Classical Mechanics Problems

1. A cannonball is fired from ground level at an angle of 45 degrees to the horizontal. The ball has a mass of 2 kg, and its initial velocity is 50 ms^{-1}.
   a. Sketch the trajectory of the cannonball, and on the same diagram, indicate the forces acting on the ball during its motion.
   b. Ignoring air resistance, calculate the time taken for the cannonball to complete its trajectory, and the horizontal distance travelled during this time.
2. An electron, initially travelling with speed v_0 at an angle θ to the horizontal, enters a region of length L between two parallel, horizontal charged plates, where it experiences a constant vertical acceleration, α. You may assume that the electron beam is initially directed towards the negatively charged plate. Any effects due to gravity may be neglected.
   a. Write down the equations of motion for the electron.
   b. Show that the electron will enter and exit the plates at the same vertical displacement if the vertical acceleration is given by

$$\alpha = \frac{2v_0^2}{L}\sin\theta\cos\theta$$

   c. Show that under these circumstances the electron will emerge from the plates at an angle $-\theta$ to the horizontal.
3. The potential between two argon atoms varies with the interatomic separation, r, approximately according to the equation

$$V(r) = 4\epsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$

   with $\varepsilon = 1.7 \times 10^{-21}$ J and $\sigma = 3.4 \times 10^{-10}$ m.
   a. Sketch the variation in potential energy as a function of argon atom separation.
   b. How does the force exerted on the argon atoms vary with atomic separation? At what separations is the force between the atoms attractive and at what separations is it repulsive?

   c. Two argon atoms (A and B) of mass m = 6.6 × 10^{-26} kg undergo a head on collision. Atom A has an initial speed of 400 ms^{-1} while the second atom B is stationary. You may assume that the interaction potential is as described in the previous question.
      i. What is the distance of closest approach of the two argon atoms, r_0? Use energy conservation and make the substitution $x = (\sigma/r)^6$.
      ii. What is the speed of the two argon atoms at r_0?
      iii. What is the acceleration between the atoms at r_0?
   d. An elastic collision is one which conserves kinetic energy (in addition to total energy and momentum, which are conserved in all collisions). The collision between two Ar atoms is elastic. Given this fact, what are the final velocities of the atoms after the collision?
4. A particle of mass m undergoes uniform rotational motion at a constant angular frequency ω. The particle is held fixed at a constant radius α from the axis of rotation.
   a. Assume that the particle rotates in the xy plane, and at time $t = 0$ lies along the x axis. Write down expressions for the x and y co-ordinates of the particle as a function of time, t.
   b. Show that the linear velocity of the particle, v, has magnitude $\alpha\omega$ and is directed tangentially to the orbital motion (i.e. perpendicular to its position vector r).
   c. Define the term angular momentum.
   d. What is the moment of inertia, I, of the particle?
   e. Expressing your answers in terms of I, determine the angular momentum and the angular kinetic energy of the particle. How are the two quantities related?
5. The rigid rotor model for rotation of an isolated diatomic molecule comprises two point masses, m_1 and m_2, separated at a fixed bond length, r.
   a. Explain why the rotational motion must occur about the an axis which passes through the centre-of-mass of the molecule. Why is the angular momentum of the molecule conserved?
   b. Show that the moment of inertia for the molecule is given by

$$I = \mu r^2 \; ; \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

c. Assuming that both HCl and DCl have bond lengths, $r = 1.27 \times 10^{-10}$ m, evaluate their moments of inertia.

d. The average rotational kinetic energy of a molecule at temperature T is $k_{\mathrm{B}}T$, where k_{B} is the Boltzmann constant (this comes from the equipartition principle). Use this fact to determine the average angular velocity and the average angular momentum of HCl and DCl at 300 K. Comment on the results you obtain. Take m_{Cl} = 35u, m_{H} = 1u, and m_{D} = 2u, where u is the atomic mass unit.

6. The following expression describes the time dependence of the displacement of a harmonic oscillator with force constant k and of effective mass μ from its rest position

$$x(t) = A\sin(\omega t + \phi) \qquad \text{where} \quad \omega = \sqrt{\frac{k}{\mu}}$$

ω is the angular frequency of the oscillator, and ϕ determines the phase of the oscillation.

a. Use this expression to determine how the velocity, $v(t)$, of the oscillator varies with time.

b. Using the above expression for $x(t)$, together with your result for $v(t)$, show that the total energy, E, of a harmonic oscillator is conserved.

7. The vibrations of a diatomic molecule can be described approximately using the harmonic oscillator model.

a. Show that for such a system the effective mass is given by the expression

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

where m_1 and m_2 are the masses of the two atoms.

b. The angular frequencies of the vibrations in $H^{35}Cl$ and $D^{35}Cl$ are 5.634×10^{14} rad s^{-1} and 4.041×10^{14} rad s^{-1} respectively. Determine the force constants for the two molecules. Comment on your results. Take the masses of H, D and ^{35}Cl to be 1.008u, 2.014u and 34.969u.