Name: ______________________________ **Section:** ______________

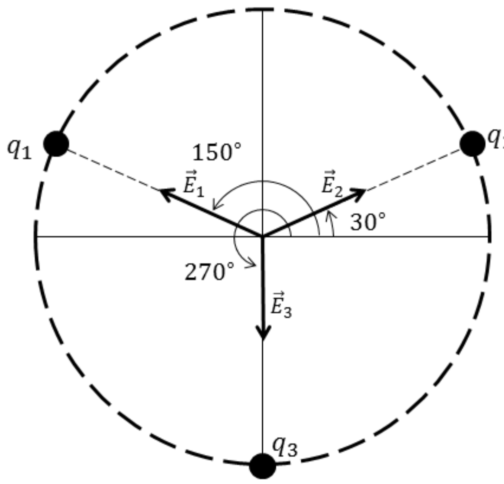
Physics 72 Problem Solving Session 02: Electric field calculations

August 15, 2019

General Instructions: Write your **name**. This is a closed–notes–quiz. You may discuss with your classmates or with your discussion class instructor. Answer **all problems**. Show your **complete solutions**. Write legibly. This exercise set is an adaptation of selected problems from College Physics by Serway and Vuille (10*th* edition) and University Physics by Young and Freedman (12*th* edition).

1. (8 points) **Steering wheel.** Three identical charges of magnitude $q = -5.0\mu C$ lie along a circle of radius 2.0 m where one charge is located 30° from the x-axis. What is the resultant electric field at the center of the circle?

   a (3 points) Draw a vector diagram (with angular and length measurements) describing the electric field due to each source at the center of the circle.



   b (3 points) Write down each electric field vector $\vec{E}_i$ in unit–vector notation ($\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$). *Show your solution.*

$$\vec{E}_1 = (9.0 \times 10^9 \left[\text{N m}^2 \text{ C}^{-2}\right])\frac{(5.0 \times 10^{-6}C)}{(2.0\ m)^2}(-\cos(30°)\hat{\imath} + \sin(30°)\hat{\jmath})$$
$$= -9.7 \times 10^3 \text{ N/C } \hat{\imath} + 5.6 \times 10^3 \text{ N/C } \hat{\jmath}$$

$$\vec{E}_2 = (9.0 \times 10^9 \left[\text{N m}^2 \text{ C}^{-2}\right])\frac{(5.0 \times 10^{-6}C)}{(2.0\ m)^2}(\cos(30°)\hat{\imath} + \sin(30°)\hat{\jmath})$$
$$= 9.7 \times 10^3 \text{ N/C } \hat{\imath} + 5.6 \times 10^3 \text{ N/C } \hat{\jmath}$$

$$\vec{E}_3 = (9.0 \times 10^9 \left[\text{N m}^2 \text{ C}^{-2}\right])\frac{(5.0 \times 10^{-6}C)}{(2.0\ m)^2}(-\hat{\jmath})$$
$$= -1.1 \times 10^4 \text{ N/C } \hat{\jmath}$$

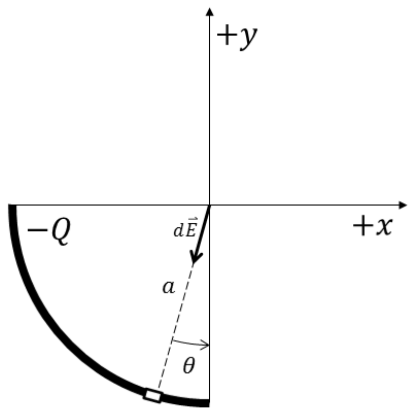
c (2 points) What is the resultant electric field at the center of the circle, in unit vector notation? If there were a negative test charge of the same magnitude at the center of the circle, where would the test charge accelerate to?

$$\begin{aligned}\vec{E}_{net} &= \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \\ &= \left(-9.7 \times 10^3 \text{ N/C } \hat{\imath} + 5.6 \times 10^3 \text{ N/C } \hat{\jmath}\right) + \left(9.7 \times 10^3 \text{ N/C } \hat{\imath} + 5.6 \times 10^3 \text{ N/C } \hat{\jmath}\right) \\ &\quad + -1.1 \times 10^4 \text{ N/C } \hat{\jmath} \\ &= 2.0 \times 10^2 \text{ N/C } \hat{\jmath}\end{aligned}$$

The test charge will accelerate towards the $-y$ direction.

2. (7 points) **Crispy Cream.** Negative charge $-Q$ is distributed uniformly around a quarter-circle of radius a that lies on the third quadrant, with the center of curvature at the origin. Find the electric field at the origin and write it in unit-vector notation.

a (2 points) Draw a vector diagram describing the electric field due to an infinitesimal charge along the semicircle at the center of its curvature.



b (3 points) Write down the infinitesimal electric field vector $d\vec{E}$ in unit–vector notation ($\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$). *Show your solution.*

$$\begin{aligned}d\vec{E} &= k\frac{dq}{r^2}\hat{r} \\ &= k\frac{(Q)\left(\frac{\pi a}{2}\right)^{-1}(a\ d\theta)}{a^2}\left(-\cos\theta\ \hat{\imath} - \sin\theta\ \hat{\jmath}\right) \\ &= -2k\frac{Q}{\pi a^2}\left(\cos\theta\ \hat{\imath} + \sin\theta\ \hat{\jmath}\right)\ d\theta\end{aligned}$$

c (2 points) Setup the integral with the appropriate bounds. [**BONUS**: Evaluate the integral. (2 points)]

$$\vec{E} = \int_0^{\frac{\pi}{2}} -2k\frac{Q}{\pi a^2}\left(\cos\theta\ \hat{\imath} + \sin\theta\ \hat{\jmath}\right)\ d\theta$$

"*Huwag mong tanungin kung mahirap, tanungin mo kung mahalaga.*"
Fr. Roque Ferriols, S.J.