Hessian2DFilters:
... A Clean Multiscale Hessian-Based Filtering Framework (2D)

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1 Overview

This document describes `hessian2DFilters`, a clean and well-tested implementation of multi-scale Hessian-based filters for 2D images.

The framework is designed for *geometric feature detection*, not classification or segmentation, and explicitly excludes tensor-based or nonlinear multiscale evolution methods such as MFAT or RLINE.

Core principles:

- Stateless per-scale evaluation
- Max-over-scales aggregation
- Scalar responses derived from Hessian eigenvalues
- Clear separation of algorithm families

2 Supported Filters

Table 1: Supported Hessian-Based Filters

FilterType	Detects	Typical Applications
vesselness	Tubular structures	Blood vessels, pipes
ridge	Line-like ridges	Curvilinear features
neuriteness	Curvilinear continuity	Neurites, filaments
blob	Isotropic blobs	Vesicles, spots
plate	Thick elongated regions	Membranes, bands

Non-goals: MFAT, RLINE, tensor FA, scale summation, logical gating.

3 Algorithm Description

For each Gaussian scale σ, the algorithm performs:

1. Gaussian smoothing of the image
2. Computation of the Hessian matrix

$$H = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}$$

3. Eigenvalue and eigenvector decomposition
4. Evaluation of a scalar response function
5. Max-over-scales aggregation

This design ensures predictable behavior and avoids inter-scale feedback.

4 Eigenvalue Convention

Eigenvalues are interpreted as:

$$|\lambda_1| \leq |\lambda_2|$$

For bright structures on a dark background, line-like geometry corresponds to:

$$\lambda_2 < 0$$

Eigenvalue ordering required by specific filters (e.g. Frangi vesselness) is enforced locally within the response function.

5 Filter-Specific Invariants

Table 2: Mathematical Invariants of Each Filter

Filter	**Guarantees**	**Does Not Guarantee**
Vesselness	Tubes > background, blob suppression	Thin > thick, binary output
Ridge	Structured regions > background	Blob suppression
Neuriteness	Orientation sensitivity	Blob suppression
Blobness	Blob centers > background	Zero response on lines
Plateness	Thick > thin elongated regions	Blob suppression

6 Parameter Tuning

6.1 Scale Selection

Rule of thumb:

$$\text{structure width} \approx 2\sigma$$

Table 3: Typical Scale Ranges

Structure	**Suggested σ Range**
Thin filaments	0.5 – 2
Vessels	1 – 6
Thick bands	3 – 8

6.2 Filter Parameters

- **Vesselness:**
  - β (tube vs blob discrimination): 0.3–0.7
  - c (noise suppression): 10–20
- **Ridge / Plate:**
  - α (anisotropy penalty): ≈ 0.5

7 Example Figures

7.1 Synthetic Test Image

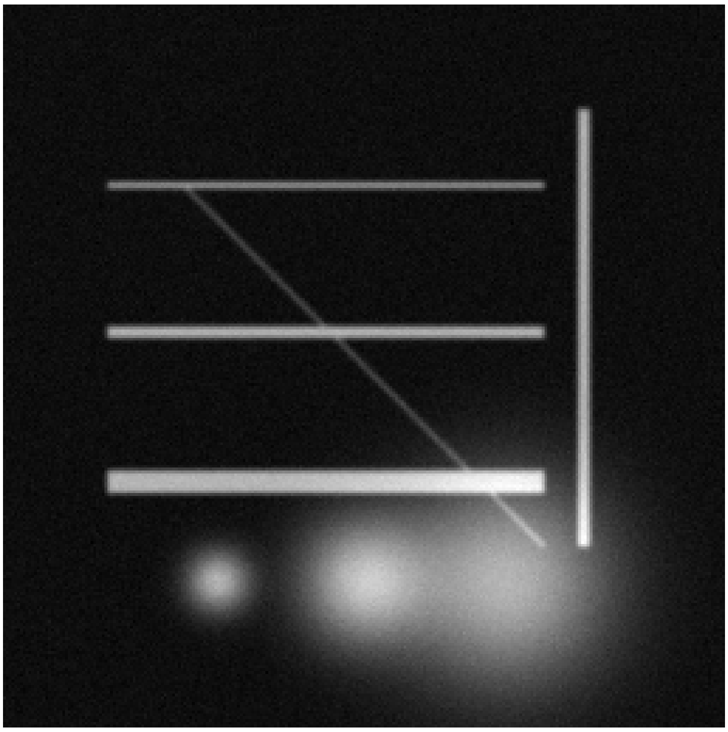
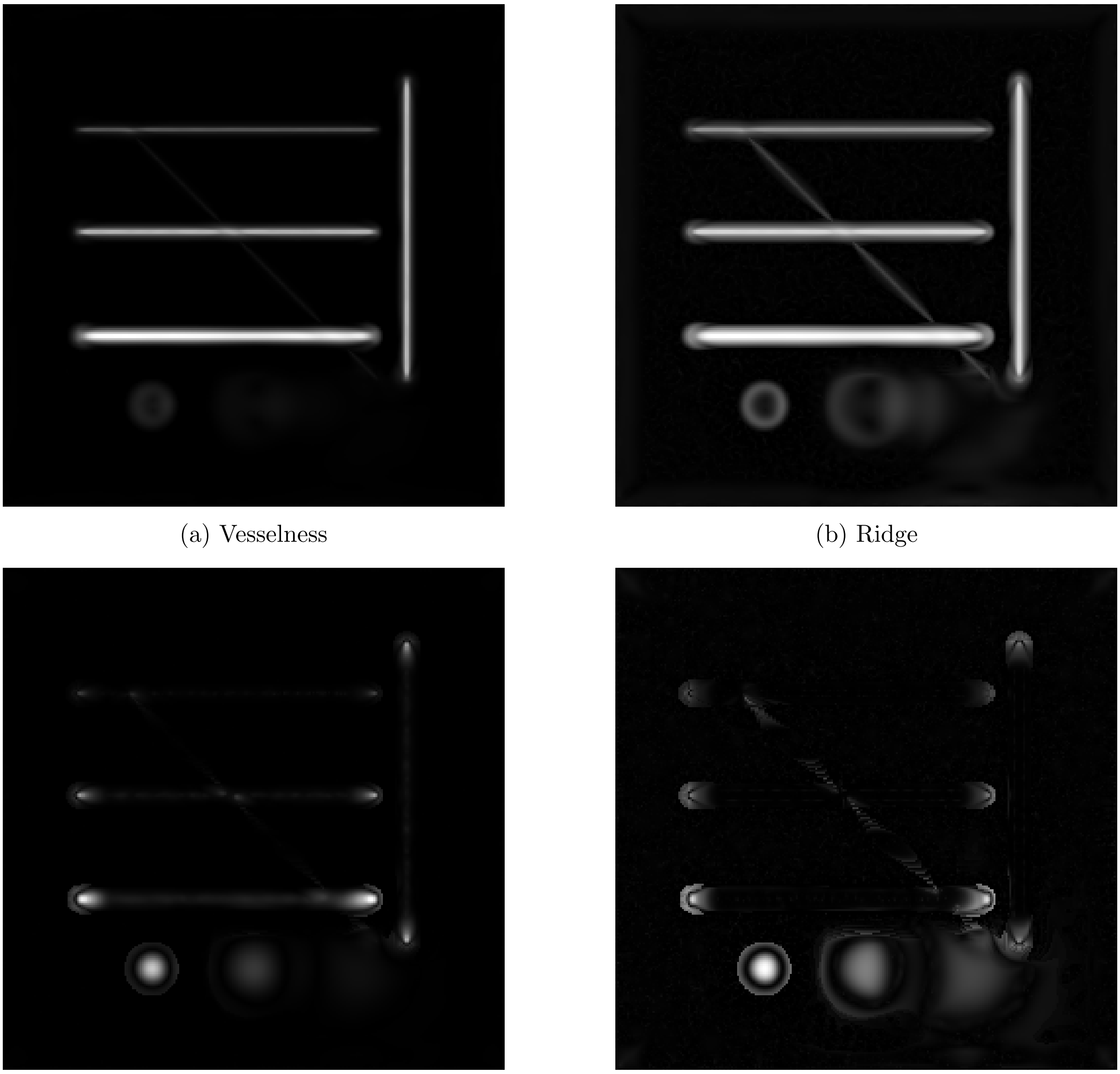


Figure 1: Synthetic test image with thin, medium, and thick lines, diagonal structures, blobs of varying size, and noise.

7.2 Filter Responses



(a) Vesselness

(b) Ridge

(c) Blobness

(d) Plateness

Figure 2: Example responses of Hessian-based filters applied to the test image. Each filter highlights different geometric structures.

8 Unit Testing

The implementation is accompanied by a comprehensive unit test suite that verifies:

- Eigenvalue ordering correctness
- Filter-specific geometric invariants
- Parameter robustness
- Performance regression bounds

Tests are executed using:

```
results = runtests('tests');
```

9 Non-Goals

This framework does **not** implement:

- MFAT (Alhasson / Obara)
- RLINE / flux-based methods
- Tensor FA evolution
- Logical classification or segmentation
- Scale summation or nonlinear inter-scale feedback

Such methods require a fundamentally different algorithmic structure.

10 References

1. A. F. Frangi et al., *Multiscale Vessel Enhancement Filtering*, MICCAI, 1998.
2. Y. Sato et al., *Three-dimensional multi-scale line filter*, Medical Image Analysis, 1998.
3. E. Meijering et al., *Design and validation of a tool for neurite tracing*, Cytometry A, 2004.
4. T. Lindeberg, *Scale-Space Theory in Computer Vision*, IJCV, 1998.