Condition-based Maintenance and Production Policies for Multi-Unit Systems

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1 Introduction

A large part of the total production costs is due to maintenance. Grber (2004) states that maintenance costs can be at much as 30 percent of production costs for power plants, while Bevilacqua and Braglia (2000) give an estimate of between 15 and 70 percent of the total production costs. Wang (2002) and Ding and Kamaruddin (2015) give a review of past research on maintenance policies. An increasingly popular maintenance strategy is condition-based maintenance (CBM), which uses conditioning information about components of a system to designs maintenance policies. Despite the increasing popularity of CBM, the implementation of CBM policies in practical applications is lagging behind (Keizer, Flapper, and Teunter, 2017). For this reason, research on CBM involves more complex systems with multiple dependent components ().

A general type of system is the k-out-of-N system, which consists of N components, and is functioning when at least k components are functioning. Keizer, Teunter, Veldman, and Babai (2018) consider a 1-out-of-N system with economic dependence by imposing a fixed cost for performing maintenance and failure dependence by load sharing between units, that is, the deterioration of the functioning units is affected by the number of functioning units. The load of the functioning units is assumed to be shared uniformly over all functioning units.

In this paper, I consider a k-out-of-N system with load sharing, which allows the load of all functioning units to depend on the current deterioration level of these units. Setting the load of the functioning units allows to cluster maintenance more effectively, due to greater control of the deterioration of the components of the total system. In the rest of the paper, I will use the terminology production instead of load, however observe that production does not assumes the production of an item. As I will show in this paper, the effectiveness of setting the production (load) increases with the economic dependence between units.

The coupling of production and maintenance for single-unit systems has been the subject of several studies in recent years. Xiang, Cassady, Jin, and Zhang (2014) present an option-based cost model for scheduling a joint production and maintenance policy for a single-period, single-product system. Cheng, Zhou, and Li (2016) study an integrated control strategy of production and maintenance for a machining system which produces a single type of product to meet the deterministic demand. The authors consider a single-unit system with a preventive maintenance policy.

[[**Next paragraph is still detached from the rest of the story. Have to fit this in somehow.**]] Offshore wind farms is one of the renewable energy solutions. Since the operation and maintenance costs represent a substantial portion of the total life cycle costs of these offshore wind farms Hau and von Renouard (2003), reliability and condition strategies have drawn increasing interest for decreasing these costs, e.g. Krokoszinski (2003). Existing CBM methods for offshore wind farm systems deal with wind turbines as individual units separately (**?**). Later maintenance strategies where economic dependence for maintenance actions between wind turbines is taken into account have gained interest (Tian, Jin, Wu, and Ding, 2011). The authors claim a maintenance costs reduction of up to 44.42% compared to maintenance strategies that do not take the economic dependence into account. This reduction is mostly due to the fact that it exploits the large fixed costs from and cluster maintenance actions. One element of the fixed costs unit is the cost of sending a maintenance team to the wind farm,, which generally requires expensive vessels.

In this paper, I consider a joint production and maintenance strategy for a multi-unit system. I formulate the system as a Markov Decision Process and analyse the structural properties of the optimal policies. The model extends the model of Keizer et al. (2018) by allowing the production (load) to be set for all units. The deterioration process of the units is modelled by a discrete-time Markov chain with a known transition probability matrix. The production and maintenance strategy are jointly optimised, while k-out-of-N models usually assume that the production is shared uniformly over all functioning units.

The remainder of the paper is organised as follows. Section 2 provides the definition and formulation of the system. Section 3 provides a description of the Markov Decision Process used to model the system introduced in Section 2. In Section 4 and 5 the structural properties of the optimal policies of a system with respectively 2 and 3 units is analysed. Conclusions and future research directions are provided in Section 6.

2 System formulation

We consider a discrete-time system consisting of N identical units, which are subject to variable production and economic dependence. We denote the set of units as $\mathcal{I} = \{1, \ldots, n\}$. The total production rate that has to be met during every time period is denoted π. Let x_i denote the deterioration state of unit i, which deteriorates according to a discrete-time Markov chain with deterioration states $X = \{0, 1, \ldots, L\}$. State 0 represents the as-good-as-new state, $L - 1$ denotes the most deteriorated state such that the unit still functions and state L the failed state. We assume that the production capacity of a unit is not affected by the deterioration state. A schematic overview of the events in one time period is presented in Figure 1.

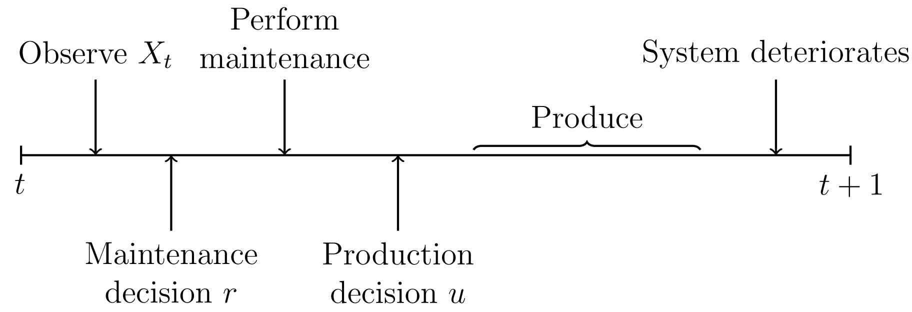

Figure 1: Order of events for time epoch t.

2.1 Maintenance and production actions

The deterioration state of each unit is known at the beginning of each time period. This assumption becomes is justified for an increasing number of applications, due to better monitoring equipment. Maintenance is assumed to take negligible time and is performed after the deterioration states of each unit is observed. Preventive maintenance is performed at a cost of c_{pm} if the unit has not reached the failed state, whereas a corrective maintenance action is performed in the failed state at a cost of c_{cm}. Generally, the cost of preventive maintenance is lower than the corrective maintenance action ($c_{\text{pm}} < c_{\text{cm}}$). Performing maintenance on a unit instantaneously brings it back to the as-good-as new state. The economic dependence is included in the form of a fixed cost C for performing maintenance for a time period. The cost is incurred when at least one unit will be maintained.

After selecting the maintenance action, we also have to set the production rate of each unit. Each unit can produce at most $m+1$ different production rates, ranging from 0 to 1 (maximum production). The possible production rates are uniformly distributed over the interval. If a unit is in the failed state, we cannot produce, so the production rate of this unit is set to 0. For all other deteriorating levels, we can select one of the $m+1$ production rates.

Since I assume that we can perform maintenance instantaneously, contrary to the work of Keizer et al. (2018), I do impose a penalty if the system does not function, i.e., the number of functioning units is less than k. Instead, I restrict the set of feasible maintenance policies to be such that we need at least k functioning units after the maintenance action.

2.2 Deterioration Process

Each unit i deteriorates according to a gamma process with rate $\lambda(u_i)$, which depends on the current production rate of the unit, and scale parameter γ. The gamma process is a rather flexible process that is applicable to model a wide variety of applications. According to Van Noortwijk (2009), the gamma process is most appropriate to model deterioration processes.

The rate for a given production rate u_i is given by

$$\lambda(u_i) = g(u_i)\lambda_{\max},$$

where $\lambda_{\max}$ is the rate parameter under the maximum production rate, and g is a non-decreasing function. We assume that $g(u_i)$ as a function of the production rate u_i for unit i is of the following form

$$g(u_i) = \beta + (1-\beta)u_i^{\alpha}.$$

We assume that $0 \leq \beta \leq 1$ and $\alpha \geq 0$, since g should be non-decreasing in u_i and positive for all production rates. If the production rate u_i is zero, then g has value β i.e., β determines the deterioration rate if the unit does not produce. The parameter α determines the curvature of the function g. In Figure 2 the function g is plotted for several settings of α and β.

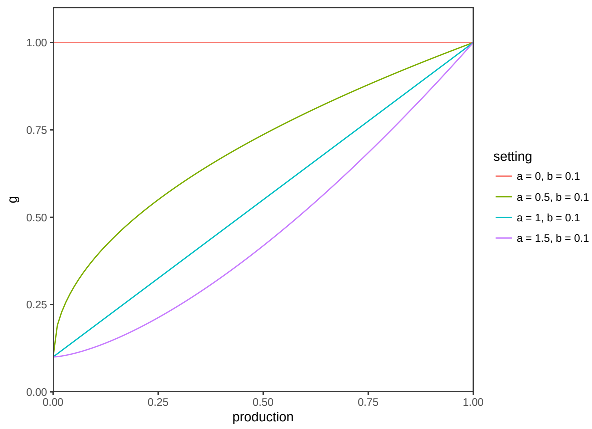

Figure 2: Function g for values $\alpha = 0, 0.5, 1, 1.5$ and $\beta = 0.1$.

The discrete jump probabilities for the gamma process are obtained by discretising the gamma process. One property of the gamma process is that the jump probabilities are independent of the current state. We denote the probability of having a jump of size j with rate parameter a and scale parameter b by $p(i; a, b)$. The derivation of the jump probabilities can be found in the manuscript by De Jonge on pages 51-53.

3 Markov Decision Process

In this section I provide the Markov Decision Process (MDP) formulation of the system as described in Section 2. The MDP consists of a state space, action space, transition probabilities, expected costs and a performance evaluation.

3.1 State space

We consider a multi-unit system that consists of n identical units. We denote the set of units as $\mathcal{I} = \{1, \ldots, n\}$. Each unit deteriorates according to a discrete-time Markov chain with deterioration states $X = \{0, 1, \ldots, L\}$. State 0 represents the as-good-as-new state and state L the failed state. The whole state of all units is denoted by

$$x = (x_1, \ldots, x_n) \in X^n.$$

3.2 Action space

At the start of each time unit we have to decide on which units to maintain (M) and do nothing (DN). Maintenance can be performed one all units, independent of the current state, i.e., the action space for the maintenance decision can be defined as

$$R = \left\{ r = (r_1, \ldots, r_n) : r_i \in \{\mathrm{DN}, \mathrm{M}\}, \ \ \forall i \in \mathcal{I} \right\}.$$

We define the set of possible production rates for unit i in state x_i $U(x_i)$ as follows

$$U(x_i) = \begin{cases} \{0, 1/m, \ldots, 1\} & \text{if } x_i \neq L, \\ \{0\} & \text{if } x_i = L. \end{cases}$$

Let $u = (u_1, \ldots, u_n)$ denote the production rate of all units in the system. The total production rate of the system is restricted to π, i.e., $\sum_{i=1}^{n} u_i = \pi$. This results in the following action space for the system in state x

$$U(x) = \left\{ u : \sum_{i=1}^{n} u_i = \pi, u_i \in U(x_i) \ \ \forall i \in \mathcal{I} \right\}.$$

Note that the production action space decreases from $(m+1)^n$ elements to $\binom{n+\pi-1}{\pi-1}$ by restriction the total production rate to the value π.

3.3 Transition probabilities

For a unit i the probability of having $q \geq 0$ jumps in the deterioration states for a time epoch under production rate u_i and scale parameter b is given by $p(q; \lambda(u_i), b)$. The transition probability of unit k for going from state i to j with production rate u_i and maintenance action DN for any time epoch is defined as

$$P_{u_k, \mathrm{DN}}(i, j) = \begin{cases} p(j - i; \lambda(u_k), b) & \text{if } i \leq j \leq L - 1, \\ 0 & \text{if } j < i, \\ 1 - \sum_{l=0}^{L-1-i} p(l; \lambda(u_k), b) & \text{if } i \leq m - 1 \text{ and } j = L, \\ 1 & \text{if } i = j = L, \end{cases}$$

and for maintenance action M by

$$P_{u_k, \mathrm{M}}(i, j) = \begin{cases} p(j; \lambda(u_k), b) & \text{if } j \leq L - 1, \\ 1 - \sum_{l=0}^{L-1} p(l; \lambda(u_k), b) & \text{if } j = L. \end{cases}$$

Note that for the maintenance action M the probability of going from state i to j does not depend on i, since we assume that the system will go to state 0 immediately after the maintenance action.

The transition probability of a unit in the system is independent of the other units. Therefore, the transition probability that the system goes from state $x = (x_1, \ldots, x_n)$ to state $\tilde{x} = (\tilde{x}_1, \ldots, \tilde{x}_n)$ by selecting production rates u and maintenance actions m is given by

$$p(x, \tilde{x} \mid u, m) = \prod_{i=1}^{n} P_{u_i, m_i}(x_i, \tilde{x}_i),$$

due to independence of the deterioration process of the units.

3.4 Expected costs

To define the cost for a time unit, we define a function

$$c(x_i, r_i) = \begin{cases} c_{\text{pm}} & \text{if } r_i = \text{M and } x_i \neq L, \\ c_{\text{cm}} & \text{if } r_i = \text{M and } x_i = L, \\ 0 & \text{elsewhere.} \end{cases}$$

for all states $x_i \in X$ and maintenance actions $r_i \in \{\text{DN}, \text{M}\}$ of unit $i \in \mathcal{I}$. This means that if unit i is in the failed state and we decide to perform maintenance, we incur the corrective maintenance costs c_{cm}, and if the unit has not failed, then the preventive maintenance costs c_{pm} is incurred.

The expected costs for a time unit by selecting maintenance action r and in state x is given by

$$\bar{c}(x, r) = \begin{cases} C + \sum_{i=1}^{n} c(x_i, r_i) & \text{if } r \text{ contains a maintenance action,} \\ 0 & \text{otherwise.} \end{cases}$$

In words, this means that if we do not perform maintenance on any unit for a given time period, we do not incur any costs. However, if we do perform maintenance, we incur the fixed cost C and some corrective or preventive costs depending on the state of the maintained unit.

3.5 Performance criteria

The performance criteria is to minimise the long-run average (maintenance) cost per time unit. This way, we can compare different condition-based maintenance and production strategies over the life cycle of the system.

4 Numerical results two-unit system

In this section I will discuss the key features of the optimal maintenance and production policy for a two-unit system using the MDP as described in Section 3. Similar to Keizer et al. (2018) we select preventive maintenance costs $c_{\text{pm}} = 5$, set-up cost $C = 4$, and a corrective maintenance cost $c_{\text{cm}} = 11$. The rate parameter under the maximum production rate is $\lambda_{\max} = 0.7$ and the scale parameter $\gamma = 1$. The failure level L and the number of possible production rates are set to 25 and 26, respectively. We vary the total production π with values 20, 30, 40 and 48. For $\pi = 20$, one unit can handle all the production, whereas for $\pi = 48$ we do not have much overcapacity in the system. We set $\alpha = 0.1$, such that the deterioration rate is low when the unit is not producing. We consider three different values for β: 0 (deterioration rate does not depend on production), 1 (deterioration rate depends linearly on production) and 1.5 (deterioration rate is convex in the production).

Figures 3-6 show the optimal policies systems with total production rates of 20, 30, 40 and 48. For every time period we first decide on the maintenance action and subsequently set a production rate for both units. For this reason we do not have to consider the production policies for states in which we perform maintenance. When the system can meet demand with one unit (Figure 3) we observe that maintenance is only performed if either both machines are in a highly deteriorated state, or one unit is a highly deteriorated state and the other is not. The last phenomenon could be explained by the fact that we want to bring the highly deteriorated unit back to the as-good-as-new state, such that, by setting its production rate higher than the other unit, we can bring the deterioration states close to each other to end up clustering the maintenance of both units. Furthermore, we observe that it is optimal to bring the deterioration state of the units close to each other by adjusting the production rate accordingly.

As we increase the total production, we observe a different optimal production policy. When unit 1 (2) is in a 'medium' deterioration state and unit 2 (1) is slightly higher (stated marked by the blue (pink) colour), it is optimal to try to bring the deterioration states close to each other by setting the production rate of unit 1 higher. However, when the difference in deterioration in sufficiently large, it is optimal to keep the expected difference in the deterioration state the same.

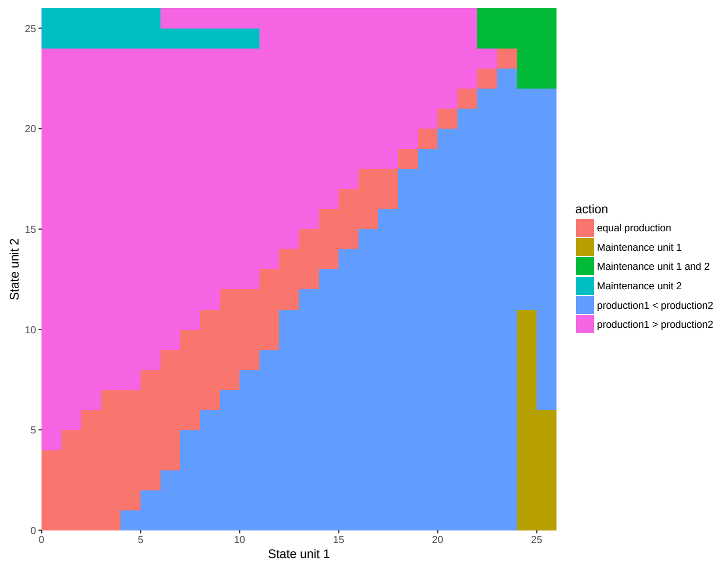

Figure 3: Optimal maintenance and production policy for a system with $N = 2, L = 25, m = 25, c_{\text{pm}} = 5, c_{\text{cm}} = 11, C = 4, \beta = 0.1, \alpha = 1.5, \pi = 20$

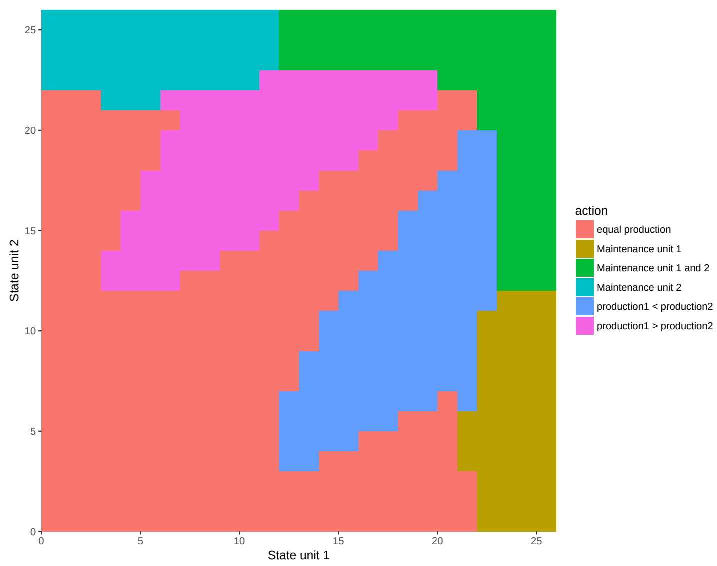

Figure 4: Optimal maintenance and production policy for a system with $N = 2, L = 25, m = 25, c_{\text{pm}} = 5, c_{\text{cm}} = 11, C = 4, \beta = 0.1, \alpha = 1.5, \pi = 30$

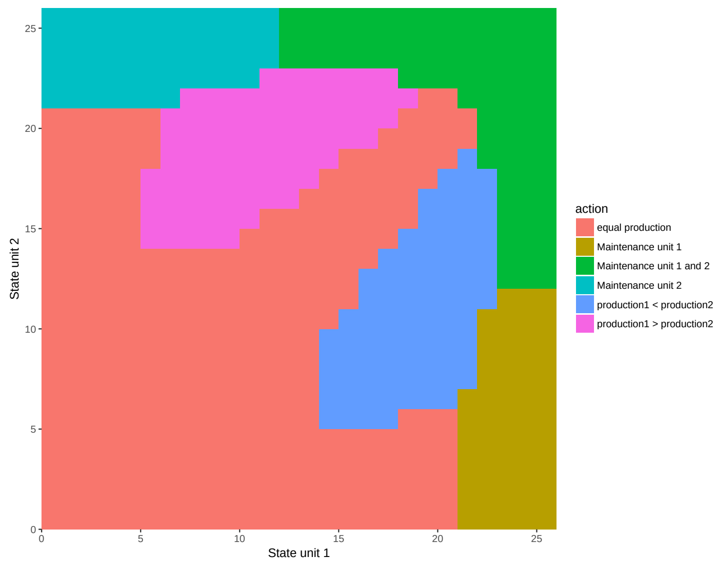

Figure 5: Optimal maintenance and production policy for a system with $N = 2, L = 25, m = 25, c_{\text{pm}} = 5, c_{\text{cm}} = 11, C = 4, \beta = 0.1, \alpha = 1.5, \pi = 40$

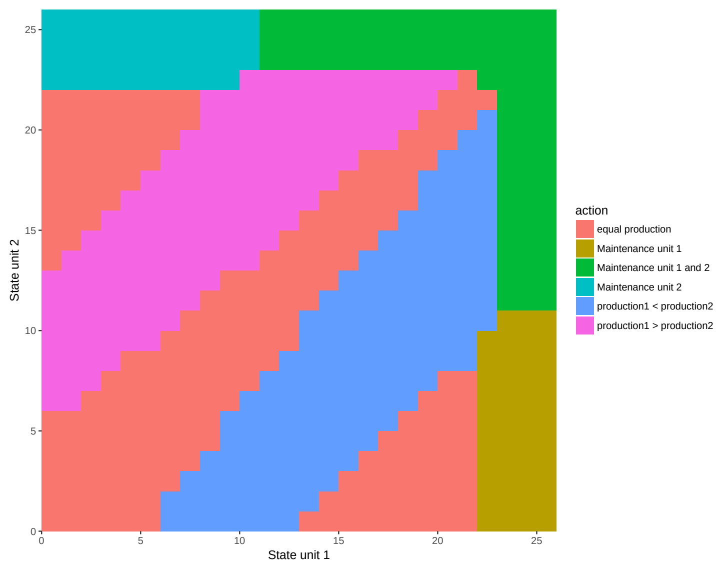

Figure 6: Optimal maintenance and production policy for a system with $N = 2, L = 25, m = 25, c_{\text{pm}} = 5, c_{\text{cm}} = 11, C = 4, \beta = 0.1, \alpha = 1.5, \pi = 48$

4.1 Benchmark policies

Since the action space of our problem consists of a maintenance action and a production action, we distinguish between a benchmark for the maintenance action and the production action.

In the literature, condition-based maintenance is often implemented in the form of an opportunistic threshold policy (Zhang and Zeng, 2015; Zhou, Xi, and Lee, 2009). For each time period t this heuristic works as follows:

1) Observe the deterioration states of all units, $i = 1, \dots, n$.

2) Maintain all units which have a deterioration state that exceed or equal a threshold T_r, i.e., if $X_i(t) \geq T_r$, $i = 1, \dots, n$.

3) If at least one maintenance action is initiated in step 2), also maintain all units for which $X_i(t) \geq T_o$, $i = 1, \dots, n$,

where $T_o \leq T_r$. The optimal threshold values for T_r and T_o are found using a grid search procedure, where the objective is to minimise the long-run average cost. Due to the economic dependence (fixed cost of maintenance) and the ability to set the production rate of the units, this policy is probably not optimal. From Figure 3, observe that the optimal maintenance policy is not an opportunistic policy.

Additionally, we need a benchmark policy for the production policy. Inspired by the load sharing formulation in Keizer et al. (2018), we will use the following benchmark for the production policy: For all system states determine the optimal selection of functioning units, where the selected units share the total production π uniformly. The optimal selection is found by minimising the long-run average cost per time unit.

To compare the optimal policy of the maintenance and production policy to the heuristic algorithms, we define the following combinations of heuristics:

- *Heuristic 1*: maintenance policy determined by the heuristic algorithm and an optimal production policy;
- *Heuristic 2*: an optimal maintenance policy and a heuristic production policy;
- *Heuristic 3*: heuristics for both the maintenance and production policy.

Heuristic 2 is used to quantifies the benefit of allowing for a variable production rate, whereas heuristic 3 is combination of popular policies that are widely used in practice.

Table 1 shows that for the different total productions the additional average costs can be as much as 3.5%. The additional average costs for heuristic 1 is lower than heuristic 2, [[**So....**]].

Table 1: The minimal long-run average cost per time unit for the heuristic policies with different values for π

Total production		Average cost		Optimal thresholds
	Heuristic 1	Heuristic 2	Heuristic 3	Heuristic 1
$\pi = 20$	0.1307 (+0.06%)	0.1320 (+1.80%)	0.1351 (+3.46%)	$T_r = 24$, $T_o = 16$
$\pi = 30$	0.6378 (+0.34%)	0.6404 (+0.76%)	0.6417 (+0.96%)	$T_r = 23$, $T_o = 15$
$\pi = 40$	0.6521 (+0.19%)	0.6543 (+0.56%)	0.6552 (+0.66%)	$T_r = 23$, $T_o = 13$
$\pi = 48$	0.5976 (+0.11%)	0.6047 (+1.28%)	0.6052 (+1.38%)	$T_r = 23$, $T_o = 11$

4.2 Sensitivity analysis with respect to the maintenance set-up cost

To investigate how the optimal policy and relative effectiveness of the optimal policy compared to the heuristics varies for different levels of economic dependence

4.3 Sensitivity analysis with respect to variation in the deterioration process

5 Numerical results three-unit system

6 Conclusion

Directions for future research in the direction of joint production and maintenance policies for multi-unit systems could be: 1) include dependence between the production capacity of units and its deterioration state, 2) include lead time for planning maintenance and 3) model dependence between the deterioration process of all units due to external shocks. The first extension could be realistic in some applications where the production of a unit is affected by the deterioration of that unit, e.g., semiconductor manufacturing (Kazaz and Sloan, 2013). The second direction is relevant for systems that have a high frequency of maintenance actions relative to the life cycle of the system. The last extension could be relevant when units are affected by, for example, weather conditions. It is realistic to assume that the deterioration of identical units is impacted in a similar way due to bad weather.

7 Notes

- I have not figured out yet how to set figures next to each other. My apologies.
- The colours of the optimal policy plots are disturbing. I do have ideas for different visualisations, however if you have any ideas, feel free to write them in your review.
- The practical relevance of heuristic 1 is still unclear for me. I might leave it out.
- I do not show absolute values of production in the optimal policy plot to not clutter the message, however I feel like key features of the optimal policy are missing in this way.

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A Appendix

A.1 Discretising the gamma process

See Maintenance planning and optimisation literature of Bram de Jonge.