Q1

Friday, September 25, 2020 4:30 PM

Differentiate the function.

$f(x) = 3x \ln(7x) - 3x$

$f'(x) = \boxed{3\ln(7x)}$ ✓

The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Logarithmic Differentiation

$$\frac{d}{dx}(\log_b x) = \frac{1}{x\ln b}\frac{d}{dx}(x), \quad \frac{d}{dx}(\ln x) = \frac{1}{x}\frac{d}{dx}(x)$$

$$f(x) = 3x\ln(7x) - 3x$$

$$f'(x) = \frac{d}{dx}\left[3x\ln(7x) - 3x\right]$$

$$= 3\frac{d}{dx}\left[x\ln(7x)\right] - \frac{d}{dx}(3x)$$

$$= 3\left[x\frac{d}{dx}(\ln(7x)) + \ln(7x)\frac{d}{dx}(x)\right] - 3$$

$$= 3\left[x\frac{1}{7x}\frac{d}{dx}(7x) + \ln(7x)(1)\right] - 3$$

$$= 3\left[\frac{1}{7}(7) + \ln(7x)\right] - 3$$

$$= 3\left[1 + \ln(7x)\right] - 3$$

$$= 3 + 3\ln(7x) - 3$$

$$\boxed{f'(x) = 3\ln(7x)}$$

Q2

Friday, September 25, 2020 5:04 PM

Differentiate the function.

$f(x) = \sin(5\ln(x))$

$f'(x) = \dfrac{5\cos(5\ln(x))}{x}$

Logorithmic Differentiation

$\frac{d}{dx}(\log_b x) = \frac{1}{x\ln b}\frac{d}{dx}(x)$, $\frac{d}{dx}(\ln x) = \frac{1}{x}\frac{d}{dx}(x)$

$f(x) = \sin(5\ln(x))$

$f'(x) = \frac{d}{dx}\left[\sin(5\ln(x))\right]$

Chain Rule

$u = 5\ln(x)$

$f(u) = \sin(u) = y$

$\frac{dy}{du}$ $\frac{du}{dx}$

$= \frac{dy}{du}\left[\sin(u)\right]\frac{du}{dx}(u)$

$= \cos(u)\frac{d}{dx}\left[5\ln(x)\right]$

$= \cos(5\ln(x))\, 5\frac{d}{dx}\left[\ln(x)\right]$

$= \cos(5\ln(x))\, 5\left(\frac{1}{x}\right)\frac{d}{dx}(x)$

$= \cos(5\ln(x))\,\frac{5}{x}(1)$

$$\boxed{f'(x) = \frac{5\cos(5\ln(x))}{x}}$$

Q3

Friday, September 25, 2020 5:18 PM

Differentiate the function.

$f(x) = \ln(49 \sin^2(x))$

$f'(x) = 2\cot(x)$

Logorithmic Differentiation

$$\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}\frac{d}{dx}(x), \quad \frac{d}{dx}(\ln x) = \frac{1}{x}\frac{d}{dx}(x)$$

$$f(x) = \ln(49\sin^2(x))$$

$$f'(x) = \frac{d}{dx}\left[\ln(49\sin^2(x))\right]$$

$$= \left[\frac{1}{49\sin^2(x)}\right]\frac{d}{dx}\left[49\sin^2(x)\right]$$

$$= \left[\frac{1}{49\sin^2(x)}\right]49\frac{d}{dx}\left[(\sin(x))^2\right]$$

Chain Rule

$u = \sin(x)$

$f(x) = u^2 = y$

$\frac{dy}{du} \quad \frac{du}{dx}$

$$= \left[\frac{1}{49\sin^2(x)}\right]49\frac{dy}{du}(u^2)\frac{du}{dx}(u)$$

$$= \left[\frac{1}{49\sin^2(x)}\right]49(2u)\frac{d}{dx}\left[\sin(x)\right]$$

$$= \left[\frac{1}{49\sin^2(x)}\right]49\left[(2)\sin(x)\right]\cos(x)$$

$$= \left[\frac{1}{49\sin^2(x)}\right]49(2)\sin(x)\cos(x)$$

$$= \frac{\cancel{49}(2)\cancel{\sin(x)}\cos(x)}{\cancel{49}\sin^{\cancel{2}}(x)}$$

$$= \frac{2\cos(x)}{\sin(x)}$$

$$\boxed{f'(x) = 2\cot(x)}$$

Q4

Friday, September 25, 2020 5:38 PM

Differentiate the function.

$$y = \frac{6}{\ln(x)}$$

$$y' = \boxed{\frac{-6\ln(x)^{-2}}{x}}$$

Logarithmic Differentiation

$$\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}\frac{d}{dx}(x) \quad , \quad \frac{d}{dx}(\ln x) = \frac{1}{x}\frac{d}{dx}(x)$$

$$y = \frac{6}{\ln(x)}$$

$$= 6[\ln(x)]^{-1}$$

$$= 6\ln(x)^{-1}$$

$$y' = \frac{d}{dx}\left[6\ln(x)^{-1}\right]$$

$$= 6\frac{d}{dx}\left[\ln(x)^{-1}\right]$$

Chain Rule

$u = \ln(x)$

$f(u) = u^{-1} = y$

$\frac{dy}{du} \quad \frac{du}{dx}$

$$= 6\frac{dy}{du}(u^{-1})\frac{du}{dx}(u)$$

$$= 6(-u^{-2})\frac{d}{dx}[\ln(x)]$$

$$= 6(-\ln(x)^{-2})\left(\frac{1}{x}\right)\frac{d}{dx}(x)$$

$$= -6\ln(x)^{-2}\left(\frac{1}{x}\right)$$

$$\boxed{y' = \frac{-6\ln(x)^{-2}}{x}}$$

Q5

Friday, September 25, 2020 5:47 PM

Differentiate the function.

$$g(x) = \ln(xe^{-4x})$$

$$g'(x) = \boxed{\frac{-4x+1}{x}}$$

The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Logarithmic Differentiation

$$\frac{d}{dx}(\log_b x) = \frac{1}{x\ln b}\frac{d}{dx}(x), \quad \frac{d}{dx}(\ln x) = \frac{1}{x}\frac{d}{dx}(x)$$

$$g(x) = \ln(xe^{-4x})$$

$$g'(x) = \ln(xe^{-4x})$$

$$= \left(\frac{1}{xe^{-4x}}\right)\frac{d}{dx}\left(xe^{-4x}\right)$$

$$= \left(\frac{1}{xe^{-4x}}\right)\left[x\frac{d}{dx}(e^{-4x}) + e^{-4x}\frac{d}{dx}(x)\right]$$

$$= \left(\frac{1}{xe^{-4x}}\right)\left[x(e^{-4x})\frac{d}{dx}(-4x) + e^{-4x}(1)\right]$$

$$= \left(\frac{1}{xe^{-4x}}\right)\left[xe^{-4x}(-4) + e^{-4x}\right]$$

$$= \frac{-4xe^{-4x} + e^{-4x}}{xe^{-4x}}$$

$$= \frac{\cancel{e^{-4x}}(-4x+1)}{x\cancel{e^{-4x}}}$$

$$\boxed{g'(x) = \frac{-4x+1}{x}}$$

Q6

Saturday, September 26, 2020 1:56 PM

Differentiate the function.

$F(s) = \ln(\ln(4s))$

$F'(s) = \frac{1}{s\ln(4s)}$

Logarithmic Differentiation

$\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b} \frac{d}{dx}(x)$, $\frac{d}{dx}(\ln x) = \frac{1}{x} \frac{d}{dx}(x)$

$$F(s) = \ln(\ln(4s))$$

$$F'(s) = \frac{d}{dx}\left[\ln(\ln(4s))\right]$$

$$= \left[\frac{1}{\ln(4s)}\right] \frac{d}{dx}\left[\ln(4s)\right]$$

$$= \left[\frac{1}{\ln(4s)}\right]\left(\frac{1}{4s}\right) \frac{d}{dx}(4s)$$

$$= \left[\frac{1}{\ln(4s)}\right]\left(\frac{1}{4s}\right) 4$$

$$= \frac{4}{4s[\ln(4s)]}$$

$$\boxed{F'(s) = \frac{1}{s\ln(4s)}}$$

Q7

Saturday, September 26, 2020 2:45 PM

Differentiate the function.

$$y = \tan(\ln(ax + b))$$

$$y' = \boxed{\frac{a\sec^2(\ln(ax+b))}{ax+b}}$$

$$y = \tan(\ln(ax+b))$$

$$y' = \frac{d}{dx}\left[\tan(\ln(ax+b))\right]$$

Chain Rule

$u = \ln(ax+b)$

$f(u) = \tan(u) = y$

$\frac{dy}{du} \cdot \frac{du}{dx}$

$$= \frac{dy}{du}\left[\tan(u)\right]\frac{du}{dx}(u))$$

$$= \sec^2(\ln(ax+b))\frac{du}{dx}\left[\ln(ax+b)\right]$$

$$= \sec^2(\ln(ax+b))\frac{1}{ax+b}\frac{d}{dx}(ax+b$$

$$= \sec^2(\ln(ax+b))\frac{1}{ax+b}(a+0)$$

$$= \sec^2(\ln(ax+b))\frac{a}{ax+b}$$

$$\boxed{y' = \frac{a\sec^2(\ln(ax+b))}{ax+b}}$$

Q8

Saturday, September 26, 2020 3:34 PM

The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Logorithmic Differentiation

$$\frac{d}{dx}(\log_b x) = \frac{1}{x\ln b}\frac{d}{dx}(x), \quad \frac{d}{dx}(\ln x) = \frac{1}{x}\frac{d}{dx}(x)$$

$$y = \sqrt{x}\ln(x)$$

$$y' = \frac{d}{dx}\left[\sqrt{x}\ln(x)\right]$$

$$= \sqrt{x}\frac{d}{dx}[\ln(x)] + \ln(x)\frac{d}{dx}\left(x^{1/2}\right)$$

$$= \sqrt{x}\left(\frac{1}{x}\right)\frac{d}{dx}(x) + \ln(x)\left(\frac{1}{2}x^{-1/2}\right)$$

$$= \sqrt{x}\left(\frac{1}{x}\right) + \frac{1}{2}x^{-1/2}\ln(x)$$

$$= \sqrt{x}\frac{1}{x} + \frac{1}{2}\cdot\frac{1}{\sqrt{x}}\ln(x)$$

$$\boxed{y' = \sqrt{x}\frac{1}{x} + \frac{1}{2\sqrt{x}}\ln(x)}$$

$$y'' = \frac{d}{dx}\left[\sqrt{x}\frac{1}{x} + \frac{1}{2\sqrt{x}}\ln(x)\right]$$

$$= \frac{d}{dx}\left[\sqrt{x}\frac{d}{dx}\left(x^{-1}\right) + \frac{1}{x}\frac{d}{dx}\left(x^{1/2}\right)\right] + \frac{d}{dx}\left[\frac{1}{2\sqrt{x}}\frac{d}{dx}(\ln(x)) + \ln(x)\frac{d}{dx}\left(\frac{1}{2}x^{-1/2}\right)\right]$$

$$= \sqrt{x}\left(-x^{-2}\right) + \frac{1}{x}\left(\frac{1}{2}x^{-1/2}\right) + \frac{1}{2\sqrt{x}}\left(\frac{1}{x}\right)\frac{d}{dx}(x) + \ln(x)\left(-\frac{1}{4}x^{-3/2}\right)$$

$$= \sqrt{x}\left(-\frac{1}{x^2}\right) + \frac{1}{x}\left(\frac{1}{2\sqrt{x}}\right) + \frac{1}{2\sqrt{x}}\left(\frac{1}{x}\right) + \ln(x)\left(-\frac{1}{4x^{3/2}}\right)$$

$$= -\frac{\sqrt{x}}{x^2} + \frac{1}{2x\sqrt{x}} + \frac{1}{2x\sqrt{x}} - \frac{\ln(x)}{4x\sqrt{x}}$$

$$\boxed{y'' = -\frac{\sqrt{x}}{x^2} + \frac{2}{2x\sqrt{x}} - \frac{\ln(x)}{4x\sqrt{x}}}$$

Q9

Saturday, September 26, 2020 4:10 PM

Differentiate f and find the domain of f. (Enter the domain in interval notation.)

$$f(x) = \ln(x^2 - 6x)$$

derivative $f'(x) = \boxed{\dfrac{2(x-3)}{x(x-6)}}$

domain $\boxed{(-\infty,0) \cup (6,\infty)}$

$$f(x) = \ln(x^2 - 6x)$$

$$f'(x) = \frac{d}{dx}\left[\ln(x^2 - 6x)\right]$$

$$= \frac{1}{x^2 - 6x} \frac{d}{dx}(x^2 - 6x)$$

$$= \frac{1}{x^2 - 6x}(2x - 6)$$

$$= \frac{2x - 6}{x^2 - 6x}$$

$$\boxed{f'(x) = \frac{2(x-3)}{x(x-6)}}$$

$$\text{Domain of } f(x) = \{x \mid x(x-6) > 0\}$$

$$= \boxed{(-\infty, 0) \cup (6, \infty)}$$

Q10

Saturday, September 26, 2020 4:22 PM

Find an equation of the tangent line to the curve at the given point.

$y = \ln(x^2 - 4x + 1), \quad (4, 0)$

$y = \boxed{4x - 16}$

$$y = \ln(x^2 - 4x + 1) \quad , (4, 0)$$

$$y' = \frac{d}{dx}\left[\ln(x^2 - 4x + 1)\right]$$

$$= \frac{1}{x^2 - 4x + 1} \frac{d}{dx}(x^2 - 4x + 1)$$

$$= \frac{1}{x^2 - 4x + 1}(2x - 4 + 0)$$

$$y'(4) = \frac{2x - 4}{x^2 - 4x + 1}$$

$$= \frac{2(4) - 4}{(4)^2 - 4(4) + 1}$$

$$= \frac{8 - 4}{16 - 16 + 1}$$

$$= \frac{4}{1} = 4 = m$$

$$y - y_1 = m(x - x_1)$$

$$y - (0) = 4[x - (4)]$$

$$\boxed{y = 4x - 16}$$