Classical Mechanics
&
Properties of Gases

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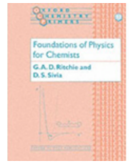
Learning Resources

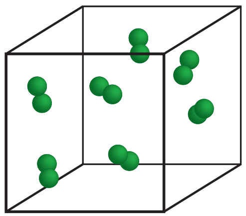
- Handout, Slides, Questions
- wallace.chem.ox.ac.uk



- Foundations of Physics for Chemists *Ritchie & Sivia*
- Elements of Physical Chemistry *Atkins & de Paula*
- Physics, *Alonso & Finn*

Classical Mechanics

- Translation
  - Newton's laws of motion
  - Momentum, work & energy
- Rotation
  - Angular momentum
  - Moments of inertia
- Vibration
  - Simple harmonic motion

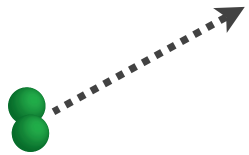


Equations of Motion

$$r(t)$$

$$v(t) = \frac{dr}{dt} \equiv \dot{r}$$

$$a(t) = \frac{dv}{dt} = \frac{d^2r}{dt^2} \equiv \ddot{r}$$



An Easy Example

$$r(t) = At^3$$

$$v(t) = \frac{dr}{dt} = 3At^2$$

$$a(t) = \frac{dv}{dt} = 6At$$

$$r(t) = \int_0^t v(t)\,dt = \int_0^t 3At^2\,dt = At^3$$

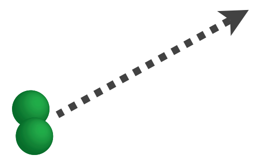
$$v(t) = \int_0^t a(t)\,dt = \int_0^t 6At\,dt = 3At^2$$

Equations of Motion

$$r(t)$$

$$v(t) = \frac{dr}{dt} \equiv \dot{r}$$

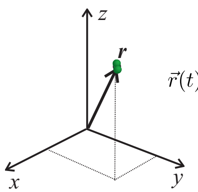
$$a(t) = \frac{dv}{dt} = \frac{d^2r}{dt^2} \equiv \ddot{r}$$



So, for constant acceleration

$$r(t) = r_0 + v_0 t + \frac{1}{2}at^2$$

Equations of Motion in 3D



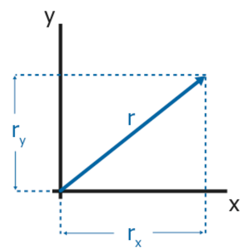
$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$a_x(t) = a_x \qquad v_x(t) = v_{x0} + a_x t \qquad r_x(t) = r_{x0} + v_{x0}t + \tfrac{1}{2}a_x t^2$$

$$a_y(t) = a_y \qquad v_y(t) = v_{y0} + a_y t \qquad r_y(t) = r_{y0} + v_{y0}t + \tfrac{1}{2}a_y t^2$$

$$a_z(t) = a_z \qquad v_z(t) = v_{z0} + a_z t \qquad r_z(t) = r_{z0} + v_{z0}t + \tfrac{1}{2}a_z t^2$$

Vectors



Components

$$r_x = |r|\cos\theta \qquad r_y = |r|\sin\theta$$

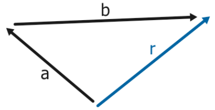
Magnitude

$$|r| = \sqrt{(r_x^2 + r_y^2)}$$

Direction

$$\tan\theta = \frac{r_y}{r_x}$$

Vector Addition



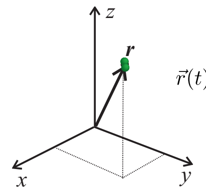
Vector Sum

$\vec{r} \;=\; \vec{a} + \vec{b}$

Components

$r_x \;=\; a_x + b_x \qquad r_y \;=\; a_y + b_y$

Equations of Motion in 3D



$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$a_x(t) = a_x \qquad v_x(t) = v_{x0} + a_x t \qquad r_x(t) = r_{x0} + v_{x0}t + \tfrac{1}{2}a_x t^2$$

$$a_y(t) = a_y \qquad v_y(t) = v_{y0} + a_y t \qquad r_y(t) = r_{y0} + v_{y0}t + \tfrac{1}{2}a_y t^2$$

$$a_z(t) = a_z \qquad v_z(t) = v_{z0} + a_z t \qquad r_z(t) = r_{z0} + v_{z0}t + \tfrac{1}{2}a_z t^2$$

Force

Any influence that changes the motion of an object

- Units, Newton $1 \text{ N} = 1 \text{ kg ms}^{-2}$

Fundamental Force	Relative Strength	Range	Comments
Strong	1	10^{-15} m	Holds the nucleus together
Electromagnetic	**10^{-2}**	**∞**	**Chemistry!**
Weak	10^{-6}	10^{-17} m	Associated with radioactivity
Gravitational	10^{-38}	∞	Causes apples to fall

Gravitational Forces

- The weight of an object is the net gravitational force acting on it.
- The gravitational force between point or spherical masses, m_1 and m_2, is

$$F = -\frac{Gm_1m_2}{r^2} \qquad G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$$

- For objects close to the earth's surface

$$F = mg \qquad g = \frac{Gm_\mathrm{E}}{R_\mathrm{e}^2} \simeq 9.8\ \mathrm{m\,s^{-2}}$$

g = acceleration due to gravity
r = R_e the radius of the earth

Electromagnetic Forces

- The Coulomb force between point or spherical charges, q_1 and q_2, is

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \qquad \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \,\mathrm{N\,m^2\,C^{-2}}$$

- The Coulomb force can be either attractive or repulsive depending on the sign of the charges.
- The Physical Basis of Chemistry: The role of charge. Dr N.J.B. Green F.10 (wks 1-3)

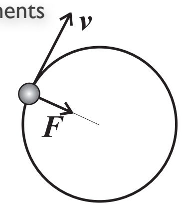
Newton's Laws of Motion

- An object in motion will remain in motion unless acted upon by a net force.
- A force acting on a body is directly proportional to its rate of change of momentum. (F = ma)
- To every action there is an equal and opposite reaction.

Newton's 1st Law

An object in motion will remain in motion unless acted upon by a net force

- Provides definition of force
- If an object isn't moving then the forces on the object must add up to zero
- Dependent on direction,
  - Use vectors / resolve into orthogonal components



Newton's 2nd Law

A force acting on a body is directly proportional to its rate of change of momentum (F = ma)

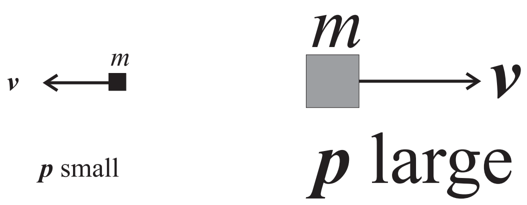
$$\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$$

- Units of kg ms^{-2} or Newtons

Linear Momentum

Product of an object's mass times its velocity

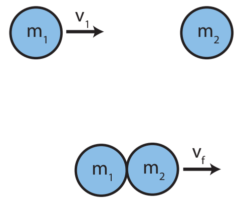
- $\vec{p} = m\vec{v}$
- Units of kg ms^{-1}



Conservation of Momentum

The total momentum of an isolated system of particles is constant

$$\vec{P} = \sum_i \vec{p_i} = \vec{p_1} + \vec{p_2} + \vec{p_3} + \ldots = \text{constant}$$

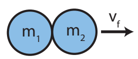


Inelastic Collisions

- Momentum before collision?

$$\sum_i p_i \;=\; m_1 v_1 + m_2 v_2 = mv + 0 = mv$$

- Momentum after collision?



$$\sum_i p_i \;=\; (m_1 + m_2)\, v_f = 2mv_f$$

- Applying conservation of momentum

$$mv = 2mv_f \qquad v_f = \frac{v}{2}$$

Newton's 2nd Law

A force acting on a body is directly proportional to its rate of change of momentum (F = ma)

- $\vec{F} = \dfrac{\mathrm{d}\vec{p}}{\mathrm{d}t}$
- Units of kg ms^{-2} or Newtons

$$\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = \frac{\mathrm{d}(m\vec{v})}{\mathrm{d}t} = m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} + \vec{v}\frac{\mathrm{d}m}{\mathrm{d}t} \approx m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = m\vec{a}$$

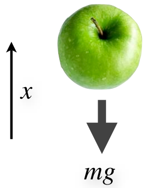
- $\boxed{\vec{F} = m\vec{a}}$

Newton's 2nd Law

A force acting on a body is directly proportional to its rate of change of momentum (F = ma)

$$\sum_i \vec{F}_i = m\vec{a}$$

$$F = -mg$$



$$r(t) = r_0 + v_0 t + \frac{1}{2}at^2$$

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

$$= 0 + 0 - \frac{1}{2}gt^2 \qquad t = \sqrt{\frac{-2x}{g}}$$

An Aside: Dimensional Analysis

$$t = \sqrt{\frac{-2x}{g}}$$

$$s = m^{1/2}(ms^{-2})^{-1/2}$$

An Aside: Dimensional Analysis

$$t \propto x^\alpha m^\beta g^\gamma$$

$$s^1 \propto m^\alpha kg^\beta (ms^{-2})^\gamma$$

$$\beta = 0 \qquad \alpha + \gamma = 0 \qquad 1 = -2\gamma$$

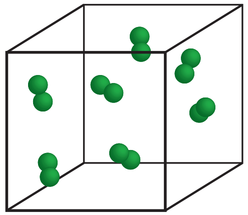
$$\alpha = \frac{1}{2} \qquad \gamma = -\frac{1}{2}$$

$$s^{-1} \propto m^{1/2} kg^0 (ms^{-2})^{-1/2}$$

$$t \propto \sqrt{\frac{x}{g}}$$

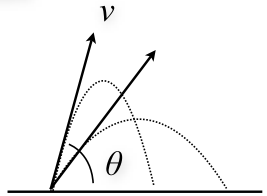
Newton's 3rd Law

To every action there is an equal and opposite reaction



Example Questions

Rocks at the beach

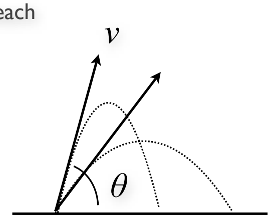


$$v_{x1} = v\cos\theta_1$$

$$v_{x2} = v\cos\theta_2$$

Example Questions

Rocks at the beach



Only consider x-components

$$x_1 = x_{1,0} + v_x t + \frac{1}{2}at^2$$

$$x_1 = vt\cos\theta_1$$

$$x_2 = v(t-\Delta t)\cos\theta_2$$

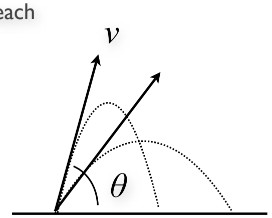
So when they collide:

$$vt\cos\theta_1 = v(t-\Delta t)\cos\theta_2$$

$$\frac{\cos\theta_1}{\cos\theta_2} = \frac{t-\Delta t}{t} \qquad \frac{\Delta t}{t} = 1 - \frac{\cos\theta_1}{\cos\theta_2}$$

Example Questions

Rocks at the beach



Let's pick some angles

$$\cos 60 = 1/2$$

$$\cos 30 = \sqrt{3}/2$$

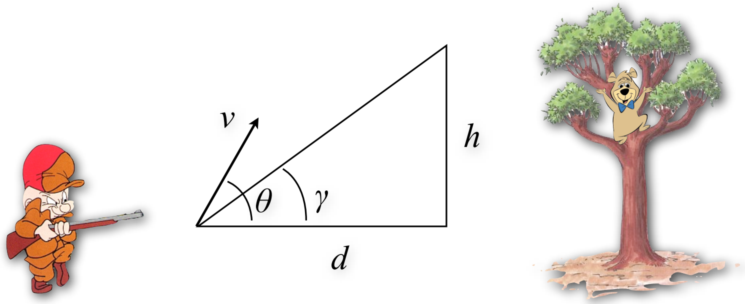
So when will they hit?

$$\frac{\Delta t}{t} = 1 - \frac{\cos\theta_1}{\cos\theta_2} = 1 - \frac{1}{\sqrt{3}} \qquad t = \approx 2.3\Delta t$$

Q: Same angle?

Example Questions

Bear hunting



Example Questions

Bear hunting

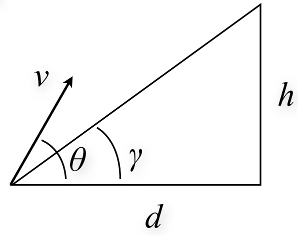
Consider x components of Bullet

$$v_{x,0}(t) = s_0 \cos\theta$$

$$x(t) = \cancelto{0}{x_0} + v_{0,x}t + \frac{1}{2}\cancelto{0}{a_{0,x}}t^2$$

$$x(t) = v_{0,x}t$$

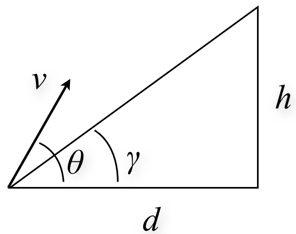
$$t_{death} = \frac{d}{v_{0,x}}$$



Example Questions

Bear hunting

Consider x components of Bear



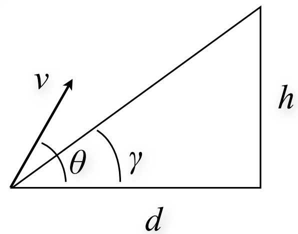
Example Questions

Bear hunting

Consider y components of Bear

$$y(t) = y_0 + v_0 t + \frac{1}{2}a_0 t^2$$

$$y(t) = h + 0 - \frac{1}{2}gt^2$$



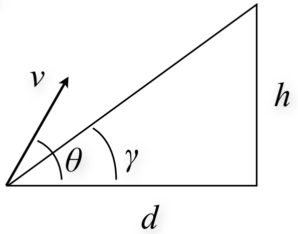
Example Questions

Bear hunting

Consider y components of Bullet

$$y(t) = y_0 + v_{0,y}t + \frac{1}{2}a_{0,y}t^2$$

$$y(t) = 0 + v_{0,y}t - \frac{1}{2}gt^2$$



Example Questions

Bear hunting

For a collision to occur

$$y_{bullet} = y_{bear}$$

$$v_{0,y}t - \frac{1}{2}gt^2 = h - \frac{1}{2}gt^2$$

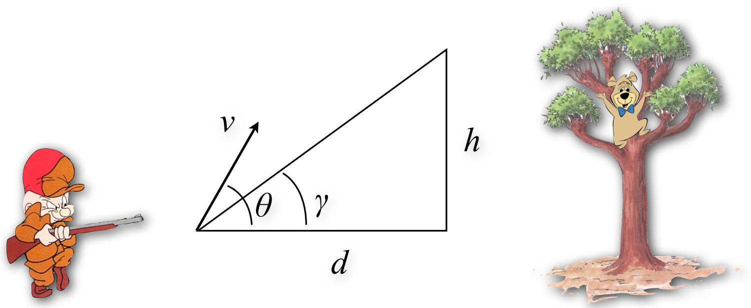
$$v_{0,y}t = h$$

We can substitute for the time of collision

$$t_{death} = \frac{d}{v_{0,x}} \qquad \frac{h}{v_{0,y}} = \frac{d}{v_{0,x}} \qquad \frac{h}{d} = \frac{v_{0,y}}{v_{0,x}}$$

Example Questions

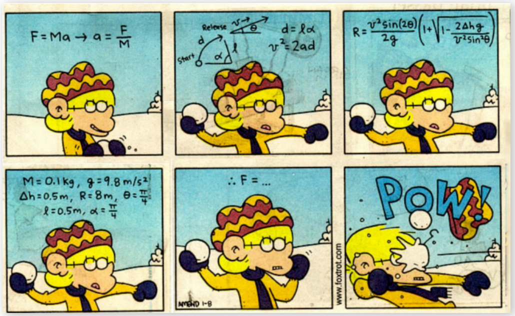
Bear hunting



$$\tan\gamma = \frac{h}{d} = \frac{v_{0,y}}{v_{0,x}} = \frac{s_0 \sin\theta}{s_0 \cos\theta} = \tan\theta$$

Summary

- Equations of Motion
- Newton's laws
- Conservation of Momentum
- Inelastic Collisions
- Frames of Reference


F = Ma → a = F/M
Release
v
θ
d
Start
α
l
d = lα
v² = 2ad
R = v² sin(2θ)/2g (1 + √(1 − 2Δhg/(v² sin²θ)))
M = 0.1 kg, g = 9.8 m/s²
Δh = 0.5 m, R = 8 m, θ = π/4
l = 0.5 m, α = π/4
∴ F = ...
AMEND 1-8
www.foxtrot.com
POW!