# Electromagnetics without equations

Why isn't studying EM more popular?

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At the first meeting of a required undergraduate fields course I was assigned to teach as a visiting Professor, I asked the 35 or so students, "How many of you would be taking this course if it were not required?" When not a single hand was raised, I knew this would not likely be a "fun" course to teach, nor evidently, to take.

Given the ubiquity of electrostatic, magnetostatic and electromagnetic (EM) fields, why isn't studying EM more popular? The answer can probably be found early on in most texts introducing this subject that arguably forms the foundation of electrical and computer engineering. Calculus, not just differential and integral, but its vector form too, soon engulfs anyone having the apparent misfortune of being led to the electromagnetics water trough. Given that most formative discoveries in electromagnetics were made from various laboratory experiments, this mathematically intensive teaching approach to the subject seems unnecessary. This is especially true for those who don't intend on majoring in the discipline. In fact, it can be counterproductive.

This brief discussion here demonstrates that at least one fundamental property of EM fields, the radiation and propagation of power through space, can be explained without complicated mathematics. Since EM fields would not exist without radiation, this means that understanding EM need not be mathematically intimidating.

This demonstration depends on two basic observations. First, *the propagation speed of EM fields is finite*—EM fields propagate at a finite speed of approximately 3 x 108 m/s in free space, a quantity conventionally denoted as "c." This means that information about any "disturbance" of the position of the charges that produce electric fields, or of the velocity of charges in motion that produce magnetic fields, cannot be detected by an observer at a distance R away until a time t = R/c has passed.

In EM parlance, this delay is called "retarded time." The distant observer "finds out" about changes in charge position and/or velocity at a time retarded by the propagation delay. For example, an eruption on the sun's surface cannot be observed on earth until about +8 minutes later. This is because the earth is about eight minutes away at the speed of light.

Second, *electric lines of force are continuous*—Electric charges produce continuous lines of force that in turn define electric fields. A positive test charge placed in an electric field moves along, and in the direction of, the electric fields lines. By convention, electric fields originate on plus- and terminate on minus-signed charges.

For charge distributions that are stationary or moving at a constant velocity, the field lines are spatially unchanging. There is no radiation. It's only when the charges are changed in velocity, i.e. accelerated, that radiation occurs. This fundamental property of EM fields once appreciated can make this topic conceptually must easier to understand.

## The "kink" model

That radiation only occurs because of charge acceleration can be demonstrated by combining these two simple facts: finite propagation speed and continuous electric-field lines. Consider a stationary, isolated charge at the origin, O, of a rectangular coordinate system, whose electric field lines therefore lie along radii terminating at O (Fig. 1a). If the charge is abruptly accelerated to a velocity v = 0.3c along the +x axis, then at time $t = t_1$ the field lines will be as shown in Fig. 1b.

An observer in the x-y plane at point $R = \sqrt{(x^2 + y^2)}$ won't be aware of the charge movement until a time t = R/c has passed when the electric field at the observer's position changes. This change occurs because the E-field lines that terminated initially on the charge at the origin must "shift" over time with the changing charge position. This process is not instantaneous because c has a finite value.

Also, due to the continuity in the E-field lines, the outer part of a given line will continue to point to the origin while the part near the charge will point to its changing location. Thus, the old and new field lines are joined by a line segment (or kink) that is parallel to neither, but lies along the circumference of the sphere (a circle in this two-dimensional plot) defined by R = ct and which moves outward at the speed of light. These kinks constitute the electric components of an electromagnetic radiation field that are accompanied by magnetic-field components as well (not shown here).

If the charge is abruptly stopped at time $t_1$, a second spherical surface of E-field kinks is formed as shown in Fig. 1c at a time $t_2$ later. This new surface propagates outward from the position $x = vt_1$ about which it has expanded to the radius $R = ct_2$ while the original radiation pulse has expanded further to a radius $R = c(t_1 + t_2) = ct_3$.

There are some other interesting phenomena to observe in Fig. 1. Note that the E-field lines joining the two kinks continue to move in the +x direction at the speed v to continue pointing at the location where the charge would be had it not been stopped. In addition, observe that the kink portions lengthen as their respective spheres propagate further away from their origins. This indicates a decreasing value of their electric fields, which must decay with distance, r, as 1/r.

Finally, note that the distance between the acceleration and decelera-


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tion kinks in Fig. 1c is shorter in the +x direction than in the -x direction. This demonstrates the well-known Doppler shift exhibited by a moving source of electromagnetic or acoustic energy, i.e., a "squeezing" together of the waveform in the direction of motion and its "stretching" out in the opposite direction.

*Propagating fields are vectorial*—It's a fact, perhaps unfortunate from a vector calculus viewpoint, that EM radiation is a vector phenomenon. To see how this is so, consider an observer located at x = X, y = z = 0 and retrace the steps just taken. Because the charge is moved along the x-axis, the old and new field lines at the observer's position also lie along the x-axis. This means no "joining" non-radial segment is needed to connect the old and new lines. Thus, there is no propagating, or radiating, component. As a matter of fact, a short, straight (compared with the wavelength) segment of current has a maximum in its radiated electric field perpendicular to its axis and a zero along its axis. By a principle known as reciprocity, its receiving characteristics will have a similar orientation dependence. Furthermore, the radiation components of the electric field lie in the plane of the current segment. Being tangential to the expanding circle, they are vectors.

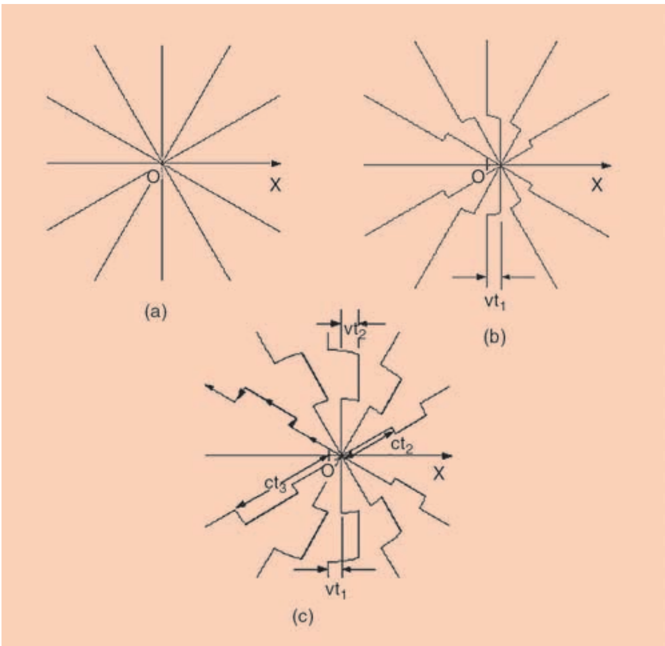


*Fig. 1 Depiction of the E-field lines for (a) charge that's abruptly accelerated from the origin to a speed v along the positive x-axis (b) and after a time $t_1$ is abruptly stopped (c). Information about the changed charge positions propagates outward from its old and new positions at the speed of light. This is shown by the circular line segments joining the original field lines and those moving with the charge (b) and the moving field lines and those that terminate at the new stationary position (c). The direction of the radiated field caused by charge acceleration is opposite that due to deceleration as can be seen by the reversed directions of the kinks on the field line with the arrows in this plot. Results shown here were obtained from combining two sequences of Cabrera's program, one for impulse acceleration and one for impulse deceleration.*

A practical consequence of this vector nature of EM radiation is that an antenna made of a straight piece of wire—typified by a "whip" antenna on a car—will receive a maximum signal when oriented parallel to an AM broadcast antenna. Conversely, it will produce a "null" or zero signal when it points at the broadcast antenna. These whip antennas are normally mounted vertically on a vehicle since the stations from which they receive their signals are installed perpendicular to the earth's surface.

While this conceptual demonstration of the "kink" radiation model is qualitatively correct, it ignores relativity. Relativity must be taken into account to produce a more accurate picture of the E-field lines. The results shown in Fig. 1 do not explicitly account for the time required to move the charge from x = 0 to x = d, a time that would be greater than d/c since no physical object can set the charge into motion. The following results do include relativistic effects, although for the cases shown these effects are not very dramatic.

## Some simple charge motions

The radiation kink model, although not useful for solving boundary-value problems in EM (whose solutions are needed to determine the scattering properties of radar targets or the current distributions on actual antennas), can help illustrate various radiation phenomena. A computer program based on the kink model was developed at Stanford University about 1985 (Blas Cabrera, "Physics Simulations 11: Electromagnetism, Academic Version"). This program develops a series of E-field plots for a charge undergoing certain kinds of motion selected by the user. These plots can then be viewed as a movie to display the evolution of the field as a function of time in a manner similar to a continuous "film loop." Some sample plots made by that program are presented here (B. Cabrera and

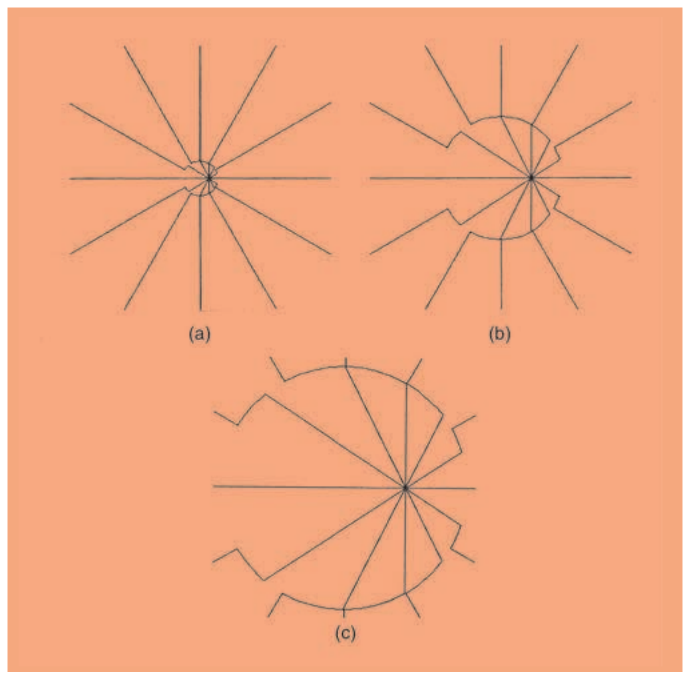


*Fig. 2 E-field lines about a charge undergoing impulse acceleration. The charge starts from rest and reaches a maximum speed of 0.5c to produce the "bubble" of radiation shown expanding from (a) to (c).*

E. K. Miller, "Macintosh Movies for Teaching Undergraduate Electricity and Magnetism"). In the example figures shown, time advances proceed from part (a) to (b) to (c). They were all generated by Cabrera's program.

*A charge given an abrupt push*—An abrupt push is perhaps the simplest kind of radiation-producing acceleration to visualize, see Fig. 2. This example differs from that depicted in Fig. 1 since here, the charge continues in motion after receiving an impulse of acceleration.

A single burst of radiation is produced in this case. But because the final charge speed chosen here is 0.5c, a relativistic effect is seen as a "bunching" of the field lines towards a line perpendicular (the y-axis) to the direction of motion. The faster the charge speed, the more the field lines bunch around the perpendicular.

A practical use of impulsive acceleration occurs when an electron beam impinges on a physical object, producing what is known as Bremsstrahlung (breaking) radiation. High-energy electrons smashing into a dense material, such as tungsten, are used to produce X-rays through this phenomenon.

*A charge given a constant push*—Suppose that instead of being impulsively accelerated to some maximum velocity, a charge is continuously accelerated. While relativity theory shows that the charge's final speed can't exceed that of light, its speed can approach c, increasing in mass as it does so.

As seen in Fig. 3 where the final speed is 0.99c, the radiation has the appearance of an EM shock-wave with a zone of nearly overlapping field lines (the density of the lines is also a measure of the field strength) which is most concentrated towards the forward direction. This kind of radiation would be associated with a linear particle accelerator where charged particles are accelerated to speeds near c over extended distances.

*A charge moving at constant speed around a circle*—Results are shown in Fig. 4 for a charge moving around a circle at a constant speed of 0.9c. The strongest radiation field can be seen as a spiral of coalescing field lines synchronized with the motion of the charge around the circle. This kind of radiation is produced in circular particle accelerators, and is called "Synchotron" radiation. It's also sometimes called searchlight radiation: a burst of radiation is seen at a given observation point with every orbit of the charge, much like the light produced by a continuously rotating searchlight beam.

The radiation takes place continuously here since the charge is constantly inwardly accelerated as it moves around the circular path. This is in contrast with the linear acceleration of the previous examples.

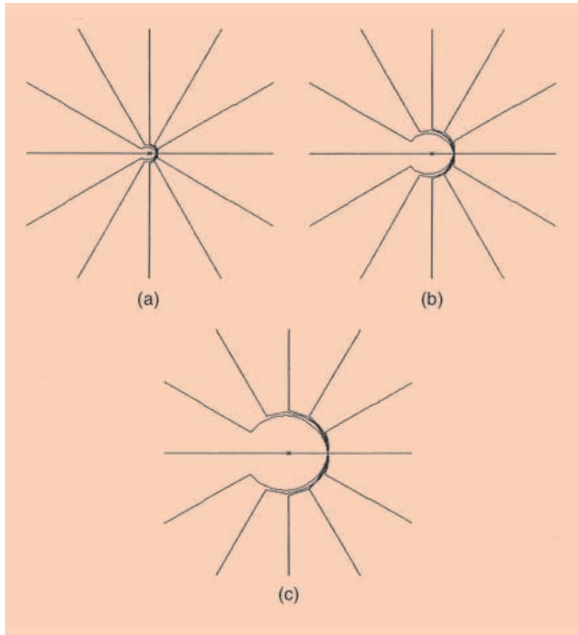


***Fig. 3** E-field lines for a charge accelerated to 0.99c by a constant force. Because its speed is so near c, the field lines at the charge are concentrated in a small angular sector about the perpendicular, creating an EM shock wave.*

*A charge moving at constant speed around a square*—Results for a charge moving at a constant speed of 0.5c around a square path are shown in Fig. 5. This kind of charge motion can be especially illuminating since the radiation occurs in a series of pulses as the charge moves around each right-angle corner of the square. During the time the charge moves along the straight side of the square, there is no radiation. However, a circular radiation field can be seen centered on each corner as the charge changes direction there. The vectorial nature of this kind of radiation is demonstrated by these radiation pulses not being closed circles. There are periodic nulls in the transverse electric field as the charge goes around the corner.

*A charge undergoing oscillatory motion*—The motion shown in Fig. 5 is most relevant to electromagnetic communications and is oscillatory. That is, a charge moves back-and-forth along a straight line with speed that varies as $V\cos(\omega t)$, with $\omega = 2\pi f$, where f is frequency of the radiation that it produces and $V = 0.5c$. This is not exactly analogous to how the charge motion varies on an actual wire antenna such as the whip mentioned before. But it suffices to illustrate the production of a time-harmonic radiation field as shown in Fig. 6. Not surprisingly, the radiation field is seen to be oscillatory in space and, therefore in time as well at a fixed observation point, as the wave propagates by.

The motion of a point charge and the E-field associated with that motion as illustrated here is quite different from the behavior of charge and current on a real conductor such as a straight wire. For example, on a wire assumed to be perfectly conducting and excited at its center by a time-harmonic voltage—a structure known as a dipole antenna—the effective speed of the E-fields along the wire is c. The reason is because EM waves propagate at light speed in free space. Since the EM wave E-field lines terminate perpendicularly at charges on the wire, the effective charge speed is also c. This speed is in contrast with the sinusoidal speed assumed here for the charge. But physical charges in the wire can't move at light speed, and neither does the same physical charge move from one of the wire to the other. Instead, the charges interact somewhat as a row of falling dominoes to constitute an equivalent current that moves much faster than the charge speeds.

## Some observations

Keep in mind, that a kink model demonstrates that EM radiation is essentially produced by "wriggling" a charge so as to wriggle its E-field lines. As shown in Fig. 6, the outward-propagating waves carried by a line of electric field is tantalizingly similar to the waves caused on a rope tied on one end and wriggled at the other end. This analogy is not entirely superficial. Both waves are transverse, i.e., the displacement of the field line and the rope is perpendicular or transverse, to the direction of wave travel. Also, a field line in the direction of the wriggling charge motion carries no wave and neither does

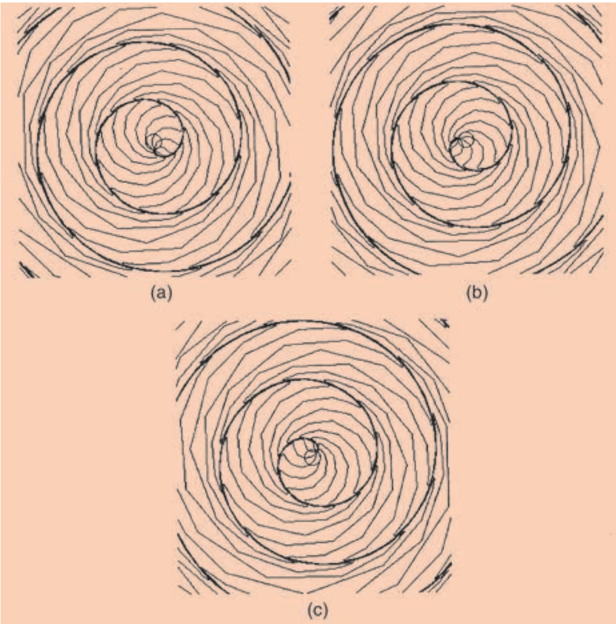


***Fig. 4 E-field lines for a charge moving in a circular path at a speed 0.9c. High values of electric field are indicated by the dark bands in this figure producing an effect called Synchotron radiation.***

a rope when its end is pushed or pulled on. Thus, any time a charge or current, which is comprised of moving charges, is wriggled, radiation can be expected.

Electromagnetic radiation also involves power flow. The greater the power provided to an antenna, the larger the field at a distant point where its strength varies as the square root of the antenna power. The radiation kink model does not include any consideration of the power needed to make physical charges move in the various arbitrary ways chosen for the previous examples. Also, the power in the EM field is carried in the interacting electric and magnetic fields, in an amount and direction given by **ExH** where **H** is the magnetic field and "x" signifies a vector cross product. The magnetic field has been neglected in this discussion for simplicity.

Finally, EM radiation and scattering problems normally begin without knowledge of what the charge and current distributions may be on objects of interest, say that vehicular whip antenna previously mentioned. Before the radiation fields can be obtained, the currents and charges flowing on the whip antenna and the vehicle on which it's mounted must be found. This, in turn, requires solving a "boundary-value" problem. Then the fact that an electric field tangential to a good conductor, such as aluminum or copper, is zero is used. When this "boundary condition" is combined with a specification of how the antenna is "excited" by either an incident EM wave when used in a receiving system, or by the voltage from a generator when used in a transmitting system, the currents and charges can be obtained. These in turn permit evaluation of the receiving or transmitting properties of the antenna. This procedure can be complicated. Thus, the kink model—though unsuited for quantitative solution of such problems—can yield insight into electromagnetic radiation that a more rigorous approach might not provide.

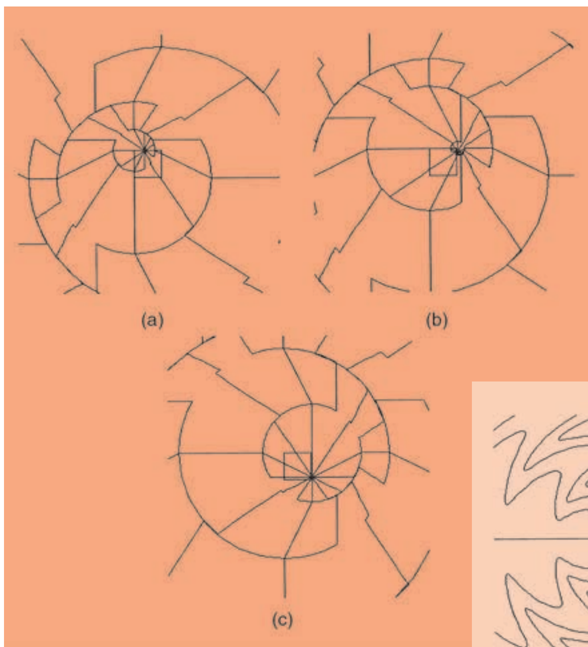


***Fig. 5 E-field lines for a charge moving around a square path at a constant speed of 0.5c. The charge produces an expanding circle of radiation as it goes around each of the right-angle comers of the square.***

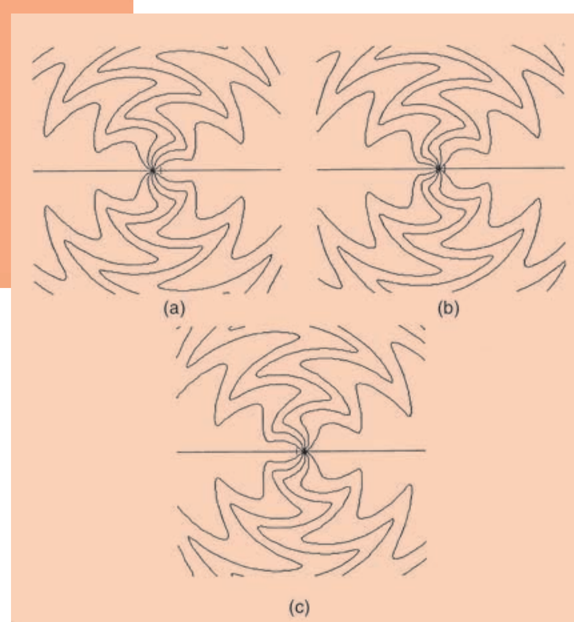


***Fig. 6 The E-field lines about a point charge undergoing oscillatory motion along a straight line with a maximum speed of 0.5c. The generation of outward propagating, time-harmonic waves is clearly discernible, in a fashion somewhat analogous to how an actual wire antenna works.'***

## Read more about it

• B. Cabrera and E. K. Miller, "Macintosh Movies for Teaching Undergraduate Electricity and Magnetism," 1986 International IEEE AP-S Symposium, Philadelphia, PA, June 9-13.

• Blas Cabrera, "Physics Simulations 11: Electromagnetism, Academic Version," Intellimation Library for the Macintosh, PO Box 1530, Santa Barbara, CA 93116-1530, 1990.

• E. K. Miller, R. Merrill, and R. J. Cole, "Computer Movies for Education," *IEEE Transactions on Education*, May, pp. 58-68, 1988.

## About the author

Dr. Edmund K. Miller is an associate editor of *IEEE Potentials*. When he is not doing field research for his upcoming book, "Cruisin' and trippin' thru retirement," he manages to find time to squeeze out a few words for *Potentials*' "On the job."