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ECE478 Financial Signal Processing
Problem Set I
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1. **Be a Financial Engineer!**

Here we will study mixes of European calls and puts to craft different types of payouts. Here assume maturity date T, with underlying security price S_T at T. The payout for a European call and put, respectively, at time T with strike price K is, respectively:

$$\begin{aligned} c(T,T) &= (S_T - K)^+ \\ p(T,T) &= (K - S_T)^+ \end{aligned}$$

Define a *digital* call or put option to have respective payout as follows:

$$\begin{aligned} d_C(T,T) &= 1 \text{ if } S_T > K, \text{ 0 otherwise} \\ d_p(T,T) &= 1 \text{ if } \; S_T < K \text{ 0 otherwise} \end{aligned}$$

A *long* position in one unit of a derivative with payout D_T means the value of the asset in the portfolio is D_T. A *short* position in one unit means the value is $-D_T$.

Combining various amounts of long and short positions in call and put options can yield a general piecewise linear but overall continuous value $V(S_T)$ at the time of maturity. Including digital options allows discontinuities. Here we will see several important combinations.

Use a computer to generate plots by assigning reasonable values to the parameters K, etc. You just need to produce one example of each. In all cases, horizontal axis is $S_T > 0$ and the vertical axis is $V(S_T)$.

Note: As you graph these, keep in mind you are looking at the "payout" $V(S_T)$. If you purchase one of these derivatives at a price say V_t at t, then your net "profit" or "loss" at time T would be $V(S_T) - V_t/Z(t,T)$ (i.e., subtract off the money you would have had in your pocket if you never purchased the derivative). This would be a more useful graph, perhaps, e.g., to determine the range of S_T for which you end up with a net positive, but that would require determining the price at t, V_t, which we are not dealing with here. One thing to look for in any of these instruments: is the potential loss unbounded? In other words, is $\min V(S_T)$ finite?

Also, in what follows below, except for one case I'm not asking you to derive an analytic formula for $V(S_T)$. If you write core code to compute the payout for the call, put, digital call, digital put, then just write code to combine these as suggested to obtain the graphs. Your code should be general purpose, e.g., with parameters such as K adjustable. When you generate graphs, plug in reasonable values, e.g., $K = 1$ or whatever. Scale your horizontal and vertical axes accordingly.

(a) Graph $V(S_T)$ for each of the following: long call, long put, short call, short put.

(b) A *straddle* is a combination of one unit of a call and one unit of a put for the same security at the same strike price K. Verify $V(S_T) = |S_T - K|$, and graph this for one choice of K.

(c) A *call-put spread* is to long a call at K_1 and short a call at K_2 with $K_1 < K_2$. Graph $V(S_T)$ for one choice of the pair K_1, K_2.

(d) A *butterfly* is a combination of the following. Let $K_1 < K_2$, and $0 < \lambda < 1$. Let $K^* = \lambda K_1 + (1 - \lambda) K_2$. Then:

- long λ calls at strike price K_1
- long $(1 - \lambda)$ calls at strike price K_2.
- short 1 call at strike price K^*.

Fix one choice for $K_1 < K_2$. Graph a butterfly $V(S_T)$ for each of the following cases: $\lambda = 1/3,\ 1/2,\ 2/3$.

(e) *Call ladder:* with $K_1 < K_2 < K_3$, long one K_1 call, short one K_2 call, short one K_3 call.

(f) A *digital call spread:* with $K_1 < K_2$, a long a digital call with strike K_1 and short a digital call with strike K_2.