

TOPIC 1: Permutations

Permutations – determines the number of possible arrangements in a set when the order of the arrangement matters.

Factorial Notation- If n is a positive integer, $n!$ (n factorial) is a product of all positive integers less than or equal to n .

Example: $3! = (3)(2)(1) = 6$

Remember: $0! = 1$

Permutation Formulas:

1. The number of permutations of objects taken all at a time in $n!$ where n is the number of objects taken.

Formula: $P(n,n) = n!$

Example: How many possible ways can we arrange 5 students in a classroom for limited face to face classes.

Solution: $n=5$

$$\begin{aligned}P(n,n) &= n! \\&= 5! \\&= (5)(4)(3)(2)(1) \\&= \mathbf{120 \text{ possible ways}}\end{aligned}$$

2. **Formula: $P(n,r) = \frac{n!}{(n-r)!}$** where n is the number of object and r is the total number of object taken at a time.

Example: There are 12 volleyball varsity players in a team, but only 6 can play the game inside the court. How many possible arrangements that the coach can do so that all the players can play the game?

Solution: $n=12$, $r=6$

$$\begin{aligned}
 P(n,r) &= \frac{n!}{(n-r)!} \\
 &= \frac{12!}{(12-6)!} \\
 &= \frac{12!}{6!} \\
 &= \frac{(12)(11)(10)(9)(8)(7)\cancel{6!}}{\cancel{6!}} \\
 &= (12)(11)(10)(9)(8)(7) \\
 &= \mathbf{665,280 \text{ possible ways}}
 \end{aligned}$$

3. $P = \frac{n!}{p! \, q! \, r!}$

Example: In how many distinguishable permutations are possible with the letters of the word "MISSISSIPPI"?

Solution: In the MISSISSIPPI, there are 4 I's, 4 S and 2 P's.

$n=11$, $p=4$, $q=4$ and $r=2$

$$\begin{aligned}
 P &= \frac{n!}{p! \, q! \, r!} \\
 &= \frac{11!}{4! \, 4! \, 2!} \\
 &= \frac{(11)(10)(9)(8)(7)(6)(5)\cancel{4!}}{(\cancel{4})(\cancel{3})(\cancel{2})(1)(2)(1)\cancel{4!}} \\
 &= \mathbf{34,650 \text{ ways}}
 \end{aligned}$$

4. Circular Permutation: $P=(n-1)!$

Example: In a classroom, the class was divided into 5 groups with 7 members in each. The groups are asked to form a circle. How many possible ways can the 7 members of the team be arranged?

Solution: $n=7$

$$\begin{aligned}
 P &= (n-1)! \\
 &= (7-1)! \\
 &= 6! \\
 &= (6)(5)(4)(3)(2)(1) \\
 &= \mathbf{720 \text{ possible ways}}
 \end{aligned}$$