

Regular Languages

1. Draw out *DFA*s (not *NFA*s) for each of the following languages. For some of these, a small hint is provided. Your goal is to construct a *DFA* with as few states as possible (just like how we prefer to write succinct code when possible). For all of these, let $\Sigma = \{a, b\}$
 - $\{w \mid w \text{ does not contain the substring } abba\}$ (**Hint: Draw out the DFA for a simpler language that DOES contain abba and then try to change that machine slightly.**)
 - $\{w \mid w \text{ contains BOTH the substrings } ab \text{ and } ba\}$
 - $\{w \mid w \in a^*b^*a^*\}$
 - $\{w \mid w \neq ab \wedge w \neq bb\}$
 - $\{w \mid w \in a^i w' \mid i \in \mathbb{N}, w' \in \{a, b\}^*, w' \text{ contains at least } i \text{ a's}\}$ (**Hint: This one LOOKS not regular but it actually is. Can you figure out why?**)
2. Prove that regular languages are closed under *intersection*. Do this by starting with *DFA*s for two regular languages A and B , and describe how to construct a new *DFA* for $A \cap B$
3. Prove that regular languages are closed under *complement*. Do this by starting with a *DFA* for a regular language A , and describe how to construct a new *DFA* for \bar{A} .
4. For any string $w = w_1 w_2, \dots, w_n$, let w^R be the reverse of string w (i.e., $w^R = w_n, \dots, w_2, w_1$). Prove that if a language A is regular, then the language $A^R = \{w^R \mid w \in A\}$ is also regular.
5. Use the pumping lemma to show that the following languages are not regular OR argue that they are regular.
 - $A = \{0^* 0^n 1^n 1^* \mid n \geq 0\}$
 - $B = \{www \mid w \in \{0, 1\}^*\}$
6. Find and describe the error that exists in the following proof. The proof attempts to show that $0^* 1^*$ is not regular, when in fact it is:

Assume, for sake of contradiction, that $0^ 1^*$ is regular. We select an element from this language that is greater than the pumping length p . We select $0^p 1^p$. In class, when proving that $0^n 1^n$ was not regular, we showed that $0^p 1^p$ cannot be pumped. Therefore, $0^* 1^*$ is not regular.*