Problem (3), P. 29

Subgroups of the quaternion group

The group is $1, -1, i, -i, j, -j, k, -k$

It is of order 8

Subgroup of order 1 : 1 H_1

" " " 2 : $1, -1$ H_2

" " " 4 : $1, -1, i, -i$ H_3

$1, -1, j, -j$ H_4

$1, -1, k, -k$ H_5

Note that these subgroups are abelian while the group is not. i.e. the subgroups are invariant. Some specific examples:

H_1 is invariant

H_2 is invariant

$i H_3$ is $i, -i, -1, 1$ which is H_3