Q1

Wednesday, November 18, 2020 4:30 PM

Find the area of the shaded region.

$$\frac{45}{4} - \ln(8)$$

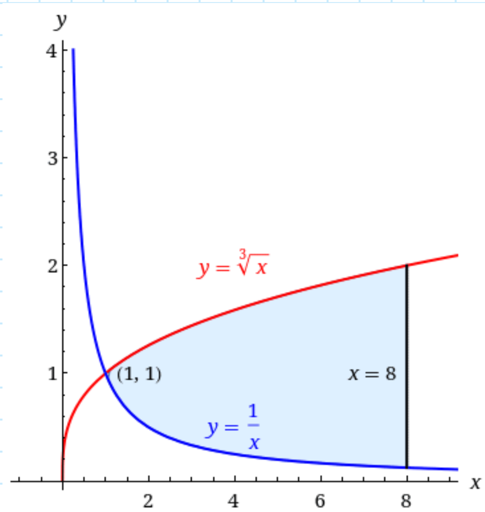
$$\int_1^8 \left(\sqrt[3]{x} - \frac{1}{x}\right) dx = \int_1^8 \left(x^{1/3} - x^{-1}\right) dx$$

$$= \left[\frac{3}{4}x^{4/3} - \ln(x)\right]_1^8$$

$$= F(8) - F(1)$$

$$= \left[\frac{3}{4}(8)^{4/3} - \ln(8)\right] - \left[\frac{3}{4}(1)^{4/3} - \ln(1)\right]$$

$$= 12 - \ln(8) - \frac{3}{4} + 0 = \boxed{\frac{45}{4} - \ln(8)}$$

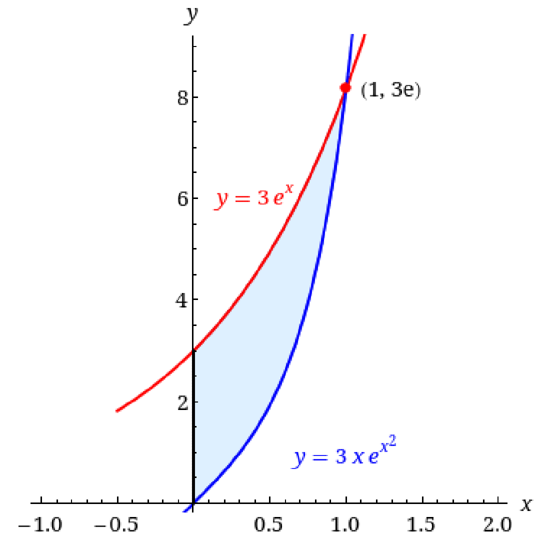


Q2

Friday, November 20, 2020 9:37 PM

Find the area of the shaded region.

$$3e - \frac{3e}{2} - \frac{3}{2}$$



$$\int 3e^x = \underline{\underline{3e^x + C}}$$

$$\int \left(3xe^{x^2}\right) dx \quad , u = x^2 \qquad \frac{du}{dx} = 2x \qquad du = 2x\,dx$$

$$= 3\int \frac{1}{2} \cdot 2x\,dx\, e^{x^2} = \frac{3}{2}\int e^u\, du$$

$$= \frac{3}{2}e^u + C = \underline{\underline{\frac{3}{2}e^{x^2} + C}}$$

$$F(1) - F(0) = 3e^x - \frac{3}{2}e^{x^2} \Big|_0^1$$

$$= \left[3e^1 - \frac{3}{2}e^{1^2}\right] - \left[3e^0 - \frac{3}{2}e^0\right]$$

$$= 3e - \frac{3e}{2} - 3 + \frac{3}{2}$$

$$= 3e - \frac{3e}{2} - \frac{3}{2}$$

Q3

Friday, November 20, 2020 10:15 PM

Find the area of the shaded region.

$$e - \frac{1}{e} + \frac{28}{3}$$

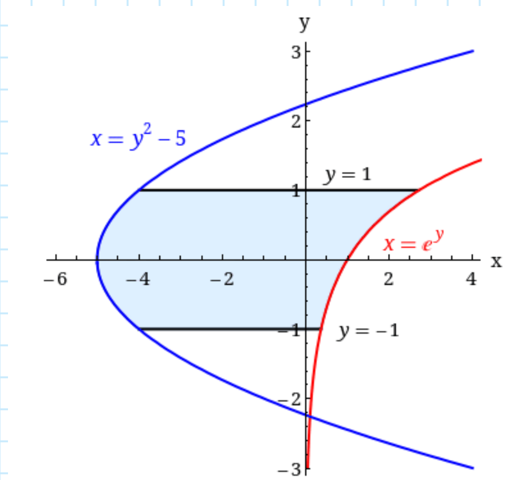
$$\int_{y=-1}^{y=1} [x_R - x_L]\, dy = \int_{-1}^{1} [e^y - (y^2 - 5)]\, dy$$

$$= \int_{-1}^{1} (e^y - y^2 + 5)\, dy = e^y - \frac{y^3}{3} + 5y \Big|_{-1}^{1}$$

$$= F(1) - F(-1)$$

$$= \left(e^1 - \frac{1^3}{3} + 5(1)\right) - \left(e^{-1} - \frac{-1^3}{3} + 5(-1)\right)$$

$$= e - \frac{1}{3} + 5 - \frac{1}{e} - \frac{1}{3} + 5 = \boxed{e - \frac{1}{e} + \frac{28}{3}}$$



Q4

Saturday, November 21, 2020 8:42 AM

Find the area of the shaded region.

$\frac{243}{8}$

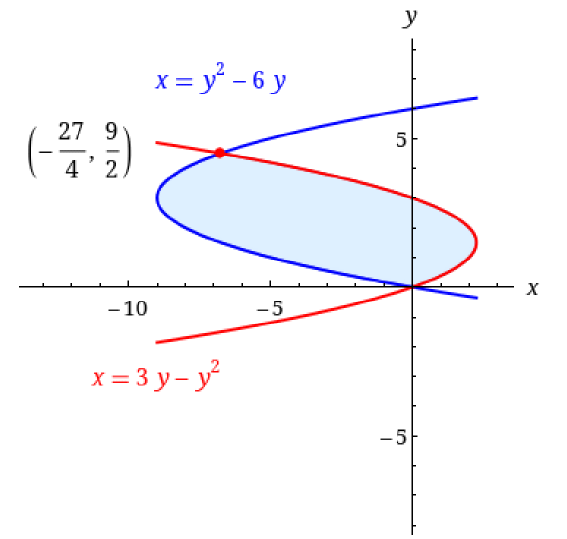
$$\int_{y=0}^{y=\frac{9}{2}} [x_R - x_L]\,dy = \int_0^{\frac{9}{2}} [3y - y^2 - (y^2 - 6y)]\,dy$$

$$= \int_0^{\frac{9}{2}} (-2y^2 + 9y)\,dy = -\frac{2y^3}{3} + \frac{9y^2}{2} \Big|_0^{\frac{9}{2}}$$

$$= F(\tfrac{9}{2}) - F(0)$$

$$= \left[-\frac{2(\frac{9}{2})^3}{3} + \frac{9(\frac{9}{2})^2}{2}\right] - \left[\frac{2(0)^3}{3} + \frac{9(0)^2}{2}\right]$$

$$= -\frac{243}{4} + \frac{729}{8} = \boxed{\frac{243}{8}}$$



Q5

Saturday, November 21, 2020 8:57 AM

Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle.

$y = e^x, \ \ y = x^2 - 1, \ \ x = -1, \ \ x = 1$

$y = e^x, \ y = x^2 - 1, \ x = -1, \ x = 1$

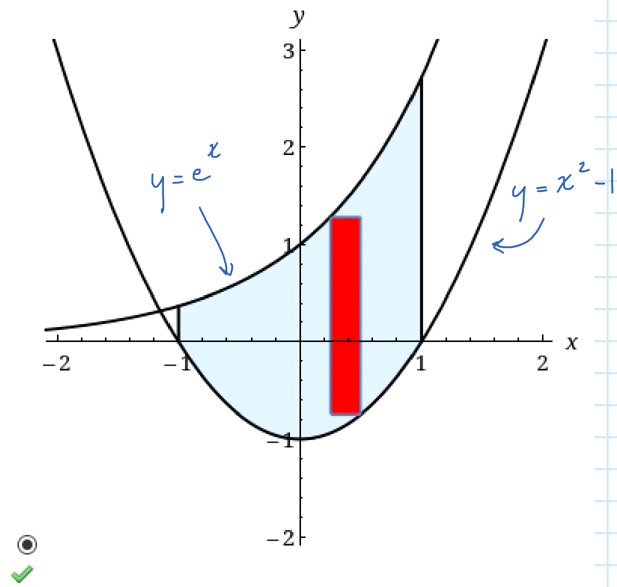
$$\int_{-1}^{1} \left[e^x - (x^2 - 1)\right] dx$$

$$= \int_{-1}^{1} (e^x - x^2 + 1)\, dx = e^x - \frac{x^3}{3} + x \Big|_{-1}^{1}$$

$$= F(1) - F(-1)$$

$$= \left[e^1 - \frac{1^3}{3} + 1\right] - \left[e^{-1} - \frac{-1^3}{3} + (-1)\right]$$

$$= e - \frac{1}{3} + 1 - \frac{1}{e} - \frac{1}{3} + 1 \ = \boxed{e - \frac{1}{e} + \frac{4}{3}}$$



Find the area of the region.

$e - \frac{1}{e} + \frac{4}{3}$

Q6

Saturday, November 21, 2020 9:12 AM

Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle.

$y = \sin(x)$, $y = 3x$, $x = \pi/2$, $x = \pi$

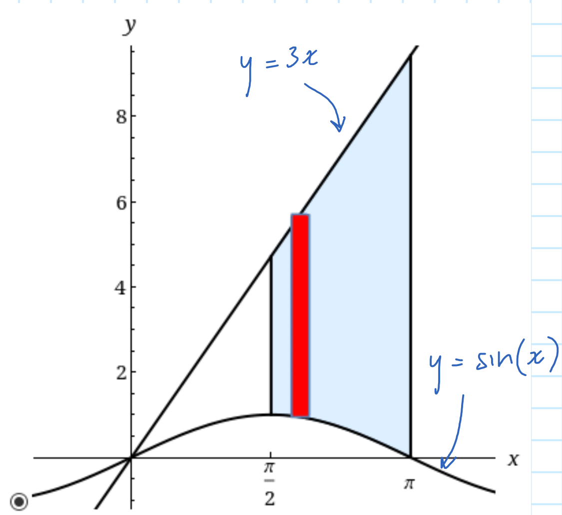
$y = \sin(x)$, $y = 3x$, $x = \pi/2$, $x = \pi$

$$\int_{\frac{\pi}{2}}^{\pi} [3x - \sin(x)]\, dx = \frac{3x^2}{2} + \cos(x) \Big|_{\frac{\pi}{2}}^{\pi}$$

$$= F(\pi) - F\left(\frac{\pi}{2}\right)$$

$$= \left[\frac{3(\pi)^2}{2} + \cos(\pi)\right] - \left[\frac{3\left(\frac{\pi}{2}\right)^2}{2} + \cos\left(\frac{\pi}{2}\right)\right]$$

$$= \frac{3\pi^2}{3} - 1 - \frac{3\pi^2}{8} = \boxed{\frac{9\pi^2}{8} - 1}$$



Find the area of the region.

$\frac{9\pi^2}{8} - 1$

Q7

Saturday, November 21, 2020 9:29 AM

Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle.

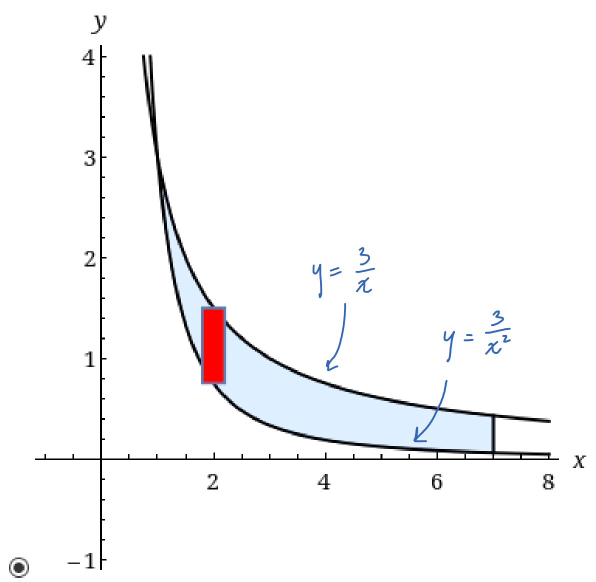
$y = 3/x, \quad y = 3/x^2, \quad x = 7$

Get the point of intersection

$$\frac{3}{x} = \frac{3}{x^2}$$

$$3x = 3$$

$$x = \frac{3}{3} = 1$$



$$\int_1^7 \left(\frac{3}{x} - \frac{3}{x^2}\right) dx = \int_1^7 \left(3\frac{1}{x} - 3x^{-2}\right) dx$$

$$= 3\ln(x) - \frac{3}{x} \Big|_1^7$$

$$= F(7) - F(1)$$

$$= \left[3\ln(7) - \frac{3}{7}\right] - \left[3\ln(1) - \frac{3}{1}\right] = 3\ln(7) - \frac{3}{7} - 3\ln(1) - 3$$

$$= 3\ln(7) - 3\ln(1) - \frac{24}{7}$$

Find the area of the region.

$3\ln(7) - \frac{18}{7}$

Q8

Saturday, November 21, 2020 11:00 AM

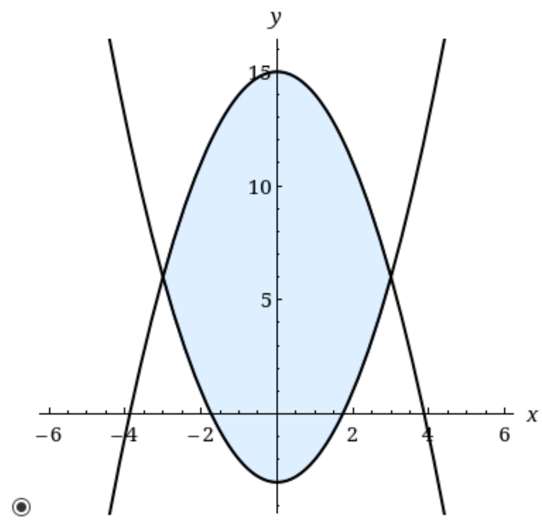
Sketch the region enclosed by the given curves.

$y = 15 - x^2, \quad y = x^2 - 3$

Get points of intersection

$$15 - x^2 = x^2 - 3$$
$$0 = 2x^2 - 18$$
$$= 2(x^2 - 9)$$
$$= 2(x+3)(x-3)$$

$x = -3$ $\qquad$ $x = 3$



$$\int_{-3}^{3} \left[15 - x^2 - (x^2 - 3)\right] dx = \int_{-3}^{3} (-2x^2 + 18)\, dx$$

$$= -\frac{2x^3}{3} + 18x \Big|_{-3}^{3}$$

$$= F(3) - F(-3)$$

$$= \left[-\frac{2(3)^3}{3} + 18(3)\right] - \left[-\frac{2(-3)^3}{3} + 18(-3)\right]$$

$$= 36 + 36 = \boxed{72}$$

Find its area.

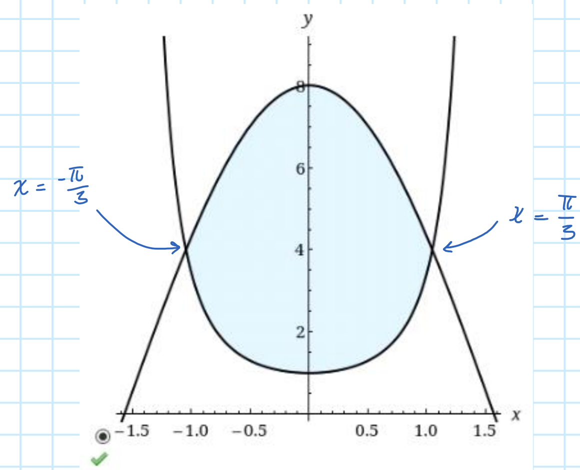
72

Q9

Saturday, November 21, 2020 11:21 AM

Sketch the region enclosed by the given curves.

$y = \sec^2(x), \quad y = 8\cos(x), \quad -\pi/3 \le x \le \pi/3$



$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left[8\cos(x) - \sec^2(x)\right] dx = 8\sin(x) - \tan(x) \Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$$

$$= F\left(\frac{\pi}{3}\right) - F\left(-\frac{\pi}{3}\right)$$

$$= \left[8\sin\left(\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{3}\right)\right] - \left[8\sin\left(-\frac{\pi}{3}\right) - \tan\left(-\frac{\pi}{3}\right)\right]$$

$$= \boxed{8\sin\left(\frac{\pi}{3}\right) - 8\sin\left(-\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{3}\right) + \tan\left(-\frac{\pi}{3}\right)}$$

Find its area.

$$8\sin\left(\frac{\pi}{3}\right) - 8\sin\left(-\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{3}\right) + \tan\left(-\frac{\pi}{3}\right)$$

Q10

Saturday, November 21, 2020 12:29 PM

Sketch the region enclosed by the given curves.

$y = 4\cos(5x), \quad y = 4 - 4\cos(5x), \quad 0 \le x \le \pi/5$

Get point of intersection

$$4\cos(5x) = 4 - 4\cos(5x)$$

$$8\cos(5x) - 4 = 0$$

$$4\left(2\cos(5x) - 1\right) = 0$$

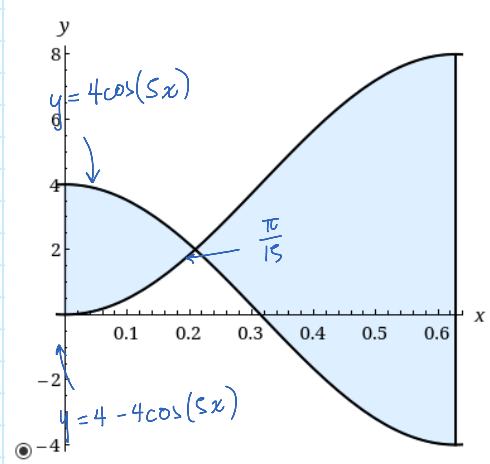
$$\downarrow$$

$$2\cos(5x) - 1 = 0$$

$$2\cos(5x) = 1$$

$$\cos(5x) = \frac{1}{2}$$

$$x = \frac{\pi}{15} + \frac{2\pi n}{5}, \quad \frac{\pi}{3} + \frac{2\pi n}{5}$$



$$\int_0^{\frac{\pi}{15}} \left[4\cos(5x) - (4 - 4\cos(5x))\right] dx + \int_{\frac{\pi}{15}}^{\frac{\pi}{5}} \left[4 - 4\cos(5x) - (4\cos(5x))\right] dx$$

$$= \int_0^{\frac{\pi}{15}} \left[8\cos(5x) - 4\right] dx + \int_{\frac{\pi}{15}}^{\frac{\pi}{5}} \left[-8\cos(5x) + 4\right] dx, \quad u = 5x \qquad \frac{du}{dx} = 5, \quad du = 5dx$$

$$= \frac{1}{5}\int_0^{\frac{\pi}{15}} 8\cos(u) - 4\, du + \frac{1}{5}\int_{\frac{\pi}{15}}^{\frac{\pi}{5}} -8\cos(u) + 4\, du$$

$$= \frac{1}{5} 8\sin(u) - 4x \Big|_0^{\frac{\pi}{15}} + -\frac{1}{5} 8\sin(u) + 4x \Big|_{\frac{\pi}{15}}^{\frac{\pi}{5}}$$

$$= \frac{8\sin(5x)}{5} - 4x \Big|_0^{\frac{\pi}{15}} + \frac{-8\sin(5x)}{5} + 4x \Big|_{\frac{\pi}{15}}^{\frac{\pi}{5}}$$

$$= F\left(\frac{\pi}{15}\right) - F(0) + G\left(\frac{\pi}{5}\right) - G\left(\frac{\pi}{15}\right)$$

$$= \left[\left(\frac{8\sin(5\frac{\pi}{15})}{5} - 4\left(\frac{\pi}{15}\right)\right) - \underbrace{\cancel{\left(\frac{8\sin(0)}{5} - 4(0)\right)}}_{=0}\right] + \left[\left(\underbrace{\cancel{\frac{-8\sin(5\frac{\pi}{5})}{5}}}_{=0} + 4\left(\frac{\pi}{5}\right)\right) - \left(\frac{-8\sin(5\frac{\pi}{15})}{5} + 4\left(\frac{\pi}{15}\right)\right)\right]$$

$$= \left[\frac{8\sin(\frac{\pi}{3})}{5} - \frac{4\pi}{15}\right] + \left[\frac{4\pi}{5} + \frac{8\sin(\frac{\pi}{3})}{5} - \frac{4\pi}{15}\right] = \boxed{\frac{4\pi}{15} + \frac{16\sin(\frac{\pi}{3})}{5}}$$

Find its area.

$$\boxed{\frac{4\pi}{15} + \frac{16\sin\left(\frac{\pi}{3}\right)}{5}}$$

Q11

Saturday, November 21, 2020 1:40 PM

Sketch the region enclosed by the given curves.

$y = 3/x, \quad y = 12x, \quad y = \frac{1}{3}x, \quad x > 0$

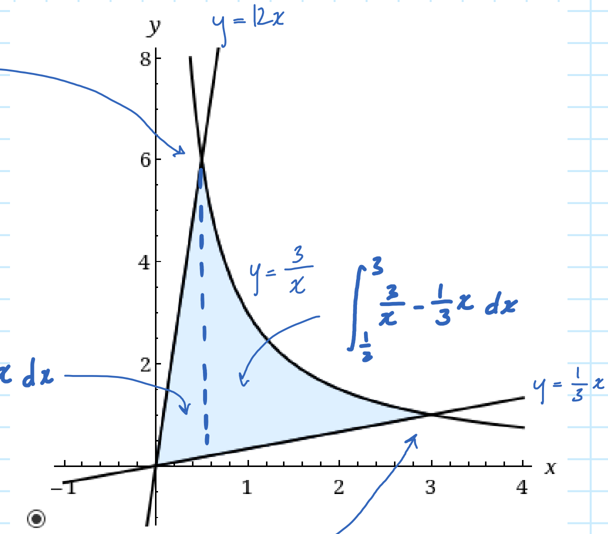
Get points of intersection

$12x = \frac{3}{x}$

$12x^2 = 3$ $\qquad x = -\frac{1}{2}, \boxed{x = \frac{1}{2}}$

$x^2 = \frac{3}{12}$

$x = \pm\sqrt{\frac{1}{4}}$



$\frac{3}{x} = \frac{1}{3}x$

$3 = \frac{1}{3}x^2$ $\qquad \boxed{x = 3}$

$9 = x^2$

$$\int_0^{\frac{1}{2}} \left(12x - \frac{1}{3}x\right)dx + \int_{\frac{1}{2}}^{3} \left(\frac{3}{x} - \frac{1}{3}x\right)dx$$

$$= 6x^2 - \frac{x^2}{6} \Big|_0^{\frac{1}{2}} + 3\ln(x) - \frac{x^2}{6} \Big|_{\frac{1}{2}}^{3}$$

$$= F\left(\tfrac{1}{2}\right) - F(0) + G(3) - G\left(\tfrac{1}{2}\right)$$

$$= \left[6\left(\tfrac{1}{2}\right)^2 - \frac{\left(\frac{1}{2}\right)^2}{6}\right] - \left[6(0)^2 - \frac{0^2}{6}\right] + \left[3\ln(3) - \frac{3^2}{6}\right] - \left[3\ln\left(\tfrac{1}{2}\right) - \frac{\left(\frac{1}{2}\right)^2}{6}\right]$$

$$= \frac{35}{24} - 0 + 3\ln(3) - \frac{3}{2} - 3\ln\left(\tfrac{1}{2}\right) + \frac{1}{24} = \boxed{3\ln(3) - 3\ln\left(\tfrac{1}{2}\right)}$$

Find its area.

$3\ln(3) - 3\ln\left(\frac{1}{2}\right)$

Q12

Saturday, November 21, 2020 2:19 PM

The birth rate of a population is $b(t) = 2500e^{0.022t}$ people per year and the death rate is $d(t) = 1420e^{0.017t}$ people per year, find the area between these curves for $0 \le t \le 10$. (Round your answer to the nearest integer.)

12485 people

What does this area represent?

- ○ This area represent the number of children through high school over a 10-year period.
- ○ This area represents the number of deaths over a 10-year period.
- ○ This area represents the number of births over a 10-year period.
- ○ This area represents the decrease in population over a 10-year period.
- ◉ This area represents the increase in population over a 10-year period.

$u = 0.022t$ $\qquad$ $v = 0.017t$

$\frac{du}{dt} = 0.022$ $\qquad$ $\frac{dv}{dt} = 0.017$

$du = 0.022\,dt$ $\qquad$ $dv = 0.017\,dt$

$$\int_0^{10}\left(2500e^{0.022t} - 1420e^{0.017t}\right)dt = \int_0^{10} 2500e^{0.022t}\,dt - \int_0^{10} 1420e^{0.017t}\,dt$$

$$= \frac{2500}{0.022}\int_0^{10} e^{u}\,dt - \frac{1420}{0.017}\int_0^{10} e^{v}\,dt$$

$$= \frac{2500}{0.022}e^{0.022t}\Big|_0^{10} - \frac{1420}{0.017}e^{0.017t}\Big|_0^{10}$$

$$= \left[F(10) - F(0)\right] - \left[G(10) - G(0)\right]$$

$$= \left[\frac{2500}{0.022}e^{0.022(10)} - \frac{2500}{0.022}e^{0.022(0)}\right] - \left[\frac{1420}{0.017}e^{0.017(10)} - \frac{1420}{0.017}e^{0.017(0)}\right]$$

$$= 27963.26484 - 15478.40523$$

$$\approx \boxed{12485}$$