# Classical Mechanics
# &
# Properties of Gases

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# Other Resources

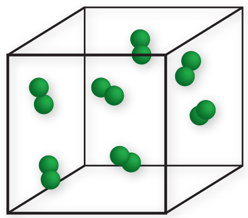
- Handouts / Questions / Slides
  - wallace.chem.ox.ac.uk
- Problem Classes
- Divisional Tutorial Provision
- tinyurl.com/MITPhysics

# Summary of Last Lecture

- Frames of Reference
- Work
- Kinetic Energy
- Inelastic & Elastic Collisions
- Conservation of Energy

# Classical Mechanics

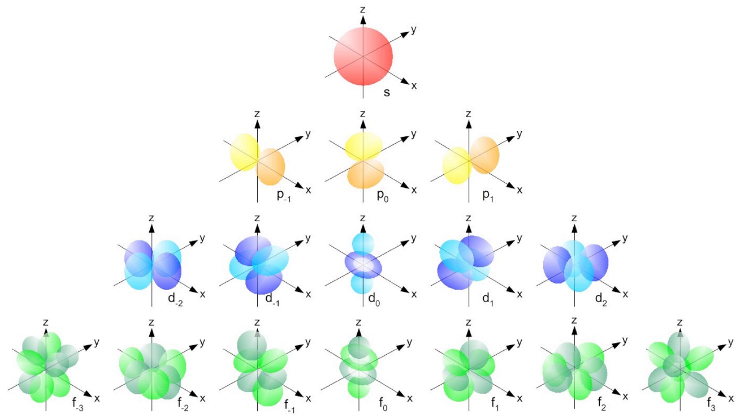
- Translation
  - Newton's laws of motion
  - Momentum, work & energy
- Rotation
  - Angular momentum
  - Moments of inertia
- Vibration
  - Simple harmonic motion



# Angular Momentum

The angular momentum of an object rotating about some reference point is the measure of the extent to which the object will continue to rotate about that point unless acted on by some external force.

# Angular Momentum

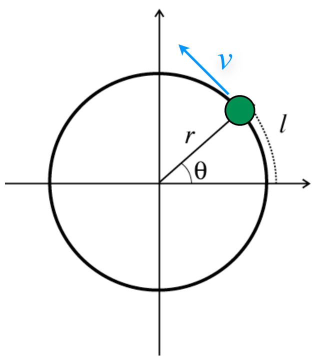


# Uniform Circular Motion

$$l = r\theta$$

$$v = r\frac{\theta}{t}$$

$$v = r\omega$$




Angular velocity

$$\omega = \frac{\theta}{t}$$

Frequency

$$f = \frac{1}{T}$$

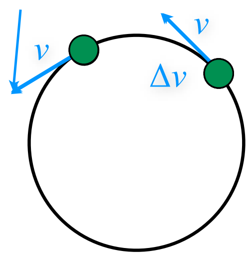
$$\omega = 2\pi f$$

# Uniform Circular Motion

$$l = r\theta$$

$$v = r\frac{\theta}{t}$$

$$v = r\omega$$




Angular velocity

$$\omega = \frac{\theta}{t}$$

Frequency

$$f = \frac{1}{T}$$

$$\omega = 2\pi f$$

# Centripetal Acceleration

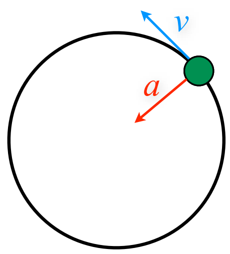
$$v = r\omega$$

$$a = v\omega$$

$$a = r\omega^2 = \frac{v^2}{r}$$

$$F = ma$$

$$\boxed{F = \frac{mv^2}{r}}$$

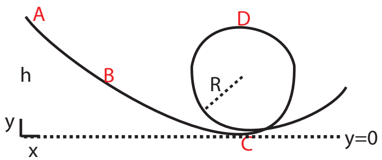



# An example

Rollercoasters

# Example Questions

Rollercoasters




Centripetal acceleration, $a = v^2/R$ (Next lecture in detail)

Consider point D here centripetal acceleration must be greater than gravitational acceleration or else ....

$$a = v^2/R \geq g \qquad \frac{2g(h-y)}{R} \geq g$$

$$\begin{aligned} 2g(h-2R) &\geq gR \\ 2h - 4R &\geq R \\ h &\geq \frac{5}{2}R = 2\frac{1}{2}R \end{aligned}$$

# An example

Rotation of the Earth

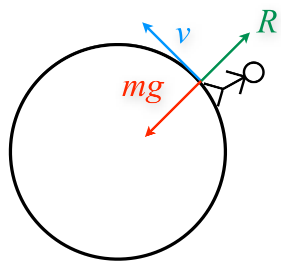
$$F_{net} = mg - R = \frac{mv^2}{r}$$

$$mg = \frac{mv^2}{r}$$

$$v = \sqrt{rg}$$

$$v = \sqrt{6378100 \times 9.8}$$

$$v = 7906$$




$$T = \frac{2\pi}{\omega} = \frac{2\pi r}{v} = 5069s = 1.4h$$

# Uniform Circular Motion

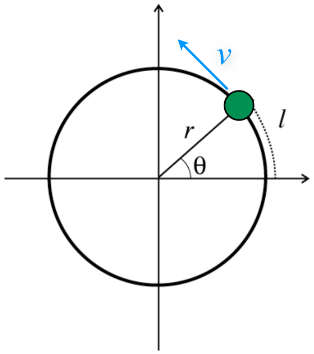
Vectors

$l = r\theta$

$v = r\dfrac{\theta}{t}$

$v = r\omega$

$\boxed{v = \omega \times r}$




## Angular velocity

$\omega = \dfrac{\theta}{t}$

## Frequency

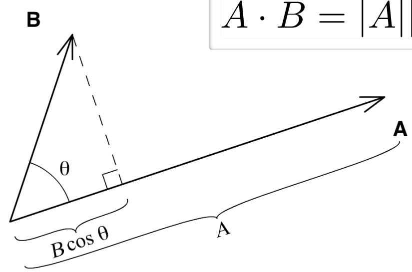
$f = \dfrac{1}{T}$

$\omega = 2\pi f$

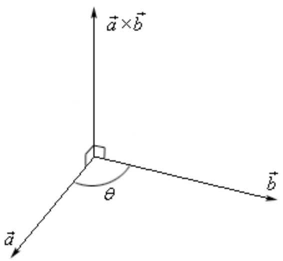
# Scalar Product

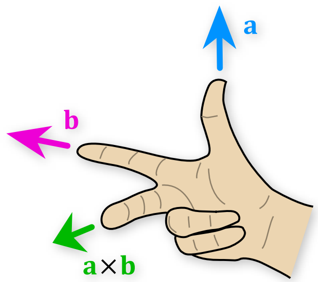
It is the product of the magnitudes of one vector ($A$), and the projection of a second vector ($B$) along the first.

$\boxed{\vec{A} \cdot \vec{B} = |A||B| \cos\theta}$




# Vector (Cross) Product

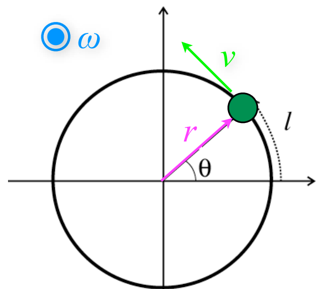


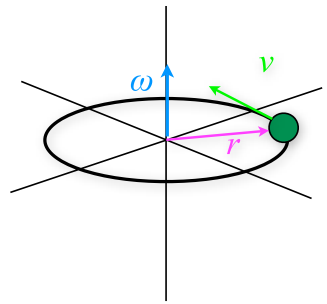




$a \times b = |a||b| \sin\phi$

# Uniform Circular Motion

$\boxed{v = \omega \times r}$

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= into board

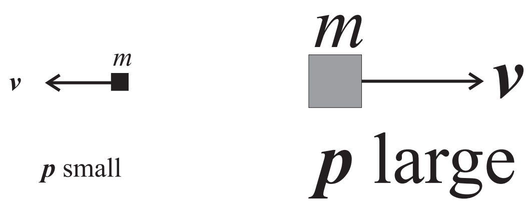
# Angular Momentum

The angular momentum of an object rotating about some reference point is the measure of the extent to which the object will continue to rotate about that point unless acted on by some external force.

# Linear Momentum Revisited

Product of an object's mass times its velocity

- $\vec{p} = m\vec{v}$
- Units of kg ms$^{-1}$




# Angular Momentum

$$\vec{l} = \vec{r} \times \vec{p}$$

The angular momentum of an object rotating about some reference point is the measure of the extent to which the object will continue to rotate about that point unless acted on by some external force.

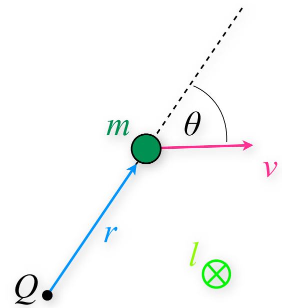
The angular momentum of a particle about an origin is a vector quantity equal to the mass of the particle multiplied by the cross product of the position vector of the particle with its velocity vector.

# Angular Momentum

The angular momentum, $l$

$$\vec{l} = \vec{r} \times \vec{p}$$
$$\vec{l} = \vec{r} \times (m\vec{v})$$
$$= (\vec{r} \times \vec{v})m$$

$$|l| = m|r||v| \sin\theta$$

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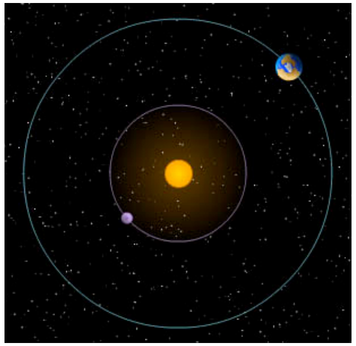
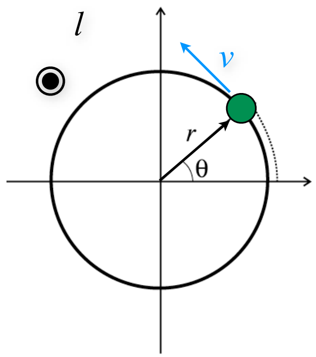
# An example

Planetary Motion

$$\vec{l} = \vec{r} \times \vec{p}$$






# Angular Momentum

Precession

http://www.surendranath.org/Applets/Dynamics/AngMom/AngMomApplet.html

# Conservation of Angular Momentum

The angular momentum of an isolated system is conserved

The angular momentum, $l$

$$\vec{l} = \vec{r} \times \vec{p}$$

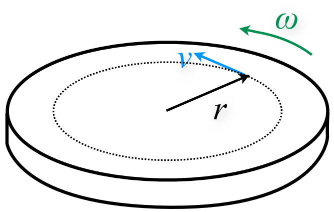
$$\vec{l} = \vec{r} \times (m\vec{v})$$

$$= (\vec{r} \times \vec{v})m$$

$$|l| = m|r||v|\sin\theta$$

# Moment of Inertia

The rotational equivalent of mass




$$K_i = \frac{1}{2} m_i v_i^2 \qquad v = r\omega$$

$$K_i = \frac{1}{2} m_i \omega_i^2 r_i^2$$

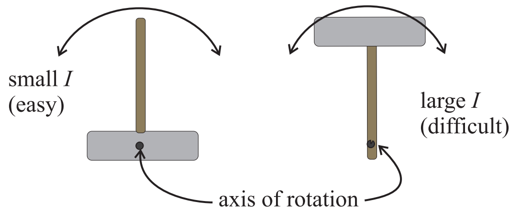
$$K = \frac{1}{2}\omega^2 \sum_i m_i r_i^2$$

$$\boxed{I = \sum_i m_i r_i^2} \qquad \boxed{K = \frac{1}{2} I\omega^2}$$

$$l = I\omega$$

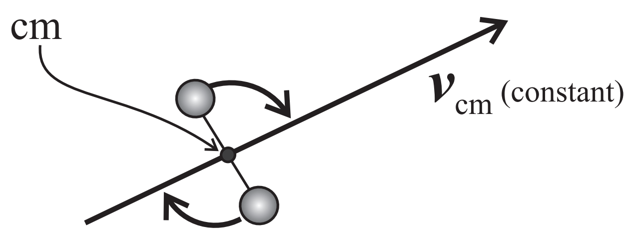
$$p = mv$$

# Moments of Inertia



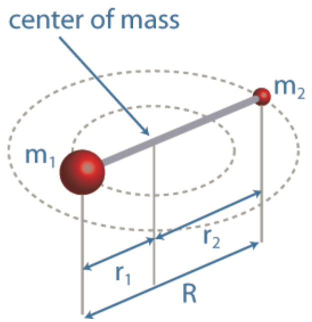

# An example

Rotation of a diatomic molecule




# An example

Rotation of a diatomic molecule




## Moment of Inertia

$$I = \sum_i m_i r_i^2$$

$$I = m_1 r_1^2 + m_2 r_2^2$$

## Centre of mass definition

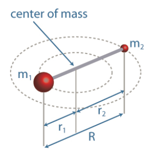
$$\sum_i m_i r_i = 0$$

$$m_1 r_1 + m_2 r_2 = 0$$

$$r_1 = -\frac{m_2}{m_1} r_2$$

# An example

Rotation of a diatomic molecule




$$m_1 r_1 + m_2 r_2 = 0$$

$$r_1 = -\frac{m_2}{m_1} r_2$$

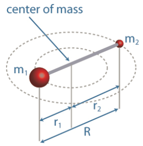
$$R = r_2 - r_1$$

$$R = r_2 + \frac{m_2}{m_1} r_2$$

$$m_1 R = m_1 r_2 + m_2 r_2$$

$$m_1 R = (m_1 + m_2) r_2$$

$$r_2 = \frac{m_1 R}{(m_1 + m_2)} \qquad r_1 = \frac{m_2 R}{(m_1 + m_2)}$$



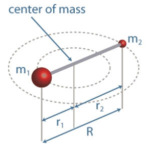

# An example

Rotation of a diatomic molecule

$$I = m_1 r_1^2 + m_2 r_2^2 \qquad r_2 = \frac{m_1 R}{(m_1 + m_2)} \qquad r_1 = \frac{m_2 R}{(m_1 + m_2)}$$

$$I = m_1 \left(\frac{m_2 R}{m_1 + m_2}\right)^2 + m_2 \left(\frac{m_1 R}{m_1 + m_2}\right)^2$$
$$= \frac{m_1 m_2 R^2}{(m_1 + m_2)^2}(m_1 + m_2)$$
$$= \frac{m_1 m_2}{m_1 + m_2} R^2$$

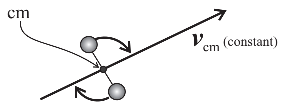
$I = \mu R^2$ $\mu$ is the reduced mass




# Reduced Mass

Rotation of a diatomic molecule

- $I = \mu R^2 \qquad \mu = \dfrac{m_1 m_2}{m_1 + m_2}$
- $\mu$ is the reduced mass
- The relative motion of two objects that are acted upon by a force can be described by Newton's 2nd Law as if they were a single mass with a value called the reduced mass.




# Lecture Summary

- Uniform Circular Motion
- Angular Momentum
- Moments of Inertia
- Reduced Mass