Determining the inertial mass through periodic oscillations

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Abstract

The study aims to investigate the viability of inertial balances for approximating objects of unknown masses. Two different inertial balances were prepared, one of which was improvised from a metal ruler and a styrofoam cup. Unit masses consisting of five 25-centavo coins were added one-by-one per run, and the period of oscillation was recorded. A linear relationship of mass and period was established. Moreover, it was found that as the mass was increased, the period also increased. From the calibration curves obtained by plotting $m - vs - T$ and $m - vs - T^2$, the inertial mass can be approximated, with the latter providing better approximations since the percent error from the true mass values were found to be small (¡5%). However, this prescription was not entirely followed, and may be due to errors introduced by the unintentional oscillation of the whole balance, and thus may be corrected.

Keywords: Inertial mass, periodic oscillations

1. Introduction

In 1687, Isaac Newton postulated three laws of motion. One of which describes the natural state of an object's trajectory without the influence of an external force. This is regarded to as the law of inertia, which relates the *natural tendency* of objects to stay in motion or at rest to how massive such an object is [1]. This is directly implied by the second law which relates the force applied on an object to its acceleration, linearly, according to an inertial frame of reference,

$$F = ma \tag{1}$$

which tells us that the more massive an object is, the harder it is to change its natural trajectory [1]. From this, the concept of an *inertial mass* was coined so as to quantify this property of an object in motion.

This experiment aimed to determine the relationship between the inertial mass of a collection of unit masses, in terms of five 25 Philippine centavo coins each, to the period of the inertial balance's motion on which the collection rested upon. This was eventually used to determine the inertial mass of the collection with additional unit masses according to the derived relationship.

2. Methodology and Scope

To demonstrate the concept of the inertial mass of an object, an inertial balance was used so as to relate the period of its translational motion to the mass that it carries. To calibrate the inertial balance, it was made to oscillate six times, each with a load that is incremented by one unit mass, composed of five 25 centavo coins. The period of each oscillation was measured by recording its motion through a Vernier motion sensor. The number of unit masses was plotted linearly against the period and the square of the period per trial using Microsoft Excel. The relationship derived experimentally was used as a calibration plot to determine the inertial mass of the load containing 7, 8, and 9 unit masses, by doing the same procedure for each.

The same procedure was done for an improvised inertial balance with a metal ruler as its lever arm and a styrofoam cup as its holding vessel. Instead of using a motion sensor, a camera capable of recording at a frame rate of 60 fps was used. The period was obtained by analyzing the motion through FFMPEG, a frame-splitting software.

It was assumed that all twenty-five centavo coins used in this experiment were uniform in mass.

3. Discussion and Analysis of Results

3..1 Analysis using Vernier Motion Sensor

The inertial balance was first calibrated using six masses amounting to six runs. The data for each run was then analyzed to calculate the period of oscillation of the inertial balance. From the data, the peak values were identified. Then, the time between two adjacent peaks was taken to be the period T. Since there are several peaks in one data run, the best value for the period T was the calculated average. The following table summarizes the average period per run:

Table 1: Average T and T^2 values for each calibration run

Run	Number of unit mass m	T	T^2
1	1	0.171	0.0291
2	2	0.188	0.0354
3	3	0.209	0.0437
4	4	0.226	0.0510
5	5	0.246	0.0604
6	6	0.260	0.0676

From Table 1, an increasing trend of the period is evident, as the unit mass was increased. This supports the proportionality of the inertial balance's period to mass [2]:

$$T \propto \sqrt{m} \tag{2}$$

which confirms that the mass of an object is proportional to the amount of effort needed to divert it from its rest state, with respect to some reference frame.

By plotting the unit mass m versus the corresponding period T, a calibration curve was produced. It can be seen in Figure 1 that the R^2 value is close to 1, which implies that the mass can be linearly correlated to T.

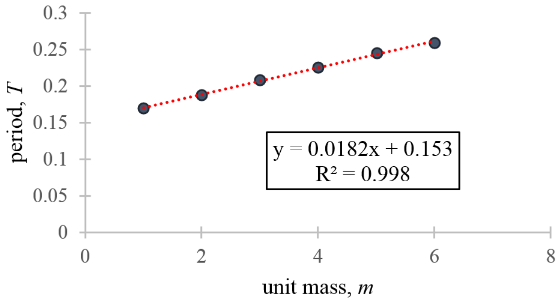


Figure 1: Graph of unit mass m versus period T

Taking y to be the mass m and x to be the period T, one can now solve for an unknown mass by using the inertial balance and plugging in measured period of oscillation via:

$$T = (0.0182)m + 0.153 \tag{3}$$

Hence, solving for m:

$$m = \frac{T - 0.153}{0.0182} \tag{4}$$

To test the calibration equation obtained, the period of three masses, each incremented by a unit mass, were noted. These period values were then plugged back to the equation above. The following table shows the result for the calculated unit mass:

Table 2: Calculated unit mass using Eq.4

Number of unit mass	T	Calculated number of unit mass	% error
7	0.275	6.70	4.29
8	0.302	8.19	2.38
9	0.315	8.90	1.11

By comparison, the calculated unit mass correspond closely to the expected number of unit mass, as supported by the minimal percent error, all of which are less than 5 %.

Another calibration curve was obtained by plotting the unit mass m against the square of the period T^2.

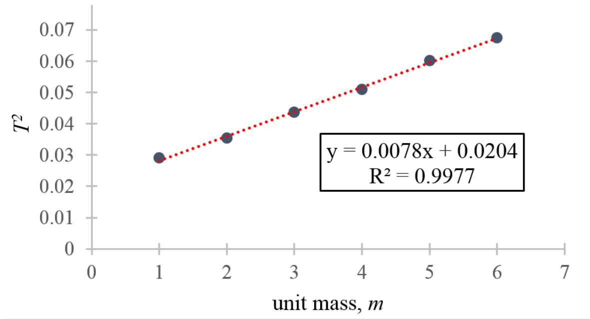


Figure 2: Graph of unit mass m versus period squared T^2

From the new calibration equation, an expression relating the unit mass and T^2 is given by:

$$m = \frac{T^2 - 0.0204}{0.0078} \tag{5}$$

Table 3 shows the calculated unit mass using the equation above.

Table 3: Calculated unit mass using Eq.5

Number of unit mass	T^2	Calculated number of unit mass	% error
7	0.0756	7.08	1.14
8	0.0909	9.04	13.0
9	0.0991	10.1	12.2

3..2 Improvised inertial balance

Similar methods of analysis were used for the improvised inertial balance except for counting the period of the balance per run. The count was performed by taking a video of the oscillation, splitting the video into frames, counting the number of frames taken per period for 32 periods per run, and taking the mean of the periods. The balance was calibrated using six masses amounting to seven runs, including the case when no unit mass is added.

Table 4: Average T and T^2 values for each calibration run of the improvised inertial balance

Run	Number of unit mass m	T	T^2
1	0	0.213	0.0457
2	1	0.266	0.0708
3	2	0.306	0.0936
4	3	0.343	0.1176
5	4	0.379	0.1436
6	5	0.421	0.1777
7	6	0.472	0.2228

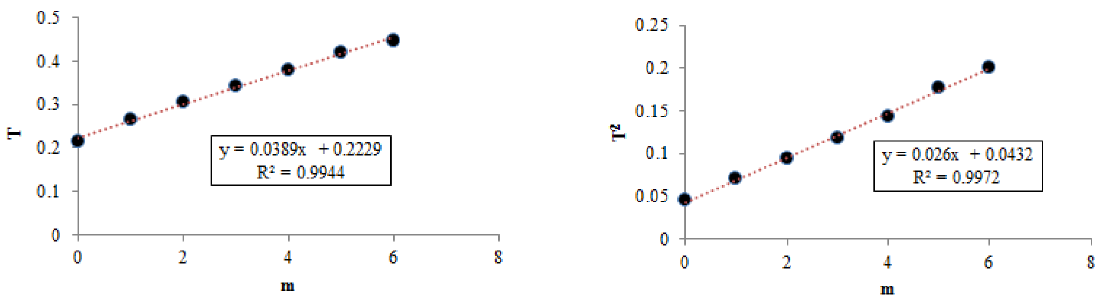
Table 2 displays an increasing trend of the period with increasing number of unit masses in the improvised inertial balance, as was also seen in the inertial balance in Table 1. It must be noted, however, that the table by itself can only show the increasing trend in the period, not *how* the trend is increasing.

Figure 3 shows the manner at which the period is increasing with unit mass. The higher correlation in T^2 vs m is in agreement with Eq. 2.

Solving for m from the linear trend line equation in Fig. 2a, we get

$$m = \frac{T - 0.2299}{0.0389} \tag{6}$$

while from the quadratic trend line equation in Fig. 2b, we get



(a) (b)

Figure 3: Graphs of unit mass m versus T (a) and T^2 (b) for the improvised inertial balance

$$m = \frac{T^2 - 0.0432}{0.026} \tag{7}$$

The above calibration equations are tested on three different masses. The corresponding period values were plugged back into the above equations to yield the calculated number of unit masses. The results are shown in the following table.

Table 5: Calculated unit mass using Eq.6

Number of unit mass	T	Calculated m	% error
7	0.472	6.40	8.52
8	0.500	7.12	11.0
9	0.524	7.74	14.0

Table 6: Calculated unit mass using Eq.7

Number of unit mass	T^2	Calculated m	% error
7	0.223	6.91	1.32
8	0.250	7.95	0.577
9	0.275	8.90	1.12

From the above tables, we see that the improvised inertial balance approximates unknown masses very well.

4. Generalization and Recommendations

A linear relationship between inertial mass and period of oscillation was obtained. Furthermore, it was shown that as the mass was increased, the period also increased.

For calculating unknown masses, it is apparent that the calibration equation obtained by plotting mass m versus T^2 is better in approximating the unit mass due to its relatively small percent error values. This is further supported by Eq. 2. However, returning to Table 3, the last two masses had large error values (¿10 %). This error values could be accounted to error introduced by the inertial balance. As more mass was put, the inertial balance may be dislodged from its foothold, so it may itself had been oscillating. This additional motion could be a source of error in measurement.

To capitulate, even if the linear regression describes a closely linear relationship for both T and T^2, it is more precise to establish a linear relationship between m and T^2. Using the calibration curve obtained by plotting such will yield better approximations for the inertial mass.

It was also found that the improvised inertial balances performs very well in approximating unknown masses. It also agrees with the relationship shown in Eq. 2.

Acknowledgments

We wish to thank Dr Wilson Garcia and Mr Lean Dasallas for their unwavering support by imparting their precious knowledge and guidance for the success of our project. This undertaking was supported by the National Institute of Physics by providing us with materials required in our methodologies.

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