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ECE300 Communication Theory
Problem Set VI: Digital Communications
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1. For binary transmission over the AWGN channel of two symbols with correlation coefficient ρ, the exact probability of error for *coherent* detection is:

$$P_e = Q\left(\sqrt{(1 - \mathrm{Re}(\rho))\,\gamma_b}\right)$$

For binary *orthogonal* transmission with *noncoherent* detection, the exact probability of error is:

$$P_e = \frac{1}{2} e^{-\gamma_b/2}$$

Note that in MATLAB, the inverse of $Q(\cdot)$ is the function *qfuncinv.*

For binary FSK, the minimum value of ρ (which achieves minimum probability of error for coherent detection) is:

$$\rho = -0.217$$

Compute γ_b (in dB) necessary to achieve $P_e = 10^{-5}$ for various situations:

(a) Coherent demodulation of binary FSK with the optimal ρ as above.

(b) Coherent demodulation of binary orthogonal FSK.

(c) Noncoherent demodulation of binary orthogonal FSK.

Remark: You should observe that the SNR requirements in the order given are increasing!

2. Assuming an initial phase of $0°$, specify the sequence of phases for encoding the following binary sequence in $\pi/4 - DQPSK$, using the diagram provided in the notes.

01 11 11 00 10

3. Reference the constellation shown. Note that:

$$d_{\min}^2 = 4\mathcal{E}_0$$

Also be careful: $\mathcal{E}_0$ is a reference value but it does *not* equal the average energy per symbol.

(a) Assign binary codes to the symbols using Gray coding rules.

(b) Express the energy per bit $\mathcal{E}_b$ in terms of $\mathcal{E}_0$.

(c) Determine all distinct distances between pairs of points, sorted as $d_1 < d_2 < \cdots$, with $d_1 = d_{\min}$, and the number of pairs k_1, k_2, etc. As a check, $\sum k_i = 8 \times 7$ (remember going from symbol i to symbol j, and symbol j to symbol i is counted twice!). [You can do this manually, or code this in MATLAB]

(d) Note that, with Gray coding, the probability of bit error P_b is related to the probability of symbol error P_e as $P_b = \frac{1}{3}P_e$ (as there are 8 symbols and we used Gray coding). Set up the union bound to obtain an expression of the form:

$$P_b \leq \sum \alpha_i Q\left(\sqrt{\beta_i \gamma_b}\right)$$

for constants α_i, β_i. Specifically, use MATLAB to compute vectors containing the list of α_i, β_i values.

(e) Obtain an approximation for the upper bound keeping only the term with the *(smallest/largest)* β_i (which should you keep, if you keep only one?).

(f) Use MATLAB to superimpose graphs of the bounds, with vertical axis probability on a log scale, and horizontal axis γ_b in dB, with $2dB \leq \gamma_b \leq 8dB$. [The difference between the curves may not be visible!] It turns out the size of the error decreases with increasing γ_b. Also, the probability approximations get very small, so the best way to compare the two would be to examine their ratio (rather than their difference):

$$1 - P_{err,\text{only one term}}/P_{err,\text{all terms}}$$

Compute this value at $2dB$ and at $8dB$.