

H subgroup of G

Is aHa^{-1} a subgroup of G ?

aHa^{-1} is the set of elements aha^{-1} where h runs over all elements of H and a is an element of G .

1) all elements of aHa^{-1} are members of G

2) The product of any two members of aHa^{-1} must be members of aHa^{-1}
eg. $b = (aha^{-1})(ah_2a^{-1})$ must be a member of aHa^{-1} .

$b = (ah_1h_2a^{-1})$ and since h_1 and h_2 are members of the subgroup H , h_1h_2 is also a member

3) aHa^{-1} must contain the identity
 e is contained in H since H is a subgroup. And $aea^{-1} = aa^{-1}e = e$ is a member of aHa^{-1} . QED.

4) Consider a given member of aHa^{-1} aha^{-1} ,
 h^{-1} is a member of H so $ah^{-1}a^{-1}$ is a member of aHa^{-1} . And

$$\text{and } (ah^{-1}a^{-1})(aha^{-1}) = ah^{-1}ha^{-1} = e$$
$$(aha^{-1})(ah^{-1}a^{-1}) = ahha^{-1} = e$$

So aHa^{-1} contains the inverse of aha^{-1}

$\therefore aHa^{-1}$ is a subgroup of G