$A = (1234)(5678)$

and

$B = (1537)(2846)$

Each a permutation.

Their product is a permutation.

What is AB^2.

$\boxed{B}$ is $1 \to 5,\ 5 \to 3,\ 3 \to 7,\ 7 \to 1,\ 2 \to 8,\ 8 \to 4,$
$4 \to 6,\ 6 \to 2$

$\boxed{A}$ is $1 \to 2,\ 2 \to 3,\ 3 \to 4,\ 4 \to 1,\ 5 \to 6,\ 6 \to 7,$
$7 \to 8,\ 8 \to 5$

$\boxed{AB}$, apply B 1^{st} followed by a

$1 \to 5 \to 6,\ 5 \to 3 \to 4,\ 3 \to 7 \to 8,\ 7 \to 1 \to 2,$
$2 \to 8 \to 5,\ 8 \to 4 \to 1,\ 4 \to 6 \to 7,\ 6 \to 2 \to 3$

$AB = (1638)(5472)$

$\boxed{A^2}$ has $1 \to 3,\ 2 \to 4,\ 3 \to 1,\ 4 \to 2, \ldots$
$5 \to 7,\ 6 \to 8,\ 7 \to 5,\ 8 \to 6$

$A^2 = (13)(24)(57)(68)$

$\boxed{B^2}$ has $1 \to 3,\ 5 \to 7,\ 3 \to 1,\ 7 \to 5,$
$2 \to 4,\ 8 \to 6,\ 4 \to 2,\ 6 \to 8$

$B^2 = (13)(24)(57)(68)$, $B^2 = A^2$ ②

$\boxed{BA}$ has $1 \to 8$, $2 \to 7$, $3 \to 6$, $4 \to 5$,

$5 \to 2$, $6 \to 1$, $7 \to 4$, $8 \to 3$

$(1836)(2745)$

$\boxed{A^3}$ has $1 \to 4$, $2 \to 1$, $3 \to 2$, $4 \to 3$,

$5 \to 8$, $6 \to 5$, $7 \to 6$, $8 \to 7$

$(1432)(5876)$

$\boxed{A^2 B}$ has $1 \to 7$, $5 \to 1$, $3 \to 5$, $7 \to 3$,

$2 \to 6$, $8 \to 2$, $4 \to 8$, $6 \to 4$

$(1735)(2648)$

$\boxed{ABA}$ has $1 \to 5$, $2 \to 8$, $3 \to 7$, $4 \to 6$,

$5 \to 3$, $6 \to 2$, $7 \to 1$, $8 \to 4$

$(1537)(2846)$

$ABA = B$

$\boxed{AB^2}$ has $1 \to 4$, $5 \to 8$, $3 \to 2$, $7 \to 6$,

$2 \to 1$, $8 \to 7$, $4 \to 3$, $6 \to 5$

$(1432)(7658)$

$AB^2 = A^3$ (should have known from $B^2 = A^2$)

$\boxed{BA^2}$ has $1 \to 7$, $2 \to 6$, $3 \to 5$, $4 \to 8$,
$5 \to 1$, $6 \to 4$, $7 \to 3$, $8 \to 2$,
$(1735)(2648)$

$BA^2 = A^2B$

$\boxed{BAB}$ has $1 \to 2$, $5 \to 6$, $3 \to 4$, $7 \to 8$,
$2 \to 3$, $8 \to 5$, $4 \to 1$, $6 \to 7$
$(1234)(5678)$

$BAB = A$

$\boxed{B^2A}$ $1 \to 4$, $2 \to 1$, $3 \to 2$, $4 \to 3$
$5 \to 8$, $6 \to 5$, $7 \to 6$, $8 \to 7$
$(1432)(5876)$ (known from $B^2 = A^2$)

$B^2A = A^3$

$\boxed{B^3}$ $1 \to 7$, $5 \to 1$, $3 \to 5$, $7 \to 3$,
$2 \to 6$, $8 \to 2$, $4 \to 8$, $6 \to 4$

$(1735)(2648)$

$B^3 = A^2B = BA^2$

(known from $B^2 = A^2$)

$\boxed{A^4} = A^2 A^2$ has

$1 \to 1,\ 2 \to 2,\ 3 \to 3,\ 4 \to 4,\ 5 - 5,\ 6 \to 6$

$7 \to 7,\ 8 \to 8$

$(1)(2)(3)(4)(5)(6)(7)(8)$

$A^4 = e$

Group is

$e = (1)(2)(3)(4)(5)(6)(7)(8)$

$A^4 = B^4 = BA^2B$

$A = (1234)(5678)$ $\quad BAB$

$B = (1527)(2846)$ $\quad ABA$

$A^2 = (13)(24)(57)(68)$ $\quad B^2 = ABAB$

$AB = (1638)(2547)$ $\quad B^3A$

$BA = (1836)(2745)$ $\quad ABA^2 \quad A^3B$

$A^3 = (1432)(5876)$ $\quad AB^2$

$A^2B = (1735)(2648)$ $\quad BA^2$

$A^3B = 1\to 8,\ 5\to 2,\ 3\to 6,\ 7\to 4,$

$2\to 7,\ 8\to 3,\ 4\to 5,\ 6\to 1$

$= (1836)(2745) \quad = BA$

$B^2AB = 1\to 8,\ 5\to 2,\ 3\to 6,\ 7\to 4,$

$2\to 7,\ 8\to 3,\ 4\to 5,\ 6\to 1$

$= (1836)(2745) \quad \checkmark = BA$

$B^3A = (A^2)(BA) \quad 1\to 6,\ 8\to 1,\ 3\to 8,\ 6\to 3,$

$= BA^3$

$2\to 5,\ 7\to 2,\ 4\to 7,\ 5\to 4$

$= (1638)(2547) = AB$

$BA^2B = BB^2B = B^2B^2 = A^2A^2 = A^4 = e$

$A^2BA = B^2BA = B^3A = AB$

$A^5 = A^4A = eA = A$

$ABA^2 = (AB)A^2 \quad 1\to 8,\ 3\to 6,\ 2\to 7,\ 4\to 5,$

$5\to 2,\ 7\to 4,\ 6\to 1,\ 8\to 3$

$(1836)(2745) = BA$

$ABAB = AB \cdot B^3A = A^2$

$ABA^3 = (AB)(B^2A)$

$= AB^3A = A(AB) = A^2B$

$ABA^2B = (ABA)(AB) = (AB)A^2B$

$= (AB)(B^2)B = AB^4 = A$

$BA^2 = A^2B$

$BA^3 = BB^2A = B^3A = AB$

$BA^2B = B^4 = e$

$BABA = B(AB)A = B(B^3A)A = B^4A^2 = A^2$

$(BA)(A^2B) = B(A^3B) = B(BA) = B^2A = A^3$

$A^2BA = B^3A = AB$

$A^2BA^2 = A^2(A^2B) = B$

$A^2BAB = A^3$

$$A^2 B BA = A^5 = A$$

$$A^2 B A^3 = A^2 B^3 A^3 = A^2 A B A^3$$

$$= A^3 B = AB$$

$$A^2 B A^2 B = A^2 (B A^2 B) = A^2 B^4$$

$$= A^2 B$$

$$A^3 \cdot A^2 B = A^4 A B = AB$$

$$A^3 B A = (A^2)(ABA) = A^2 B$$

$$A^2 B A^3 = (A^2)(B)(B^2) A \quad AB$$

$$= A^2 A^2 BA = BA$$

	A	B	A^2	AB	BA	A^3	A^2B
A	A^2	AB	A^3	A^2B	B	e	BA
B	BA	A^2	A^2B	A	A^3	AB	e
A^2	A^3	A^2B	e	BA	AB	A	B
AB	B	A^3	BA	A^2	e	A^2B	A
BA	A^2B	A	AB	e	A^2	B	A^3
A^3	e	BA	A	B	A^2B	A^2	AB
A^2B	AB	e	B	A^3	A	BA	A^2

$1, -1, i, -i, j, -j, k, -k$

$i^2 = j^2 = k^2 = -1, \quad ij = k, \; jk = i, \; ki = j$

$A^2 \to -1, \; e \to 1,$

$A, A^3 \quad B, A^2B \; , \quad AB, BA$

$i, -i \quad j, -j \quad k \;\; -k$

$A \cdot B = AB$

	i	j	-1	k	$-k$	$-i$	$-j$
i	-1	k	$-i$	$-j$	j	1	$-k$
j	$-k$	-1	j	i	$-i$	k	1
-1	$-i$	$-j$	1	$-k$	k	i	j
k	j	$-i$	$-k$	-1	1	$-j$	i
$-k$	$-j$	i	k	1	-1	j	$-i$
$-i$	1	$-k$	i	j	$-j$	-1	k
$-j$	k	1	j	$-i$	i	$-k$	-1

isomorphic ✓