

# Prošli puta: LR parser

Stanje	<i>akcija</i>						<i>novo_stanje</i>		
	<b>var</b>	+	*	(	)	$\perp$	<i>E</i>	<i>T</i>	<i>F</i>
0	s5			s4					
1		s6				<i>prihvati</i>			
2		r2	s7			r2	r2		
3		r4	r4			r4	r4		
4	s5			s4					
5		r6	r6		r6	r6			
6	s5			s4					
7	s5			s4					
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Regularna gramatika  $G_3=(V, T, P, S)$ :

$$A \rightarrow wB \text{ i } A \rightarrow w$$

ili

$$A \rightarrow Bw \text{ i } A \rightarrow w$$

$$A, B \in V, w \in T^*$$

Konačni automat:

$$M_3=(Q, \Sigma, \delta, q_0, F)$$

II

III

Regularni jezici:  $L_3=L(G_3)=L(M_3)$

Kontekstno neovisna gramatika  $G_2=(V, T, P, S)$ :

$$\begin{aligned} A &\rightarrow \alpha \\ A \in V, \alpha &\in (T \cup V)^* \end{aligned}$$

Regularna gramatika  $G_3=(V, T, P, S)$ :

$$\begin{aligned} A &\rightarrow wB \text{ i } A \rightarrow w \\ &\text{ili} \\ A &\rightarrow Bw \text{ i } A \rightarrow w \\ A, B &\in V, w \in T^* \end{aligned}$$

III

Potisni automat:

$$M_2=(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

Konačni automat:

$$M_3=(Q, \Sigma, \delta, q_0, F)$$

IV

Kontekstno-neovisni jezici:  $L_2=L(G_2)=L(M_2)$

Regularni jezici:  $L_3=L(G_3)=L(M_3)$

Kontekstno ovisna gramatika  $G_1=(V, T, P, S)$ :

$$\begin{aligned}\alpha \rightarrow \beta, |\alpha| \leq |\beta| \\ \alpha, \beta \in (T \cup V)^*, \alpha \neq \epsilon\end{aligned}$$

Kontekstno neovisna gramatika  $G_2=(V, T, P, S)$ :

$$\begin{aligned}A \rightarrow \alpha \\ A \in V, \alpha \in (T \cup V)^*\end{aligned}$$

Regularna gramatika  $G_3=(V, T, P, S)$ :

$$\begin{aligned}A \rightarrow wB \text{ i } A \rightarrow w \\ \text{ili} \\ A \rightarrow Bw \text{ i } A \rightarrow w \\ A, B \in V, w \in T^*\end{aligned}$$

Linearno ograničeni automat:

$$M_1=(Q, \Sigma, \Gamma, \delta, q_0, \epsilon, \$, F)$$

Potisni automat:

$$M_2=(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

Konačni automat:

$$M_3=(Q, \Sigma, \delta, q_0, F)$$

III

II

Kontekstno-ovisni jezici:  $L_1=L(G_1)=L(M_1)$

Kontekstno-neovisni jezici:  $L_2=L(G_2)=L(M_2)$

Regularni jezici:  $L_3=L(G_3)=L(M_3)$

Gramatika  
neograničenih produkcija  $G_0=(V, T, P, S)$ :  
 $\alpha \rightarrow \beta$   
 $\alpha, \beta \in (T \cup V)^*$ ,  $\alpha \neq \varepsilon$

Kontekstno ovisna gramatika  $G_1=(V, T, P, S)$ :  
 $\alpha \rightarrow \beta$ ,  $|\alpha| \leq |\beta|$   
 $\alpha, \beta \in (T \cup V)^*$ ,  $\alpha \neq \varepsilon$

Kontekstno neovisna gramatika  $G_2=(V, T, P, S)$ :  
 $A \rightarrow \alpha$   
 $A \in V$ ,  $\alpha \in (T \cup V)^*$

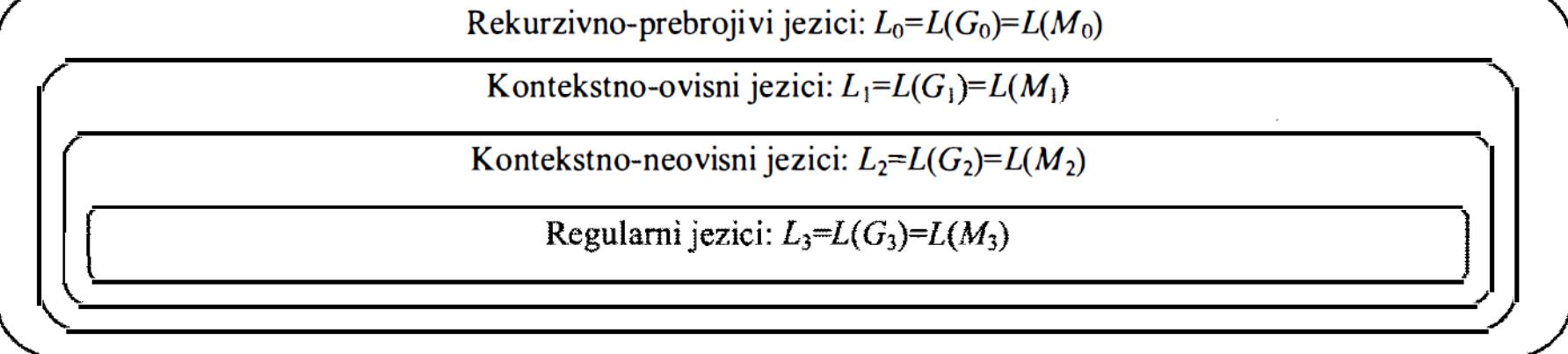
Regуларна граматика  $G_3=(V, T, P, S)$ :  
 $A \rightarrow wB$  и  $A \rightarrow w$   
или  
 $A \rightarrow Bw$  и  $A \rightarrow w$   
 $A, B \in V$ ,  $w \in T^*$

Turingov stroj:  
 $M_0=(Q, \Sigma, \Gamma, \delta, q_0, B, F)$

Linearno ograničeni automat:  
 $M_1=(Q, \Sigma, \Gamma, \delta, q_0, \emptyset, \$, F)$

Potisni automat:  
 $M_2=(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

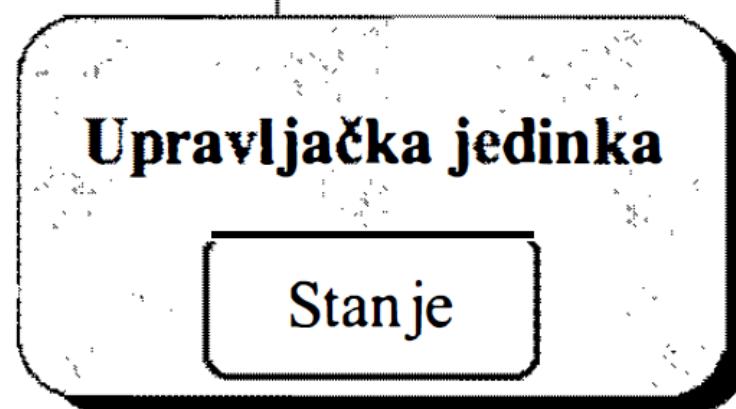
Konačni automat:  
 $M_3=(Q, \Sigma, \delta, q_0, F)$



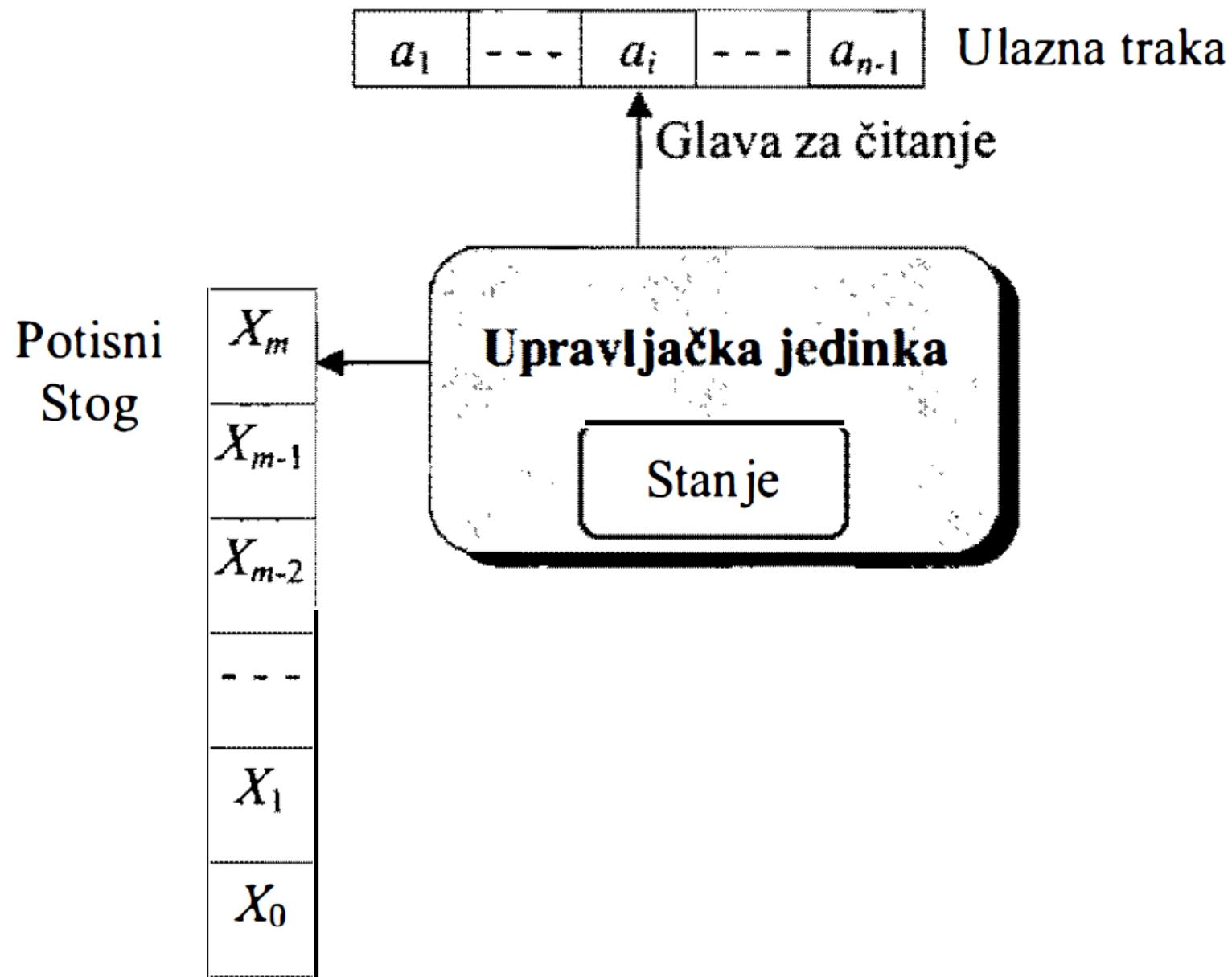
# Potisni automat (PA)

$a_1$	- - -	$a_i$	- - -	$a_{n-1}$
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Ulagana traka



Glava za čitanje



**(1)**

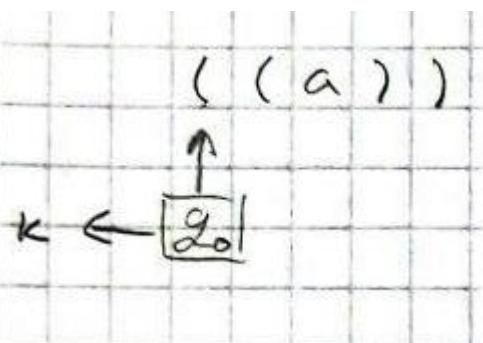
Gradi se PA  $M$  koji prihvaca praznim stogom jezik  $N(M)=\{(\overset{n}{a})^n \mid n \geq 1\}$ , gdje je znak  $a$  okruzen jednakim brojem otvorenih i zatvorenih zagrada. Ulazni znakovi PA su  $(, )$ , i  $a$ .

PA ima dva stanja:  $q_0$  i  $q_1$ . Na početku rada PA jest u stanju  $q_0$ , a na vrhu stoga je znak  $K$ . PA u stanju  $q_0$  čita redom ulazne znakove sve do znaka  $a$ . PA stavlja jedan znak  $A$  na vrh stoga za svaku pročitanu otvorenu zagradu. Nakon što pročita znak  $a$ , PA prelazi u stanje  $q_1$ . PA u stanju  $q_1$  uzima jedan znak  $A$  s vrha stoga za svaku pročitanu zatvorenu zagradu. Broj znakova  $A$  na stogu mora biti jednak broju pročitanih zatvorenih zagrada. Nema li znakova  $A$  na stogu nakon pročitanih svih znakova ulazne trake, niz se prihvaca. Skup znakova stoga jednak je  $\{A, K\}$ . Prijelazi PA određuju se na sljedeći način:

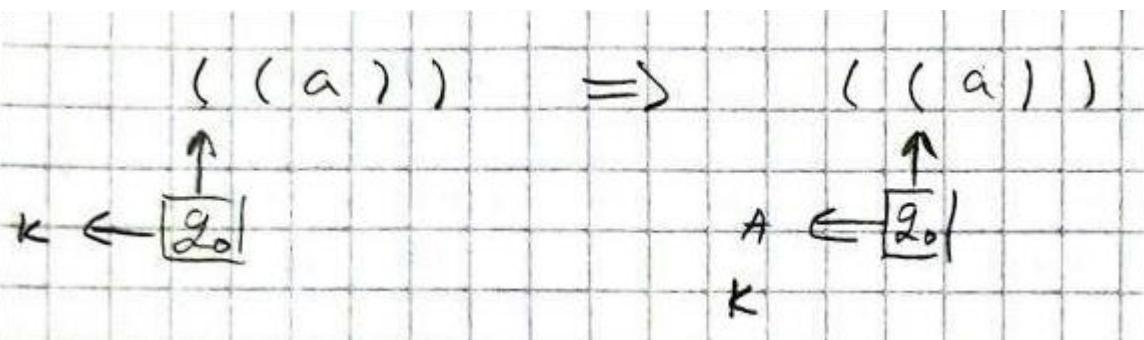
Prijelaz	Stanje	Ulaz	Stog	Novo stanje	Novi vrh stoga	Glava za čitanje
1)	$q_0$	(	$K$	$q_0$	$AK$	Pomak u desno
2)	$q_0$	(	$A$	$q_0$	$AA$	Pomak u desno
3)	$q_0$	$a$	$A$	$q_1$	$A$	Pomak u desno
4)	$q_1$	)	$A$	$q_1$	$\epsilon$	Pomak u desno
5)	$q_1$	$\epsilon$	$K$	$q_1$	$\epsilon$	Zadrži na istom mjestu

<i>Prijelaz</i>	<i>Stanje</i>	<i>Ulaz</i>	<i>Stog</i>	<i>Novo stanje</i>	<i>Novi vrh stoga</i>	<i>Glava za čitanje</i>
1)	$q_0$	(	$K$	$q_0$	$AK$	Pomak u desno
2)	$q_0$	(	$A$	$q_0$	$AA$	Pomak u desno
3)	$q_0$	$a$	$A$	$q_1$	$A$	Pomak u desno
4)	$q_1$	)	$A$	$q_1$	$\epsilon$	Pomak u desno
5)	$q_1$	$\epsilon$	$K$	$q_1$	$\epsilon$	Zadrži na istom mjestu

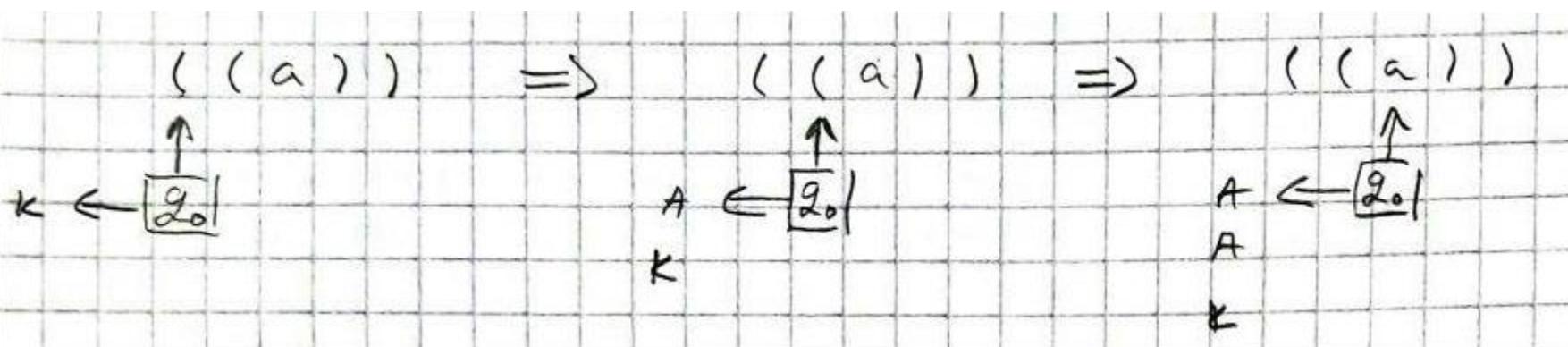
Prijelaz	Stanje	Ulaz	Stog	Novo stanje	Novi vrh stoga	Glava za čitanje
1)	$q_0$	(	$K$	$q_0$	$AK$	Pomak u desno
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3)	$q_0$	$a$	$A$	$q_1$	$A$	Pomak u desno
4)	$q_1$	)	$A$	$q_1$	$\epsilon$	Pomak u desno
5)	$q_1$	$\epsilon$	$K$	$q_1$	$\epsilon$	Zadrži na istom mjestu



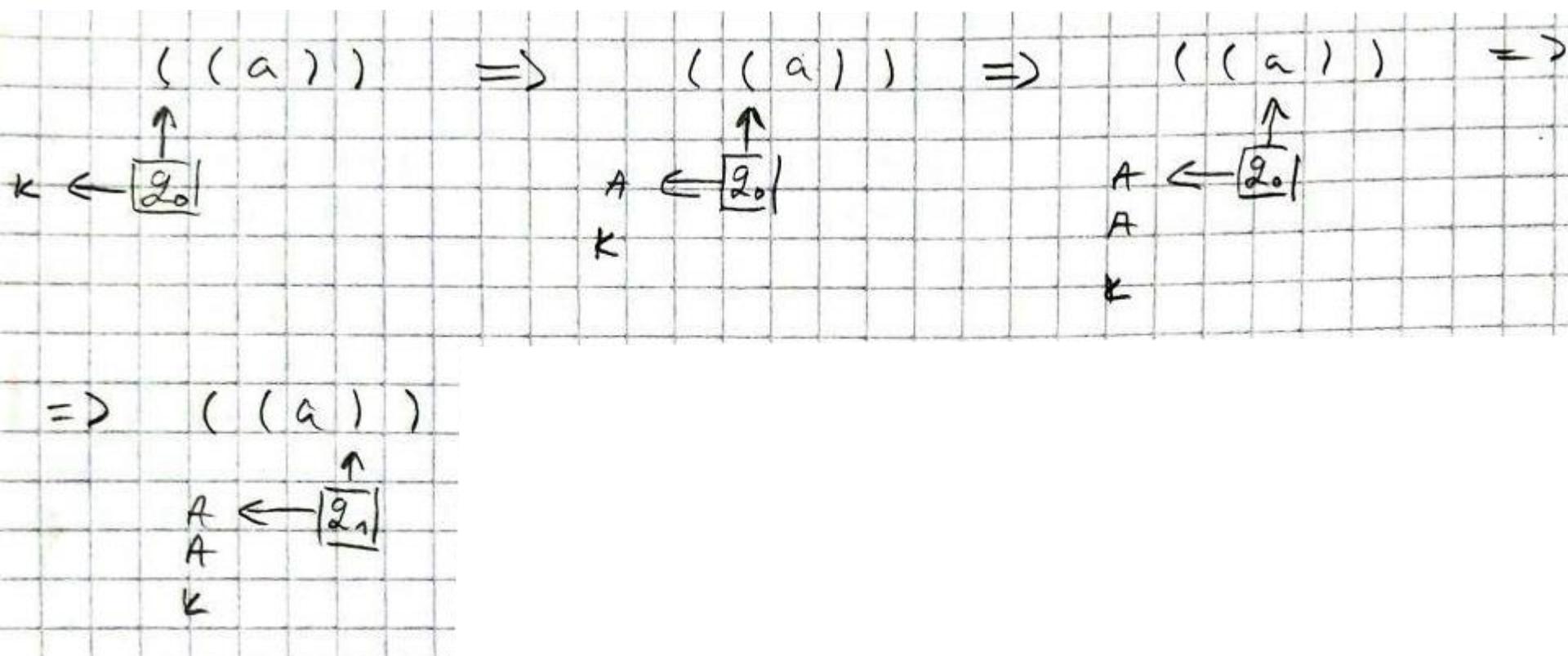
Prijelaz	Stanje	Ulas	Stog	Novo stanje	Novi vrh stoga	Glava za čitanje
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2)	$q_0$	(	$A$	$q_0$	$AA$	Pomak u desno
3)	$q_0$	$a$	$A$	$q_1$	$A$	Pomak u desno
4)	$q_1$	)	$A$	$q_1$	$\epsilon$	Pomak u desno
5)	$q_1$	$\epsilon$	$K$	$q_1$	$\epsilon$	Zadrži na istom mjestu



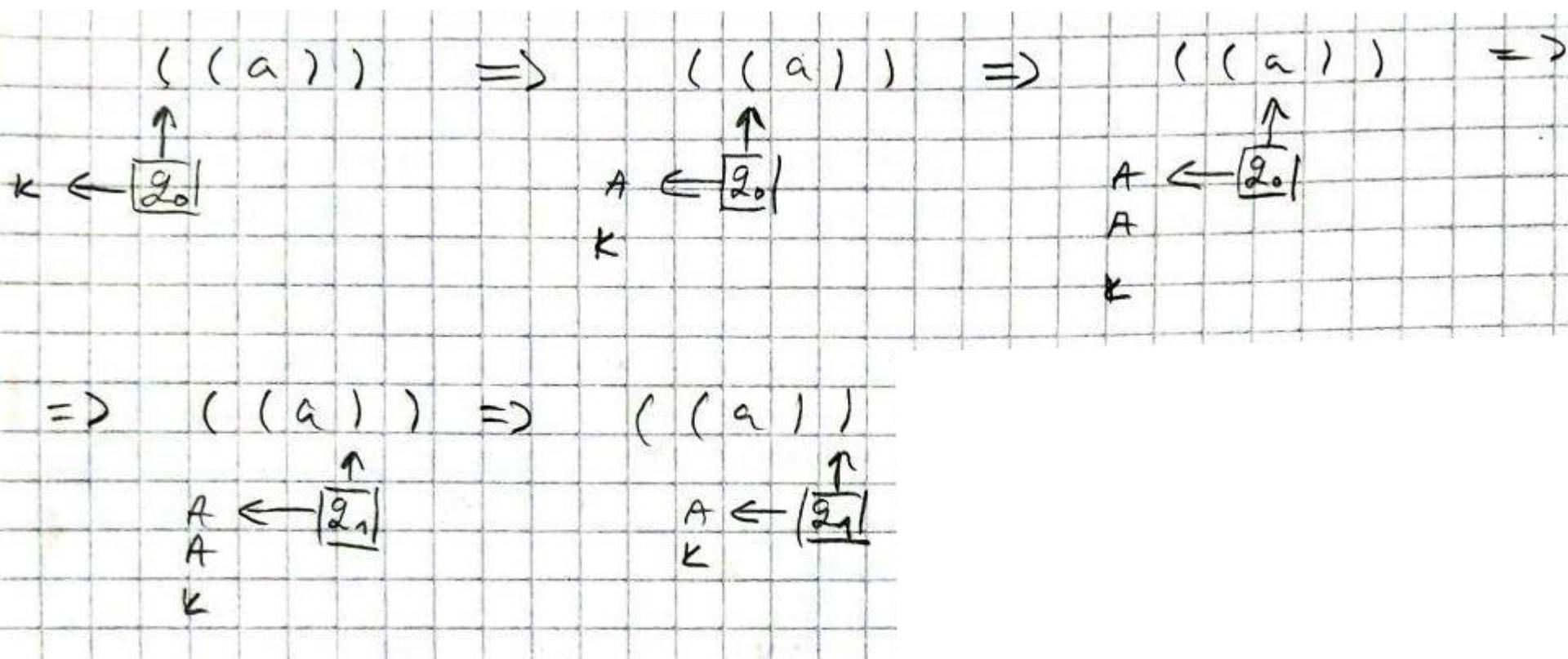
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3)	$q_0$	$a$	$A$	$q_1$	$A$	Pomak u desno
4)	$q_1$	)	$A$	$q_1$	$\epsilon$	Pomak u desno
5)	$q_1$	$\epsilon$	$K$	$q_1$	$\epsilon$	Zadrži na istom mjestu



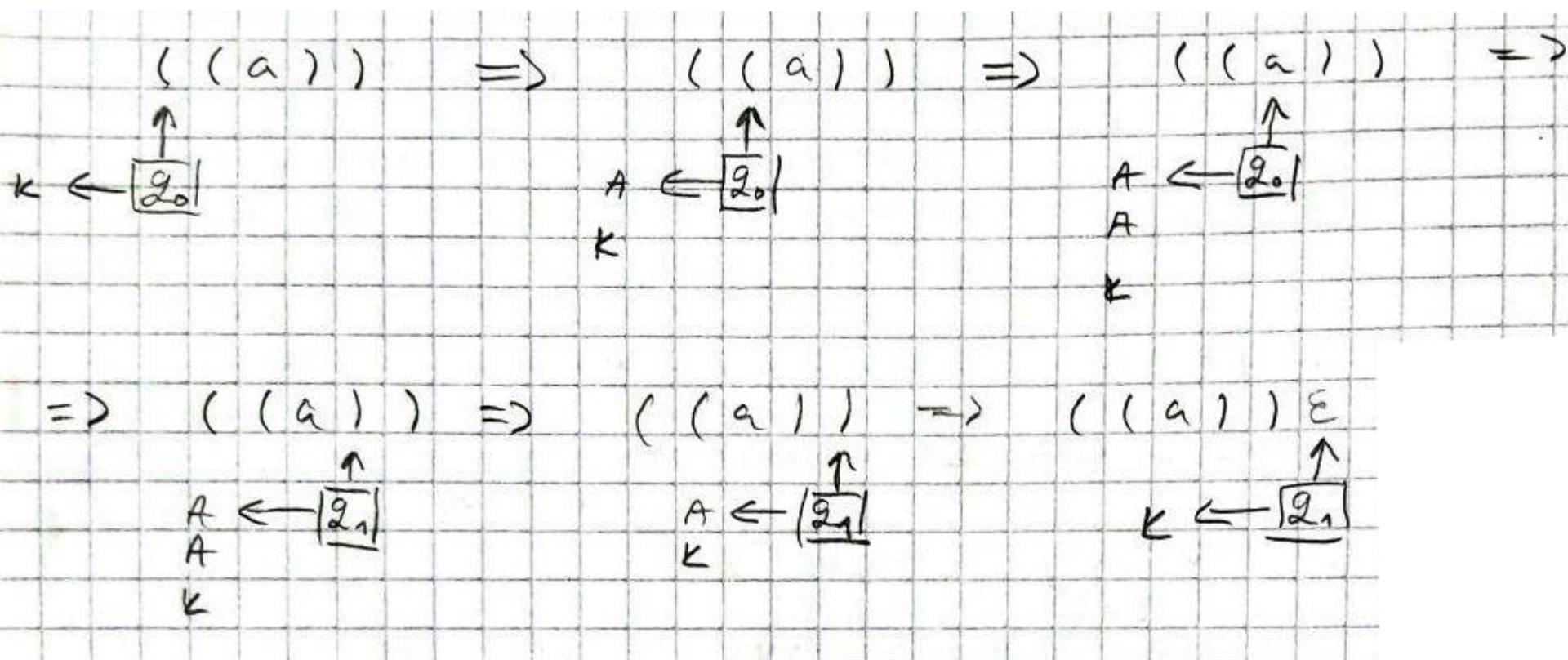
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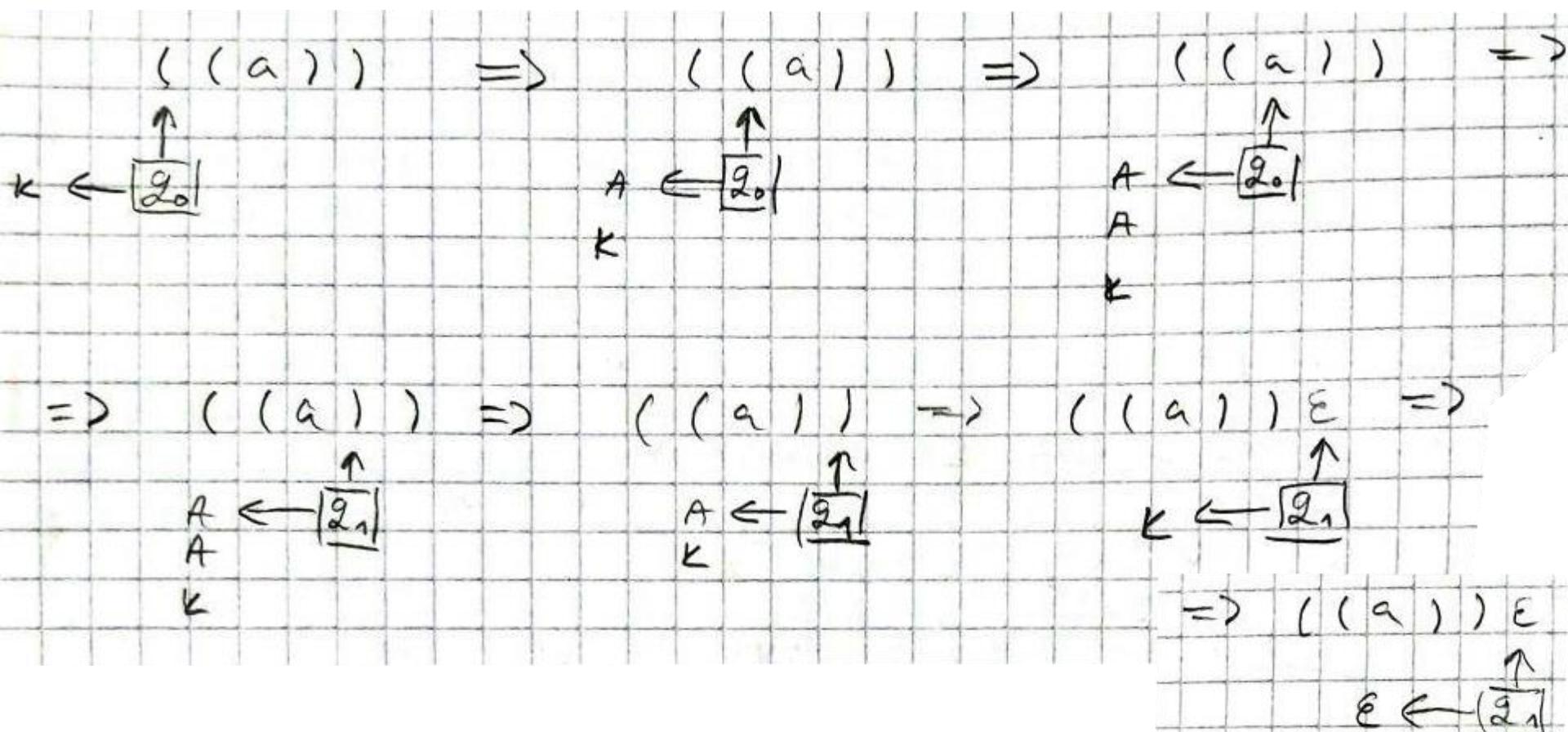
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4)	$q_1$	)	$A$	$q_1$	$\epsilon$	Pomak u desno
5)	$q_1$	$\epsilon$	$K$	$q_1$	$\epsilon$	Zadrži na istom mjestu



NIE SE  
PRIHVACAO

# Definicija potisnog automata

Potisni automat (PA) formalno se zadaje kao uređena sedmorka:

$$pa = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

gdje je:

- $Q$  - konačan skup stanja;
- $\Sigma$  - konačan skup ulaznih znakova (abeceda ulaznih znakova);
- $\Gamma$  - konačan skup znakova stoga (abeceda znakova stoga);
- $\delta$  - funkcija prijelaza  $\delta$  pridružuje trojki  $Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma$  konačan podskup skupa svih mogućih parova  $Q \times \Gamma^*$ ;
- $q_0 \in Q$  - početno stanje;
- $Z_0 \in \Gamma$  - početni znak stoga;
- $F \subseteq Q$  - skup prihvativih stanja.

(2)

Zadan je PA  $M_1 = (\{q_1, q_2, q_3\}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{q_3\})$  sa sljedećim prijelazima:

- |  |  |
|--|--|
| 1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$                     | 2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$           |
| 3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$                     | 4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$           |
| 5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$                     | 6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$           |
| 7) $\delta(q_1, 2, K) = \{(q_2, K)\}$                      |  |
| 8) $\delta(q_1, 2, N) = \{(q_2, N)\}$                      |  |
| 9) $\delta(q_1, 2, J) = \{(q_2, J)\}$                      |  |
| 10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$           | 11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$ |
| 12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$ |  |

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| 12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$ |  |

<i>Stanje</i>	<i>Nepročitani dio niza</i>	<i>Stog (vrh stoga je krajnje lijevi znak)</i>	<i>Funkcija prijelaza</i>
$q_1$	0012100	K	$\delta(q_1, 0, K) = \{(q_1, NK)\}$

Zadan je PA  $M_1 = (\{q_1, q_2, q_3\}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{q_3\})$  sa sljedećim prijelazima:

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| 1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$                     | 2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$           |
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$q_1$	0012100	$K$	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
$q_1$	012100	$NK$	$\delta(q_1, 0, N) = \{(q_1, NN)\}$

Zadan je PA  $M_1 = (\{q_1, q_2, q_3\}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{q_3\})$  sa sljedećim prijelazima:

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| 12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$ |  |

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$q_1$	012100	$NK$	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
$q_1$	12100	$NNK$	$\delta(q_1, 1, N) = \{(q_1, JN)\}$

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$q_1$	012100	$NK$	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
$q_1$	12100	$NNK$	$\delta(q_1, 1, N) = \{(q_1, JN)\}$
$q_1$	2100	$JNNK$	$\delta(q_1, 2, J) = \{(q_2, J)\}$

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| 8) $\delta(q_1, 2, N) = \{(q_2, N)\}$                      |  |
| 9) $\delta(q_1, 2, J) = \{(q_2, J)\}$                      |  |
| 10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$           | 11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$ |
| 12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$ |  |

Stanje	Nepročitani dio niza	Stog (vrh stoga je krajnje lijevi znak)	Funkcija prijelaza
$q_1$	0012100	$K$	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
$q_1$	012100	$NK$	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
$q_1$	12100	$NNK$	$\delta(q_1, 1, N) = \{(q_1, JN)\}$
$q_1$	2100	$JNNK$	$\delta(q_1, 2, J) = \{(q_2, J)\}$
$q_2$	100	$JNNK$	$\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

Zadan je PA  $M_1 = (\{q_1, q_2, q_3\}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{q_3\})$  sa sljedećim prijelazima:

- |  |  |
|--|--|
| 1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$                     | 2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$           |
| 3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$                     | 4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$           |
| 5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$                     | 6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$           |
| 7) $\delta(q_1, 2, K) = \{(q_2, K)\}$                      |  |
| 8) $\delta(q_1, 2, N) = \{(q_2, N)\}$                      |  |
| 9) $\delta(q_1, 2, J) = \{(q_2, J)\}$                      |  |
| 10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$           | 11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$ |
| 12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$ |  |

Stanje	Nepročitani dio niza	Stog (vrh stoga je krajnje lijevi znak)	Funkcija prijelaza
$q_1$	0012100	$K$	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
$q_1$	012100	$NK$	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
$q_1$	12100	$NNK$	$\delta(q_1, 1, N) = \{(q_1, JN)\}$
$q_1$	2100	$JNNK$	$\delta(q_1, 2, J) = \{(q_2, J)\}$
$q_2$	100	$JNNK$	$\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
$q_2$	00	$NNK$	$\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

Zadan je PA  $M_1 = (\{q_1, q_2, q_3\}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{q_3\})$  sa sljedećim prijelazima:

- |  |  |
|--|--|
| 1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$                     | 2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$           |
| 3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$                     | 4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$           |
| 5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$                     | 6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$           |
| 7) $\delta(q_1, 2, K) = \{(q_2, K)\}$                      |  |
| 8) $\delta(q_1, 2, N) = \{(q_2, N)\}$                      |  |
| 9) $\delta(q_1, 2, J) = \{(q_2, J)\}$                      |  |
| 10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$           | 11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$ |
| 12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$ |  |

<i>Stanje</i>	<i>Nepročitani dio niza</i>	<i>Stog (vrh stoga je krajnje lijevi znak)</i>	<i>Funkcija prijelaza</i>
$q_1$	0012100	$K$	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
$q_1$	012100	$NK$	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
$q_1$	12100	$NNK$	$\delta(q_1, 1, N) = \{(q_1, JN)\}$
$q_1$	2100	$JNNK$	$\delta(q_1, 2, J) = \{(q_2, J)\}$
$q_2$	100	$JNNK$	$\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
$q_2$	00	$NNK$	$\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
$q_2$	0	$NK$	$\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

Zadan je PA  $M_1 = (\{q_1, q_2, q_3\}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{q_3\})$  sa sljedećim prijelazima:

- |  |  |
|--|--|
| 1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$                     | 2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$           |
| 3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$                     | 4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$           |
| 5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$                     | 6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$           |
| 7) $\delta(q_1, 2, K) = \{(q_2, K)\}$                      |  |
| 8) $\delta(q_1, 2, N) = \{(q_2, N)\}$                      |  |
| 9) $\delta(q_1, 2, J) = \{(q_2, J)\}$                      |  |
| 10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$           | 11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$ |
| 12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$ |  |

Stanje	Nepročitani dio niza	Stog (vrh stoga je krajnje lijevi znak)	Funkcija prijelaza
$q_1$	0012100	$K$	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
$q_1$	012100	$NK$	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
$q_1$	12100	$NNK$	$\delta(q_1, 1, N) = \{(q_1, JN)\}$
$q_1$	2100	$JNNK$	$\delta(q_1, 2, J) = \{(q_2, J)\}$
$q_2$	100	$JNNK$	$\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
$q_2$	00	$NNK$	$\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
$q_2$	0	$NK$	$\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
$q_2$		$K$	$\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

Zadan je PA  $M_1 = (\{q_1, q_2, q_3\}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{q_3\})$  sa sljedećim prijelazima:

- |  |  |
|--|--|
| 1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$                     | 2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$           |
| 3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$                     | 4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$           |
| 5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$                     | 6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$           |
| 7) $\delta(q_1, 2, K) = \{(q_2, K)\}$                      |  |
| 8) $\delta(q_1, 2, N) = \{(q_2, N)\}$                      |  |
| 9) $\delta(q_1, 2, J) = \{(q_2, J)\}$                      |  |
| 10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$           | 11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$ |
| 12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$ |  |

Stanje	Nepročitani dio niza	Stog (vrh stoga je krajnje lijevi znak)	Funkcija prijelaza
$q_1$	0012100	$K$	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
$q_1$	012100	$NK$	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
$q_1$	12100	$NNK$	$\delta(q_1, 1, N) = \{(q_1, JN)\}$
$q_1$	2100	$JNNK$	$\delta(q_1, 2, J) = \{(q_2, J)\}$
$q_2$	100	$JNNK$	$\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
$q_2$	00	$NNK$	$\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
$q_2$	0	$NK$	$\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
$q_2$		$K$	$\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$
$q_3$			$q_3 \in F$ i niz se prihvata

Zadan je PA  $M_1 = (\{q_1, q_2, q_3\}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{q_3\})$  sa sljedećim prijelazima:

- |  |  |
|--|--|
| 1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$                     | 2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$           |
| 3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$                     | 4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$           |
| 5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$                     | 6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$           |
| 7) $\delta(q_1, 2, K) = \{(q_2, K)\}$                      |  |
| 8) $\delta(q_1, 2, N) = \{(q_2, N)\}$                      |  |
| 9) $\delta(q_1, 2, J) = \{(q_2, J)\}$                      |  |
| 10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$           | 11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$ |
| 12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$ |  |

(3)

- Niz: 1002100

Stanje	Nepročitani dio niza	Stog (vrh stoga je krajnje lijevi znak)	Funkcija prijelaza

Zadan je PA  $M_1 = (\{q_1, q_2, q_3\}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{q_3\})$  sa sljedećim prijelazima:

- |  |  |
|--|--|
| 1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$                     | 2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$           |
| 3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$                     | 4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$           |
| 5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$                     | 6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$           |
| 7) $\delta(q_1, 2, K) = \{(q_2, K)\}$                      |  |
| 8) $\delta(q_1, 2, N) = \{(q_2, N)\}$                      |  |
| 9) $\delta(q_1, 2, J) = \{(q_2, J)\}$                      |  |
| 10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$           | 11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$ |
| 12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$ |  |

- Niz: 1002100

<i>Stanje</i>	<i>Nepročitani dio niza</i>	<i>Stog (vrh stoga je krajnje lijevi znak)</i>	<i>Funkcija prijelaza</i>
q1	1002100	K	2)
q1	002100	JK	5)
q1	02100	NJK	3)
q1	2100	NNJK	8)
q2	100	NNJK	odbaci niz

(4)

Zadan je PA  $M_2 = (\{q_1, q_2\}, \{0, 1\}, \{N, J, K\}, \delta, q_1, K, \emptyset)$  koji prihvaca jezik  $N(M_2)$  praznim

stogom sa sljedećim prijelazima:

- |  |  |
|--|--|
| 1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$                     | 2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$                     |
| 3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$ | 4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$                     |
| 5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$                     | 6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$ |
| 7) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$            | 8) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$            |
| 9) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$  |  |
| 10) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$ |  |

$WW^R$

(4)

Zadan je PA  $M_2 = (\{q_1, q_2\}, \{0, 1\}, \{N, J, K\}, \delta, q_1, K, \emptyset)$  koji prihvaca jezik  $N(M_2)$  praznim

stogom sa sljedećim prijelazima:

- |  |  |
|--|--|
| 1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$                     | 2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$                     |
| 3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$ | 4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$                     |
| 5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$                     | 6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$ |
| 7) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$            | 8) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$            |
| 9) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$  |  |
| 10) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$ |  |

$(q_1, 001100, K)$

(4)

Zadan je PA  $M_2 = (\{q_1, q_2\}, \{0, 1\}, \{N, J, K\}, \delta, q_1, K, \emptyset)$  koji prihvaca jezik  $N(M_2)$  praznim

stogom sa sljedećim prijelazima:

1)  $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3)  $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$

5)  $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

7)  $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

9)  $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

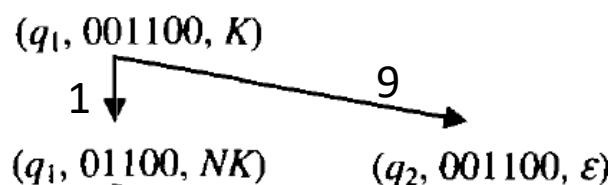
10)  $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

2)  $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4)  $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6)  $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

8)  $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$



(4)

Zadan je PA  $M_2 = (\{q_1, q_2\}, \{0, 1\}, \{N, J, K\}, \delta, q_1, K, \emptyset)$  koji prihvaca jezik  $N(M_2)$  praznim

stogom sa sljedećim prijelazima:

1)  $\delta(q_1, 0, K) = \{(q_1, NK)\}$

2)  $\delta(q_1, 1, K) = \{(q_1, JK)\}$

3)  $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$

4)  $\delta(q_1, 1, N) = \{(q_1, JN)\}$

5)  $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

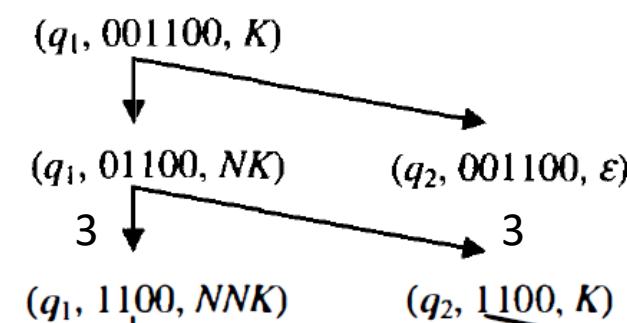
6)  $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

7)  $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

8)  $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

9)  $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

10)  $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$



(4)

Zadan je PA  $M_2 = (\{q_1, q_2\}, \{0, 1\}, \{N, J, K\}, \delta, q_1, K, \emptyset)$  koji prihvaca jezik  $N(M_2)$  praznim

stogom sa sljedećim prijelazima:

1)  $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3)  $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$

5)  $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

7)  $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

9)  $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

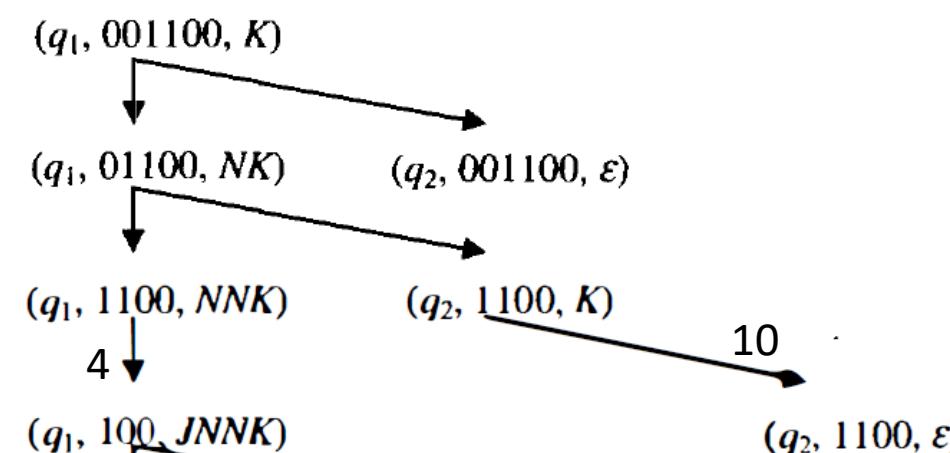
10)  $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

2)  $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4)  $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6)  $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

8)  $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$



(4)

Zadan je PA  $M_2 = (\{q_1, q_2\}, \{0, 1\}, \{N, J, K\}, \delta, q_1, K, \emptyset)$  koji prihvaca jezik  $N(M_2)$  praznim stogom sa sljedećim prijelazima:

1)  $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3)  $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$

5)  $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

7)  $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

9)  $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

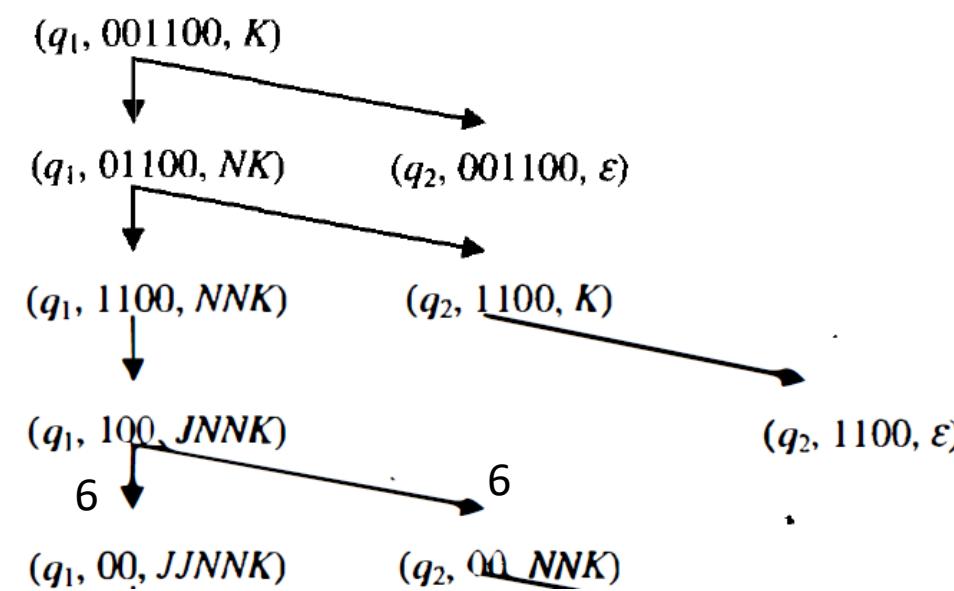
10)  $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

2)  $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4)  $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6)  $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

8)  $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$



(4)

Zadan je PA  $M_2 = (\{q_1, q_2\}, \{0, 1\}, \{N, J, K\}, \delta, q_1, K, \emptyset)$  koji prihvaca jezik  $N(M_2)$  praznim stogom sa sljedećim prijelazima:

1)  $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3)  $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$

5)  $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

7)  $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

9)  $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

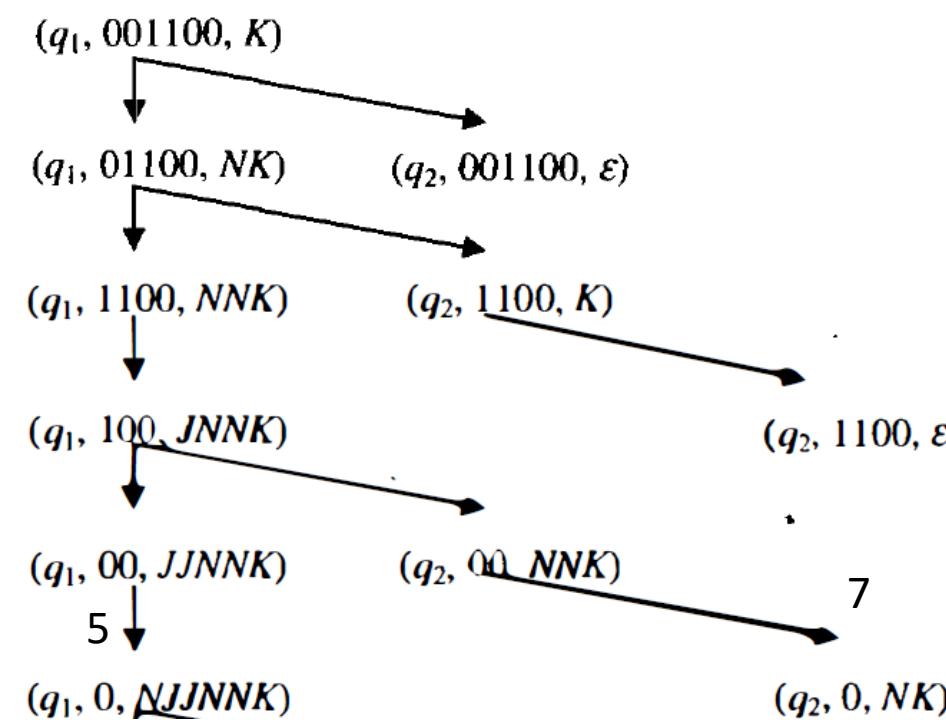
10)  $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

2)  $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4)  $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6)  $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

8)  $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$



5

7

(4)

Zadan je PA  $M_2 = (\{q_1, q_2\}, \{0, 1\}, \{N, J, K\}, \delta, q_1, K, \emptyset)$  koji prihvaca jezik  $N(M_2)$  praznim stogom sa sljedećim prijelazima:

1)  $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3)  $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$

5)  $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

7)  $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

9)  $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

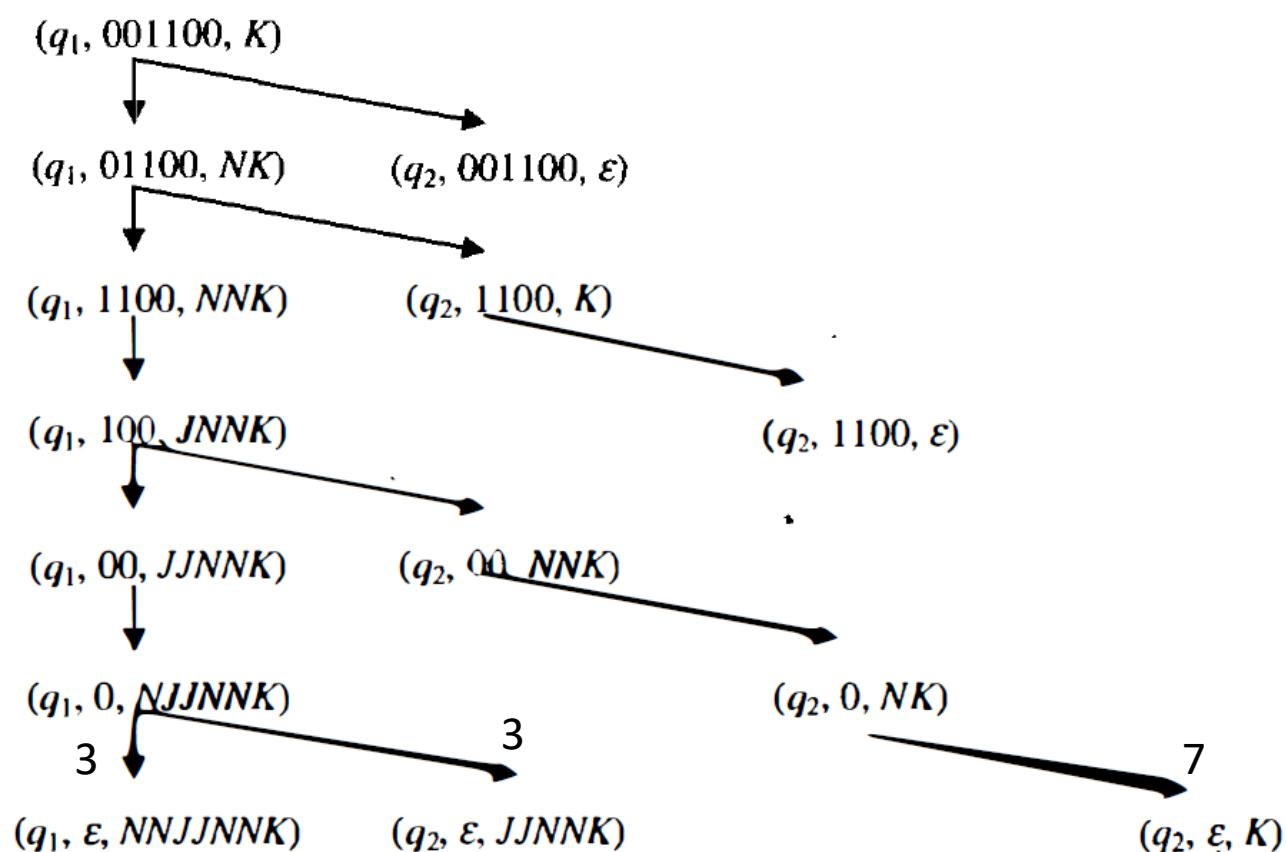
10)  $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

2)  $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4)  $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6)  $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

8)  $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$



(4)

Zadan je PA  $M_2 = (\{q_1, q_2\}, \{0, 1\}, \{N, J, K\}, \delta, q_1, K, \emptyset)$  koji prihvaca jezik  $N(M_2)$  praznim

stogom sa sljedećim prijelazima:

1)  $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3)  $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$

5)  $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

7)  $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

9)  $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

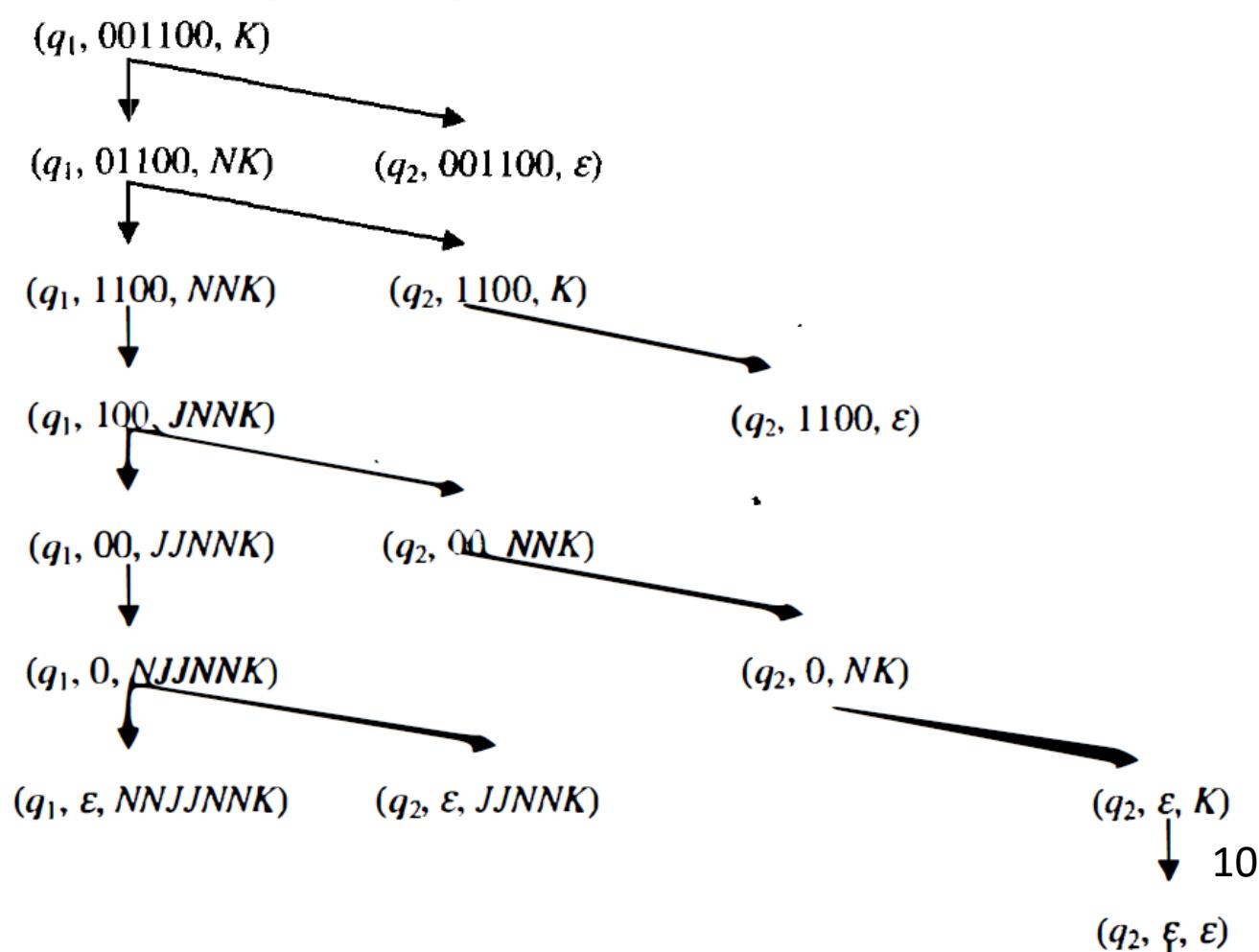
10)  $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

2)  $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4)  $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6)  $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

8)  $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$



(4)

Zadan je PA  $M_2 = (\{q_1, q_2\}, \{0, 1\}, \{N, J, K\}, \delta, q_1, K, \emptyset)$  koji prihvaca jezik  $N(M_2)$  praznim stogom sa sljedećim prijelazima:

1)  $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3)  $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$

5)  $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

7)  $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

9)  $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

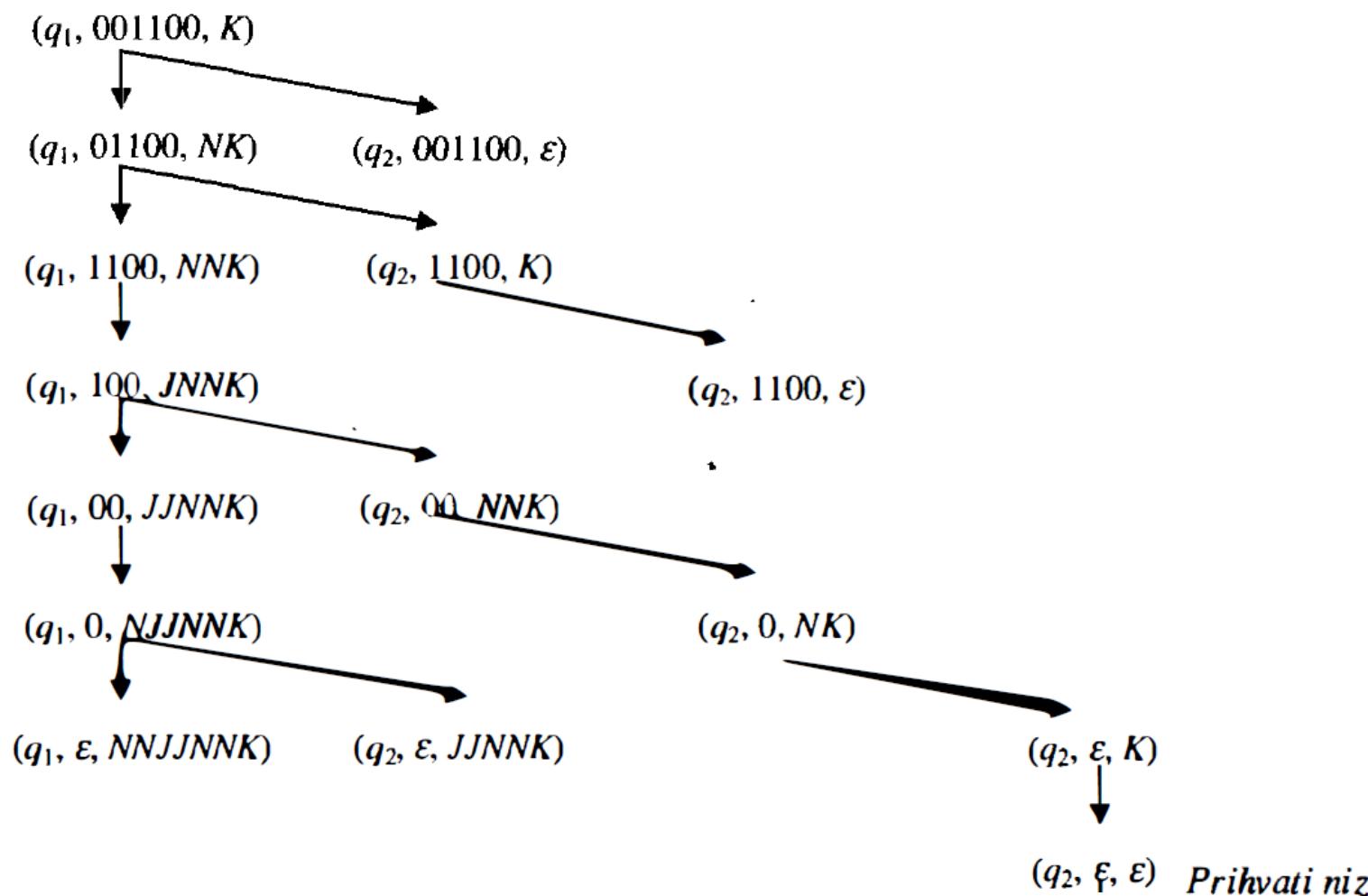
10)  $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

2)  $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4)  $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6)  $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

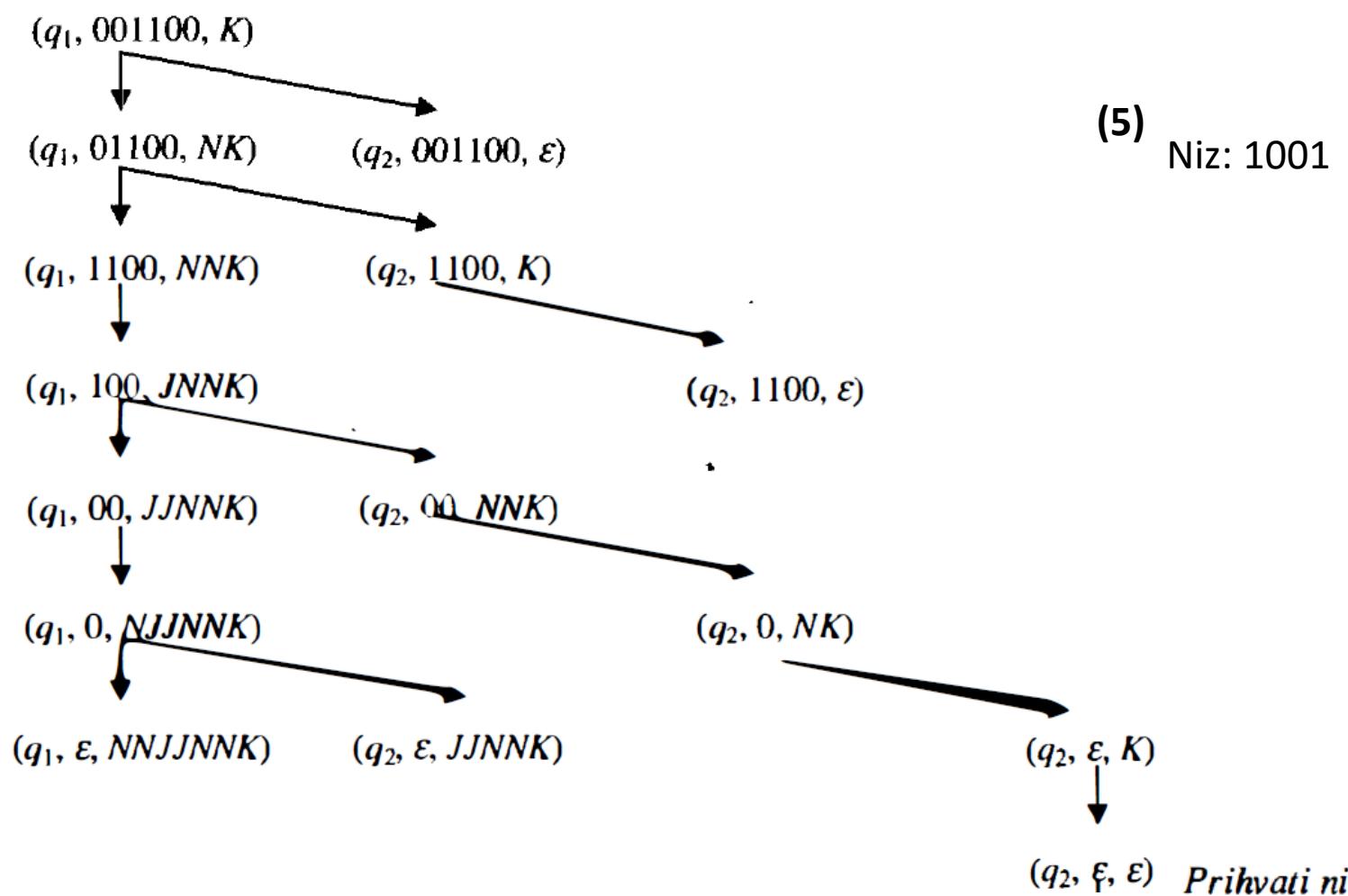
8)  $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$



**(4)**

Zadan je PA  $M_2 = (\{q_1, q_2\}, \{0, 1\}, \{N, J, K\}, \delta, q_1, K, \emptyset)$  koji prihvaca jezik  $N(M_2)$  praznim stogom sa sljedećim prijelazima:

- |  |  |
|--|--|
| 1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$                     | 2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$                     |
| 3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$ | 4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$                     |
| 5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$                     | 6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$ |
| 7) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$            | 8) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$            |
| 9) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$  |  |
| 10) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$ |  |

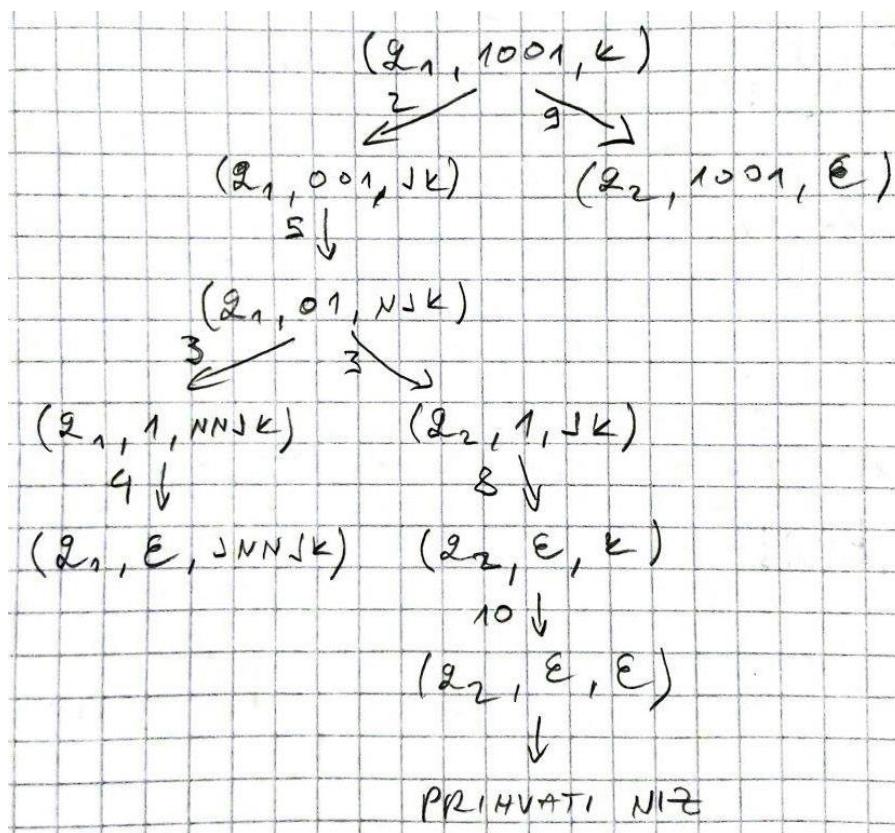


Zadan je PA  $M_2 = (\{q_1, q_2\}, \{0, 1\}, \{N, J, K\}, \delta, q_1, K, \emptyset)$  koji prihvata jezik  $N(M_2)$  praznim stogom sa sljedećim prijelazima:

- |   |   |
|---|---|
| 1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$                  | 2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$                  |
| 3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \epsilon)\}$ | 4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$                  |
| 5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$                  | 6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \epsilon)\}$ |
| 7) $\delta(q_2, 0, N) = \{(q_2, \epsilon)\}$            | 8) $\delta(q_2, 1, J) = \{(q_2, \epsilon)\}$            |
| 9) $\delta(q_1, \epsilon, K) = \{(q_2, \epsilon)\}$     |   |
| 10) $\delta(q_2, \epsilon, K) = \{(q_2, \epsilon)\}$    |   |

(5)

Niz: 1001



PA koji prihvata prihvatljivim stanjem  
-> PA koji prihvata praznim stogom

**Primjer 3.18.** Zadan je PA  $M_2 = \{q_1, q_2\}, \{0, 1\}, \{N, K\}, \delta, q_1, K, \{q_2\}$  sa sljedećim prijelazima:

- 1)  $\delta(q_1, 0, K) = \{(q_1, NK)\}$
- 2)  $\delta(q_1, 0, N) = \{(q_1, NN)\}$
- 3)  $\delta(q_1, 1, N) = \{(q_2, \epsilon)\}$
- 4)  $\delta(q_2, 1, N) = \{(q_2, \epsilon)\}$

PA  $M_2$  prihvata jezik  $L(M_2) = \{0^m 1^n \mid n \geq 1, m \geq 1, m \leq n\}$  prihvatljivim stanjem  $q_2$ . Prijelazi (1) i (2) stavljuju po jedan znak  $N$  na vrh stoga za svaki pročitan znak 0. Pojavom prvog znaka 1, PA prijelazom (3) mijenja stanje u prihvatljivo stanja  $q_2$ . Prijelaz (4) provjerava da znakova 1 nema više od znakova 0.

Istovjetni PA  $M_1 = \{q_1, q_2, q_0', q_e\}, \{0, 1\}, \{N, K, X_0\}, \delta', q_0', X_0, \emptyset\}$  koji prihvata praznim stogom konstruira se na sljedeći način. Korak konstrukcije (1) daje prijelaz u početnu konfiguraciju PA  $M_1$ :

$$1) \quad \delta'(q_0', \epsilon, X_0) = \{(q_1, KX_0)\}.$$

U koraku (2) preuzimaju se svi prijelazi PA  $M_2$ :

- 1)  $\delta'(q_1, 0, K) = \{(q_1, NK)\}$
- 2)  $\delta'(q_1, 0, N) = \{(q_1, NN)\}$
- 3)  $\delta'(q_1, 1, N) = \{(q_2, \epsilon)\}$
- 4)  $\delta'(q_2, 1, N) = \{(q_2, \epsilon)\}$

Korak (3) dodaje  $\epsilon$ -prijelaze u stanje  $q_e$ :

- 5)  $\delta'(q_2, \epsilon, N) = \{(q_e, \epsilon)\}$
- 6)  $\delta'(q_2, \epsilon, K) = \{(q_e, \epsilon)\}$
- 7)  $\delta'(q_2, \epsilon, X_0) = \{(q_e, \epsilon)\}$

Korak (4) dodaje  $\epsilon$ -prijelaze koji prazne stog:

- 8)  $\delta'(q_e, \epsilon, N) = \{(q_e, \epsilon)\}$
- 9)  $\delta'(q_e, \epsilon, K) = \{(q_e, \epsilon)\}$
- 10)  $\delta'(q_e, \epsilon, X_0) = \{(q_e, \epsilon)\}$ .

Slijed prijelaza PA  $M_1$  za niz 00011 je:

$$\begin{aligned} (q_0', 00011, X_0) &\succ (q_1, 00011, KX_0) \succ (q_1, 0011, NKX_0) \succ (q_1, 011, NNKX_0) \succ \\ &(q_1, 11, NNNKX_0) \succ (q_2, 1, NNKX_0) \succ (q_2, \epsilon, NKX_0) \succ (q_e, \epsilon, KX_0) \succ \\ &(q_e, \epsilon, X_0) \succ (q_e, \epsilon, \epsilon). \end{aligned}$$

PA  $M_1$  prihvata niz 00011, jer su pročitani svi znakovи niza, a stog je prazan.

Slijed prijelaza PA  $M_2$  za niz 00011 je:

$$\begin{aligned} (q_1, 00011, K) &\succ (q_1, 0011, NK) \succ (q_1, 011, NNK) \succ \\ &(q_1, 11, NNNK) \succ (q_2, 1, NNK) \succ (q_2, \epsilon, NK). \end{aligned}$$

PA  $M_2$  prihvata niz 00011, jer su pročitani svi znakovи niza, a stanje  $q_2$  jest prihvatljivo stanje.

## Konstrukcija PA koji prihvata praznim stogom iz zadatog PA koji prihvata prihvatljivim stanjem

Neka PA:

$$M_2 = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

prihvata jezik  $L(M_2)$  prihvatljivim stanjem. Želi se izgraditi istovjetni PA  $M_1$  koji prihvata praznim stogom. Konstrukcija PA  $M_1$  zasniva se na simulaciji PA  $M_2$ . Uđe li tijekom simulacije PA  $M_2$  u jedno od prihvatljivih stanja, PA  $M_1$  isprazni stog ako i samo ako PA  $M_2$  uđe u jedno od prihvatljivih stanja, uvodi se dodatni znak stoga  $X_0$ . Da bi se omogućilo da PA  $M_1$  isprazni stog ako i samo ako PA  $M_2$  uđe u jedno od prihvatljivih stanja, uvodi se dodatni znak stoga  $X_0$ . Na početku rada PA  $M_1$  stavi na dno stoga znak  $X_0$ . PA  $M_2$  svojim prijelazima ne može uzeti sa stoga znak  $X_0$ . Isprazni li PA  $M_2$  stog, a ne uđe u jedno od prihvatljivih stanja, na dnu stoga ostaje znak  $X_0$  koji onemogućava da PA  $M_1$  prihvati niz.

Konstruira se PA  $M_1$ :

$$M_1 = (Q \cup \{q_0'\}, \Sigma, \Gamma \cup \{X_0\}, \delta', q_0', X_0, \emptyset),$$

Funkcija prijelaza PA  $M_1$  gradi se na sljedeći način:

$$1) \quad \delta'(q_0', \epsilon, X_0) = \{(q_0, Z_0 X_0)\}.$$

Na početku rada PA  $M_1$  prelazi iz svoje početne konfiguracije u početnu konfiguraciju PA  $M_2$ . PA  $M_1$  ostavlja znak  $X_0$  na dnu stoga.

$$2) \quad \text{U skup } \delta'(q, a, Z) \text{ stave se svi elementi skupa } \delta(q, a, Z).$$

Skup  $\delta'(q, a, Z)$  računa se za sva stanja  $q$  iz  $Q$ , za sve znakove  $a$  iz  $\Sigma \cup \{\epsilon\}$  i za sve znakove stoga  $Z$  iz  $\Gamma$ . Ovi prijelazi omogućuju simuliranje rada PA  $M_2$ .

$$3) \quad \text{U skup } \delta'(q, \epsilon, Z) \text{ dodaje se } \epsilon\text{-prijelaz } (q_e, \epsilon, \epsilon), q \in F.$$

$\epsilon$ -prijelazi dodaju se za sva stanja  $q$  iz skupa prihvatljivih stanja  $F$  i za sve znakove stoga  $Z$  iz  $\Gamma \cup \{X_0\}$ . Uđe li PA  $M_2$  u jedno od prihvatljivih stanja, skup prijelaza proširuje se  $\epsilon$ -prijelazom u stanje  $q_e$ . Istodobno se s vrha stoga uzme jedan znak.

$$4) \quad \text{U skup } \delta'(q_e, \epsilon, Z) \text{ dodaje se } \epsilon\text{-prijelaz } (q_e, \epsilon, \epsilon).$$

$\epsilon$ -prijelazi dodaju se za sve znakove stoga  $Z$  iz  $\Gamma \cup \{X_0\}$ .  $\epsilon$ -prijelazi u stanju  $q_e$  prazne čitav stog, uključujući i znak  $X_0$ .

$$M_2 = (\{2_1, 2_2\}, \{0, 1\}, \{N, k\}, \mathcal{F}, 2_1, k, \{2_2\})$$

$$1) S(2_1, 0, k) = \{(2_1, Nk)\}$$

$$2) S(2_1, 0, N) = \{(2_1, NN)\}$$

$$3) S(2_1, 1, N) = \{(2_2, \epsilon)\}$$

$$4) S(2_2, 1, N) = \{(2_2, \epsilon)\}$$

$$M_2 = (\{2_1, 2_2\}, \{0, 1\}, \{N, k\}, \mathcal{F}, 2_1, k, \{2_2\})$$

$$1) S(2_1, 0, k) = \{(2_1, Nk)\}$$

$$2) S(2_1, 0, N) = \{(2_1, NN)\}$$

$$3) S(2_1, 1, N) = \{(2_2, \varepsilon)\}$$

$$4) S(2_2, 1, N) = \{(2_2, \varepsilon)\}$$

$$M_1 = (\{2_1, 2_2, 2'_1, 2'_2\}, \{0, 1\}, \{N, \varepsilon, x_0\}, \mathcal{F}', 2'_1, x_0, \emptyset)$$

$$M_2 = (\{2_1, 2_2\}, \{0, 1\}, \{N, k\}, \mathcal{F}, 2_1, k, \{2_2\})$$

- 1)  $\mathcal{S}(2_1, 0, k) = \{(2_1, Nk)\}$
- 2)  $\mathcal{S}(2_1, 0, N) = \{(2_1, NN)\}$
- 3)  $\mathcal{S}(2_1, 1, N) = \{(2_2, \varepsilon)\}$
- 4)  $\mathcal{S}(2_2, 1, N) = \{(2_2, \varepsilon)\}$

$$M_1 = (\{2_1, 2_2, 2_0, 2_e\}, \{0, 1\}, \{N, \varepsilon, x_0\}, \mathcal{F}', 2_0, x_0, \emptyset)$$

- 1)  
2)  
3)  
4)

$$M_2 = (\{z_1, z_2\}, \{0, 1\}, \{N, \epsilon\}, \mathcal{F}, z_1, K, \{z_2\})$$

- 1)  $\mathcal{S}(z_1, 0, \epsilon) = \{(z_1, Nz)\}$
- 2)  $\mathcal{S}(z_1, 0, N) = \{(z_1, NN)\}$
- 3)  $\mathcal{S}(z_1, 1, N) = \{(z_2, \epsilon)\}$
- 4)  $\mathcal{S}(z_2, 1, N) = \{(z_2, \epsilon)\}$

$$M_1 = (\{z_1, z_2, z'_0, z_e\}, \{0, 1\}, \{N, \epsilon, x_0\}, \mathcal{F}', z'_0, x_0, \emptyset)$$

$$0) \mathcal{S}'(z'_0, \epsilon, x_0) = \{(z_1, Kx_0)\}$$

- 1)
- 2)
- 3)
- 4)

$$M_2 = (\{z_1, z_2\}, \{0, 1\}, \{N, \kappa\}, \mathcal{F}, z_1, \kappa, \{z_2\})$$

- 1)  $\mathcal{S}(z_1, 0, \kappa) = \{(z_1, N\kappa)\}$
- 2)  $\mathcal{S}(z_1, 0, N) = \{(z_1, NN)\}$
- 3)  $\mathcal{S}(z_1, 1, N) = \{(z_2, \varepsilon)\}$
- 4)  $\mathcal{S}(z_2, 1, N) = \{(z_2, \varepsilon)\}$

$$M_1 = (\{z_1, z_2, z'_1, z'_2\}, \{0, 1\}, \{N, \varepsilon, x_0\}, \mathcal{F}', z'_1, x_0, \emptyset)$$

0)  $\mathcal{S}'(z'_1, \varepsilon, x_0) = \{(z_1, Kx_0)\}$

- 1)
- 2)
- 3)
- 4)

5)  $\mathcal{S}'(z'_2, \varepsilon, N) = \{(z_2, \varepsilon)\}$

6)  $\mathcal{S}'(z'_2, \varepsilon, \kappa) = \{(z_2, \varepsilon)\}$

7)  $\mathcal{S}'(z'_2, \varepsilon, x_0) = \{(z_2, \varepsilon)\}$

$$M_2 = (\{2_1, 2_2\}, \{0, 1\}, \{N, k\}, \mathcal{S}, 2_1, k, \{2_2\})$$

- 1)  $S(2_1, 0, k) = \{(2_1, Nk)\}$
- 2)  $S(2_1, 0, N) = \{(2_1, NN)\}$
- 3)  $S(2_1, 1, N) = \{(2_2, \epsilon)\}$
- 4)  $S(2_2, 1, N) = \{(2_2, \epsilon)\}$

$$M_1 = (\{2_1, 2_2, 2'_1, 2'_2\}, \{0, 1\}, \{N, \epsilon, x_0\}, \mathcal{S}', 2'_1, x_0, \emptyset)$$

0)  $\mathcal{S}'(2'_1, \epsilon, x_0) = \{(2_1, kx_0)\}$

- 1)
- 2)
- 3)
- 4)

5)  $\mathcal{S}'(2_2, \epsilon, N) = \{(2_2, \epsilon)\}$

6)  $\mathcal{S}'(2_2, \epsilon, k) = \{(2_2, \epsilon)\}$

7)  $\mathcal{S}'(2_2, \epsilon, x_0) = \{(2_2, \epsilon)\}$

8)  $\mathcal{S}'(2_2, \epsilon, N) = \{(2_2, \epsilon)\}$

9)  $\mathcal{S}'(2_2, \epsilon, k) = \{(2_2, \epsilon)\}$

10)  $\mathcal{S}'(2_2, \epsilon, x_0) = \{(2_2, \epsilon)\}$