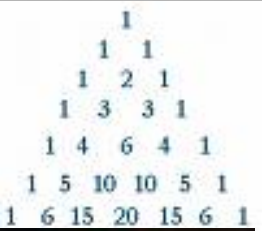
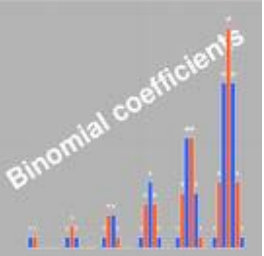
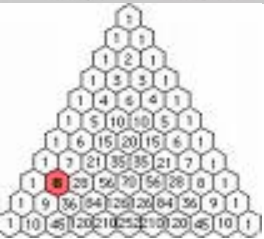
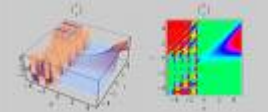


$\binom{n}{k}$  : number of ways of arranging  $k$  success among  $n$  observations  
 $0 \leq k \leq n$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

# Binomni koeficienti



$\binom{n}{k}$  : number of ways of arranging  $k$  success among  $n$  observations  
 $0 \leq k \leq n$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

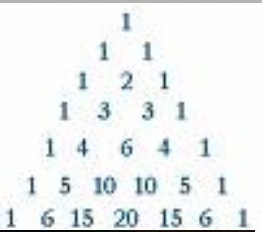
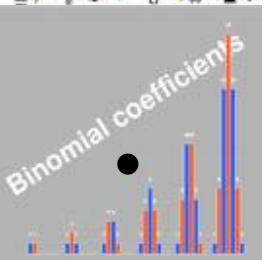
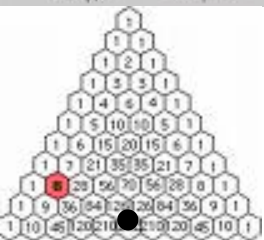
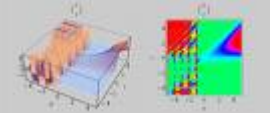
$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n \cdot (n-1) \cdot \dots \cdot (n-r+1)}{r!}$$

• Broj svih  $r$ -kombinacija  $n$ -članog skupa

označavamo sa  $\binom{n}{r}$ .

- $\binom{0}{0} := 1$
- za  $r > n$  je  $\binom{n}{r} := 0$
- $\binom{0}{r} := 0$  za  $r \in \mathbf{N}$



$\binom{n}{k}$  : number of ways of arranging  $k$  success among  $n$  observations  
 $0 \leq k \leq n$

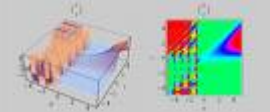
$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

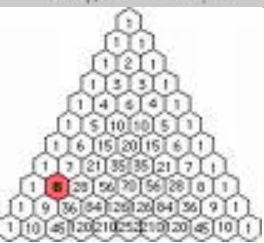
$$\binom{n}{r} = \binom{n}{n-r}$$

$$\binom{n}{r} = \frac{n}{r} \cdot \binom{n-1}{r-1}$$

$$r \cdot \binom{n}{r} = n \cdot \binom{n-1}{r-1}$$



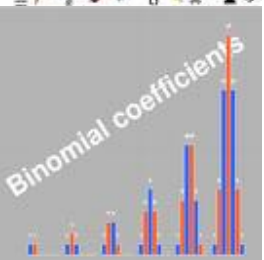
# Pascalova formula



**Teorem**

Za  $n, r \in \mathbf{N}$ ,  $1 \leq r \leq n - 1$  vrijedi

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}.$$



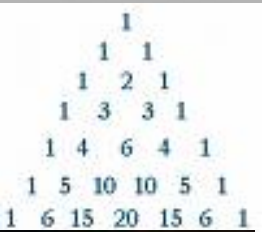
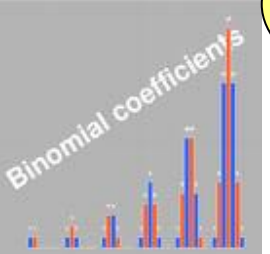
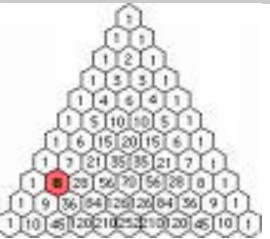
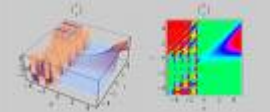
direktni dokaz teorema raspisivanjem binomnih koeficijenata



$\binom{n}{k}$  : number of ways of arranging  $k$  success among  $n$  observations  
 $0 \leq k \leq n$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

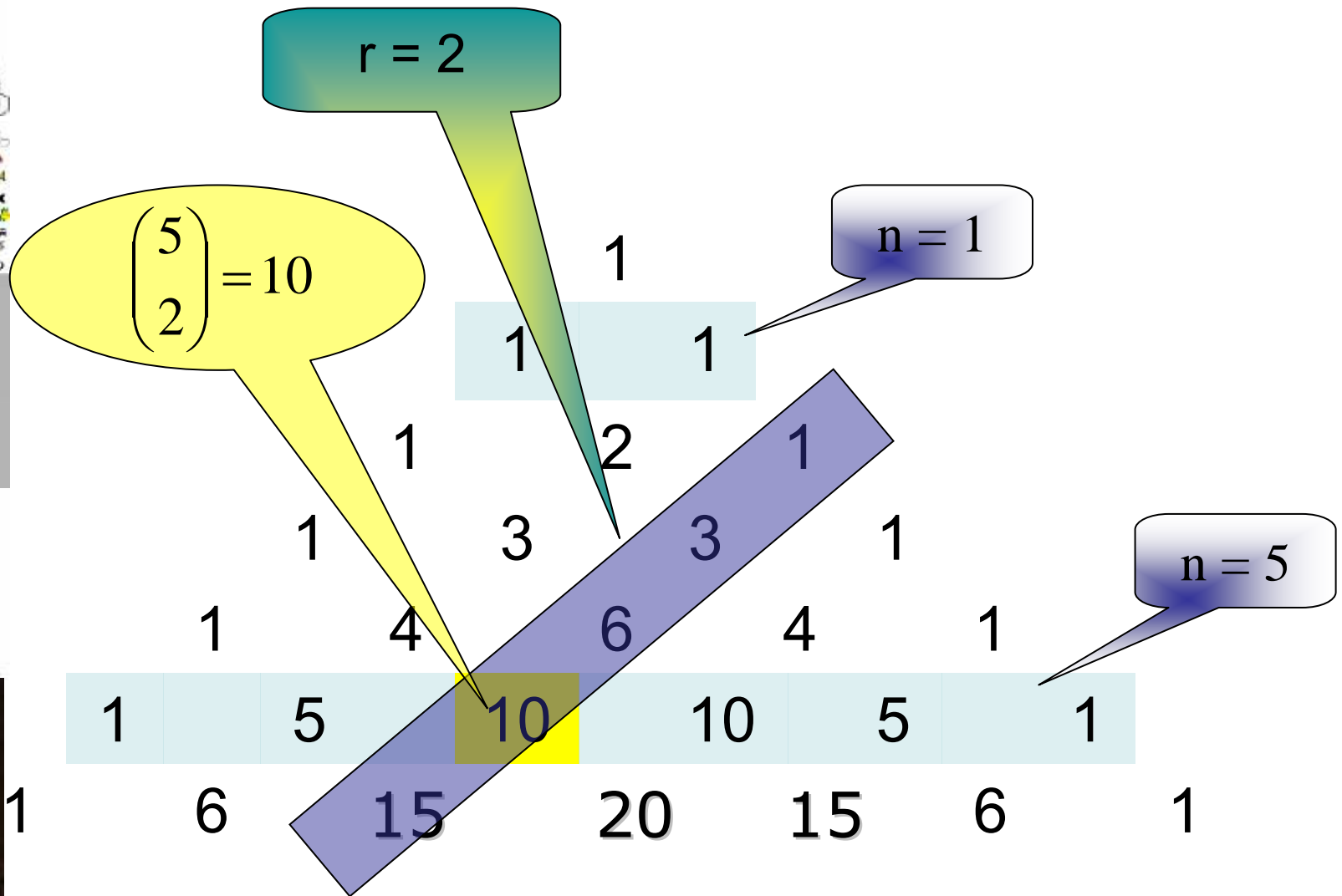


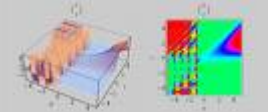
$\binom{n}{k}$  : number of ways of arranging  $k$  success among  $n$  observations  
 $0 \leq k \leq n$

$$Q(x) = \frac{d}{dx} \ln \frac{P(x)}{P(x-k)}$$

$$P(x+k) = \binom{n}{k} p^k (1-p)^{n-k}$$

# Pascalov trokut





# Teorem

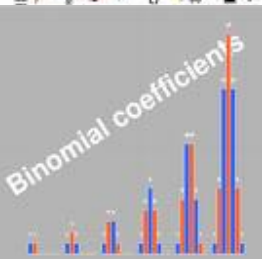


Za svako  $n \in N$  niz  $\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \dots, \binom{n}{n}$  je unimodalan.



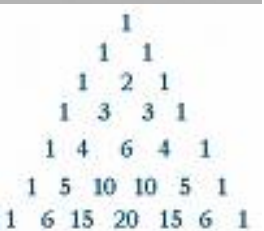
Ako je  $n$  paran, onda je

$$\binom{n}{0} < \binom{n}{1} < \dots < \binom{n}{n/2}, \binom{n}{n/2} > \dots > \binom{n}{n-1} > \binom{n}{n}.$$



Ako je  $n$  neparan, onda je

$$\binom{n}{0} < \binom{n}{1} < \dots < \binom{n}{(n-1)/2} = \binom{n}{(n+1)/2} > \dots > \binom{n}{n-1} > \binom{n}{n}.$$



U svakom slučaju medju brojevima  $\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \dots, \binom{n}{n}$

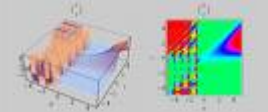


najveći je  $\binom{n}{\lfloor n/2 \rfloor} = \binom{n}{\lceil n/2 \rceil}.$

$\binom{n}{k}$  : number of ways of arranging  $k$  success among  $n$  observations  
0 ≤ k ≤ n

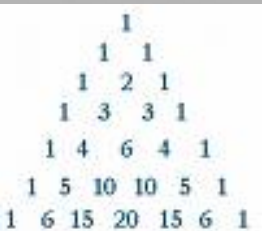
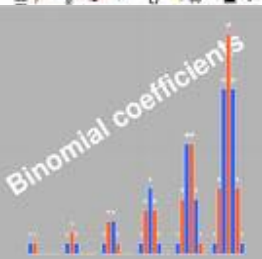
$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$



**Teorem**

# Binomni teorem



$\binom{n}{k}$  = number of ways of arranging  $k$  successes among  $n$  observations

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Za  $n \in \mathbf{N}$  i sve  $x, y \in \mathbf{C}$  vrijedi

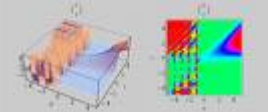
$$(x + y)^n = x^n + \binom{n}{1} x^{n-1} y + \binom{n}{2} x^{n-2} y^2 + \dots + \binom{n}{n-1} x y^{n-1} + y^n$$

$$= \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k.$$



dokaz teorema indukcijom



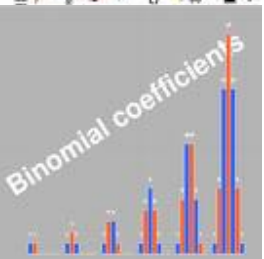


**Teorem**

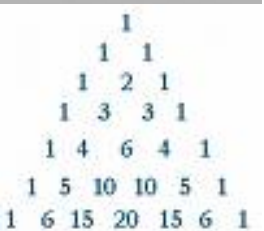
# Vandermondeova konvolucija



$$\binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \binom{n}{2}\binom{m}{r-2} + \dots + \binom{n}{r}\binom{m}{0} = \sum_{k=0}^r \binom{n}{k}\binom{m}{r-k} = \binom{m+n}{r}$$



$$\binom{n}{0}\binom{m}{0} + \binom{n}{1}\binom{m}{1} + \binom{n}{2}\binom{m}{2} + \dots + \binom{n}{n}\binom{m}{n} = \sum_{k=0}^n \binom{n}{k}\binom{m}{k} = \binom{m+n}{n}$$

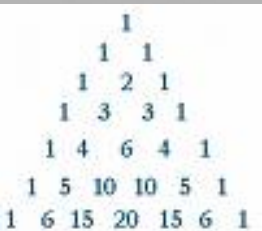
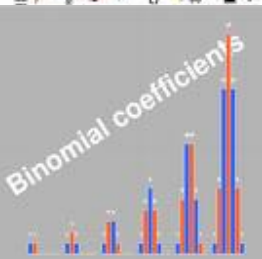
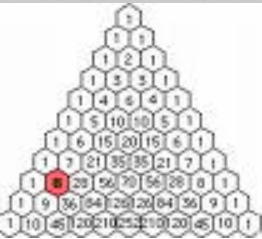
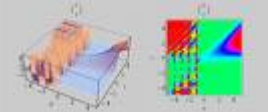


Posebno, za  $m = n$  je  $\sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n}$ .

$\binom{n}{k}$  = number of ways of arranging  $k$  successes among  $n$  observations

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$   
 $P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$





$\binom{n}{k}$  : number of ways of arranging  $k$  success among  $n$  observations  
 $0 \leq k \leq n$

$$Q(x) = \frac{d}{dx} \ln Q(x) = \frac{P(x+1) - P(x)}{P(x)}$$

