Learning Electrical Circuit Analysis - Solutions

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Networks - 20 Questions

A-LEVEL TASK 1. D has most (4).

A-LEVEL TASK 2. See figure.

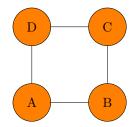


Figure 1.1: A Friends Network

A-LEVEL TASK 3. D is friends with 2 people, not 4.

A-LEVEL TASK 4. Towns

A-LEVEL TASK 5. 6 roads, some are two lane

A-LEVEL TASK 6. Roads

A-LEVEL TASK 7. 10 lanes

A-LEVEL TASK 8. Can not get to E from B

C-LEVEL TASK 1. Yes. EDBAC. Yes, if you start anywhere but E, you can't get to E.

D-LEVEL TASK 1. Can only have two nodes with odd number of roads (Konigsberg Bridge Problem).

A-LEVEL TASK 9. 28 miles

A-LEVEL TASK 10. E-town

A-LEVEL TASK 11. $200 \frac{cars}{min}$

C-LEVEL TASK 2. 150 entering from A, but 50 leaving to D, so net $+100 \frac{cars}{min}$

D-LEVEL TASK 2. Algorithm:

- Label all roads with undetermined flows with a variable.
- Write an equation for each node that the sum of flows into it sum to zero.
- Solve this simultaneous set of equations.

B-LEVEL TASK 1. 1 gall = 3.78 kg, so 37.8 kg/s.

B-LEVEL TASK 2. It would increase at 2 gal/s. The water would need to be stored inside J1 (maybe compressed?).

C-LEVEL TASK 3. $\frac{Vol}{sec} = \frac{Area*length}{s} = \frac{0.000628m^3}{s} \rightarrow 9.97gal/s$

A-LEVEL TASK 12. Neurons or little computers.

B-LEVEL TASK 3. 5*1+10*2=25>5 therefore out=1

C-LEVEL TASK 4. 5*C3+2*C2-5*C1=5-2-5=-1

C-LEVEL TASK 5. No way. Outflow will always be 1 or -1.

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B-LEVEL TASK 4. Table.

particle	charge (units of e^-)
electron	-1
proton	+1
top quark	$\frac{2}{3}$
muon	-1
neutron	0
photon	0
Sodium Ion	+1

Table 2.1: Summary of amount of charge that some particles have

A-LEVEL TASK 13. Coulombs

B-LEVEL TASK 5. $\frac{1}{1.6E-19} = 6.25E18$ electrons

A-LEVEL TASK 14. 5 C each second. 300 C each minute.

B-LEVEL TASK 6. 3.125E19 each second, 1.875E21 each minute.

B-LEVEL TASK 7. Table.

item	abbreviation	units	units abbreviation
Force	F	Newton	N
mass	m	kilogram	kg
charge	q	Coulomb	С
current	I	Amp	A
temperature	T	°Celsius	°C
Energy	E	Joule	J
Power	P	Watt	W

Table 2.2: Symbols and units

A-LEVEL TASK 15. $2.5\frac{m}{s}$

B-LEVEL TASK 8. Can't tell.

D-LEVEL TASK 3. Can't tell because we don't know the position at any time. Velocity can't determine position, only change in position.

B-LEVEL TASK 9. $\bar{a} = \frac{2.5-1}{4-0} = 0.375 \frac{m}{s^2}$

B-LEVEL TASK 10. θ

B-LEVEL TASK 11. Use tangent: $a = \frac{2.5-1}{4-2} = 0.75 \frac{m}{s^2}$

C-LEVEL TASK 6. Graph.

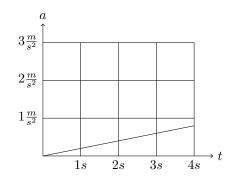


Figure 2.1: The car's acceleration as a function of time.

A-LEVEL TASK 16. 2.5C

B-LEVEL TASK 12. 0.375A

B-LEVEL TASK 13. 0.75A

C-LEVEL TASK 7. same as Figure 2.1, but with units of Amps

A-LEVEL TASK 17. $\frac{1}{s}$

C-LEVEL TASK 8. Given: i = 3q + 1

$$\frac{dq}{dt} = 3q + 1$$

$$\frac{dq}{3q + 1} = dt$$

$$\int_{5}^{Q} \frac{dq}{3q + 1} = \int_{0}^{t} dt$$

$$ln(3Q + 1) - ln(16) = 3t$$

$$Q = \frac{16e^{3t} - 1}{3}$$

C-LEVEL TASK 9. $i = 6e^{2t}$

C-LEVEL TASK 10. Table

item	relevant formula	rough value
stand up	$W = F^*d$	$(100kg)*(9.8\frac{m}{s^2})*.5m \approx 500J$
heat up cup of coffee	$Q = mc\Delta T$	0.5 * 4186 * 80 = 160000J
start running sideways	$KE = \frac{1}{2}mv^2$	50J
drag a log 20 feet	$W = \bar{F} * d$	500*6 = 3000J
create a 1kg of mass	$E = mc^2$	3E18 J
create one photon (λ =650 nm)	$E = \frac{hc}{\lambda}$	3.06E-19J
case A*	$W = \int F \bullet dx$	$W = \int_{4}^{2} \frac{-kq_eq_e}{x^2} \bullet dx = 5.76\text{e-}29\text{J}$
case B*	$W = \int F \bullet dx$	$W = \int_{2}^{4} \frac{G(1)(1)}{x^2} \bullet dx = 1.67 \text{E-}11 \text{J}$

Table 2.3: Approximate amount of energy needed to do several tasks.

B-LEVEL TASK 14. Energy can be transformed from one form to another, but not created or destroyed in a closed system.

A-LEVEL TASK 18. Trails

B-LEVEL TASK 15. Yes, their elevations above some reference point.

A-LEVEL TASK 19. θ

B-LEVEL TASK 16. Any loop starting and ending at the same spot with have a net height change of zero.

B-LEVEL TASK 17. $\Delta PE_{Comet}=24500J, \Delta PE_{Ajax}=39200J, \ but \ per \ mass \ both \ are \ 490 \ \frac{J}{kg}$

0

B-LEVEL TASK 18. Table

pts	Grav. Potential (V_G)
V_{AB}	$+294\frac{J}{kg}$
V_{AC}	$-490\frac{J}{kg}$
V_{CE}	$+490\frac{J}{kg}$
V_{CD}	0
V_{EC}	$-490\frac{J}{kg}$

Table 2.4: Values for gravitational potential energy per mass (gravitational potential)

C-LEVEL TASK 11. NO!

B-LEVEL TASK 19. $\frac{J}{ka}$

B-LEVEL TASK 20. 22.5J

A-LEVEL TASK 20. NO!

B-LEVEL TASK 21. $V = \frac{PE_{electrical}}{q}$

A-LEVEL TASK 21. No, just transforms it from one form to another.

A-LEVEL TASK 22. $\frac{Energy}{sec} = \frac{J}{s}$

B-LEVEL TASK 22. 7500J

C-LEVEL TASK 12. $\approx 67000 W$

A-LEVEL TASK 23. $\approx 746J$

B-LEVEL TASK 23. $39.2\frac{J}{kg}$ difference

B-LEVEL TASK 24. 1510 Watts

B-LEVEL TASK 25. $\frac{C}{s} * \frac{J}{C} = \frac{J}{s}$

C-LEVEL TASK 13. $\frac{60.96m*9.8*.37.8kg}{60s} = 376W$

C-LEVEL TASK 14. $\frac{0.436m*9.8*1000}{s} = 4270W$

B-LEVEL TASK 26. 1800 Joules each minute

A-LEVEL TASK 24. photon

B-LEVEL TASK 27. $\frac{300}{0.19}/1000 = 1.58m^2$

B-LEVEL TASK 28. 3240000J

B-LEVEL TASK 29. 6.25A

B-LEVEL TASK 30. about 373 panels

B-LEVEL TASK 31. Table.

which points	Voltage (Arrangement 1)	Voltage (Arrangement 2)
BC	48V	48V
СВ	-48	-48V
AB	0	0
BD	96V	48V
DE	-96V	-48V
DA	-96V	-48V
AD	+96V	+48V
FD	n/a	+48V

B-LEVEL TASK 32. *6.25A*

B-LEVEL TASK 33. Table.

which current	Current (Arrangement 1)	Current (Arrangement 2)
I1	-6.25A	-12.5A
I2	6.25A	-6.25A
I3	-6.25A	-12.5A

B-LEVEL TASK 34. First case: 96V, 6.25A, 600W. Second case: 48V, 12.5A, 600W.

C-LEVEL TASK 15. Two strings of 4 in parallel.

A-LEVEL TASK 25. 200J

B-LEVEL TASK 35. Because Watt is already a rate. Does this mean they install 500W each day?

B-LEVEL TASK 36. 36000000J

B-LEVEL TASK 37. 1.5kWhr

B-LEVEL TASK 38. about 30

B-LEVEL TASK 39. 234MJ, \$13 to charge

B-LEVEL TASK 40. 1200W, 54.2 hours

B-LEVEL TASK 41. about 20 minutes

B-LEVEL TASK 42. Table.

	network	what are nodes	what are edges	what flows
friends people		people	indications of friendship	n/a
	traffic	towns	roads	cars
	water	junctions	pipes	water
	electrical	places of equal electrical energy	wires, resistors	electrical charge, probably ele

Table 2.5: Network Summary Table

A-LEVEL TASK 26. $\frac{J}{C}$ or $\frac{Nm}{C}$

A-LEVEL TASK 27. $\frac{J}{kg}$ or $\frac{Nm}{kg}$

B-LEVEL TASK 43. $\vec{F} = 30N\hat{j}$ then $\vec{a} = 10\frac{m}{s^2}\hat{j}$

B-LEVEL TASK 44. $\vec{F} = -10N\hat{i} + 30N\hat{j}$ then $|\vec{a}| = 10.54 \frac{m}{s^2}$

B-LEVEL TASK 45. higher at (0,0) because field would pull mass to the right

C-LEVEL TASK 16. +15J

C-LEVEL TASK 17. $+1.5\frac{J}{kg}$

C-LEVEL TASK 18. +10V, higher at $(0,0)\frac{J}{kg}$

C-LEVEL TASK 19. Drawing requires a special component....

B-LEVEL TASK 46. Case 1 has person touching the case and the +wire touching case, but the breaker is already tripped.

A-LEVEL TASK 28. Light emitting diode

B-LEVEL TASK 47. A two-terminal device usually associated with assymetrical behavior, like a one-way valve.

A-LEVEL TASK 29. 500 or 600 Celsius

A-LEVEL TASK 30. Voltage

A-LEVEL TASK 31. Ohms

C-LEVEL TASK 20. $\Omega = \frac{Js}{C^2}$

C-LEVEL TASK 21. $\Omega = \frac{kg*m^2}{s*C^2}$

B-LEVEL TASK 48. hack, hack, always good (definition), hack

B-LEVEL TASK 49. Table

item	answer	hints
I_1	-3A	
V_{AB}	0	
V_{BC}	+15V	use Ohm's law
V_{AD}	+15V	
$I_{fromBtoC}$	3A	
Power absorbed by R	45W	
Power produced by current source	45W	

Table 2.6: Summary of select parameters for simple circuit.

B-LEVEL TASK 50. 12-gauge: $A=3.31mm^2,~R=\frac{1.68E-8*20}{3.31E-6}=0.101\Omega$ **B-LEVEL TASK 51.** $R=\frac{2.65E-8*.005}{1E-13}=1325\Omega,~V=132.5V,~P=13.25W$ **D-LEVEL TASK 4.**

$$\begin{split} mass &= \rho * Volume = 1.35E - 12kg \\ \Delta Q &= 13.25 * 60 = 795J \\ \Delta T &= \frac{Q}{mc} = \frac{795J}{1.35E - 12kg * 920\frac{J}{kgC}} = 6.4E11^{\circ}C \\ T_{melt} &= \frac{Q_{melt}}{P_{in}} = \frac{7.5E - 8J + 5.1E - 10J}{13.25W} \approx 6ns \end{split}$$

B-LEVEL TASK 52. $A=3.31E-6\ m^2,\ k=398\ W/mK,\ so\ Q=60s*0.044J/s=2.64\ J/min$

B-LEVEL TASK 53. $A=3.31E-6\ m^2,\ \rho=1.72E-8\ \Omega m,\ so\ Q=60s*3460\ A\ so\ Q=207,000\ C/min$

Basic Tools. 29 Questions

B-LEVEL TASK 54. 20 + 5 * 120 - 7 * 60 = 200Amps therefore out=1

D-LEVEL TASK 5. No, mass and energy can be converted back and forth. It should work for the cafeteria if food in stomachs and trash were considered

B-LEVEL TASK 55. options

- No, gasoline to exhaust.
- Yes.
- Yes, but only over time.
- Yes.

B-LEVEL TASK 56. $I_1 = 1A, I_5 = 0.5A, V_{AD} = 2.5V$

C-LEVEL TASK 22. 12.5V

C-LEVEL TASK 23. Table

component	current through it	voltage across it	power consumed
2A source	+2A	-13V (V, I in opp. dirs.)	-26Watts
3A source	3A	12.5V	-37.5W
10 Ohm (E-F)	+2A	+20V	+40W
5 Ohm (B-C)	+1A	+5V	+5W
10 Ohm (G-D)	0.7A	+7V	+4.9W
1 Ohm (A-D)	2.5A	-2.5V	+6.25W
5 Ohm (A-D)	0.5A	-2.5V	+1.25W
5V source	2.3A	5V	+11.5W
Gator Top	2A	2V	-4W
Gator Side	.7A	2V	-1.4W

Table 3.1: Table to check conservation of energy.

C-LEVEL TASK 24. $P_{Total} = 68.9W - 68.9W = 0$

C-LEVEL TASK 25. Table

component	current through it	voltage across it	power consumed
2A source	+2A	-8V (V, I in opp. dirs.)	-16Watts
3A source	3A	30V	-90W
10 Ohm (E-F)	+2A	+20V	+40W
5 Ohm (B-C)	+1A	+10V	+10W
10 Ohm (G-D)	0.7A	+7V	+4.9W
1 Ohm (A-D)	1.5A	-15V	+22.5W
5 Ohm (A-D)	1.5A	-15V	+22.5W
5V source	2.3A	5V	+11.5W
Gator Top	2A	2V	-4W
Gator Side	.7A	2V	-1.4W

Table 3.2: Table to check conservation of energy. 111.4W=111.4W

C-LEVEL TASK 26. $V_T = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3) \rightarrow V_T = IR_T$ therefore $R_T = R_1 + R_2 + R_3$

C-LEVEL TASK 27. In parallel: $I_T = \frac{V}{R_1} + \frac{V}{R_2} = V(\frac{1}{R_1} + \frac{1}{R_2}) \rightarrow I_T = \frac{V}{R_T}$ therefore $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$

C-LEVEL TASK 28. rearrange...

B-LEVEL TASK 57. $\frac{\Omega^3}{\Omega} \neq \Omega$

B-LEVEL TASK 58. resistance will go Down

D-LEVEL TASK 6. $R_T = \frac{S_N}{S_{N-1}}$

C-LEVEL TASK 29. Table

components	series, parallel or neither
X and W	parallel
Z and W	neither
Y and M	neither
N and M	series
Z and Y	parallel
X and M	neither

Table 3.3: Series or parallel?

C-LEVEL TASK 30. $30~\Omega$

B-LEVEL TASK 59. $5 + 5 \parallel 5$

A-LEVEL TASK 32. harder

C-LEVEL TASK 31.

$$R_{T} = R + R \parallel R_{T} = R + \frac{RR_{T}}{R + R_{T}}$$

$$R_{T}^{2} + R_{T}R = R_{T}R + R^{2} + RR_{T}$$

$$R_{T}^{2} - RR_{T} - R^{2} = 0$$

$$R_{T} = R(\frac{1 + \sqrt{5}}{2})$$

B-LEVEL TASK 60. 12.875 Distrusters

C-LEVEL TASK 32. 3.6 Distrusters

C-LEVEL TASK 33. Current sources would add in parallel, but different ideal current sources would be a catastrophe in series.

B-LEVEL TASK 61. It is made of copper because of the resistivity value.

B-LEVEL TASK 62.
$$R = \frac{15.46\Omega}{3} = 5.15\Omega \rightarrow R_T = 5.64\Omega$$
 $V_{SAW} = 115 - 2 * .246 * \frac{115}{5.64} = 104.96V$

C-LEVEL TASK 34. $\frac{R}{N}$

C-LEVEL TASK 35.
$$R_{SAW} = \frac{115^2}{P}, R_{cord} = L * .00807 \frac{\Omega}{m}$$
 $R = \frac{\frac{115^2}{P}}{N} \Omega \rightarrow R_T = \frac{115^2}{PN} + 2 * L * .00807 \Omega$ $V_{SAW} = 115 - 2 * L * .00807 * \frac{115}{\frac{115^2}{PN} + 2 * L * .00807}$ Should be simplified or use voltage division (next section) to get simpler result.

A-LEVEL TASK 33. $V_{R1} = 6V, V_{R2} = 12V$

B-LEVEL TASK 63.
$$V_{AB} + V_{BC} = 18V$$

$$V_{AB} + V_{BA} = 0$$
$$V_{AB} + V_{CB} = -6V$$

D-LEVEL TASK 7.
$$V_i = \frac{R_i}{R_i + R_{remaining}} = ...$$

B-LEVEL TASK 64. 1.47V

C-LEVEL TASK 36. 3.68V

C-LEVEL TASK 37. 2.21 V

Powerful Tools

A-LEVEL TASK 34. N=3 equations, M=2 unknowns

B-LEVEL TASK 65. infinite solutions

C-LEVEL TASK 38.
$$x=a, y=10-a \text{ or } \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix} + a \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

D-LEVEL TASK 8. form a matrix and check its determinant. If it equals 0, then they are not independent.

B-LEVEL TASK 66.
$$\begin{bmatrix} 0 \\ 30 \end{bmatrix}$$

B-LEVEL TASK 67.
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

B-LEVEL TASK 68.
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

B-LEVEL TASK 69. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$, yes, because that is what an inverse does..it undoes multiplying by M

B-LEVEL TASK 70.

$$= M^{-1}M^{-1}M^{-1}M^{-1}MMMMM$$

$$= M^{-1}M^{-1}M^{-1}(M^{-1}M)MMMM$$

$$= M^{-1}M^{-1}M^{-1}(I)MMMM$$

$$= M^{-1}M^{-1}M^{-1}MMMM \rightarrow= M$$

B-LEVEL TASK 71.
$$\begin{bmatrix} -6 & 7 \\ -8 & -6 \end{bmatrix} = I$$

C-LEVEL TASK 39. MxN can only be multiplied by RxC is N=R. The result will be a MxC matrix.

B-LEVEL TASK 72.
$$\begin{bmatrix} -5 & -3.5 & -2.5 \\ -3.5 & -4.55 & -3.25 \\ -2.5 & -3.25 & -3.75 \end{bmatrix}$$
C-LEVEL TASK 40.
$$\begin{bmatrix} 3V \\ 12.9V \\ 13.5V \end{bmatrix}$$

C-LEVEL TASK 40.
$$\begin{bmatrix} 3V \\ 12.9V \\ 13.5V \end{bmatrix}$$

B-LEVEL TASK 73. $I = \frac{C-B}{3} = \frac{9.9}{3} = 3.3A, P=32.7W$

 $\textbf{C-LEVEL TASK 41.} \ \textit{Table}$

component	I	ΔV	power absorbed (- if produced)
3 Ω	3.3A	9.9V	+32.67W
2 Ω	3A	6V	+0.18W
10 Ω	0.3A	3V	+0.9W
5 Ω	2.7A	13.5V	+36.45W
3A left source	3A	12.9V	-38.7W
3A center source	3A	10.5W	-31.5W

Table 4.1: Power check: -70.2W=70.2W

C-LEVEL TASK 42.
$$\begin{bmatrix} 0\\30V\\30V \end{bmatrix}$$

C-LEVEL TASK 43.
$$\begin{bmatrix} 14.7 \\ 14.07V \\ 7.67V \\ -1.67A \end{bmatrix}$$

A-LEVEL TASK 35. Easier to handle current sources.

C-LEVEL TASK 44. Equations

$$\frac{C-B}{3} + \frac{0-B}{10} - I_1 = 0 \tag{Node B}$$

$$I_2 + \frac{B-C}{3} + \frac{D-C}{2} = 0$$
 (Node C)

$$\frac{C-D}{2} + I_1 + \frac{0-D}{5} = 0$$
 (Node D)

B = D + 7(Bonus Equation due to 7V Voltage Source) C = 3(Bonus Equation due to 3V Voltage Source)

$$Answers: \begin{bmatrix} B = 6.53V \\ C = 3V \\ D = -.47V \\ I_1 = -1.83A \\ I_2 = 0.559A \end{bmatrix}$$

C-LEVEL TASK 45. 3+(9-1)=11 equations or rows

C-LEVEL TASK 46. 3+(25-1)=27 equations or rows

A-LEVEL TASK 36. 5 rectangles

B-LEVEL TASK 74. 7 polygons

C-LEVEL TASK 47. 3 primitive polygons

B-LEVEL TASK 75.
$$\begin{bmatrix} -0.22 & -0.13 & -.15 \\ -0.13 & -0.28 & -0.088 \\ -.15 & -0.088 & -0.165 \end{bmatrix}$$

C-LEVEL TASK 48.
$$\begin{bmatrix} I_1 = 0.56A \\ I_2 = 1.74A \\ I_3 = -.094A \end{bmatrix}$$

$$\begin{bmatrix} I_3 = -.094A \end{bmatrix}$$
 C-LEVEL TASK 49. when switched.. $\begin{bmatrix} I_1 = 0.76A \\ I_2 = -.94A \\ I_3 = 0.98A \end{bmatrix}$

D-LEVEL TASK 9. The 3V source is on the outer edge of the graph and therefore is in one loop, the 7V source is on an inside edge and is shared by two loops. For a planar network (assumption), it can only appear twice.

D-LEVEL TASK 10. Equations would no longer be independent.

C-LEVEL TASK 50.

First Loop (A-C-B-A):
$$+3 - 3(I_1 - I_2) - 10(I_1 - I_3) = 0$$

Second Loop (B-C-D-B): $-3(I_2 - I_1) - 2I_2 + V_x = 0$
Third Loop (A-B-D-A): $-10(I_3 - I_1) - V_x - 5I_3 = 0$
Current Source Bonus Eq: $I_3 - I_2 = 3A$

In matrix form:

$$\begin{bmatrix} (-3-10) & 3 & 10 & 0 \\ 3 & (-3-2) & 0 & 1 \\ 10 & 0 & (-10-5) & -1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_x \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 0 \\ 3 \end{bmatrix}$$
(4.0.1)

Solution (Vx could have either sign):
$$\begin{bmatrix} 0.82A \\ -1.71A \\ 1.29A \\ -11.04V \end{bmatrix}$$

C-LEVEL TASK 51. Loop analysis would be easier with more voltage sources.

C-LEVEL TASK 52. 5+1=6

B-LEVEL TASK 76. $B = \frac{\mu_o I}{2\pi r} = 3.33E - 5T$

C-LEVEL TASK 53. $P = IV = \frac{V^2}{R_1} \rightarrow lim_{R_1 \rightarrow 0} P = \infty$

B-LEVEL TASK 77. $I = \frac{V}{R_1}$

B-LEVEL TASK 78. 7.89V

B-LEVEL TASK 79. $V_1 = \frac{R_1}{R_{in} + R_1} V$

C-LEVEL TASK 54. $R_1 = R_{in}$

A-LEVEL TASK 37. $I = \frac{5}{1+R}$

C-LEVEL TASK 55. $I = \frac{5}{1+R}$

A-LEVEL TASK 38. Figure

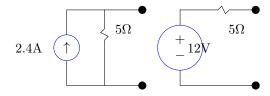


Figure 4.1: Both versions

B-LEVEL TASK 80. Table

Thévenin version	Norton Version
$W_V = (30V, 10\Omega)$	$W_N = (3A, 10\Omega)$
$W_V = (3V, 10\Omega)$	$W_N = (0.3A, 10\Omega)$
$W_V = (12V, 5\Omega)$	$W_N = (2.4A, 5\Omega)$

B-LEVEL TASK 81. $(W + Y)_T = (1V, 12\Omega)$

C-LEVEL TASK 56. $(W + Y)_T = (28V, 12\Omega)$

C-LEVEL TASK 57. $I_5 = 0.79A$ note that 7V is now backwards

A-LEVEL TASK 39. \$120 per hour

B-LEVEL TASK 82. 20.5V

C-LEVEL TASK 58. 58V

A-LEVEL TASK 40. \$180,\$180

B-LEVEL TASK 83. 5.34V

C-LEVEL TASK 59.

$$V_{out} = \underbrace{5V}_{7V} + \underbrace{-4.28V}_{3A} = 0.72V$$

B-LEVEL TASK 84. 161.5N

C-LEVEL TASK 60. $\frac{16}{5} * 24.5 + \frac{25}{10} * 68.5 = 249.7N$

C-LEVEL TASK 61. Let $R_x=5\parallel(2\parallel10+3)=2.414\Omega$ $V_2=(\frac{2\parallel10}{2\parallel10+3})(\frac{R_x}{R_x+7})5=0.458V$ So $I=\frac{0.458V}{2}=0.229A$

C-LEVEL TASK 62.
$$V_2=(\frac{2\|10}{2\|10+3+7\|5})5=1.538V$$
 So $I=\frac{1.538V}{2}=0.769A$

Dependent Sources

C-LEVEL TASK 63. Table:

item	with dependent source	if 10Ω were connected directly to AB
V_1	6V	0.0887V
$V_{10\Omega}$	6V	0.0887V
$I_{10\Omega}$	0.6A	0.00887A
$P_{10\Omega}$	3.6W	0.000786W

Table 5.1: Comparison between circuit with and without dependant source.

A-LEVEL TASK 41. vccs

B-LEVEL TASK 85. amps/volt

C-LEVEL TASK 64. $V_1 = 0$

B-LEVEL TASK 86. Table:

in+	in-	V_{out}
+5V	+7V	$-\infty$
-5V	+3V	$-\infty$
-5V	-5.1V	∞
0V	0.00001V	$-\infty$

Table 5.2: Op-amp output voltages

C-LEVEL TASK 65. Just compare the two input voltages; if in+ exceeds inthen the output is ∞ otherwise it is $-\infty$ V.

B-LEVEL TASK 87. Table:

in+	in-	V++	V-	V_{out}
+5V	+7V	10V	-10V	-10V
-5V	+3V	10V	-10V	-10V
-5V	-5.1V	10V	-10V	+10V
0V	0.00001V	10V	-10V	-10V
+5V	+7V	5V	-3V	-3V
-5V	+3V	5V	-3V	-3V
-5V	-5.1V	5V	-3V	5V
0V	0.00001V	5V	-3V	-3V

Table 5.3: Op-amp output values taking with known supply voltages

C-LEVEL TASK 66. Just compare the two input voltages; if in+ exceeds inthen the output is V++ otherwise it is V++.

A-LEVEL TASK 42. yes

A-LEVEL TASK 43. ON

B-LEVEL TASK 88. Will stabilize at T_A

A-LEVEL TASK 44. ON

B-LEVEL TASK 89. $3T_A$

A-LEVEL TASK 45. increase

B-LEVEL TASK 90. +5V

B-LEVEL TASK 91. increase, because half of 4 is less than 3

B-LEVEL TASK 92. 8V

C-LEVEL TASK 67. $V_{out} = \frac{R_1 + R_2}{R_1} V_{in}$, answers might have R1 and R2 interchanged

Time Dependent Circuits

A-LEVEL TASK 46. alternating current, direct current

A-LEVEL TASK 47. Henry

C-LEVEL TASK 68. $I = \frac{1}{L} \int V_L dt$

B-LEVEL TASK 93. $V_L = 30t$

C-LEVEL TASK 69. $V_C = t^3 + 5t + 2$

A-LEVEL TASK 48. 0.15N

A-LEVEL TASK 49. Farads

B-LEVEL TASK 94. 50 F

B-LEVEL TASK 95. 0.5V, 2.5V, easier to push charge onto a larger capacitor

C-LEVEL TASK 70. 1E11 N

C-LEVEL TASK 71. V(t=2s)=1.2V, V(t=4s)=2.4V, V(t=6s)=3.6V

B-LEVEL TASK 96. $\mathcal{E}_{br-air} = 3e6\frac{V}{m}$, so for a 1 mm gap $\rightarrow 3000V$

A-LEVEL TASK 50. Yes. Yes.

A-LEVEL TASK 51. No.

A-LEVEL TASK 52. Yes.

C-LEVEL TASK 72. Volts. $\frac{1}{s}$

C-LEVEL TASK 73. $I = 15iCe^{3it}$

A-LEVEL TASK 53. About 1E8 J

B-LEVEL TASK 97. *3000V*

B-LEVEL TASK 98. $1E8 = \frac{1}{2}C(3000)^2 \rightarrow C = 22F$

B-LEVEL TASK 99. $22 = \frac{A\epsilon}{.001} \rightarrow A = 2.5E9m^2$. No, way too big.

D-LEVEL TASK 11. See steps in textbook.

A-LEVEL TASK 54. +15C, -15C

B-LEVEL TASK 100. infinite amps

C-LEVEL TASK 74. 37.5J

A-LEVEL TASK 55. E: Volts, 5: Volts, 2: Ohms

A-LEVEL TASK 56. 5: Volts, Ei: Volts, 6: seconds

B-LEVEL TASK 101. 5 Volts

B-LEVEL TASK 102. 13.8 s

C-LEVEL TASK 75. Table:

ſ	time	V_C	V_R	I_R	extra charge delivered to C during next 0.01 s	rise in V_C during no
ſ	2s	1.42V	3.58V	1.79A	0.018C	0.00597V
ſ	20s	4.82V	0.18V	0.089A	0.00089C	0.000297V

Table 6.1: Comparison of charging rate for a capacitor

C-LEVEL TASK 76. $V = V - (V - E_i)e^{-\frac{t}{RC}}$

B-LEVEL TASK 103.
$$\vec{z} = \begin{vmatrix} 0 \\ 5 \end{vmatrix}$$
 or $\vec{z} = \begin{vmatrix} 1 \\ 2 \end{vmatrix}$ or $\vec{z} = \begin{vmatrix} 2 \\ -1 \end{vmatrix}$

A-LEVEL TASK 57. 11

A-LEVEL TASK 58. 0,10

C-LEVEL TASK 77.
$$\vec{z} = a \begin{vmatrix} 2 \\ 5 \end{vmatrix} + \begin{vmatrix} 4 \\ 0 \end{vmatrix}$$

D-LEVEL TASK 12. No, but it will capture the same space. Not necessarily same z_p .

A-LEVEL TASK 59. 1 by 2 (r by c)

C-LEVEL TASK 78. 0,10

B-LEVEL TASK 104. Goes to zero. Stays at 5V.

C-LEVEL TASK 79.
$$I_H = Ae^{-\frac{3}{2}t}, I_P = \frac{10}{3}$$

B-LEVEL TASK 105. 1. 3dx + 3dy = 0 YES

2.
$$3dx + 2dy = 0$$
 YES

3.
$$3xdx + 3ydy = 0$$
 YES

4.
$$3ydx + 3xdy = 0$$
 YES

5.
$$3x^2dx + 3y^2dy = 0$$
 YES

6.
$$3y^2dx + 3x^2dy = 0$$
 NO

7.
$$(y-5)dx + 3dy = 0$$
 NO

8.
$$k_1 dT + \frac{k_2}{V} dV = 0$$
 YES

B-LEVEL TASK 106. $\frac{d}{dE}(E-5) \neq \frac{d}{dt}6$

B-LEVEL TASK 107.
$$\frac{d}{dE}(e^{\frac{t}{6}}(E-5)) = e^{\frac{t}{6}}$$
 $\frac{d}{dt}(e^{\frac{t}{6}}*6) = e^{\frac{t}{6}}$

C-LEVEL TASK 80. $\mu = e^{\frac{3}{2}t}, I = \frac{k}{3}e^{-\frac{3}{2}t} + \frac{10}{3}$

C-LEVEL TASK 81. $V_C = 5 - 5e^{-\frac{t}{100}}$

C-LEVEL TASK 82. 0.47V, op-amp terminals reversed, resistor bigger, Vx set to 0.47V

A-LEVEL TASK 60. meters

B-LEVEL TASK 108. plug in..4.06m

C-LEVEL TASK 83. Mechanical: Force of 2N, k=1, drag=3. Electrical: C=1, V=2, R=3.

D-LEVEL TASK 13. Mechanical: Force of 2N, k=1, drag=3, start at -2m. Electrical: C=1, V=2, R=3, Vinitial=-2V.

A-LEVEL TASK 61. Tesla

B-LEVEL TASK 109. $5x10^{-5}$ Tesla

C-LEVEL TASK 84. $2x10^{-4}$ Tesla

A-LEVEL TASK 62. 0 Volts

B-LEVEL TASK 110. 5 Volts

B-LEVEL TASK 111. $I_C \rightarrow 0, I_P \rightarrow \frac{5}{R}$

B-LEVEL TASK 112. perfect wire

B-LEVEL TASK 113. s

B-LEVEL TASK 114. 20.333, 0

B-LEVEL TASK 115. 20 m/s (no change)

B-LEVEL TASK 116. $V_L = 500000V$ so $\frac{dI}{dt} = \frac{500000}{L} \frac{Amps}{s}$

C-LEVEL TASK 85.

$$V - 1000010I - L\frac{dI}{dt} = 0 KVL$$

$$I_L = I_C + I_P = Ae^{-\frac{1000010}{L}t} + \frac{V}{100010}$$

$$V_L = L\frac{dI}{dt} = 500000e^{-\frac{1000010}{L}t}$$

C-LEVEL TASK 86. 26 microseconds

C-LEVEL TASK 87. Table:

object	$I(t=0_{-})$	$V(t=0_{-})$	$I(t=0_+)$	$V(t=0_+)$
5V source	0	5V	0	5V
switch	0	5V	0	0V
R	0	0	0	0
L	0	0	0	5V

Table 6.2: Initial condition table. Switch is initially open, then closes at t=0s.

A-LEVEL TASK 63. $u(3)=1, u(-1)=0, u(5)=1, u(\pi)=1$

B-LEVEL TASK 117. shifted right 2, flipped horizontally, stretched up 3 and shifted left 1

C-LEVEL TASK 88. sin function that starts at 0, unit pulse

D-LEVEL TASK 14. ramp, delta function

Second Order Circuits

B-LEVEL TASK 118. $e^x(5e^{-2y} - 4e^{2y})$

A-LEVEL TASK 64. $x = -\frac{5}{2} \pm \frac{\sqrt{29}}{2}$

B-LEVEL TASK 119.

$$x^{2} + 6x - 2 = x^{2} + 6x + 9 - 11 = 0$$
$$(x+3)^{2} = 11$$
$$x = \pm \sqrt{11} - 3$$

C-LEVEL TASK 89.

$$x^{2} + bx + c = x^{2} + bx + (\frac{b}{2})^{2} - (\frac{b}{2})^{2} + c = 0$$
$$(x + \frac{b}{2})^{2} = (\frac{b}{2})^{2} - c$$
$$x = \pm \sqrt{(\frac{b}{2})^{2} - c} - \frac{b}{2}$$

B-LEVEL TASK 120. Not A, Yes B, Not C

C-LEVEL TASK 90. Table:

object	$I(t=0_{-})$	$V(t=0_{-})$	$I(t=0_+)$	$V(t=0_+)$
5V source	0	5V	0	5V
switch	0	5V	0	0
R	0	0	0	0
L	0	0	0	5V
С	0	0	0	0

Table 7.1: Initial condition table.

B-LEVEL TASK 121. $\alpha \to \frac{1}{s}, \omega \to \frac{1}{s}$ or $\omega \to \frac{rad}{s}$

A-LEVEL TASK 65. True

A-LEVEL TASK 66. False

B-LEVEL TASK 122. Yes

B-LEVEL TASK 123. No. 1/2

B-LEVEL TASK 124. Yes.

B-LEVEL TASK 125. No. 1-2

C-LEVEL TASK 91. Even Numbers: Subtraction: Yes. Addition: Yes. Multiplication: Yes. Division: No.

C-LEVEL TASK 92. Rational Numbers: Subtraction: Yes. Addition: Yes. Multiplication: Yes. Division: Yes.

C-LEVEL TASK 93. Rational Numbers: Integer Exponents: Yes. Rational Exponents: No, $2^{\frac{1}{2}}$.

C-LEVEL TASK 94. Real Numbers: Integer Exponents: Yes. Rational Exponents: No, $(-1)^{\frac{1}{2}}$.

A-LEVEL TASK 67. 3-2i,-1+8i,17+i

A-LEVEL TASK 68. 3-5i

B-LEVEL TASK 126. $-1,1,i,i^502 = -1$

C-LEVEL TASK 95. -1

C-LEVEL TASK 96. $A*A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. The A matrix is acting like i.

A-LEVEL TASK 69. $\sqrt{10}$

B-LEVEL TASK 127. Table

rectangular form	polar form
1+5i	$5.1e^{i79^o}$
-2+i	$3e^{i30^{o}}$
2.6-1.5i	$3e^{-i30^{o}}$
2-5i	$5.4e^{-i68^o}$
0.34-1.97	$-2e^{i100^o}$
i	e^{i90^o}
-i	$e^{-i90^{o}}$

Table 7.2: Convert rectangular and polar forms.

C-LEVEL TASK 97.

$$sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$$

C-LEVEL TASK 98.

$$(\frac{e^{ix}-e^{-ix}}{2i})^2 + (\frac{e^{ix}+e^{-ix}}{2})^2 = ?$$

$$\frac{e^{2ix}+e^{-2ix}-2}{-4} + \frac{e^{2ix}+e^{-2ix}+2}{4} = ?$$

$$\frac{1}{2} + \frac{1}{2} = 1$$

B-LEVEL TASK 128. i

B-LEVEL TASK 129. $i^i = e^{i\frac{\pi}{2}i} = e^{-\frac{\pi}{2}} = 0.208$

C-LEVEL TASK 99. Multiplying by i adds 90 degrees to the angle.

A-LEVEL TASK 70. -2.1 Amps

B-LEVEL TASK 130. Amps

B-LEVEL TASK 131. The decay rate would be more gradual. The graph would oscillate more rapidly.

C-LEVEL TASK 100. Alternate method:

$$V_C = 5 - iR - V_L$$

$$V_C = 5 - iR - L\frac{dI}{dt}$$

$$V_C = 5 - i1 - 1\frac{dI}{dt}$$

$$V_C = 5 + 5.77 * e^{-\frac{1}{2}t} (sin(0.866t)) - 2.885 * e^{-\frac{1}{2}t} (sin(0.866t)) + 4.997 * e^{-\frac{1}{2}t} (cos(0.866t))$$
$$V_C = 5 + e^{-\frac{1}{2}t} (2.885 * sin(0.866t) + 4.997 * cos(0.866t))$$

A-LEVEL TASK 71. 4s

C-LEVEL TASK 101. 5.82 V

A-LEVEL TASK 72. $\frac{rad}{s}$

D-LEVEL TASK 15. Solve $V_C = 4.5$. Around 4.71 seconds.

C-LEVEL TASK 102. R changed to 0.5 instead of 1 Ohm.

$$I_L = 5.165e^{-\frac{1}{4}t}(sin(0.968t))$$

$$V_C = 5 - 1.29e^{-\frac{1}{4}t}sin(0.968t) - 5e^{-\frac{1}{4}t}cos(0.968t)$$

C-LEVEL TASK 103. $m=-\frac{3}{2}\pm\frac{\sqrt{5}}{2}=-.382,-2.618$

C-LEVEL TASK 104. $I_L = 2.236e^{-0.382t} - 2.236e^{-2.618t}$

C-LEVEL TASK 105. $R^2 = \frac{4L}{C} \rightarrow R = 2\Omega$

A-LEVEL TASK 73. $\frac{Ns}{m}$

B-LEVEL TASK 132. RLC Circuit with L=7H, R=3 Ohms and $C=\frac{1}{2}F$

C-LEVEL TASK 106. $m = -\frac{3}{14} \pm i \frac{\sqrt{47}}{14}$ Imaginary means it will oscillate.

C-LEVEL TASK 107. $x(t) = 2.5 - 1.092e^{-0.214t}sin(0.49t) - 2.5e^{-.214t}cos(0.49t)$

D-LEVEL TASK 16. spring k=1 N/m, mass=1 kg, drag b=1 Ns/m and force of N

C-LEVEL TASK 108. Table

object	$I(t=0_{-})$	$V(t=0_{-})$	$I(t=0_+)$	$V(t=0_+)$
5V source	0	10	5	10
switch	0	10	5	0
R_1	0	0	0	0
R_2	0	0	5	10
L	0	0	0	10
С	0	0	5	0

Table 7.3: Initial condition table.

B-LEVEL TASK 133. $\frac{10-V_{out}}{R1} + C\frac{d(10-V_{out})}{dt} + \frac{1}{L}\int{(0-V_{out})dt} + \frac{0-V_{out}}{R_2} = 0$

 $\textbf{C-LEVEL TASK 109.} \ \ V_{out} = e^{-0.1875t} (3.42 sin(0.363t) + 10 cos(0.363t))$

B-LEVEL TASK 134. yes, it does oscillate

B-LEVEL TASK 135. θ

AC Circuits

A-LEVEL TASK 74. $\frac{1}{12} = 0.083 \ Hz$

B-LEVEL TASK 136. $f = \frac{1}{12*60*60} = 2.3E-5 Hz$

B-LEVEL TASK 137. f=8~Hz

A-LEVEL TASK 75. 2π

B-LEVEL TASK 138. $T = \frac{1}{30}s$, $\omega = 60\pi \frac{rad}{s}$

A-LEVEL TASK 76. red comes first

A-LEVEL TASK 77. $V_1 = 2cos(2t), V_2 = 3cos(2t - 45^o),$ answers may vary so long as they are 45 degrees out of phase

A-LEVEL TASK 78. Period π s, Amplitude=2A

B-LEVEL TASK 139. Table:

source	amplitude	period	angular frequency	frequency	units of bolded value
$V=3\cos(5t)$	3	$\frac{2\pi}{5}$	5	$\frac{5}{2\pi}$	Volts
I=5cos(3 t)	5	$\frac{2\pi}{3}$	3	$\frac{3}{2\pi}$	$\frac{rads}{s}$
$I=5\cos(3(t+1.2))$	5	$\frac{2\pi}{3}$	3	$\frac{3}{2\pi}$	S
$I=5\cos(3t+1.2)$	5	$\frac{2\pi}{3}$	3	$\frac{3}{2\pi}$	radians

Table 8.1: Practice with AC source terminology

A-LEVEL TASK 79. θ , θ

B-LEVEL TASK 140. 0.33333 Hz

B-LEVEL TASK 141. rms=1.22 A

A-LEVEL TASK 80. rms=5.32 V

B-LEVEL TASK 142. $rms = \sqrt{\frac{0+.5+1+.5+0+.5+1+.5}{8}} = \frac{1}{\sqrt{2}} = 0.707$

D-LEVEL TASK 17. $rms = \frac{1}{2\pi} \int_0^{2\pi} sin^2(t) dt = \frac{1}{\sqrt{2}}$

B-LEVEL TASK 143. surface roughness = 0.0148 mm

B-LEVEL TASK 144. wall outlet A=170V, $T=\frac{1}{60}=0.0167s$

C-LEVEL TASK 110.

$$5cos(10t) + 7sin(10t + 55^{o}) = 10.733 - 4.015i$$
$$= 11.46cos(10t - 20.5^{o})Volts$$

C-LEVEL TASK 111. multiply matrices to get identity. Part b:

$$\frac{1}{a^2+1} \begin{vmatrix} -a & 1\\ 1 & a \end{vmatrix}$$

A-LEVEL TASK 81. Volts

A-LEVEL TASK 82. Z_P is the steady-state solution

D-LEVEL TASK 18. m must not be negative in order for Z_P to go to zero.

A-LEVEL TASK 83. Volts

C-LEVEL TASK 112. The cos term goes away faster because it's coeff goes as $\frac{1}{\omega^2}$.

$$Z(\tilde{highf}) = \frac{A}{RC\omega} sin(\omega t)$$

A-LEVEL TASK 84. Ohms, Ohms, Ohms

C-LEVEL TASK 113. Table:

component	Z formula	Z at high freq (high or low)	Z at low freq
resistor	R	R	R
capacitor	$\frac{1}{Ci\omega}$	Low	High
inductor	$Li\omega$	High	Low

A-LEVEL TASK 85. $Z \rightarrow -\frac{i}{36}\Omega$

A-LEVEL TASK 86. 8i Ohms

B-LEVEL TASK 145.

Node B:
$$\frac{3e^{4it} - B}{-\frac{i}{12}} - I_1 + \frac{0 - B}{10} = 0$$

Node D: $\frac{3e^{4it} - D}{8i} + I_1 + \frac{0 - D}{5} = 0$
Bonus: $D = B + 2e^{4it}$

C-LEVEL TASK 114. $2.02 + 1.09i = 2.295cos(4t + 28^{\circ})$

C-LEVEL TASK 115. $\pi E - 4T$

A-LEVEL TASK 87. 170 V

B-LEVEL TASK 146. 120V, f=0.78 Hz V

A-LEVEL TASK 88. I(t = 1s) = 3.57 Amps - watch degrees to radians

A-LEVEL TASK 89. peak voltage comes first

B-LEVEL TASK 147. $V_L = -0.1*5*82.5sin(5t-14^o) = -41.25sin(5t-14^o)$ Volts, rms=29.2V

B-LEVEL TASK 148. 90° , yes, always the case

B-LEVEL TASK 149. $V_R = 165cos(5t - 14^o)$ Volts, rms=117V

C-LEVEL TASK 116. show by adding phasors, or trig

A-LEVEL TASK 90. no, rms voltages do not add to the total voltage

C-LEVEL TASK 117. L=0.4 H for phase diff or 45 degree

A-LEVEL TASK 91. 7.96 Hz

C-LEVEL TASK 118. $Z_{parallel} = \infty$

C-LEVEL TASK 119. Table:

component	peak voltage	rms voltage
source		
resistor		
inductor		
capacitor		

Table 8.2: Table summarizing rms and peak values.