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Class : BSCS - 4A

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MVC
Assignment #4

Q.1 $\iint_R \frac{1}{1+x^2+y^2} \, dA$

$y=0 \quad y=x, \; x^2+y^2=4$

In polar cordinates
$x = r\cos\theta$
$y = r\sin\theta$

$x^2+y^2=r^2$

$dA = r\,dr\,d\theta$

$$\iint_R \frac{1}{1+r^2} r.\,dr\,d\theta$$

So Now we have:

$r \rightarrow (0, 2)$

$\underbrace{y=0, \; y=x}$ → In the first quadrant
$\theta = \frac{\pi}{4}$

So: $\int_0^{\pi/4} \int_0^2 \frac{1}{1+r^2} r.\,dr\,d\theta$

we solve w.r.t.

$\int_0^2 \frac{1}{1+r^2} r\,dr$

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Now by using Substitution

$$u = 1 + r^2$$
$$du = 2r\,dr$$
$$r\,dr = \frac{du}{2}$$

Now when:

$r = 0,\ r = 1 \Rightarrow u = 1 + (0^2) = 1$

$r = 2,\ r = 5\ \ u = 1 + (2)^2 = 5$

$$\int_1^5 \frac{1}{u}\frac{du}{2} = \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2} \cdot \ln u \Big|_1^5$$

$$= \frac{1}{2}\ln(5-1)$$

$$\int_0 \frac{1}{2}\ln 5\, d\theta$$

$$= \frac{1}{2}\ln 5\,[\theta]_0^{\pi/4}$$

$$= \frac{1}{2}\ln(5)$$

$$= \int_0^{\pi/4} \frac{1}{2}\ln(5)\,d\theta$$

$$= \frac{1}{2}\ln(5)\int_0^{\pi/4} d\theta$$

$$\frac{1}{2}\ln(5)\ \theta\Big|_0^{\pi/4}$$

$$1/2\ \ln(5)\ (\pi/4)$$

$$\boxed{= \pi/8\ \ln(5)}$$

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Q2) $z = 9 - x^2$, $z = 0$, $y^2 = 3x$

$y^2 = 3x$

so Knowing this we know that parabolla opens right

as $\frac{y^2}{3} = x$ $\quad$ $x \geq 0$ so $y^2 \leq 3x$

$y^2 = 3x$

so supposing $y = 3$

$3^2 = 3x$

$x = 3$

Now we have

$$\int_0^3 \int_0^{\sqrt{3x}} 9 - x^2 \, dy \, dx$$

$$\int_0^3 9y - x^2 y \Big|_0^{\sqrt{3x}} dx$$

$$\int_0^3 9\sqrt{3x} - x^2\sqrt{3x}$$

$$\int_0^3 (9 - x^2) \sqrt{3x} \, dx$$

$$\sqrt{3} \int_0^3 (9 - x^2) \, x^{1/2} \, dx$$

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$$\sqrt{3}\left[\int_0^3 9x^{1/2}\,dx + \int_0^3 -x^2 \cdot x^{1/2}\,dx\right]$$

$$\sqrt{3}\left[9\int_0^3 x^{1/2}\,dx\right.$$

$$\sqrt{3}\left[\int_0^3 9x^{1/2}\,dx + \int_0^3 -x^2 \cdot x^{1/2}\,dx\right]$$

$$\sqrt{3}\left[9\int_0^3 x^{1/2}\,dx - \int_0^3 x^{5/2}\,dx\right]$$

$$\sqrt{3}\left[9\,\frac{x^{3/2}}{\frac{3}{2}} - \frac{x^{7/2}}{7/2}\right]_0^3$$

$$= \sqrt{3}\left[6x^{2} - \frac{2x^{7/2}}{7}\right]_0^3$$

$$= \sqrt{3}\left[6(3)^{3/2} - \frac{2}{7}(3)^{7/2}\right]$$

$$= \sqrt{3}\left(18\sqrt{3} - \frac{54}{7}\sqrt{3}\right)$$

$$= \sqrt{3}\cdot\sqrt{3}\left(18 - \frac{54}{7}\right)$$

$$= 3\left(\frac{72}{7}\right)$$

$$\boxed{= \frac{216}{7}}$$

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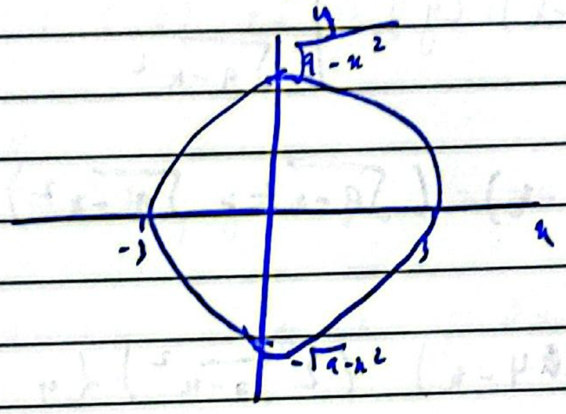
Q3 $z = 1$, $x + z = 5$ $x^2 + y^2 = 9$

$z = 5 - x$ $y = \sqrt{9 - x^2}$



$$\text{volume of } G = \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{1}^{5-x} dz\,dy\,dx$$

$$= \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} z \Big|_{1}^{5-x} dy\,dx$$

$$= \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} 5 - x - 1 \,dy\,dx$$

$$= \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} 4 - x \,dy\,dx$$

$$= \int_{-3}^{3} 4y - xy \Big|_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \, dx$$

$$= \int_{-3}^{3} (4-x)(y) \Big|_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}$$

$$= \int_{-3}^{3} (4-x)\left(\sqrt{9-x^2} + \sqrt{9-x^2}\right)$$

$$= \int_{-3}^{3} (4-x)\left(2\sqrt{9-x^2}\right) dx$$

$$= \int_{-3}^{3} (8-2x)\sqrt{9-x^2} \, dx$$

$$= 8\int_{-3}^{3} \sqrt{9-x^2} \, dx - \int_{-3}^{3} 2x\sqrt{9-x^2} \, dx$$

$\sqrt{9-x^2}$

= with $r \to 3$

$$\frac{1}{2}\pi r^2 = \frac{9\pi}{2}$$

$$= 8\left(\frac{9\pi}{2}\right)$$

$$\boxed{= 36\pi}$$

Q4

$$\int x^2\, dx - y x^2\, dy + 3\, dz$$

A $(0,0,0)$ to B $(1,0,0)$
B $(1,0,0)$ to C $(1,1,0)$
C $(1,1,0)$ to D $(1,1,1)$

$A \to B$

param $x = t$, $y = 0$, $z = 0$ $t(0,1)$

$dx = dt$, $dy = 0$, $dz = 0$

$x^2\, dx - y x^2\, dy + 3\, dz = t^2 (0)\, dt$

$B \to C$

param $x = 1$, $y = t$, $z = 0$

$dx = 0$, $dy = dt$, $dz = 0$

$x^2\, dx - y x^2\, dy + 3\, dz = (1)^2 (0) \cdot 0$

$- t \cdot 1^2\, dt + 0$

$= -t\, dt$

$$\int_{-1/2}^{1} -t\, dt$$

$= -\dfrac{1}{2}$

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$C \to D$

path $x=1, y=1, z=t$ $t \in (0,1)$

$dx = dt$ $dy = 0$ $dz = 0$

$x^2 z\,dx - yx^2 dy + 3dz = t^2(0)\,dt - t^2$
$= 0$

$B \to C$

path $x=1$ $y=t$ $z=0$
$dx=0$ $dy=dt$ $dz=0$

$x^2 z\,dx - yx^2 dy + 3dz = 0 - 0 + 3dt$

$\int_0^1 3dt = 3$

Total $= 0 - 1/2 + 3$
$= 5/2$

36

Segment C_1 from $(0,0,0)$ to $(1,1,0)$
$x=t$ $y=t$ $z=0$ where $0 \le t < 1$
$dx=dt$ $dy=dt$ $dz=0$

so this segment gives
$x^2 z\,dx = t^2 \cdot 0 \cdot dt = 0$
$-yx^2 dy = -t(1^2) \cdot dt = 0$
~~$3dz=0$~~ $3dz = 0$

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$$\int_0^1 t^3 dt = \frac{-t^4}{4}\Big|_1^1 = \frac{-1}{4}$$

Segment C_2 from $(1,1,0)$ to $(1,1,1)$

$x=1,\ y=1,\ z=t \qquad 0 \le t \le 1$

$dx=0,\ dy=0,\ dz=dt$

$x^2 dx=0,\ -yx^2dy=0,\ 3dz=3dt$

$$\int_0^1 3dt = 3$$

Segment C_3 from $(1,1,1)$ to $(0,0,0)$

$x=1-t,\ y=1-t,\ z=1-t$

$dx=-dt,\ dy=-dt,\ dz=-dt$

$x^2 dx = (1-t)^2(1-t)(-dt) = -(1-t)^3 dt$

$-yx^2dy = -(1-t)(1-t)^2(-dt) = (1-t)^3$

$3dz=-3dt$

$$= \int_0^1 [-(1-t)^3 + (1-t^3) - 3]\,dt$$

$$= \int_0^1 -3dt$$

$= -3$

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so total from all segment

$= -1/4 + 3 - 3$

$\boxed{= -1/4}$