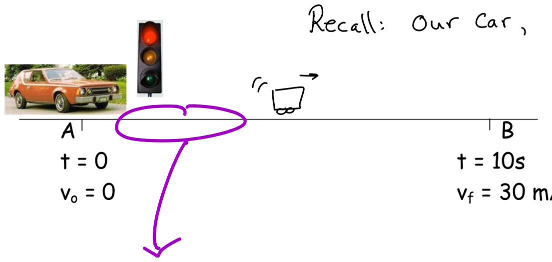
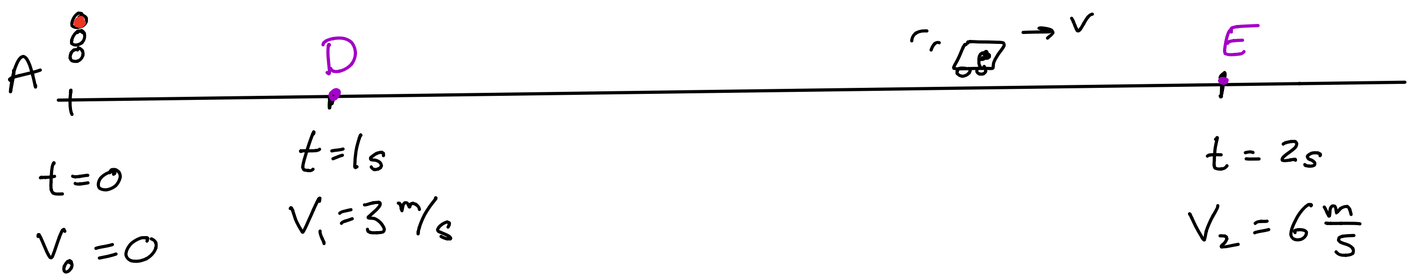
Recall: Our car, $a = 3\frac{m}{s^2}$

$a = 3\frac{m}{s^2} = 3\frac{\frac{m}{s}}{s}$





For the 1-second interval $\overline{DE}$, since $a =$ constant:

$$\bar{V} = \frac{V_0 + V_f}{2} = \frac{V_1 + V_2}{2} = \frac{3\frac{m}{s} + 6\frac{m}{s}}{2} = 4.5\frac{m}{s}$$

Does the car ever have an instantaneous velocity of 4.5 m/s?

When?? Guess: Maybe at $t = 1.5s$?

Check:

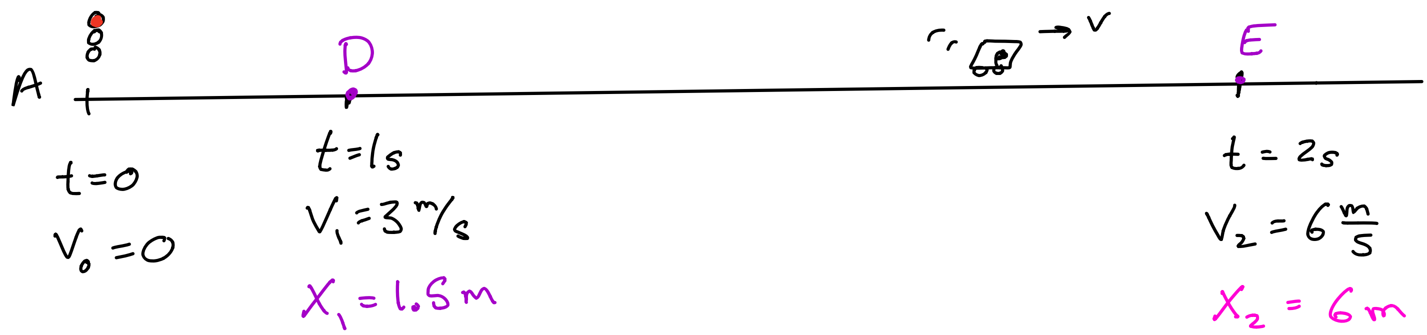
① $V_f = V_0 + at$, $V_f = 0 + 3\frac{m}{s^2}(1.5s)$

$\therefore V_f = 4.5\,m/s$ at $t = 1.5s$ ☺

For some time interval ($\overline{DE}$ example), the instantaneous velocity of the car at the middle of that time interval is equal to the average velocity of the car during that interval.

So that means that the car actually does have a value of the instantaneous velocity that's the same as the average velocity, and this occurs at at the midpoint in time, but not the midpoint in space.

$V_{\text{instantaneous}} = \bar{V}$ at the middle of the time interval, but not in the midpoint of the ROAD. * why?



* Eq ② $X_f = X_0 + V_0 t + \frac{1}{2} a t^2$

From A → D:

$$X_{f_D} = 0 + 0 + \frac{1}{2}\left(3\,\frac{m}{s^2}\right)\left(1^2 s^2\right)$$

$$X_{f_D} = 1.5\,m$$

Eq ②:

From A → E

$$X_{f_E} = 0 + 0 + \frac{1}{2}\left(3\,\frac{m}{s^2}\right)\left(2^2 s^2\right)$$

$$X_{f_E} = 6\,m$$

* where is the car at this 1.5s point in which $V_{inst} = \overline{V}_{DE}$??

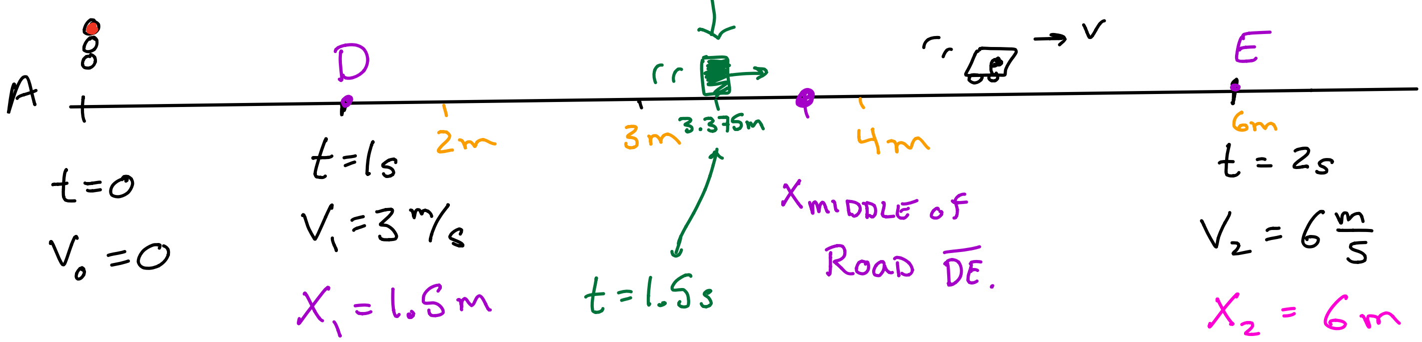
$$X_{car} = 0 + 0 + \frac{1}{2}\left(3\frac{m}{s^2}\right)\left(1.5^2 s^2\right) = \underline{3.375\,m} \ *$$

$A \rightarrow t = 1.5s$

* $X_{car} = 3.375m$

Car is here when $V_{inst} = \overline{V}_{DE}$

(* This is NOT in the middle of X_{DE}.)



$$* \quad X_{middle_{DE}} = \frac{X_1 + X_2}{2} = \frac{1.5m + 6m}{2} = 3.75m$$

For interval $\overline{DE}$, the midpoint in time does not occur at the midpoint in position.

This is occurs because the position as

a function of time is nonlinear in t :

From EQ #2 : $X_f = X_0 + V_0 t + \frac{1}{2} a t^2$

Proof of $\bar{V} = \frac{V_0 + V_f}{2}$ when a = CONSTANT.

$\bar{V} = \frac{\Delta X}{t}$, subs for ΔX using EQ #2.

(2) $\Delta X = V_0 t + \frac{1}{2} a t^2$

$\bar{V} = \frac{\left(V_0 t + \frac{1}{2} a t^2\right)}{t} \Rightarrow \bar{V} = \frac{V_0 t}{t} + \frac{\frac{1}{2} a t^2}{t}$

$\bar{V} = V_0 + \frac{1}{2} a t$, subs for "at" using EQ #1

$V_f = V_0 + a t$, so $a t = V_f - V_0$

$\bar{V} = V_0 + \frac{1}{2}\left(V_f - V_0\right)$

$$\bar{V} = V_0 + \frac{1}{2}V_f - \frac{1}{2}V_0$$

$$\bar{V} = V_0 - \frac{1}{2}V_0 + \frac{1}{2}V_f$$

$$\bar{V} = \frac{1}{2}V_0 + \frac{1}{2}V_f \quad , \quad \therefore \quad \bar{V} = \frac{V_0 + V_f}{2}$$

* when $a =$ CONSTANT !!