

Tutorial 1

COMP 3601

Design and Analysis of Algorithms

Question 1

Use mathematical induction to prove the following

a)

$$\sum_{i=1}^n (2i - 1) = n^2 \quad \text{for all positive integers } n \geq 1$$

b)

$$\sum_{i=1}^n i(i + 1) = \frac{n(n + 1)(n + 2)}{3} \quad \text{for all positive integers } n \geq 1$$

Question 2

Compute the time complexity of the following code fragments.

a)

```
int sum = 0, i, j;
for (i = 0; i < n; i++)
    for (j = 0; j < n; j++)
        sum++;
```

b)

```
int sum = 0, i, j;
for (i = 1; i <= n; i++)
    for (j = 1; j <= i; j++)
        sum++;
```

Question 3

- a) Use the definition of the big-Oh notation, O , to show that

$$100n + 5 \in O(n)$$

- b) Use the definition of the big-Omega notation, Ω , to show that

$$100n + 5 \in \Omega(n)$$

- c) Parts (a) and (b) give an upper and a lower bound of the same order. Determine the tightest asymptotic bound for $100n + 5$. Support your answer by stating suitable constants and a value of n_0 .