The procedure and its inherent dangers are nicely illustrated by trying to determine if the real GDP data shown in Figure 4.1 has a unit root. It is a good idea to replicate the results reported below using the data in RGDP.XLS. Using quarterly data over the 1947Q1–2008Q2 period, the correlogram of the logarithm of real GDP exhibits slow decay. At the end of Section 2, the logarithmic first difference of the series was estimated as

$$\Delta lrgdp_t = \underset{(7.14)}{0.0055} + \underset{(5.47)}{0.3309}\Delta lrgdp_{t-1}$$

The model is well estimated in that the residuals appear to be white noise and all coefficients are of high quality. For our purposes, the interesting point is that the $\{\Delta lrgdp_t\}$ series appears to be a stationary process. Integrating suggests that $\{lrgdp_t\}$ has a stochastic and a deterministic trend. The issue here is to determine whether it was appropriate to difference the log of real GDP. Toward this end, consider the augmented Dickey–Fuller equation with t-statistics in parentheses:

$$\Delta lrgdp_t = \underset{(3.52)}{0.0246} + \underset{(2.49)}{0.00028}t - \underset{(-2.59)}{0.0360}\, lrgdp_{t-1} + \underset{(5.66)}{0.3426}\Delta lrgdp_{t-1} \quad \text{(A2.1)}$$

As in Step 1, we estimate the least restrictive model; as such, (A2.1) contains an intercept and a deterministic trend. The point estimates of (A2.1) suggest that the real GDP is trend-stationary. However, the issue is to formally test the statistical significance of the null hypothesis $\gamma = 0$. The t-statistic for the null hypothesis $\gamma = 0$ is –2.59. Critical values with exactly 244 usable observations are not reported in the Dickey–Fuller table. However, with 250 observations, the critical value of τ_τ at the 10 percent and 5 percent significance levels are –3.13 and –3.43, respectively. At the 5 and 10 percent levels, we cannot reject the null of a unit root. However, the power of the test may have been reduced due to the presence of an unnecessary time trend and/or drift term. In Step 2, we use the ϕ_3 statistic to test the joint hypothesis $a_2 = \gamma = 0$. The sample value of F for the restriction is 4.12. Since the critical value of ϕ_3 is 6.34 at the 5 percent significance level, it is possible to conclude that the restriction $a_2 = \gamma = 0$ is not binding. Thus, we can proceed to Step 3 and estimate the model without the trend. Consider the following equation:

$$\Delta lrgdp_t = \underset{(4.39)}{0.0077} - \underset{(-1.42)}{0.0014}\, lrgdp_{t-1} + \underset{(5.33)}{0.3233}\Delta lrgdp_{t-1} \quad \text{(A2.2)}$$

In (A2.2), the t-statistic for the null hypothesis $\gamma = 0$ is –1.42. Since the critical value of the τ_μ statistic is –2.88 at the 5 percent significance level, the null hypothesis of a unit root is not rejected at conventional significance levels. Again, the power of this test will have been reduced if the drift term does not belong in the model. To test for the presence of the drift, use the ϕ_1 statistic. The sample value of F for the restriction $a_0 = \gamma = 0$ is 26.63. Since 26.63 exceeds the critical value of 4.63, we conclude that the restriction is binding. Either $\gamma = 0$ (so there is not a unit root), $a_0 = 0$ (so there is not an intercept term), or both γ and a_0 are zero. In reality, any sensible researcher would stop at this point since it is not plausible to believe that real U.S.