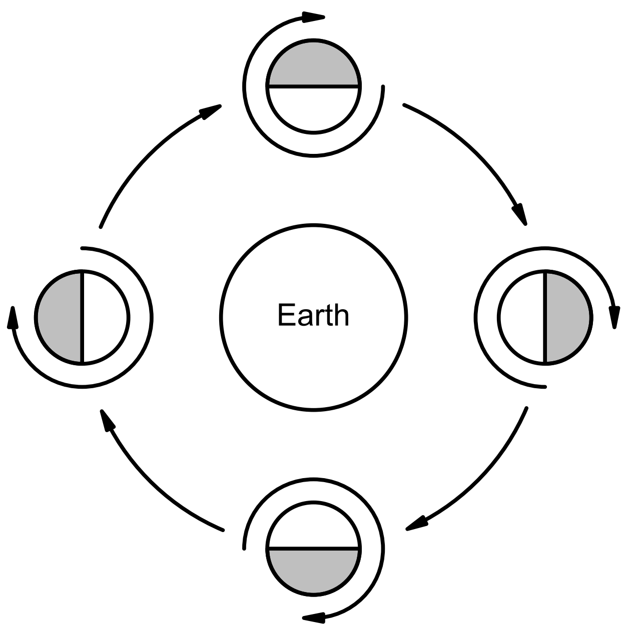
Appendix: Days and Nights on Patra-Bannk (Revised 2020)

On Earth, because its period of rotation is so short compared to its period of revolution around the sun (24 hours versus 365 days) we tend to forget that the apparent motion of the sun is, in actuality, caused by *both* the rotation and revolution of the Earth. To be more precise, the apparent motion of the sun is due to the differential rotation of the Earth, i.e., the sum or difference of the two motions.

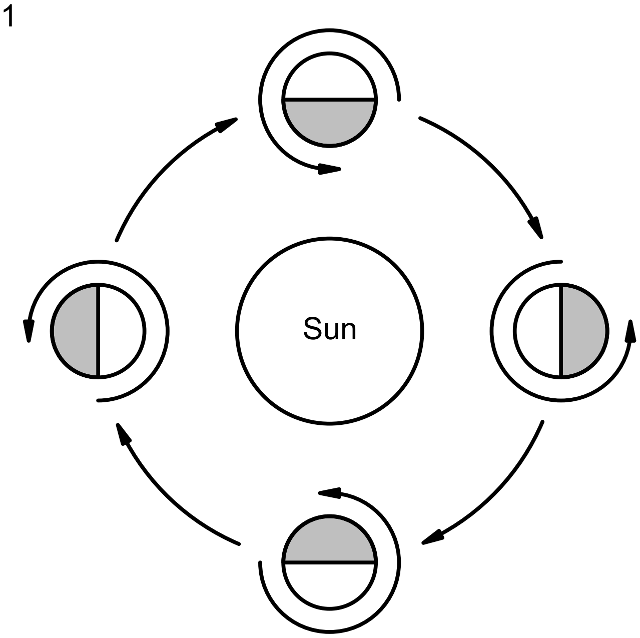
This effect would become quite visible if the Earth were slowed down until its period of rotation was comparable to its period of revolution, or until its day was comparable to its year. This is the case of the moon, where both periods are equal. Since the moon rotates in the same direction it revolves, and with equal rates, we see only one face pointing toward Earth. If the Earth is considered as the moon's sun, the moon has no days and nights (see Figure 1). Venus also rotates with a period roughly equal to its period of revolution around the sun (243 days versus 225 days). In this case, however, Venus rotates in the opposite direction from the direction of revolution and, as a result, gets days and nights of about ½ the periods, approximately 117 days. Here we see the effect of the differential rotation very well. Since Venus rotates in the opposite direction than it revolves, the sun seems to be moving twice as fast as it would if Venus did not rotate and thus the days are ½ as long.

Figure 1



Moon

Direction of rotation and revolution the same, rates equal, yields one face of moon always pointed to Earth.

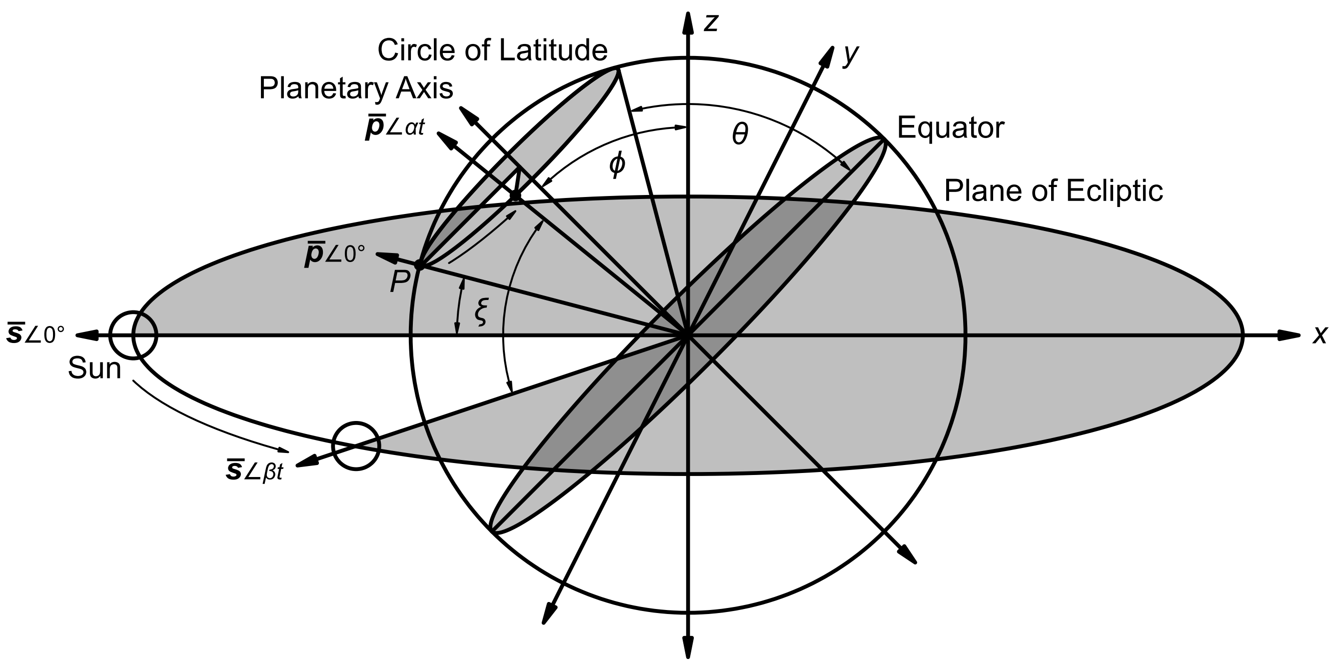


Venus

Direction of rotation and revolution opposite, rates equal, yields two days and nights on Venus per revolution.

While writing *The World is Round* I became interested in the daylength for a planet whose relative rates of rotation and revolution were arbitrary. I also wanted to take the tilt of the planet into account. To keep matters fairly simple, the orbit of the planet was assumed to be circular, not elliptical. The final diagram used to set up the problem is reproduced in Figure 2.

Figure 2



P = observation point, position of observer on given circle of latitude
$\bar{p}$ = vector passing through center of planet and observer to zenith of observer, rotates around planetary axis at circle of latitude
θ = latitude of observer
α = rate that planet spins around on axis in degrees per sidereal year, same as rate of observer spinning around circle of latitude
ϕ = inclination of planet to plane of ecliptic (23.5° on Earth, 20° on Patra-Bannk)
$\bar{s}$ = vector from center of planet to sun
β = rate at which $\bar{s}$ moves around plane of ecliptic (360° per sidereal year by definition)
ξ = angle between $\bar{p}$ and $\bar{s}$ for any time t, or angle between sun and zenith

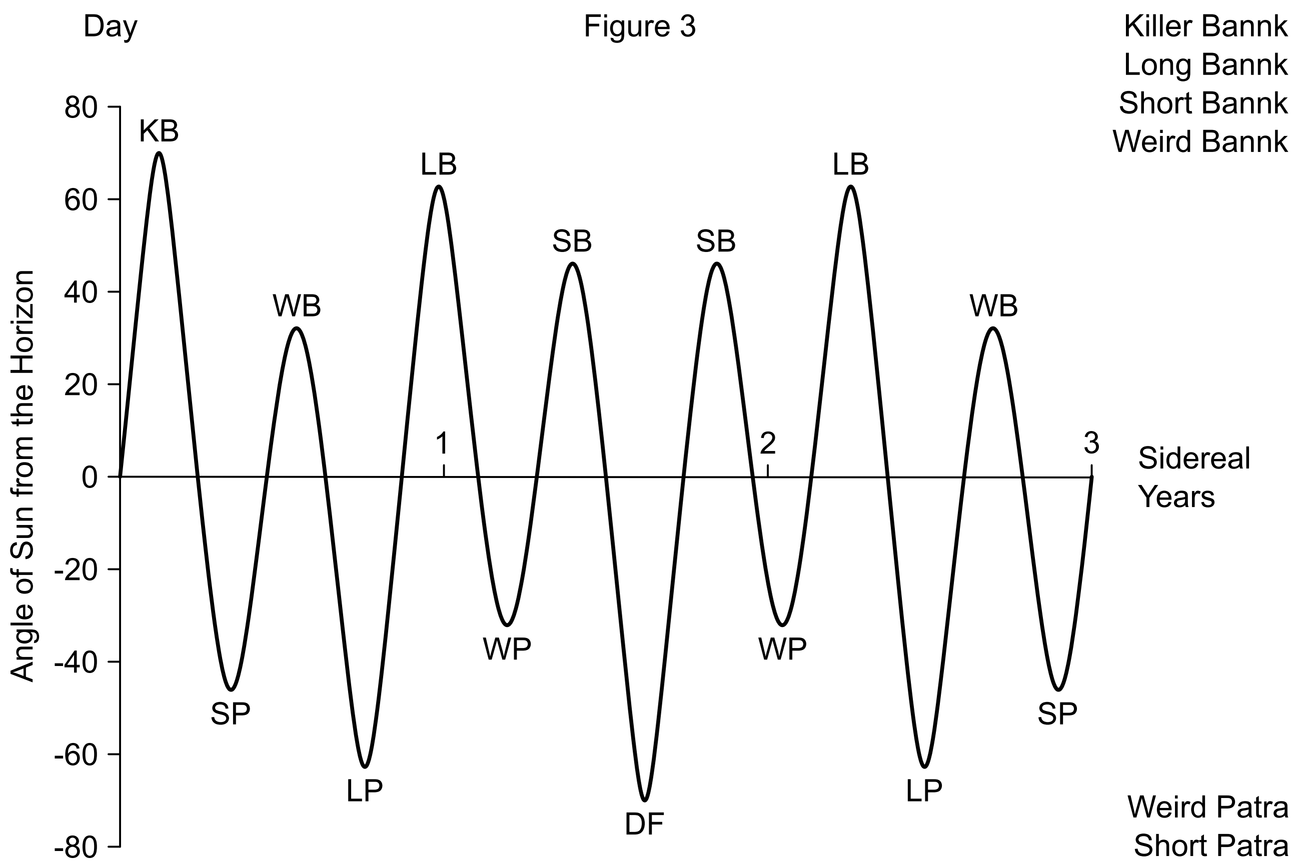
The trick in doing the problem is to pretend the sun is revolving around the planet rather than vice-versa. (In fact, an easier way to set up the problem than shown here is to assume the planetary rotation axis is vertical, along the z-axis, and that the sun's orbital plane, the plane of the ecliptic, is tilted by an angle ϕ.)

In either case, you can see by examining Figure 2 that an observer at point P rotates around his or her circle of latitude at rate α. The sun rotates around the planet at rate β, which is by definition 360 degrees per sidereal year. (A sidereal year is the time for one revolution of the sun around a point fixed in space from the point of view at the planet. For the story I assumed that the sun was moving in the opposite direction from the planet's rotation, and so β = -360.)

We must find ξ, the angle between the observer's zenith and the sun, and we need to do this for any time t. We want ξ for any rate of rotation α; any tilt of planet ϕ; and any latitude θ. By definition, cos ξ is the dot product between the unit vectors $\bar{p}$ and $\bar{s}$, and so one must find the components of these vectors in terms of the required angles and simply multiply them together. Using standard spherical trigonometry the result is:

$$\cos\xi = (\cos\theta\cos\phi\cos\alpha t + \sin\theta\sin\phi)\cos\beta t + \cos\theta\sin\alpha t\sin\beta t$$

To get ξ, finally, we take the inverse cosine of this equation, and then plot ξ versus t. This should be done numerically with whatever app you prefer, and the result is shown in Figure 3. We have incremented t from 0 upward in small values until the day-night cycle began repeating itself. For each value of t we have actually plotted $90° - \xi$ which is the angle of sun above the horizon instead of the angle from the zenith. Time is in sidereal years which was just an easy way to solve the problem. For the purposes of the story you can take one sidereal year as 1000 Earth days. Thus, on the graph, .2 sidereal years is 200 Earth days, etc. With a finer scale you would see that the Weird Bannk lasts 181 Earth days, while the Killer Bannk lasts 240.



Daylight and Angle of Sun on Patra-Bannk
α=480°/year β=-360°/year ϕ=20° θ=40°

« Prev Appendix: Days and Nights on Patra-Bannk (Revised 2020) Next »