

Fundamentals (Numbers, Sets, Words, Functions, and Relations)

Exercise 1. How many numbers are there between 100 and 1000 that are

- (a) divisible by 3?
- (b) divisible by 5?
- (c) divisible by 15?

Exercise 2. Prove that $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$

Exercise 3. Consider the relation $R \subseteq \mathbb{R} \times \mathbb{R}$ defined by aRb if, and only if, $b + 0.5 \geq a \geq b - 0.5$. Is R

- (a) reflexive?
- (b) antireflexive?
- (c) symmetric?
- (d) antisymmetric?
- (e) transitive?

Exercise 4. For each of the following statements, provide a valid proof if it is true for all sets S and all relations $R_1 \subseteq S \times S$ and $R_2 \subseteq S \times S$. If the statement is not always true, provide a counterexample.

- (a) If R_1 and R_2 are symmetric, then $R_1 \cap R_2$ is symmetric.
- (b) If R_1 and R_2 are antisymmetric, then $R_1 \cup R_2$ is antisymmetric.