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ECE478 Financial Signal Processing
Problem Set II: Portfolio Analysis
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Data

Our primary source of data in this experiment will be S&P 500, USD LIBOR rate (which we will take as the risk-free rate), and stock data you can get from online sources such as Yahoo finance.

Select 5 stocks that you expect to have some correlation, e.g., from a common sector. [Make sure these companies have been publicly traded since 2000]

Take data over the period 2000 through 2018. We will keep data over each year separate. For purposes of computing the risk-free return, take the average LIBOR over the calendar year. Note that the definition of the *daily return* is the annual return (i.e., for a one year time horizon) divided by 360; this is the daycount conversion for interest rates, not stocks! If you look at monthly returns, then divide by 12.

Pull daily date for S&P500 and your 5 stocks. Use the *adjusted closing price.* This compensates for effects such as dividend payments.

Compute the daily log returns.

Compute the monthly log returns by comparing the first (trading) day of a month to the first (trading) day of the prior month.

For the first day (month) of a calendar year, use the final day (month) of the previous calendar year. (E.g., you will need some data from 1999 to do this for the year 2000).

In what follows, you can use "canned" routines to compute mean vectors, covariance matrices, and linear algebra operations such as eigenanalysis. However, the rest you should build up yourself. Don't use higher level economic/financial functions.

There are potentially many graphs generated. You have to decide how to organize things intelligently. I also want to see comments, especially when data doesn't seem to match theory.

Analysis

Write code to take the daily data over a period of one calendar year. Consider a portfolio comprised of your 5 stocks plus the risk-free return derived from USD LIBOR.

When you do this, there may be years when the standard theory fails because the USD LIBOR is too high. Even if it is close to the assumed threshold, though technically below the level, results can look weird. Have your code identify those conditions, as an exception. If the exception occurs, then just pick an artificial but convenient value for the risk-free return, just to push through the theory. But you should clearly indicate when that happens. You should perform the following operations for several years (no, not for EVERY year).

1. Compute the mean vector $\mathbf{m}$ and covariance matrix C for the stocks.

2. Compute the weight vectors of the MVP and Market portfolios ($\vec{w}_{MVP}$, $\vec{w}_M$, respectively), and the respective points $(\sigma_{MVP}, \mu_{MVP})$ and (σ_M, μ_M).

3. Generate the points on the efficient frontier. Graph it, and also place markers at the MVP and market portfolio points. Also superimpose a graph of the CAPM.

4. Identify the portion of the efficient frontier that involves no short selling at all, and highlight it with a different color.

5. Look at the MVP and Market Portfolio weight vectors. Some of them may be negative. If this is the case, zero those terms out, and adjust the other weights (rescale them) to sum to 1. We can take this as an approximation to the optimal solution subject to the constraint of no short-selling. Place markers for these "modified" MVP and MP points.

6. Pick two stocks out of 5 and compute the MP for just this pair. Compute the correlation coefficient between this portfolio and the MP (for the full set of 5 stocks), the β, the systematic risk and diversifiable risk.

7. Now study the covariance matrix C:

   (a) Compute the eigenvalues, and show that are all positive.

   (b) Find the condition number, $\lambda_{\max}/\lambda_{\min}$. This is one way to quantify how numerically "unstable" manipulating C is, e.g., how close to nonsingular it is (the larger this is, the worse for us).

   (c) Graph the eigenvalues on a log scale in descending order.

8. Now we look at the S&P 500. Compute its (μ, σ) and place a marker for it on the above graph, to compare it to the other portfolios you have been looking at.

9. Compare the S&P500 with the MP for the full 5 stocks, and the MP for just the 2 stock subset you took, and also for the 'modified" MP in which you removed any short selling. . In particular, do one of these dominate some of the others? Compute the correlation coefficient between the S&P500 and these others.