

Tutorial 4

COMP 3601

Design and Analysis of Algorithms

Question 1

The average-case number of key comparisons made by the bubble sort algorithm is

$$C(n) = \frac{1}{n-1} \sum_{i=1}^{n-1} C(i), \text{ where } C(i) = \sum_{j=n-i}^{n-1} j$$

Show that $C(n) = \frac{n^2}{3} - \frac{n}{6} \in \Theta(n^2)$.

Question 2

Use the Master Theorem to determine the asymptotic order of growth for each recurrence. Assume $n = 2^k$ and $T(1) = 1$.

- a) $T(n) = 4T(n/2) + n$
- b) $T(n) = 8T(n/2) + n^2$
- c) $T(n) = 4T(n/2) + n^3$

Question 3

Use the Master Theorem to determine the asymptotic order of growth for the recurrence $M(n) = 143640M(n/70)$. Assume $n = 70^k$ and $M(1) = 1$.

Question 4

For successful search, the average number of key comparisons made by the binary search algorithm is $C(n) = \frac{1}{n} \sum_{i=1}^k i 2^{i-1}$. Show that

$$C(n) = k - 1 + \frac{k}{n} \approx \log_2(n+1) - 1 \approx \log_2 n \in \Theta(\log_2 n). \text{ for } n = 2^k - 1.$$