$$\boxed{\int u\,dv = uv - \int v\,du}$$

We want to integrate $\int \sin^m(x)\cos^n(x)\,dx$.

Case 1: If both m & n are even, use the half-angle identities to reduce:

$$\boxed{\begin{aligned} \sin^2 x &= \tfrac{1}{2}\left(1-\cos(2x)\right) \\ \cos^2(x) &= \tfrac{1}{2}\left(1+\cos(2x)\right) \end{aligned}}$$

Case 2: If only one of m/n is odd and the other is even, use a substitution: u = base of even power and trig identities to get a polynomial.

Case 3: Both m & n are odd, let u = base of the higher power and proceed as in case 2.

Case 4: If one power isn't an integer, try letting u = base of non-integer power and try to cancel stuff out.

To integrate $\int \tan^m x \sec^n x \, dx$:

<u>Case 1</u>: If the power of secant is <u>even</u> (and tan's is anything) let $u = \tan x$ and use $\sec^2 x = 1 + \tan^2 x$

<u>Case 2</u>: If the power of tan is <u>odd</u> (and sec is anything) let $u = \sec x$ and use $\tan^2 x = \sec^2 x - 1$

<u>Case 3</u>: If $m = \text{even}$ & $n = \text{odd}$, use $\tan^2 x = \sec^2 x - 1$ to write the whole integral as powers of secant and evaluate those instead using the reduction formula (on assign #1).

1) $\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

2) $\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

3) $\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$.

§7.5 - Trigonometric Substitution

Sometimes, changing x into a trig function can make certain integrals easier.

There are 3 ~~cases~~ situations where this is useful:

1) $\sqrt{a^2-x^2} \to x = a\sin\theta \quad (-\pi/2 < \theta < \pi/2)$

2) $\sqrt{a^2+x^2} \to x = a\tan\theta \quad (-\pi/2 < \theta < \pi/2)$

3) $\sqrt{x^2-a^2} \to x = a\sec\theta \quad (0 \leq \theta < \pi/2 \text{ or } \pi \leq \theta < 3\pi/2)$

If the denominator has...	Then we write....
1) Distinct Linear factors $(a_1x+b_1)(a_2x+b_2)\cdots(a_nx+b_n)$	One constant per factor $\frac{A_1}{(a_1x+b_1)} + \frac{A_2}{(a_2x+b_2)} + \cdots + \frac{A_n}{(a_nx+b_n)}$
2) A repeated linear factor: $(ax+b)^n$	One constant per power $\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \cdots + \frac{A_n}{(ax+b)^n}$