

# Tutorial 4

## COMP 3601

### Design and Analysis of Algorithms

#### Question 1

The average-case number of key comparisons made by the bubble sort algorithm is

$$C(n) = \frac{1}{n-1} \sum_{i=1}^{n-1} C(i), \text{ where } C(i) = \sum_{j=n-i}^{n-1} j$$

Show that  $C(n) = \frac{n^2}{3} - \frac{n}{6} \in \Theta(n^2)$ .

#### Question 2

Use the Master Theorem to determine the asymptotic order of growth for each recurrence. Assume  $n = 2^k$  and  $T(1) = 1$ .

- a)  $T(n) = 4T(n/2) + n$
- b)  $T(n) = 8T(n/2) + n^2$
- c)  $T(n) = 4T(n/2) + n^3$

#### Question 3

Use the Master Theorem to determine the asymptotic order of growth for the recurrence  $M(n) = 143640M(n/70)$ . Assume  $n = 70^k$  and  $M(1) = 1$ .

#### Question 4

For successful search, the average number of key comparisons made by the binary search algorithm is  $C(n) = \frac{1}{n} \sum_{i=1}^k i2^{i-1}$ . Show that

$$C(n) = k - 1 + \frac{k}{n} \approx \log_2(n+1) - 1 \approx \log_2 n \in \Theta(\log_2 n). \text{ for } n = 2^k - 1.$$