Mark Kaiser Assignment 1 Written
V00884677

Written assignment 1

1a) 5 C++ , 4 Java

No restrictions : 9! arrangements = 362880

b) Languages should alternate
-Must start w/ C++

C J C J C J C J C J

Ways to arrange the C++ books: 5!
Ways to arrange Java books : 4!

∴ Ways to arrange all books : 5! 4! = 2880

c) C++ next to each other
-Ways to arrange C++ books = 5! = 120
- Consider all C++ books to take up one spot of 5

C J J J J
-Ways to arrange above = 5! = 120
∴ 5! × 5! = 14400 ways

d) All C++ next to each other, Java next to each other :
C J or J C (2 ways)
C++ : 5! arrangements
Java: 4! arrangements

∴ ~~ways~~ 2 × 5! × 4! = 5760 ways

Mark Kaiser Assignment 1 Written
V00884677

Assignment 1

2a) 2 spades of 13 : $\binom{13}{2}$
3 non-spades of 39: $\binom{39}{3}$

$\therefore \binom{13}{2}\binom{39}{3} = 712\,842$

b) 3 cases:

1) 0 spades of 13 : $\binom{13}{0}$
5 non-spades of 39: $\binom{39}{5}$

2) 1 spade of 13 : $\binom{13}{1}$
4 non-spades of 39: $\binom{39}{4}$

3) 2 spades of 13 : $\binom{13}{2}$
3 nonspades of 39 : $\binom{39}{3}$

$\therefore \binom{13}{0}\binom{39}{5} + \binom{13}{1}\binom{39}{4} + \binom{13}{2}\binom{39}{3} = 2357862$

c) 3 spades of 13 : $\binom{13}{3}$
2 hearts of 13 : $\binom{13}{2}$

S H H S S $\quad \therefore \binom{13}{3}\binom{13}{2} = 22308$

Mark Kaiser Assignment 1 Written
V00884677

Assuming kings can be ~~[illegible]~~ any suit

2d) Case 1 : 1 Spade, K
2 Hearts, K, K'
2 CD, K', K'

$\binom{1}{1}\binom{1}{1}\binom{12}{1}\binom{24}{2} = 3312$
S,K H,K H,K' CD,K',K'

~~Case 2 : 1 spade, K'~~
~~2 Hearts, K', K~~
~~2 CD, K', K'~~

~~$\binom{12}{1}$ (~~
S,K'

Case 3 : 1 spade, K'
2 Hearts, K', K'
2 CD, K, K

$\binom{12}{1}\binom{12}{2}\binom{2}{2} = 792$
S,K' H,K',K' CD,K,K

Case 3 : 1 spade, K'
2 Hearts, K, K'
2 CD, K, K'

$\binom{12}{1}\binom{1}{1}\binom{12}{1}\binom{2}{1}\binom{24}{1} = 6912$
S,K' H,K H,K' CD,K CD,K'

Case 4 : 1 spade, K
2 Hearts, K', K'
2 CD, K, K'

$\binom{1}{1}\binom{12}{2}\binom{2}{1}\binom{24}{1} = 3168$
S,K H,K',K' CD,K CD,K'

<u>14184 ways</u>

Mark Kaiser Assignment 1 Written
V00884677

3a) If $n > 2$, $n \in \mathbb{Z}^+$, prove $\binom{n}{2} + \binom{n-1}{2}$ is a perfect square

$$\frac{n!}{2!(n-2)!} + \frac{(n-1)!}{2!(n-3)!}$$

$$= \frac{n(n-1)!}{2(n-2)(n-3)!} + \frac{(n-1)!}{2(n-3)!}$$

$$= \frac{n}{n-2}\left[\frac{(n-1)!}{2(n-3)!}\right] + \frac{(n-1)}{2(n-3)!}$$

$$= \frac{(n-1)!}{2(n-3)!}\left(\frac{n}{n-2} + 1\right)$$

$$= \frac{(n-1)!}{2(n-3)!}\left(\frac{2n-2}{n-2}\right)$$

$$= \frac{(n-1)\cancel{(n-2)}}{2}\left(\frac{2n-2}{\cancel{n-2}}\right)$$

$$= (n-1)^2$$

$\therefore$ for $n>2$, $n \in \mathbb{Z}^+$, $\binom{n}{2} + \binom{n-1}{2}$ is a perfect square with root $n-1$

3b) For $x \in \mathbb{R}$, $n \in \mathbb{Z}^+$, prove

$$1 = (1+x)^n - \binom{n}{1}x(1+x)^{n-1} + \binom{n}{2}x^2(1+x)^{n-2} - \cdots + (-1)^n\binom{n}{n}x^n$$

Consider $y = 1+x$, $z = -x$

$$RHS = y^n + \binom{n}{1}zy^{n-1} + \binom{n}{2}z^2y^{n-2} + \cdots + \binom{n}{n}z^n$$

$$= (y+z)^n \quad \text{(Binomial Theorem)}$$

$$= [1+x+(-x)]^n$$

$$= 1^n = 1 = LHS \quad \square$$

4a) $x_1 + x_2 + x_3 + x_4 = 32$

$x_i \geq 0$, $1 \leq i \leq 4$

$$\left.\binom{n+k-1}{k-1}\right|_{\substack{n=32\\k=4}} = \binom{32+4-1}{4-1} = \binom{35}{3} = 6545$$

b) $x_1 > 0$, $x_2 > 1$, $x_3 > 2$, $x_4 > 3$

$\Rightarrow x_1 \geq 1$, $x_2 \geq 2$, $x_3 \geq 3$, $x_4 \geq 4$

$\Rightarrow x_1 - 1 \geq 0$, $x_2 - 2 \geq 0$, $x_3 - 3 \geq 0$, $x_4 - 4 \geq 0$

$\Rightarrow x_1' = x_1 - 1$, $x_2' = x_2 - 2$, $x_3' = x_3 - 3$, $x_4' = x_4 - 4$

$\Rightarrow x_1' + 1 + x_2' + 2 + x_3' + 3 + x_4' + 4 = 32$, $x_i' \geq 0$, $1 \leq i \leq 4$

$\Rightarrow x_1' + x_2' + x_3' + x_4' = 22$

$$\left.\binom{n+k-1}{k-1}\right|_{\substack{n=22\\k=4}} = \binom{22+4-1}{4-1} = \binom{25}{3} = 2300$$

5) $A = \{a_1 \ldots a_m\}$ $R_1 = \{(a_1, a_1) \ldots (a_n, a_n)\}$
$B = \{b_1 \ldots b_n\}$ $R_2 = \{(b_1, b_1) \ldots (b_n, b_n)\}$

1) Since $(a_i, a_i) \in (A, R_1)$ and $(b_i, b_i) \in (B, R_2)$ for all a_i, b_i (poset : symmetric), this results in $((a_i, b_i), (a_i, b_i)) \in R$ for all i, thus R is reflexive

2) If $(a_i, a_j) \in R_1$, then $(a_j, a_i) \notin R_1$.
Similarly, if $(b_i, b_j) \in R_2$, then $(b_j, b_i) \notin R_2$.
Thus since $((a_i, b_i), (a_j, b_j)) \in R$ implies $(a_i, a_j) \in R_1$ and $(b_i, b_j) \in R_2$, then $((a_j, b_j), (a_i, b_i)) \notin R$, unless $i = j$, so R is antisymmetric

3) Since R_1, R_2 are posets, they are transitive.
Thus for $((a_i, b_i), (a_j, b_j))$ and $((a_j, b_j), (a_k, b_k)) \in R$, this implies $(a_i, a_j) \in R_1$, $(a_j, a_k) \in R_1$ and equivalently for b, R_2. This means $(a_i, a_k) \in R_1$ and $(b_i, b_k) \in R_2$, so also $((a_i, b_i), (a_k, b_k)) \in R$, so R is transitive

Thus R is a partial order