

APPLIED MECHANISM DESIGN FOR SOCIAL GOOD

JOHN P DICKERSON

Lecture #13 – 3/10/2020

**CMSC828M
Tuesdays & Thursdays
2:00pm – 3:15pm**



COMPUTER SCIENCE
UNIVERSITY OF MARYLAND

PROJECT PROPOSALS

I'd like you to submit a 1-2 pager covering an initial plan for your course project by the end of the week.

How to submit:

- Make a channel on Slack (public or private)
- Invite all group members + @John Dickerson
- Upload the PDF of your initial course project plan
- “@ me”

You will get 100% for this if you submit something “okay” – this is just to kickstart (i) movement and (ii) discussion between us



PROJECT PROPOSALS: A SUGGESTION

Consider a 75%/100%/125% set of goalposts:

Project Plan:

75% goals

- Create and train 3 regressor system for electrical energy consumption dataset.
- Design the adaptive learning algorithm.

100% goals

- Implement the adaptive learning algorithm.
- Apply the algorithm to forecasting electrical energy consumption in the United States problem.
- Compare its performance with baselines which are:
 - Single regressor agent.
 - Multi-agents with equal weights.

125% goals

- Compare this algorithm performance against other techniques used to improve long horizon forecast.
- Test this algorithm performance on other forecasting problems including a forecasting brain ventricular volume as a biomarker for neurodegenerative disease progression.
- Test performance on other decision making problems that are unrelated to forecasting.

THIS CLASS: STACKELBERG & SECURITY GAMES

SIMULTANEOUS PLAY

Previously, assumed players would play **simultaneously**

- Two drivers simultaneously decide to go straight or divert
- Two prisoners simultaneously defect or cooperate
- Players simultaneously choose rock, paper, or scissors
- Etc ...

No knowledge of the other players' chosen actions

What if we allow **sequential** action selection ...?

LEADER-FOLLOWER GAMES



Heinrich von
Stackelberg

Two players:

- The **leader** commits to acting in a specific way
- The **follower** observes the leader's mixed strategy

NE, iterated strict dominance

What is the Nash equilibrium ??????????

- Social welfare: 2
- Utility to row player: 1

Row player = leader; what to do ??????????

- Social welfare: 3
- Utility to row player: 2

Commit to "Bottom"	
0, 0	2, 1

ASIDE: FIRST-MOVER ADVANTAGE (FMA)

From the econ side of things ...

- Leader is sometimes called the **Market Leader**
- Some advantage allows a firm to move first:
 - Technological breakthrough via R&D
 - Buying up all assets at low price before market adjusts

By committing to a strategy (some amount of production), can effectively force other players' hands.

Things we won't model:

- Significant cost of R&D, uncertainty over market demand, initial marketing costs, etc.

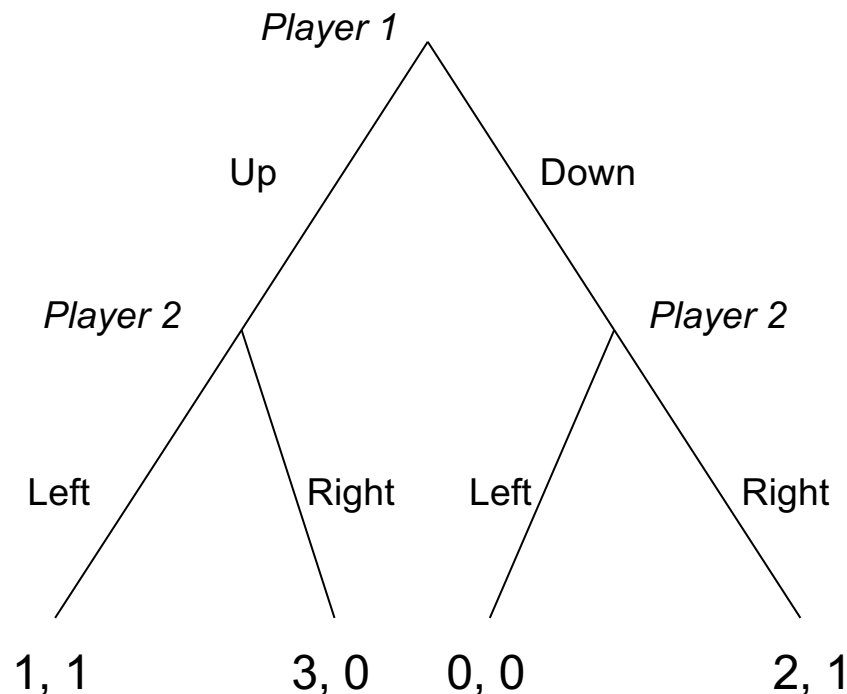
These can lead to **Second-Mover Advantage**

- Atari vs Nintendo, MySpace (or earlier) vs Facebook

COMMITMENT AS AN EXTENSIVE-FORM GAME

For the case of committing to a **pure** strategy:

1, 1	3, 0
0, 0	2, 1



COMMITMENT TO MIXED STRATEGIES

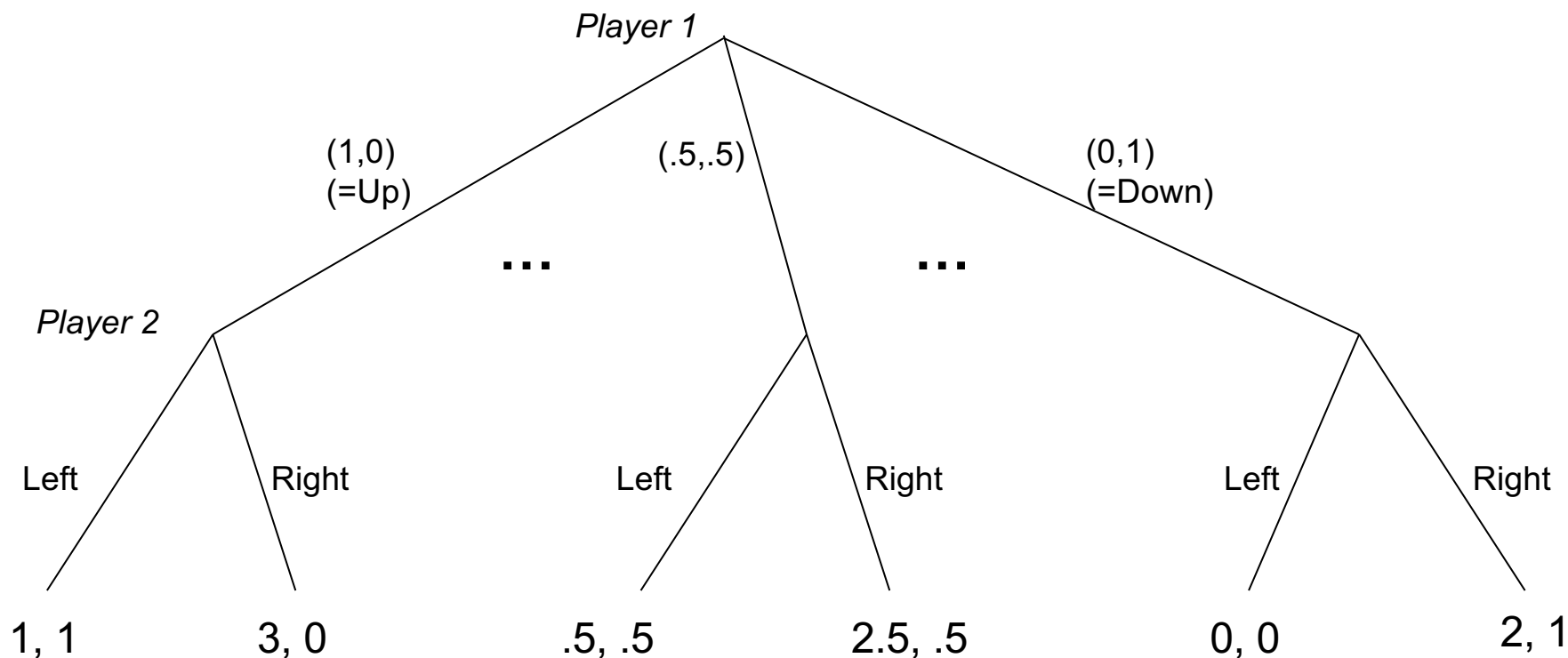
	0	1
.49	1, 1	3, 0
.51	0, 0	2, 1

What should Column do ????????

Sometimes also called a **Stackelberg (mixed) strategy**

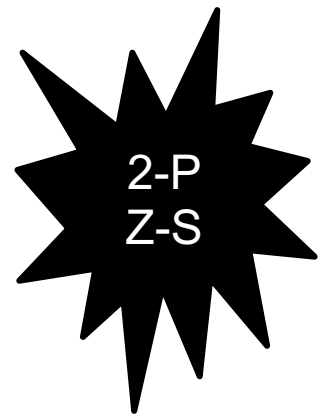
COMMITMENT AS AN EXTENSIVE-FORM GAME...

For the case of committing to a mixed strategy:



- Economist: Just an extensive-form game ...
- Computer scientist: **Infinite-size game!** Representation matters

WHAT SHOULD THE LEADER COMMIT TO?



Special case: 2-player zero-sum normal-form games

Recall: Row player plays Minimax strategy

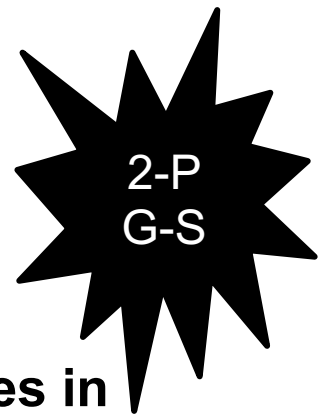
- Minimizes the maximum expected utility to the Col
- Minimax utility: $\min_{\sigma_{-i}} \max_{s_i} u_i(s_i, \sigma_{-i})$

Doesn't matter who commits to what, when

Minimax strategies = Nash Equilibrium
 = **Stackelberg Equilibrium**
 (not the case for general games)

Polynomial time computation via LP – earlier lectures

WHAT SHOULD THE LEADER COMMIT TO?



Strong Stackelberg Equilibrium (SSE): follower breaks ties in favor of the **leader**

Theorem [Conitzer & Sandholm]: In 2-player, general-sum normal-form games, an SSE can be found in polytime

- ?????????????

Idea:

- Iterate over every **follower** pure strategy aka column **c**
- Compute a mixed strategy **r** for **leader** such that playing pure strategy **c** is a best response for **follower**
- Choose **r***, the best (aka highest value for **leader**) mixed strategy amongst those strategies!

WHAT SHOULD THE LEADER COMMIT TO?



Separate LP for every column c^* :

maximize $\sum_r p_r u_R(r, c^*)$ Row utility

s.t.

for all c , $\sum_r p_r u_C(r, c^*) \geq \sum_r p_r u_C(r, c)$

Column optimality
aka Col best response

$\sum_r p_r = 1$

Distributional
constraints

for all r , $p_r \geq 0$

Choose strategy from LP with highest objective

RUNNING EXAMPLE

x	1, 1	3, 0
y	0, 0	2, 1

maximize $1x + 0y$

s.t.

$$1x + 0y \geq 0x + 1y$$

$$x + y = 1$$

$$x \geq 0$$

$$y \geq 0$$

maximize $3x + 2y$

s.t.

$$0x + 1y \geq 1x + 0y$$

$$x + y = 1$$

$$x \geq 0$$

$$y \geq 0$$

IS COMMITMENT ALWAYS GOOD FOR THE LEADER?

Yes, if we allow commitment to mixed strategies

- Always weakly better to commit [von Stengel & Zamir, 2004] ??????
- If (r^*, c) is Nash, then Row can always commit to $r^* \rightarrow$ Col will play c^* , can achieve value of that equilibrium

What about only pure strategies?

Expected utility to Row
by playing mixed Nash:
????????????

$$E_R[\langle 1/3, 1/3, 1/3 \rangle] = 0$$

Expected utility to Row by
any pure commitment:
????????????

$$E_R[\langle 1, 0, 0 \rangle] = -1$$

$$E_R[\langle 0, 1, 0 \rangle] = -1$$

$$E_R[\langle 0, 0, 1 \rangle] = -1$$

	Rock	Paper	Scissors
Rock			
Paper	+1, -1	0, 0	-1, +1
Scissors			

WHAT SHOULD THE LEADER COMMIT TO?



Bayesian games: player i draws type θ_i from Θ

Special case: **follower has only one type**, leader has type θ

Like before, solve a separate LP for every column c^* :

$$\text{maximize } \sum_{\theta} \pi(\theta) \sum_r p_{r,\theta} u_{R,\theta}(r, c^*)$$

s.t.

$$\text{for all } c, \sum_{\theta} \pi(\theta) \sum_r p_{r,\theta} u_C(r, c^*) \geq \sum_{\theta} \pi(\theta) \sum_r p_{r,\theta} u_C(r, c)$$

$$\text{for all } \theta, \sum_r p_{r,\theta} = 1$$

$$\text{for all } r, \theta, p_{r,\theta} \geq 0$$

Choose strategy from LP with highest objective

WHAT SHOULD THE LEADER COMMIT TO?



So, we showed **polynomial-time** methods for:

- 2-Player, zero-sum
- 2-Player, general-sum
- 2-Player, general-sum, Bayesian with 1-type follower

In general, **NP-hard** to compute:

- 2-Player, general-sum, Bayesian with 1-type leader
 - Arguably more interesting (“I know my own type”)
- 2-Player, general-sum, Bayesian general
- N -Player, for $N > 2$:
 - 1st player commits, $N-1$ -Player leader-follower game, 2nd player commits, recurse until 2-Player leader-follower

STACKELBERG SECURITY GAMES

Leader-follower → Defender-attacker

- Defender is interested in protecting a set of targets
- Attacker wants to attack the targets

The defender is endowed with a set of resources

- Resources protect the targets and prevent attacks

Utilities:

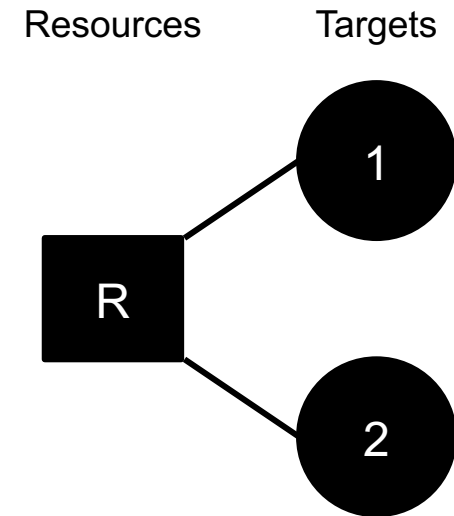
- Defender receives positive utility for preventing attacks, negative utility for “successful” attacks
- Attacker: positive utility for successful attacks, negative otherwise
- Not necessarily zero-sum

SECURITY GAMES: A FORMAL MODEL

Defined by a 3-tuple (N, U, M) :

- **N : set of n targets**
- **U : utilities associated with defender and attacker**
- **M : all subsets of targets that can be simultaneously defended by deployments of resources**
 - A schedule $S \subseteq 2^N$ is the set of target defended by a single resource r
 - Assignment function $A : R \rightarrow 2^S$ is the set of all schedules a specific resource can support
- **Then we have m pure strategies, assigning resources such that the union of their target coverage is in M**
- **Utility $u_{c,d}(i)$ and $u_{u,d}(i)$ for the defender when target i is attacked and is covered or defended, respectively**

SIMPLE EXAMPLE



Targets	Defender		Attacker Type θ_1		Attacker Type θ_2	
i	$u_{c,d}(i)$	$u_{u,d}(i)$	$u_{c,a}(i)$	$u_{u,a}(i)$	$u_{c,a}(i)$	$u_{u,a}(i)$
1	0	-1	0	+1	0	+1
2	0	-2	0	+5	0	+1

REAL-WORLD SECURITY GAMES



Lots of deployed applications!

- Checkpoints at airports
- Patrol routes in harbors
- Scheduling Federal Air Marshalls
- Patrol routes for anti-poachers



Carnegie Mellon

Typically solve for **strong** Stackelberg Equilibria:

- Tie break in favor of the defender; always exists
- Can often “nudge” the adversary in practice

Two big practical problems: **computation** and uncertainty

OVERVIEW OF AN IMPACTFUL PAPER IN THIS SPACE

[Kiekintveld et al. 2009]

Computing Optimal Randomized Resource Allocations for Massive Security Games (linked on course webpage)

- Motivated first by resource assignment for checkpoints at LAX, e.g., multiple canine units assigned to cover multiple terminals ...
- ... and later by much larger games such as Federal Air Marshals Service assignments and port inspection.

m resources to cover n targets, $m < n$

Defender (leader) commits to a mixed strategy

Attacker (follower) observes the probabilities for each coverage set

- Surveillance, insider threat, etc – maybe not perfectly realistic

Attacker chooses a pure strategy

Equilibrium concept not ex post

OVERVIEW OF AN IMPACTFUL PAPER IN THIS SPACE

[Kiekintveld et al. 2009]

Initially assume interchangeable resources (extended in paper, won't cover here)

Assume players are **risk neutral**

One type of follower (attacker)

- Recall: one type of follower \rightarrow PTIME solvable, one LP solved for each pure strategy of follower ...
- ... but the number of pure strategies in some games might be large, e.g., with 100 targets and 10 resources, 1.7×10^{13} !

RUNNING EXAMPLE

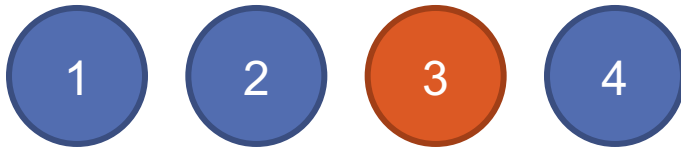
4 targets, 2 resources

Qualitatively:

- Defender values all 4 targets equally (and prefers a covered attack to an uncovered attack).
- Attacker gets twice as much utility for successful attack on target 3. All failed attacks get the same (lower) utility.



MOTIVATION AND INTRODUCTION



Targets {1, 2, 4}		
	Covered	Uncovered
Defender	4	1
Attacker	0	1

“Utility for follower Ψ if attacks target 3 and it is covered (c) / uncovered (u)”

“Utility for leader Θ if the target 3 is attacked and it is covered (c) or uncovered (u)”

$$u_{\Theta}^c(3) \quad u_{\Theta}^u(3)$$

Target 3		
	Covered	Uncovered
Defender	4	1
Attacker	0	2

$$u_{\Psi}^c(3) \quad u_{\Psi}^u(3)$$

COMPACT REPRESENTATIONS OF SECURITY GAMES—EXTENSIVE FORM IS TOO BIG!

Defender commits to a mixed strategy (one of uncountably many, i.e., EFG tree will be infinite size)

$$\begin{aligned} \Delta = (\delta_{12}, \delta_{13}, \delta_{14}, \delta_{23}, \delta_{24}, \delta_{34}) \\ \forall i, j \quad 0 \leq \delta_{ij} \leq 1 \\ \sum_{i,j} \delta_{ij} = m \end{aligned} \quad \left. \vphantom{\begin{aligned} \Delta = (\delta_{12}, \delta_{13}, \delta_{14}, \delta_{23}, \delta_{24}, \delta_{34}) \\ \forall i, j \quad 0 \leq \delta_{ij} \leq 1 \\ \sum_{i,j} \delta_{ij} = m \end{aligned}} \right\} \text{In general, size } \binom{n}{m}$$

Attacker strategy is an efficient algorithm, which given **any** mixed strategy, Δ , computes target

$$\arg \max_{t \in \Gamma(\Delta)} U_{\Theta}(\Delta, t)$$

Where optimization is taken over the **attack set** $\Gamma(\Delta)$, the set of targets yielding max expected payoff for attacker given Δ

$$\Gamma(\Delta) = \{t : t \in \arg \max U_{\Psi}(\Delta, t)\}$$

COMPACT REPRESENTATIONS OF SECURITY GAMES

Key insight: the only information needed to represent the defender strategy is the probabilities a target is covered

$$\delta_{\Theta}^{1,2} + \delta_{\Theta}^{1,3} + \delta_{\Theta}^{1,4} = c_1$$

$$\delta_{\Theta}^{1,2} + \delta_{\Theta}^{2,3} + \delta_{\Theta}^{2,4} = c_2$$

$$\delta_{\Theta}^{1,3} + \delta_{\Theta}^{2,3} + \delta_{\Theta}^{3,4} = c_3$$

$$\delta_{\Theta}^{1,4} + \delta_{\Theta}^{2,4} + \delta_{\Theta}^{3,4} = c_4$$

← In our 2 resources, 4 targets example: probability c_1 that target 1 is covered is sum of all pure strategies that cover 1

This gives us a coverage vector C

- Running example: $C = [c_1, c_2, c_3, c_4]$

ERASER (Efficient Randomized Allocation of SEcurity Resources) **takes security game & computes C that is SSE for defender**

ERASER FORMULATION

$$\begin{array}{llll}
 \max & d & & \\
 a_t \in & \{0, 1\} & \forall t \in T & \\
 \sum_{t \in T} a_t = & 1 & & \\
 c_t \in & [0, 1] & \forall t \in T & \\
 \sum_{t \in T} c_t \leq & m & & \\
 d - U_{\Theta}(t, C) \leq & (1 - a_t) \cdot Z & \forall t \in T & \\
 0 \leq k - U_{\Psi}(t, C) \leq & (1 - a_t) \cdot Z & \forall t \in T & \\
 U_{\Theta}(t, C) = & c_t U_{\Theta}^c(t) + (1 - c_t) U_{\Theta}^u(t) & &
 \end{array}$$

Attacker can assign mass to exactly one target

Defender applies valid (aka at most m) probability mass over targets

↓

(Theorem in paper states how to convert coverage vector to mixed strategy)

ERASER FORMULATION

$$\max \quad d$$

$$a_t \in \{0, 1\} \quad \forall t \in T$$

$$\sum_{t \in T} a_t = 1$$

$$c_t \in [0, 1] \quad \forall t \in T$$

$$\sum_{t \in T} c_t \leq m$$

$$d - U_{\Theta}(t, C) \leq (1 - a_t) \cdot Z \quad \forall t \in T$$

$$0 \leq k - U_{\Psi}(t, C) \leq (1 - a_t) \cdot Z \quad \forall t \in T$$

Expected utility to leader given attack on t
and coverage vector with coverage c_t

$$U_{\Theta}(t, C) = c_t U_{\Theta}^c(t) + (1 - c_t) U_{\Theta}^u(t)$$

Determine the defender's expected payoff d , given the target attacked (a_t)

- **For unattacked targets ($a_t=0$), RHS is huge (i.e., Z)**
- **For attacked target ($a_t=1$), RHS is 0 \rightarrow d = utility of defender given t attacked, and coverage vector C**

Objective: maximize d

ERASER FORMULATION

$$\begin{aligned} \max \quad & d \\ a_t \in \quad & \{0, 1\} \quad \forall t \in T \\ \sum_{t \in T} a_t = \quad & 1 \\ c_t \in \quad & [0, 1] \quad \forall t \in T \\ \sum_{t \in T} c_t \leq \quad & m \end{aligned}$$

$$\begin{aligned} d - U_{\Theta}(t, C) &\leq (1 - a_t) \cdot Z \quad \forall t \in T \\ 0 \leq k - U_{\Psi}(t, C) &\leq (1 - a_t) \cdot Z \quad \forall t \in T \end{aligned}$$

Two bottom sets of constraints imply that defender's coverage vector C is best response to attack vector A , & vice versa

→ Strong Stackelberg Equilibrium

“Big M” (or in this case “Big Z”) style of constraints are a common way to encode if statements

ERASER: RUNNING EXAMPLE (2 RESOURCES, 4 TARGETS)

$$\max d$$

$$s.t.$$

$$a_1 + a_2 + a_3 + a_4 = 1$$

$$c_1 + c_2 + c_3 + c_4 \leq m$$

$$d - 4c_1 + (c_1 - 1) \leq (1 - a_1)Z$$

$$d - 4c_2 + (c_2 - 1) \leq (1 - a_2)Z$$

$$d - 4c_3 + (c_3 - 1) \leq (1 - a_3)Z$$

$$d - 4c_4 + (c_4 - 1) \leq (1 - a_4)Z$$

$$0 \leq k + c_1 - 1 \leq (1 - a_1)Z$$

$$0 \leq k + c_2 - 1 \leq (1 - a_2)Z$$

$$0 \leq k + 2c_3 - 2 \leq (1 - a_3)Z$$

$$0 \leq k + c_4 - 1 \leq (1 - a_4)Z$$

$$c_t \in [0, 1]$$

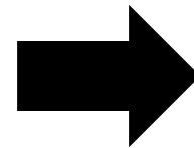
$$a_t \in \{0, 1\}$$

ERASER: RUNNING EXAMPLE (2 RESOURCES, 4 TARGETS)

```
Elapsed time = 0.01 sec. (0.26 ticks, tree = 0.01 MB, solutions = 3)

Root node processing (before b&c):
  Real time      = 0.01 sec. (0.26 ticks)
Parallel b&c, 4 threads:
  Real time      = 0.00 sec. (0.00 ticks)
  Sync time (average) = 0.00 sec.
  Wait time (average) = 0.00 sec.
-----
Total (root+branch&cut) = 0.01 sec. (0.26 ticks)

Solution status = 101 : MIP_optimal
Solution value  = 3.14285714286
Row 0: Slack = 0.000000
Row 1: Slack = 0.000000
Row 2: Slack = 99.142857
Row 3: Slack = 99.142857
Row 4: Slack = 0.000000
Row 5: Slack = 99.142857
Row 6: Slack = 0.000000
Row 7: Slack = 0.000000
Row 8: Slack = 0.000000
Row 9: Slack = 0.000000
Row 10: Slack = 100.000000
Row 11: Slack = 100.000000
Row 12: Slack = 0.000000
Row 13: Slack = 100.000000
Column 0: Value = 3.142857
Column 1: Value = -0.000000
Column 2: Value = -0.000000
Column 3: Value = 1.000000
Column 4: Value = 0.000000
Column 5: Value = 0.428571
Column 6: Value = 0.428571
Column 7: Value = 0.714286
Column 8: Value = 0.428571
Column 9: Value = 0.571429
Coverage vector: [0.428571428571, 0.428571428571, 0.714285714286, 0.428571428571]
Adversary attack vector: [-0.0, -0.0, 1.0, 0.0]
mb_pro_umd:mech ngupta$
```



$$c_1 = c_2 = c_4 = 3/7$$
$$c_3 = 5/7$$

ERASER – RUNNING EXAMPLE

$$\delta_{12} + \delta_{13} + \delta_{14} = 3/7$$

$$\delta_{12} + \delta_{23} + \delta_{24} = 3/7$$

$$\delta_{13} + \delta_{23} + \delta_{34} = 5/7$$

$$\delta_{14} + \delta_{24} + \delta_{34} = 3/7$$

$$0 \leq \delta_{12} \leq 1$$

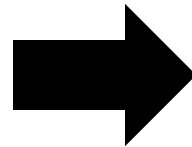
$$0 \leq \delta_{13} \leq 1$$

$$0 \leq \delta_{14} \leq 1$$

$$0 \leq \delta_{23} \leq 1$$

$$0 \leq \delta_{24} \leq 1$$

$$0 \leq \delta_{34} \leq 1$$



$$\delta_{12} = \delta_{14} = \delta_{24} = 2/21$$

$$\delta_{13} = \delta_{23} = \delta_{34} = 5/21$$