Classical Mechanics
&
Properties of Gases

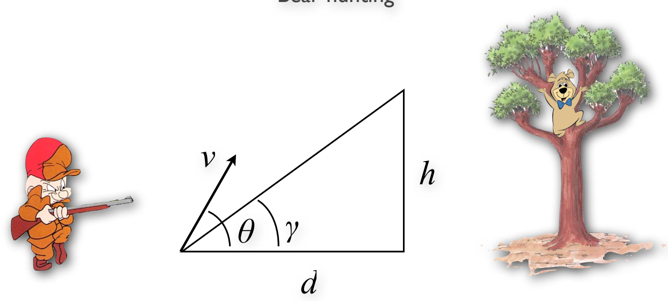
mark.wallace@chem.ox.ac.uk

Summary of Last Lecture

- Equations of Motion
- Newton's laws
- Conservation of Momentum
- Inelastic Collisions
- Frames of Reference

Example Questions

Bear hunting



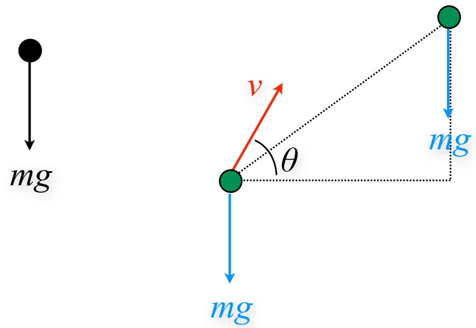
For a collision to occur

$$y_{bullet} = y_{bear}$$

$$v_{0,y}t - \frac{1}{2}gt^2 = h - \frac{1}{2}gt^2$$

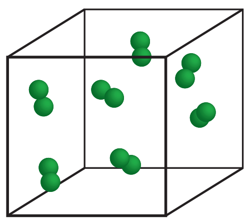
Frames of Reference

Bear hunting



Classical Mechanics

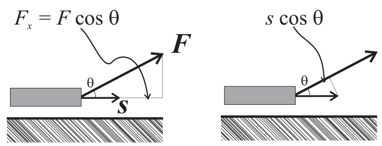
- Translation
  - Newton's laws of motion
  - Momentum, work & energy
- Rotation
  - Angular momentum
  - Moments of inertia
- Vibration
  - Simple harmonic motion



Work

Provides the link between force and energy

- The mechanical work, W, done by a constant force, F, is the force times the total displacement, s, in the direction of the force.
- Work is a scalar
- Units, Nm or Joules



$$W = |F||s| \cos \theta$$

Work

The mechanical work, W, done by a constant force, F, is the force times the total displacement, s, in the direction of the force.

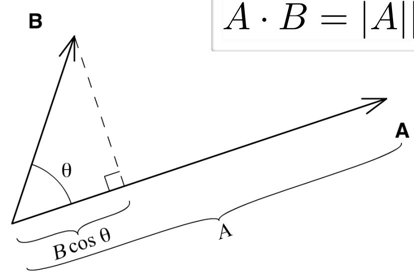
$$W = 0 \qquad W = |F||s|$$

$$\boxed{W = \vec{F} \cdot \vec{s}}$$

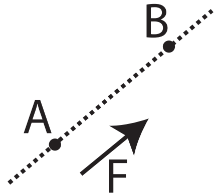
Scalar Product

It is the product of the magnitudes of one vector (A), and the projection of a second vector (B) along the first.

$$\boxed{\vec{A} \cdot \vec{B} = |A||B| \cos \theta}$$



Work-Energy Theorem



$$W_{AB} = Fx_{AB}$$

$$W_{AB} = \int_A^B F dx$$

$$W_{AB} = \int_A^B m\frac{dv}{dt}dx \qquad dx = vdt$$

$$W_{AB} = \int_{v_A}^{v_B} mvdv = [\frac{1}{2}mv^2]_{v_A}^{v_B} = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

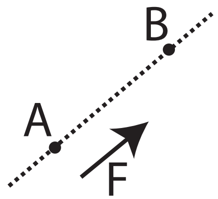
Kinetic Energy

- The kinetic energy, K, of a particle is the energy a particle possesses by virtue of its motion.
- For a particle of mass m moving along x with velocity v_x

$$K = \tfrac{1}{2}mv_x^2$$

Work-Energy Theorem

The change in the kinetic energy of an object is equal to the net work done on the object

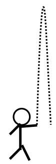


$$W_{AB} = Fx_{AB}$$

$$W_{AB} = \int_A^B F dx$$

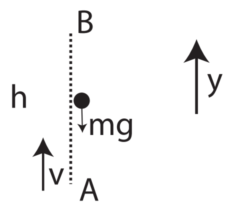
$$W_{AB} = \int_A^B m\frac{dv}{dt}dx \qquad dx = vdt$$

$$W_{AB} = \int_{v_A}^{v_B} mvdv = [\frac{1}{2}mv^2]_{v_A}^{v_B} = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = \Delta K$$



An example

Throwing a ball straight up



$$W_{AB} = -mgh = K_B - K_A$$

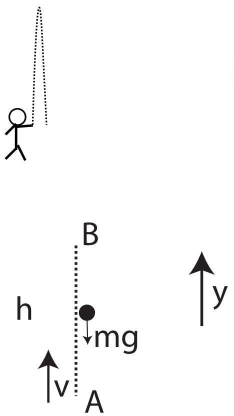
$$W_{AB} = -mgh = 0 - K_A$$

$$-mgh = -\frac{1}{2}mv^2$$

$$h = \frac{v^2}{2g}$$

An example

Throwing a ball straight up



$$y(t) = 0 + v_0 t - \frac{1}{2} g t^2$$

$$h = v_0 t - \frac{1}{2} g t^2$$

$$v_y(t) = v_0 - gt$$

$$0 = v_0 - gt$$

$$t = v_0 / g$$

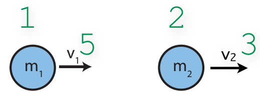
$$h = \frac{{v_0}^2}{g} - \frac{1}{2} g \left(\frac{v_0}{g} \right)^2$$

$$gh = v_0^2 (1 - \frac{1}{2})$$

$$h = \frac{v_0^2}{2g}$$

Inelastic Collisions

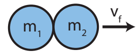
- Momentum before collision?



$$\sum_i p_i = m_1 v_1 + m_2 v_2$$

1*5 + 2*3 =11

- Momentum after collision?



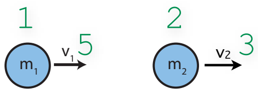
$$\sum_i p_i = (m_1 + m_2) v_f$$

(1+2)*(11/3) =11

Inelastic Collisions

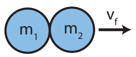
Kinetic energy is not conserved in an inelastic collision

$$K_i = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$



$1/2*1*5^2 + 1/2*2*3^2 = 21.5$

$$K_f = \frac{1}{2} (m_1 + m_2) v_f^2$$



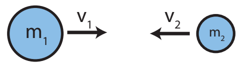
1/2*(1+2)*(11/3)^2 =20.2

Elastic Collisions

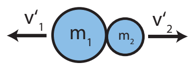
Kinetic energy is conserved in an elastic collision

An example

- Consider the head-on collision between two atoms.



- Let us derive an expression for the final velocity of both particles assuming collision is elastic.



Elastic Collisions

Kinetic energy is conserved in an elastic collision

- Initial momentum of system

$$p_i \;=\; m_1 v_{i1} + m_2 v_{i2}$$

- Final momentum of the system

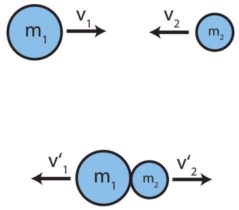
$$p_f \;=\; m_1 v_{f1} + m_2 v_{f2}$$

- Initial kinetic energy of system

$$K_i \;=\; \tfrac{1}{2} m_1 v_{i1}^2 + \tfrac{1}{2} m_2 v_{i2}^2$$

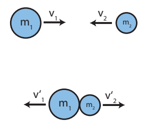
- Final kinetic energy of the system

$$K_f \;=\; \tfrac{1}{2} m_1 v_{f1}^2 + \tfrac{1}{2} m_2 v_{f2}^2$$



Elastic Collisions

Kinetic energy is conserved in an elastic collision



- Conservation of Momentum

$$\begin{aligned} m_1 v_{i1} + m_2 v_{i2} &= m_1 v_{f1} + m_2 v_{f2} \\ m_1 (v_{i1} - v_{f1}) &= m_2 (v_{f2} - v_{i2}) \end{aligned}$$

- Conservation of Kinetic Energy

$$\begin{aligned} m_1 v_{i1}^2 + m_2 v_{i2}^2 &= m_1 v_{f1}^2 + m_2 v_{f2}^2 \\ m_1 (v_{i1}^2 - v_{f1}^2) &= m_2 (v_{f2}^2 - v_{i2}^2) \\ m_1 (v_{i1} + v_{f1})(v_{i1} - v_{f1}) &= m_2 (v_{f2} + v_{i2})(v_{f2} - v_{i2}) \\ v_{i1} + v_{f1} &= v_{f2} + v_{i2} \end{aligned}$$

$$\ldots. \quad v_{f2} = \frac{2m_1}{m_1 + m_2} v_{i1} + \frac{m_2 - m_1}{m_1 + m_2} v_{i2}$$

Centre of Mass Frame

- The centre of mass of a system is the point where the system responds as if it were a point with a mass equal to the sum of the masses of its constituent parts.
- In one-dimension for N particles in a system this is described by

$$M = \sum_{i=1}^{N} m_i \qquad\qquad M x_{CM} = \sum_{i=1}^{N} m_i x_i$$

Elastic Collisions

Kinetic energy is conserved in an elastic collision

- Initial momentum of system

$$p_i \;=\; m_1 v_{i1} + m_2 v_{i2}$$

- Final momentum of the system

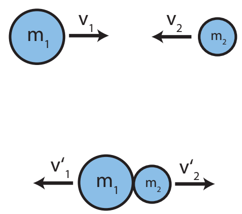
$$p_f \;=\; m_1 v_{f1} + m_2 v_{f2}$$

- Initial kinetic energy of system

$$K_i \;=\; \tfrac{1}{2} m_1 v_{i1}^2 + \tfrac{1}{2} m_2 v_{i2}^2$$

- Final kinetic energy of the system

$$K_f \;=\; \tfrac{1}{2} m_1 v_{f1}^2 + \tfrac{1}{2} m_2 v_{f2}^2$$

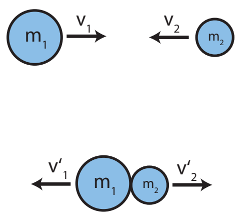


Elastic Collisions

Kinetic energy is conserved in an elastic collision

$v' = v - v_{COM}$

- Initial momentum of system
$$0 \; = \; m_1 v'_{i1} + m_2 v'_{i2}$$
- Final momentum of the system
$$0 \; = \; m_1 v'_{f1} + m_2 v'_{f2}$$
- Initial kinetic energy of system
$$K_i \; = \; \tfrac{1}{2} m_1 v'^2_{i1} + \tfrac{1}{2} m_2 v'^2_{i2}$$
- Final kinetic energy of the system
$$K_f \; = \; \tfrac{1}{2} m_1 v'^2_{f1} + \tfrac{1}{2} m_2 v'^2_{f2}$$



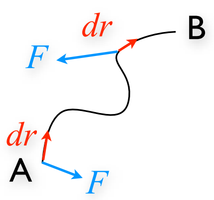
Elastic Collisions

Kinetic energy is conserved in an elastic collision

- Initial momentum of system
$$0 \; = \; m_1 v'_{i1} + m_2 v'_{i2}$$
- Final momentum of the system
$$0 \; = \; m_1 v'_{f1} + m_2 v'_{f2}$$
- Initial kinetic energy of system
$$K_i \; = \; \tfrac{1}{2} m_1 v'^2_{i1} + \tfrac{1}{2} m_2 v'^2_{i2}$$
- Final kinetic energy of the system
$$K_f \; = \; \tfrac{1}{2} m_1 v'^2_{f1} + \tfrac{1}{2} m_2 v'^2_{f2}$$

$$\begin{aligned}
m_1 {v'_{i1}}^2 + m_2 {v'_{i2}}^2 &= m_1 {v'_{f1}}^2 + m_2 {v'_{f2}}^2 \\
m_1 {v'_{i1}}^2 + m_2 \left(-\frac{m_1}{m_2} v_{i1}\right)^2 &= m_1 {v'_{f1}}^2 + m_2 \left(-\frac{m_1}{m_2} v_{f1}\right)^2 \\
{v'_{i1}}^2 \left(m_1 + \frac{m_1^2}{m_2}\right) &= {v'_{f1}}^2 \left(m_1 + \frac{m_1^2}{m_2}\right) \\
{v'_{i1}}^2 &= {v'_{f1}}^2
\end{aligned}$$

3D work



$$W_{AB} = \int_A^B \vec{F} \cdot \vec{dr}$$

$$dW = F_x dx + F_y dy + F_z dz$$

$$\vec{F} = F_x \hat{x} + F_y \hat{y} + F_z \hat{z}$$
$$\vec{dr} = dx \hat{x} + dy \hat{y} + dz \hat{z}$$

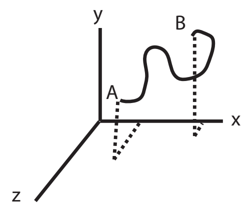
$$W = \int_A^B F_x dx + \int_A^B F_y dy + \int_A^B F_z dz$$

$$W = \frac{1}{2} m (v_{xB}^2 - v_{xA}^2) + \frac{1}{2} m (v_{yB}^2 - v_{yA}^2) + \frac{1}{2} m (v_{zB}^2 - v_{zA}^2) = \frac{1}{2} m (v_B^2 - v_A^2)$$

3D Work example

$$F_y = -mg$$

$$y_B - y_A = h$$



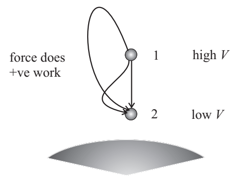
$$W = \int_A^B F_y dy = -mg(y_B - y_A) = -mgh$$

Conservative Forces

- For a conservative force, the work done in moving between two points is independent of the path taken between the two points
- Electromagnetic Force is a conservative force
  - Chemistry deals with conservative forces
- Gravity is a conservative force
- Friction is not a conservative force

Conservation of Energy

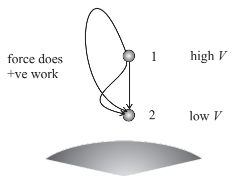
- The potential energy, V, is the energy associated with the position of a particle.
- Potential energy may be thought of as stored energy, or the capacity to do work.
- The sum of kinetic and potential energy is constant.



$$W = \int_A^B F_y dy = -mg(y_B - y_A) = K_B - K_A$$

$$K_B + mgy_B = K_A + mgy_A$$

Conservation of Energy



$$W = \int_A^B F_y dy = -mg(y_B - y_A) = K_B - K_A$$

$$K_B + mgy_B = K_A + mgy_A$$

$$V(x_2) - V(x_1) \;=\; \Delta V \;=\; -\int_1^2 F_x \, \mathrm{d}x \;=\; -W$$

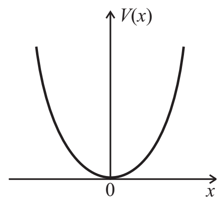
$$F = -\frac{\mathrm{d}V}{\mathrm{d}x}$$

An example

The harmonic spring potential

$$V(x) = \frac{1}{2}kx^2$$

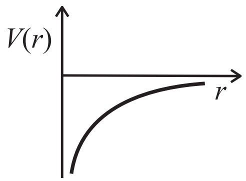
$$F(x) = -\frac{\mathrm{d}V}{\mathrm{d}x} = -kx$$



An example
Gravitational potential energy

$$V(r) = -\frac{Gm_1m_2}{r}$$

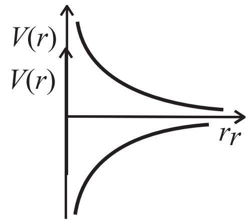
$$F(r) = -\frac{\mathrm{d}V}{\mathrm{d}r} = -\frac{Gm_1m_2}{r^2}$$



An example
Electrostatic potential energy

$$V(r) = \frac{q_1q_2}{4\pi\epsilon_0 r}$$

$$F(r) = -\frac{\mathrm{d}V}{\mathrm{d}r} = \frac{q_1q_2}{4\pi\epsilon_0 r^2}$$

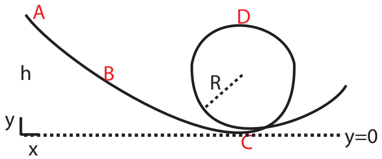


Here q_1 and q_2 have the opposite sign. / same sign.

An example
Rollercoasters

Example Questions

Rollercoasters



Conservation of Energy?

$$V_A + K_A = V_B + K_B = V_C + K_C = V_D + K_D$$

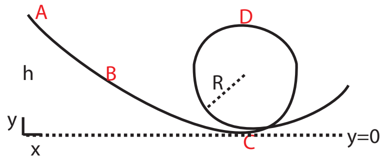
Compare initial energy (A) with an arbitrary point (B)

$$mgh = mgy + \frac{1}{2}mv^2$$

$$v^2 = 2g(h - y)$$

Example Questions

Rollercoasters



Centripetal acceleration, $a = v^2/R$ (Next lecture in detail)

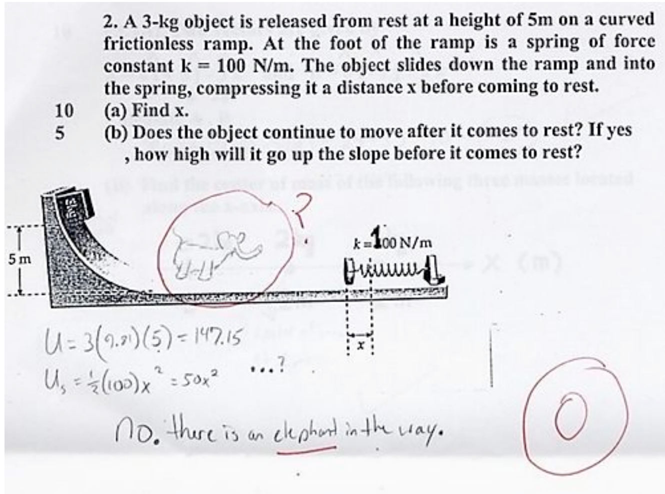
Consider point D here centripetal acceleration must be greater than gravitational acceleration or else ....

$$a = v^2/R \geq g \qquad \frac{2g(h-y)}{R} \geq g$$

$$\begin{aligned} 2g(h-2R) &\geq gR \\ 2h - 4R &\geq R \\ h &\geq \frac{5}{2}R = 2\frac{1}{2}R \end{aligned}$$

Lecture Summary

- Frames of Reference
- Work
- Kinetic Energy
- Elastic Collisions
- Conservation of Energy



2. A 3-kg object is released from rest at a height of 5m on a curved frictionless ramp. At the foot of the ramp is a spring of force constant k = 100 N/m. The object slides down the ramp and into the spring, compressing it a distance x before coming to rest.

10 (a) Find x.

5 (b) Does the object continue to move after it comes to rest? If yes, how high will it go up the slope before it comes to rest?