Determining the acceleration due to gravity using physical pendulums

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Abstract

The measurement of the acceleration due to gravity g was done via the analysis of the motion of two pendula, namely the bar and the compound pendula. The resulting g values for each pendulum were found to closely correspond to the value of $g = 9.81\frac{m}{s^2}$, with $g = 10.41\frac{m}{s^2}$ for the bar pendulum and $g = 9.72\frac{m}{s^2}$ for the compound pendulum. The percent errors were calculated to be 0.06% and 0.008%, respectively. Next, the period of the cylindrical pendulum was found to be $0.666 \pm 0.0124s$, which closely correspond to the theoretical value of 0.663 ± 0.008. g was also calculated from the cylindrical pendulum to be 9.72 ± 0.43 m/s^2, with an error of 0.9%.

Keywords: Acceleration due to gravity, bar pendulum, cylindrical pendulum, compound pendulum

1. Introduction

Measuring the acceleration due to gravity g can be indirectly done through the analysis of the motion of a pendulum. This experiment aims to calculate g by analyzing the motion of a bar pendulum and a compound pendulum.

As the name suggests, the bar pendulum is a metallic bar, with holes throughout its length. The holes are equally spaced from each other. For a bar pendulum of mass M oscillating at small angles, the period is given by [1]:

$$T = \sqrt{\frac{L}{g}} \tag{1}$$

where

$$L = \frac{k^2}{l} + l \tag{2}$$

The factor k is called the radius of gyration and l is the length from the center of gravity to the center of suspension.

From Eq. (2), the roots of l can be solved from the quadratic equation [1]:

$$l^2 - lL + k^2 = 0 \tag{3}$$

Using Eqs. (2) and (3), the following equation can be derived [1]:

$$l^2 = \frac{g}{4\pi^2} lT^2 - k^2 \tag{4}$$

Hence, by plotting l^2 and lT^2, the value of g can be obtained by multiplying the slope of the resulting linear fit by $4\pi^2$, which is otherwise known as Ferguson's method [1].

On the other hand, a compound pendulum consists of a cylindrical mass and a bar pendulum [2], wherein the mass of the cylinder is much greater than the mass of the bar.For a compound pendulum, the acceleration due to gravity g can be calculated by the same token.

In addition to calculating g by analyzing the motion of the bar and compound pendula, the period of a cylindrical physical pendulum was studied. The theoretical period is given by [3]:

$$T = 2\pi\sqrt{\frac{3R_1^2 + R_2^2}{2gR_1}} \tag{5}$$

where R_1 is the inner radius and R_2 is the outer radius. The resulting period obtained from Eq. (5) is then compared to the value obtained experimentally. The period experimentally obtained can also be used to calculate the value of g. Solving for g from Eq. (5),

$$g = 4\pi^2\left(\frac{3R_1^2 + R_2^2}{2R_1T^2}\right) \tag{6}$$

2. Methodology and Scope

The value of the acceleration due to gravity, g, was measured by recording the motion of physical pendulums through their period of oscillation. This experiment consists of three parts, which pertains to three different physical pendulums which has a different characteristic dynamics for each.

The first part of the experiment concerns a bar pendulum with holes located on increments of 6 *cm* from each preceding hole, starting from either end. The center of mass (*com*) was determined by balancing the metal bar onto a knife edge, until little to no significant inclination is apparent to either side. It was made to oscillate by suspending the metal bar onto a lubricated rod that protrudes from a wall and releasing it from a fixed angle that is small enough to make valid zeroth order approximations regarding its period, T. The lubrication was applied to reduce static friction between the metal and the protruded rod. This enabled the observation of later oscillations, on the 19th onwards on an appreciable degree. The time taken for it to undergo twenty oscillations was recorded and the procedure was done for all holes from either end up to the hole closest to the *com*. At the *com*, it was expected that releasing the pendulum from any release angle will not yield an oscillation, based on the formula for the period. The data set was plotted on a spreadsheet program and was analyzed by doing linear regression. The slope of the line was then used to compute for the acceleration due to gravity.

The same procedure was done for the compound pendulum, which consists of a metal bar with a massive solid cylindrical object attached at its center. However, the setup differs in terms of the location of the pivots (holes) from the *com*, which was assumed to be within the massive attachment. The distances of the holes from the *com* were measured to be, 55.2, 50.0, 45.1, 36.0, 27.9, 20.7, 17.4, 14.4, 11.5, 9 and 6.7 cm ($\pm$0.05 cm).

The oscillation of a cylindrical pendulum , which has an inner radius $R_1 = 5.2$ cm ± 0.1 cm and an outer radius $R_2 = 5.7$ cm ± 0.1 cm, onto a wedged protruded platform was examined by recording its period, from a fixed release angle that is small enough to make valid zeroth order approximations. The period was measured by recording the oscillation using a phone camera capable of taking 60 frames per second. The video was analyzed using Avidemux, a video processing software, by splitting the video into frames. The period was taken from the average of all periods recorded in one recording.

It was assumed that the material's mass distribution is uniform; all throughout the metal for the bar and compound, and radially, for the solid attachment on the compound pendulum, and the cylindrical pendulum. The validity of such approximations will be discussed on a later section.

3. Discussion and Analysis of Results

3..1 Bar pendulum

In order to compute for the acceleration due to gravity, g, the square of the distances of the pivot of oscillation from the center of mass, l^2, was plotted against lT^2, which was theoretically established as linearly related with slope $m = \frac{g}{4\pi^2}$ and y-intercept corresponding to the square of the radius of gyration of the pendulum. Shown in Figure 1 below is the plot of l^2 against lT^2.

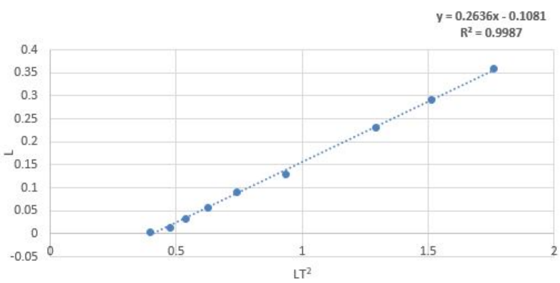

Figure 1: l^2 vs. lT^2. The R^2 value being close to 1 indicates that the relationship between the parameters is most probably linear.

The slope of the linear fit is $m = 0.2636$. Plugging this into the theoretical expression for the slope, the experimental acceleration due to gravity g is equal to 10.41 $\frac{m}{s^2}$ which deviates from the theoretical by 0.06%.

3..2 Compound pendulum

The same procedure was done for the data set gathered from observing the oscillation of a compound pendulum at pivot points with varying distances from the center of mass. The l^2 vs lT^2 plot for the compound pendulum is shown in Figure 2 below.

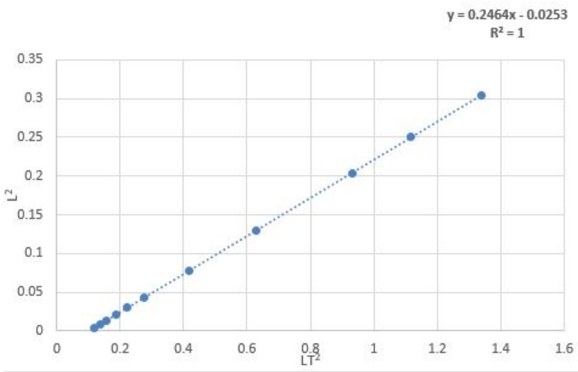

Figure 2: l^2 vs. lT^2. The R^2 value being equal to 1 indicates that the relationship between the parameters is linear.

The slope of the linear fit is $m = 0.2464$. The acceleration due to gravity, g, corresponding to this slope value is 9.72 $\frac{m}{s^2}$ which deviates from the theoretical by 0.008%.

3..3 Cylindrical physical pendulum

The average experimental period was found to be $T = 0.666 \pm 0.0124s$, where the standard deviation is taken to be the uncertainty. The period obtained via Eq. (5) was calculated to be $T = 0.663 \pm 0.008$. The uncertainty was solved via rules of error propagation, as shown in the Appendix*. Hence, the reported period is highly acceptable as it falls within the range of the theoretical values. The accuracy of the reported period could be accounted to the use of a wedged pivot, instead of a cylindrical pivot. Doing so reduces the introduction of systematic errors, since the pivot could be thought of as a fixed point.

The value for g was also calculated using the average experimental period and Eq. (6), and was found to be 9.72 ± 0.43 m/s^2, deviating from the theoretical value by 0.9%. The theoretical value for g is well within the uncertainty of the experimental value for g.

4. Generalization and Recommendations

By obtaining the slope of the linear fit of l^2-lT^2 plot, the acceleration of gravity was found to be equal to $g = 10.41\frac{m}{s^2}$ for the bar pendulum and $g = 9.72\frac{m}{s^2}$ for the compound pendulum. These value correspond to percent errors of 0.06 % and 0.008 %, respectively.

For the period of the cylindrical physical pendulum, an experimental value of $T = 0.666 \pm 0.0124$ s was obtained. This result agrees with the one obtained via Eq.(5), $T = 0.663 \pm 0.008$. An experimental value for $g = 9.72 \pm 0.43$ m/s^2 was also obtained using Eq. (6). This deviates from the theoretical value for g by only 0.9%. By using a wedge-shaped pivot instead of a cylindrical pivot, the introduction of systematic errors were greatly reduced.

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References

[1] Laboratory manual on Bar Pendulum. UKIERI.

[2] *The Compound Pendulum.* Overbeck et al. Sargent-Welch Scientific Company, 2003.

[3] *A Cylindrical Physical Pendulum.* Millersville University, 2012. Accessed from: http://www.millersville.edu/physics/experiments/050 /index.php

A Appendix 1: Error Propagation

The following is the derivation of the uncertainty for the period of the cylindrical pendulum and g as calculated using said period. First, Eq. (5) can be rewritten as:

$$T = \sqrt{m} \tag{7}$$

where $m = \frac{3R_1^2+R_2^2}{2gR_1}$. Solving first for the fractional uncertainty of the numerator $q = 3R_1^2 + R_2^2$:

$$\frac{\delta q}{q} = \frac{\sqrt{(2R_1\delta R_1)^2 + (2R_2\delta R_2)^2}}{3R_1^2 + R_2^2} \tag{8}$$

Then the fractional uncertainty of m is given by:

$$\frac{\delta m}{m} = \sqrt{\left(\frac{\delta q}{q}\right)^2 + \left(\frac{\delta R_1}{R_1}\right)^2} \tag{9}$$

Finally, the uncertainty of T can be solved via:

$$\delta T = \frac{1}{2}T\frac{\delta m}{m} = 0.008\ s \tag{10}$$

For g, Eq. (6) can be written as:

$$g = \frac{m}{T^2} \tag{11}$$

The fractional uncertainty of g can therefore be written as:

$$\frac{\delta g}{g} = \sqrt{\left(\frac{\delta q}{q}\right)^2 + \left(\frac{\delta R_1}{R_1}\right)^2 + \left(2\frac{\delta T}{T}\right)^2} \tag{12}$$

Using above equation, δg was calculated to be 0.43 m/s^2.