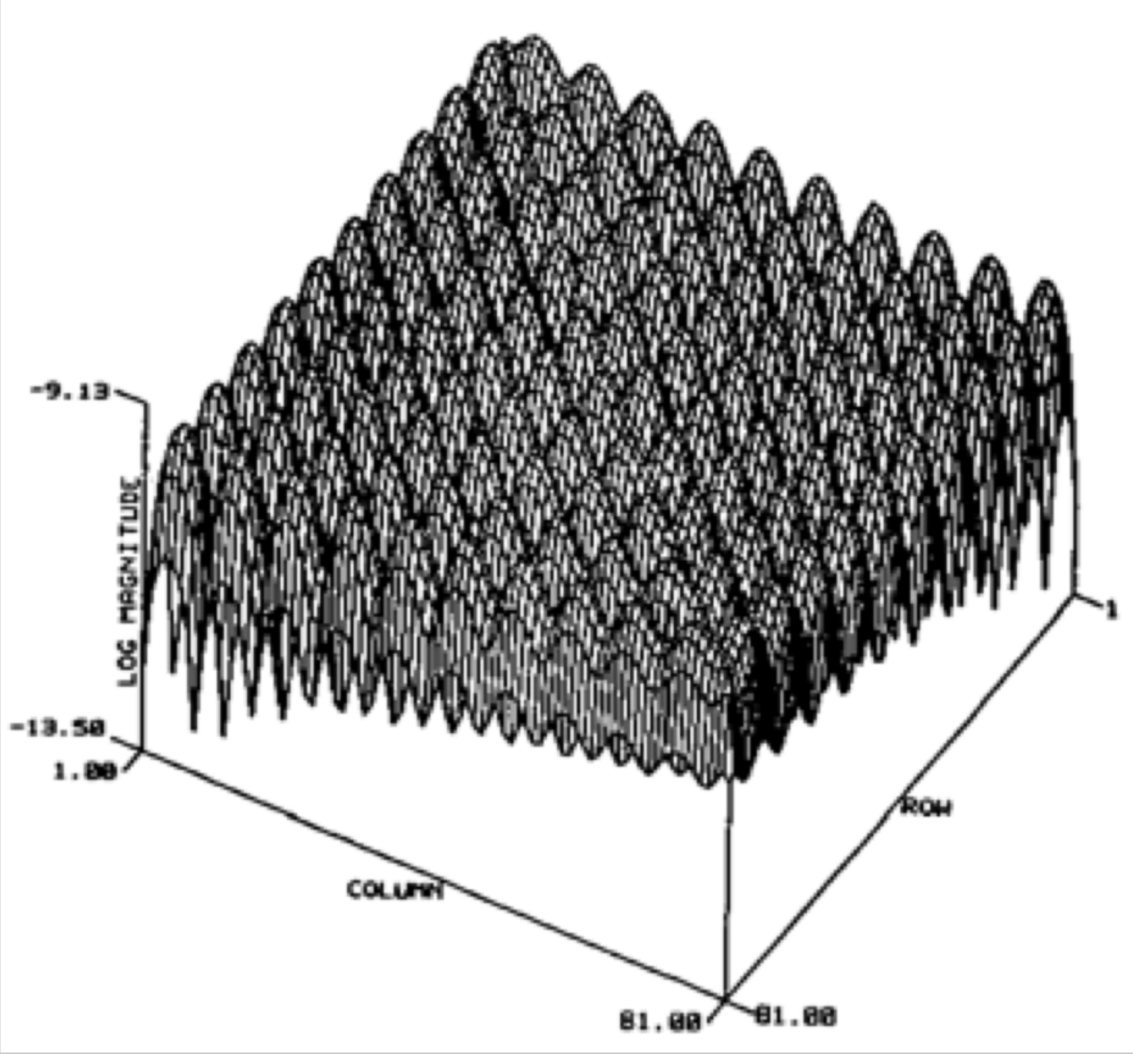
THE WD MBPE MODEL NEEDS TO TAKE MEDIUM PROPERTIES INTO ACCOUNT

Surface plot of $|Y_{m,n}|$ for a two-wavelength (in free-space) wire parallel to a dielectric half-space of ε_r = 10 when 10-4 wavelengths above the interface.



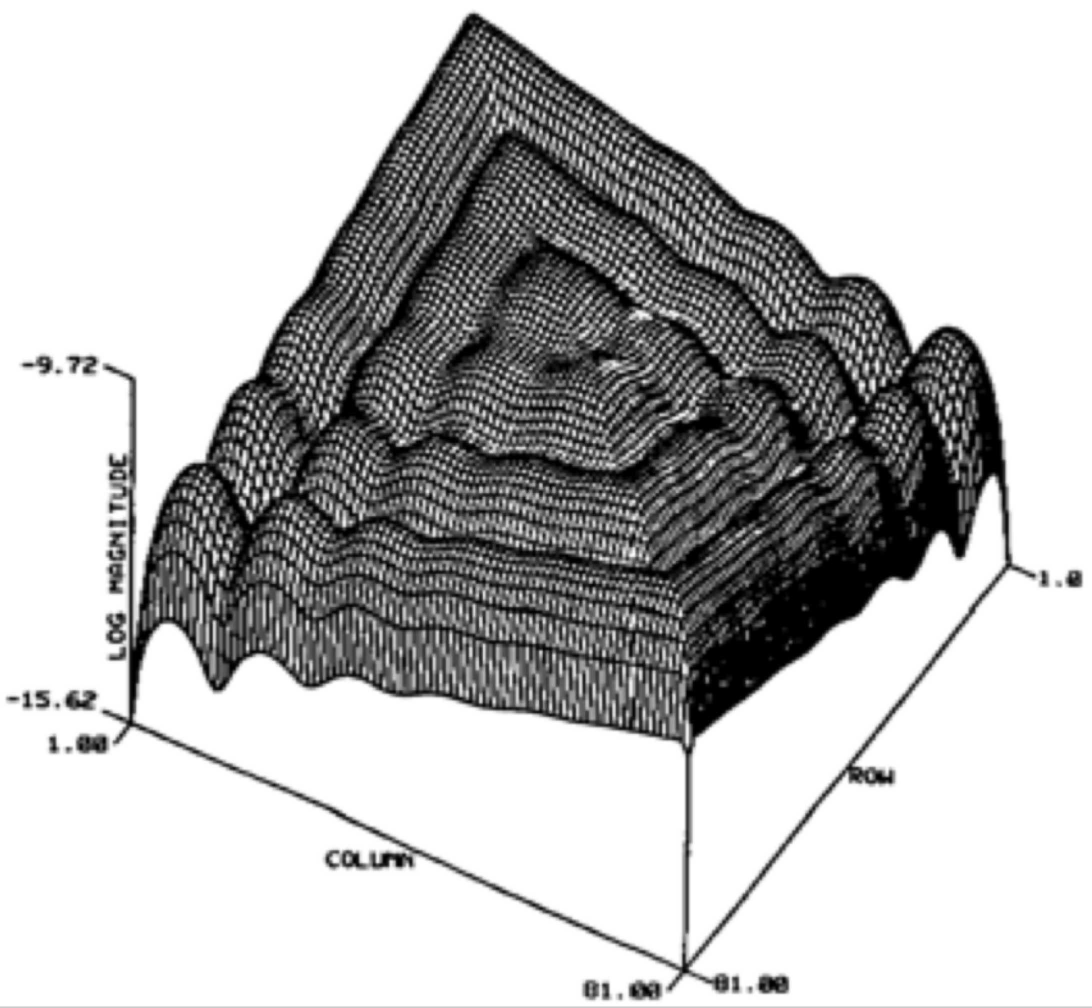
THE WD MBPE MODEL NEEDS TO TAKE MEDIUM PROPERTIES INTO ACCOUNT

Surface plot of $|Y_{m,n}|$ for a two-wavelength (in free-space) wire parallel to a dielectric half-space of ε_r = 10 when 10^{-4} wavelengths beneath the interface.



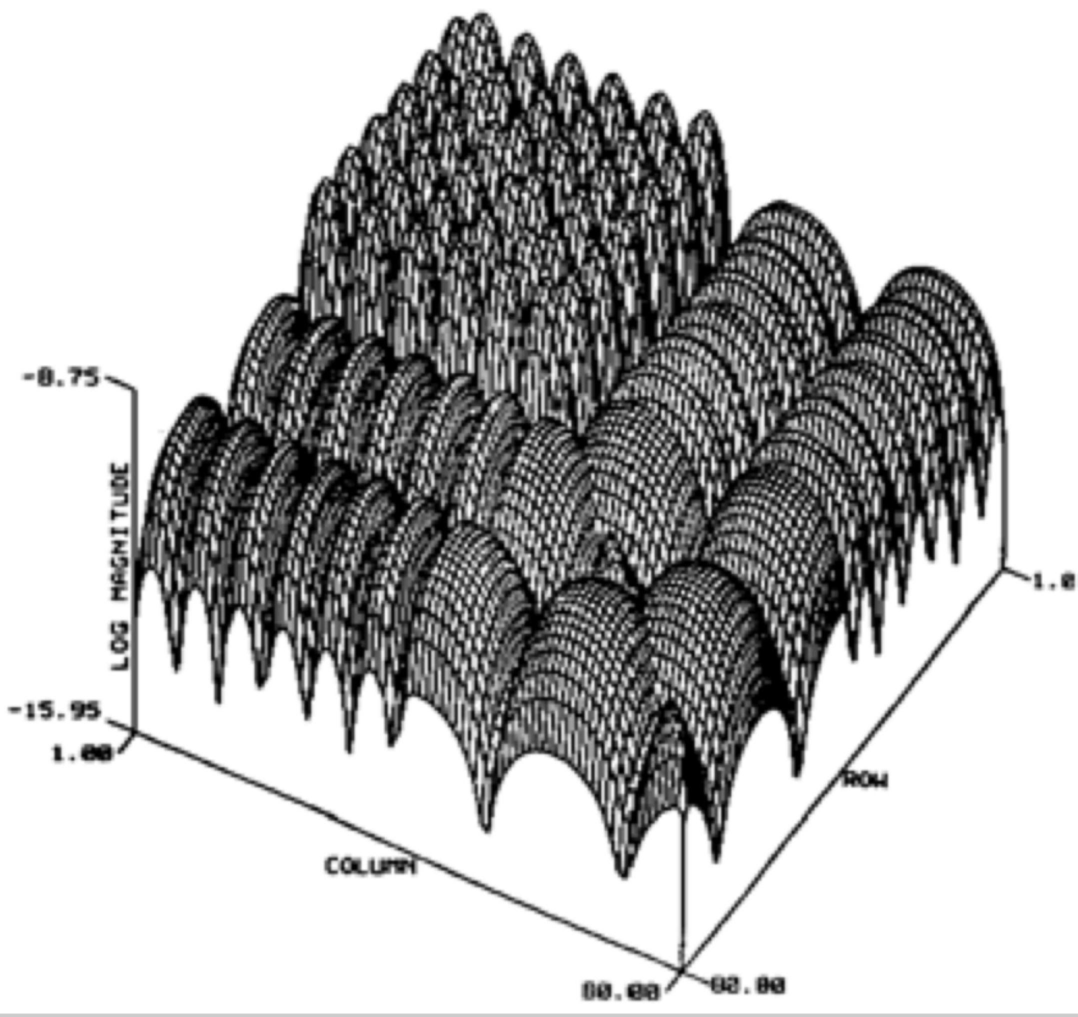
EFFECTS OF LOSSY HALFSPACE OR CROSSING INTERFACE FURTHER SHOW PROBLEM WAVELENGTH DEPENDENCE

Surface plot of $|Y_{m,n}|$ for a two λ wire in free space when parallel to and $10^{-4}\,\lambda_o$ above a half space with ε_r = 10 - j10.



EFFECTS OF LOSSY HALFSPACE OR CROSSING INTERFACE FURTHER SHOW PROBLEM WAVELENGTH DEPENDENCE

Surface plot of $|Y_{m,n}|$ for a two λ wire perpendicular to a dielectric half space of $\varepsilon_r = 10$ with its center at the interface.



A GENERIC MBPE MODEL FOR THE $Y_{m,n}$ MATRIX AS A FUNCTION OF FREQUENCY MIGHT LOOK LIKE:

- OBSERVABLES GENERATED FROM $Y_{m,n}$ ARE WELL-APPROXIMATED BY POLE SERIES

- THUS IN GENERAL

$$Y(s)=\frac{1}{D(s)}\begin{bmatrix} X & \mathrm{L} & X & n_{1a}(s) & \mathrm{L} & n_{1b}(s) & \mathrm{L} \\ X & \mathrm{L} & X & n_{2a}(s) & \mathrm{L} & n_{2b}(s) & \mathrm{L} \\ X & \mathrm{L} & X & n_{3a}(s) & \mathrm{L} & n_{3b}(s) & \mathrm{L} \\ \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} \\ X & \mathrm{L} & X & n_{X_s a}(s) & \mathrm{L} & n_{X_s b}(s) & \mathrm{L} \end{bmatrix}$$

- $n_{i,j}(s)$ IS NUMERATOR POLYNOMIAL FOR COEFFICIENT i,j
- $D(s)$ IS THE COMMON DENOMINATOR POLYNOMIAL
- a AND b ARE INDICES OF EXCITATION PORT WHOSE CURRENT RESPONSE HAS BEEN MODELD

PRESENTATION HAS EXPLORED SOME ISSUES IN ESTIMATING & REPRESENTING EM OBSERVABLES

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