Q1

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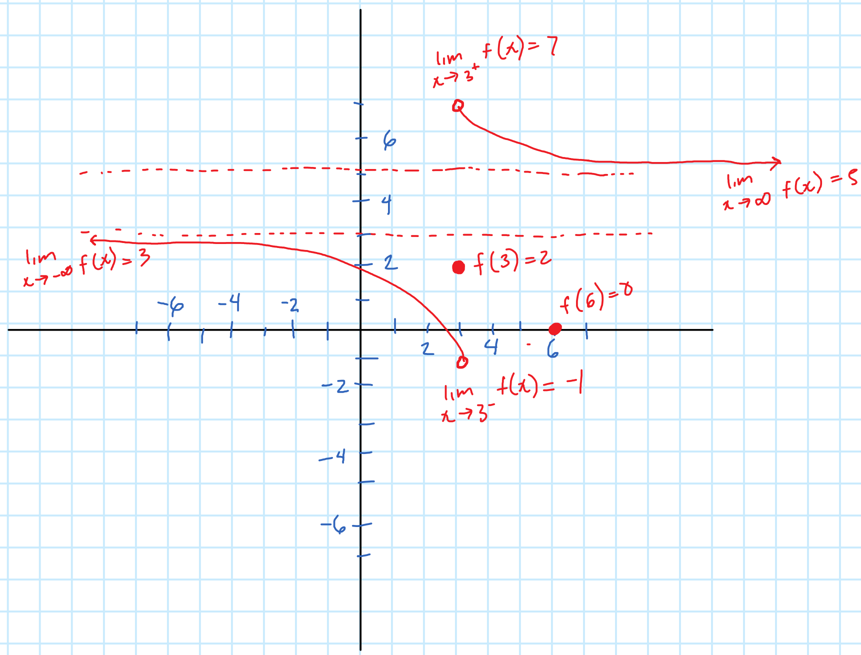
1) Sketch a graph of a function with all of the following properties:

$\lim_{x \to 3+} f(x) = 7$ $\lim_{x \to 3-} f(x) = -1$ f(3)=2

$\lim_{x \to \infty} f(x) = 5$ $\lim_{x \to -\infty} f(x) = 3$ f(6)=0

11 points



Q2

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2) Evaluate $\lim_{x \to 9} \frac{9-x}{3-\sqrt{x}}$. You must show all of your algebra. 11 points

$$\lim_{x \to 9} \frac{9-x}{3-\sqrt{x}} = \frac{9-(9)}{3-\sqrt{(9)}} = \frac{0}{0} = \emptyset$$

The limit as x approaches 9, DOES NOT EXIST

Q3

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3) Let $f(x) = \begin{cases} x^2 + 1 \; for \; x \geq 0 & \text{— Right} \\ \cos(x) \; for \; x < 0 & \text{— Left} \end{cases}$

Is f(x) continuous at x=0? Why or why not? Your explanation must include an <u>algebraic</u> argument. 11 points

$$\lim_{x \to 0^-} f(x) = \cos(x)$$
$$= \cos(0)$$
$$= 1$$

$$\boxed{\lim_{x \to 0^-} f(x) = 1}$$

$$\lim_{x \to 0^+} f(x) = x^2 + 1$$
$$= (0)^2 + 1$$
$$= 1$$

$$\boxed{\lim_{x \to 0^+} f(x) = 1}$$

$$\lim_{x \to 0^-} = \lim_{x \to 0^+}$$

Therefore, f(x) is continuous at x=0

Q4

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4) Evaluate $\lim_{x \to \infty} \frac{\sqrt{4x^2 + x}}{x - 3}$. You must show all of your algebra.

11 points

$$\lim_{x \to \infty} \frac{\sqrt{4x^2+x}}{x-3}$$

Take the highest term from both the numerator and denominator

$$\frac{\sqrt{4x^2}}{x} = \frac{2x}{x} = \boxed{2}$$

Q5

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5) Find $\lim_{x \to b} \frac{(x-b)^{5000} - x + b}{x-b}$ 11 points

$$\lim_{x \to b} \frac{(x-b)^{5000} - x + b}{x-b}$$

$$\frac{\cancel{(x-b)}(x-b)^{4999} - x + b}{\cancel{x-b}}$$

$$= (x-b)^{4999} - \frac{x+b}{x-b}$$

$$= (x-b)^{4999} - \frac{\cancel{x+b}}{-1\cancel{(x+b)}}$$

$$= (x-b)^{4999} + 1$$

$$\lim_{x \to b} (x-b)^{4999} + 1$$

$$= (b-b)^{4999} + 1 = (0)^{4999} + 1$$

$$= 1$$

$$\boxed{\lim_{x \to b} = 1}$$

Q6

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6) Let f(x)= $x^2 + 1$ 12 points

Find $lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ You must show all of your algebra. No using more advanced short cuts from later chapters.

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \qquad f(x) = x^2+1$$

$$\lim_{h \to 0} \frac{(x+h)^2+1-(x^2+1)}{h}$$

$$= \frac{\cancel{x^2}+2xh+h^2 \cancel{+1} \cancel{-x^2} \cancel{-1}}{h}$$

$$= \frac{2xh+h^2}{h} = \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$\lim_{h \to 0} = 2x+h$$

$$= 2x+(0)$$

$$\boxed{\lim_{h \to 0} = 2x}$$

Q7

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7) Evaluate $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{x^2+x}\right)$

11 points

$$\lim_{x\to 0} \frac{1}{x} - \frac{1}{x^2+x}$$

$$= \frac{1}{x}\frac{(x^2+x)}{(x^2+x)} - \frac{1}{x^2+x}\frac{(x)}{(x)}$$

$$= \frac{x^2+x}{x^3+x^2} - \frac{x}{x^3+x^2}$$

$$= \frac{x^2}{x^3+x^2}$$

$$= \frac{\cancel{x^2}}{\cancel{x^2}(x+1)}$$

$$\lim_{x\to 0} = \frac{1}{x+1} = \frac{1}{0+1} = \frac{1}{1} = \boxed{1}$$

Q8

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8) Let $f(x) = \frac{x^2-x-6}{x^2-16}$ 11 points

Find all horizontal and vertical asymptotes.

$$f(x) = \frac{x^2 - x - 6}{x^2 - 16}$$

Take the highest term from both the numerator and denominator

$$\frac{x^2}{x^2} = 1$$

$$\boxed{y = 1}$$

$$\frac{x^2 - x - 6}{x^2 - 16} = \frac{x^2 - x - 6}{(x-4)(x+4)}$$

$$\boxed{x = 4,\ x = -4}$$

Q9

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9) Let $f(x) = \frac{1}{x}$ 12 points

Find $\lim_{x\to 2} \frac{f(x)-f(a)}{x-a}$ You must show all of your algebra. No using more advanced short cuts from later chapters.

$$\lim_{x\to 2} \frac{f(x)-f(a)}{x-a}, \quad f(x) = \frac{1}{x}$$

$$\lim_{x\to 2} \frac{\frac{1}{x}-\frac{1}{a}}{x-a} = \frac{\frac{1}{x}\frac{(a)}{(a)} - \frac{1}{a}\frac{(x)}{(x)}}{x-a}$$

$$= \frac{\frac{a}{xa}-\frac{x}{xa}}{x-a} = \frac{a-x}{xa}\cdot\frac{1}{x-a}$$

$$= \frac{a-x}{xa(x-a)} = \frac{-1\cancel{(-a+x)}}{xa\cancel{(x-a)}}$$

$$= \frac{-1}{xa}$$

$$\lim_{x\to 2} \frac{-1}{xa} = \frac{-1}{(2)a} = \boxed{-\frac{1}{2}a}$$