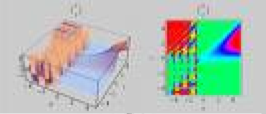
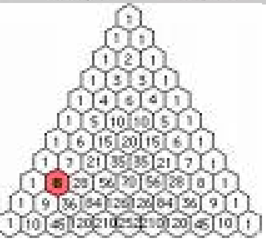
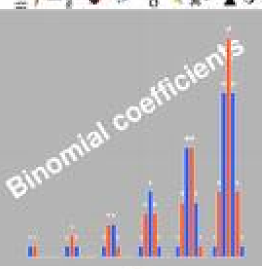
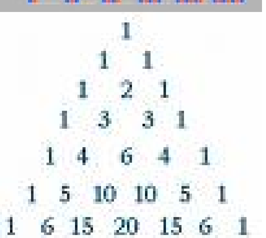
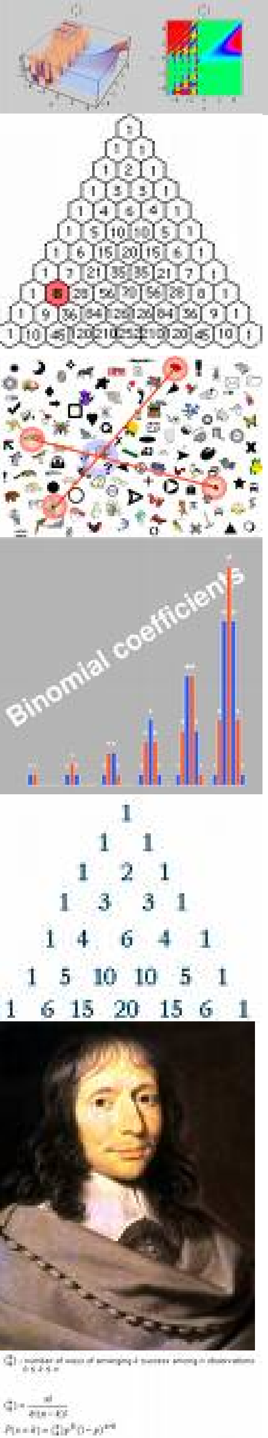
Binomni koeficijenti









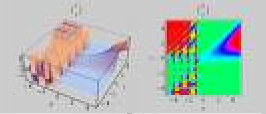


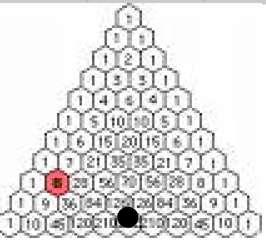
$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n\cdot(n-1)\cdot\ldots\cdot(n-r+1)}{r!}$$

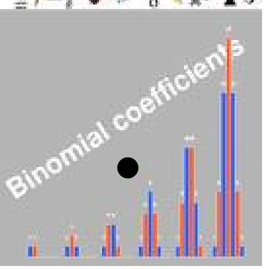
- Broj svih *r*-kombinacija *n*-članog skupa

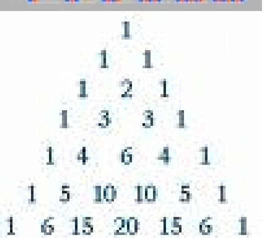
označavamo sa $\binom{n}{r}$.

- $\binom{0}{0} := 1$
- za *r* > *n* je $\binom{n}{r} := 0$
- $\binom{0}{r} := 0$ za $r \in \mathbf{N}$





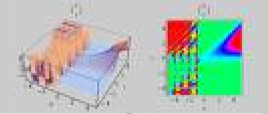


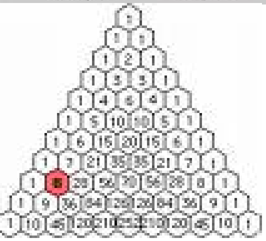


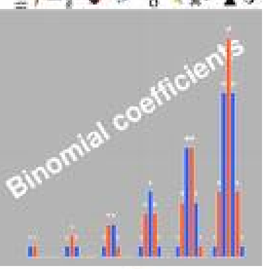
$$\binom{n}{r} = \binom{n}{n-r}$$

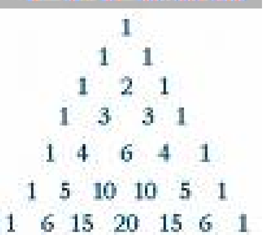
$$\binom{n}{r} = \frac{n}{r} \cdot \binom{n-1}{r-1}$$

$$r \cdot \binom{n}{r} = n \cdot \binom{n-1}{r-1}$$









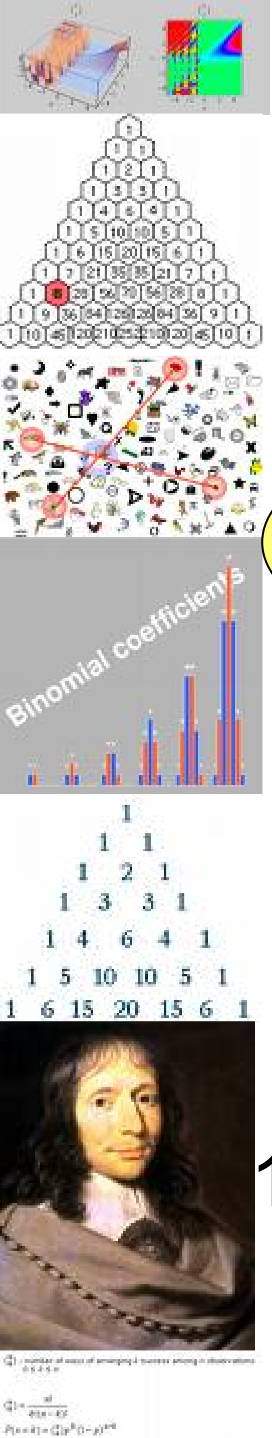
Pascalova formula

Teorem

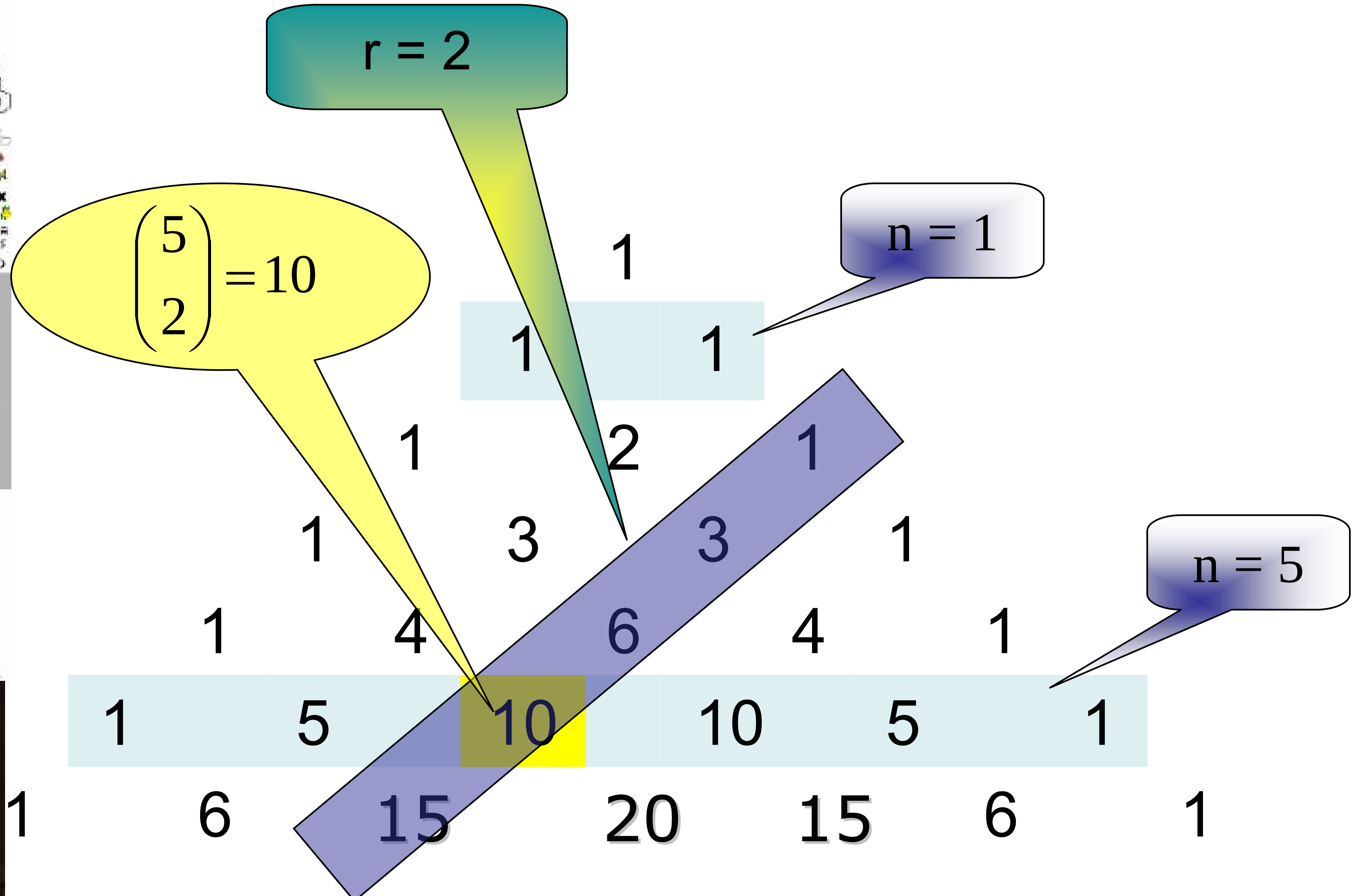
Za $n, r \in \mathbf{N}$, $1 \leq r \leq n - 1$ vrijedi

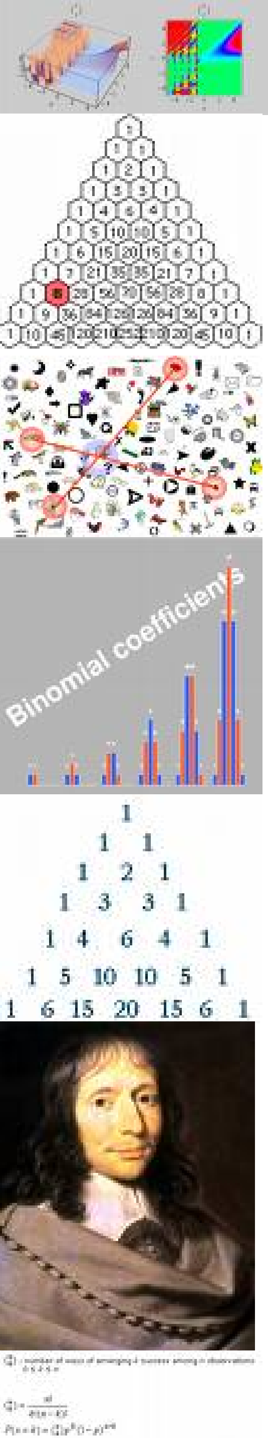
$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}.$$

direktni dokaz teorema raspisivanjem binomnih koeficijenata



Pascalov trokut





Teorem

Za svako $n \in N$ niz $\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \ldots, \binom{n}{n}$ je unimodalan.

Ako je n paran, onda je

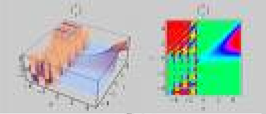
$$\binom{n}{0} < \binom{n}{1} < \ldots < \binom{n}{n/2}, \binom{n}{n/2} > \ldots > \binom{n}{n-1} > \binom{n}{n}.$$

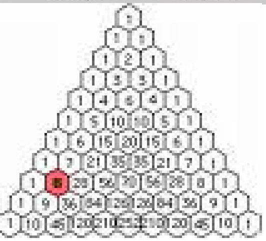
Ako je n neparan, onda je

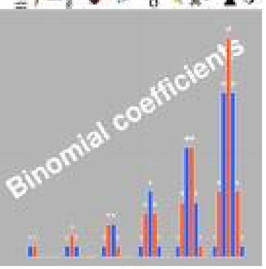
$$\binom{n}{0} < \binom{n}{1} < \ldots < \binom{n}{(n-1)/2} = \binom{n}{(n+1)/2} > \ldots > \binom{n}{n-1} > \binom{n}{n}.$$

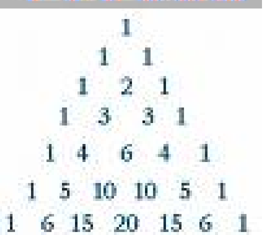
U svakom slučaju medju brojevima $\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \ldots, \binom{n}{n}$

najveći je $\binom{n}{\lfloor n/2 \rfloor} = \binom{n}{\lceil n/2 \rceil}.$









Teorem

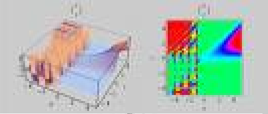
Binomni teorem

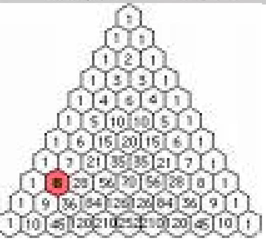
Za $n \in \mathbf{N}$ i sve x, $y \in \mathbf{C}$ vrijedi

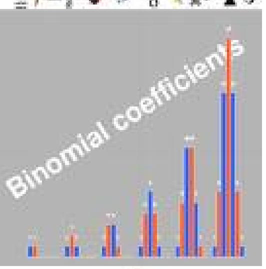
$$(x+y)^n = x^n + \binom{n}{1} x^{n-1} y + \binom{n}{2} x^{n-2} y^2 + \ldots + \binom{n}{n-1} x y^{n-1} + y^n$$

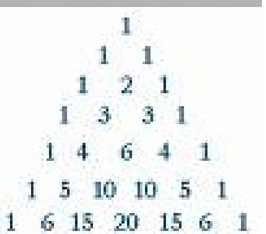
$$= \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k.$$

dokaz teorema indukcijom









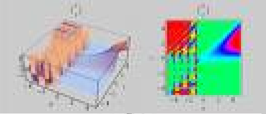
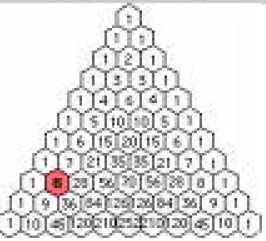
Teorem

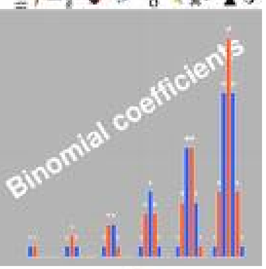
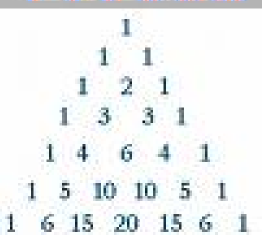
Vandermondeova konvolucija

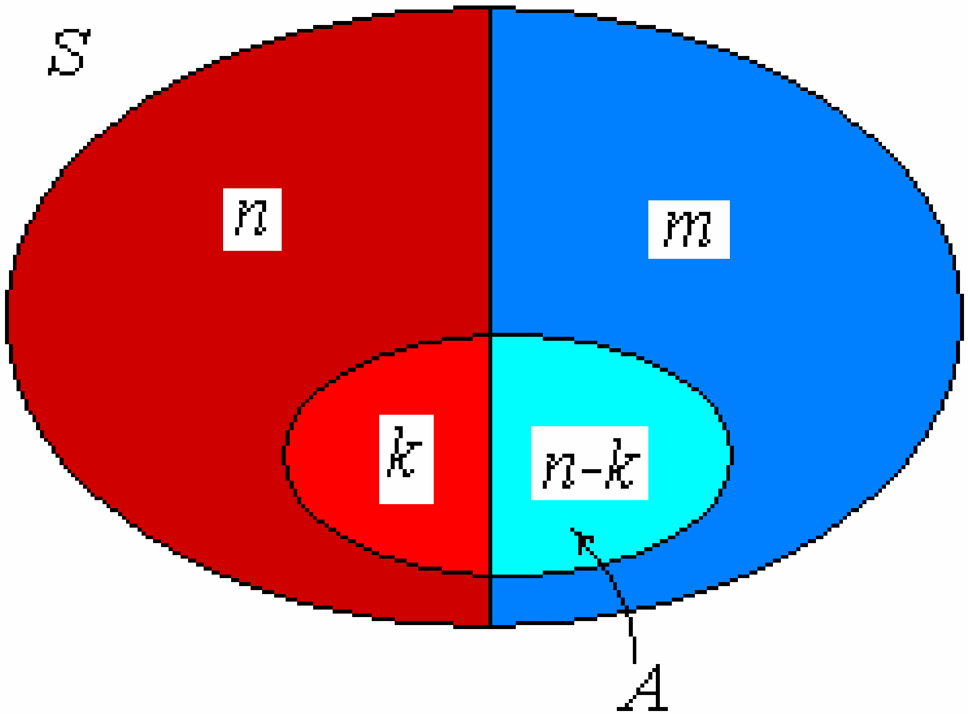
$$\binom{n}{0}\binom{m}{r}+\binom{n}{1}\binom{m}{r-1}+\binom{n}{2}\binom{m}{r-2}+\ldots+\binom{n}{r}\binom{m}{0}=\sum_{k=0}^{r}\binom{n}{k}\binom{m}{r-k}=\binom{m+n}{r}$$

$$\binom{n}{0}\binom{m}{0}+\binom{n}{1}\binom{m}{1}+\binom{n}{2}\binom{m}{2}+\ldots+\binom{n}{n}\binom{m}{n}=\sum_{k=0}^{n}\binom{n}{k}\binom{m}{k}=\binom{m+n}{n}$$

Posebno, za $m = n$ je $\sum_{k=0}^{n}\binom{n}{k}^2=\binom{2n}{n}$.


Binomial coefficients




S
n
m
k
n-k
A