Homework 3 (Due on Oct 7th, 2022)

1. This set of exercise uses data from the file entitled QUARTERLY.xls in order to estimate the dynamic interrelationships among the level of industrial production, the unemployment rate, and interest rate. Create the interest rate spread (y_t) as the difference between the 10-year rat and the T-bill rate, and create the logarithmic change in the index of industrial production (ip) as $\Delta lip_t = \ln(ip_t) - \ln(ip_{t-1})$ and the seasonal difference of the unemployment rate as $\Delta_4 ur_t = ur_t - ur_{t-4}$.

a. Plot Δlip_t, $\Delta_4 ur_t$, and y_t and create summary statistics. Are they stationary? Apply formal unit root tests to confirm.

b. Estimate the three-variable VAR using eight lags of each variable and a constant and save the residuals. Explain why the estimation cannot be begin earlier than 1963Q1. What are the potential advantages of using the variables Δlip_t and $\Delta_4 ur_t$ instead of ip_t and ur_t?

c. Verify that $\ln(|\Sigma_8|) = -13.968$ and (assuming normality) that the log of the likelihood function is 493.647. Calculate the multivariate AIC and SBC using the formulas $AIC = Tln(|\Sigma|) + 2N$ and $SBC = Tln(|\Sigma|) + Nln(T)$. Calculate the multivariate AIC and SBC using $AIC^* = -2ln(L)/T + 2n/T$ and $SBC^* = -2ln(L)/T + nln(T)/T$.

d. Estimate the model using three lags of each variable and save the residuals. Show that the AIC selects the eight-lag model and that the SBC selects the three-lag model. Show that the same ambiguity applies to the AIC* and SBC*. Why is it important to estimate the three-variable VAR beginning with 1963Q1?

e. Conduct the likelihood ratio test for the null hypothesis of three lags against the alternative of eight lags. How many restrictions are there in the system? How many regressors are there in each of the unrestricted equations? If you answer correctly, you should find that the calculated value χ^2 with 45 degrees of freedom is 95.20, with a significance level smaller than 0.0001. Hence, the restriction of three lags is binding.

f. Now estimate the model with six lags. You should find that the likelihood ratio test selects the eight-lag model, the AIC selects the six-lag model, and the SBC selects the three-lag model..

2. Question 1 indicates that a three-lag VAR seems reasonable for the variable Δlip_t, $\Delta_4 ur_t$, and y_t. Estimate the three-VAR beginning in 1961Q4 and use the ordering such that Δlip_t is causally prior to $\Delta_4 ur_t$ and that $\Delta_4 ur_t$ is causally prior to y_t.

a. If you perform a test to determine whether y_t Granger causes Δlip_t, you should find that the F-statistic is 3.09 with a *prob*-value of 0.0098. How do you interpret this result? (Note: Some software package will report that the sample χ^2 statistics is 11.71.)

b Verify that $\Delta_4 ur_t$ does not Granger cause Δlip_t . You should find that the F-statistic is 1.7018 with a *prob*-value of 0.168.

c. It turns out that the correlation coefficient between e_{1t} and e_{2t} is -0597. The correlation between e_{1t} and e_{3t} is -0.231 and between e_{2t} and e_{3t} is 0.223. Explain why the ordering in a Choleski decomposition is likely to be important for obtaining the impulse responses.

d. Verify that the forecast error variance decompositions are:

Horizon	Response to Δlip_t shock			Response to $\Delta_4 ur_t$ shock			Response to y_t shock		
	Δlip_t	$\Delta_4 ur_t$	y_t	Δlip_t	$\Delta_4 ur_t$	y_t	Δlip_t	$\Delta_4 ur_t$	y_t
1	100.00%	35.63%	5.37%	0.00%	64.36%	1.12%	0.00%	0.00%	93.51%
4	93.60	71.53	20.77	1.66	27.46	1.35%	4.78	1.01	77.88
8	89.15	58.05	37.57	1.86	20.31	1.25%	8.99	21.64	61.18

e. Obtain the impulse response functions. Show that a positive shock to industrial production induces a decline in the unemployment rate and in the long-term interest rate relative to the short-term rate.