On the kinematics of coupled pendulums over varying parameters

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Abstract

Oscillations are one of the fundamental dynamical behavior of systems found in nature. Real physical systems interact and give rise to an effective dynamics depending on how strong individual units of action are coupled. The aim of this experiment is to examine the kinematics of two coupled pendulums by taking note of how energy is transferred from one to another through the imposed coupling, with respect to parameters that may affect the coupling strength. It was found out that the coupling strength is inversely related to tension, separation and string length of the pendulums.

Keywords: Coupled oscillations, pendulum

1. Introduction

Oscillations are very much observed in several natural phenomena studied in physics. In its most simplistic case, an oscillation can be described as a back and forth movement from an equilibrium point to that of a relative maximum, which occurs repeatedly over time. The most practical example would be a simple pendulum of length L and mass m. The period of its oscillation is given by the relation [1]

$$T = 2\pi\sqrt{\frac{L}{g}}. \tag{1}$$

However, it is more useful to note how these simple systems act when they are coupled to each other. Most realistic physical systems are virtually coupled in which exchange of energy happens proportionally to the strength of the coupling. In mechanical systems, this is directly related to the properties of the materials used in coupling and how it was effectively assembled. In this study, a two-pendulum system coupled using a rigid string was observed against varied parameters, which is discussed in the next section. The aim is to observe the mechanics of the oscillation of the system in terms of its resonant frequencies. Moreover, the resonance will be analyzed in terms of the factors specified in the next section.

2. Methodology

In order to observe the kinematics of two coupled pendulums, of the same mass, a setup described in Figure 1 was assembled. The weights that were hung on the free side of the string from which the pendulums were suspended are significantly higher than that of the masses of the pendulum bobs. This experiment was divided into three parts depending on which parameter was varied in increments.

The first part of the experiment deals with the effect of the separation d between the two pendulums on its period of energy exchange, which implies an effect on the coupling constant. The separation distance values used are 5.0, 5.2, 7.9, 10.7, and 11.9 centimeters (cm).The lengths a, b, d, c_1, c_2, as shown in Figure 1, were measured and recorded. One pendulum was released from a small angle from its equilibrium point, away from the normal of the string from the top view, and its motion was recorded using a video camera with a 60 frames per second (fps) capability for two cycles of energy exchange. One cycle of energy exchange happens when the first pendulum that was set into motion moved again. The period was measured and recorded for each separation distance. The coupling constant was plotted against the percent separation between the two pendulums, which is given by,

$$\%separation = \frac{100d}{a+b+d} \tag{2}$$

The same procedure was done for two other parameters, tensile force and pendulum length. The tensile force was adjusted by changing the mass attached on the free side of the string from which the pendulums were suspended. The masses used are 478.4, 670.3, 863.9, and 1063.9 grams (g). The expression for the coupling constant in this case is

$$K = \left(\frac{4\pi m}{P}\right) / \left(\sqrt{\frac{g}{l+\frac{ac}{T}}}\right) \tag{3}$$

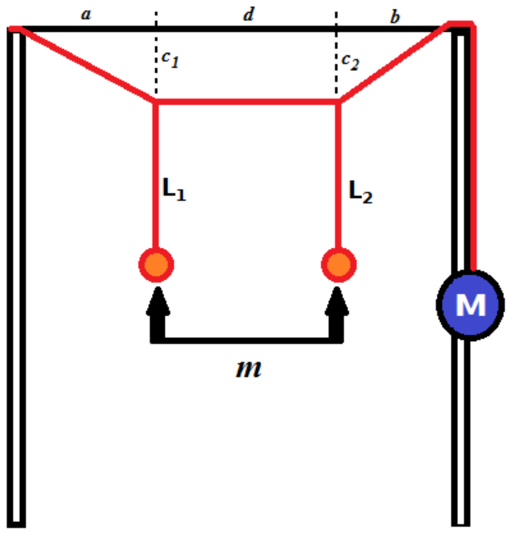

Figure 1: General set-up used for the experiment.

However, the coupling constants calculated was plotted against the values of tensile force, instead. Lastly, the length of the pendulums was varied, with the lengths of both pendulums being equal for each variation. The lengths used are 21.5, 27, 31.8 and 42.9 centimeters (cm). The expression for the coupling constant in terms of the length l of a pendulum is given by

$$K = \left(\frac{4\pi m}{P}\right) / \left(\sqrt{\frac{g}{l}}\right) \tag{4}$$

The coupling constants were plotted against $P\sqrt{l}$, where P is the period of energy exchange and l is the length of the pendulums.

3. Results and Discussion

Setting the length of the pendulums to be 42.9 cm, the uncoupled frequency was determined experimentally to be 0.74 Hz. This deviated from the theoretical value of 0.76 Hz by 2.6 %.This value for frequency would be used for the next two parts of the experiment.

The dependence of the coupling coefficient, K_3, to the tension experienced by the string is shown in Figure 2.

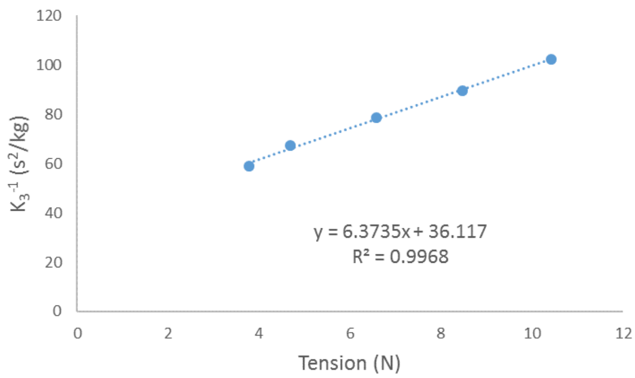

Figure 2: The coupling coefficient was determined to increase inversely with the tension.

The high R^2 value validates the linearity of the dependence between the inverse of K_3 and tension. As the tension decreases, the inclination of the string near the metal stands increases, increasing the distance of the pivot points of the pendulums with respect to the horizontal. This causes the whole system to swing more as response to the swinging of the individual pendulums. Since this is the mechanism in which the two pendulums exchange energy, the coupling coefficient should then increase. This explains the inverse relationship obtained. The linearity should have arisen from the fact that the weights used were much more massive than the pendulums which makes the inclination small enough for linear approximations to be valid.

Figure 3 shows the inverse relationship between K_3 and percent separation between the pendulums.

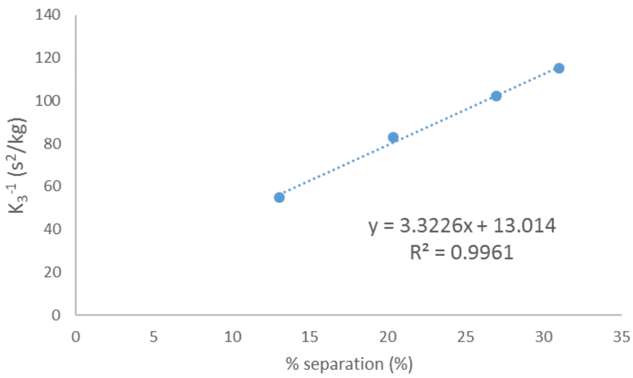

Figure 3: The coupling coefficient was determined to increase inversely with the percent separation.

Decreasing the separation distance between the two pendulums increases the inclination of the string. This results to increase in K_3 as explained in the previous part and as such, the inverse relationship was observed.

Lastly, the response of the value of K_3 with variations in the lengths of the pendulums was demonstrated in Figure 4. The reciprocal of the coupling coefficient increases as the string used for the pendulums lengthens.

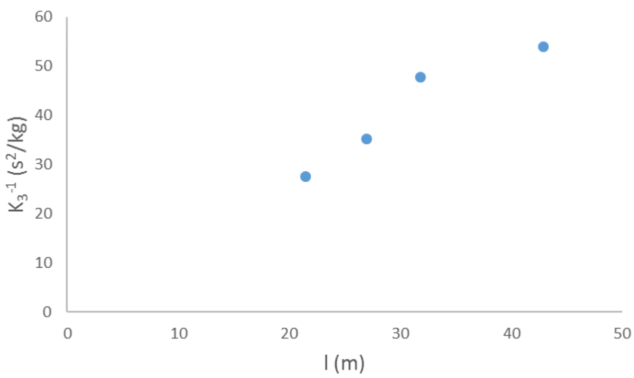

Figure 4: The coupling coefficient was shown to decrease with length.

To further demonstrate their relationship, the inverse of K_3 was plotted against the product of the period of energy exchange and the square root of the length which is shown in Figure 5.

Similar to the first two graphs, a high degree of linearity was observed for the graph of K_3 vs $Pl^{1/2}$. This result was expected since the said relationship automatically arises mathematically when short angle approximation shown in equation 1 was used for the frequency in the formula for the coupling coefficient.

Sources of error for the experiment include limited precision of the measuring devices used, probable error in judgment on when the system was said to undergo one cycle, and slippage in the set-up. Also, for the determination of the dependence of K_3 on the length of the string, the frequency used for the calculation of the coupling coefficient was measured from the video where the two pendulums were undergoing coupled oscillation and thus, may not be their actual uncoupled frequency.

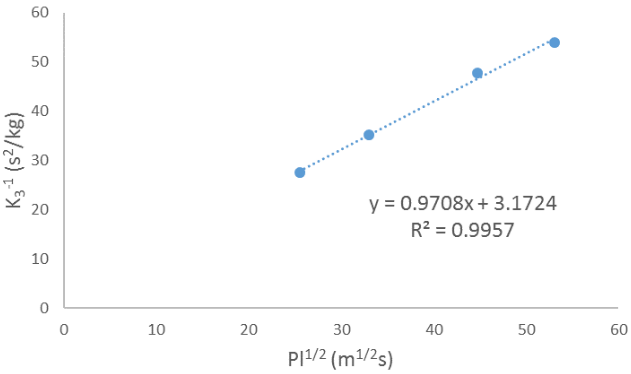

Figure 5: The coupling coefficient was determined to increase inversely with $Pl^{1/2}$.

4. Conclusion and Recommendation

The coupling coefficient is found to decrease with the increase in tension, separation, and string length of the pendulums. Specifically, the inverse of the coupling coefficient was found to be linearly dependent on tension, percent separation, and product of period of exchange and square root of the string length for the pendulums.

For future replication of this study, one recommendation is to explore the critical values of different variables wherein beyond those values, the linear relationships observed in this experiment break down.

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References

[1] P. Tipler and G. Mosca. *Physics for Scientists and Engineers*, 6th ed. New York: W.H. Freeman. (2003)