Factor completely the given polynomial expressions.

1)
$$x^6 - 64$$

2)
$$-8x^6 - 63x^3 + 8$$

Factored Form:
$$(x-2)(x+2)(x^2-2x+4)(x^2+2x+4)$$

Factored Form:
$$-(x+2)(2x-1)(x^2-2x+4)(4x^2+2x+1)$$

Remainder Theorem

Give the remainder of each of the following expressions using remainder theorem. В.

1)
$$(x^2 - 3x + 2) \div (2x - 1)$$

2)
$$(2x^3 + 9x^2 + 13x + 6) \div (2x - 3)$$

Remainder: 12

Remainder: $\frac{105}{2}$

Factor Theorem

 $\mathbf{C}.$ State if the given binomial is a factor of the given polynomial.

1)
$$(4x^3 + 2x^2 - 14x - 12) \div (-x - 2)$$

2)
$$(-2x^5 + 5x^4 + 9x^3 - 19x^2 + x + 6) \div (2x + 2)$$

Answer: Not a Factor

Answer: Not a Factor

Rational Root Theorem

Identify the nature of the roots (table of variations), the number of roots (FTA), possible D. roots, actual roots and the factored form of the given polynomial.

1)
$$f(x) = -x^4 - 3x^3 - x^2 + 3x + 2$$

2)
$$f(x) = x^4 - 2x^3 - 4x^2 + 2x + 3$$

FTA: Atmost 4 Factored form: $-(x-1)(x+1)^2(x+2)$ Actual roots: -2, -1 mul. 2, 1

FTA: Atmost 4

Factored form: $(x-3)(x-1)(x+1)^2$

Actual roots: -1 mul. 2, 1, 3

Graphing Polynomial

Give the possible roots (RRT), nature of roots (DRS), number of roots (FTA), factored $\mathbf{E}.$ form, actual roots, end behavior and graph of the given polynomial.

1)
$$f(x) = x^3 + 3x^2 - x - 3$$

2) $f(x) = x^5 - 7x^4 + 18x^3 - 22x^2 + 13x - 3$

FTA: Atmost 3

Factored form: (x-1)(x+1)(x+3)

Actual roots: -3, -1, 1

End Behavior:

$$\begin{array}{l} f(x) \to -\infty \ as \ x \to -\infty \\ f(x) \to \infty \ as \ x \to \infty \end{array}$$

Graph:

FTA: Atmost 5

Factored form: $(x-3)(x-1)^4$ Actual roots: 1 mul. 4, 3

End Behavior:

$$f(x) \to -\infty \ as \ x \to -\infty$$

$$f(x) \to \infty \ as \ x \to \infty$$

Graph: