The dynamics of an underwater pendulum

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Abstract

This paper aimed to examine the dynamics of a pendulum submerged underwater and compare it with the motion of a pendulum in air. A pendulum of mass m was submerged underwater, with its period T_w analyzed. The value for T_w was found to be in the range bounded by the values 1.457 ± 0.014 s and 1.458 ± 0.014 s, with percent deviation between 1.6-1.7% as compared to theoretical value of 1.481 s. Furthermore, the values for T_w differed from the theoretical value of the period in air T_a by 11.3-11.5%.

Keywords: buoyancy, underwater pendulum

1. Introduction

Objects moving under water are subject to another force, aptly called the buoyant force. Buoyancy arises due to fluid pressure changing according to depth, resulting to an unbalanced upward force on the object [1].

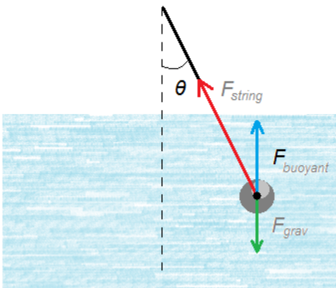

Figure 1: l^2 vs. lT^2. The FBD of a simple pendulum of mass m oscillating underwater

For a simple pendulum of mass m oscillating underwater, the buoyant force affects its period, as described by the following equation:

$$T = 2\pi\sqrt{\frac{m^*L}{mg^*}} \tag{1}$$

where, $m^* = m(1 + \frac{1}{2}\frac{\rho_{water}}{\rho_{bob}})$ is the effective mass and $g^* = (1 - \frac{\rho_{water}}{\rho_{bob}})g$ is the effective gravitational strength [2].

For this experiment, the period for the underwater pendulum was noted and then compared to the theoretical result obtained when the pendulum oscillates in air. The variation of T_w with θ_r is also determined, if any.

2. Methodology and Scope

A simple pendulum set-up was prepared using a 50.0 g mass and string of length 42.5 cm. Said pendulum was made to oscillate underwater. A video recording of the oscillation was taken using a cellphone camera which records at a rate of 60 frames per second. This procedure was repeated for three different angles of release, all of which small enough so that the small angle approximation holds.

Analysis of the periods of the oscillations was then done by decomposing the videos into separate frames using FFMPEG. The period was taken by counting the number of frames spanned by one oscillation for 20 oscillations per video. The mean of the obtained values is taken to be the period of the oscillation in question.

3. Discussion and Analysis of Results

The effect of buoyant force and viscous drag of water on the period of the pendulum is analyzed. Using Eq. (1), the period of the underwater pendulum T_w is calculated to be 1.481 s. This value assumes that the mass used as the pendulum bob is composed of steel, with mass density $\rho_{steel} = 8.05$ g/cm^3 and the mass density of water is 0.998 g/cm^3 at 20^{o}C.

The period of the underwater pendulum T_w is compared to the period of the pendulum in air T_a, which is computed using the following equation:

$$T_a = 2\pi\sqrt{\frac{L}{g}} \tag{2}$$

From Eq. (2) a value of $T_a = 1.308$ s is obtained. T_w differs from the value of T_a by 11.7%. This result implies that the period of the pendulum should increase when submerged in water.

The period of the pendulum underwater is then experimentally obtained. Three oscillations were recorded, each corresponding to different angles of release. Only small angles were used so as to make the small angle approximation and, consequently, Eq. (1) valid. The following table summarizes the data.

Table 1: T_w for different angles of release θ_r and comparison with T_a

θ_r, ^{o}	T_w, s	% error	% difference from T_a
10.0	1.457 ± 0.014	1.7	11.3
11.9	1.457 ± 0.014	1.7	11.3
15.5	1.458 ± 0.014	1.6	11.5

From Table 1, the small error of the period can be seen, which means that the period of the underwater pendulum obtained in the experiment agrees well with the theoretical result. Consequently, the T_w values deviate largely from T_a, as expected.

It can also be seen from Table 1 that T_w has little to no variation with respect to θ_r. This result reflects the lack of dependence in θ_r of T_w as seen in Eq. (1). This in turn further implies that the experiment agrees with theory. It should be noted, however, that this result holds only for small angle oscillations where the higher orders are too small to be worthy of consideration, at least in the levels of precision of this experiment. For higher θ_r, said approximations may not hold anymore and higher order terms may have to be taken into consideration.

4. Generalization and Recommendations

The period of oscillation of the pendulum submerged underwater T_w was analyzed through this experiment. The value for T_w was found in the experiment to be in the range bounded by the values 1.457 ± 0.014 s and 1.458 ± 0.014 s, deviating from the theoretical value of 1.481 s by only 1.6-1.7%. The above values for T_w differed from T_a by 11.3-11.5%. All these results agree with theoretical expectations.

As a recommendation, further inquiry in the dynamics of underwater pendulums may involve making θ_r large such that the small angle approximation no longer hold. Object tracking may also be used in the future so that the oscillation can be mapped in a θ vs. t curve. Curve fitting may then be used to compare the compare the obtained curve with the expected curve resulting from the equation of motion describing the oscillation of the underwater pendulum. These extensions require set-ups allowing better precision in measurements and/or more sophisticated methods of data analysis.

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