SETS

Sets

- A set is a collection of objects.
- Objects are sometimes referred to as elements or members.
- If a set is finite and not too large, we can describe it by listing the elements in it.

Example, the equation

$$A = \{1, 2, 3, 4\}$$

Describes a set A made up of the four elements 1, 2, 3, and 4.

Sets

- A set is determined by its elements and not by any particular order in which the elements might be listed.

Thus, the set A might just as well be specified as A = { 1, 3, 4, 2} **(1.1.1)**

- The elements making up a set are assumed to be distinct, and for some reasons may have duplicates in the list.

describe the set as A = {1, 2, 2, 3, 4}.

Sets

- If a set is a large finite set or an infinite set, we can describe it by listing a property necessary for membership.

Example, the equation

$$B = \{ x | x \text{ is a positive, even integer}\} \qquad \textbf{(1.1.2)}$$

- Describes the set B is made up of all positive, even integers; that is, B consists of the integers 2, 4, 6, and so on. B = { 2, 4, 6,8, ….}
- The vertical bar "|" is read "such that".
- Equation (1.1.2) would be read "B equals the set of all x such that x is a positive, even integer. The property necessary for membership is "is a positive, even integer." Note that the property appears after the vertical bar.

Sets

- The notation in (1.1.2) is called **set-builder notation.**
- A set may contain any kind of elements whatsoever, and they need not be of the same "type".

Example: $\{4.5$, Lady Gaga, π, $14\}$

It consist of four elements: the number 4.5, the person Lady Gaga, the number π(=3.1415…), and the number 14.

- A set may contain elements that are themselves sets.

for example, the set $\{3, \{5,1\}, 12, \{\pi, 4.5, 40, 16\}$, Henry Cavill$\}$

Consist of five elements: the number 3, the set $\{5,1\}$, the number 12, the set $\{\pi, 4.5, 40, 16\}$, and the person Henry Cavill.

Sets

- Some sets of numbers that occur frequently in mathematics generally, and in discrete mathematics in particular, are shown in Figure 1.1.1.
- The symbol Z comes from the German word, Zahlen, for integer.
- Rational numbers are quotients of integers, thus Q for quotient.

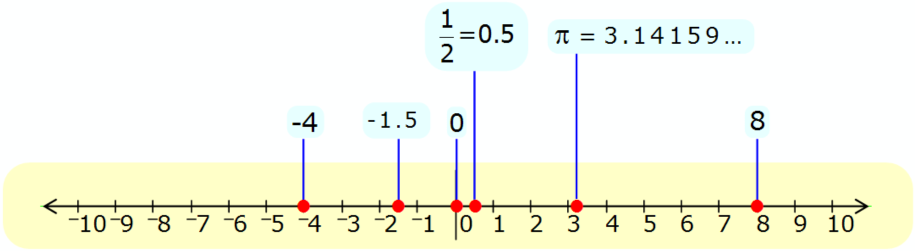
Symbol	Set	Example of members
Z	Integers	-3, 0, 2, 145
Q	Rational Numbers	-1/3, 0, 24/15
R	Real Numbers	-3, -1.766, 0, 4/15, $\sqrt{2}$, 2.666…, π

FIGURE 1.1.1

- The set of real numbers R can be depicted as consisting of all points on a straight line extending indefinitely in either direction Figure 1.1.2.

Sets

- Figure 1.1.2 The real number line



- To denote the negative numbers that belong to one of Z, Q, or R, we use the superscript minus.

  For example: Z^- denotes the set of negative integers, namely -1, -2, -3, ….

- To denote the positive numbers that belong to one of the three sets, we use the superscript plus.

  For example: Q^+ denotes the set of positive rational numbers

- To denote the nonnegative numbers that belong to one of the three sets we use the superscript *nonneg.*

for example: Z^{nonneg} denotes the set of nonnegative integers, namely 0, 1, 2, 3, ...

Sets

- If X is a finite set, we let |X| = number of elements in X.
- We call |X| the **cardinality** of X

For example: The cardinality of the integers, Z, is denoted $\aleph_0$ read "aleph null." Aleph is the first letter of the Hebrew alphabet.

- In mathematics, the cardinality of a set is a measure of the "number of elements" of the set.

Example 1.1.1 For the set in 1.1.1, we have |A|= 4, the cardinality of A is 4.

- The cardinality of the set {R,Z} is 2 since it contains two elements, namely the two sets R and Z.

Sets

- Given a description of a set X such as (1.1.1) or (1.1.2) and an element of x
- Determine whether or not x belong to X.
- If the members of X are listed as in (1.1.1), we inspect if whether or not x appears in the listing.
- As (1.1.2), we check to see whether the element x has the property listed.
- If x is in the set X, we write $x \in X$, and if x is not in X, we write $x \notin X$

Sets

- For example, $3 \in \{1, 2, 3, 4\}$, but $3 \in \{ x \mid x \text{ is a positive, even integer}\}$. The set with no elements is called the empty (or null or void) set and is denoted $\emptyset$. Thus $\emptyset = \{\}$.
- Two sets X and Y are equal and we write $X = Y$ if X and Y have the same elements. To put it another way, $X = Y$ if the following two conditions hold:

■ For every x, if $x \in X$, then $x \in Y$,

and

■ For every x, if $x \in Y$, then $x \in X$.

The first condition ensures that every element of X is an element of Y, and the second condition ensures that every element of Y is an element of X.

Sets

- Example 1.1.2

  If A = {1, 3, 2} and B = {2, 3, 2, 1}, by inspection, A and B have the same elements. Therefore A = B.

Sets

- Study/Review examples on chapter 1, pages 3-6.

- Reference :

  R. Johnsonbaugh, Discrete Mathematics (2018) eight edition.

SETS (Examples)

- Example 1.1.3

Show that if $A = \{x \mid x^2 + x - 6 = 0\}$ and $B = \{2, -3\}$, then $A = B$.

SOLUTION According to the criteria in the paragraph immediately preceding Example 1.1.2, we must show that for every x,

$$\text{if } x \in A, \text{ then } x \in B, \tag{1.1.3}$$

and for every x,

$$\text{if } x \in B, \text{ then } x \in A \tag{1.1.4}$$

SETS (Examples)

- To verify condition (1.1.3), suppose that $x \in A$. Then

$$x^2 + x - 6 = 0.$$

Solving for x, we find that $x = 2$ or $x = -3$. In either case, $x \in B$. Therefore, condition (1.1.3) holds.

To verify condition (1.1.4), suppose that $x \in B$. Then $x = 2$ or $x = -3$.

If $x = 2$, then

$$x^2 + x - 6 = 0$$

$$(2)^2 + 2 - 6 = 0$$

$$4 + 2 - 6 = 0$$

$$6 - 6 = 0$$

SETS (Examples)

- Therefore, $x \in A$. If $x = -3$, then

$$x^2 + x - 6 = 0$$

$$(-3)^{2} + (-3) - 6 = 0$$

$$9+(-3)-6 = 0$$

$$6-6 = 0$$

Again, $x \in A$. Therefore, condition (1.1.4) holds. We conclude that $A = B$.

- For a set X to not be equal to a set Y (written $X \neq Y$), X and Y must not have the same elements: There must be at least one element in X that is not in Y or at least one element in Y that is not in X (or both).

SETS (Examples)

- Example 1.1.4

Let $A = \{1, 2, 3\}$ and $B = \{2, 4\}$. Then $A \neq B$ since there is at least one element in A

(1 for example) that is not in B. [Another way to see that $A \neq B$ is to note that there is at least one element in B (namely 4) that is not in A.]

- Suppose that X and Y are sets. If every element of X is an element of Y, we say that X is a subset of Y and write $X \subseteq Y$. In other words, X is a subset of Y if for every x, if $x \in X$, then $x \in Y$.

SETS (Examples)

- Example 1.1.5

If $C = \{1, 3\}$ and $A = \{1, 2, 3, 4\}$, by inspection, every element of C is an element of A. Therefore, C is a subset of A and we write $C \subseteq A$.

- Example 1.1.6

Let $X = \{x \mid x^2 + x - 2 = 0\}$. Show that $X \subseteq Z$.

SOLUTION We must show that for every x, if $x \in X$, then $x \in Z$. If $x \in X$, then $x^2 + x - 2 = 0$. Solving for x, we obtain $x = 1$ or $x = -2$. In either case, $x \in Z$. Therefore, for every x, if $x \in X$, then $x \in Z$. We conclude that X is a subset of Z and we write $X \subseteq Z$.

SETS (Examples)

- EXAMPLE 1.1.7 Consider the sets:

$$A = \{1, 3, 4, 7, 8, 9\},\ B = \{1, 2, 3, 4, 5\},\ C = \{1, 3\}.$$

Then $C \subseteq A$ and $C \subseteq B$ since 1 and 3, the elements of C, are also members of A and B. But $B \nsubseteq A$ since some of the elements of B, e.g., 2 and 5, do not belong to A. Similarly, $A \nsubseteq B$.

Property 1: It is common practice in mathematics to put a vertical line "|" or slanted line "/" through a symbol to indicate the opposite or negative meaning of a symbol.

Property 2: The statement $A \subseteq B$ does not exclude the possibility that $A = B$. In fact, for every set A we have $A \subseteq A$ since, trivially, every element in A belongs to A. However, if $A \subseteq B$ and $A \neq B$, then we say A is a proper subset of B (sometimes written $A \subset B$).

Property 3: Suppose every element of a set A belongs to a set B and every element of B belongs to a set C. Then clearly every element of A also belongs to C. In other words, if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

The above remarks yield the following theorem.

Theorem 1.1: Let A, B, C be any sets. Then:

(i) $A \subseteq A$

(ii) (ii) If $A \subseteq B$ and $B \subseteq A$, then $A = B$

(iii) (iii) If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$

SETS

Special symbols

- Some sets will occur very often in the text, and so we use special symbols for them. Some such symbols are:
- N = the set of natural numbers or positive integers: 1, 2, 3,...
- Z = the set of all integers: ..., −2, −1, 0, 1, 2,...
- Q = the set of rational numbers
- R = the set of real numbers
- C = the set of complex numbers

Observe that $N \subseteq Z \subseteq Q \subseteq R \subseteq C$.

- Universal Set, Empty Set

All sets under investigation in any application of set theory are assumed to belong to some fixed large set called the universal set which we denote by **U**

Such a set with no elements is called the empty set or null set and is denoted by **∅**

There is only one empty set. That is, if S and T are both empty, then S = T , since they have exactly the same elements, namely, none.

The empty set ∅ is also regarded as a subset of every other set. Thus we have the following simple result which we state formally.

Theorem 1.2: For any set A, we have $\emptyset \subseteq A \subseteq U$.

Sets

Disjoint Sets

Two sets A and B are said to be disjoint if they have no elements in common. For example, suppose

$$A = \{1, 2\},\ B = \{4, 5, 6\},\ \text{and}\ C = \{5, 6, 7, 8\}$$

Then A and B are disjoint, and A and C are disjoint. But B and C are not disjoint since B and C have elements in common, e.g., 5 and 6. We note that if A and B are disjoint, then neither is a subset of the other (unless one is the empty set).