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CSI4105 Type 1 Project

Weighted Set Covering Problem

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1. Introduction

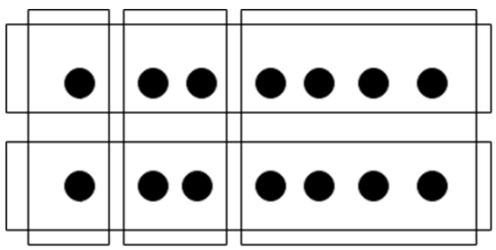
Weighted Set Cover is a very popular NP-Hard problem which has a few different forms that include just set cover, where all weights are set to one and other forms such as vertex cover. There are many important algorithmic problems that can be formulated as set cover and that's why an approximation algorithm found will be used for many problems (Kleinberg, 2005). There have been many attempts at finding a feasible solution to the problem but it remains unsolved. It has proven to be NP-Complete in decision form; Richard Manning Karp showed in his 1972 paper "Reducibility Among Combinatorial Problems" that there is a polynomial time reduction from SAT to Set Cover, and thus it is NP-Complete. Solving the Weighted Set Cover in P time means that P = NP. In the pursuit of such solutions, approximation algorithms and heuristics have been developed. Theses approximation algorithms and heuristics are used in real life applications of problems that can be transformed to weighted set cover. Real world applications of weighted set cover include choosing the smallest possible set of short sentences to tune all the features in a speech recognition software. In our presentation, we talked about how IBM found that searching only so many subsets of strings would lead them to finding all the viruses instead of searching the large number of viruses. Numerous other examples include finding a good location for facilities to set up, different scheduling problems, and resource allocation. The study of weighted set cover led to the development of fundamental techniques for the entire field of approximation algorithms (Kleinberg, 2005). Our report will discuss three algorithms for solving the Set Cover problem: greedy approximation, greedy heuristic, and simulated annealing.

The greedy approximation algorithm is efficient and provides a guaranteed cost when compared with the optimal solution. The greedy heuristic is extremely efficient and is used as the basis for the Simulated Annealing algorithm. Simulated Annealing is based on a method called annealing where in physical systems one guides a material to a crystalline state by gradually cooling an object from a high temperature state giving it time time to reach an equilibrium (Kleinberg 2005). Simulated annealing generates a random solution with a cost c(s) and then generates a solution from the neighbour and if that cost is less we take that solution and if not we may take the solution. We only take the solution of the cost if its higher depending on a certain probability.

For the experimental results, our group is using the OR-Library as our source of test data to run our algorithms against. The library contains and extensive list of different types of sets, that when tested on different algorithms will work better or worse. The library contains many different instances of weighted set cover problems. We will use the results of these tests to compare the effectiveness of the algorithms in terms of solutions given and time to arrive at those solutions. We will now define formally weighted set cover and show three methods for solving an instance of set cover.

2. Problem Definition

We first define the problem of set cover as Given a Set $E = \{e_1, e_2, \ldots, e_n\}$ of n elements, a collection of m subsets $S_1, S_2, \ldots, S_m$ of E with weights $w_1, w_2, \ldots, w_m$. The goal to find a set $I \subset \{S_i, \ldots, S_m\}$ such that all elements covered by I and the sum of the weights in I is minimized. In decision form the problem is defined as: can we find a set cover with weight at most W. The optimal solution is two subsets, the two subsets chosen are the two horizontal subsets.
Example:
Assume weight of each subset is 1
Find the minimal set cover.
In decision form it is: Does there exist a set cover of weight W?



2.1. Greedy Approximation Algorithm

The greedy approximation algorithm provides a guarantee when compared with the optimal solution to the problem. Unlike most approximation algorithms, the guarantee is not a constant multiple of the optimal but instead a it is a function multiple of the optimal based on the size of the input for the instance. The greedy approximation algorithm is guaranteed provide a solution that is less than or equal to $\alpha \cdot OPT(I)$ where α is given by

$$\alpha = H_k = \sum_{i=1}^{k} \frac{1}{i} \leq 1 + \log(k)$$

where k is the size of the largest subset and H_k is the kth partial sum of the harmonic series. Since k will always be less than n, we can re-frame α in terms of n if so desired. The proof of this is given by David S. Johnson but is too complicated for the scope of this report

The guarantee of $H_k \cdot OPT(I)$ seems weak compared to algorithms for other NP-Hard problems where there are 2-approximation guarantees and even 3/2-approximations. The harmonic series quickly increases past 2 (passing it at $k = 4$) and surpasses a guarantee of $3 \times OPT$ at $k = 11$. However, it has been shown that the efficiency of the greedy algorithm for general set cover problems cannot be improved to a constant nor to anything significantly more efficient than the greedy algorithm

The implementation of this algorithm is fairly simple and is given below in pseudo code. The actual implementation can be examined in the included code or github source repository (the greedy_set_cover function in main.py within the GreedySet-Cover directory). In the pseudo code, C represents the set of covered elements, X represents the list of chosen subsets.
Initialize $C = \{\}$, $X = []$
While $|C| \neq |U|$
Find sub-set S with smallest cost effectiveness (cost of S divided by uncovered elements)
Set $C = C \cup S$
Add S to X
Output X

Referring to the example given previously, there are 14 elements and 5 subsets where the weights of each subset is one. The optimal solution is to choose the two horizontal subsets while the greedy approximation algorithm will choose the three vertical subsets as they will consistently have better cost effectiveness at each iteration. The guarantee of the greedy approximation algorithm on this problem is $5.4 \times OPT$ which is more than the total number of subsets in the problem. This example does not give much hope for the reliability of the guarantee for the approximation algorithm.

The greedy approximation algorithm also works on unweighted set cover where the weights of each subset is simply 1. The algorithm has a worst-case running time of $O(N \log(N))$ where N is the actual size of the input including all subsets.

2.2. Greedy Heuristic

Our heuristic algorithm is intended to quickly find an initial feasible solution and to construct neighbouring solutions for Simulated Annealing in a randomized manner. To this end we employ a fast greedy method proposed by Balas and Ho in 1987. cite

This algorithm is intended for weighted set covering problems in particular, and aims to exploit the array of set costs – and little else. It first greedily seeks the cheapest sets which help it to complete the cover by randomly choosing uncovered elements and selecting precisely the cheapest set which covers it. Usually, a combination of sets chosen later will include earlier sets as a subset, so that the latter become redundant. In order to capitalize on redundancies the algorithm greedily seeks expensive sets which can be removed without making the cover infeasible. It does this by iterating over the sets which were added to construct the cover in reverse order of their addition, and removes them if possible. This is summarized in the pseudocode below.

- Initialize $C = \{\}$ and $X = []$
- While $|C| \neq |U|$
  - Pick a random element in C, and find the set S of minimal cost which covers the element
  - Let $C = C \cup S$ and append S to X
- Examine each selected set, in order by recency of selection. If the set is redundant – that is, it can be removed without leaving an element uncovered – remove it.

- Return the cover X.

The order of addition of sets is important to this algorithm. Cheap sets which cover many elements are statistically more likely to be added earlier, since there are more elements whose random selection prescribes the selection of these sets. Thus the algorithm discerns some statistical information about the cost effectiveness of the sets without performing any calculation, which is expressed in the order of the sets and exploited in the reduction by iterating in reverse. In essence, since the later sets added to build a complete cover are less likely to have a high cost-effectiveness, we are more likely to reduce to a cost-effective cover if we judge the later sets first.

While this algorithm has no performance guarantee, it scales better than the approximation algorithm since it does not compute cost-effectiveness across sets at each addition; yet it still exploits cost effectiveness in an implicit statistical manner, and performs better than the "take the cheapest" greedy idea would lead one to expect.

Our implementation of this algorithm has been written with its application as a subroutine of Simulated Annealing firmly in mind. As such, we have presumed a preprocessing step precedes the algorithm which sorts the array of costs in nondecreasing order, and indexes the sets accordingly; this is referred to in literature, *e.g.*, as by Brusco, Jacobs, and Thompson 1999, as the "natural order" of the weighted set covering problem specification, and is the format used by our test problems from the OR Library. In fact, the OR library format can be described as a column-wise list-of-lists sparse representation of the problem's unweighted hypergraph adjacency matrix in a natural-order basis, such that we know the set of sets which cover each element in order of their cost. These problems are then already in the optimal format for application of the heuristic algorithm, and our implementation benefits from this – while the approximation algorithm does not.

2.3. Simulated Annealing

NP-Hard optimization problems with NP-Complete decision problem analogs are characterized by an objective function which is easy to evaluate at a point but a "search space" of feasible solutions which is exponentially large in the size of the problem. Local search is a strategy which attempts to find a good solution quickly by iteratively evaluating feasible solutions which are in some heuristic sense similar to the present solution – in its "neighbourhood" – and choosing solutions which are an improvement on the present. An ideal local search algorithm thereby works toward optimality while evaluating only an exponentially smaller subset of the search space—but inevitably will converge to a a point which is only optimal with respect to its immediate neighbours. Our Heuristic and Approximation algorithms apply different local search strategies to the minimum weighted set covering problem, and in our tests find results fairly close to the global minimum, but very rarely attain it.

Metaheuristics are algorithms which augment local search with global information, progressively divining the structure of the search space from local search results and guiding the local searches towards the optimal. In order to improve on the performance of our algorithms, we guide the local search with an annealing metaheuristic; that is, one which allows selection of inferior neighbouring solutions prescribed according to a "computational temperature" parameter whose magnitude corresponds to a "smoothing" of the neighbourhood structure which the local search algorithm imposes on the search space. Of the varieties of annealing metaheuristics, we choose Simulated Annealing (SA) due Kirkpatrick, Gelatt, and Vecchi 1983, which was the first annealing method used in optimization and has remained the most popular. The SA pseudocode follows:

- GENERATE an initial solution X
- for T in schedule:
  - SEARCH for a neighbour solution X'
  - let $\Delta = cost(X) - cost(X')$
  - if $\Delta < 0$ or rand(0, 1] $< \exp\left(-\frac{\Delta}{T}\right)$
    - let $X = X'$
- return X

The GENERATE and SEARCH subprocedures respectively define the initial state and neighbourhood of states. In our SA algorithm we base both on the heuristic algorithm outlined in 2.2. The GENERATE subprocedure just runs the heuristic from $C = \{\}$ and $X = []$. The SEARCH subprocedure randomly removes sets from a feasible cover X, creating a partial cover which is then completed by the heuristic.

In our implementation, we use the fact that Δ is a cost difference and can be very easily evaluated in terms of only the set removals and additions in a single SEARCH iteration. Our algorithm has been designed to allow random swapping of the SEARCH algorithm to a different local-search heuristic at fixed state, in order to expand the search neighbourhood; as well as swapping state between simultaneous annealing runs – and integration of ideas from Parallel Tempering, a different annealing method due Swendsen and Wang 1986 – to allow "tunneling" between regions of the search space and make the algorithm parallelizable. However, the investigation of these capabilities is not within the scope of this project, and we restrict our analysis to one annealer with the above-defined SEARCH subprocedure based on our heuristic.

Without augmentation, SA with this SEARCH subprocedure enjoys an expanded search space versus the heuristic *per se*, but is not sufficient to enable exploration of the entire space – and our SA implementation cannot in general find the optimum.

This initial implementation also faces the problem that the SEARCH procedure can often yield a state which is equivalent state to the current state. This is not of itself a problem; in terms of the mathematical foundations of the algorithm, it is expected, even necessary. When the neighbourhood is small, the cooling rate is in effect increased by the increased frequency of non-moves. However, the entire cooling schedule can be exhausted in this way for certain problems, making tweaking necessary. We've devised additional parameters which serve as a sort of second-order temperature, controlling the distribution of the SEARCH neighbourhood distribution directly: the number of drops to make in transforming a cover to a partial cover within the SEARCH routine, and an "activity" parameter which demands of the SEARCH procedure a minimum number of attempts towards a unique neighbour before allowing the proposition of a non-move.

Through experimentation, we have found that the standard exponential cooling schedule works well for the problems we consider, and have set it to run in stages with different (increasing) cooling rates to target temperature ratios to improve the speed of convergence. The activity has been defined to change proportionally to the temperature, to conserve the concept of "cooling": we want the neighbourhoods to sharpen on both scales. We've set the temperature scale, the number of drops in SEARCH, and initial activity as functions in the number of elements and sets and total size, as well as average set cost, of arbitrary problems to correspond roughly to our intuitive expectations for a reasonable annealing run for arbitrary problems. This is one of the more interesting aspects of our work, but unfortunately the details do not suit the scope of this report. However, it should be noted that all runs in our analysis were executed for very different problems with very different, adaptively determined parameters; and we were unable to significantly improve results by hand-tuning. In order to improve our algorithm further we must expand the SEARCH subroutine beyond the simple heuristic such that the algorithm can explore a larger subspace of feasible solutions.

3. Experimental Data

For our project, we used the OR-Library's set of set covering problems and ran our algorithms against it. We used 80 of the data sets that were in the same format for ease of importing, but the OR-Library also provides a small data set of interesting looking problems regarding real life rail-roads, but the data sets were formatted differently so we did not have time to use them in our analysis. Of the 80 data sets, 50 of them included the optimal solution which greatly enhanced our analysis against these problems.

For our analysis, we will compare the results of the algorithms against the guarantee (where applicable), against the optimal (where available), against each other, and the running time to achieve the results. All of the raw results for our experiments can be found in the appendix in an easily comparable table. We will go on to explain what these results mean and compare the different methods against each other. In the experimental results below, Simulated Annealing will be discussed separately. Note that, in the table, only the average results for the heuristic and Simulated Annealing but the best and worst cases for the algorithms were not far from the average and thus the average works as an acceptable number to report on.

3.1. Harmonic Guarantee Results

As mentioned earlier, we compared the results of the greedy approximation algorithm with the guarantee it provides. As surmised earlier, the results show that the guarantee of the approximation are not a good indication of the result that the greedy approximation algorithm will return nor are they very good when compared with the optimal result. The comparison can only be made on data sets where an optimal solution has been provided, but this is the case for fifty of the data sets so it makes for a decent appraisal. On average, the guarantee is 204% larger than the actual result provided by the greedy approximation algorithm. In the worst case, the guarantee was 277% larger and in the best case it was 156% larger than the actual results. Thus it seems that the guarantee provided by the harmonic series is a poor indication of the result that the greedy approximation algorithm will return, but at least a guarantee is given which cannot be said for other algorithms.

3.2. Greedy Approximation Algorithm vs. Optimal

Contrary to the guarantee, the actual results of the approximation algorithm (when compared against the optimal) are quite strong. On average, the greedy approximation algorithm result was only 12% larger than that of the optimal solution and in the worst case it was only 23% larger for all the data sets with optimal solutions provided. For the SCPE data sets, the greedy approximation algorithm managed to find the optimal solution for four out of the five data sets. With only a 1/4 size difference compared with the optimal, the experimental results show that the greedy algorithm is a quite reliable algorithm for the time it takes to come to a solution.

3.3. Greedy Heuristic Algorithm vs. Optimal

The greedy heuristic had roughly similar results to that of the greedy approximation when compared with optimal solutions but did differ in certain respects. It should be noted that the results in the table are averages of multiple results since the algorithm uses randomness in it's solution, but that the results of the heuristic did not differ much from the average case. The greedy heuristic was larger than the optimal by 13% on average and had a worst case of being 53% larger than the optimal. Unlike the approximation, the greedy heuristic was never able to reach the optimal solution for any data set but did manage to get to 4% larger than the optimal in the best case. While the numbers look larger when compared with those of the greedy approximation, especially the worst case, it is also true that the greedy heuristic managed to find a better solution in 35 of the 50 data sets. This means that the approximation managed to be more consistent but the heuristic managed to find a better result more frequently.

3.4. Algorithm Results with no Optimal Solution Available

Of the 80 data sets, 30 of them had no optimal solution provided for them. For these data sets, the greedy approximation algorithm significantly outperformed that of the heuristic. Out of the 30 data sets, the approximation algorithm gave a better result for 19 of them. The key element that these data sets have in common, besides having no optimal solution attached to them, is that they all have uniformly weighted subsets (i.e., they are unweighted set covering problems). Out of the data sets with optimal solutions provided, only the SCPE sets had uniform weights and the approximation algorithm performed better on all of them (typically finding the optimal solution). Thus, the approximation algorithm found the better solution on 24 out of 35 unweighted set covering problems. We can conclude from this that the approximation algorithm is the clear winner when it comes to solving unweighted set covering problems.

3.5. Simulated Annealing

The experimental results for Simulated Annealing showed improvements on the greedy heuristic (which it uses as a starting point), but the improvements were not very significant. The biggest improvements over the heuristic were in the SCPCYC data sets where Simulated Annealing improved the heuristic result by 10-14%, but for all other problems the the difference between the heuristic Simulated Annealing result were typically between 0-2%. Even with the improvements made by Simulated Annealing for the SCPCYC data set, the approximation algorithm fared significantly better (the CYC problems being unweighted). The Simulated Annealing algorithm requires multiple parameters to be chosen and we chose general parameters to apply across all the data sets. With more time and focusing on specific problems, it might be possible to produce better results using Simulated Annealing by tweaking the parameters to fit the specific problem. The algorithm may also perform better if a different method (other than the heuristic) was used as a starting point. As it is, our current implementation did not fare better overall than the approximation algorithm.

3.6. Running Times

An important factor in evaluating the algorithms is comparing the time it takes to solve the problem sets and how well the algorithms scale. The experimental data shows that the running time for the approximation algorithm is significantly longer than that of the heuristic algorithm and that, as the problem size grows, the approximation algorithm's running time grows significantly faster. The approximation algorithm runs several order of magnitudes longer than the heuristic. The running time of Simulated Annealing is difficult to discuss as it is easy to change the parameters so that it runs in similar time to the other algorithms, or significantly longer to get better results. Note that since the algorithm uses the heuristic as a component, it will necessarily run longer. However, the Simulated Annealing algorithm could potentially be tweaked in such a way as to run in similar time to the approximation algorithm. For weighted set covering problems, it is possible that Simulated Annealing could be tweaked to generally provide better results than the approximation algorithm in similar time given the fact that the heuristic outperformed the approximation in more weighted problem sets and Simulated Annealing will only improve on these results. This is not the case in the parameters and runtime we chose for our experiments, but the avenue is open for further experimentation.

4. Conclusion

In this report we have discussed and defined the set covering problem along with multiple algorithms that provide efficient solutions that are not optimal but tend to be very close to optimal. The discussed and outlined the greedy approximation, greedy heuristic, and Simulated Annealing algorithms. In addition to defining the algorithms, we ran experiment runs against data sets from the OR-Library to see how they compared against optimal solutions as well as compared against each other.

The Greedy algorithm and the heuristic returned similar cost results on average when run against the library, but the approximation algorithm performed significantly better on unweighted set covering problems and was more consistent while the heuristic varied wildly. The execution speed of the heuristic is significantly faster than that of the greedy algorithm and scales much better. Since both the approximation and the heuristic are roughly equivalent for weighted set cover problems but the approximation algorithm is significantly better for unweighted set cover, we consider the approximation algorithm to be the better choice overall for the general set cover problem. The greedy algorithm also provides a cost guarantee for the solution, though it is not particularly useful according to our results. However, if speed is more important than accuracy and consistency, the greedy heuristic provides a suitable solution in a very short time frame. Simulated Annealing results are found to be closely tied to how the heuristic performs and thus cannot be recommended over the approximation algorithm with the implementation used for our experiments, but further testing may improve upon the method.

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A. Tables of Results

Below are the experimental results for all 80 OR-Library data sets. Each table lists on type of data set. Where applicable, the optimal result and the guarantee for the approximation algorithm (Harmonic*OPT) are given as well. Problem size is defined as the number of elements in all of the subsets.

				Approximation		Heuristic		SA	
Problem	Size	OPT	Harmonic	Cost	Runtime	Cost	Runtime	Cost	Runtime
scp41	4009	429	1295	463	0.044	450	0.003	449	0.904
scp410	3905	514	1595	556	0.043	579	0.003	573	0.452
scp42	3982	512	1499	582	0.043	571	0.003	561	0.326
scp43	3984	516	1558	598	0.044	542	0.003	537	0.861
scp44	4009	494	1446	548	0.043	544	0.003	534	0.290
scp45	3939	512	1546	577	0.042	531	0.003	527	0.369
scp46	4083	560	1640	615	0.041	625	0.003	612	0.334
scp47	3920	430	1334	476	0.037	458	0.002	448	0.228
scp48	4017	492	1441	533	0.038	518	0.003	512	0.466
scp49	3955	641	1935	747	0.045	706	0.003	698	0.408

				Approximation		Heuristic		SA	
Problem	Size	OPT	Harmonic	Cost	Runtime	Cost	Runtime	Cost	Runtime
scp51	7995	253	741	289	0.083	283	0.003	280	0.656
scp510	8001	265	842	293	0.082	282	0.003	280	0.374
scp52	7997	302	960	348	0.081	325	0.003	318	0.247
scp53	8015	226	661	246	0.076	248	0.002	247	0.649
scp54	7935	242	769	265	0.082	255	0.003	251	0.626
scp55	7855	211	637	236	0.079	228	0.003	225	0.304
scp56	7995	213	643	251	0.081	249	0.003	247	0.275
scp57	8058	293	884	326	0.081	321	0.003	315	0.308
scp58	7921	288	843	323	0.081	320	0.003	317	0.374
scp59	7871	279	817	312	0.078	316	0.003	315	1.914

Problem	Size	OPT	Harmonic	Approximation Cost	Approximation Runtime	Heuristic Cost	Heuristic Runtime	SA Cost	SA Runtime
scp61	9836	138	496	159	0.033	155	0.002	153	0.327
scp62	10002	146	517	170	0.033	162	0.002	160	0.322
scp63	9922	145	514	161	0.030	163	0.002	162	0.337
scp64	9857	131	464	149	0.033	142	0.002	140	0.863
scp65	9943	161	562	196	0.033	185	0.002	186	0.396

Problem	Size	OPT	Harmonic	Approximation Cost	Approximation Runtime	Heuristic Cost	Heuristic Runtime	SA Cost	SA Runtime
scpa1	18091	253	870	288	0.159	265	0.003	261	0.392
scpa2	18073	252	851	284	0.158	281	0.003	278	0.652
scpa3	18077	232	797	270	0.161	254	0.003	253	0.637
scpa4	18084	234	804	278	0.157	254	0.004	252	0.252
scpa5	18072	236	811	271	0.159	246	0.003	242	0.425

Problem	Size	OPT	Harmonic	Approximation Cost	Approximation Runtime	Heuristic Cost	Heuristic Runtime	SA Cost	SA Runtime
scpb1	44921	69	273	77	0.113	86	0.002	86	0.862
scpb2	44869	76	301	86	0.119	90	0.002	88	0.517
scpb3	44906	80	316	89	0.118	87	0.002	85	0.473
scpb4	44893	79	312	89	0.128	85	0.002	85	0.492
scpb5	44883	72	285	78	0.114	81	0.002	79	0.432

Problem	Size	OPT	Harmonic	Approximation Cost	Approximation Runtime	Heuristic Cost	Heuristic Runtime	SA Cost	SA Runtime
scpc1	32041	227	827	258	0.272	239	0.004	235	0.504
scpc2	31954	219	787	258	0.271	240	0.004	236	0.405
scpc3	31969	243	849	276	0.274	279	0.004	271	0.579
scpc4	31971	219	787	257	0.264	250	0.004	247	0.998
scpc5	31955	215	773	233	0.252	227	0.004	223	0.515

Problem	Size	OPT	Harmonic	Approximation Cost	Approximation Runtime	Heuristic Cost	Heuristic Runtime	SA Cost	SA Runtime
scpd1	80143	60	255	74	0.203	65	0.002	63	0.233
scpd2	80105	66	279	74	0.201	75	0.002	74	0.545
scpd3	80163	72	300	83	0.207	80	0.002	79	0.662
scpd4	80034	62	258	71	0.214	68	0.002	68	0.650
scpd5	80072	61	254	69	0.201	66	0.002	66	0.700

Problem	Size	OPT	Harmonic	Approximation Cost	Runtime	Heuristic Cost	Runtime	SA Cost	Runtime
scpe1	4914	5	17	5	0.002	8	0.000	7	0.087
scpe2	5013	5	17	5	0.002	8	0.000	8	0.279
scpe3	5040	5	17	5	0.002	6	0.000	6	0.249
scpe4	4952	5	17	6	0.002	7	0.000	7	0.294
scpe5	5017	5	17	5	0.002	6	0.000	6	0.269

Problem	Size	Approximation Cost	Runtime	Heuristic Cost	Runtime	SA Cost	Runtime
scpclr10	13230	33	0.012	35	0.003	35	1.307
scpclr11	41910	30	0.032	35	0.006	35	1.743
scpclr12	126225	32	0.092	35	0.011	35	2.702
scpclr13	365365	32	0.262	35	0.026	35	4.831

Problem	Size	Approximation Cost	Runtime	Heuristic Cost	Runtime	SA Cost	Runtime
scpcyc06	960	60	0.006	86	0.004	80	0.962
scpcyc07	2688	148	0.043	210	0.011	192	1.110
scpcyc08	7168	364	0.253	496	0.038	448	1.065
scpcyc09	18432	816	1.386	1148	0.155	1028	1.192
scpcyc10	46080	1928	7.583	2610	0.664	2309	2.357
scpcyc11	112640	4304	40.469	5854	2.993	5138	5.443

Problem	Size	Approximation Cost	Runtime	Heuristic Cost	Runtime	SA Cost	Runtime
scpnre1	249448	30	0.267	30	0.002	30	1.105
scpnre2	249367	36	0.285	35	0.002	35	0.530
scpnre3	249371	31	0.256	34	0.002	34	0.457
scpnre4	249341	32	0.280	33	0.002	33	1.011
scpnre5	249393	33	0.283	31	0.002	30	0.368

Problem	Size	Approximation Cost	Runtime	Heuristic Cost	Runtime	SA Cost	Runtime
scpnrf1	499314	16	0.270	17	0.001	17	0.410
scpnrf2	499275	16	0.257	18	0.001	18	0.563
scpnrf3	499250	17	0.279	18	0.002	17	0.495
scpnrf4	499302	17	0.281	17	0.001	18	0.378
scpnrf5	499336	16	0.266	16	0.001	16	0.374

Problem	Size	Approximation Cost	Approximation Runtime	Heuristic Cost	Heuristic Runtime	SA Cost	SA Runtime
scpnrg1	199471	203	1.333	198	0.007	197	0.881
scpnrg2	199451	182	1.306	171	0.007	169	0.467
scpnrg3	199498	192	1.312	182	0.007	180	0.845
scpnrg4	199456	191	1.298	189	0.007	184	0.434
scpnrg5	199450	194	1.303	191	0.007	189	0.508

Problem	Size	Approximation Cost	Approximation Runtime	Heuristic Cost	Heuristic Runtime	SA Cost	SA Runtime
scpnrh1	499163	76	1.206	74	0.004	73	0.454
scpnrh2	499167	74	1.156	72	0.004	68	0.530
scpnrh3	499126	65	1.088	70	0.004	69	0.501
scpnrh4	499149	69	1.116	68	0.004	68	0.449
scpnrh5	499179	63	1.082	62	0.004	61	0.760