1) We need to show that for every left-leaning red black tree (LLRBT), there is an ~~equivalently~~ equivalent right-leaning red black tree (RLRBT), and vice versa.

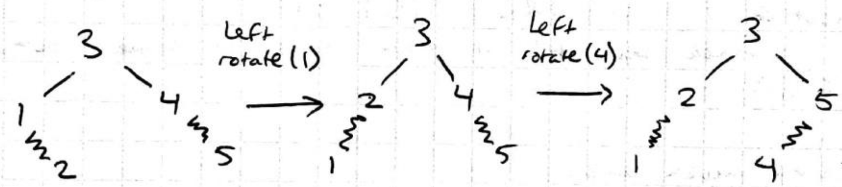
Given a ~~set~~ LLRBT, we can construct a RLRBT by rotating each red node's parent to the ~~left~~ right.

Ex.

~~Then~~ So we have obtained the corresponding RLRBT

Given a RLRBT, we can construct a LLRBT by rotating each red node's parent to the left.

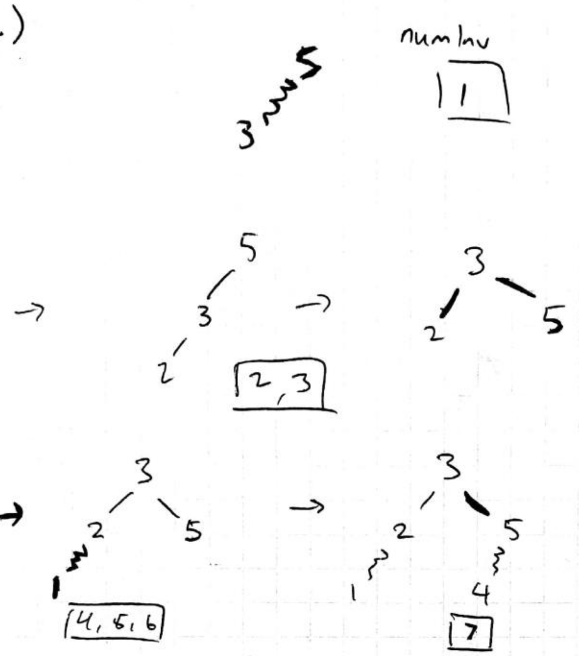
Ex.



So we have obtained the corresponding LLRBT.

By construction, we have shown that there is a correspondance between RLRBTs and LLRBTs.

2)

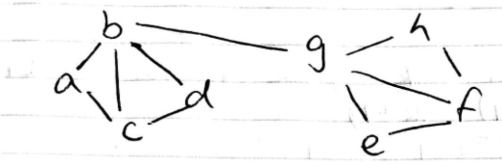



```
// numInv += 1 + size(h.right)

private int numInv = 0;

private Node put(Node h, Key key, Value val) {
    if (h==null) return new Node(key, 1, val, RED);
    int cmp = key.compareTo(h.key);
    if (cmp<0) {
        h.left = put(h.left, key, val);
        numInv += 1 + size(h.right);
    }
    if (cmp>0) h.right = put(h.right, key, val);
    else h.val = val;
    if (isRed(h.right) && !isRed(h.left))     h = rotateLeft(h);
    if (isRed(h.left) && isRed(h.left.left))  h = rotateRight(h);
    if (isRed(h.left) && isRed(h.right))      flipColors(h);
    h.N = size(h.left) + size(h.right) + 1;
    return h;
}
```


By updating the put method to increment the inverse counter by (1 + size(h.right)), we are able to use left leaning red black trees to count the number of inverses in a set. This works because as we add node x to the tree, we only recurse left if x < h (the comparison node). From this, we see that x comes after h in the set, but x < h. Also, x < h.right. Due to this, we add to the counter: 1 (as x < h) + size(h.right) (as x < h.right).

3a) 

~~[illegible]~~ gh

b) 3 from a to g
3 from g to h
$\therefore$ $3 \times 3 = 9$ paths from a to ~~g~~ h

c) a b g h
a c b g h
a b g f h
] 3 paths with length < 5

4) Consider $n=1$, $\therefore$ 1 vertex, no edges
$n=1$ $m=0$

$\therefore 2m = 0$
$0 \leq 1^2 - 1 = 0$
$\therefore$ True for $n=1$

Assume true for $n=k$
$\therefore 2m \leq k^2 - k$

Given true for $n=k$, prove true for $n=k+1$
$RHS = (k+1)^2 - (k+1)$
$= k^2 + 2k + 1 - k - 1$
$= k^2 + k$

Given $2m \leq k^2 - k$
~~$2m + 2k \leq k^2 - k + 2k$~~
For a simple graph, ~~[illegible]~~) $m \leq n - 1$
~~[illegible]~~ $\therefore m \leq (k+1) - 1 = k$
~~$4k \leq k^2 + k$~~
$\therefore 2m \leq 2k \leq k^2 + k$
as needed.

$\therefore$ For $G = (V, E)$, $n = |V|$ and $m = |E|$,
$2m \leq n^2 - n$ for $n \geq 1$

5a) We require all vertices.
There are 9 edges.
$\therefore 2^9 = 512$ spanning subgraphs

b) 3 ~~complete~~ connected spanning subgraphs, since there is a cycle of 3 edges

c) 3 edges into 0.
$\therefore 2^{9-3} = 2^6 = 64$ spanning subgraphs that contain 0 as an isolated vertex.