

## Context Free Languages

- For each of the languages below, provide a context-free grammar that generates it (Note that some of these might also be regular languages, but we still want a grammar for each). For all parts,  $\Sigma = \{0, 1\}$ :
  - Strings that contain exactly two 1's OR exactly two 0's
  - Strings of even length that contain 1100 directly in the center (i.e.,  $w1100u \mid |w| = |u|$ )
  - $ww^Ruu^R \mid w \in \Sigma^* \wedge u \in \Sigma^*$
- Draw PDAs for each of the languages in the previous exercise (note that you can draw a DFA / NFA if the language happens to be regular).
- For this question, you will prove that context-free languages are NOT closed under intersection. The alphabet for all languages in this question is  $\Sigma = \{a, b, c\}$ . Do this by showing the following:
  - Part 1:** First, show that  $A = \{a^m b^n c^n \mid m, n \geq 0\}$  is context-free by producing a context-free grammar that generates it.
  - Part 2:** Do the same, but for language  $B = \{a^n b^n c^m \mid m, n \geq 0\}$
  - Part 3:** Lastly, find the intersection of these two sets and use the pumping lemma to show that the intersection language is not context-free.
- Let us define a new operation using the  $\diamond$  symbol as such: if  $A$  and  $B$  are languages, then  $A \diamond B = \{xy \mid x \in A, y \in B, |x| = |y|\}$ . Prove that if  $A$  and  $B$  are regular languages, then  $A \diamond B$  must be a context-free language.
- Suppose you have a context-free language  $C$  and a regular language  $R$ . Prove that  $C \cap R$  is context-free.
- Consider the language  $A = \{w \mid w \in \{a, b, c\}^* \wedge F(w, a) = F(w, b) = F(w, c)\}$  where  $F(w, a)$  counts the number of occurrences of character  $a$  in string  $w$ . Prove that  $A$  is not context-free by using the result of the previous question (*Hint: Assume this language is a CFL and intersect it with a regular language of your choice!*)
- Prove that the language  $A$  from the previous question is not context-free again, but this time do so by utilizing the *Pumping Lemma for Context-Free Languages*.