Full Wave Analysis of RF Signal Attenuation in a Lossy Cave using a High Order Time Domain Vector Finite Element Method

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Abstract

We present a computational study of signal propagation and attenuation of a 200 MHz dipole antenna in a cave environment. The cave is modeled as a straight and lossy random rough wall. To simulate a broad frequency band, the full wave Maxwell equations are solved directly in the time domain via a high order vector finite element discretization using the massively parallel CEM code EMSolve. The simulation is performed for a series of random meshes in order to generate statistical data for the propagation and attenuation properties of the cave environment. Results for the power spectral density and phase of the electric field vector components are presented and discussed.

1 Introduction

The study of electromagnetic wave propagation in caves and tunnels is of great practical interest to antenna engineers due to the increasing demands for reliable wireless communications systems in such environments. Current wireless radio frequency (RF) communication systems were not designed to operate reliably in enclosed environments such as caves and tunnels, and signal quality is severely compromised due to the rough and lossy surfaces of the cave. Today there is limited ability to maintain communications in cave-like structures, tunnels or subways, prohibiting the quick deployment of wireless systems in caves and tunnels. Military or rescue personnel are potentially at risk when in these environments. If the electromagnetic properties of the tunnel could be characterized (e.g. dissipation, dispersion, fading, etc.), then a more robust communication system could be designed specifically for operation in such environments, hence full wave EM simulations of propagation in this type of environment are very useful.

Much theoretical work in this field has been done in order to develop a better understanding of the RF propagation channel. Dudley recently studied models for propagation in circular tunnels [1]. He produced expressions for the fields in terms of a Fourier transform over the axial variables, and presented the numerical results for the field intensity both as a function of axial distance and as a function of radial distance. However, this work only involved smooth tunnel walls and not the more realistic situation of rough wall tunnels. In this case, the electromagnetic fields are no longer deterministic, but random variables that need to be solved using statistical methods. Recently, Pao and Kasey have investigated the statistical properties of wave propagation in straight, rough-walled tunnels [2], [3]. This work assumes a perfect electrical conductor (PEC) boundary at the rough wall / air interface. A more realistic model needs to take into account the lossy nature of the rough walls and the cave material (typically granite or some sort of earth like material with electrical conductivities on the order of $0.1 S/m$).

In this paper we use a high order finite element discretization to solve the full wave Maxwell equations directly in the time domain for the case of a dipole antenna source placed at the mouth of a straight, lossy rough walled tunnel. We chose a time domain simulation in order to efficiently compute the response over a broad frequency band. The electromagnetic wave equation for the electric field is discretized over a sequence of randomly generated meshes to determine statistical properties for the power spectral density (PSD) and phase of the three components of the electric field. The problem is solved directly in the time domain using a leap-frog method, and results are converted into the frequency domain via FFT algorithms. We begin with a brief description of the numerical method employed for this problem (as implemented in the EMSolve code [4]) followed by a presentation and description of the numerical results.

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[§] This work was performed under the auspices of the U.S. Department of Energy by the University of California, Lawrence Livermore National Laboratory under contract No. W-7405-Eng-48, UCRL-CONF-208481

2 Numerical Formulation

We begin with the second order time dependent wave equation for the electric field in a 3 dimensional domain Ω

$$\begin{array}{rcll} \epsilon \frac{\partial^2}{\partial t^2}\mathbf{E} & = & -\nabla \times (\mu^{-1}\nabla \times \mathbf{E}) - \sigma \frac{\partial}{\partial t}\mathbf{E} - \frac{\partial}{\partial t}\mathbf{J} & \text{in } \Omega \\ \nabla \cdot (\epsilon \mathbf{E}) & = & 0 & \text{in } \Omega \\ \hat{\mathbf{n}} \times \mathbf{E} & = & \mathbf{E}_{bc} & \text{on } \partial\Omega \end{array} \tag{1}$$

where $\partial\Omega$ is the two dimensional boundary of the domain, $\hat{\mathbf{n}}$ is the outwardly directed unit normal of this boundary and $\mathbf{J}$ is a free current density source that can be added to drive the problem. The value $\mathbf{E}_{bc}$ represents an arbitrary boundary condition imposed on the electric field intensity while ϵ, μ and σ denote the dielectric permittivity, magnetic permeability and electrical conductivity of the materials contained in the domain Ω respectively.

Applying an arbitrary order Galerkin finite element discretization to the wave equation (the details of which can be found in [5], [6]) yields the following semi-discrete system of ordinary differential equations

$$M_\epsilon \frac{\partial^2}{\partial t^2} e = -S_\mu e - M_\sigma \frac{\partial}{\partial t} e - \frac{\partial}{\partial t} j \tag{2}$$

where M_ϵ, M_σ are finite element mass matrices, S_μ is a finite element stiffness matrix and e, j are discrete arrays of finite element degrees of freedom. Applying a backward difference approximation for the first order time derivative in (2) and a central difference approximation for the second order time derivative yields the following fully discrete linear system of equations

$$M_\epsilon \, e_{n+1} = (2M_\epsilon - \Delta t^2 S_\mu - \Delta t M_\sigma) e_n + (\Delta t M_\sigma - M_\epsilon) e_{n-1} - j' \tag{3}$$

where Δt is the discrete time step, the integer n denotes the current time step and the time derivative of the free current source has been directly incorporated into the new source term j'.

For the results shown in section 3 below we employ second-order interpolatory $H(curl)$ basis functions along with custom quadrature rules that yield a diagonal "mass" matrix M_ϵ. The details of this discretization are presented in [7] [8]. This method is much more accurate than standard FDTD, the numerical dispersion for this method is $O(h^4)$ rather than $O(h^2)$ as it is for FDTD. Compared to higher-order FDTD schemes, this method is better at modeling the jump discontuity of fields across the air-earth interface.

3 Computational Results

For our calculations we choose a tunnel with an average diameter of $2m$ and a total length of $75m$. The random rough surface is generated as follows. Step 1: Generate a cylindrical surface of appropriate radius and length, Step 2: Add a random perturbation of specified standard deviation to the surface, Step 3: Smooth the random surface (low pass filter) to introduce a surface correlation of a given length, Step 4: Generate a 3D Cartesian mesh, where the electrical conductivity of each element depends upon whether the element is inside the random surface (air) or outside the random surface (earth). For mesh elements that straddle the random surface, a volume-fraction is used to determine the conductivity. To model the dielectric and conducting properties of earth, we choose a uniformly varying electrical conductivity of 0.1-0.02S/m and a dielectric permittivity of 5 times the free space permittivity ϵ_0 (a typical value for granite). Each computational mesh consists of $583,200$ hexahedral elements, an example of which is shown in Figure 1. Note that the portion of the mesh representing the air has been removed to illustrate the random rough surface.

A z-oriented dipole antenna is placed at one end of the tunnel, modeled by applying a time dependent current source (represented by the term j') to one of the mesh edges , while a simple absorbing boundary condition is placed at the other. A PEC boundary condition is applied to the outer most walls of the cave to fully define the problem. The temporal dependence of the current source is a Gaussian pulsed sine wave centered at 200 MHz with a 20% bandwidth. The simulation is performed using high order $p = 2$ basis functions to mitigate the effects of numerical dispersion. The various simulation parameters are summarized in Table 1

Nine random caves were simulated. A time history was recorded at each x, for all time steps. This data was used to find the spectrum at every spatial step. For each simulation the mean and variance of the PSD and phase were extracted over the bandwidth of the signal. The run was normalized by dividing by the total PSD magnitude at the first x-data point, thereby removing the characteristics of the input signal, but preserving the

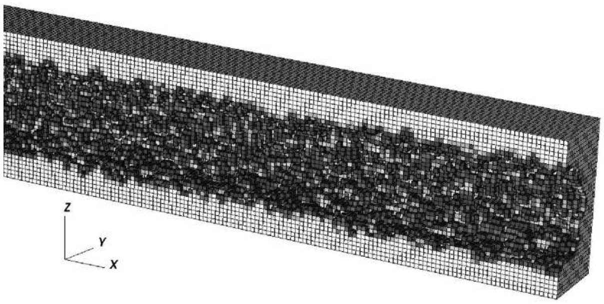

Figure 1: Example of randomly generated cave mesh with interior removed (close-up view).

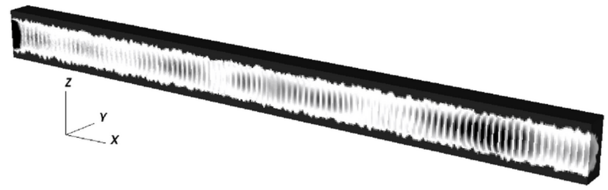

Figure 2: Example of computed electric field magnitude at the final time step.

Avg. Cave Radius	2 m (1.333 λ)
Cave Length	75 m (50 λ)
Element Size (Δx)	0.167 m (0.111 λ)
Surface Roughness Variance	0.3 m (0.2 λ), Gauss. dist. centered at 0
Conductivity Variation	Uniform, 0.1 – 0.02 S/m
Signal Type	Modulated Gaussian pulse, z-oriented dipole
Pulse Frequency	200 MHz, 20% Bandwidth
Gaussian Width, Delay	4.67e-9s, 2.50e-8 s
No. Unknowns per Trial	$\sim$ 14 million
No. Parallel CPU's per Trial	192

Table 1: Summary of computational statistics for 9 random cave simulations.

relative magnitudes vs. polarization. The last 100 spatial samples were removed to avoid reflections from the end of the cave. The results for this are shown in Figure 3. For these plots, The x-axis is frequency, and the y-axis is distance into the cave. No significant variation was found in the result across the spectrum. In Figure 4 we plot the mean of the PSD for E_x, E_y, and E_z at 200 MHz. It is compared to the corresponding PSD in a cave with no roughness. Note that for the primary transmitted field (E_z), the scattered field fills in nulls which are created by destructive interference in the smooth case. However, due to cross-polarization scattering, the PSD of E_z is in general lower. Also note that in the E_x and E_y lines, the cross-polarization scattering significantly increases their energy further into the cave, as would be expected.

4 Conclusions

We have applied the high order time domain vector finite element methods described in [5], [6] and [8] to the case of RF electric field propagation in a lossy rough wall tunnel. This particular calculation has proved difficult to solve using direct theoretical analysis. We have presented statistical data for the power spectral density and phase of a 200 MHz dipole antenna source and have compared our data in a rough walled cave to one with a smooth surface. Future research will involve the use of a planar ring shaped antenna for direct comparison to the work of [1]. Once the full wave simulation technique is verified against theory, it will allow modeling of cave structures such as bends and forks that are too complex for theoretical techniques.

References

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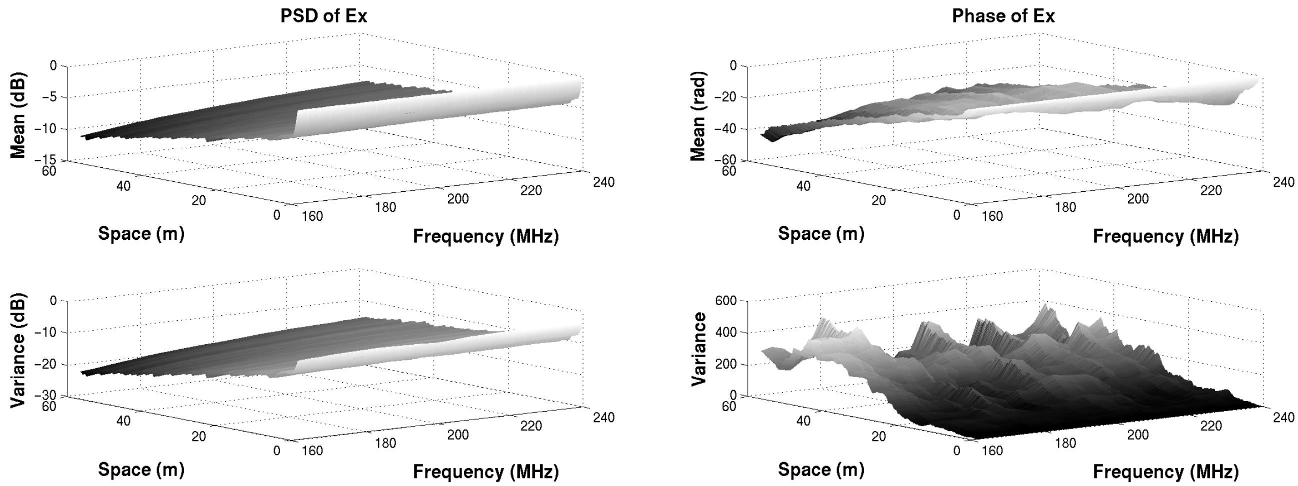

Figure 3: Mean and variance of computed power spectral density (*left*) and phase (*right*) for the electric field vector, normalized with respect to the initial field strength.

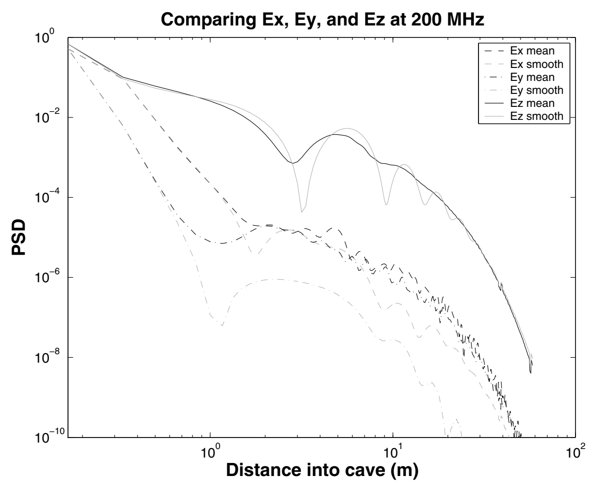

Figure 4: Comparison of power spectral density as a function of propagation distance for a smooth walled cave and a rough walled cave.

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