PROBLEM SET 2

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1. Problem 1

Given graph $G = (V, E, W)$ consider a random walk on V with transition probabilities $M_{ij} = PX(t+1) = j|X(t) = i = \frac{w_{ij}}{\deg(i)}$.

Partition the vertex set as $V = V_+ \cup V_- \cup V_*$. Suppose that every node in V_* is connected to at least one node in either V_+ or V_-. Given a node $i \subset V$ let $g(i)$ be the probability that a random walker starting at i reaches a node in V_+ before reaching one in V_-. If $i \in V_+$, then $g(i) = 1$ and if $i \in V_-$, then $g(i) = 0$. Find $g(i)$ for $i \in V_*$.

There are two scenarios in which the random walker will reach a node in V_+ before one in V_-: Moving from V_* to V_+ immediately, or moving from V_* to V_* repeatedly until moving to V_+. We essentially view leaving V_* as an absorbing barrier, and want the probability that we end in V_+, which is the sum of all probabilities that will end in that scenario:

$$\sum_{j \in V_+} M_{ij} + \sum_{j \in V_*} \sum_{k \in V_+} M_{ij} M_{jk} + \sum_{j \in V_*} \sum_{k \in V_*} \sum_{l \in V_+} M_{ij} M_{jk} M_{kl} + \dots$$