

The Cooper Union Department of Electrical Engineering
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ECE300 Communication Theory
Problem Set VI: Digital Communications
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1. For binary transmission over the AWGN channel of two symbols with correlation coefficient ρ , the exact probability of error for *coherent* detection is:

$$P_e = Q\left(\sqrt{(1 - \text{Re}(\rho))\gamma_b}\right)$$

For binary *orthogonal* transmission with *noncoherent* detection, the exact probability of error is:

$$P_e = \frac{1}{2}e^{-\gamma_b/2}$$

Note that in MATLAB, the inverse of $Q(\cdot)$ is the function *qfuncinv*.

For binary FSK, the minimum value of ρ (which achieves minimum probability of error for coherent detection) is:

$$\rho = -0.217$$

Compute γ_b (in dB) necessary to achieve $P_e = 10^{-5}$ for various situations:

- (a) Coherent demodulation of binary FSK with the optimal ρ as above.
- (b) Coherent demodulation of binary orthogonal FSK.
- (c) Noncoherent demodulation of binary orthogonal FSK.

Remark: You should observe that the SNR requirements in the order given are increasing!

2. Assuming an initial phase of 0° , specify the sequence of phases for encoding the following binary sequence in $\pi/4 - DQPSK$, using the diagram provided in the notes.

01 11 11 00 10

3. Reference the constellation shown. Note that:

$$d_{\min}^2 = 4\mathcal{E}_0$$

Also be careful: \mathcal{E}_0 is a reference value but it does *not* equal the average energy per symbol.

- (a) Assign binary codes to the symbols using Gray coding rules.
- (b) Express the energy per bit \mathcal{E}_b in terms of \mathcal{E}_0 .
- (c) Determine all distinct distances between pairs of points, sorted as $d_1 < d_2 < \dots$, with $d_1 = d_{\min}$, and the number of pairs k_1, k_2 , etc. As a check, $\sum k_i = 8 \times 7$ (remember going from symbol i to symbol j , and symbol j to symbol i is counted twice!). [You can do this manually, or code this in MATLAB]

- (d) Note that, with Gray coding, the probability of bit error P_b is related to the probability of symbol error P_e as $P_b = \frac{1}{3}P_e$ (as there are 8 symbols and we used Gray coding). Set up the union bound to obtain an expression of the form:

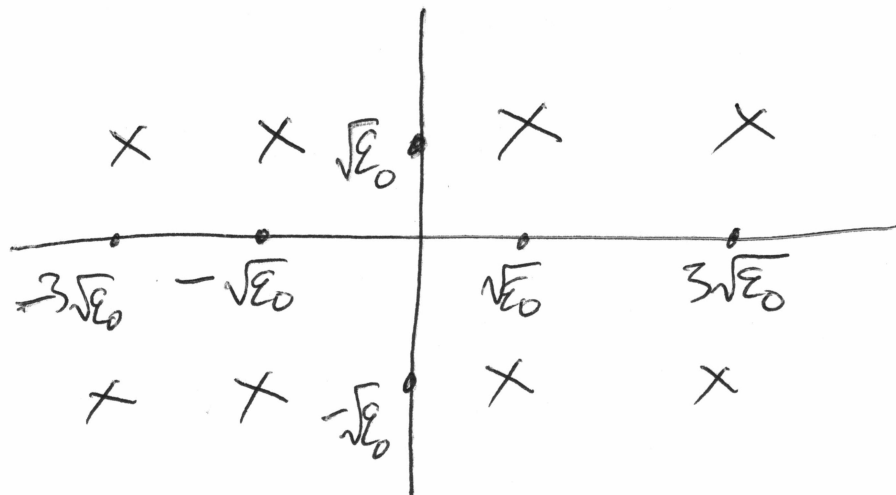
$$P_b \leq \sum \alpha_i Q\left(\sqrt{\beta_i \gamma_b}\right)$$

for constants α_i, β_i . Specifically, use MATLAB to compute vectors containing the list of α_i, β_i values.

- (e) Obtain an approximation for the upper bound keeping only the term with the (*smallest/largest*) β_i (which should you keep, if you keep only one?).
- (f) Use MATLAB to superimpose graphs of the bounds, with vertical axis probability on a log scale, and horizontal axis γ_b in dB, with $2dB \leq \gamma_b \leq 8dB$. [The difference between the curves may not be visible!] It turns out the size of the error decreases with increasing γ_b . Also, the probability approximations get very small, so the best way to compare the two would be to examine their ratio (rather than their difference):

$$1 - P_{err, \text{only one term}} / P_{err, \text{all terms}}$$

Compute this value at $2dB$ and at $8dB$.



8 point constellation