Homework 1 Mark Garas CMSC448

Problem 1

a).

$$\overset{A}{\begin{bmatrix}1 & 2\\ 2 & 4\end{bmatrix}} \times \overset{B}{\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}} = \begin{bmatrix}7 & 10\\ 14 & 20\end{bmatrix}$$

$1\cdot1+2\cdot3=7$

$2\cdot1+2\cdot4=10$

$2\cdot1+12=14$

$2\cdot2+4\cdot4=20$

b).

$$x^T=[2\ \ 1] * \begin{bmatrix}1 & 2\\ 2 & 4\end{bmatrix} = [4\ \ 8] = x^T A$$

$$2+2 \quad 4+4 = [4\ \ 8]\begin{bmatrix}2\\1\end{bmatrix} = [16]$$

$8+8$

c).

$$[2\ \ 1]\begin{bmatrix}2\\1\end{bmatrix} = [5]$$

d).

$$\begin{bmatrix}2\\1\end{bmatrix}[2\ \ 1] = \begin{bmatrix}4 & 2\\ 2 & 1\end{bmatrix}$$

e).

$$\frac{v\cdot v_1}{v_1\cdot v_1}v_1 + \frac{v\cdot v_2}{v_2\cdot v_2}v_2 + \cdots +$$

$$\frac{(2,1)\cdot(1,2)}{(1,2)\cdot(1,2)}\cdot(1,2) + \frac{(2,1)\cdot(2,4)}{(2,4)\cdot(2,4)}\cdot(2,4)$$

$$\frac{4}{5}(1,2) + \frac{8}{20}(2,4)$$

$$\left(\frac{4}{5}, \frac{8}{5}\right) + \left(\frac{4}{5}, \frac{8}{5}\right)$$

$$\left(\frac{8}{5}, \frac{16}{5}\right)$$

f).

$$f(z) = z^T A z$$

$$(z_1, z_2)\begin{pmatrix}a_{11} & a\\ a & a_{22}\end{pmatrix}\begin{pmatrix}z_1\\ z_2\end{pmatrix} =$$

$$\underbrace{(z_1, z_2)\begin{pmatrix}a_{11}z_1 + a z_2\\ a z_1 + a_{22}z_2\end{pmatrix}}$$

$$a_{11}z_1^2 + 2a z_1 z_2 + a_{22}z_2^2 = f(z_1, z_2)$$

$$\begin{pmatrix}df/dz_1\\ df/dz_2\end{pmatrix} = \begin{pmatrix}2a_{11}z_1 + 2a z_2\\ 2a z_1 + 2a_{22}z_2\end{pmatrix}$$

$$2\begin{pmatrix}a_{11} & a\\ a & a_{22}\end{pmatrix}\begin{pmatrix}z_1\\ z_2\end{pmatrix}$$

$$2Az$$

g).

Problem 2

a). $X \sim N(1,2)$

$E[X] = 1$

$E[X^2] - E[X]^2 = 2$

b). Berroulli

c). Using berroulli, which is $p^{x}(1-p)^{1-x_i}$, $i=1,2,\dots,n$

its a function of, $f(x_1 \dots n) = p^{\sum_{i=1}^{n} x_i}(1-p)^{n-\sum_{i=1}^{n} x_i}$

$$l(p) = \ln\left(p^{\sum_{i=1}^{n} x_i}(1-p)^{n-\sum_{i=1}^{n} x_i}\right) \rightarrow \sum_{i=1}^{n} x_i \ln(p) + \left(n - \sum_{i=1}^{n} x_i\right) \cdot \ln(1-p)$$

$$\frac{d}{dp} l(p) = \sum_{i=1}^{n} x_i \cdot \frac{1}{p} + \left(n - \sum_{i=1}^{n} x_i\right) \cdot \left(\frac{-1}{1-p}\right)$$

$$\left(\frac{\sum_{i=1}^{n} x_i}{p}\right) - \left(\frac{n - \sum_{i=1}^{n}}{1-1}\right) = 0$$

$$(1-p)\left(\sum_{i=1}^{n} x_i\right) - (p)\left(n - \sum_{i=1}^{n} x_i\right) = 0$$

$$\sum_{i=1}^{n} x_i - \cancel{p \sum_{i=1}^{n} x_i} - pn + \cancel{p \sum_{i=1}^{n} x_i} = 0$$

$$\sum_{i=1}^{n} x_i - pn = 0$$

$$p = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{8}{14}$$

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Homework 1

Problem 3

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 1 & 1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 0 & -1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & -2 & 2 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

rank=2

Problem 4

see photo

```
In [2]: import numpy as npLib
        a = [[3, 1, 1], [2, 4, 2], [-1, -1, 1]]
        eValue, eVector = npLib.linalg.eig(a)
        eVectorInverse = npLib.linalg.inv(eVector)
        Lambda = npLib.diag(eValue)
        print(a)
        print(eValue)
        print(eVector)
        print(eVectorInverse)
        print(Lambda)

[[3, 1, 1], [2, 4, 2], [-1, -1, 1]]
[4. 2. 2.]
[[-0.40824829 -0.78064311 -0.40288743]
 [-0.81649658  0.18308436 -0.41358579]
 [ 0.40824829  0.59755874  0.81647322]]
[[-1.22474487 -1.22474487 -1.22474487]
 [-1.53716959  0.52138102 -0.49440756]
 [ 1.73741049  0.23080269  2.19901587]]
[[4. 0. 0.]
 [0. 2. 0.]
 [0. 0. 2.]]
```

Problem 5

$$f(x) = \ln(1+e^{-2x})$$

chain rule

$$\frac{1}{1+e^{-2x}} \underbrace{\frac{d}{dx}\left(1+e^{-2x}\right)}$$

$$\frac{1}{1+e^{-2x}} \cdot -2e^{-2x} = \frac{-2e^{-2x}}{1+e^{-2x}}$$

Problem 6

As shown in Problem 1 f, $(f(x) = x^T A x)$'s gradient is $2Ax$, so $(\frac{1}{2}$ of $f(x))$'s gradient would be Ax. $b^T x$, after deriving in terms of x simply becomes b after the transposition and dropping the x.

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Problem 7

a). $[-4,4] \to R$ $\qquad g(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 6x + \frac{27}{2}$

$\bigstar \to g'(x) = \frac{3}{2}x^2 - x - 6$

$0 = \frac{3}{2}x^2 - x - 6 \qquad 6 = \frac{3}{2}x^2 - x$

$6 = x\left(\frac{3}{2}x - 1\right)$

b). $\int_0^1 \left(\frac{3}{2}x^2 - x - 6\right) dx =$

$x = 2.361$
-1.694

both between $[-4, 4]$

$\left(\frac{3}{2}1^2 - 1 - 6\right) - (-6)$

-4	function	-2.5	local min
-1.694	⇨	19.799	local max
2.361		3.127	
4		13.5	