Exponential Distribution Simulation

Mark Culp March 25, 2017

Overview:

This paper explores simulations on an Exponential Distribution, a distribution that describes the time between events in a Poisson process. We use R's rexp(n, lambda) function to create samples from an Eponential Distribution, and compare it with the Central Limit Theorem (CLT). The CLT is one of the most important theorems in statistics which states that the distribution or sum of a large number of independent random variables tends toward a normal distribution regardless of the underlying distribution.

First, let's create a small exponential distribution sample, and examine its properties:

```
# Set parameters for distribution
n <- 40
lambda <- 0.2

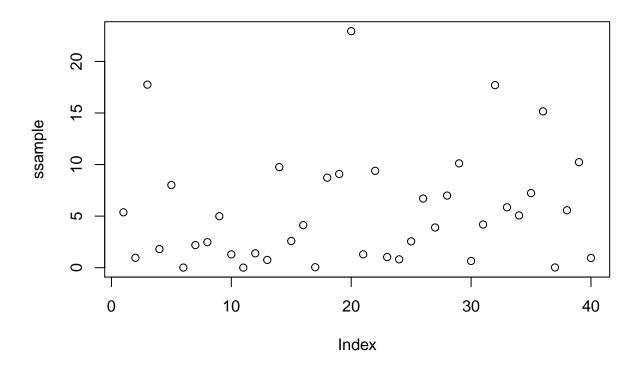
# Create small sample
ssample <- rexp(n, lambda)

dim(ssample)

## NULL

# An exploratory plot

plot(ssample)</pre>
```



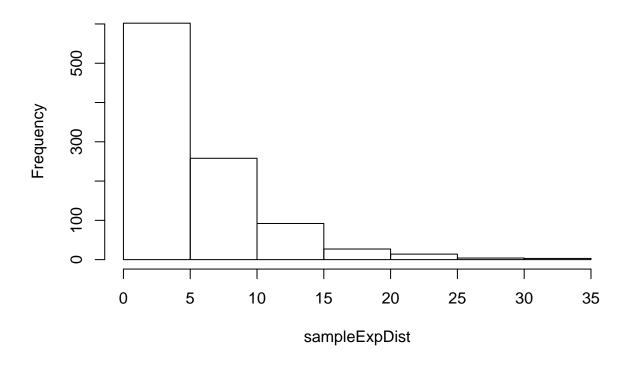
```
# abline(0,..)
```

Simulations:

Now let's run a couple of simulations involving ...

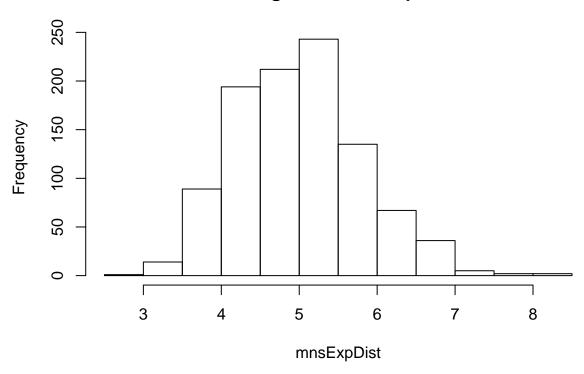
```
set.seed(0)
# Generate 1,000 random deviates for an exponential distribution
sampleExpDist <- rexp(1000, lambda)</pre>
# Generate a distribution of 1000 averages of 40
# random deviates for an exponential distribution
# and calculate the variances in each sample
expSample = NULL
mnsExpDist = NULL
varExpDist = NULL
for (i in 1 : 1000) {
  expSample = rexp(n, lambda)
  mnsExpDist = c(mnsExpDist, mean(expSample))
  varExpDist = c(varExpDist, var(expSample))
}
# Create a histogram of the distribution of exponentials
hist(sampleExpDist)
```

Histogram of sampleExpDist



Create histogram of the distribution of exponential averages hist(mnsExpDist)

Histogram of mnsExpDist



Sample Mean versus Theoretical Mean

This distribution is centered at ... The theoretical center, or mean of the distribution is centered at ...

```
# Load libraries
library(knitr)

# Calculate the mean of the sample exponential distribution
sampleExpDistMean <- round(mean(sampleExpDist),3)

# Calculate mean of the distribution of exponential averages
mnsDistMean <- round(mean(mnsExpDist),3)

# Calculate hypothetical mean for the exponential distribution
hypoDistMean <- 1/lambda

# Combine means in a row
rowMeans <- rbind(c(sampleExpDistMean, mnsDistMean, hypoDistMean))

# Display table of calculated means
kable(rowMeans, caption = "Comparison of Sample and Hypothetical Means", col.names = c("Sample", "Average")</pre>
```

Table 1: Comparison of Sample and Hypothetical Means

Sample	Averages	Hypothetical
5.148	4.994	5

Sample Variance versus Theoretical Variance

This distribution's variance is ... The theoretical variance of this distribution would be ...

```
# Calculate variance of the sample exponential distribution
sampleExpDistVariance <- round(var(sampleExpDist),3)

# Calculate variance of the distribution of exponential averages
mnsExpDistVariance <- round(mean(varExpDist),3)

# Calculate hypothetical mean for the exponential distribution
hypoDistVariance <- (1/lambda)^2

# Combine variances in a row
rowVar <- rbind(c(sampleExpDistVariance, mnsExpDistVariance, hypoDistVariance))

# Display table of calculated variances
kable(rowVar, caption = "Comparison of Sample and Hypothetical Variances", col.names = c("Sample","Aver.</pre>
```

Table 2: Comparison of Sample and Hypothetical Variances

Sample	Averages	Hypothetical
24.418	25.386	25

Distribution

We know this distribution is normal because ...

Our conclusions and assumptions ...