

## CS 3120 Exam 1

This packet contains exam 1. This **cover sheet** is here to provide instructions, and to cover the questions until the quiz begins. **do not open this quiz packet** until your proctor instructs you to do so.

You will have 1 hour and 45 minutes to complete this exam. Each quiz is two pages (front and back of one sheet of paper) worth of questions. Make sure to **write your name and computing id at the top of each individual page**.

When you are finished, simply submit this packet at the front of the classroom.

This quiz is CLOSED text book, closed-notes, closed-calculator, closed-cell phone, closed-computer, closed-neighbor, etc. Questions are worth different amounts, so be sure to look over all the questions and plan your time accordingly. Please sign the honor pledge below.

*A crash reduces  
Your expensive computer  
To a simple stone.*

**Module 1: Computers and Cardinality****Name** \_\_\_\_\_

1. [6 points] Answer the following True/False questions.

$ \{0, 1\}^\infty  >  \mathbb{N} $	<b>True</b>	<b>False</b>
Computers (as we defined them) can only understand <i>binary strings</i>	<b>True</b>	<b>False</b>
For a computing model, it is allowed that $\Sigma = \mathbb{Z}$	<b>True</b>	<b>False</b>
It is not possible to implement every function (from strings to strings) in <i>python</i>	<b>True</b>	<b>False</b>
All finite sets have an <i>injection</i> to $\mathbb{N}$	<b>True</b>	<b>False</b>
All finite sets have a <i>surjection</i> to $\mathbb{N}$	<b>True</b>	<b>False</b>

2. [2 points] In class we said the following: For any set  $A$ ,  $|A| < |\mathbb{N}|$  if there is a bijection between  $A$  and  $[k] = \{n : n \in \mathbb{N} \wedge n < k\}$ . Explain what this means in your own words.
3. [2 points] Suppose I take an uncountably infinite set  $A$  and a countably infinite set  $B$ . What can I say about the size of  $A \cap B$ ? Explain your answer.

**Module 2: Regular Languages****Name** \_\_\_\_\_

4. [6 points] Answer the following True/False questions.

$\{1^*0^n1^n0^* \mid n \geq 0\}$  is a regular language because we can always set  $n = 0$ , reducing the language to just  $1^*0^*$  **True** **False**

It is possible for a *DFA* to have at least one accept state ( $|F| > 0$ ) and for the language of that *DFA* to be the empty set ( $\emptyset$ ) **True** **False**

The *regular languages* are closed under *intersection* **True** **False**

Any *regular language* can be expressed using an *NFA*, but some *regular languages* cannot be expressed with a *DFA* (an *NFA* is required) **True** **False**

All regular expressions can be represented using only the *union*, *concatenation*, and *star* operators **True** **False**

*DFAs* are required to have a *finite* number of states ( $|Q| < |\mathbb{N}|$ ) **True** **False**

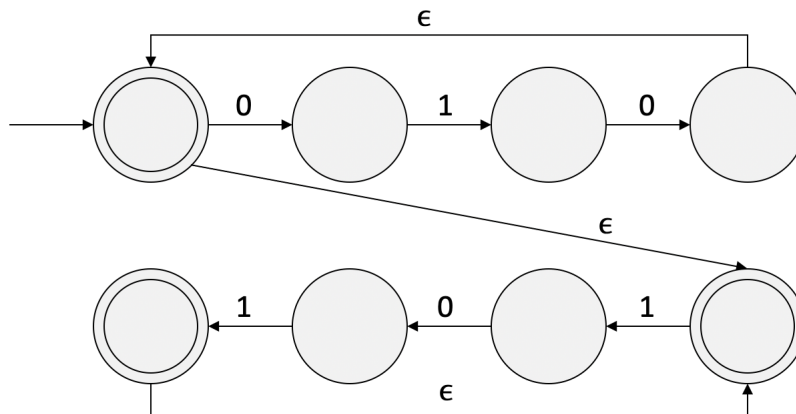
5. [2 points] Draw a working *DFA* or *NFA* for the language represented by the following regular expression:  
 $0^*110^* \cup 1^*001^*$

6. [2 points] Prove that the regular languages are closed under complement. Recall that if  $A$  is a language of strings, then  $\bar{A} = \{w \in \Sigma^* \mid w \notin A\}$

**Miscellaneous**

7. [3 points] Consider the following claim regarding the pumping lemma: *If a language  $A$  is regular then it has some pumping length  $p$ . Given a string  $w \in A$  such that  $|w| > p$ , any substring of  $w$  that has length  $p$  must contain a substring  $y$  that can be pumped.* Is this claim true or false. Explain your answer.

8. [3 points] Look at the following NFA and give a regular expression for the language recognized by this NFA.



9. [3 points] Consider the following set: *The Real numbers strictly between 0 and 1.* Is this set countably or uncountably infinite? If the former, show a bijection to  $\mathbb{N}$ . If the latter, prove by diagonalization.