Due: 26th of August 2018 at 11:59pm

COMP 9020 – Assignment 1

Note: In your assignment, *how* you arrived at your solution is as important (if not more so) than the solution itself and will be assessed accordingly. There may be more than one way to find a solution, and your approach should contain enough detail to justify its correctness. Lecture content can be assumed to be common knowledge.

1. (a) Compute $\gcd(132, 84)$.
   (b) Suppose $a, b \in \mathbb{N}$ are co-prime. What is $\gcd(a, a + b)$?

2. For sets A and B, define $A * B$ to be $(A \cup B)^c$ (the complement of $A \cup B$).
   (a) Simplify $(A * B) * (A * B)$. Justify your answer (e.g. using a Venn diagram or some other technique).
   (b) Express A^c using A and $*$. Justify your answer.
   (c) Express $A \cap B$ using A, B, and $*$. Justify your answer.

3. (a) List all possible functions $f : \{a, b, c\} \to \{0, 1\}$
   (b) Describe a connection between your answer for (a) and $\mathrm{Pow}(\{a, b, c\})$.
   (c) In general, if $\mathrm{card}(A) = m$ and $\mathrm{card}(B) = n$, how many:
      (i) functions are there from A to B?
      (ii) relations are there between A and B?

4. Let $\Sigma = \{a, b\}$ and $L = \{w \in \Sigma^* \ : \ 3|\mathrm{length}(w)\}$.
   (a) List the elements of $L^{\leq 3}$ in lexicographic order.

   Define $R \subseteq \Sigma^* \times \Sigma^*$ as follows: $(w, w') \in R$ if there is a $v \in \Sigma^*$ such that: either $wv \in L$ and $w'v \notin L$, or $wv \notin L$ and $w'v \in L$. For example $(a, bbb) \in R$ because for $v = \lambda$, $av = a \notin L$ and $bbbv = bbb \in L$. On the other hand, $(a, b) \notin R$ because for any $v \in \Sigma^*$, $\mathrm{length}(av) = \mathrm{length}(bv)$; so whenever $av \in L$, $bv \in L$ and vice-versa.

   (b) Which of the following are elements of R:
      (i) $(abab, baba)$?
      (ii) $(ab, abab)$?
      (iii) (λ, b)?
      (iv) (λ, bb)?
      (v) (λ, bbb)?

Now define $S \subseteq \Sigma^* \times \Sigma^*$ as the complement of R. That is $(w, w') \in S$ if $(w, w') \notin R$.

(b) State a simple rule for determining if $(w, w') \in S$.

(c) Show that S is an equivalence relation. That is, show that S is reflexive, symmetric, and transitive.

(d) How many equivalence classes does S have?

Advice on how to do the assignment

All submitted work must be done individually without consulting someone else's solutions in accordance with the University's "Academic Dishonesty and Plagiarism" policies.

- Assignments are to be submitted via WebCMS (or give) as a pdf
- Be careful with giving multiple or alternative answers. If you give multiple answers, then we will give you marks only for "your worst answer", as this indicates how well you understood the question.
- Some of the questions are very easy (with the help of the lecture notes or book). You can use the material presented in the lecture or book (without proving it). You do not need to write more than necessary (see comment above).
- When giving answers to questions, we always would like you to prove/explain/motivate your answers.
- If you use further resources (books, scientific papers, the internet,...) to formulate your answers, then add references to your sources.