Classical Mechanics
&
Properties of Gases

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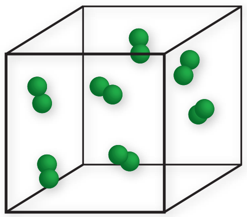
Summary of Last Lecture

- Uniform Circular Motion
- Angular Momentum
- Moments of Inertia
- Reduced Mass

Classical Mechanics

- Translation
  - Newton's laws of motion
  - Momentum, work & energy
- Rotation
  - Angular momentum
  - Moments of inertia
- Vibration
  - Simple harmonic motion



The wave equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi$$

$$\frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial x^2}$$

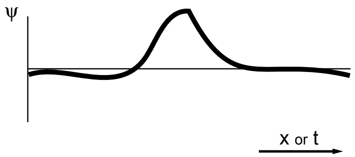
$$\frac{\partial^2\psi}{\partial t^2} = c^2\frac{\partial^2\psi}{\partial x^2}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\frac{\partial [A]}{\partial t} = D\frac{\partial^2 [A]}{\partial x^2}$$

The wave equation

$$\frac{\partial^2\psi}{\partial t^2} = c^2\frac{\partial^2\psi}{\partial x^2}$$



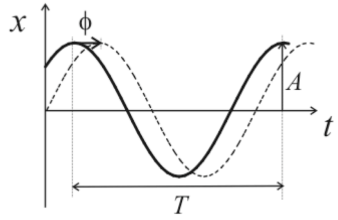
$$\psi(x,t) = f_1(x - vt) + f_2(x + vt)$$

$$\psi(x,t) = A\sin(x - ct) + B\sin(x + ct)$$

Sinusoidal waves

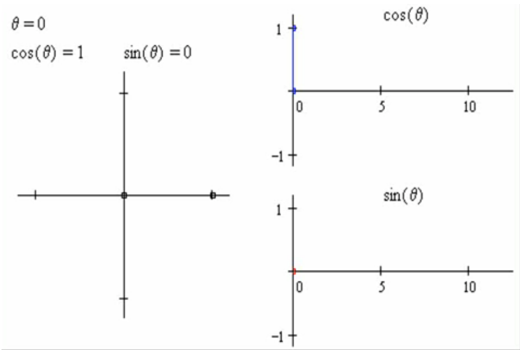
$$\psi = A\sin k(x - ct) + B\sin k(x + ct)$$

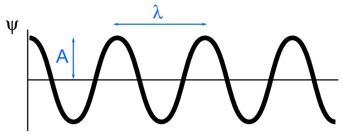
- Period, T
  The time required to complete a full cycle
- Frequency, f
  The number of cycles per second (Hz)
  Angular frequency, $\omega = 2\pi f$
- Amplitude, A
  The maximum displacement from equilibrium
- Wavelength, λ
  Repeat distance of wave
- Velocity of propagation, v
  $v = f\lambda$
- Angular wavenumber, k
  The number of wavelengths in the distance 2π
  $k = 2\pi/\lambda$.
- Initial phase, φ



Sinusoidal waves

As a projection of Uniform Circular Motion





$$\frac{\partial^2\psi}{\partial t^2} = c^2\frac{\partial^2\psi}{\partial x^2}$$

$$\psi(x,t) = A\sin(kx - \omega t + \phi)$$
$$\frac{\partial\psi}{\partial x} = Ak\cos[kx - \omega t + \phi]$$
$$\frac{\partial^2\psi}{\partial x^2} = -Ak^2\sin[kx - \omega t + \phi] = -k^2\psi(x,t)$$

$$\psi(x,t) = A\sin(kx - \omega t + \phi)$$
$$\frac{\partial\psi}{\partial t} = -A\omega\cos(kx - \omega t + \phi)$$
$$\frac{\partial^2\psi}{\partial t^2} = -A\omega^2\sin(kx - \omega t + \phi) = -\omega^2\psi(x,t)$$

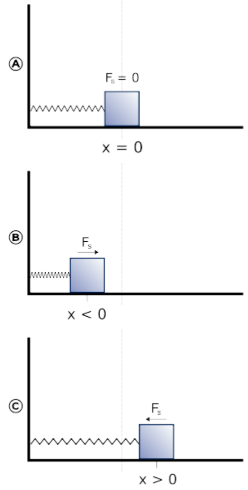
$$\frac{1}{\omega^2}\frac{\partial^2\psi}{\partial t^2} = \frac{1}{k^2}\frac{\partial^2\psi}{\partial x^2}$$
$$\frac{\partial^2\psi}{\partial t^2} = \frac{\omega^2}{k^2}\frac{\partial^2\psi}{\partial x^2}$$
$$\frac{\partial^2\psi}{\partial t^2} = \left(\frac{2\pi f}{2\pi/\lambda}\right)^2\frac{\partial^2\psi}{\partial x^2}$$

Simple harmonic motion

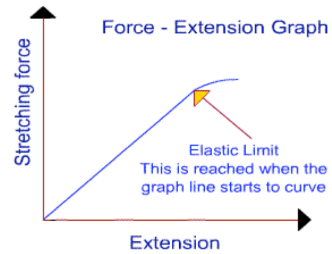
- SHM requires a restoring force that is proportional to the displacement of the system
- e.g., springs, a pendulum, sound waves, molecular vibrations...

Simple harmonic motion

Springs & Hooke's law

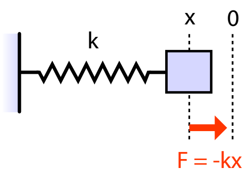


$$F = -kx \qquad T = 2\pi\sqrt{\frac{m}{k}}$$



Equations of motion

Harmonic spring



$$\psi(x,t) = A\sin(kx - \omega t + \phi)$$

$$x(t) = A\cos(\omega t + \phi)$$

$$ma = -kx$$

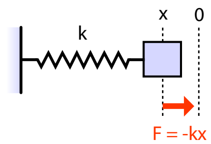
$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -kx$$

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + kx = 0$$

$$\boxed{\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \frac{k}{m}x = 0}$$

Equations of motion

Harmonic spring



$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \frac{k}{m}x = 0 \qquad x(t) = A\cos(\omega t + \phi)$$

$$x(t) = A\cos(\omega t + \phi)$$

$$\frac{d^2x}{dt^2} = -\omega^2 A\cos[\omega t + \phi]$$

$$-\omega^2 A\cos[\omega t + \phi] + \frac{k}{m}A\cos[\omega t + \phi] = 0$$

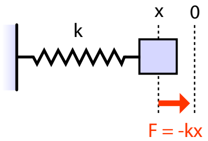
$$-\omega^2 + \frac{k}{m} = 0$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$x(t) = A\cos[\sqrt{\frac{k}{m}}t + \phi]$$

Equations of motion

Harmonic spring



$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$x(t) = A\cos(\omega t + \phi)$$

$$x(t) = A\cos[\sqrt{\frac{k}{m}}t + \phi]$$

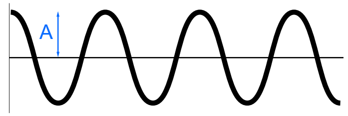
$$x(t) = x_0\cos[\sqrt{\tfrac{k}{m}}t]$$

Boundary conditions

$t = 0 \qquad x = x_0 \qquad \phi = 0$

$$x(0) = x_0\cos[0]$$

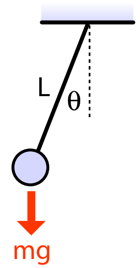
$$x(t) = x_0\cos[\sqrt{\tfrac{k}{m}}t]$$



$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

An example

Simple pendulum



$$-mg\sin\theta = ma$$ — $\sin\theta \approx \theta$

$$-mg\theta = ma$$

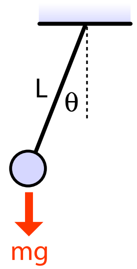
$$m\frac{d^2x}{dt^2} + mg\theta = 0$$ — $x = l\sin\theta$, $x = l\theta$, $\frac{d^2x}{dt^2} = l\frac{d^2\theta}{dt^2}$

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0$$

$$\theta(t) = A\sin(\omega t + \phi)$$

An example

Simple pendulum



$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0$$

$$-\omega^2\theta + \frac{g}{l}\theta = 0$$

$$\omega = \sqrt{\frac{g}{l}}$$

$$\theta(t) = A\sin(\omega t + \phi)$$

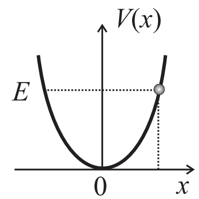
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

$$\frac{d^2\theta}{dt^2} = -\omega^2 A\sin(\omega t + \phi)$$

Energy in SHM

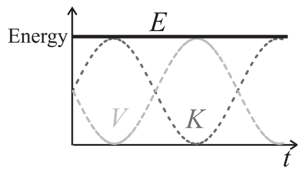
Potential

$$\begin{aligned} V(t) &= \tfrac{1}{2}kx^2 \\ x(t) &= A\sin(\omega t + \phi) \\ V(t) &= \tfrac{1}{2}kA^2\sin^2(\omega t + \phi) \end{aligned}$$



Kinetic

$$\begin{aligned} K(t) &= \tfrac{1}{2}mv^2 \\ v(t) &= A\omega\cos(\omega t + \phi) \\ K(t) &= \tfrac{1}{2}mA^2\omega^2\cos^2(\omega t + \phi) \\ K(t) &= \tfrac{1}{2}kA^2\cos^2(\omega t + \phi) \end{aligned}$$

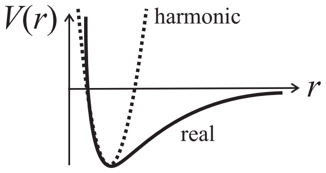


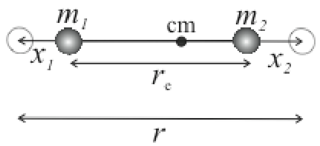
Total

$$\begin{aligned} E &= K + V \\ E &= \tfrac{1}{2}kA^2\left[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)\right] = \tfrac{1}{2}kA^2 \end{aligned}$$

An example

Molecular vibrations

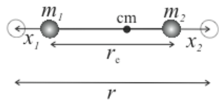




- Molecular vibration approximated as simple harmonic motion
- Interested purely in vibrational motion, consider vibration about the centre of mass (CM)
- We're going to need to use reduced mass again

An example

Molecular vibrations



$$F = m_1 a_1 = -m_2 a_2$$

$$m_1 a_1 + m_2 a_2 = 0$$

$$a_1 = -\frac{m_2}{m_1} a_2$$

$$a_2 = -\frac{m_1}{m_2} a_1$$

$$a = a_1 - a_2$$

$$a = a_1 + \frac{m_1}{m_2} a_1$$

$$a = \frac{m_2 + m_1}{m_2} a_1$$

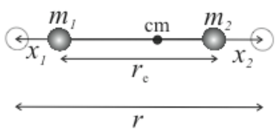
$$a = \frac{m_2 + m_1}{m_1 m_2} m_1 a_1$$

$$F = \frac{m_2 m_1}{m_1 + m_2} a$$

$$F = \mu a$$

An example

Molecular vibrations

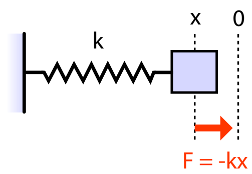


$$\mu a = -kx$$

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \frac{k}{\mu} x = 0$$

An example

Molecular vibrations



$$ma = -kx$$

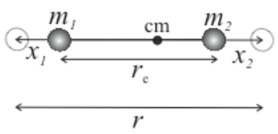
$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \frac{k}{m} x = 0$$

$$x(t) = A \cos(\omega t + \phi)$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}$$

An example

Molecular vibrations



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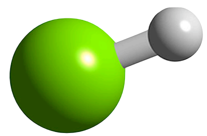
$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \frac{k}{\mu}x = 0$$

$$x(t) = A\cos(\omega t + \phi)$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\mu}{k}}$$

An example

Molecular vibrations of $H^{35}Cl$

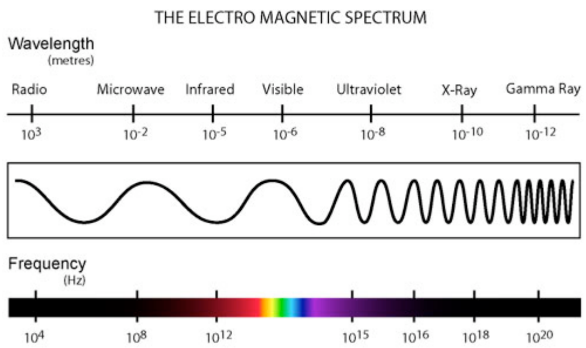


$$k = 481 Nm^{-1}$$

$$\begin{aligned} \mu &= \frac{(1.0078)(34.9688)}{1.0078 + 34.9688} \\ &= 0.9796 \quad amu \\ &= 0.9796 \quad (1.66 \times 10^{-27})kg \end{aligned}$$

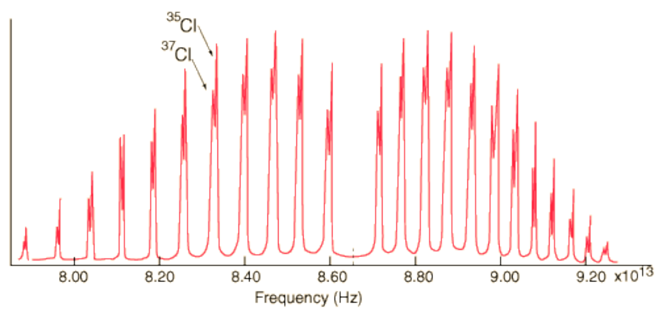
$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} = 8.66 \times 10^{13} Hz$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{8.66 \times 10^{13}} = 3.46 \times 10^{-6} m$$



An example

Molecular vibrations of $H^{35}Cl$



http://hyperphysics.phy-astr.gsu.edu/hbase/molecule/vibrot2.html#c1

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