Dynamics of a bifilar pendulum under different modes of oscillation

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Abstract

The period of several bifilar suspensions were studied and then compared to the theoretical values. For bifilar pendula of equal pendulum lengths, the period calculated from formula was found to closely correspond to that of the period obtained from experimental data for the three modes of oscillation studied, namely swing-like, lateral and torsional. As such, the provided equations are said to be effective approximations for the period of a particular mode.
Keywords: Bifilar pendulum, oscillations

1. Introduction

The dynamics of a cylindrical rod suspended using two strings may have several modes depending on how the rod was made to oscillate about a pivot. Each configuration can be characterized by a totally different characteristic equation of motion. For example, a rod of length L and radius R, and suspended at a height H may be made to oscillate to and from its equilibrium plane, which is a swing-like motion. The period of its oscillation is given by the formula [1],

$$T = 2\pi\sqrt{\frac{R^2 + 2H^2}{2gH}} \tag{1}$$

Also, the same setup may be made to oscillate laterally, within the equilibrium plane. This has a period of oscillation given by [1],

$$T = 2\pi\sqrt{\frac{H - R}{g}} \tag{2}$$

The setup can also be made to rotate about an axis perpendicular to the rod length, so as to characterize a torsional oscillation. Both points of suspension are expected to have a period of oscillation given by [1],

$$T = 2\pi\sqrt{\frac{H - R}{g}\frac{L/d}{\sqrt{12}}} \tag{3}$$

as a zeroth order (small angle) approximation, where d is the distance between the points of attachment. Note that this expression is for the case when the central pivot of oscillation runs through the center of mass (com) of the rod. A torsional oscillation for a bifilar pendulum which is inclined at an angle θ from its horizontal, but with the same length L as before is expected to have the same result as that of a normal torsional mode with a factor of $\cos\theta$ multiplied to Equation 3.

The aim of this experiment is to confirm the validity of the expressions for small-angle oscillations ($\theta << 1$) above, which were calculated theoretically based on the equilibrium dynamics of a uniform cylindrical rod suspended at two points along its axis. Also, the effect of varying the height of suspension H was also examined so as to further establish the relationship between the period of oscillation T and H.

2. Methodology and Scope

In order to examine the motion of a bifilar pendulum, a metal rod of length $L = 0.277$ m, assumed to be uniform throughout, was suspended from a rigid pole using two yarn strings of the same length $H = 0.323$ m. The rod was made to oscillate from a small angle away from its equilibrium plane, so as to verify the small angle approximation made in Equation 1. The motion was recorded using a cellphone camera, capable of capturing 60 frames per second, that was placed directly above the setup. The video was split into frames using a video processing software, Avidemux, to measure the period of one oscillation for multiple oscillations. The average of the period values was recorded and compared with the theoretical values.

The same procedure was also done for lateral oscillations, or those within the equilibrium plane, for torsional oscillations and inclined torsional oscillations, with release angle that can be approximated as small angles. The rod used in this experiment was assumed to be uniform throughout.

3. Discussion and Analysis of Results

For the bifilar suspension with equal lengths of string, the period was measured experimentally for three types of motion, as mentioned in the introduction.

For the swinging motion, the average period was found to be $T_{ave,swing} = 1.14 \pm 0.03$ s. From Eq. (1), the expected period was calculated to be $T_{theo,swing} = 1.15$ s, corresponding to a percent error of 0.752 %. This value of percent error indicates that Eq. (1) is an effective description of the period of a swinging bifilar suspension of equal pendulum lengths.

Next, for the lateral motion, the period was averaged to be $T_{ave,lat} = 1.132 \pm 0.076$ s. When compared to the value obtained from Eq. (2) $T_{theo,lat} = 1.134$ s, a percent error of 0.247% was obtained. Likewise, Eq. (2) is said to be an effective approximation for the period of the lateral motion.

For the torsional motion, the average period was found to be $T_{ave,tor} = 0.712 \pm 0.014$ s. Yet, according to Eq. (3), the period was expected to be $T_{theo,tor} = 0.733$ s. A percent error of 2.90% suggests that Eq. (3) is also a workable approximation.

Next, for the torsional motion of an inclined bifilar suspension, the period was found to closely correspond to the period that of an uninclined one. The equation for the period is similar to that of Eq. (3), albeit with an additional term of $cos\theta$ multiplied to it. By invoking the small angle approximation, the second term of the moment of inertia was eliminated.

The average experimental period was found to be $T_{ave,inc} = 0.712 \pm 0.011$ s. Comparing to the theoretical value $T_{theo,inc} = 0.722$ s upon considering small-angle approximation, a percent error of 1.2 % was calculated. The period for the inclined and uninclined bifilar pendula differed only by approximately 0.01 s.

4. Generalization and Recommendations

The dynamics of a bifilar pendulum was analyzed by confirming the validity of Eqs. (1-3) through experiment. The pendulum was made to oscillate in three different manners as described by the aforementioned equations. For each oscillation of the pendulum, the value of the period was found to be 1.14 s, 1.132 s, 0.712 s, and 0.712 s with margins of error of 0.752%, 0.247%, 2.9%, and 1.2% for the swing, lateral, torsional, and inclined torsional oscillations, respectively. It is therefore concluded that the experiment was able to confirm the validity of the aforementioned equations in describing the motion of the bifilar pendulum for different modes of oscillation.

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References

[1] All pertinent documents provided by the instructor to be used in Physics 191 laboratory class