

APPLIED MECHANISM DESIGN FOR SOCIAL GOOD

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Lecture #11 – 3/3/2020

CMSC828M

Tuesdays & Thursdays

2:00pm – 3:15pm



COMPUTER SCIENCE
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THIS CLASS: STACKELBERG & SECURITY GAMES

SIMULTANEOUS PLAY

Previously, assumed players would play **simultaneously**

- Two drivers simultaneously decide to go straight or divert
- Two prisoners simultaneously defect or cooperate
- Players simultaneously choose rock, paper, or scissors
- Etc ...

No knowledge of the other players' chosen actions

What if we allow **sequential** action selection ...?

LEADER-FOLLOWER GAMES



Heinrich von
Stackelberg

Two players:

- The **leader** commits to acting in a specific way
- The **follower** observes the leader's mixed strategy

NE, iterated strict dominance

What is the Nash equilibrium ??????????

- Social welfare: 2
- Utility to row player: 1

Row player = leader; what to do ??????????

- Social welfare: 3
- Utility to row player: 2

Commit to "Bottom"	
0, 0	2, 1

ASIDE: FIRST-MOVER ADVANTAGE (FMA)

From the econ side of things ...

- Leader is sometimes called the **Market Leader**
- Some advantage allows a firm to move first:
 - Technological breakthrough via R&D
 - Buying up all assets at low price before market adjusts

By committing to a strategy (some amount of production), can effectively force other players' hands.

Things we won't model:

- Significant cost of R&D, uncertainty over market demand, initial marketing costs, etc.

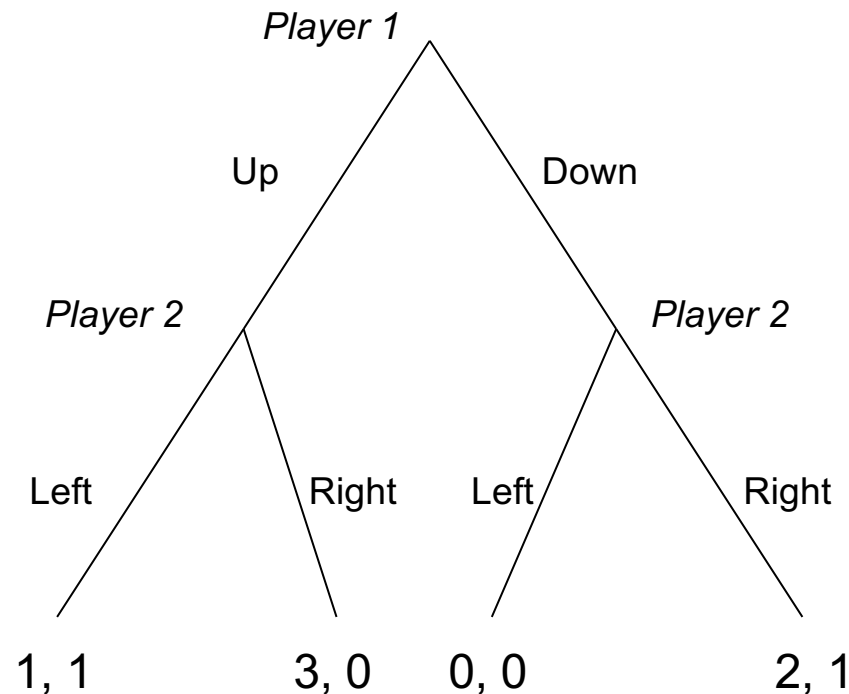
These can lead to **Second-Mover Advantage**

- Atari vs Nintendo, MySpace (or earlier) vs Facebook

COMMITMENT AS AN EXTENSIVE-FORM GAME

For the case of committing to a **pure** strategy:

1, 1	3, 0
0, 0	2, 1



COMMITMENT TO MIXED STRATEGIES

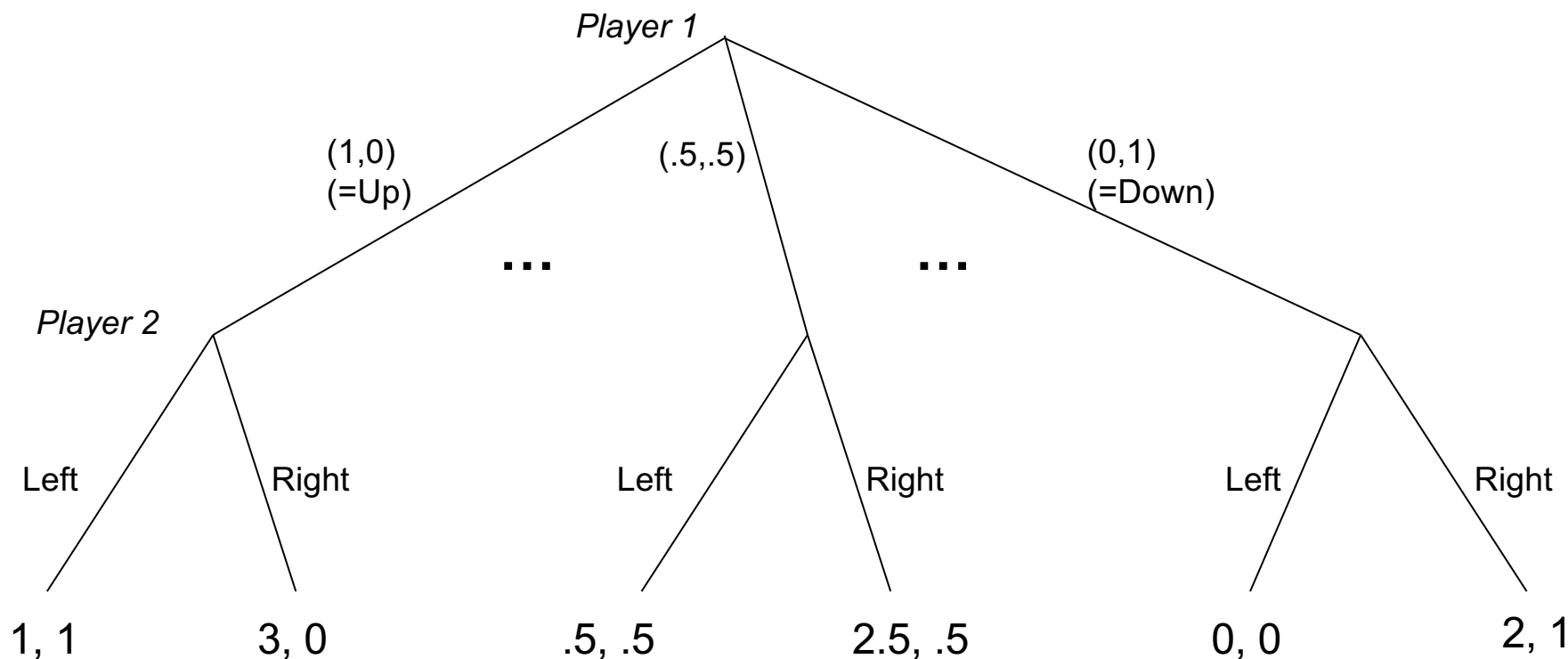
	0	1
.49	1, 1	3, 0
.51	0, 0	2, 1

What should Column do ????????

Sometimes also called a **Stackelberg (mixed) strategy**

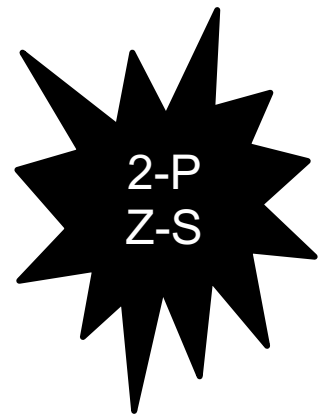
COMMITMENT AS AN EXTENSIVE-FORM GAME...

For the case of committing to a mixed strategy:



- Economist: Just an extensive-form game ...
- Computer scientist: **Infinite-size game!** Representation matters

WHAT SHOULD THE LEADER COMMIT TO?



Special case: 2-player zero-sum normal-form games

Recall: Row player plays Minimax strategy

- Minimizes the maximum expected utility to the Col

Doesn't matter who commits to what, when

Minimax strategies = Nash Equilibrium
 = **Stackelberg Equilibrium**
 (not the case for general games)

Polynomial time computation via LP – earlier lectures

WHAT SHOULD THE LEADER COMMIT TO?



Separate LP for every column c^* :

maximize $\sum_r p_r u_R(r, c^*)$ **Row utility**

s.t.

for all c , $\sum_r p_r u_C(r, c^*) \geq \sum_r p_r u_C(r, c)$ **Column optimality**

$\sum_r p_r = 1$

for all r , $p_r \geq 0$

**Distributional
constraints**

Choose strategy from LP with highest objective

RUNNING EXAMPLE

x	1, 1	3, 0
y	0, 0	2, 1

maximize $1x + 0y$

s.t.

$$1x + 0y \geq 0x + 1y$$

$$x + y = 1$$

$$x \geq 0$$

$$y \geq 0$$

maximize $3x + 2y$

s.t.

$$0x + 1y \geq 1x + 0y$$

$$x + y = 1$$

$$x \geq 0$$

$$y \geq 0$$

IS COMMITMENT ALWAYS GOOD FOR THE LEADER?

Yes, if we allow commitment to mixed strategies

- Always weakly better to commit [von Stengel & Zamir, 2004]

What about only pure strategies?

Expected utility to Row
by playing mixed Nash:

???????????

$$E_R[\langle 1/3, 1/3, 1/3 \rangle] = 0$$

Expected utility to Row by
any pure commitment:

???????????

$$E_R[\langle 1, 0, 0 \rangle] = -1$$

$$E_R[\langle 0, 1, 0 \rangle] = -1$$

$$E_R[\langle 0, 0, 1 \rangle] = -1$$

	Rock	Paper	Scissors
Rock			
Paper	+1, -1	0, 0	-1, +1
Scissors			

WHAT SHOULD THE LEADER COMMIT TO?



Bayesian games: player i draws type θ_i from Θ

Special case: **follower has only one type**, leader has type θ

Like before, solve a separate LP for every column c^* :

$$\text{maximize } \sum_{\theta} \pi(\theta) \sum_r p_{r,\theta} u_{R,\theta}(r, c^*)$$

s.t.

$$\text{for all } c, \sum_{\theta} \pi(\theta) \sum_r p_{r,\theta} u_C(r, c^*) \geq \sum_{\theta} \pi(\theta) \sum_r p_{r,\theta} u_C(r, c)$$

$$\text{for all } \theta, \sum_r p_{r,\theta} = 1$$

$$\text{for all } r, \theta, p_{r,\theta} \geq 0$$

Choose strategy from LP with highest objective

WHAT SHOULD THE LEADER COMMIT TO?



So, we showed **polynomial-time** methods for:

- 2-Player, zero-sum
- 2-Player, general-sum
- 2-Player, general-sum, Bayesian with 1-type follower

In general, **NP-hard** to compute:

- 2-Player, general-sum, Bayesian with 1-type leader
 - Arguably more interesting (“I know my own type”)
- 2-Player, general-sum, Bayesian general
- N -Player, for $N > 2$:
 - 1st player commits, $N-1$ -Player leader-follower game, 2nd player commits, recurse until 2-Player leader-follower

STACKELBERG SECURITY GAMES

Leader-follower → Defender-attacker

- Defender is interested in protecting a set of targets
- Attacker wants to attack the targets

The defender is endowed with a set of resources

- Resources protect the targets and prevent attacks

Utilities:

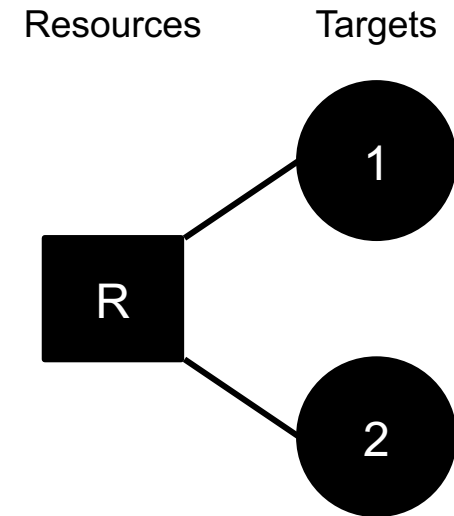
- Defender receives positive utility for preventing attacks, negative utility for “successful” attacks
- Attacker: positive utility for successful attacks, negative otherwise
- Not necessarily zero-sum

SECURITY GAMES: A FORMAL MODEL

Defined by a 3-tuple (N, U, M) :

- **N : set of n targets**
- **U : utilities associated with defender and attacker**
- **M : all subsets of targets that can be simultaneously defended by deployments of resources**
 - A schedule $S \subseteq 2^N$ is the set of target defended by a single resource r
 - Assignment function $A : R \rightarrow 2^S$ is the set of all schedules a specific resource can support
- **Then we have m pure strategies, assigning resources such that the union of their target coverage is in M**
- **Utility $u_{c,d}(i)$ and $u_{u,d}(i)$ for the defender when target i is attacked and is covered or defended, respectively**

SIMPLE EXAMPLE



Targets	Defender		Attacker Type θ_1		Attacker Type θ_2	
i	$u_{c,d}(i)$	$u_{u,d}(i)$	$u_{c,a}(i)$	$u_{u,a}(i)$	$u_{c,a}(i)$	$u_{u,a}(i)$
1	0	-1	0	+1	0	+1
2	0	-2	0	+5	0	+1

REAL-WORLD SECURITY GAMES



Lots of deployed applications!

- Checkpoints at airports
- Patrol routes in harbors
- Scheduling Federal Air Marshalls
- Patrol routes for anti-poachers



Carnegie Mellon

Typically solve for **strong** Stackelberg Equilibria:

- Tie break in favor of the defender; always exists
- Can often “nudge” the adversary in practice

Two big practical problems: **computation** and uncertainty