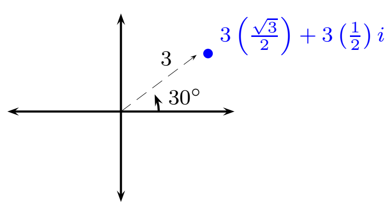
Euler's Identity

1. **Convert** Convert $3e^{i30^\circ}$ to standard form, $a + bi$

Solution: According to Euler's amazing identity..$e^{i\theta} = \cos\theta + i\sin\theta$..thus..

$$\begin{aligned} 3e^{i30^\circ} &= 3[\cos 30^\circ + i\sin 30^\circ] \\ &= 3\left[\frac{\sqrt{3}}{2} + i\frac{1}{2}\right] \\ &= \frac{3\sqrt{3}}{2} + i\frac{3}{2} \end{aligned}$$

Alternatively, you could convert using the picture... and convert in very much the same way we convert from polar to cartesian...



2. **Convert** Convert $3e^{-i30^\circ}$ to standard form, $a + bi$

Solution: According to Euler's amazing identity..$e^{i\theta} = \cos\theta + i\sin\theta$..thus..

$$\begin{aligned} 3e^{-i30^\circ} &= 3[\cos -30^\circ + i\sin -30^\circ] \\ &= 3\left[\frac{\sqrt{3}}{2} + i\frac{-1}{2}\right] \\ &= \frac{3\sqrt{3}}{2} + \frac{-3}{2}i \end{aligned}$$

Or use the picture... Or use polar conversion ideas.. $(r, \theta) = (3, -30^\circ)$

3. **Convert** Convert $e^{i\frac{\pi}{2}}$ to standard form, $a + bi$

Solution: According to Euler's amazing identity..$e^{i\theta} = \cos\theta + i\sin\theta$..thus..

$$\begin{aligned} e^{i\frac{\pi}{2}} &= \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} \\ &= 0 + i(1) \\ &= i \end{aligned}$$

4. **Convert** Convert $e^{\frac{3\pi}{2}i}$ to standard form, $a + bi$
5. **Convert** Convert $e^{\frac{7\pi}{6}i}$ to standard form, $a + bi$
6. **Convert** Convert $3e^{2i}$ to standard form, $a + bi$
7. **Convert** Convert $5e^{3.14i}$ to standard form, $a + bi$
8. **Convert** Convert $e^{6.28i}$ to standard form, $a + bi$
9. **Convert** Convert $2 + i$ to Euler form, $re^{i\theta}$

Solution: $2 + i = \sqrt{5}e^{i26.57^\circ}$ many other correct possibilities for theta..

10. **Convert** Convert $2 - 2i$ to Euler form, $re^{i\theta}$

Solution: $2 - 2i = (2\sqrt{2})e^{-i45^\circ}$ OR $(2\sqrt{2})e^{i315^\circ}$ or $(2\sqrt{2})e^{i765^\circ}$ or infinite many more possibilities for θ

11. **Convert** Convert $-2 - 2i$ to Euler form, $re^{i\theta}$

Solution: $-2 - 2i = (2\sqrt{2})e^{i225^\circ}$ many other correct possibilities for theta..

12. **Convert** Convert $-3 + 2i$ to Euler form, $re^{i\theta}$
13. **Convert** Convert i to Euler form, $re^{i\theta}$

Solution: $i = e^{i90^\circ}$ many other correct possibilities for theta.. note $r = 1$

14. **Convert** Convert $3i$ to Euler form, $re^{i\theta}$

Solution: $i = 3e^{i90^\circ}$ many other correct possibilities for theta.. note $r = 1$

15. **Convert** Convert $.5 + 3i$ to Euler form, $re^{i\theta}$
16. **Convert** Convert -1 to Euler form, $re^{i\theta}$

Solution: $i = e^{i180^\circ}$ many other correct possibilities for theta.. note $r = 1$

17. **Convert** Convert $-i$ to Euler form, $re^{i\theta}$

> **Solution:** $i = e^{i270^\circ}$ many other correct possibilities for theta.. note $r = 1$

18. **Euler's World** Multiply or divide as indicated

    (a) Multiply the old fashion way...

$$\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) \cdot \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

    (b) Multiply the Euler way, and simplify answer.. (note these are the same numbers..)

$$e^{30^\circ i} \cdot e^{60^\circ i}$$

> **Solution:** $e^{i90^\circ} = i$

19. **Euler's World** Multiply or divide as indicated

    (a) Multiply the old fashion way...

$$\left(\frac{3\sqrt{3}}{2} + \frac{3}{2}i\right) \cdot \left(\frac{2}{2} + \frac{2\sqrt{3}}{2}i\right)$$

    (b) Multiply the Euler way, and simplify answer.. (note these are the same numbers..)

$$3e^{30^\circ i} \cdot 2e^{60^\circ i}$$

> **Solution:** $6e^{i90^\circ} = 6i$

20. **Euler's World** Multiply or divide as indicated

    (a) Multiply the old fashion way...

$$\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) \cdot \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$$

    (b) Multiply the Euler way, and simplify answer.. (note these are the same numbers..)

$$e^{30^\circ i} \cdot e^{30^\circ i}$$

> **Solution:** e^{i60°

21. **Euler's World** Multiply or divide as indicated

    (a) Divide the old fashion way...

$$\frac{\frac{\sqrt{3}}{2} + \frac{1}{2}i}{\frac{1}{2} + \frac{\sqrt{3}}{2}i}$$

    (b) Divide the Euler way, and simplify answer.. (note these are the same numbers..)

$$\frac{e^{30^\circ i}}{e^{60^\circ i}}$$

> **Solution:** e^{-i30°

22. **Euler's World** Multiply or divide as indicated

    (a) Expand the binomial, the old fashion way...

$$\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^3$$

    (b) Calculate, the Euler way, and simplify answer.. (note these are the same numbers..)

$$\left(e^{30^\circ i}\right)^3$$

Solution: $e^{i90^\circ} = i$ compare this problem with #57 from two sections ago.

23. **Euler's World** Multiply or divide as indicated

(a) Expand the binomial, the old fashion way (or not..) ...

$$\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^7$$

(b) Calculate, the Euler way, and simplify answer.. (note these are the same numbers..)

$$\left(e^{30^\circ i}\right)^7$$

Solution: e^{i210°

24. **Euler's World** Convert the numbers to Euler form, then compute.

(a)

$$(3+4i)(-2-5i)$$

(b)

$$\frac{3+4i}{-2-5i}$$

25. **Euler's World** to find Roots

(a) Try to find a square root of i, i.e. a number x such that $x^2 = i$

$$\sqrt{i}$$

(b) Find a root, the Euler way, and simplify answer.. (note these are the same numbers..)

$$\left(e^{90^\circ i}\right)^{\frac{1}{2}}$$

Solution: e^{i45°

26. **Euler's World** to find Roots

(a) Try to find a square root: (after a good honest attempt, go to part b.)

$$\sqrt{\frac{1}{2} + \frac{\sqrt{3}}{2}i}$$

(b) Find a root, the Euler way, and simplify answer.. (note these are the same numbers..)

$$\left(e^{60^\circ i}\right)^{\frac{1}{2}}$$

Solution: e^{i30°

27. **Euler's World** to find Roots

(a) Try to find a fourth root: (after a good honest attempt, go to part b.)

$$\sqrt[4]{\frac{16}{2} + \frac{16\sqrt{3}}{2}i}$$

(b) Find a fourth root, the Euler way, and simplify answer.. (note these are the same numbers..)

$$\left(16e^{60^\circ i}\right)^{\frac{1}{4}}$$

Solution: $2e^{i15^\circ}$

28. **FTA and Unity Roots** The two solutions to $x^2 = 1$ are $x = 1, -1$. Show how one can derive these solutions.

29. **FTA and Unity Roots** The three solutions to $x^3 = 1$ are $x = 1$, $\frac{-1}{2} + \frac{\sqrt{3}}{2}i$, $\frac{-1}{2} - \frac{\sqrt{3}}{2}i$. Show how one can derive these solutions.

30. **FTA and Unity Roots** The four solutions to $x^4 = 1$ are $x = 1, i, -1, -i$. Show how one can derive these solutions.

31. **FTA and Unity Roots** To find the five solutions to $x^5 = 1$, start with writing $1 = e^{k360^\circ i}$, then start finding 5th roots, every value of k should produce one...

Solution: for integers k, e^{ik72°

32. **Euler's World** Another Proof of MOTA

(a) Multiply the old fashion way

$$(\cos x + i \sin x)(\cos y + i \sin y)$$

(b) multiply, the Euler way, and simplify answer.. (note these are the same numbers..)

$$e^{ix} \cdot e^{iy}$$

(c) Compare the real part of the answer for a, with the real part of the answer for part b.

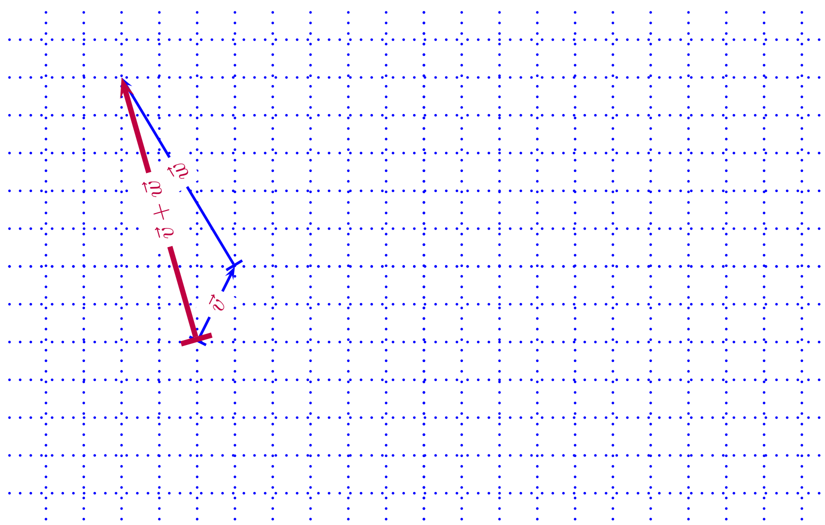
1. **Vector Arithmetic**

   (a) **Vector ADDITION** Suppose $\vec{v} = <1,2>$ and $\vec{w} = <-3,5>$, compute

$$\vec{v} + \vec{w}$$

Solution:

$$\begin{aligned} \vec{v} + \vec{w} &= <1,2> + <-3,5> && \text{(given)} \\ &= <1 + -3, \quad 2 + 5> && \text{(definition of addition on vectors)} \\ &= <-2,7> && \text{(by inspection)} \end{aligned}$$

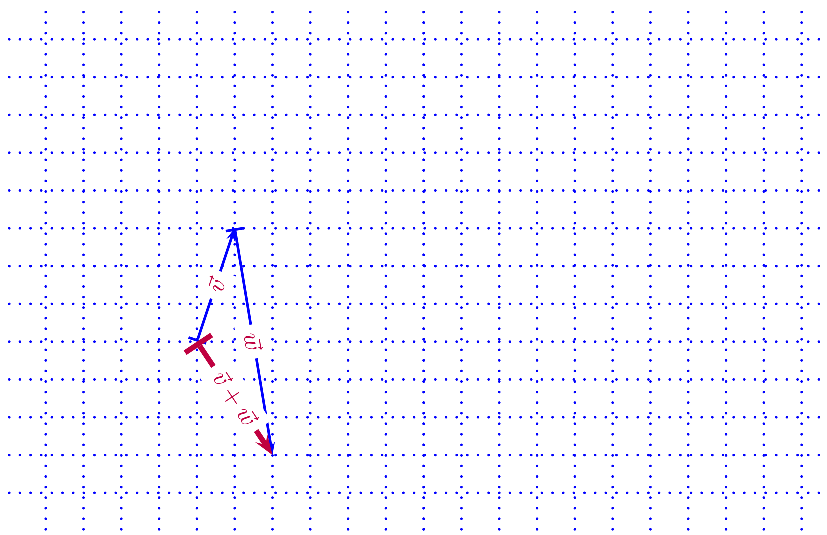


   (b) **Vector ADDITION** Suppose $\vec{v} = <1,3>$ and $\vec{w} = <1,-6>$, compute

$$\vec{v} + \vec{w}$$

Solution:

$$\begin{aligned} \vec{v} + \vec{w} &= <1,3> + <1,-6> && \text{(given)} \\ &= <1 + 1, \quad 3 + -6> && \text{(definition of addition on vectors)} \\ &= <2, -3> && \text{(by inspection)} \end{aligned}$$

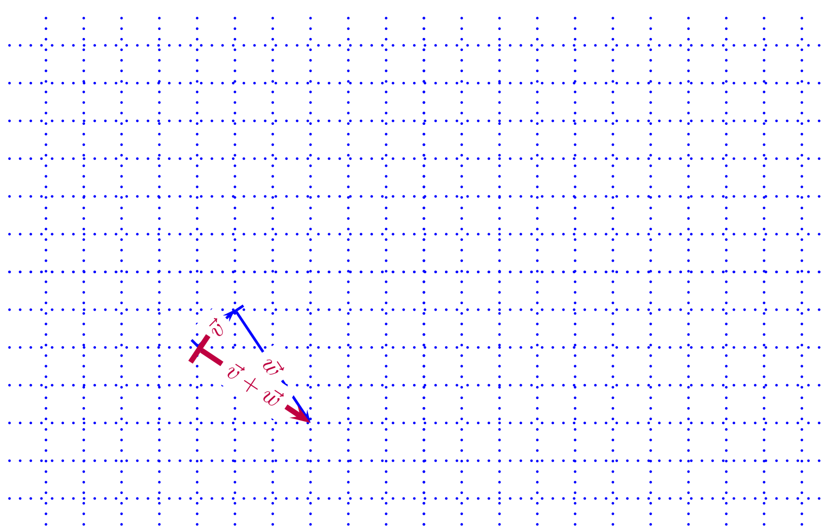


(c) **Vector ADDITION** Suppose $\vec{v} = <1,1>$ and $\vec{w} = <2,-3>$, compute

$$\vec{v}+\vec{w}$$

Solution:

$$\begin{aligned}\vec{v}+\vec{w} &= <1,1> + <2,-3> && \text{(given)}\\ &= <1+2, \quad 1+-3> && \text{(definition of addition on vectors)}\\ &= <3, \ -2> && \text{(by inspection)}\end{aligned}$$

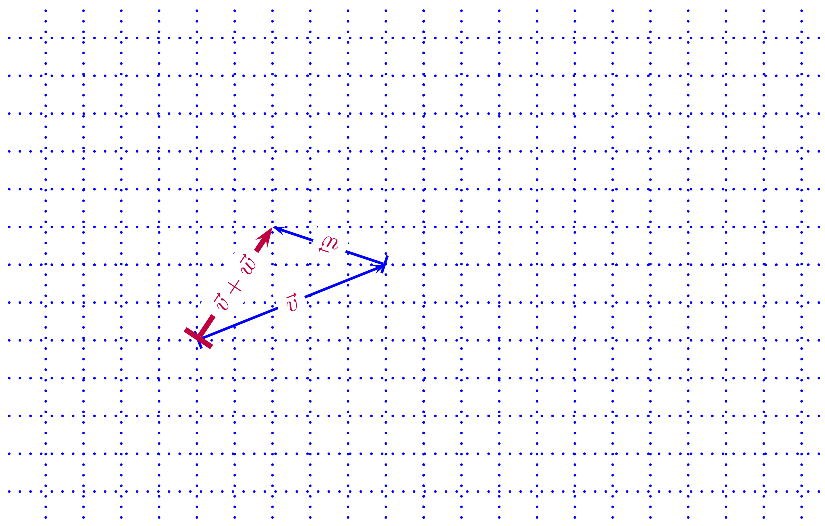


(d) **Vector ADDITION** Suppose $\vec{v} = <5,2>$ and $\vec{w} = < -3,1>$, compute

$$\vec{v}+\vec{w}$$

Solution:

$$\begin{aligned}\vec{v}+\vec{w} &= <5,2> + < -3,1> && \text{(given)}\\ &= <5+-3, \quad 2+1> && \text{(definition of addition on vectors)}\\ &= <2,3> && \text{(by inspection)}\end{aligned}$$

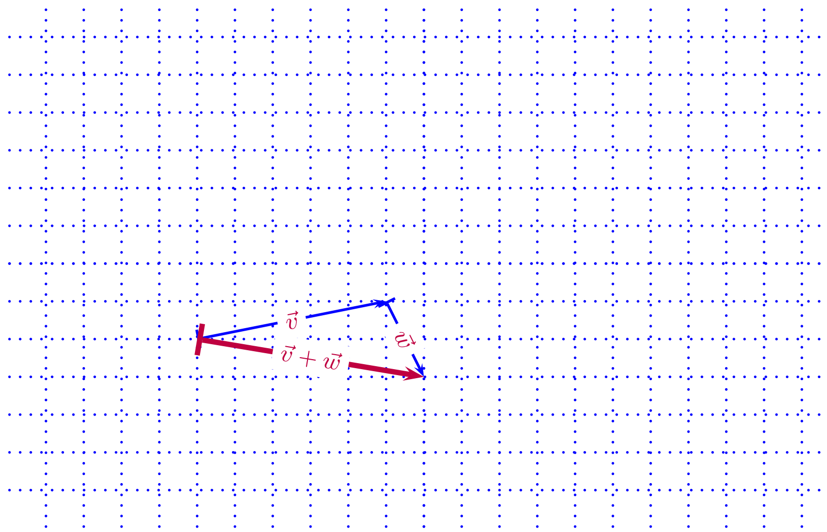


(e) **Vector ADDITION** Suppose $\vec{v} = <5,1>$ and $\vec{w} = <1,-2>$, compute

$$\vec{v}+\vec{w}$$

Solution:

$$\begin{aligned}\vec{v}+\vec{w} &= <5,1> + <1,-2> && \text{(given)}\\ &= <5+1, \quad 1+-2> && \text{(definition of addition on vectors)}\\ &= <6, -1> && \text{(by inspection)}\end{aligned}$$

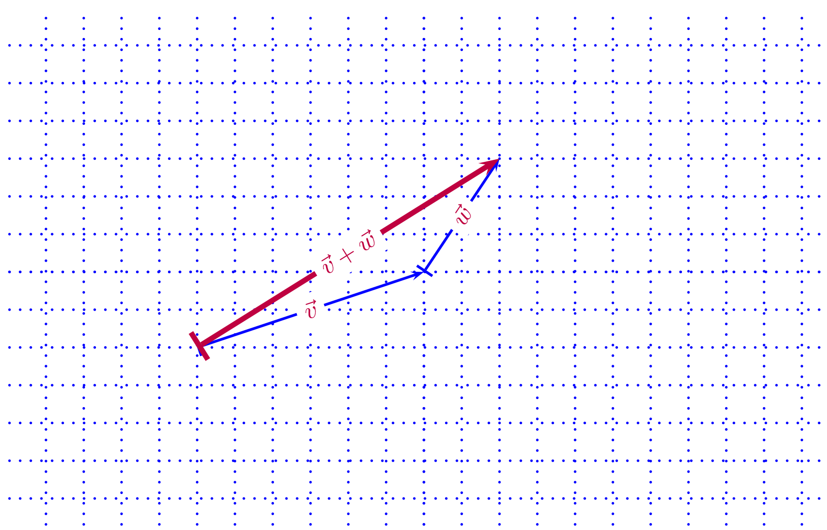


(f) **Vector ADDITION** Suppose $\vec{v} = <6,2>$ and $\vec{w} = <2,3>$, compute

$$\vec{v}+\vec{w}$$

Solution:

$$\begin{aligned} \vec{v}+\vec{w} &= <6,2> + <2,3> && \text{(given)} \\ &= <6+2, \quad 2+3> && \text{(definition of addition on vectors)} \\ &= <8,5> && \text{(by inspection)} \end{aligned}$$

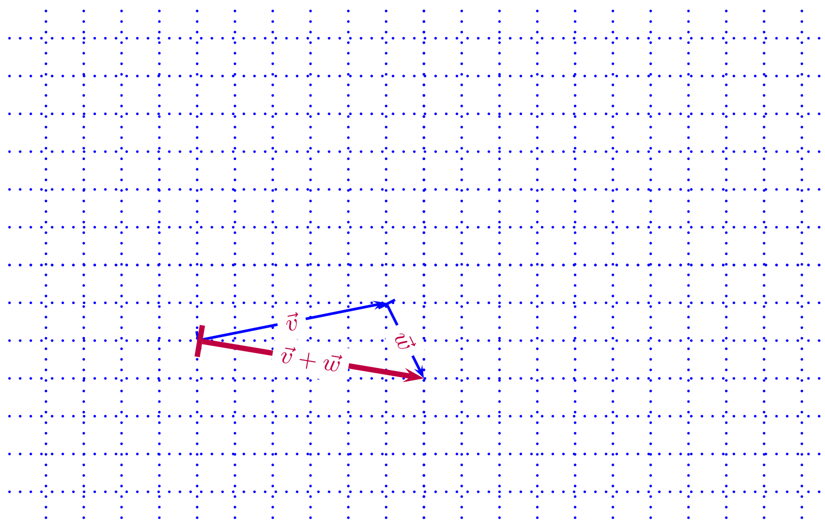


(g) **Vector ADDITION** Suppose $\vec{v} = <5,1>$ and $\vec{w} = <1,-2>$, compute

$$\vec{v}+\vec{w}$$

Solution:

$$\begin{aligned}
\vec{v} + \vec{w} &= <5, 1> + <1, -2> && \text{(given)}\\
&= <5 + 1, \quad 1 + -2> && \text{(definition of addition on vectors)}\\
&= <6, -1> && \text{(by inspection)}
\end{aligned}$$

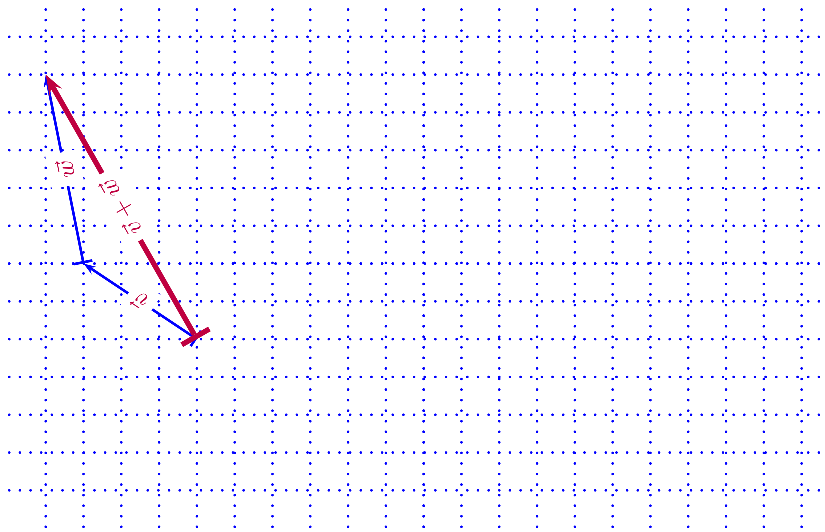


(h) **Vector ADDITION** Suppose $\vec{v} = <-3, 2>$ and $\vec{w} = <-1, 5>$, compute

$$\vec{v} + \vec{w}$$

Solution:

$$\begin{aligned}
\vec{v} + \vec{w} &= <-3, 2> + <-1, 5> && \text{(given)}\\
&= <-3 + -1, \quad 2 + 5> && \text{(definition of addition on vectors)}\\
&= <-4, 7> && \text{(by inspection)}
\end{aligned}$$



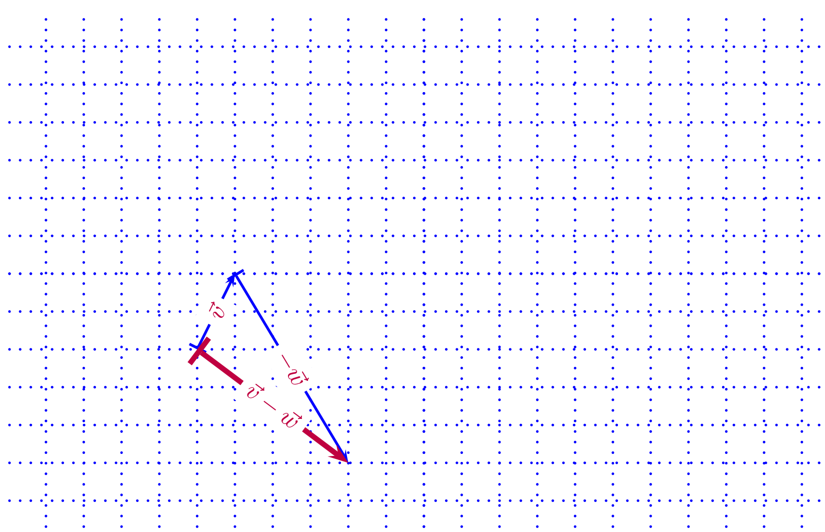
2. **Vector Arithmetic**

   (a) **Vector SUBTRACTION** Suppose $\vec{v} = <1, 2>$ and $\vec{w} = < -3, 5>$, compute

$$\vec{v} - \vec{w}$$

Solution:

$$\begin{aligned} \vec{v} - \vec{w} &= <1, 2> - < -3, 5> && \text{(given)} \\ &= <1 - -3, \quad 2 - 5> && \text{(definition of subtraction on vectors)} \\ &= <4, -3> && \text{(by inspection)} \end{aligned}$$

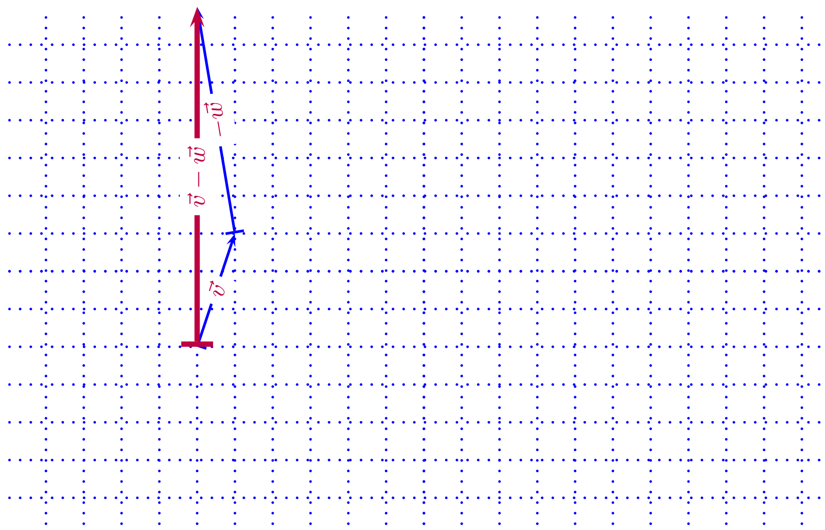


   (b) **Vector SUBTRACTION** Suppose $\vec{v} = <1, 3>$ and $\vec{w} = <1, -6>$, compute

$$\vec{v} - \vec{w}$$

Solution:

$$\begin{aligned} \vec{v} - \vec{w} &= <1,3> - <1,-6> && \text{(given)} \\ &= <1-1, \quad 3 - -6> && \text{(definition of subtraction on vectors)} \\ &= <0,9> && \text{(by inspection)} \end{aligned}$$

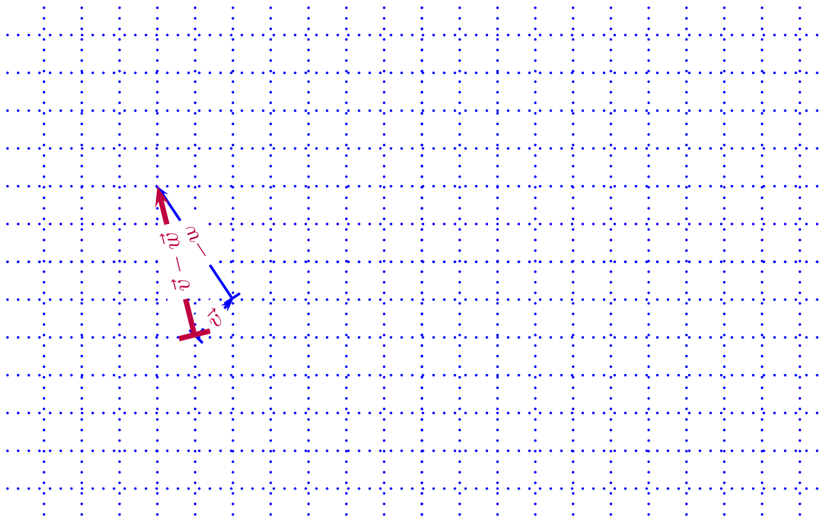


(c) **Vector SUBTRACTION** Suppose $\vec{v} = <1,1>$ and $\vec{w} = <2,-3>$, compute

$$\vec{v} - \vec{w}$$

Solution:

$$\begin{aligned} \vec{v} - \vec{w} &= <1,1> - <2,-3> && \text{(given)} \\ &= <1-2, \quad 1 - -3> && \text{(definition of subtraction on vectors)} \\ &= <-1,4> && \text{(by inspection)} \end{aligned}$$

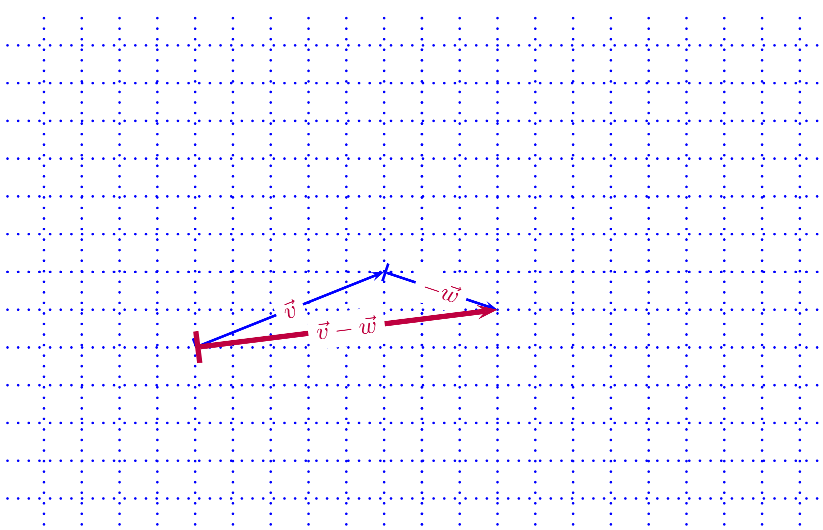


(d) **Vector SUBTRACTION** Suppose $\vec{v} = <5, 2>$ and $\vec{w} = < -3, 1>$, compute

$$\vec{v} - \vec{w}$$

Solution:

$$\begin{aligned} \vec{v} - \vec{w} &= <5, 2> - < -3, 1> && \text{(given)} \\ &= <5 - -3, \quad 2 - 1> && \text{(definition of subtraction on vectors)} \\ &= <8, 1> && \text{(by inspection)} \end{aligned}$$

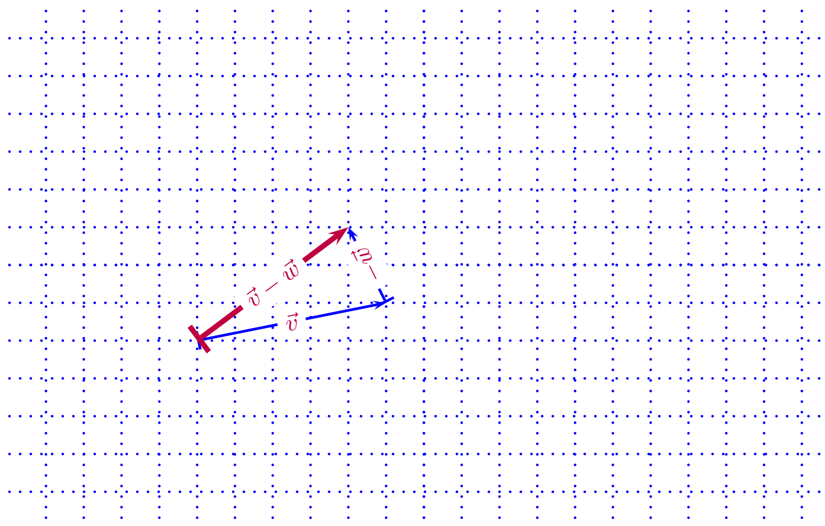


(e) **Vector SUBTRACTION** Suppose $\vec{v} = <5, 1>$ and $\vec{w} = <1, -2>$, compute

$$\vec{v} - \vec{w}$$

Solution:

$$\begin{aligned}
\vec{v} - \vec{w} &= <5, 1> - <1, -2> && \text{(given)}\\
&= <5 - 1, \quad 1 - -2> && \text{(definition of subtraction on vectors)}\\
&= <4, 3> && \text{(by inspection)}
\end{aligned}$$

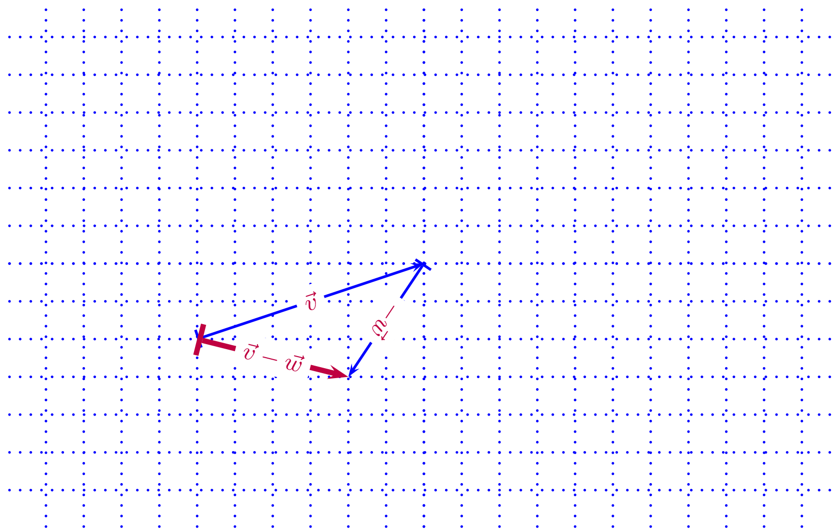


(f) **Vector SUBTRACTION** Suppose $\vec{v} = <6, 2>$ and $\vec{w} = <2, 3>$, compute

$$\vec{v} - \vec{w}$$

Solution:

$$\begin{aligned}
\vec{v} - \vec{w} &= <6, 2> - <2, 3> && \text{(given)}\\
&= <6 - 2, \quad 2 - 3> && \text{(definition of subtraction on vectors)}\\
&= <4, -1> && \text{(by inspection)}
\end{aligned}$$

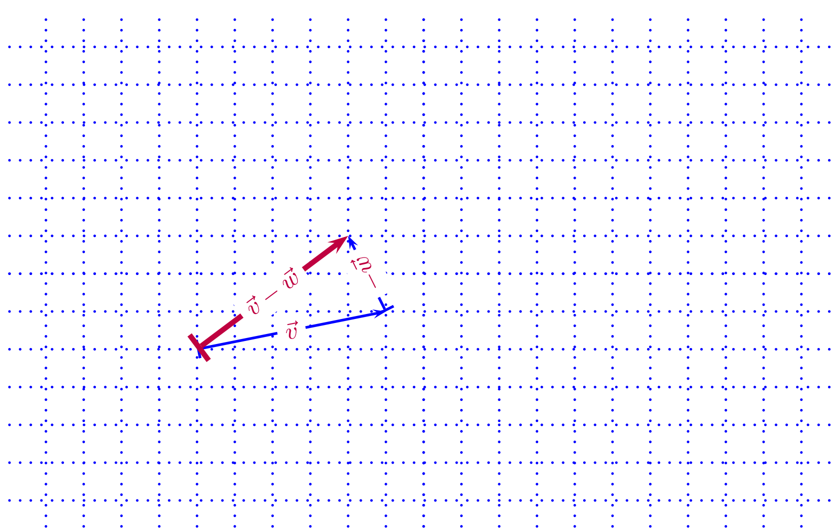


(g) **Vector SUBTRACTION** Suppose $\vec{v} = <5, 1>$ and $\vec{w} = <1, -2>$, compute

$$\vec{v} - \vec{w}$$

Solution:

$$
\begin{aligned}
\vec{v} - \vec{w} &= <5, 1> - <1, -2> && \text{(given)}\\
&= <5 - 1, \quad 1 - -2> && \text{(definition of subtraction on vectors)}\\
&= <4, 3> && \text{(by inspection)}
\end{aligned}
$$

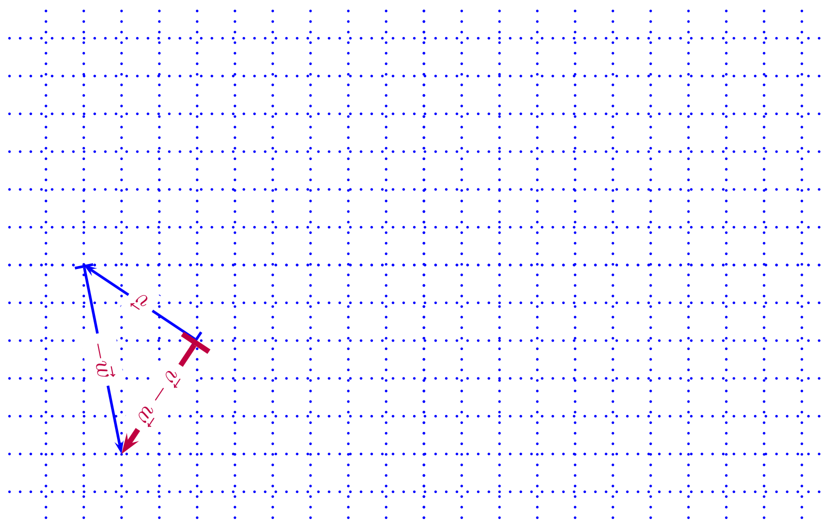


(h) **Vector SUBTRACTION** Suppose $\vec{v} = < -3, 2>$ and $\vec{w} = < -1, 5>$, compute

$$\vec{v} - \vec{w}$$

Solution:

$$\begin{aligned}
\vec{v} - \vec{w} &= <-3,2> - <-1,5> && \text{(given)}\\
&= <-3 - -1, \quad 2-5> && \text{(definition of subtraction on vectors)}\\
&= <-2,-3> && \text{(by inspection)}
\end{aligned}$$



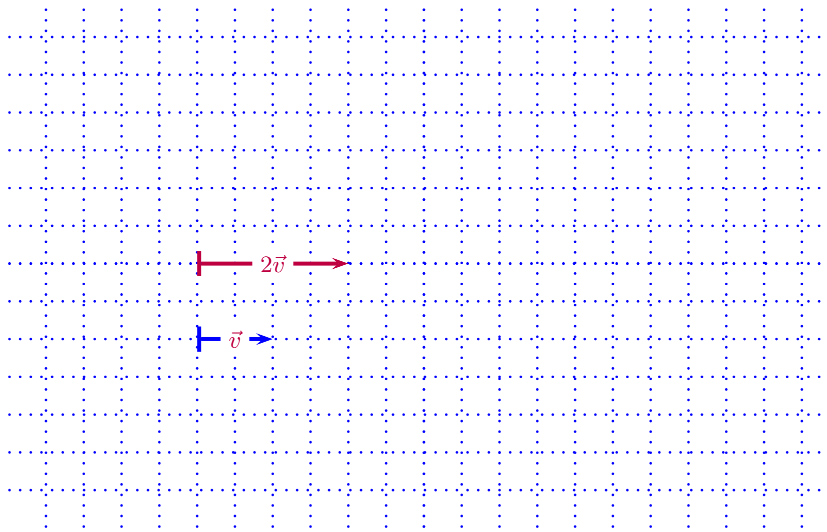
3. **Vector Arithmetic**

(a) **Vector SCALARS** Suppose $\vec{v} = <2,0>$compute

$$2\vec{v}$$

Solution:

$$\begin{aligned}
2\vec{v} &= 2<2,0> && \text{(given)}\\
&= <2(2), 2(0)> && \text{(def of scalar multiplication)}\\
&= <4,0> && \text{(by inspection)}
\end{aligned}$$

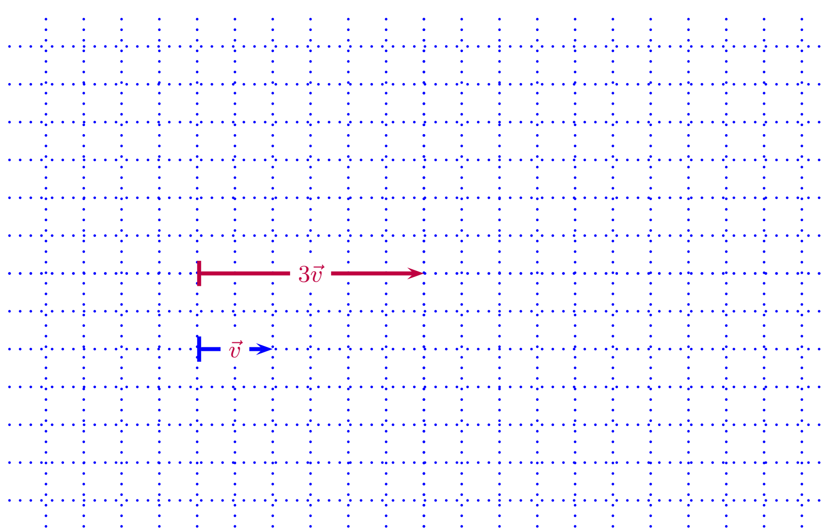


(b) **Vector SCALARS** Suppose $\vec{v} = <2, 0>$compute

$$3\vec{v}$$

Solution:

$$\begin{aligned} 3\vec{v} &= 3<2, 0> && \text{(given)} \\ &= <3(2), 3(0)> && \text{(def of scalar multiplication)} \\ &= <6, 0> && \text{(by inspection)} \end{aligned}$$

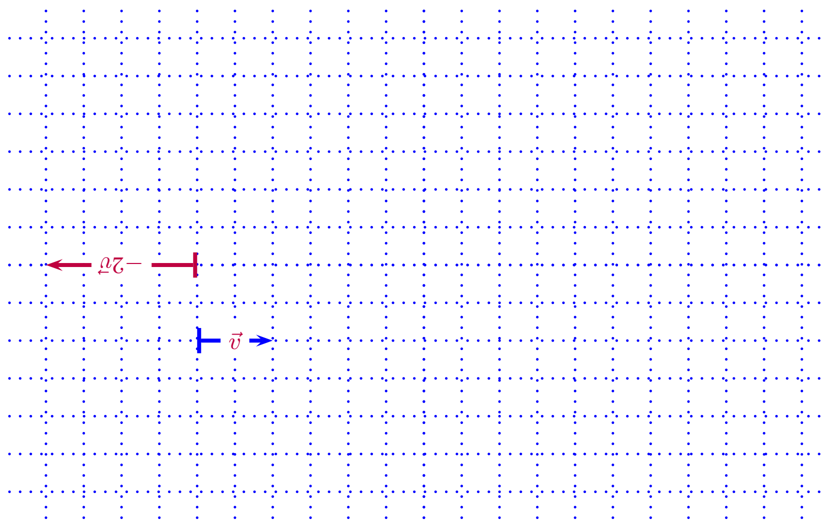


(c) **Vector SCALARS** Suppose $\vec{v} = <2, 0>$compute

$$-2\vec{v}$$

Solution:

$$\begin{aligned} -2\vec{v} &= -2{<}2, 0{>} && \text{(given)} \\ &= {<} -2(2), -2(0){>} && \text{(def of scalar multiplication)} \\ &= {<} -4, 0{>} && \text{(by inspection)} \end{aligned}$$

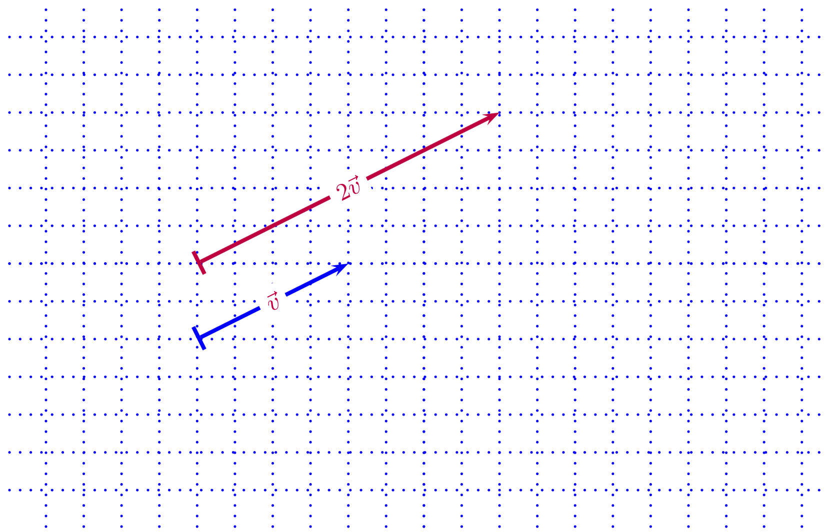


(d) **Vector SCALARS** Suppose $\vec{v} = {<}4, 2{>}$compute

$$2\vec{v}$$

Solution:

$$\begin{aligned} 2\vec{v} &= 2{<}4, 2{>} && \text{(given)} \\ &= {<}2(4), 2(2){>} && \text{(def of scalar multiplication)} \\ &= {<}8, 4{>} && \text{(by inspection)} \end{aligned}$$

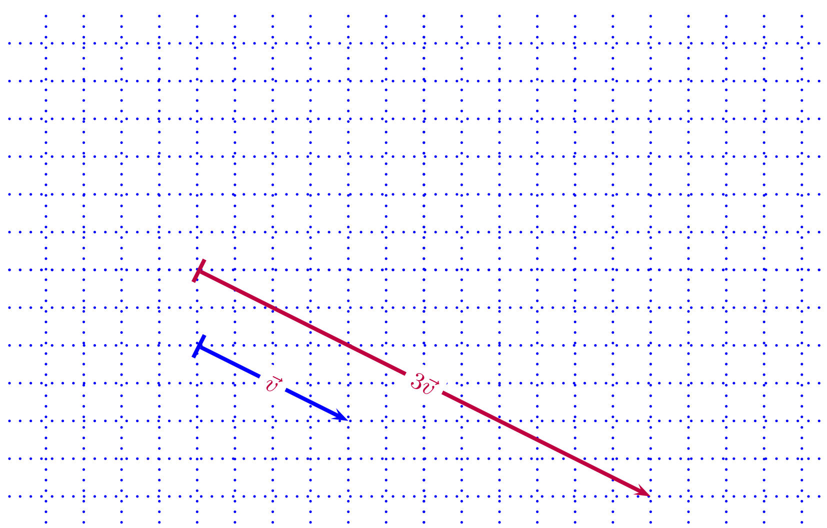


(e) **Vector SCALARS** Suppose $\vec{v} = <4, -2>$compute

$$3\vec{v}$$

Solution:

$$\begin{aligned} 3\vec{v} &= 3<4, -2> && \text{(given)} \\ &= <3(4), 3(-2)> && \text{(def of scalar multiplication)} \\ &= <12, -6> && \text{(by inspection)} \end{aligned}$$

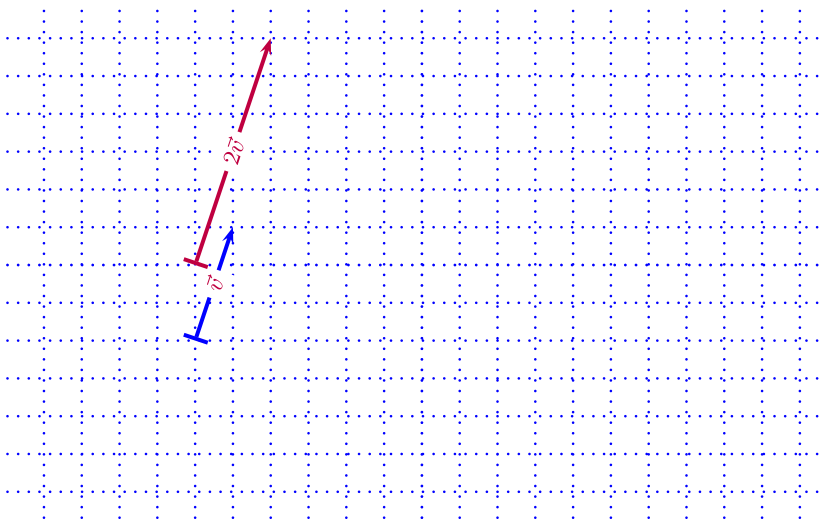


(f) **Vector SCALARS** Suppose $\vec{v} = <1, 3>$compute

$$2\vec{v}$$

Solution:

$$\begin{aligned} 2\vec{v} &= 2<1,3> && \text{(given)} \\ &= <2(1),2(3)> && \text{(def of scalar multiplication)} \\ &= <2,6> && \text{(by inspection)} \end{aligned}$$

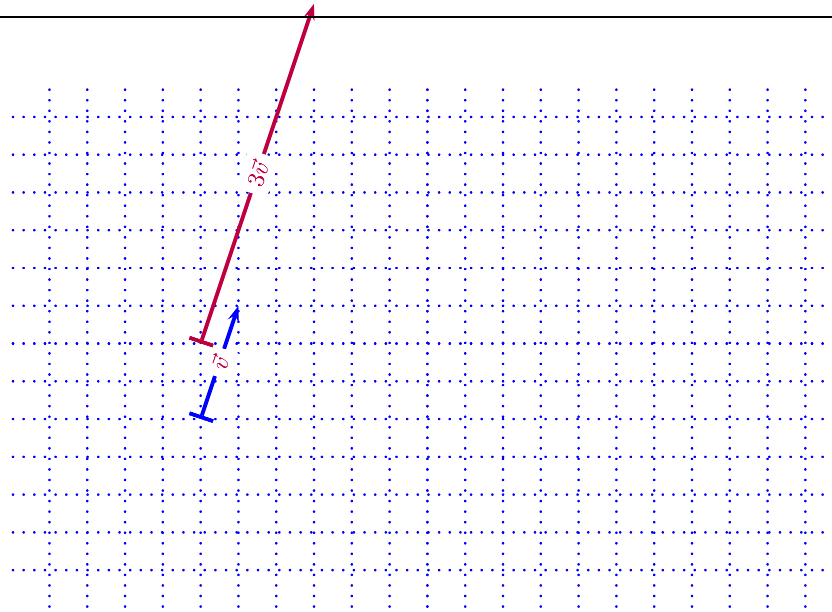


(g) **Vector SCALARS** Suppose $\vec{v} = <1,3>$compute

$$3\vec{v}$$

Solution:

$$\begin{aligned} 3\vec{v} &= 3<1,3> && \text{(given)} \\ &= <3(1),3(3)> && \text{(def of scalar multiplication)} \\ &= <3,9> && \text{(by inspection)} \end{aligned}$$

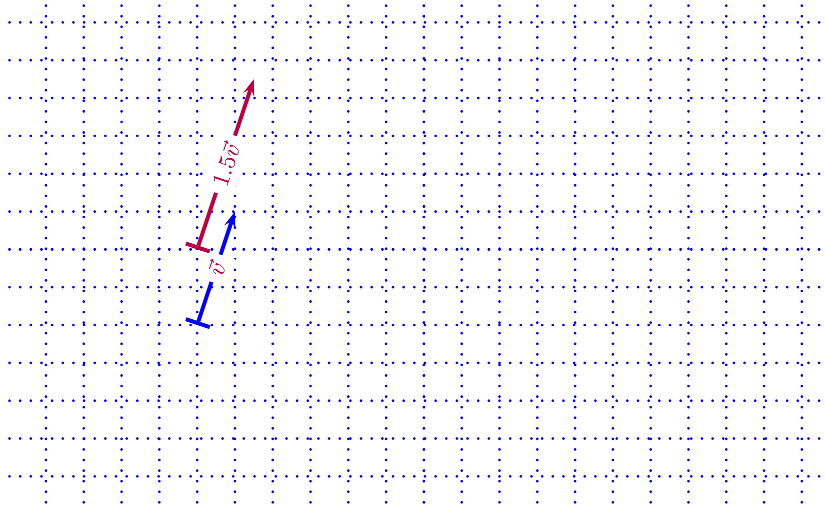


(h) **Vector SCALARS** Suppose $\vec{v} = <1, 3>$compute

$$1.5\vec{v}$$

Solution:

$$\begin{aligned} 1.5\vec{v} &= 1.5<1, 3> && \text{(given)} \\ &= <1.5(1), 1.5(3)> && \text{(def of scalar multiplication)} \\ &= <1.5, 4.5> && \text{(by inspection)} \end{aligned}$$

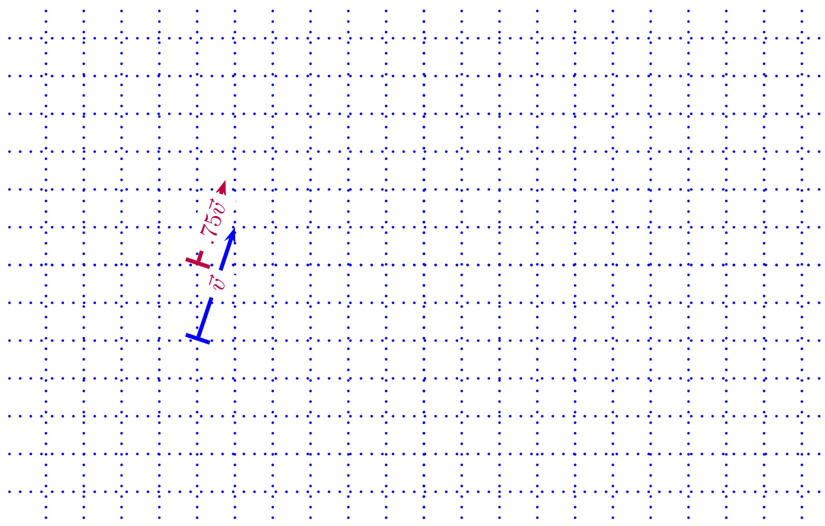


(i) **Vector SCALARS** Suppose $\vec{v} = <1, 3>$compute

$$.75\vec{v}$$

Solution:

$$\begin{aligned} .75\vec{v} &= .75{<}1,3{>} && \text{(given)} \\ &= {<}.75(1), .75(3){>} && \text{(def of scalar multiplication)} \\ &= {<}0.75, 2.25{>} && \text{(by inspection)} \end{aligned}$$

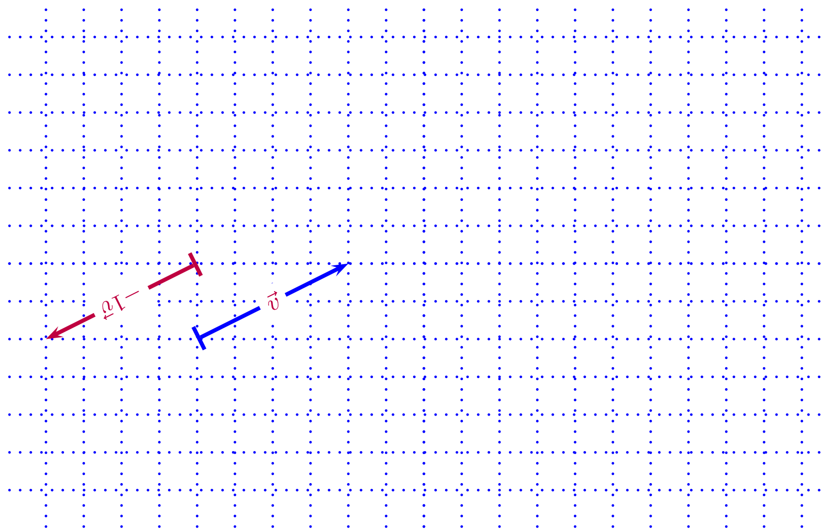


(j) **Vector SCALARS** Suppose $\vec{v} = {<}4,2{>}$compute

$$-1\vec{v}$$

Solution:

$$\begin{aligned} -1\vec{v} &= -1{<}4,2{>} && \text{(given)} \\ &= {<}-1(4), -1(2){>} && \text{(def of scalar multiplication)} \\ &= {<}-4, -2{>} && \text{(by inspection)} \end{aligned}$$

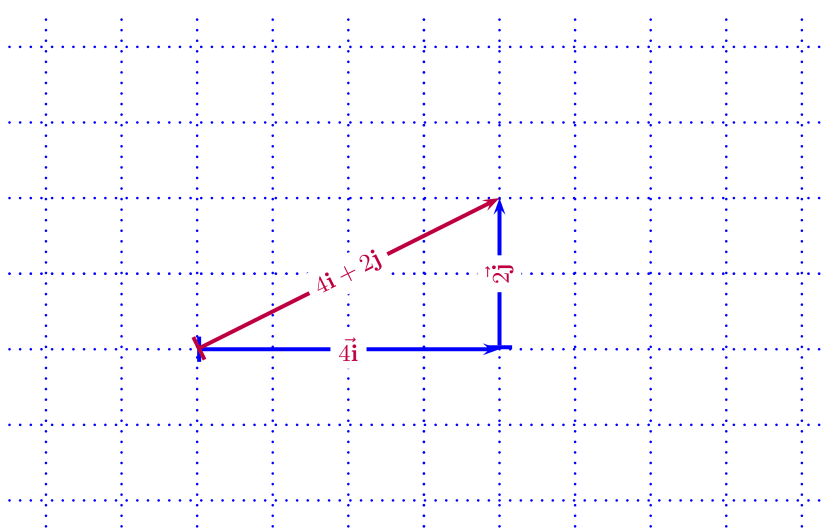


4. **Vector Arithmetic: Famous Vectors** There are two very famous vectors, there are: $\mathbf{i} =< 1, 0 >$ and $\mathbf{j} =< 0, 1 >$.
   (a) **FAMOUS VECTORS i and j** Compute and draw the following vectors

$$4\mathbf{i} + 2\mathbf{j}$$

Solution: let

$$\begin{aligned} \vec{v} &= 4\mathbf{i} + 2\mathbf{j} \\ &= 4 < 1, 0 > +2 < 0, 1 > && \text{(given)} \\ &=< 4, 0 > + < 0, 2 > && \text{(def of scalar multiplication)} \\ &=< 4, 2 > \end{aligned}$$

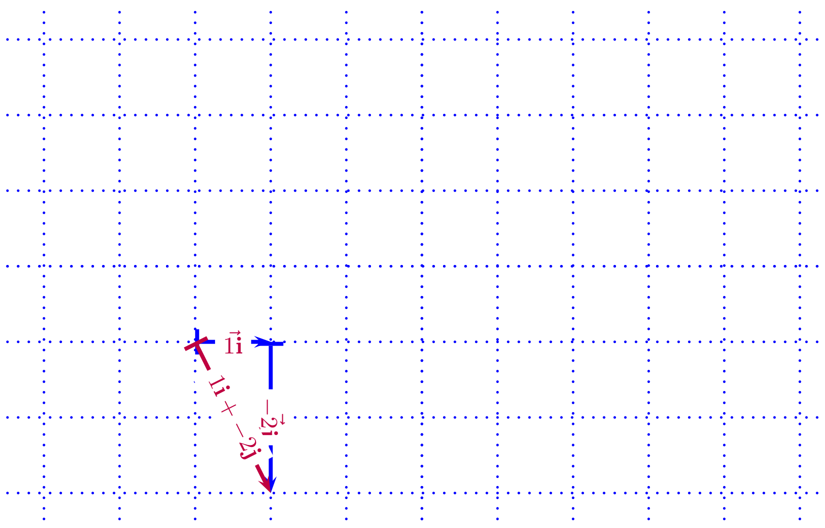


(b) **FAMOUS VECTORS i and j** Compute and draw the following vectors

$$1\mathbf{i} + -2\mathbf{j}$$

Solution: let

$$\begin{aligned}
\vec{v} &= 1\mathbf{i} + -2\mathbf{j} \\
&= 1 < 1, 0 > + - 2 < 0, 1 > && \text{(given)} \\
&= < 1, 0 > + < 0, -2 > && \text{(def of scalar multiplication)} \\
&= < 1, -2 >
\end{aligned}$$

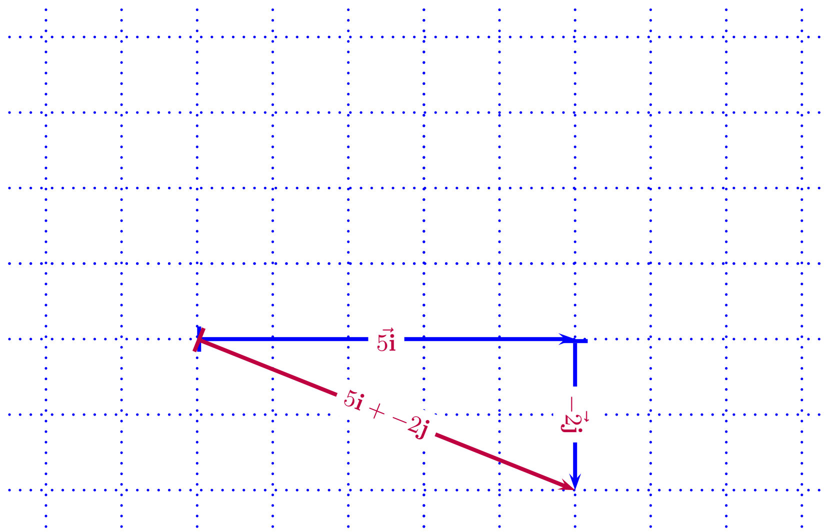


(c) **FAMOUS VECTORS i and j** Compute and draw the following vectors

$$5\mathbf{i} + -2\mathbf{j}$$

Solution: let

$$\begin{aligned}
\vec{v} &= 5\mathbf{i} + -2\mathbf{j} \\
&= 5 < 1, 0 > + - 2 < 0, 1 > && \text{(given)} \\
&= < 5, 0 > + < 0, -2 > && \text{(def of scalar multiplication)} \\
&= < 5, -2 >
\end{aligned}$$

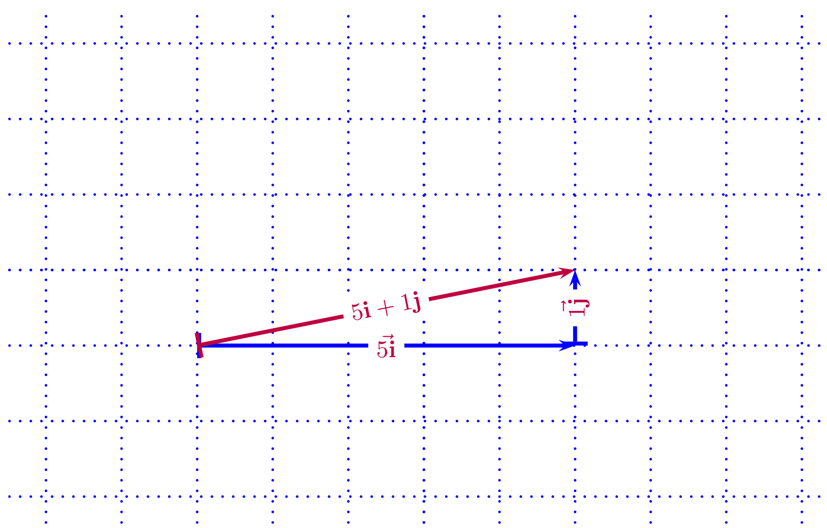


(d) **FAMOUS VECTORS i and j** Compute and draw the following vectors

$$5\mathbf{i} + 1\mathbf{j}$$

Solution: let

$$\begin{aligned}
\vec{v} &= 5\mathbf{i} + 1\mathbf{j} \\
&= 5 < 1, 0 > +1 < 0, 1 > && \text{(given)} \\
&= < 5, 0 > + < 0, 1 > && \text{(def of scalar multiplication)} \\
&= < 5, 1 >
\end{aligned}$$

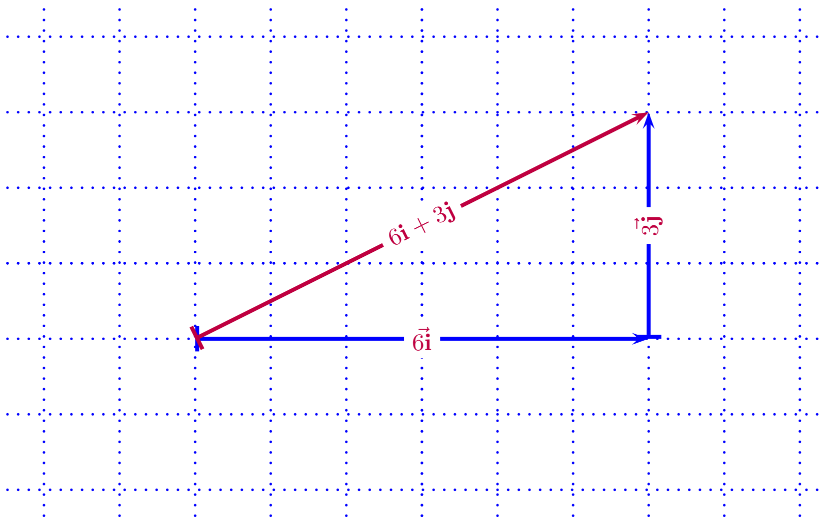


(e) **FAMOUS VECTORS i and j** Compute and draw the following vectors

$$6\mathbf{i} + 3\mathbf{j}$$

Solution: let

$$\begin{aligned}\vec{v} &= 6\mathbf{i} + 3\mathbf{j} \\ &= 6 < 1, 0 > +3 < 0, 1 > && \text{(given)} \\ &=< 6, 0 > + < 0, 3 > && \text{(def of scalar multiplication)} \\ &=< 6, 3 >\end{aligned}$$

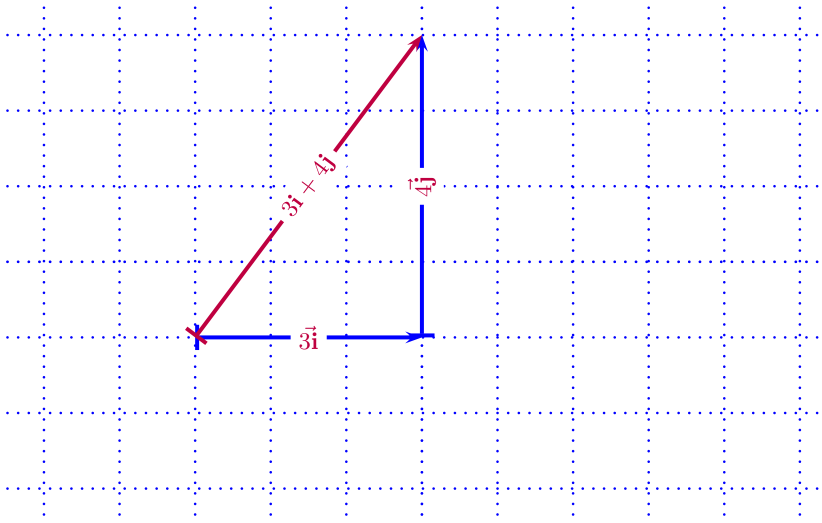


(f) **FAMOUS VECTORS i and j** Compute and draw the following vectors

$$3\mathbf{i} + 4\mathbf{j}$$

Solution: let

$$\begin{aligned}\vec{v} &= 3\mathbf{i} + 4\mathbf{j} \\ &= 3 < 1, 0 > +4 < 0, 1 > && \text{(given)} \\ &=< 3, 0 > + < 0, 4 > && \text{(def of scalar multiplication)} \\ &=< 3, 4 >\end{aligned}$$



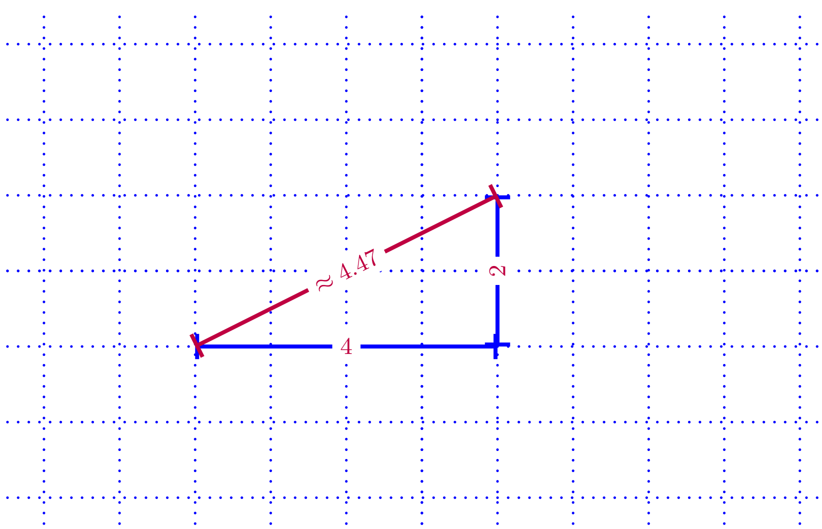
5. **NORM of a VECTOR** Find the norm of the indicated vector.

   (a) Compute:

$$|| < 4, 2 > ||$$

Solution:

$$\begin{aligned} || < 4,2 > || &= \sqrt{(4)^2 + (2)^2} && \text{(def of norm)} \\ &= \sqrt{20} && \text{(by Inspection)} \\ &\approx 4.47 && \text{(by Calc)} \end{aligned}$$

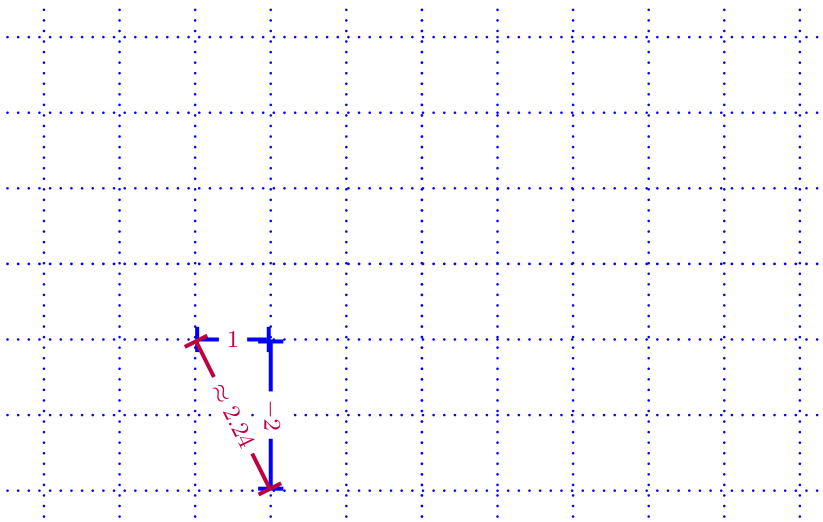


   (b) Compute:

$$|| < 1, -2 > ||$$

Solution:

$$\begin{aligned} || < 1, -2 > || &= \sqrt{(1)^2 + (-2)^2} && \text{(def of norm)} \\ &= \sqrt{5} && \text{(by Inspection)} \\ &\approx 2.24 && \text{(by Calc)} \end{aligned}$$

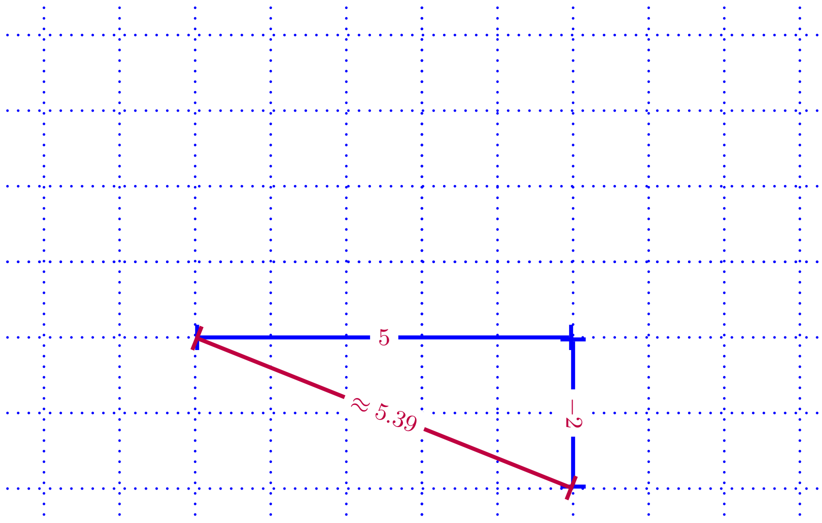


(c) Compute:

$$|| < 5, -2 > ||$$

Solution:

$$\begin{aligned} || < 5, -2 > || &= \sqrt{(5)^2 + (-2)^2} && \text{(def of norm)} \\ &= \sqrt{29} && \text{(by Inspection)} \\ &\approx 5.39 && \text{(by Calc)} \end{aligned}$$

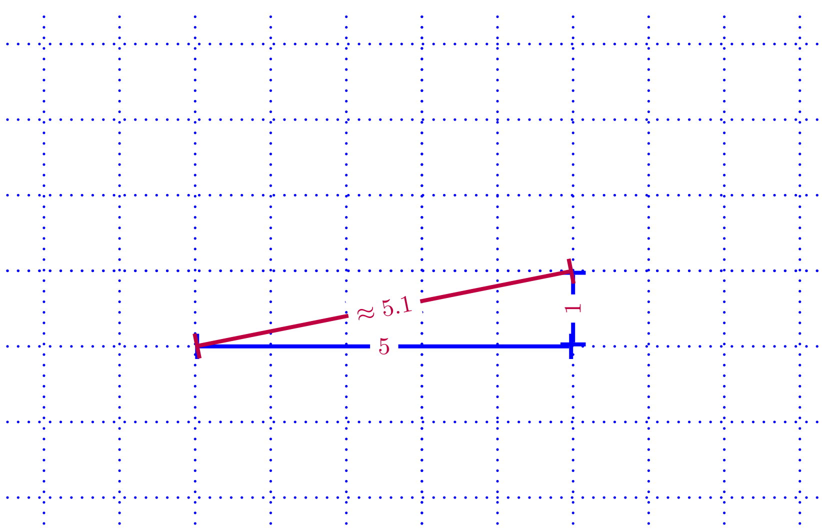


(d) Compute:

$$|| < 5, 1 > ||$$

Solution:

$$\begin{aligned} || < 5, 1 > || &= \sqrt{(5)^2 + (1)^2} && \text{(def of norm)} \\ &= \sqrt{26} && \text{(by Inspection)} \\ &\approx 5.1 && \text{(by Calc)} \end{aligned}$$

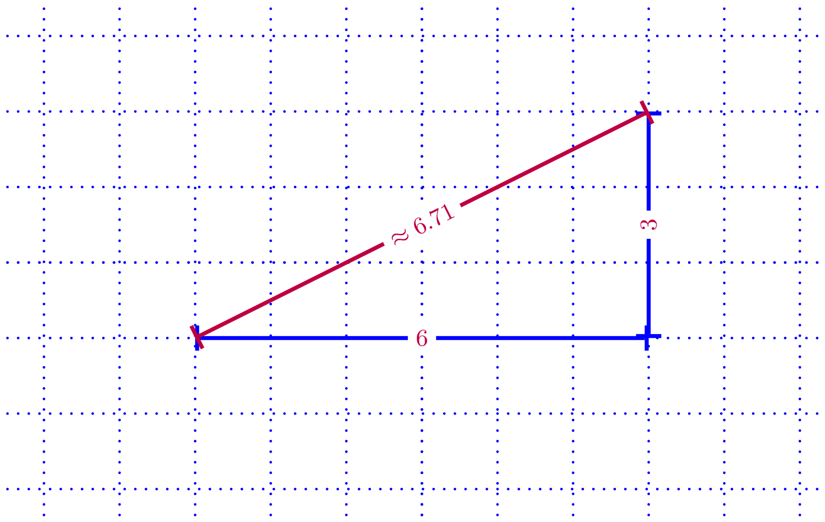


(e) Compute:

$$|| < 6, 3 > ||$$

Solution:

$$\begin{aligned} || < 6,3 > || &= \sqrt{(6)^2 + (3)^2} && \text{(def of norm)} \\ &= \sqrt{45} && \text{(by Inspection)} \\ &\approx 6.71 && \text{(by Calc)} \end{aligned}$$

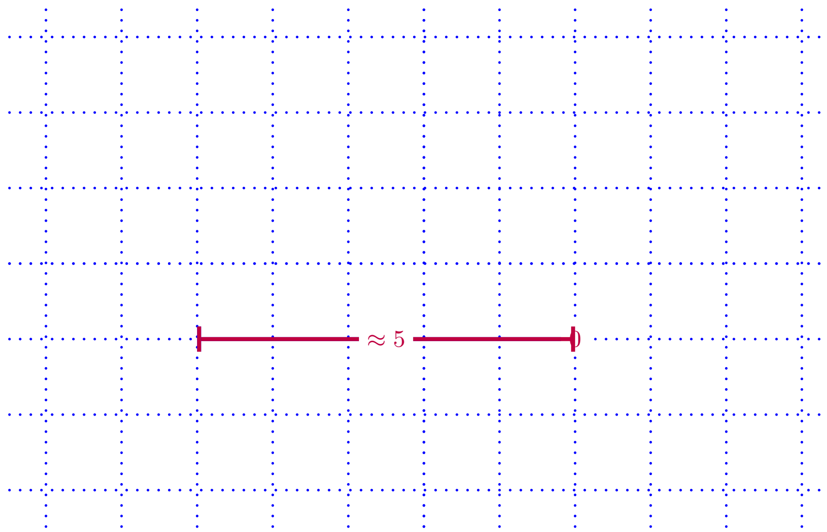


(f) Compute:

$$|| < 5,0 > ||$$

Solution:

$$\begin{aligned} || < 5,0 > || &= \sqrt{(5)^2 + (0)^2} && \text{(def of norm)} \\ &= \sqrt{25} && \text{(by Inspection)} \\ &\approx 5 && \text{(by Calc)} \end{aligned}$$



6. **Which is Bigger?**

(a) Determine which number is larger: $||4\mathbf{i} + 2\mathbf{j}||$ OR $(||4\mathbf{i}|| + ||2\mathbf{j}||)$

Solution:

$$\begin{aligned}
||4\mathbf{i} + 2\mathbf{j}|| &= || < 4, 2 > || && \text{(def of } \mathbf{i} \text{ and } \mathbf{j}\text{)}\\
&= \sqrt{(4)^2 + (2)^2} && \text{(def of norm)}\\
&= \sqrt{20} && \text{(by Inspection)}\\
&\approx 4.47 && \text{(by Calc)}
\end{aligned}$$

Meanwhile

$$\begin{aligned}
||4\mathbf{i}|| + ||2\mathbf{j}|| &= || < 4, 0 > || + || < 0, 2 > || && \text{(def of } \mathbf{i} \text{ and } \mathbf{j}\text{)}\\
&= 4 + 2 && \text{(def of norm)}\\
&= 6 && \text{(by Inspection)}
\end{aligned}$$

Therefore, $||4\mathbf{i}|| + ||2\mathbf{j}||$ is larger.

(b) Determine which number is larger: $||1\mathbf{i} + -2\mathbf{j}||$ OR $(||1\mathbf{i}|| + || - 2\mathbf{j}||)$

Solution:

$$\begin{aligned}
||1\mathbf{i} + -2\mathbf{j}|| &= || < 1, -2 > || && \text{(def of } \mathbf{i} \text{ and } \mathbf{j}\text{)}\\
&= \sqrt{(1)^2 + (-2)^2} && \text{(def of norm)}\\
&= \sqrt{5} && \text{(by Inspection)}\\
&\approx 2.24 && \text{(by Calc)}
\end{aligned}$$

Meanwhile

$$\begin{aligned}
||1\mathbf{i}|| + || - 2\mathbf{j}|| &= || < 1, 0 > || + || < 0, -2 > || && \text{(def of } \mathbf{i} \text{ and } \mathbf{j}\text{)} \\
&= 1 + 2 && \text{(def of norm)} \\
&= 3 && \text{(by Inspection)}
\end{aligned}$$

Therefore, $||1\mathbf{i}|| + || - 2\mathbf{j}||$ is larger.

(c) Determine which number is larger: $||5\mathbf{i} + -2\mathbf{j}||$ OR $(||5\mathbf{i}|| + || - 2\mathbf{j}||)$

Solution:

$$\begin{aligned}
||5\mathbf{i} + -2\mathbf{j}|| &= || < 5, -2 > || && \text{(def of } \mathbf{i} \text{ and } \mathbf{j}\text{)} \\
&= \sqrt{(5)^2 + (-2)^2} && \text{(def of norm)} \\
&= \sqrt{29} && \text{(by Inspection)} \\
&\approx 5.39 && \text{(by Calc)}
\end{aligned}$$

Meanwhile

$$\begin{aligned}
||5\mathbf{i}|| + || - 2\mathbf{j}|| &= || < 5, 0 > || + || < 0, -2 > || && \text{(def of } \mathbf{i} \text{ and } \mathbf{j}\text{)} \\
&= 5 + 2 && \text{(def of norm)} \\
&= 7 && \text{(by Inspection)}
\end{aligned}$$

Therefore, $||5\mathbf{i}|| + || - 2\mathbf{j}||$ is larger.

(d) Determine which number is larger: $||5\mathbf{i} + 1\mathbf{j}||$ OR $(||5\mathbf{i}|| + ||1\mathbf{j}||)$

Solution:

$$\begin{aligned}
||5\mathbf{i} + 1\mathbf{j}|| &= || < 5, 1 > || && \text{(def of } \mathbf{i} \text{ and } \mathbf{j}\text{)} \\
&= \sqrt{(5)^2 + (1)^2} && \text{(def of norm)} \\
&= \sqrt{26} && \text{(by Inspection)} \\
&\approx 5.1 && \text{(by Calc)}
\end{aligned}$$

Meanwhile

$$\begin{aligned}
||5\mathbf{i}|| + ||1\mathbf{j}|| &= || < 5, 0 > || + || < 0, 1 > || && \text{(def of } \mathbf{i} \text{ and } \mathbf{j}\text{)} \\
&= 5 + 1 && \text{(def of norm)} \\
&= 6 && \text{(by Inspection)}
\end{aligned}$$

Therefore, $||5\mathbf{i}|| + ||1\mathbf{j}||$ is larger.

(e) Determine which number is larger: $||6\mathbf{i} + 3\mathbf{j}||$ OR $(||6\mathbf{i}|| + ||3\mathbf{j}||)$

Solution:

$$\begin{aligned} ||6\mathbf{i}+3\mathbf{j}|| &= ||<6,3>|| && \text{(def of } \mathbf{i} \text{ and } \mathbf{j}\text{)} \\ &= \sqrt{(6)^2+(3)^2} && \text{(def of norm)} \\ &= \sqrt{45} && \text{(by Inspection)} \\ &\approx 6.71 && \text{(by Calc)} \end{aligned}$$

Meanwhile

$$\begin{aligned} ||6\mathbf{i}||+||3\mathbf{j}|| &= ||<6,0>||+||<0,3>|| && \text{(def of } \mathbf{i} \text{ and } \mathbf{j}\text{)} \\ &= 6+3 && \text{(def of norm)} \\ &= 9 && \text{(by Inspection)} \end{aligned}$$

Therefore, $||6\mathbf{i}||+||3\mathbf{j}||$ is larger.

(f) Determine which number is larger: $||5\mathbf{i}+0\mathbf{j}||$ OR $(||5\mathbf{i}||+||0\mathbf{j}||)$

Solution:

$$\begin{aligned} ||5\mathbf{i}+0\mathbf{j}|| &= ||<5,0>|| && \text{(def of } \mathbf{i} \text{ and } \mathbf{j}\text{)} \\ &= \sqrt{(5)^2+(0)^2} && \text{(def of norm)} \\ &= \sqrt{25} && \text{(by Inspection)} \\ &\approx 5 && \text{(by Calc)} \end{aligned}$$

Meanwhile

$$\begin{aligned} ||5\mathbf{i}||+||0\mathbf{j}|| &= ||<5,0>||+||<0,0>|| && \text{(def of } \mathbf{i} \text{ and } \mathbf{j}\text{)} \\ &= 5+0 && \text{(def of norm)} \\ &= 5 && \text{(by Inspection)} \end{aligned}$$

Therefore, $||5\mathbf{i}||+||0\mathbf{j}||$ is larger.

(g) $||\mathbf{u}+\mathbf{v}||$ OR $(||\mathbf{u}||+||\mathbf{v}||)$

Solution: the idea is to study the above pattern and understand that it will always be the case that the sum of the individual norms is larger or equal to the norm of the sum of the vectors. In some sense, this is equivalent to saying that the sum of the lengths of any two sides of a [Euclidean] triangle have to be larger than the size of the length of the the third side of the triangle. This is very famous, it is called the triangle inequality.

7. **Normalize this..** Find the normalized vector for each:

   (a) **NORMALIZE them** Compute and draw the corresponding normalized vector:

$$\vec{v} =< 5,2 >$$

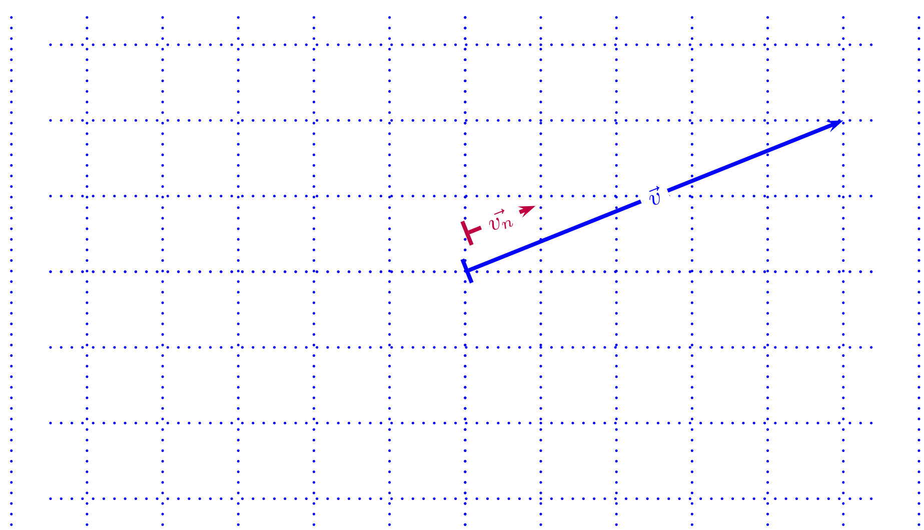
Solution: First we find the norm of the vector:

$$
\begin{aligned}
||\vec{v}|| = ||<5,2>|| &= \sqrt{(5)^2 + (2)^2} && \text{(def of norm)}\\
&= \sqrt{29} && \text{(by Inspection)}\\
&\approx 5.39 && \text{(by Calc)}
\end{aligned}
$$

Then we scale the original vector $\vec{v}$ by multiplying by $\frac{1}{||\vec{v}||}$. Let us denote the normalized vector $\vec{v}$ as "$\vec{v_n}$"

$$
\begin{aligned}
\vec{v_n} &= \frac{1}{||\vec{v}||}<5,2> && \text{(given)}\\
&= \left\langle \frac{5}{||\vec{v}||}, \frac{2}{||\vec{v}||} \right\rangle && \text{(def of scalar multiplication)}\\
&\approx \left\langle \frac{5}{5.39}, \frac{2}{5.39} \right\rangle && \text{(approximate)}\\
&\approx \langle 0.93, 0.37 \rangle && \text{(by inspection)}
\end{aligned}
$$

Now, NOTE: $\vec{v}$ and $\vec{v_n}$ have the same direction, BUT, $\vec{v_n}$ has norm "1" as intended. (also keep in mind position does not matter for vectors.. only direction and size, thus nothing should be interpreted from their position, only their size and direction)



(b) **NORMALIZE them** Compute and draw the corresponding normalized vector:

$$\vec{v} = <5, -2>$$

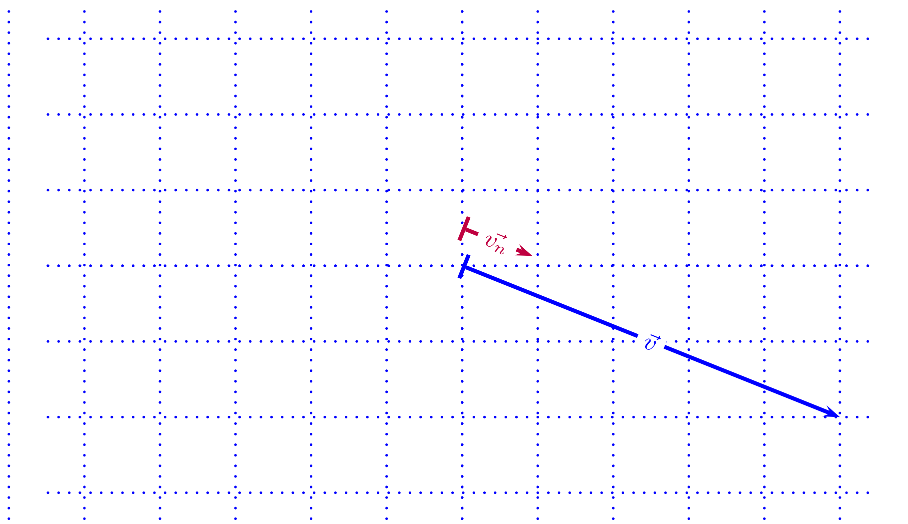
Solution: First we find the norm of the vector:

$$
\begin{aligned}
||\vec{v}|| = ||<5,-2>|| &= \sqrt{(5)^2 + (-2)^2} && \text{(def of norm)}\\
&= \sqrt{29} && \text{(by Inspection)}\\
&\approx 5.39 && \text{(by Calc)}
\end{aligned}
$$

Then we scale the original vector $\vec{v}$ by multiplying by $\frac{1}{||\vec{v}||}$. Let us denote the normalized vector $\vec{v}$ as "$\vec{v_n}$"

$$\begin{aligned}
\vec{v_n} &= \frac{1}{||\vec{v}||}{<}5,-2{>} && \text{(given)}\\
&= \left\langle \frac{5}{||\vec{v}||}, \frac{-2}{||\vec{v}||} \right\rangle && \text{(def of scalar multiplication)}\\
&\approx \left\langle \frac{5}{5.39}, \frac{-2}{5.39} \right\rangle && \text{(approximate)}\\
&\approx \langle 0.93, -0.37 \rangle && \text{(by inspection)}
\end{aligned}$$

Now, NOTE: $\vec{v}$ and $\vec{v_n}$ have the same direction, BUT, $\vec{v_n}$ has norm "1" as intended. (also keep in mind position does not matter for vectors.. only direction and size, thus nothing should be interpreted from their position, only their size and direction)



(c) **NORMALIZE them** Compute and draw the corresponding normalized vector:

$$\vec{v} = < -3, 4 >$$

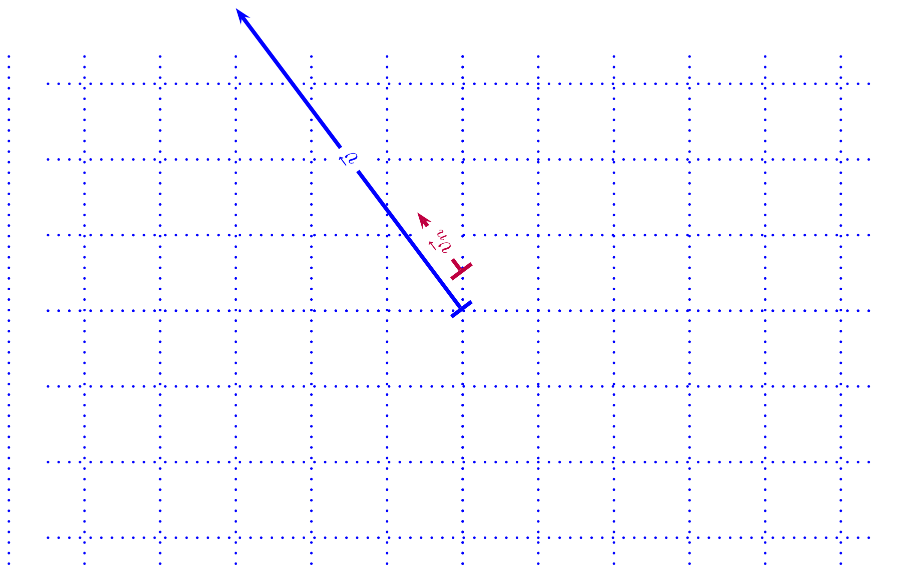
Solution: First we find the norm of the vector:

$$\begin{aligned}
||\vec{v}|| = || < -3, 4 > || &= \sqrt{(-3)^2 + (4)^2} && \text{(def of norm)}\\
&= \sqrt{25} && \text{(by Inspection)}\\
&\approx 5 && \text{(by Calc)}
\end{aligned}$$

Then we scale the original vector $\vec{v}$ by multiplying by $\frac{1}{||\vec{v}||}$. Let us denote the normalized vector $\vec{v}$ as "$\vec{v_n}$"

$$\begin{aligned}
\vec{v_n} &= \frac{1}{||\vec{v}||}<-3,4> && \text{(given)} \\
&= \left\langle \frac{-3}{||\vec{v}||}, \frac{4}{||\vec{v}||} \right\rangle && \text{(def of scalar multiplication)} \\
&\approx \left\langle \frac{-3}{5}, \frac{4}{5} \right\rangle && \text{(approximate)} \\
&\approx \langle -0.6, 0.8 \rangle && \text{(by inspection)}
\end{aligned}$$

Now, NOTE: $\vec{v}$ and $\vec{v_n}$ have the same direction, BUT, $\vec{v_n}$ has norm "1" as intended. (also keep in mind position does not matter for vectors.. only direction and size, thus nothing should be interpreted from their position, only their size and direction)



(d) **NORMALIZE them** Compute and draw the corresponding normalized vector:

$$\vec{v} =< -3, -4 >$$

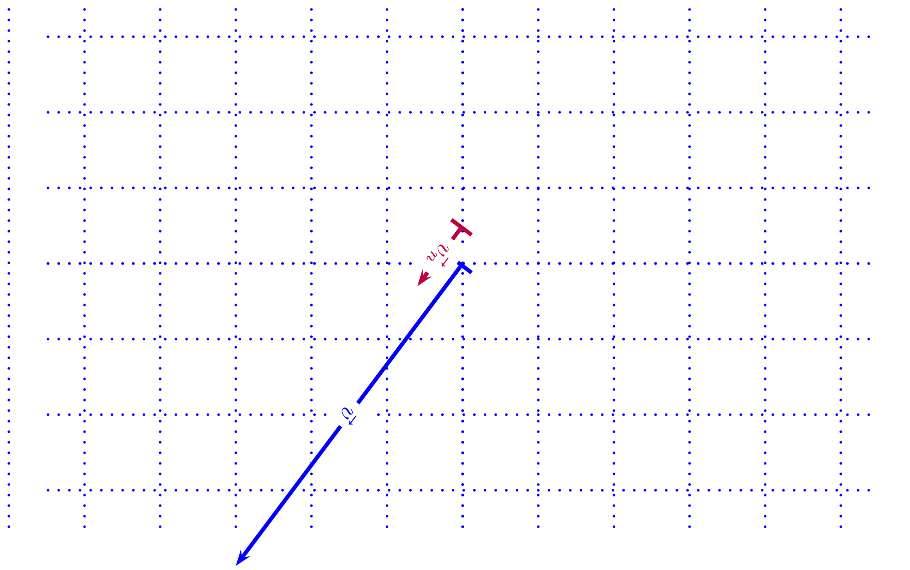
Solution: First we find the norm of the vector:

$$\begin{aligned}
||\vec{v}|| = ||< -3, -4 >|| &= \sqrt{(-3)^2 + (-4)^2} && \text{(def of norm)} \\
&= \sqrt{25} && \text{(by Inspection)} \\
&\approx 5 && \text{(by Calc)}
\end{aligned}$$

Then we scale the original vector $\vec{v}$ by multiplying by $\frac{1}{||\vec{v}||}$. Let us denote the normalized vector $\vec{v}$ as "$\vec{v_n}$"

$$\begin{aligned}
\vec{v_n} &= \frac{1}{||\vec{v}||} < -3, -4> && \text{(given)} \\
&= \left\langle \frac{-3}{||\vec{v}||}, \frac{-4}{||\vec{v}||} \right\rangle && \text{(def of scalar multiplication)} \\
&\approx \left\langle \frac{-3}{5}, \frac{-4}{5} \right\rangle && \text{(approximate)} \\
&\approx \langle -0.6, -0.8 \rangle && \text{(by inspection)}
\end{aligned}$$

Now, NOTE: $\vec{v}$ and $\vec{v_n}$ have the same direction, BUT, $\vec{v_n}$ has norm "1" as intended. (also keep in mind position does not matter for vectors.. only direction and size, thus nothing should be interpreted from their position, only their size and direction)



(e) **NORMALIZE them** Compute and draw the corresponding normalized vector:

$$\vec{v} = < -3, -1 >$$

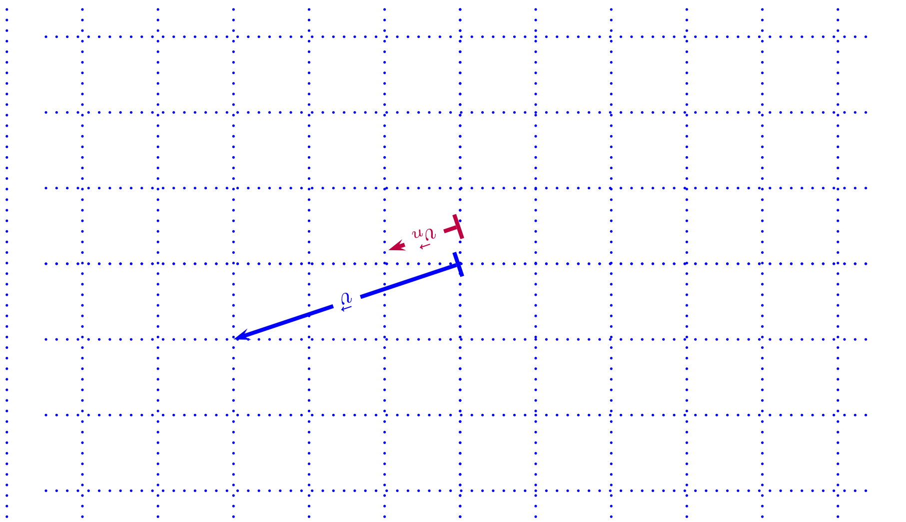
Solution: First we find the norm of the vector:

$$\begin{aligned}
||\vec{v}|| = || < -3, -1 > || &= \sqrt{(-3)^2 + (-1)^2} && \text{(def of norm)} \\
&= \sqrt{10} && \text{(by Inspection)} \\
&\approx 3.16 && \text{(by Calc)}
\end{aligned}$$

Then we scale the original vector $\vec{v}$ by multiplying by $\frac{1}{||\vec{v}||}$. Let us denote the normalized vector $\vec{v}$ as "$\vec{v_n}$"

$$\begin{aligned}
\vec{v_n} &= \frac{1}{||\vec{v}||} < -3, -1 > && \text{(given)}\\
&= \left\langle \frac{-3}{||\vec{v}||}, \frac{-1}{||\vec{v}||} \right\rangle && \text{(def of scalar multiplication)}\\
&\approx \left\langle \frac{-3}{3.16}, \frac{-1}{3.16} \right\rangle && \text{(approximate)}\\
&\approx \langle -0.95, -0.32 \rangle && \text{(by inspection)}
\end{aligned}$$

Now, NOTE: $\vec{v}$ and $\vec{v_n}$ have the same direction, BUT, $\vec{v_n}$ has norm "1" as intended. (also keep in mind position does not matter for vectors.. only direction and size, thus nothing should be interpreted from their position, only their size and direction)



(f) $\vec{v} =< a, b >$ (not both zero..)

Solution:

$$\begin{aligned}
\vec{v_n} &= \frac{1}{||\vec{v}||} < a, b > && \text{(given)}\\
&= \left\langle \frac{a}{||\vec{v}||}, \frac{b}{||\vec{v}||} \right\rangle && \text{(def of scalar multiplication)}\\
&= \left\langle \frac{a}{\sqrt{a^2 + b^2}}, \frac{b}{\sqrt{a^2 + b^2}} \right\rangle && \text{(def of norm)}
\end{aligned}$$

1. **Vector Dot Product**

   (a) Compute the following DOT product:

$$< 1, 2 > \cdot < 3, 5 >$$

Solution:

$$\begin{aligned} < 1, 2 > \cdot < 3, 5 > &= (1)(3) + (2)(5) && \text{(def of \textit{the} DOT)} \\ &= 3 + 10 && \text{(by Calc)} \\ &= 13 && \text{(by Calc)} \end{aligned}$$

   (b) Compute the following DOT product:

$$< 3, 5 > \cdot < 3, 5 >$$

Solution:

$$\begin{aligned} < 3, 5 > \cdot < 3, 5 > &= (3)(3) + (5)(5) && \text{(def of \textit{the} DOT)} \\ &= 9 + 25 && \text{(by Calc)} \\ &= 34 && \text{(by Calc)} \end{aligned}$$

   (c) Compute the following DOT product:

$$< -3, 2 > \cdot < 1, 2 >$$

Solution:

$$\begin{aligned} < -3, 2 > \cdot < 1, 2 > &= (-3)(1) + (2)(2) && \text{(def of \textit{the} DOT)} \\ &= -3 + 4 && \text{(by Calc)} \\ &= 1 && \text{(by Calc)} \end{aligned}$$

   (d) Compute the following DOT product:

$$< 1, 4 > \cdot < -3, -5 >$$

Solution:

$$\begin{aligned} < 1, 4 > \cdot < -3, -5 > &= (1)(-3) + (4)(-5) && \text{(def of \textit{the} DOT)} \\ &= -3 + -20 && \text{(by Calc)} \\ &= -23 && \text{(by Calc)} \end{aligned}$$

(e) Compute the following DOT product:

$$<2,3>\cdot<3,-3>$$

Solution:

$$\begin{aligned}
<2,3>\cdot<3,-3> &= (2)(3)+(3)(-3) && \text{(def of \textit{the} DOT)}\\
&= 6+-9 && \text{(by Calc)}\\
&= -3 && \text{(by Calc)}
\end{aligned}$$

(f) Compute the following DOT product:

$$<1,-4>\cdot<3,7>$$

Solution:

$$\begin{aligned}
<1,-4>\cdot<3,7> &= (1)(3)+(-4)(7) && \text{(def of \textit{the} DOT)}\\
&= 3+-28 && \text{(by Calc)}\\
&= -25 && \text{(by Calc)}
\end{aligned}$$

(g) $<2,1,3>\cdot<1,2,1>$

Solution: $2+2+3=7$

2. **Vector Dot Product**

(a) Compute the following DOT product:

$$(1\mathbf{i}+2\mathbf{j})\cdot(0\mathbf{i}+5\mathbf{j})$$

Solution:

$$\begin{aligned}
(1\mathbf{i}+2\mathbf{j})\cdot(0\mathbf{i}+5\mathbf{j}) &= (1<1,0>+2<0,1>)\cdot(0<1,0>+5<0,1>) && \text{(def of } \mathbf{i} \text{ and } \mathbf{j}\text{)}\\
&=<1,2>\cdot<0,5> && \text{(by Inspection)}\\
&= (1)(0)+(2)(5) && \text{(def of \textit{the} DOT)}\\
&= 0+10 && \text{(by Calc)}\\
&= 10 && \text{(by Calc)}
\end{aligned}$$

(b) Compute the following DOT product:

$$(3\mathbf{i}+0\mathbf{j})\cdot(3\mathbf{i}+5\mathbf{j})$$

Solution:

$$\begin{aligned}
(3\mathbf{i}+0\mathbf{j})\cdot(3\mathbf{i}+5\mathbf{j}) &= (3<1,0>+0<0,1>)\cdot(3<1,0>+5<0,1>) && \text{(def of } \mathbf{i} \text{ and } \mathbf{j})\\
&=<3,0>\cdot<3,5> && \text{(by Inspection)}\\
&= (3)(3)+(0)(5) && \text{(def of \textit{the} DOT)}\\
&= 9+0 && \text{(by Calc)}\\
&= 9 && \text{(by Calc)}
\end{aligned}$$

(c) Compute the following DOT product:

$$(-3\mathbf{i}+2\mathbf{j})\cdot(1\mathbf{i}+2\mathbf{j})$$

Solution:

$$\begin{aligned}
(-3\mathbf{i}+2\mathbf{j})\cdot(1\mathbf{i}+2\mathbf{j}) &= (-3<1,0>+2<0,1>)\cdot(1<1,0>+2<0,1>) && \text{(def of } \mathbf{i} \text{ and } \mathbf{j})\\
&=<-3,2>\cdot<1,2> && \text{(by Inspection)}\\
&= (-3)(1)+(2)(2) && \text{(def of \textit{the} DOT)}\\
&= -3+4 && \text{(by Calc)}\\
&= 1 && \text{(by Calc)}
\end{aligned}$$

(d) Compute the following DOT product:

$$(1\mathbf{i}+4\mathbf{j})\cdot(-3\mathbf{i}+-5\mathbf{j})$$

Solution:

$$\begin{aligned}
(1\mathbf{i}+4\mathbf{j})\cdot(-3\mathbf{i}+-5\mathbf{j}) &= (1<1,0>+4<0,1>)\cdot(-3<1,0>+-5<0,1>) && \text{(def of } \mathbf{i} \text{ and } \mathbf{j})\\
&=<1,4>\cdot<-3,-5> && \text{(by Inspection)}\\
&= (1)(-3)+(4)(-5) && \text{(def of \textit{the} DOT)}\\
&= -3+-20 && \text{(by Calc)}\\
&= -23 && \text{(by Calc)}
\end{aligned}$$

3. **Vector Dot Product *to find magnitude***

(a) Use the dot product to find the magnitude of the following vector:

$$\vec{v} =< 1, 2 >$$

Solution:

$$
\begin{aligned}
||\vec{v}||^2 &= \vec{v} \cdot \vec{v} && \text{(famous DOT product property)}\\
&= < 1,2 > \cdot < 1,2 > && \text{(given)}\\
&= (1)(1) + (2)(2) && \text{(def of } \textit{the} \text{ DOT)}\\
&= 1 + 4 && \text{(by Calc)}\\
\text{....thus....} \quad ||\vec{v}||^2 &= 5 && \text{(by Calc)}\\
\text{....then.. (assuming norm is positive)....} \quad ||\vec{v}|| &= \sqrt{5}\\
||\vec{v}|| &\approx 2.24
\end{aligned}
$$

(b) Use the dot product to find the magnitude of the following vector:

$$\vec{v} = < 3,5 >$$

Solution:

$$
\begin{aligned}
||\vec{v}||^2 &= \vec{v} \cdot \vec{v} && \text{(famous DOT product property)}\\
&= < 3,5 > \cdot < 3,5 > && \text{(given)}\\
&= (3)(3) + (5)(5) && \text{(def of } \textit{the} \text{ DOT)}\\
&= 9 + 25 && \text{(by Calc)}\\
\text{....thus....} \quad ||\vec{v}||^2 &= 34 && \text{(by Calc)}\\
\text{....then.. (assuming norm is positive)....} \quad ||\vec{v}|| &= \sqrt{34}\\
||\vec{v}|| &\approx 5.83
\end{aligned}
$$

(c) Use the dot product to find the magnitude of the following vector:

$$\vec{v} = < -3,2 >$$

Solution:

$$
\begin{aligned}
||\vec{v}||^2 &= \vec{v} \cdot \vec{v} && \text{(famous DOT product property)}\\
&= < -3,2 > \cdot < -3,2 > && \text{(given)}\\
&= (-3)(-3) + (2)(2) && \text{(def of } \textit{the} \text{ DOT)}\\
&= 9 + 4 && \text{(by Calc)}\\
\text{....thus....} \quad ||\vec{v}||^2 &= 13 && \text{(by Calc)}\\
\text{....then.. (assuming norm is positive)....} \quad ||\vec{v}|| &= \sqrt{13}\\
||\vec{v}|| &\approx 3.61
\end{aligned}
$$

(d) Use the dot product to find the magnitude of the following vector:

$$\vec{v} = < 1,4 >$$

Solution:

$$\begin{aligned}
||\vec{v}||^2 &= \vec{v}\cdot\vec{v} && \text{(famous DOT product property)}\\
&= <1,4>\cdot<1,4> && \text{(given)}\\
&= (1)(1)+(4)(4) && \text{(def of \textit{the} DOT)}\\
&= 1+16 && \text{(by Calc)}\\
\text{....thus....} \quad ||\vec{v}||^2 &= 17 && \text{(by Calc)}\\
\text{....then.. (assuming norm is positive)....} \quad ||\vec{v}|| &= \sqrt{17}\\
||\vec{v}|| &\approx 4.12
\end{aligned}$$

(e) Use the dot product to find the magnitude of the following vector:

$$\vec{v} = <2,3>$$

Solution:

$$\begin{aligned}
||\vec{v}||^2 &= \vec{v}\cdot\vec{v} && \text{(famous DOT product property)}\\
&= <2,3>\cdot<2,3> && \text{(given)}\\
&= (2)(2)+(3)(3) && \text{(def of \textit{the} DOT)}\\
&= 4+9 && \text{(by Calc)}\\
\text{....thus....} \quad ||\vec{v}||^2 &= 13 && \text{(by Calc)}\\
\text{....then.. (assuming norm is positive)....} \quad ||\vec{v}|| &= \sqrt{13}\\
||\vec{v}|| &\approx 3.61
\end{aligned}$$

(f) Use the dot product to find the magnitude of the following vector:

$$\vec{v} = <1,-4>$$

Solution:

$$\begin{aligned}
||\vec{v}||^2 &= \vec{v}\cdot\vec{v} && \text{(famous DOT product property)}\\
&= <1,-4>\cdot<1,-4> && \text{(given)}\\
&= (1)(1)+(-4)(-4) && \text{(def of \textit{the} DOT)}\\
&= 1+16 && \text{(by Calc)}\\
\text{....thus....} \quad ||\vec{v}||^2 &= 17 && \text{(by Calc)}\\
\text{....then.. (assuming norm is positive)....} \quad ||\vec{v}|| &= \sqrt{17}\\
||\vec{v}|| &\approx 4.12
\end{aligned}$$

(g) Use the dot product to find the magnitude of the following vector:

$$\vec{v} = <3,-4>$$

Solution:

$$
\begin{aligned}
||\vec{v}||^2 &= \vec{v}\cdot\vec{v} && \text{(famous DOT product property)}\\
&= <3,-4> \cdot <3,-4> && \text{(given)}\\
&= (3)(3)+(-4)(-4) && \text{(def of \textit{the} DOT)}\\
&= 9+16 && \text{(by Calc)}\\
\text{....thus....}\quad ||\vec{v}||^2 &= 25 && \text{(by Calc)}\\
\text{....then.. (assuming norm is positive)....}\quad ||\vec{v}|| &= \sqrt{25}\\
||\vec{v}|| &\approx 5
\end{aligned}
$$

4. **Vector Dot Product *to find 'distance'***

The DOT product provides a way to define the 'distance' between two vectors, $\vec{v}$ and $\vec{w}$. So long as we can define subtraction of the vectors we can define the distance between them as follows:

$$dist(\vec{v},\vec{w}) = ||\vec{v}-\vec{w}|| = \sqrt{(\vec{v}-\vec{w})\cdot(\vec{v}-\vec{w})}$$

(a) Use the dot product to find the 'distance' between the indicated vectors.

$$\vec{w} = <1,2> \qquad \vec{v} = <3,5>$$

Solution: First note that $\vec{v}-\vec{w} = <2,3>$

$$
\begin{aligned}
||\vec{v}-\vec{w}||^2 &= (\vec{v}-\vec{w})\cdot(\vec{v}-\vec{w})\\
& && \text{(famous DOT product property)}\\
&= <2,3> \cdot <2,3> && \text{(from given)}\\
&= (2)(2)+(3)(3) && \text{(def of \textit{the} DOT)}\\
&= 4+9 && \text{(by Calc)}\\
\text{....thus....}\quad ||\vec{v}-\vec{w}||^2 &= 13 && \text{(by Calc)}\\
\text{....then.. (assuming the distance is positive)....}\quad ||\vec{v}-\vec{w}|| &= \sqrt{13}\\
||\vec{v}-\vec{w}|| &\approx 3.61
\end{aligned}
$$

(b) Use the dot product to find the 'distance' between the indicated vectors.

$$\vec{w} = <-3,2> \qquad \vec{v} = <1,2>$$

Solution: First note that $\vec{v} - \vec{w} = < 4, 0 >$

$$
\begin{aligned}
||\vec{v} - \vec{w}||^2 &= (\vec{v} - \vec{w}) \cdot (\vec{v} - \vec{w}) && \text{(famous DOT product property)} \\
&= < 4, 0 > \cdot < 4, 0 > && \text{(from given)} \\
&= (4)(4) + (0)(0) && \text{(def of \textit{the} DOT)} \\
&= 16 + 0 && \text{(by Calc)} \\
\text{....thus....} \quad ||\vec{v} - \vec{w}||^2 &= 16 && \text{(by Calc)} \\
\text{....then.. (assuming the distance is positive)....} \quad ||\vec{v} - \vec{w}|| &= \sqrt{16} \\
||\vec{v} - \vec{w}|| &\approx 4
\end{aligned}
$$

(c) Use the dot product to find the 'distance' between the indicated vectors.

$$\vec{w} = < 1, 4 > \qquad \vec{v} = < -3, -5 >$$

Solution: First note that $\vec{v} - \vec{w} = < -4, -9 >$

$$
\begin{aligned}
||\vec{v} - \vec{w}||^2 &= (\vec{v} - \vec{w}) \cdot (\vec{v} - \vec{w}) && \text{(famous DOT product property)} \\
&= < -4, -9 > \cdot < -4, -9 > && \text{(from given)} \\
&= (-4)(-4) + (-9)(-9) && \text{(def of \textit{the} DOT)} \\
&= 16 + 81 && \text{(by Calc)} \\
\text{....thus....} \quad ||\vec{v} - \vec{w}||^2 &= 97 && \text{(by Calc)} \\
\text{....then.. (assuming the distance is positive)....} \quad ||\vec{v} - \vec{w}|| &= \sqrt{97} \\
||\vec{v} - \vec{w}|| &\approx 9.85
\end{aligned}
$$

(d) Use the dot product to find the 'distance' between the indicated vectors.

$$\vec{w} = < 2, 3 > \qquad \vec{v} = < 3, -3 >$$

Solution: First note that $\vec{v} - \vec{w} = < 1, -6 >$

$$
\begin{aligned}
||\vec{v} - \vec{w}||^2 &= (\vec{v} - \vec{w}) \cdot (\vec{v} - \vec{w}) && \text{(famous DOT product property)} \\
&= < 1, -6 > \cdot < 1, -6 > && \text{(from given)} \\
&= (1)(1) + (-6)(-6) && \text{(def of \textit{the} DOT)} \\
&= 1 + 36 && \text{(by Calc)} \\
\text{....thus....} \quad ||\vec{v} - \vec{w}||^2 &= 37 && \text{(by Calc)} \\
\text{....then.. (assuming the distance is positive)....} \quad ||\vec{v} - \vec{w}|| &= \sqrt{37} \\
||\vec{v} - \vec{w}|| &\approx 6.08
\end{aligned}
$$

(e) Use the dot product to find the 'distance' between the indicated vectors.

$$\vec{w} =< 1, -4 > \qquad \vec{v} =< 3, 7 >$$

Solution: First note that $\vec{v} - \vec{w} =< 2, 11 >$

$$
\begin{aligned}
||\vec{v} - \vec{w}||^2 &= (\vec{v} - \vec{w}) \cdot (\vec{v} - \vec{w}) && \text{(famous DOT product property)} \\
&=< 2, 11 > \cdot < 2, 11 > && \text{(from given)} \\
&= (2)(2) + (11)(11) && \text{(def of \textit{the} DOT)} \\
&= 4 + 121 && \text{(by Calc)} \\
\text{....thus....} \quad ||\vec{v} - \vec{w}||^2 &= 125 && \text{(by Calc)} \\
\text{....then.. (assuming the distance is positive)....} \quad ||\vec{v} - \vec{w}|| &= \sqrt{125} \\
||\vec{v} - \vec{w}|| &\approx 11.18
\end{aligned}
$$

(f) Use the dot product to find the 'distance' between the indicated vectors.

$$\vec{w} =< 3, -4 > \qquad \vec{v} =< 3, 7 >$$

Solution: First note that $\vec{v} - \vec{w} =< 0, 11 >$

$$
\begin{aligned}
||\vec{v} - \vec{w}||^2 &= (\vec{v} - \vec{w}) \cdot (\vec{v} - \vec{w}) && \text{(famous DOT product property)} \\
&=< 0, 11 > \cdot < 0, 11 > && \text{(from given)} \\
&= (0)(0) + (11)(11) && \text{(def of \textit{the} DOT)} \\
&= 0 + 121 && \text{(by Calc)} \\
\text{....thus....} \quad ||\vec{v} - \vec{w}||^2 &= 121 && \text{(by Calc)} \\
\text{....then.. (assuming the distance is positive)....} \quad ||\vec{v} - \vec{w}|| &= \sqrt{121} \\
||\vec{v} - \vec{w}|| &\approx 11
\end{aligned}
$$

(g) Use the dot product to find the 'distance' between the indicated vectors.

$$\vec{w} =< 1, -3, 5, 0, 6 > \qquad \vec{v} =< 5, 2, -10, 3, 1 >$$

Solution: don't be afraid, don't google it, don't ask anyone.. just you and the problem.. its on..., if it knocks you down, just get up... don't let it beat you..

5. **Vector Dot Product *to find 'angle'* between two vectors**

(a) Use the dot product to find the 'angle' between the indicated vectors.

$$\vec{w} =< 1, 2 > \qquad \vec{v} =< 3, -5 >$$

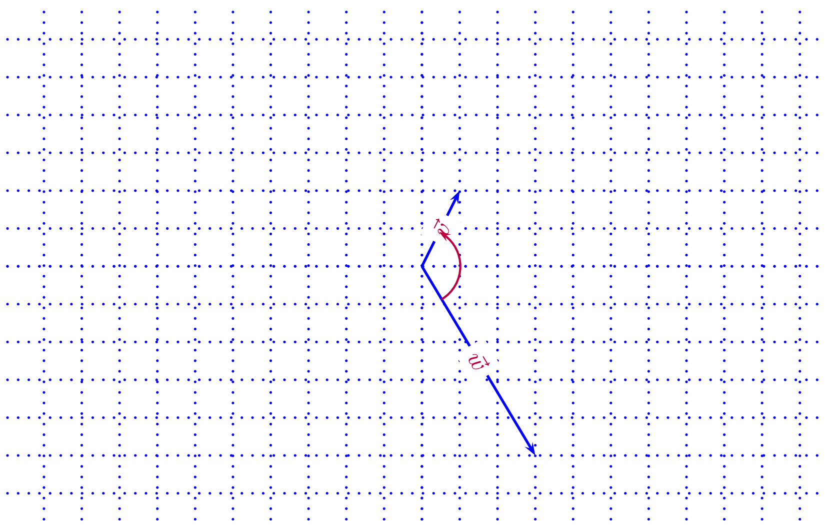
Solution: Let us assume the angle between the vectors is between 0 and 180° Suppose we call such angle 'θ', then....

$$\cos\theta = \frac{\vec{v}\cdot\vec{w}}{||\vec{v}||||\vec{w}||} \quad \text{(famous DOT product property)}$$
$$= \frac{<1,2>\cdot<3,-5>}{||<1,2>||||<3,-5>||} \quad \text{(given)}$$
$$\approx \frac{-7}{(2.236)(5.831)} \approx \frac{-7}{13.038} \quad \text{(by Calc)}$$

....thus.... $\cos\theta = -0.54$ (by Calc)

....then.. (assume the sought angle is $0 \leq \theta \leq 180°$).... $\theta \approx \cos^{-1}(-0.54)$

$$\theta \approx 2.14 \text{ radians}$$
$$\approx 122.6°$$



(b) Use the dot product to find the 'angle' between the indicated vectors.

$$\vec{w} = < -3, 2 > \qquad \vec{v} = < 3, 5 >$$

Solution: Let us assume the angle between the vectors is between 0 and 180° Suppose we call such angle

'θ', then....

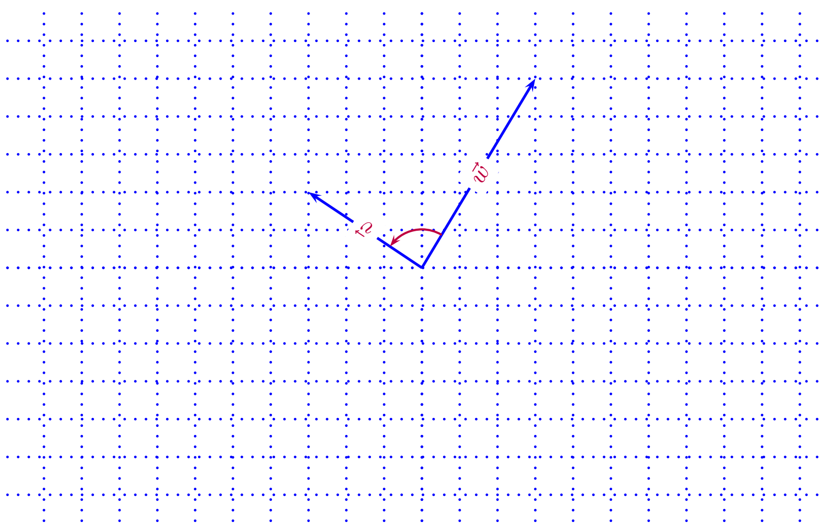
$$\cos\theta = \frac{\vec{v}\cdot\vec{w}}{||\vec{v}||||\vec{w}||} \quad \text{(famous DOT product property)}$$
$$= \frac{< -3, 2 > \cdot < 3, 5 >}{|| < -3, 2 > |||| < 3, 5 > ||} \quad \text{(given)}$$
$$\approx \frac{1}{(3.606)(5.831)} \approx \frac{1}{21.027} \quad \text{(by Calc)}$$

....thus.... $\cos\theta = 0.05$ (by Calc)

....then.. (assume the sought angle is $0 \le \theta \le 180°$).... $\theta \approx \cos^{-1}(0.05)$

$\theta \approx 1.52$ radians

$\approx 87.1°$



(c) Use the dot product to find the 'angle' between the indicated vectors.

$$\vec{w} =< 1, 4 > \qquad \vec{v} =< -3, -5 >$$

Solution: Let us assume the angle between the vectors is between 0 and 180° Suppose we call such angle 'θ', then....

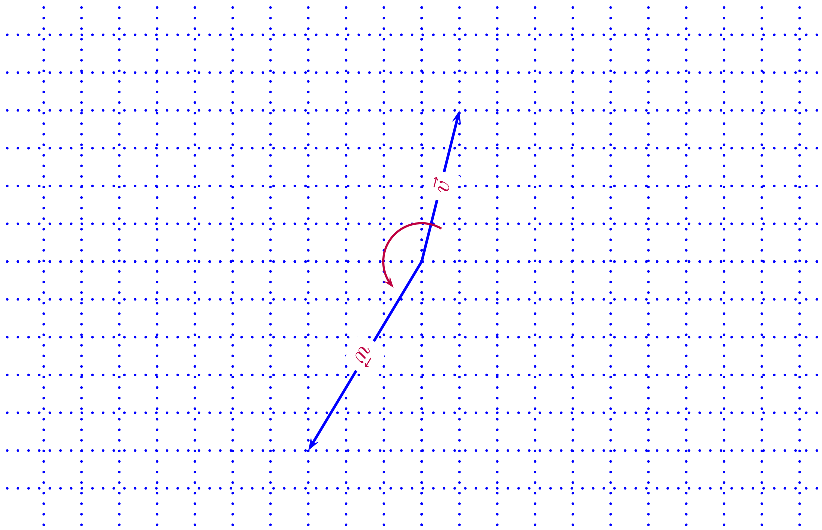
$$\cos\theta = \frac{\vec{v}\cdot\vec{w}}{||\vec{v}||||\vec{w}||} \quad \text{(famous DOT product property)}$$
$$= \frac{< 1, 4 > \cdot < -3, -5 >}{|| < 1, 4 > |||| < -3, -5 > ||} \quad \text{(given)}$$
$$\approx \frac{-23}{(4.123)(5.831)} \approx \frac{-23}{24.041} \quad \text{(by Calc)}$$

....thus.... $\cos\theta = -0.96$ (by Calc)

....then.. (assume the sought angle is $0 \le \theta \le 180°$).... $\theta \approx \cos^{-1}(-0.96)$

$\theta \approx 2.86$ radians

$\approx 163.9°$



(d) Use the dot product to find the 'angle' between the indicated vectors.

$$\vec{w} =< 2, 3 > \qquad \vec{v} =< 3, -3 >$$

Solution: Let us assume the angle between the vectors is between 0 and 180° Suppose we call such angle 'θ', then....

$$\cos\theta = \frac{\vec{v}\cdot\vec{w}}{||\vec{v}||||\vec{w}||} \quad \text{(famous DOT product property)}$$

$$= \frac{< 2, 3 > \cdot < 3, -3 >}{|| < 2, 3 > |||| < 3, -3 > ||} \quad \text{(given)}$$

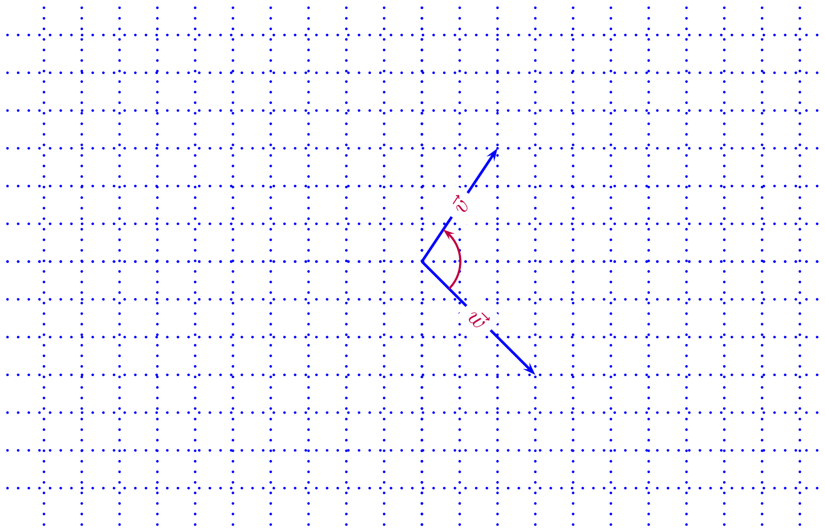
$$\approx \frac{-3}{(3.606)(4.243)} \approx \frac{-3}{15.3} \quad \text{(by Calc)}$$

....thus.... $\cos\theta = -0.2$ (by Calc)

....then.. (assume the sought angle is $0 \leq \theta \leq 180°$).... $\theta \approx \cos^{-1}(-0.2)$

$$\theta \approx 1.77 \text{ radians}$$

$$\approx 101.4°$$



(e) Use the dot product to find the 'angle' between the indicated vectors.

$$\vec{w} = < 3, 7 > \qquad \vec{v} = < 1, -4 >$$

Solution: Let us assume the angle between the vectors is between 0 and 180° Suppose we call such angle 'θ', then....

$$\cos\theta = \frac{\vec{v} \cdot \vec{w}}{||\vec{v}||||\vec{w}||} \quad \text{(famous DOT product property)}$$

$$= \frac{< 3, 7 > \cdot < 1, -4 >}{|| < 3, 7 > |||| < 1, -4 > ||} \quad \text{(given)}$$

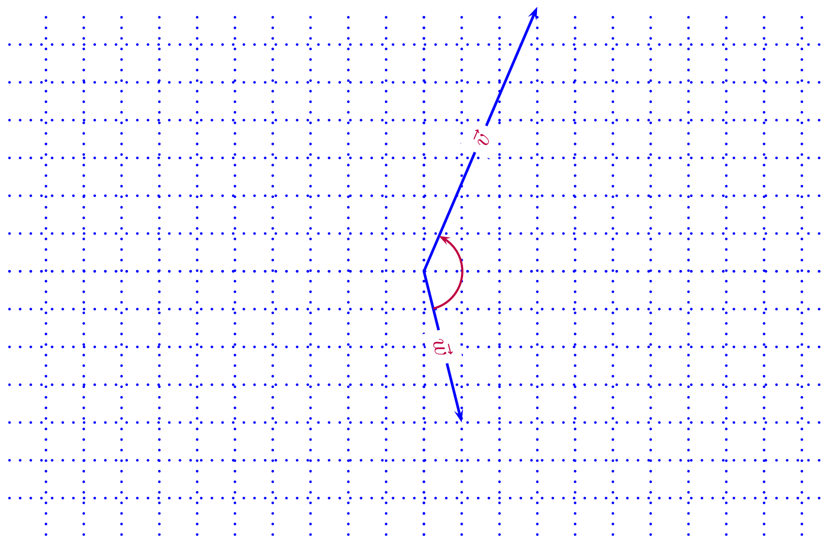
$$\approx \frac{-25}{(7.616)(4.123)} \approx \frac{-25}{31.401} \quad \text{(by Calc)}$$

....thus.... $\cos\theta = -0.8$ (by Calc)

....then.. (assume the sought angle is $0 \leq \theta \leq 180°$).... $\theta \approx \cos^{-1}(-0.8)$

$$\theta \approx 2.5 \text{ radians}$$

$$\approx 143.2°$$



(f) Use the dot product to find the 'angle' between the indicated vectors.

$$\vec{w} =< 3, 9 > \qquad \vec{v} =< 7, 1 >$$

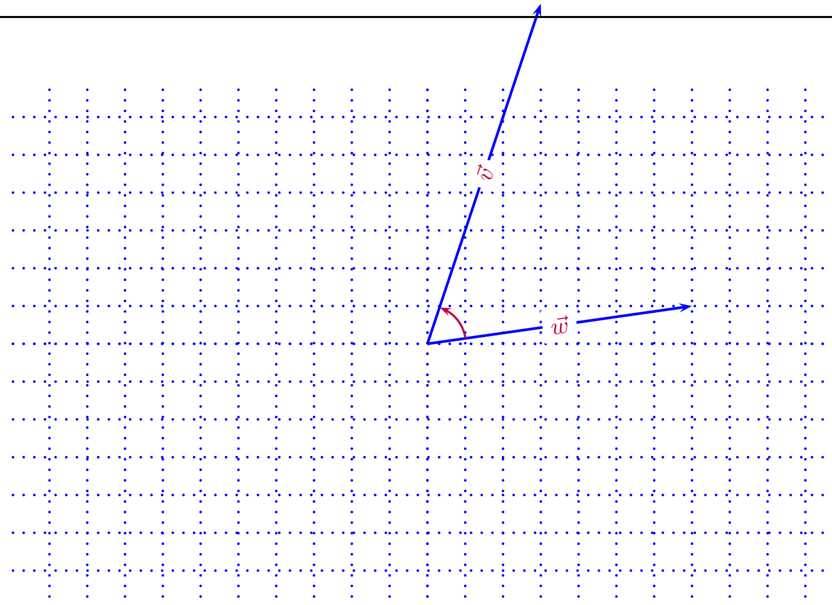
Solution: Let us assume the angle between the vectors is between 0 and 180° Suppose we call such angle 'θ', then....

$$\cos\theta = \frac{\vec{v}\cdot\vec{w}}{||\vec{v}||||\vec{w}||} \quad \text{(famous DOT product property)}$$
$$= \frac{< 3, 9 > \cdot < 7, 1 >}{|| < 3, 9 > |||| < 7, 1 > ||} \quad \text{(given)}$$
$$\approx \frac{30}{(9.487)(7.071)} \approx \frac{30}{67.083} \quad \text{(by Calc)}$$

....thus.... $\cos\theta = 0.45$ (by Calc)

....then.. (assume the sought angle is $0 \leq \theta \leq 180°$).... $\theta \approx \cos^{-1}(0.45)$

$$\theta \approx 1.1 \text{ radians}$$
$$\approx 63°$$



(g) Use the dot product to find the 'angle' between the indicated vectors.

$$\vec{w} =< -5, 2 > \qquad \vec{v} =< 2, 5 >$$

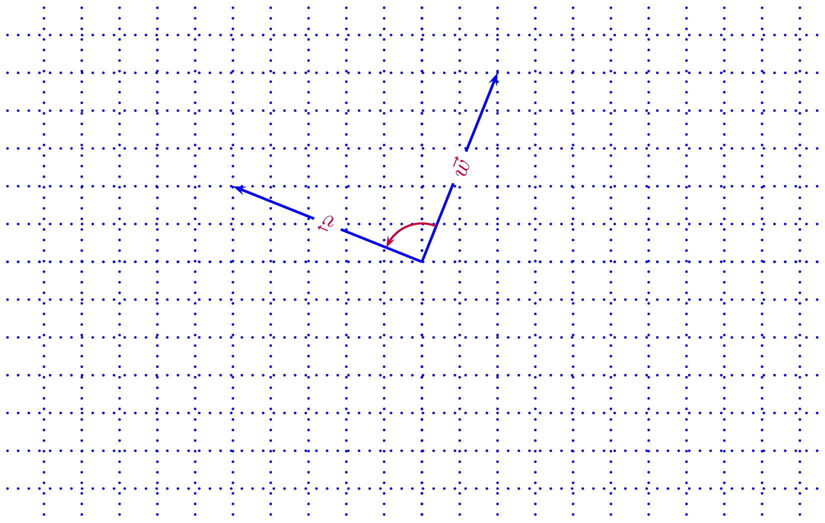
Solution: Let us assume the angle between the vectors is between 0 and 180° Suppose we call such angle 'θ', then....

$$\cos\theta = \frac{\vec{v}\cdot\vec{w}}{||\vec{v}||||\vec{w}||} \quad \text{(famous DOT product property)}$$
$$= \frac{< -5, 2 > \cdot < 2, 5 >}{|| < -5, 2 > |||| < 2, 5 > ||} \quad \text{(given)}$$
$$\approx \frac{0}{(5.385)(5.385)} \approx \frac{0}{28.998} \quad \text{(by Calc)}$$

....thus.... $\cos\theta = 0$ (by Calc)

....then.. (assume the sought angle is $0 \le \theta \le 180°$).... $\theta \approx \cos^{-1}(0)$

$$\theta \approx 1.57 \text{ radians}$$
$$\approx 90°$$



(h) Use the dot product to find the 'angle' between the indicated vectors.

$$\vec{w} = < 1, 7 > \qquad \vec{v} = < 7, -1 >$$

Solution: Let us assume the angle between the vectors is between 0 and 180° Suppose we call such angle 'θ', then....

$$\cos\theta = \frac{\vec{v}\cdot\vec{w}}{||\vec{v}||||\vec{w}||} \quad \text{(famous DOT product property)}$$

$$= \frac{< 1, 7 > \cdot < 7, -1 >}{|| < 1, 7 > |||| < 7, -1 > ||} \quad \text{(given)}$$

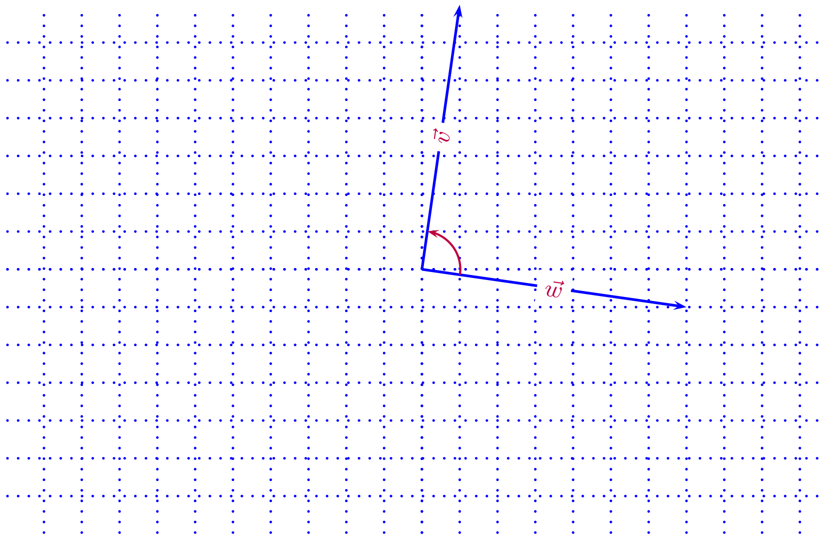
$$\approx \frac{0}{(7.071)(7.071)} \approx \frac{0}{49.999} \quad \text{(by Calc)}$$

....thus.... $\cos\theta = 0$ (by Calc)

....then.. (assume the sought angle is $0 \le \theta \le 180°$).... $\theta \approx \cos^{-1}(0)$

$$\theta \approx 1.57 \text{ radians}$$

$$\approx 90°$$

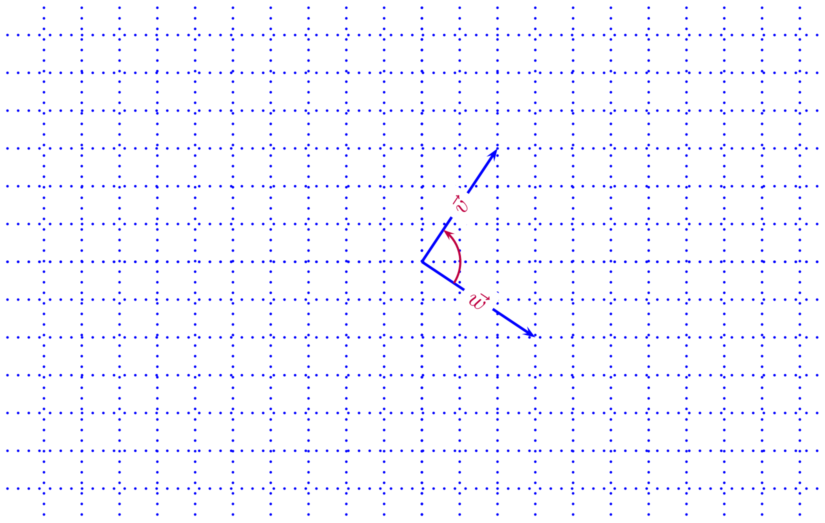


(i) Use the dot product to find the 'angle' between the indicated vectors.

$$\vec{w} =< 2, 3 > \qquad \vec{v} =< 3, -2 >$$

Solution: Let us assume the angle between the vectors is between 0 and 180° Suppose we call such angle 'θ', then....

$$\begin{aligned}
\cos\theta &= \frac{\vec{v}\cdot\vec{w}}{||\vec{v}||||\vec{w}||} && \text{(famous DOT product property)}\\
&= \frac{< 2,3 > \cdot < 3,-2 >}{|| < 2,3 > |||| < 3,-2 > ||} && \text{(given)}\\
&\approx \frac{0}{(3.606)(3.606)} \approx \frac{0}{13.003} && \text{(by Calc)}\\
\text{....thus....} \quad \cos\theta &= 0 && \text{(by Calc)}\\
\text{....then.. (assume the sought angle is } 0 \leq \theta \leq 180°\text{)....} \quad \theta &\approx \cos^{-1}(0)\\
\theta &\approx 1.57 \text{ radians}\\
&\approx 90°
\end{aligned}$$



(j) Use the dot product to find the 'angle' between the indicated vectors.

$$\vec{w} = < 1, -3, 5, 0, 6 > \qquad \vec{v} = < 5, 2, -10, 3, 1 >$$

Solution: don't be afraid, don't google it, don't ask anyone.. just you and the problem.. its on..., if it knocks you down, just get up... don't let it beat you..

6. **Perpendicular Test by *the DOT*** Find the Dot product for each pair of vectors, then determine if they are perpendicular.

(a) Use the dot product to find test if the indicated vectors are 'perpendicular'.

$$\vec{w} = < 1, 2 > \qquad \vec{v} = < 3, -5 >$$

Solution: The perpendicular test is quite simple and elegant. Vector $\vec{v}$ is perpendicular to $\vec{w}$ if and only if $\vec{v} \cdot \vec{w} = 0$, or more concisely,

$$\vec{v} \perp \vec{w} \iff \vec{v} \cdot \vec{w} = 0$$

Thus we check..

$$\begin{aligned} \vec{v} \cdot \vec{w} &= < 3, -5 > \cdot < 1, 2 > && \text{(given)} \\ &= -7 && \text{(by inspection)} \end{aligned}$$

....therefore....$\vec{v}$ and $\vec{w}$ are not perpendicular.

(b) Use the dot product to find test if the indicated vectors are 'perpendicular'.

$$\vec{w} = < 3, 2 > \qquad \vec{v} = < 2, -3 >$$

Solution: The perpendicular test is quite simple and elegant. Vector $\vec{v}$ is perpendicular to $\vec{w}$ if and only if $\vec{v} \cdot \vec{w} = 0$, or more concisely,

$$\vec{v} \perp \vec{w} \iff \vec{v} \cdot \vec{w} = 0$$

Thus we check..

$$\begin{aligned} \vec{v} \cdot \vec{w} &= < 2, -3 > \cdot < 3, 2 > && \text{(given)} \\ &= 0 && \text{(by inspection)} \end{aligned}$$

....therefore....$\vec{v}$ and $\vec{w}$ are perpendicular.

(c) Use the dot product to find test if the indicated vectors are 'perpendicular'.

$$\vec{w} = < 1, 4 > \qquad \vec{v} = < -3, -5 >$$

Solution: The perpendicular test is quite simple and elegant. Vector $\vec{v}$ is perpendicular to $\vec{w}$ if and only if $\vec{v} \cdot \vec{w} = 0$, or more concisely,

$$\vec{v} \perp \vec{w} \iff \vec{v} \cdot \vec{w} = 0$$

Thus we check..

$$\begin{aligned} \vec{v} \cdot \vec{w} &= < -3, -5 > \cdot < 1, 4 > && \text{(given)} \\ &= -23 && \text{(by inspection)} \end{aligned}$$

....therefore....$\vec{v}$ and $\vec{w}$ are not perpendicular.

(d) Use the dot product to find test if the indicated vectors are 'perpendicular'.

$$\vec{w} = < 2, 3 > \qquad \vec{v} = < 3, -3 >$$

Solution: The perpendicular test is quite simple and elegant. Vector $\vec{v}$ is perpendicular to $\vec{w}$ if and only if $\vec{v} \cdot \vec{w} = 0$, or more concisely,

$$\vec{v} \perp \vec{w} \iff \vec{v} \cdot \vec{w} = 0$$

Thus we check..

$$\begin{aligned} \vec{v} \cdot \vec{w} &= < 3, -3 > \cdot < 2, 3 > && \text{(given)} \\ &= -3 && \text{(by inspection)} \end{aligned}$$

....therefore....$\vec{v}$ and $\vec{w}$ are not perpendicular.

(e) Use the dot product to find test if the indicated vectors are 'perpendicular'.

$$\vec{w} = < 3, 6 > \qquad \vec{v} = < 2, -1 >$$

Solution: The perpendicular test is quite simple and elegant. Vector $\vec{v}$ is perpendicular to $\vec{w}$ if and only if $\vec{v} \cdot \vec{w} = 0$, or more concisely,

$$\vec{v} \perp \vec{w} \iff \vec{v} \cdot \vec{w} = 0$$

Thus we check..

$$\begin{aligned} \vec{v} \cdot \vec{w} &= < 2, -1 > \cdot < 3, 6 > && \text{(given)} \\ &= 0 && \text{(by inspection)} \end{aligned}$$

....therefore....$\vec{v}$ and $\vec{w}$ are perpendicular.

(f) Use the dot product to find test if the indicated vectors are 'perpendicular'.

$$\vec{w} = < 3, 12 > \qquad \vec{v} = < -4, 1 >$$

Solution: The perpendicular test is quite simple and elegant. Vector $\vec{v}$ is perpendicular to $\vec{w}$ if and only if $\vec{v} \cdot \vec{w} = 0$, or more concisely,

$$\vec{v} \perp \vec{w} \iff \vec{v} \cdot \vec{w} = 0$$

Thus we check..

$$\begin{aligned} \vec{v} \cdot \vec{w} &= < -4, 1 > \cdot < 3, 12 > && \text{(given)} \\ &= 0 && \text{(by inspection)} \end{aligned}$$

....therefore....$\vec{v}$ and $\vec{w}$ are perpendicular.

(g) Use the dot product to find test if the indicated vectors are 'perpendicular'.

$$\vec{w} = < -5, 2 > \qquad \vec{v} = < 3, 5 >$$

Solution: The perpendicular test is quite simple and elegant. Vector $\vec{v}$ is perpendicular to $\vec{w}$ if and only if $\vec{v} \cdot \vec{w} = 0$, or more concisely,

$$\vec{v} \perp \vec{w} \iff \vec{v} \cdot \vec{w} = 0$$

Thus we check..

$$\begin{aligned} \vec{v} \cdot \vec{w} &= < 3, 5 > \cdot < -5, 2 > && \text{(given)} \\ &= -5 && \text{(by inspection)} \end{aligned}$$

....therefore....$\vec{v}$ and $\vec{w}$ are not perpendicular.

(h) Use the dot product to find test if the indicated vectors are 'perpendicular'.

$$\vec{w} = < 1, 7 > \qquad \vec{v} = < 7, -2 >$$

Solution: The perpendicular test is quite simple and elegant. Vector $\vec{v}$ is perpendicular to $\vec{w}$ if and only if $\vec{v} \cdot \vec{w} = 0$, or more concisely,

$$\vec{v} \perp \vec{w} \qquad \Longleftrightarrow \qquad \vec{v} \cdot \vec{w} = 0$$

Thus we check..

$$\begin{aligned} \vec{v} \cdot \vec{w} &= < 7, -2 > \cdot < 1, 7 > && \text{(given)} \\ &= -7 && \text{(by inspection)} \end{aligned}$$

....therefore....$\vec{v}$ and $\vec{w}$ are not perpendicular.

(i) Use the dot product to find test if the indicated vectors are 'perpendicular'.

$$\vec{w} = < 2, 3 > \qquad \vec{v} = < 3, -2 >$$

Solution: The perpendicular test is quite simple and elegant. Vector $\vec{v}$ is perpendicular to $\vec{w}$ if and only if $\vec{v} \cdot \vec{w} = 0$, or more concisely,

$$\vec{v} \perp \vec{w} \qquad \Longleftrightarrow \qquad \vec{v} \cdot \vec{w} = 0$$

Thus we check..

$$\begin{aligned} \vec{v} \cdot \vec{w} &= < 3, -2 > \cdot < 2, 3 > && \text{(given)} \\ &= 0 && \text{(by inspection)} \end{aligned}$$

....therefore....$\vec{v}$ and $\vec{w}$ are perpendicular.

(j) test to see if perpendicular...

$$\vec{w} = < 1, -3, 5, -2, 6 > \qquad \vec{v} = < 5, 0, -1, 3, 1 >$$

Solution: don't be afraid, don't google it, don't ask anyone.. just you and the problem.. its on..., if it knocks you down, just get up... don't let it beat you..

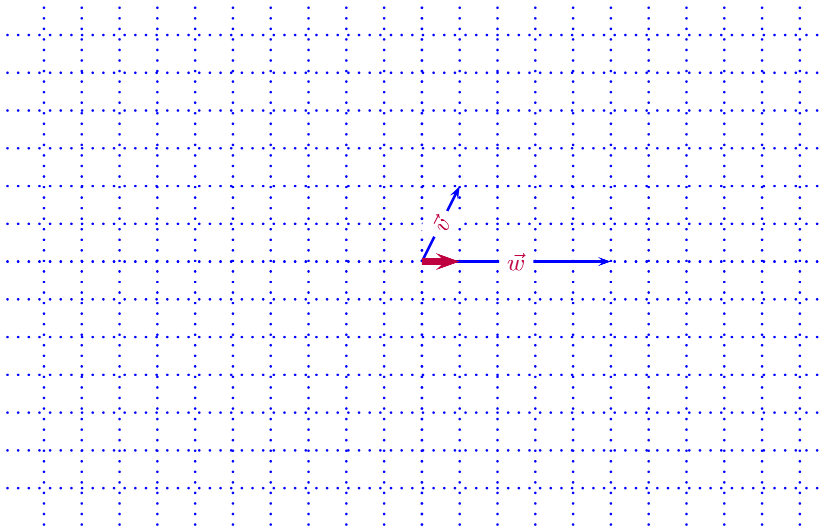
7. **Projections by *the DOT***

(a) Use the dot product to find the 'projection' of $\vec{v}$ onto $\vec{w}$ then draw such projection.

$$\vec{v} = < 1, 2 > \qquad \vec{w} = < 5, 0 >$$

Solution:

$$\begin{aligned} \mathbf{proj}_{\vec{w}}\vec{v} &= \frac{\vec{v} \cdot \vec{w}}{||\vec{w}||^2}\vec{w} && \text{(famous DOT product property)} \\ &= \frac{< 1, 2 > \cdot < 5, 0 >}{|| < 5, 0 > ||^2} < 5, 0 > && \text{(given)} \\ &\approx \frac{5}{25} < 5, 0 > && \text{(by Calc)} \\ &\approx 0.2 < 5, 0 > && \text{(by Calc)} \\ &\approx < 1, 0 > && \text{(by Calc)} \end{aligned}$$



(b) Use the dot product to find the 'projection' of $\vec{v}$ onto $\vec{w}$ then draw such projection.

$\vec{v} =< 3, 2 >$ $\vec{w} =< 5, 0 >$

Solution:

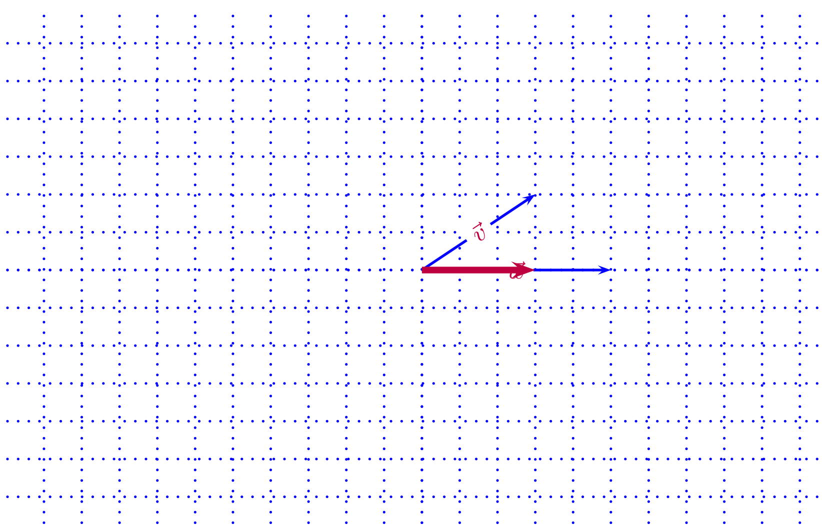
$$\mathbf{proj}_{\vec{w}}\vec{v} = \frac{\vec{v}\cdot\vec{w}}{||\vec{w}||^2}\vec{w} \quad \text{(famous DOT product property)}$$

$$= \frac{< 3, 2 > \cdot < 5, 0 >}{|| < 5, 0 > ||^2} < 5, 0 > \quad \text{(given)}$$

$$\approx \frac{15}{25} < 5, 0 > \quad \text{(by Calc)}$$

$$\approx 0.6 < 5, 0 > \quad \text{(by Calc)}$$

$$\approx < 3, 0 > \quad \text{(by Calc)}$$

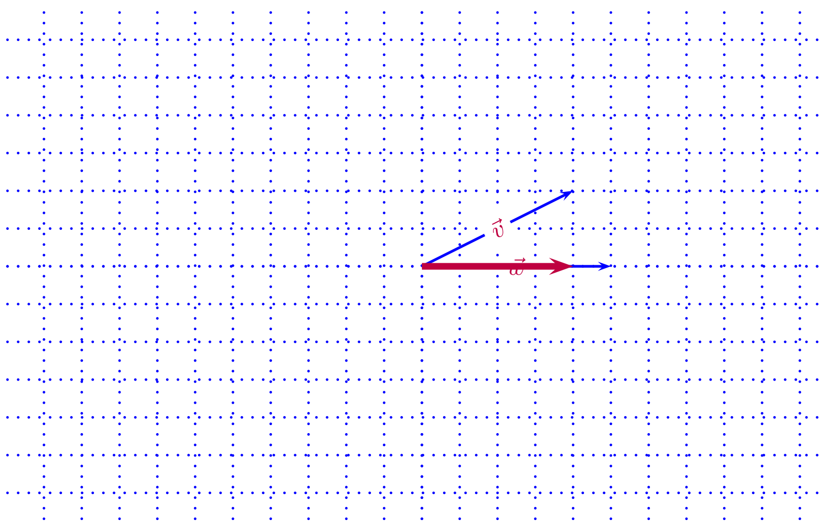


(c) Use the dot product to find the 'projection' of $\vec{v}$ onto $\vec{w}$ then draw such projection.

$$\vec{v} = < 4, 2 > \qquad \vec{w} = < 5, 0 >$$

Solution:

$$\begin{aligned}
\mathbf{proj}_{\vec{w}}\vec{v} &= \frac{\vec{v}\cdot\vec{w}}{||\vec{w}||^2}\vec{w} && \text{(famous DOT product property)}\\
&= \frac{< 4,2 > \cdot < 5,0 >}{|| < 5,0 > ||^2} < 5,0 > && \text{(given)}\\
&\approx \frac{20}{25} < 5,0 > && \text{(by Calc)}\\
&\approx 0.8 < 5,0 > && \text{(by Calc)}\\
&\approx < 4,0 > && \text{(by Calc)}
\end{aligned}$$

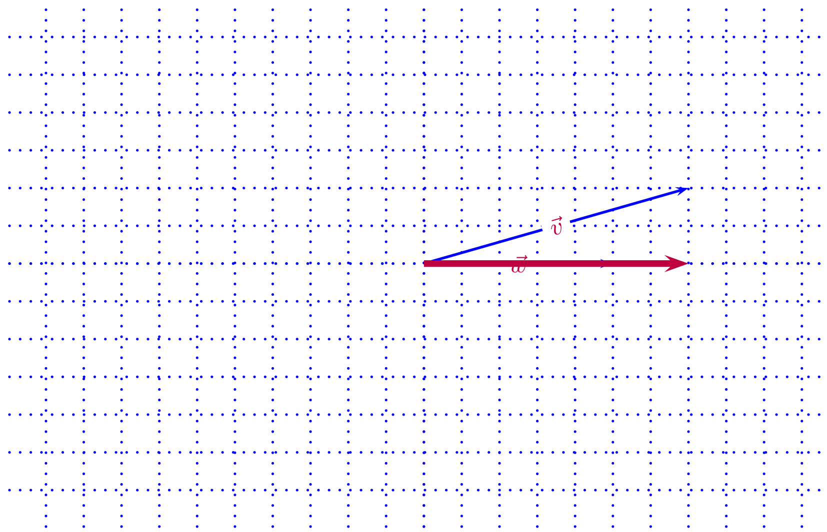


(d) Use the dot product to find the 'projection' of $\vec{v}$ onto $\vec{w}$ then draw such projection.

$$\vec{v} = < 7, 2 > \qquad \vec{w} = < 5, 0 >$$

Solution:

$$\begin{aligned}
\mathbf{proj}_{\vec{w}}\vec{v} &= \frac{\vec{v}\cdot\vec{w}}{||\vec{w}||^2}\vec{w} && \text{(famous DOT product property)}\\
&= \frac{< 7,2 > \cdot < 5,0 >}{|| < 5,0 > ||^2} < 5,0 > && \text{(given)}\\
&\approx \frac{35}{25} < 5,0 > && \text{(by Calc)}\\
&\approx 1.4 < 5,0 > && \text{(by Calc)}\\
&\approx < 7,0 > && \text{(by Calc)}
\end{aligned}$$

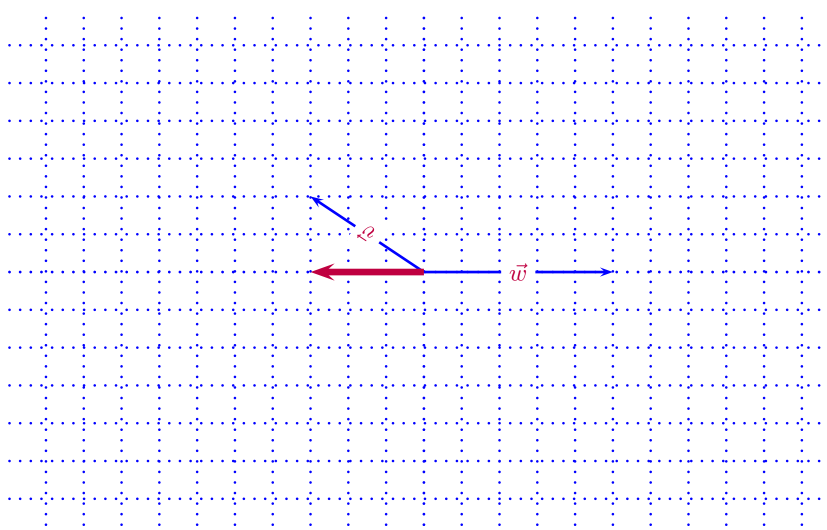


(e) Use the dot product to find the 'projection' of $\vec{v}$ onto $\vec{w}$ then draw such projection.

$$\vec{v} =< -3, 2 > \qquad \vec{w} =< 5, 0 >$$

Solution:

$$
\begin{aligned}
\mathbf{proj}_{\vec{w}}\vec{v} &= \frac{\vec{v}\cdot\vec{w}}{||\vec{w}||^2}\vec{w} && \text{(famous DOT product property)}\\
&= \frac{< -3, 2 > \cdot < 5, 0 >}{|| < 5, 0 > ||^2}< 5, 0 > && \text{(given)}\\
&\approx \frac{-15}{25}< 5, 0 > && \text{(by Calc)}\\
&\approx -0.6< 5, 0 > && \text{(by Calc)}\\
&\approx < -3, 0 > && \text{(by Calc)}
\end{aligned}
$$

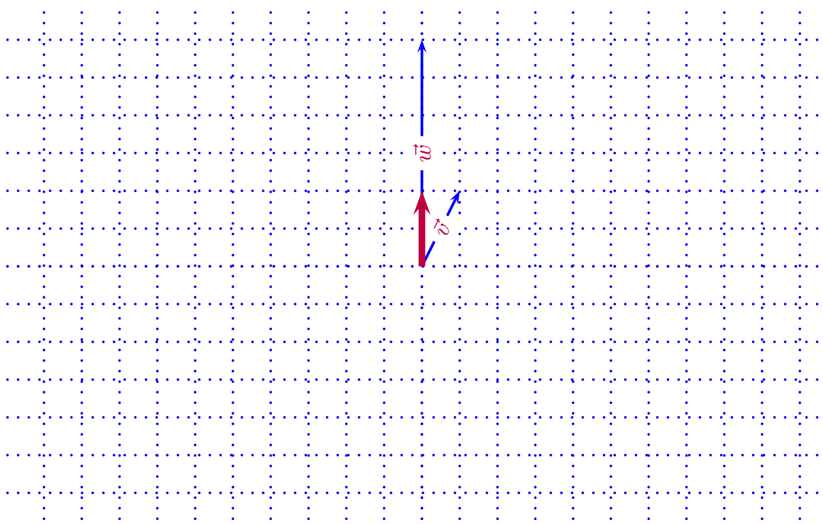


(f) Use the dot product to find the 'projection' of $\vec{v}$ onto $\vec{w}$ then draw such projection.

$$\vec{v} =< 1, 2 > \qquad \vec{w} =< 0, 6 >$$

Solution:

$$\begin{aligned}
\mathbf{proj}_{\vec{w}}\vec{v} &= \frac{\vec{v}\cdot\vec{w}}{||\vec{w}||^2}\vec{w} && \text{(famous DOT product property)}\\
&= \frac{< 1, 2 > \cdot < 0, 6 >}{|| < 0, 6 > ||^2} < 0, 6 > && \text{(given)}\\
&\approx \frac{12}{36} < 0, 6 > && \text{(by Calc)}\\
&\approx 0.333 < 0, 6 > && \text{(by Calc)}\\
&\approx < 0, 2 > && \text{(by Calc)}
\end{aligned}$$

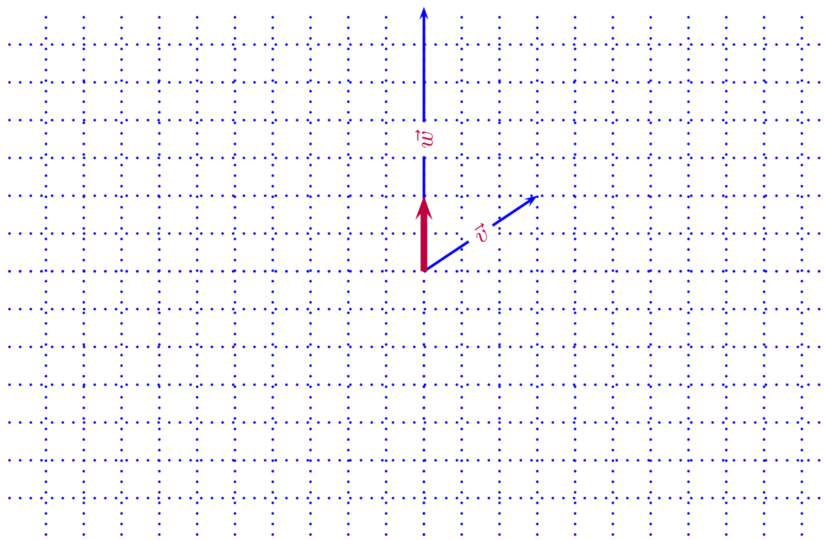


(g) Use the dot product to find the 'projection' of $\vec{v}$ onto $\vec{w}$ then draw such projection.

$$\vec{v} =< 3, 2 > \qquad \vec{w} =< 0, 7 >$$

Solution:

$$\begin{aligned}
\mathbf{proj}_{\vec{w}}\vec{v} &= \frac{\vec{v}\cdot\vec{w}}{||\vec{w}||^2}\vec{w} && \text{(famous DOT product property)}\\
&= \frac{< 3, 2 > \cdot < 0, 7 >}{|| < 0, 7 > ||^2} < 0, 7 > && \text{(given)}\\
&\approx \frac{14}{49} < 0, 7 > && \text{(by Calc)}\\
&\approx 0.286 < 0, 7 > && \text{(by Calc)}\\
&\approx < 0, 2 > && \text{(by Calc)}
\end{aligned}$$

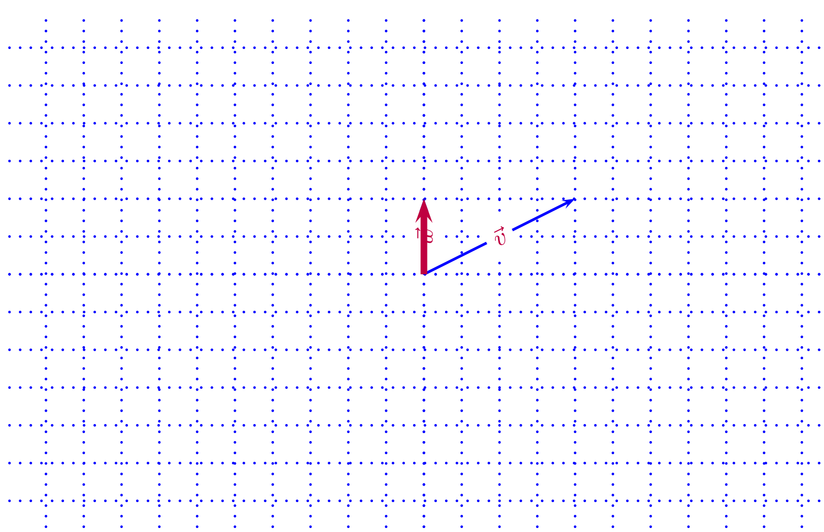


(h) Use the dot product to find the 'projection' of $\vec{v}$ onto $\vec{w}$ then draw such projection.

$$\vec{v} = <4, 2> \qquad \vec{w} = <0, 2>$$

Solution:

$$\begin{aligned}
\mathbf{proj}_{\vec{w}}\vec{v} &= \frac{\vec{v}\cdot\vec{w}}{||\vec{w}||^2}\vec{w} && \text{(famous DOT product property)}\\
&= \frac{<4,2>\cdot<0,2>}{||<0,2>||^2}<0,2> && \text{(given)}\\
&\approx \frac{4}{4}<0,2> && \text{(by Calc)}\\
&\approx 1<0,2> && \text{(by Calc)}\\
&\approx <0,2> && \text{(by Calc)}
\end{aligned}$$

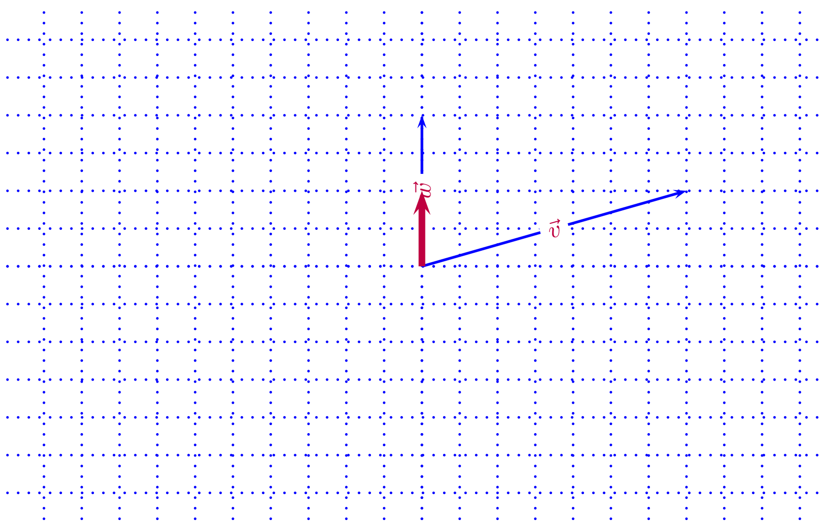


(i) Use the dot product to find the 'projection' of $\vec{v}$ onto $\vec{w}$ then draw such projection.

$$\vec{v} = <7,2> \qquad \vec{w} = <0,4>$$

Solution:

$$\begin{aligned}
\mathbf{proj}_{\vec{w}}\vec{v} &= \frac{\vec{v}\cdot\vec{w}}{||\vec{w}||^2}\vec{w} && \text{(famous DOT product property)}\\
&= \frac{<7,2>\cdot<0,4>}{||<0,4>||^2}<0,4> && \text{(given)}\\
&\approx \frac{8}{16}<0,4> && \text{(by Calc)}\\
&\approx 0.5<0,4> && \text{(by Calc)}\\
&\approx <0,2> && \text{(by Calc)}
\end{aligned}$$

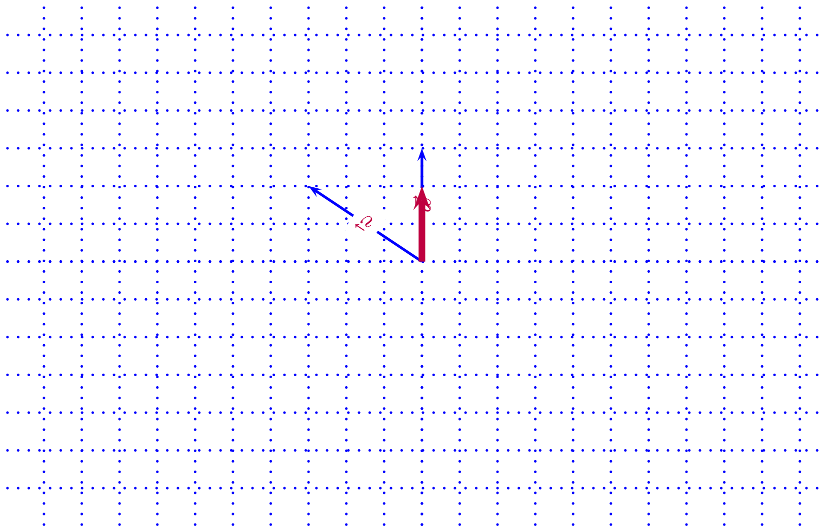


(j) Use the dot product to find the 'projection' of $\vec{v}$ onto $\vec{w}$ then draw such projection.

$$\vec{v} = <-3,2> \qquad \vec{w} = <0,3>$$

Solution:

$$\begin{aligned}
\mathbf{proj}_{\vec{w}}\vec{v} &= \frac{\vec{v}\cdot\vec{w}}{||\vec{w}||^2}\vec{w} && \text{(famous DOT product property)}\\
&= \frac{<-3,2>\cdot<0,3>}{||<0,3>||^2}<0,3> && \text{(given)}\\
&\approx \frac{6}{9}<0,3> && \text{(by Calc)}\\
&\approx 0.667<0,3> && \text{(by Calc)}\\
&\approx <0,2> && \text{(by Calc)}
\end{aligned}$$

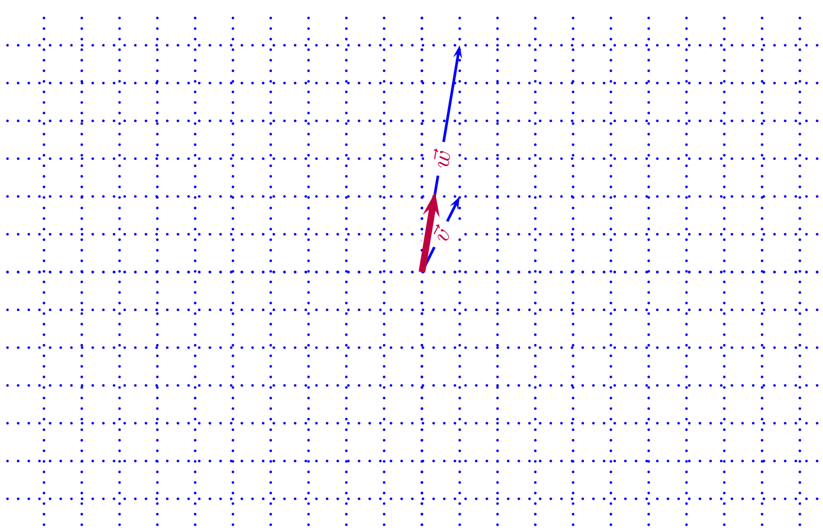


(k) Use the dot product to find the 'projection' of $\vec{v}$ onto $\vec{w}$ then draw such projection.

$$\vec{v} = <1, 2> \qquad \vec{w} = <1, 6>$$

Solution:

$$\begin{aligned}
\mathbf{proj}_{\vec{w}}\vec{v} &= \frac{\vec{v}\cdot\vec{w}}{||\vec{w}||^2}\vec{w} && \text{(famous DOT product property)}\\
&= \frac{<1,2>\cdot<1,6>}{||<1,6>||^2}<1,6> && \text{(given)}\\
&\approx \frac{13}{37}<1,6> && \text{(by Calc)}\\
&\approx 0.351<1,6> && \text{(by Calc)}\\
&\approx <0.35, 2.11> && \text{(by Calc)}
\end{aligned}$$

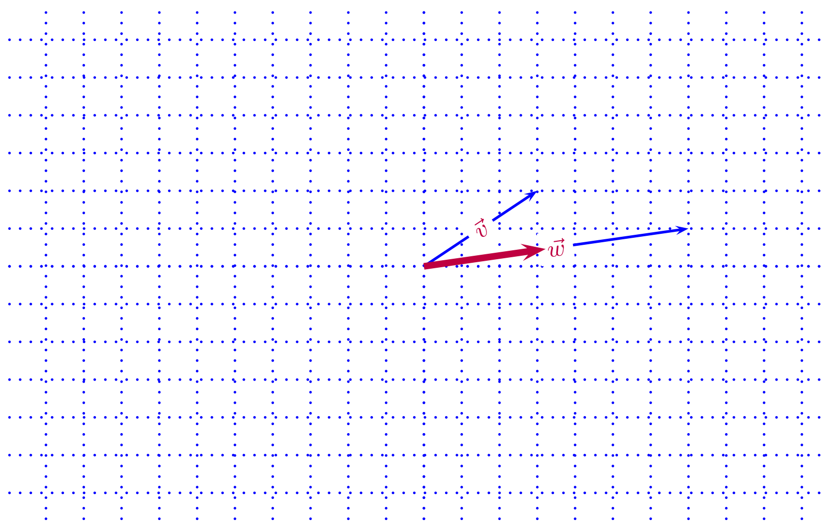


(l) Use the dot product to find the 'projection' of $\vec{v}$ onto $\vec{w}$ then draw such projection.

$$\vec{v} = <3, 2> \qquad \vec{w} = <7, 1>$$

Solution:

$$\begin{aligned}
\textbf{proj}_{\vec{w}}\vec{v} &= \frac{\vec{v}\cdot\vec{w}}{||\vec{w}||^2}\vec{w} && \text{(famous DOT product property)}\\
&= \frac{<3,2>\cdot<7,1>}{||<7,1>||^2}<7,1> && \text{(given)}\\
&\approx \frac{23}{50}<7,1> && \text{(by Calc)}\\
&\approx 0.46<7,1> && \text{(by Calc)}\\
&\approx <3.22, 0.46> && \text{(by Calc)}
\end{aligned}$$

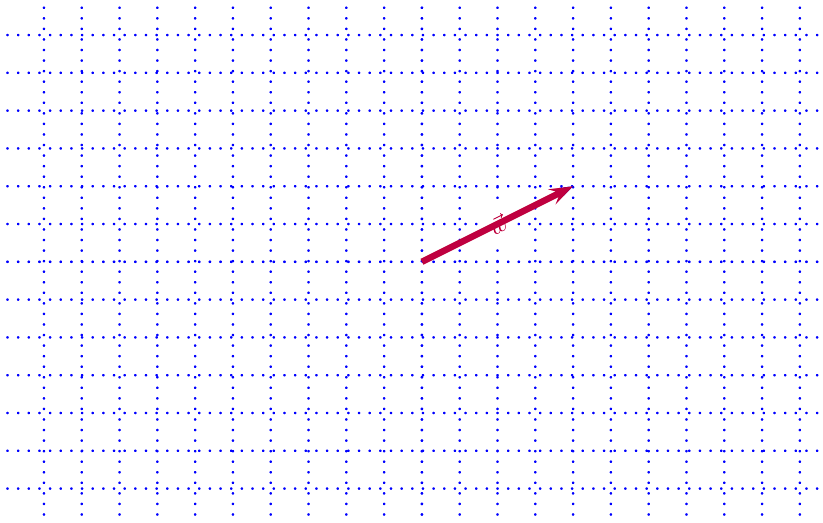


(m) Use the dot product to find the 'projection' of $\vec{v}$ onto $\vec{w}$ then draw such projection.

$$\vec{v} = <4, 2> \qquad \vec{w} = <4, 2>$$

Solution:

$$\begin{aligned}
\textbf{proj}_{\vec{w}}\vec{v} &= \frac{\vec{v}\cdot\vec{w}}{||\vec{w}||^2}\vec{w} && \text{(famous DOT product property)}\\
&= \frac{<4,2>\cdot<4,2>}{||<4,2>||^2}<4,2> && \text{(given)}\\
&\approx \frac{20}{20}<4,2> && \text{(by Calc)}\\
&\approx 1<4,2> && \text{(by Calc)}\\
&\approx <4, 2> && \text{(by Calc)}
\end{aligned}$$

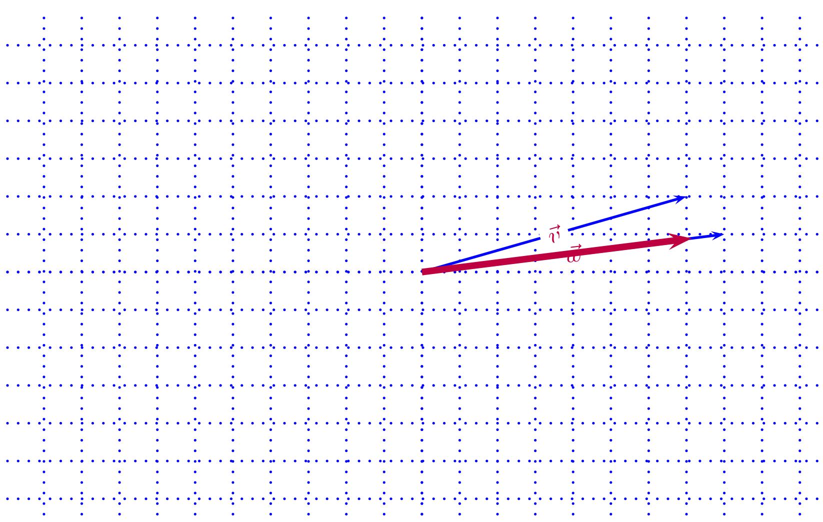


(n) Use the dot product to find the 'projection' of $\vec{v}$ onto $\vec{w}$ then draw such projection.

$$\vec{v} =< 7, 2 > \qquad \vec{w} =< 8, 1 >$$

Solution:

$$\begin{aligned}
\mathbf{proj}_{\vec{w}}\vec{v} &= \frac{\vec{v}\cdot\vec{w}}{||\vec{w}||^2}\vec{w} && \text{(famous DOT product property)}\\
&= \frac{< 7, 2 > \cdot < 8, 1 >}{|| < 8, 1 > ||^2}< 8, 1 > && \text{(given)}\\
&\approx \frac{58}{65}< 8, 1 > && \text{(by Calc)}\\
&\approx 0.892< 8, 1 > && \text{(by Calc)}\\
&\approx < 7.14, 0.89 > && \text{(by Calc)}
\end{aligned}$$

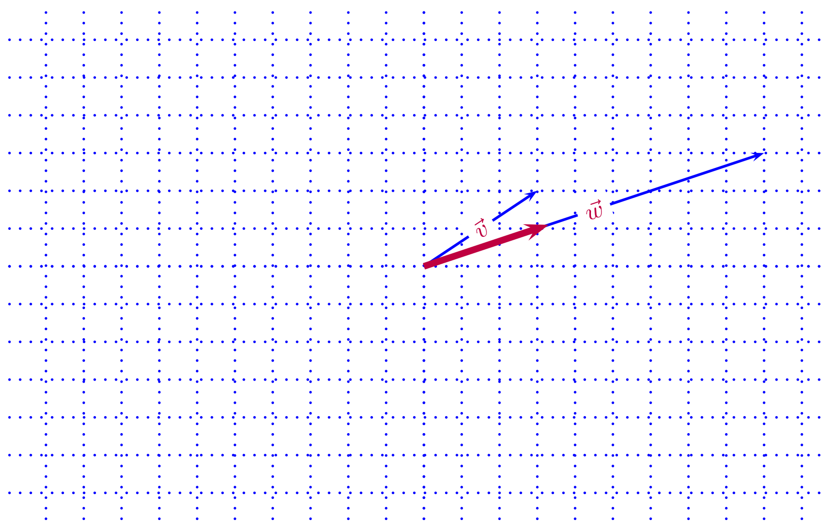


(o) Use the dot product to find the 'projection' of $\vec{v}$ onto $\vec{w}$ then draw such projection.

$$\vec{v} = < 3, 2 > \qquad \vec{w} = < 9, 3 >$$

Solution:

$$\begin{aligned}
\mathbf{proj}_{\vec{w}}\vec{v} &= \frac{\vec{v}\cdot\vec{w}}{||\vec{w}||^2}\vec{w} && \text{(famous DOT product property)}\\
&= \frac{< 3,2 > \cdot < 9,3 >}{|| < 9,3 > ||^2} < 9,3 > && \text{(given)}\\
&\approx \frac{33}{90} < 9,3 > && \text{(by Calc)}\\
&\approx 0.367 < 9,3 > && \text{(by Calc)}\\
&\approx < 3.3, 1.1 > && \text{(by Calc)}
\end{aligned}$$



(p) find projection

$$\vec{v} = < 1, -3, 5, -2, 6 > \qquad \text{onto} \vec{w} = < 5, 0, -1, 3, 1 >$$

Solution: don't be afraid, don't google it, don't ask anyone.. just you and the problem.. its on..., if it knocks you down, just get up... don't let it beat you..