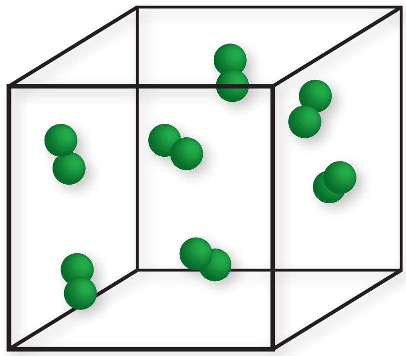
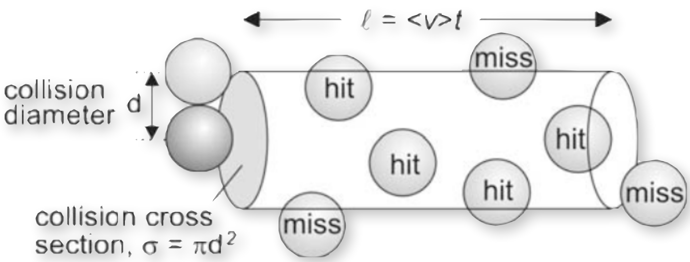
Properties of gases

- Ideal & real gases
- Kinetic theory of gases
- Maxwell-Boltzmann distribution
- Applications of kinetic theory



Collision Frequency, z



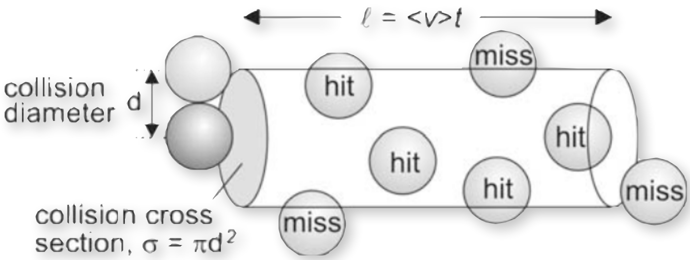
- Volume swept-out by a single particle

$$\pi d^2 \langle v_{\mathrm{rel}} \rangle t$$

- Number of collisions for one particle = volume × number density
- Collision frequency for one particle = number of collisions in unit time

$$z = \sigma \langle v_{\mathrm{rel}} \rangle \frac{N}{V} = \sigma \langle v_{\mathrm{rel}} \rangle \frac{p}{k_B T}$$

Collision Density, Z



- Collision density is the total number of collisions occurring per unit volume

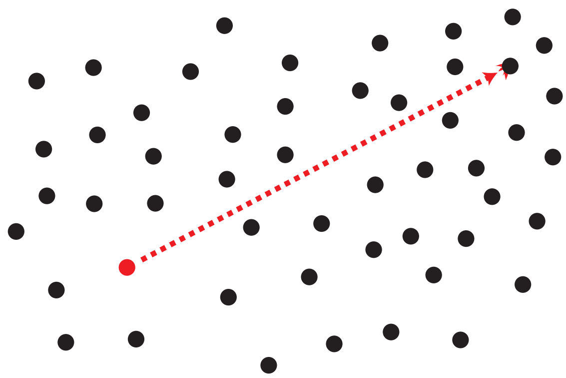
$$Z = \frac{1}{2}\sigma \langle v_{\mathrm{rel}} \rangle \left(\frac{N}{V}\right)^2$$

$$z = \sigma \langle v_{\mathrm{rel}} \rangle \frac{N}{V}$$

$$Z = \frac{1}{2}\sigma\sqrt{2}\sqrt{\frac{8k_BT}{\pi m}}\left(\frac{N}{V}\right)^2$$

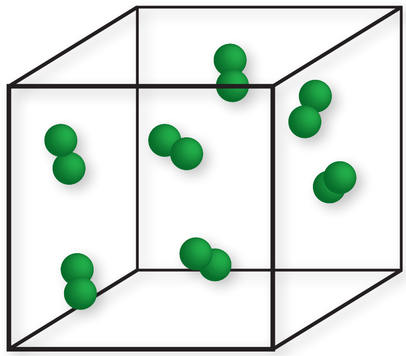
Mean free path

$$\lambda = \frac{\langle v \rangle}{z}$$



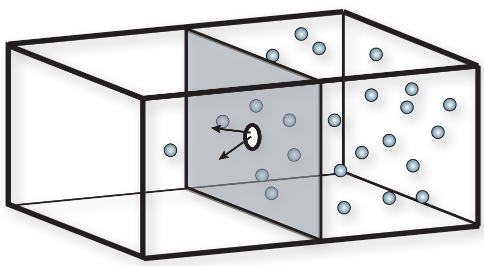
Applications of kinetic theory

- Effusion
- Diffusion
- Thermal Conductivity
- Viscosity



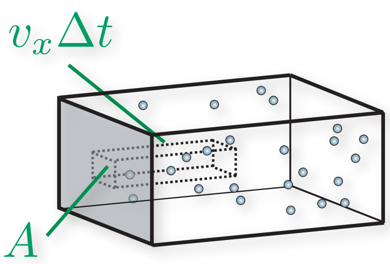
Effusion

- Gas escapes into a vacuum through a small hole
  - No collisions occur after the particles pass through the hole.
- The rate of effusion will be proportional to the rate at which particles strike the area defining the hole
- The diameter of the hole is smaller than the mean free path in the gas
  - No collisions occur as the particles pass through the hole



Effusion

Collisions with container walls



$$P(v_x)dv_x = \left(\frac{m}{2\pi k_B T}\right)^{1/2} e^{-\frac{mv_x^2}{2k_BT}} dv_x$$

$$\begin{aligned}\langle V\rangle &= \int_0^\infty V P(v_x) dv_x \\ &= A\Delta t \int_0^\infty v_x \sqrt{\frac{m}{2\pi k_B T}} \exp\left(\frac{-mv_x^2}{2k_BT}\right) dv_x \\ &= A\Delta t \sqrt{\frac{m}{2\pi k_B T}}\left(\frac{1}{2}\frac{2k_BT}{m}\right) \\ &= A\Delta t\left(\frac{k_BT}{2\pi m}\right)^{1/2}\end{aligned}$$

$$\int_0^\infty x e^{-Bx^2} dx = \frac{1}{2B}$$

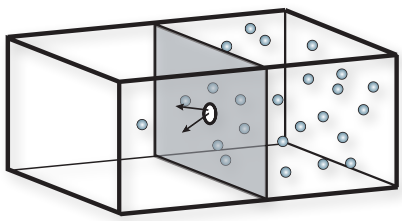
Collision frequency with wall (per unit area, per unit time)

$$z_{\text{wall}} = \rho_N \langle V\rangle = \frac{p}{k_BT}\left(\frac{k_BT}{2\pi m}\right)^{1/2} = \frac{p}{(2\pi m k_B T)^{1/2}}$$

$$\rho_N = \frac{N}{V} = \frac{p}{k_BT}$$

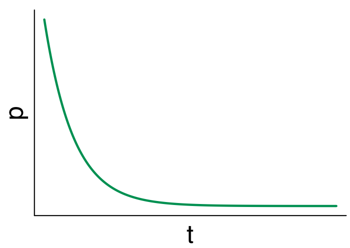
Effusion

Collisions with aperture



$$\text{Rate} = \frac{dN}{dt} = z_{\text{wall}} a = \frac{pa}{(2\pi m k_B T)^{1/2}}$$

$$\frac{dp}{dt} = \frac{d}{dt}\left(\frac{Nk_BT}{V}\right) = \frac{k_BT}{V}\frac{dN}{dt}$$



$$\frac{dp}{dt} = \frac{k_BT}{V}\left(\frac{pa}{(2\pi m k_B T)^{1/2}}\right)$$

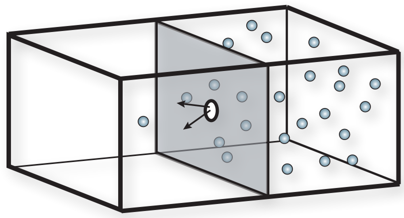
$$\int \frac{1}{p} dp = \int \left(\frac{k_BTa}{V(2\pi m k_B T)^{1/2}}\right) dt$$

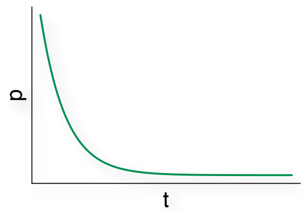
$$p = p_0 \exp -t/\tau \qquad \text{where} \qquad \tau = \left(\frac{2\pi m}{k_BT}\right)^{1/2}\frac{V}{a}$$

Effusion

$$\text{Rate} = \frac{dN}{dt} = z_{\text{wall}}a = \frac{pa}{(2\pi m k_B T)^{1/2}}$$

$$p = p_0 \exp -t/\tau$$





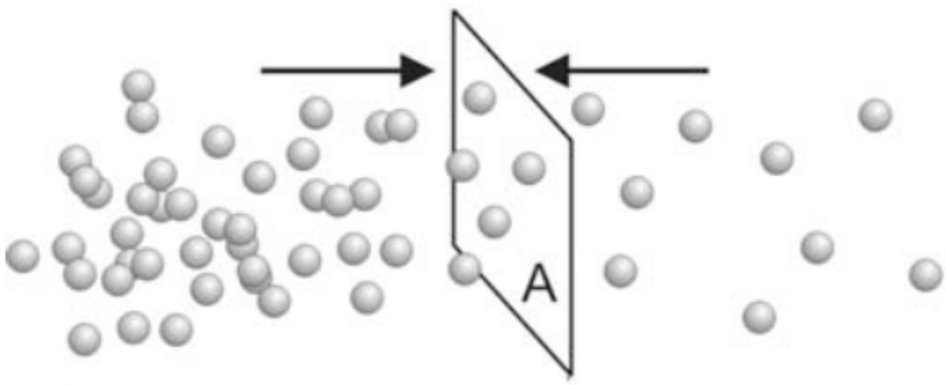
- Graham's law of effusion
  - Rate of effusion of a gas is inversely proportional to the square root of the mass of its particles

Transport Properties

Property	Transported Quantity	Kinetic Theory	Units
Diffusion	Matter	$D = \frac{1}{3}\lambda \langle v \rangle$	$m^2 s^{-1}$
Thermal Conductivity	Energy	$\kappa = \frac{1}{3}\lambda \langle v \rangle C_V [X]$	$J\ K^{-1}\ m^{-1}\ s^{-1}$
Viscosity	Momentum	$\eta = \frac{1}{3}\rho_N m \lambda \langle v \rangle$	$kg\ m^{-1}\ s^{-1}$

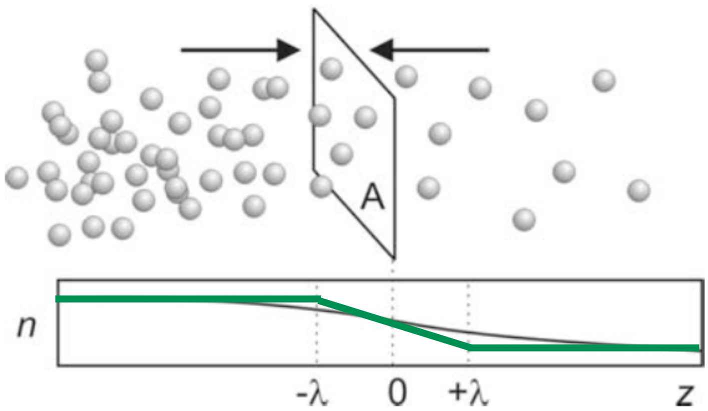
Flux

Describes the amount of matter, energy, charge... passing through a unit area per unit time



For matter flux, $J_z \propto \frac{d\rho_N}{dz}$ where ρ_N is the number density.

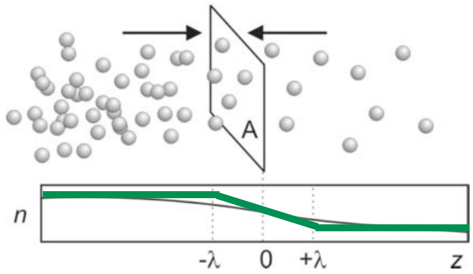
Fick's first law of diffusion



$$J_z = -D\frac{d\rho_N}{dz}$$

$$D = \frac{1}{3}\lambda \langle v \rangle$$

$$\langle v \rangle = \sqrt{\frac{8k_BT}{\pi m}}$$



$$\rho_N(-\lambda) = \rho_N(0) - \lambda\left(\frac{d\rho_N}{dz}\right)_0 \qquad \rho_N(+\lambda) = \rho_N(0) + \lambda\left(\frac{d\rho_N}{dz}\right)_0$$

$$z_{\text{wall}} = \rho_N \langle V\rangle = \frac{N}{V}\left(\frac{k_B T}{2\pi m}\right)^{1/2} = \rho_N \frac{\langle v\rangle}{4}$$

$$\langle v\rangle = \int_0^\infty v f(v) dv = \left(\frac{8k_B T}{\pi m}\right)^{1/2}$$

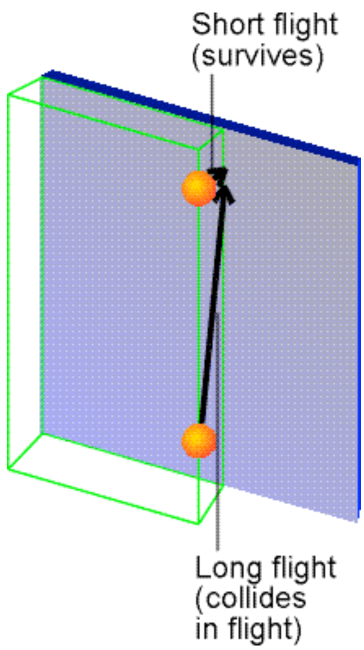
$$J_{\text{left}} = \frac{\langle v\rangle \rho_N(-\lambda)}{4} = \frac{\langle v\rangle}{4}\left(\rho_N(0) - \lambda\left(\frac{d\rho_N}{dz}\right)_0\right) \qquad J_{\text{right}} = \frac{\langle v\rangle \rho_N(+\lambda)}{4} = \frac{\langle v\rangle}{4}\left(\rho_N(0) + \lambda\left(\frac{d\rho_N}{dz}\right)_0\right)$$

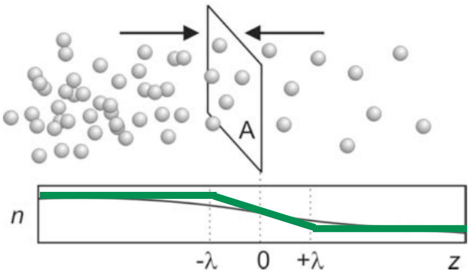
$$J_z = J_{\text{left}} - J_{\text{right}} = -\frac{1}{2}\left(\frac{d\rho_N}{dz}\right)_0 \lambda\langle v\rangle$$

For matter flux, $J_z \propto \dfrac{d\rho_N}{dz}$ where ρ_N is the number density.

$$D = \frac{1}{2}\lambda\langle v\rangle$$

Diffusion in a Gas





$$\rho_N(-\lambda) = \rho_N(0) - \lambda\left(\frac{d\rho_N}{dz}\right)_0 \qquad \rho_N(+\lambda) = \rho_N(0) + \lambda\left(\frac{d\rho_N}{dz}\right)_0$$

$$z_{\text{wall}} = \frac{N}{V}\left(\frac{k_B T}{2\pi m}\right)^{1/2} = \frac{N}{V}\frac{\langle v\rangle}{4}$$

$$\langle v\rangle = \int_0^\infty v f(v)dv = \left(\frac{8k_BT}{\pi m}\right)^{1/2}$$

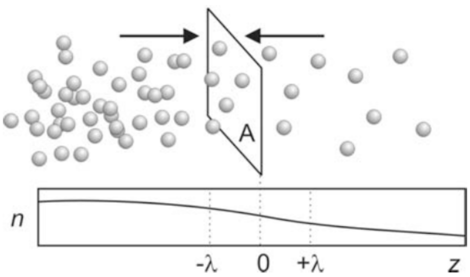
$$J_{\text{left}} = \frac{\langle v\rangle\,\rho_N(-\lambda)}{4} = \frac{\langle v\rangle}{4}\left(\rho_N(0) - \lambda\left(\frac{d\rho_N}{dz}\right)_0\right) \qquad J_{\text{right}} = \frac{\langle v\rangle\,\rho_N(+\lambda)}{4} = \frac{\langle v\rangle}{4}\left(\rho_N(0) + \lambda\left(\frac{d\rho_N}{dz}\right)_0\right)$$

$$J_z = J_{\text{left}} - J_{\text{right}} = -\frac{1}{2}\left(\frac{d\rho_N}{dz}\right)_0 \lambda\langle v\rangle$$

For matter flux, $J_z \propto \dfrac{d\rho_N}{dz}$ where ρ_N is the number density.

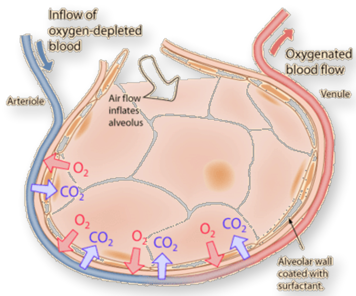
$$D = \frac{1}{3}\lambda\langle v\rangle$$

Diffusion

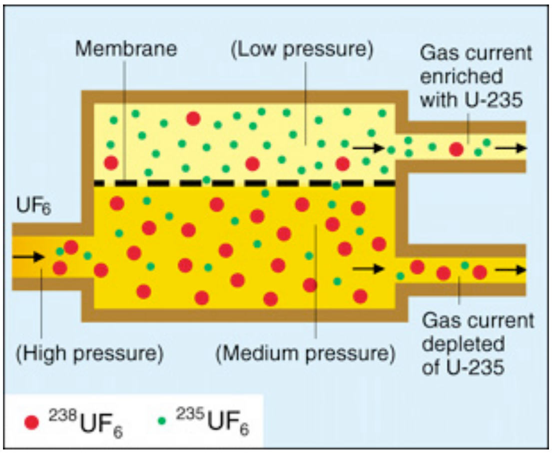


$$J_z = -D\frac{d\rho_N}{dz}$$

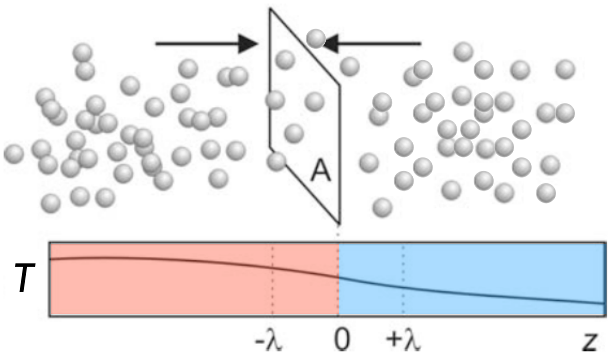
$$D = \frac{1}{3}\lambda\langle v\rangle$$



Diffusion in a Gas



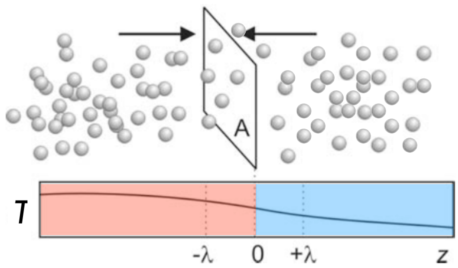
Thermal Conductivity in a Gas



$$J_z = -\kappa \frac{dT}{dz}$$

$$\kappa = \frac{1}{3}\lambda \langle v \rangle C_V \, [A]$$

$$\epsilon = \alpha k_B T$$



Previously $\rho_N(-\lambda) = \rho_N(0) - \lambda \left(\frac{d\rho_N}{dz}\right)_0$ $\rho_N(+\lambda) = \rho_N(0) + \lambda \left(\frac{d\rho_N}{dz}\right)_0$

Now $\epsilon(-\lambda) = \alpha k_B \left(T - \lambda \left(\frac{dT}{dz}\right)_0\right)$ $\epsilon(+\lambda) = \alpha k_B \left(T + \lambda \left(\frac{dT}{dz}\right)_0\right)$

$$z_{\text{wall}} = \frac{N}{V}\left(\frac{k_B T}{2\pi m}\right)^{1/2} = \frac{N}{V}\frac{\langle v \rangle}{4}$$

$$J_{left} = \frac{1}{4} < v > \rho_N \epsilon(-\lambda) \qquad J_{right} = \frac{1}{4} < v > \rho_N \epsilon(+\lambda)$$

$$J_z = J_{\text{left}} - J_{\text{right}} = -\frac{1}{2}\left(\frac{dT}{dz}\right)_0 \alpha k_B \rho_N \lambda \langle v \rangle$$

$$\kappa = \frac{1}{3}\alpha k_B \rho_N \lambda \langle v \rangle \qquad \kappa = \frac{1}{3}\lambda \langle v \rangle C_V [X]$$

Transport Properties

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Thermal Conductivity	Energy	$\kappa = \frac{1}{3}\lambda \langle v \rangle C_V [X]$	$J\ K^{-1}\ m^{-1}\ s^{-1}$
Viscosity	Momentum	$\eta = \frac{1}{3}\rho_N m \lambda \langle v \rangle$	$kg\ m^{-1}\ s^{-1}$

Transport Properties

Property	Transported Quantity	Kinetic Theory	Units
Diffusion	Matter	$D = \frac{1}{3}\lambda \langle v \rangle$	m^2s^{-1}
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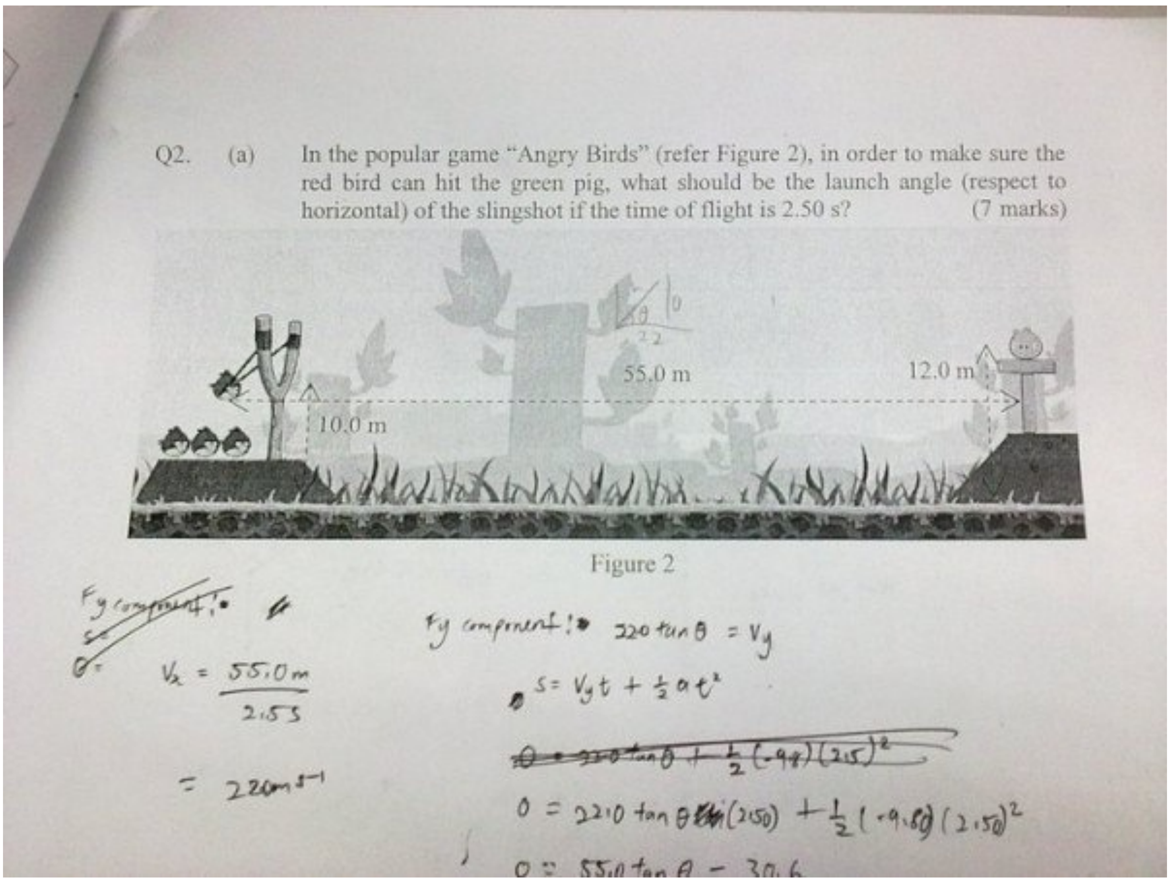
Classical Mechanics Summary

- Translation
  - Newton's laws
  - Equations of motion
  - Linear momentum
  - Frames of reference & reduced mass
- Rotation
  - Angular momentum
  - Moments of inertia
- Vibration
  - Simple harmonic motion
  - The wave equation
- Work & Energy
  - Elastic & inelastic Collisions
  - Centre of Mass

Properties of Gases Summary

- Ideal & real gases
  - pV=nRT
  - Virial expansion
  - Compression factor
- Kinetic theory of gases
  - Derivation
  - Classical equipartition
  - Predicting Cv
- Maxwell Boltzmann
  - (Derivation)
  - Collision frequencies
  - Mean free path
- Applications
  - Effusion
  - Diffusion



Q2. (a) In the popular game "Angry Birds" (refer Figure 2), in order to make sure the red bird can hit the green pig, what should be the launch angle (respect to horizontal) of the slingshot if the time of flight is 2.50 s? (7 marks)

Figure 2

$V_x = \frac{55.0\,m}{2.5\,s}$

$= 22.0\,ms^{-1}$

Fy component: $22.0 \tan\theta = V_y$

$s = V_y t + \frac{1}{2} a t^2$

$0 = 22.0 \tan\theta (2.50) + \frac{1}{2}(-9.80)(2.50)^2$

$0 = 55.0 \tan\theta - 30.6$