Name: ______________________________ **Section:** ______________

Physics 72 Problem Solving Session 5: Electric potential, potential energy [KEY]

September 5, 2019 Score: ____/20

General Instructions: Write your **name**. This is a open–notes–quiz. You may discuss with your classmates or with your discussion class instructor. Answer **all problems**. Show your **complete solutions**. Write legibly. This exercise set is an adaptation of selected problems from College Physics by Serway and Vuille (10*th* edition) and University Physics by Young and Freedman (12*th* edition).

1. (10 points) The potential in a region of space is given by the function

$$V(x, y, z) = A\ln(-xy) - B\cos(x + yz) + Ce^{xy} \qquad \sim (1)$$

Find the electric field anywhere in this region.

a How is the electric field defined in terms of the potential? Write down the formula. (1 point)

$$\vec{E} = -\vec{\nabla}V$$

b How is the gradient of some scalar field $V(x, y, z)$ defined? Write down the formula. (1 point)

$$\vec{\nabla}V = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}$$

c Evaluate the rate of change along each direction. (2 points per coordinate)

Along the x-direction:

$$\frac{\partial V}{\partial x} = A\left(-\frac{1}{xy}\right)(-y) + B\sin(x + yz) + Cye^{xy}$$

Along the y-direction:

$$\frac{\partial V}{\partial y} = A\left(-\frac{1}{xy}\right)(-x) + Bz\sin(x + yz) + Cxe^{xy}$$

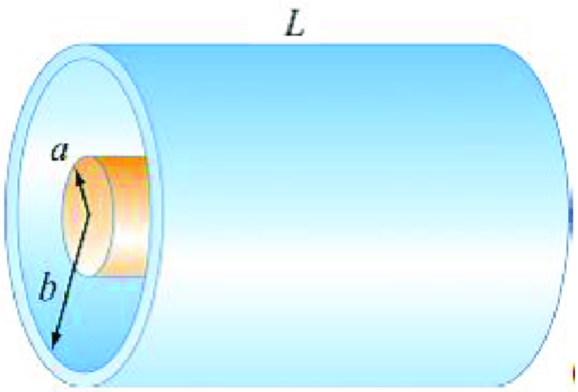
Along the z-direction:

$$\frac{\partial V}{\partial z} = By\sin(x + yz)$$

d What is the expression for the electric field in this region? (2 points)

$$\vec{E} = -\left(\frac{A}{x} + B\sin(x + yz) + Cye^{xy}\right)\hat{x} - \left(\frac{A}{y} + Bz\sin(x + yz) + Cxe^{xy}\right)\hat{y} \\ - By\sin(x + yz)\hat{z}$$

2. (10 points) Consider a cylindrical capacitor of length L as shown in the figure, with an inner conductor having a charge Q, and an outer conductor with some charge $-Q$. Let $d = b - a$ be the spacing between the inner and outer conductors.



Let the radii of the two conductors be only slightly different, such that $d << a$. Show that the capacitance of a cylindrical capacitor reduces to the capacitance of a parallel-plate capacitor with A being the surface area of each cylinder. Use the result that $ln(1+z) \approx z$ for $|z| << 1$.

a Using Gauss's law, find the electric field between the two conductors. (3 points)

$$\begin{aligned} E(2\pi r\ell) &= \frac{Q_{\text{enclosed}}}{\epsilon_0} \\ &= \frac{\left(\dfrac{Q}{2\pi aL}\right)(2\pi a\ell)}{\epsilon_0} \\ E &= \frac{Q}{2\pi\epsilon_0 rL} \end{aligned}$$

b Find the potential difference between the two conductors by using the equation (5 points)

$$\begin{aligned} V_b - V_a &= \int_a^b \vec{E}\cdot d\vec{\ell} \\ &= \int_a^b \frac{Q}{2\pi\epsilon_0 rL} dr \\ &= \frac{Q}{2\pi\epsilon_0 L}\ln(r)\bigg|_a^b \\ &= \frac{Q}{2\pi\epsilon_0 L}\ln\left(\frac{b}{a}\right) \end{aligned}$$

c Compute for the capacitance, C, of the system composed of two cylindrical conductors. (2 points)

$$\begin{aligned} C = \frac{Q}{V_{ab}} &= \frac{Q}{\frac{Q}{2\pi\epsilon_0 L}\ln\left(\frac{b}{a}\right)} \\ &= \frac{2\pi\epsilon_0 L}{\ln\left(\frac{b}{a}\right)} \end{aligned}$$

BONUS Consider the case where $d << a$. Show that the capacitance of a cylindrical capacitor reduces to the capacitance of a parallel-plate capacitor with A being the surface area of each cylinder. (2 points)

$$\begin{aligned} C &= \frac{2\pi\epsilon_0 L}{\ln\left(\frac{b}{a}\right)} \\ &= \frac{2\pi\epsilon_0 L}{\ln\left(\frac{d+a}{a}\right)} \\ &\approx \frac{2\pi\epsilon_0 L}{\frac{d}{a}} = \frac{\epsilon_0(2\pi a L)}{d} \\ &= \epsilon_0 \frac{A}{d} \end{aligned}$$

"*Huwag mong tanungin kung mahirap, tanungin mo kung mahalaga.*"
Fr. Roque Ferriols, S.J.