**The Cooper Union Department of Electrical Engineering**
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**ECE300 Communication Theory**
**Problem Set I: Fourier Transforms and Constellations**
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1. Consider 16-QAM versus 8-PSK. For transmission through an AWGN channel with a given noise power, the probability of symbol error in the two constellations will be roughly the same if the minimum Euclidean distance $d_{\min}$ between signal points are the same. For purposes of comparison, assume the 16-QAM constellation is comprised of points $I_k + jQ_k$ where $I_k, Q_k \in \{\pm 1, \pm 3\}$, while the 8-PSK constellation consists of points on a circle of radius $A$. In this problem, assume equiprobable transmission.

   (a) Find the average energy per **symbol** for each constellation, and then the average energy per **bit.** For 8-PSK, these answers depend on $A$, of course.

   (b) Find the amplitude $A$ for 8-PSK so that the two constellations have the same $d_{\min}$.

   (c) Let $\gamma_{16-QAM}$ be the SNR per bit required for 16-QAM, and $\gamma_{8-PSK}$ be the SNR per bit required for 8-PSK, both in decibels, assuming the same $d_{\min}$ found above. Which requires more SNR per bit? Find the difference between these two parameters.

   (d) As suggested in the notes, let us define the spectral efficiency $\eta$ as the number of bits per symbol, divided by the number of (real) dimensions. Compute $\eta_{16-QAM}$ and $\eta_{8-PSK}$, and identify the constellation that is more spectrally efficient.

2. Continue the previous problem. Now consider 8-ary orthogonal signalling, where each symbol has equal energy $\mathcal{E}_o$.

   (a) Express the $d_{\min}$ for this constellation in terms of $\mathcal{E}_o$.

   (b) If $d_{\min}$ is now set to be the same as for the constellations considered in the previous problem, find $\gamma_{8-orth}$, the energy *per bit* for this orthogonal signalling scheme.

   (c) Compute $\eta_{8-orth}$ for the 8-orthogonal signalling scheme.

   (d) Finally, list the 3 constellations in order of power efficiency (most power efficient first), and spectral efficiency (most spectrally efficient first).

3. The following is called a *raised-cosine spectrum* with 100% rolloff:

$$X(f) = \frac{A}{2W}\left[1 + \cos(\pi f/W)\right]\Pi\left(\frac{f}{2W}\right)$$

We will study this later in the context of intersymbol interference (ISI). Here:

$$sinc(\xi) = \frac{sin(\pi\xi)}{\pi\xi}$$

(a) Sketch $X(f)$, in particular indicating the values at $f = 0, \pm W/2, \pm W$.

(b) Find $x(t)$ without integrating. Instead, use $\mathcal{F}^{-1}\left(\frac{1}{2W}\Pi(f/2W)\right) = sinc(2Wt)$ and standard properties of the Fourier transform. Specifically, express it as the sum of several terms of the type $C_1 sinc[C_2(t - C_3)]$ for appropriate constants $C_k$. Do not attempt to sketch $x(t)$.

(c) Just to help you with the rest of the problem, what is $sinc(0)$ and what are the values of $sinc(n)$ for integers $n \neq 0$?

(d) Evaluate $x(t)$ for $t = 0, -T$ and $T$ where $T = \frac{1}{2W}$.

(e) Show that $x(kT) = 0$ (with $T = \frac{1}{2W}$) for all integers $k \neq 0, \pm 1$.

(f) *Now* you may use MATLAB to sketch $X(f)$ and $x(t)$ (taking, say, $A = 1$, $W = 1$ for normalization purposes) and confirm your result. You have to show work in the previous parts of the problem (i.e., justify your answers) to indicate you did not get your answers simply by looking at these sketches.