Q1

Monday, September 14, 2020 12:08 PM

Differentiate.

$f(x) = (5x^2 - 7x)e^x$

$f'(x) = e^x(5x^2 + 3x - 7)$

The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

$$f(x) = (5x^2 - 7x)e^x$$

$$f'(x) = (5x^2 - 7x)\frac{d}{dx}(e^x) + e^x\frac{d}{dx}[(5x^2 - 7x)]$$

$$= (5x^2 - 7x)e^x + e^x[(2)5x^{(2-1)} - (1)7x^{(1-1)}]$$

$$= (5x^2 - 7x)e^x + e^x(10x - 7)$$

$$= e^x(5x^2 - 7x + 10x - 7)$$

$$f'(x) = \boxed{e^x(5x^2 + 3x - 7)}$$

Q2

Monday, September 14, 2020 2:06 PM

Differentiate.

$$y = \frac{4x}{e^x}$$

$$y' = \frac{4 - 4x}{e^x}$$

The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

$$y = \frac{4x}{e^x}$$

$$y' = \frac{e^x \frac{d}{dx}(4x) - 4x\frac{d}{dx}(e^x)}{(e^x)^2}$$

$$= \frac{e^x (1)4x^{(1-1)} - 4x(e^x)}{e^{2x}}$$

$$= \frac{4e^x - 4xe^x}{e^{2x}}$$

$$= \frac{4e^x}{e^{2x}} - \frac{4xe^x}{e^{2x}}$$

$$= \frac{4}{e^x} - \frac{4x}{e^x}$$

$$y' = \boxed{\frac{4 - 4x}{e^x}}$$

Q3

Monday, September 14, 2020 2:18 PM

Differentiate.

$$F(y) = \left(\frac{1}{y^2} - \frac{5}{y^4}\right)(y + 9y^3)$$

The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

$$F(y) = \left(\frac{1}{y^2} - \frac{5}{y^4}\right)(y + 9y^3)$$

$$F(y)' = (y^{-2} - 5y^{-4})\frac{d}{dx}[(y+9y^3)] + (y+9y^3)\frac{d}{dx}[(y^{-2} - 5y^{-4})]$$

$$= (y^{-2} - 5y^{-4})[(1)y^{(1-1)} + (3)9y^{(3-1)}] + (y+9y^3)[(-2)y^{(-2-1)} - (-4)5y^{(-4-1)}]$$

$$= (y^{-2} - 5y^{-4})(1 + 27y^2) + (y+9y^3)(-2y^{-3} + 20y^{-5})$$

$$= (y^{-2} + 27 - 5y^{-4} - 135y^{-2}) + (-2y^{-2} + 20y^{-4} - 18 + 180y^{-2})$$

$$F(y)' = \boxed{9 + 44y^{-2} + 15y^{-4} \text{ or } 9 + \frac{44}{y^2} + \frac{15}{y^4}}$$

Q4

Monday, September 14, 2020 2:37 PM

Differentiate.

$$y = \frac{\sqrt{x}}{4+x}$$

$$y' = \frac{(4+x)\frac{1}{2}x^{-\left(\frac{1}{2}\right)} - \sqrt{x}}{x^2+8x+16}$$

The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

$$y = \frac{\sqrt{x}}{4+x}$$

$$y' = \frac{(4+x)\frac{d}{dx}(\sqrt{x}) - \sqrt{x}\frac{d}{dx}(4+x)}{(4+x)^2}$$

$$= \frac{(4+x)(x^{1/2}) - \sqrt{x}\left[0+(1)x^{(1-1)}\right]}{(4+x)^2}$$

$$= \frac{(4+x)\left[\left(\frac{1}{2}\right)x^{(1/2-1)}\right] - \sqrt{x}(0+1)}{(4+x)^2}$$

$$= \frac{(4+x)\frac{1}{2}x^{-1/2} - \sqrt{x}}{(4+x)^2}$$

$$y' = \boxed{\frac{(4+x)\frac{1}{2}x^{-1/2} - \sqrt{x}}{x^2+8x+16}}$$

Q5

Monday, September 14, 2020 2:56 PM

Differentiate.

$$y = e^p(p + p\sqrt{p})$$

$$y' = \boxed{e^p\left(1 + \frac{3}{2}p^{\left(\frac{1}{2}\right)} + p + p^{\left(\frac{3}{2}\right)}\right)}$$

The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

$$y = e^p(p + p\sqrt{p})$$

$$y' = e^p \frac{d}{dx}[(p + p\sqrt{p})] + (p + p\sqrt{p})\frac{d}{dx}(e^p)$$

$$= e^p\left[(1)p^{(1-1)} + \frac{d}{dx}p(p^{1/2})\right] + (p + p\sqrt{p})e^p$$

$$= e^p\left[1 + \frac{d}{dx}(p^{3/2})\right] + (p + p\sqrt{p})e^p$$

$$e^p = [1 + (\tfrac{3}{2})p^{(3/2 - 1)}] + (p + p\sqrt{p})e^p$$

$$= e^p(1 + \tfrac{3}{2}p^{1/2}) + (p + p \cdot p^{1/2})e^p$$

$$= e^p(1 + \tfrac{3}{2}p^{1/2}) + (p + p^{3/2})e^p$$

$$y' = \boxed{e^p(1 + \tfrac{3}{2}p^{1/2} + p + p^{3/2})}$$

Q6

Monday, September 14, 2020 3:10 PM

Differentiate.

$$f(t) = \frac{\sqrt[3]{t}}{t-6}$$

$$f'(t) = \frac{(t-6)\frac{1}{3}t^{-\left(\frac{2}{3}\right)} - t^{\left(\frac{1}{3}\right)}}{t^2 - 12t + 36}$$

The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

$$f(t) = \frac{\sqrt[3]{t}}{t-6}$$

$$f'(t) = \frac{(t-6)\frac{d}{dx}\left(\sqrt[3]{t}\right) - \sqrt[3]{t}\,\frac{d}{dx}(t-6)}{(t-6)^2}$$

$$= \frac{(t-6)\frac{d}{dx}\left(t^{1/3}\right) - \sqrt[3]{t}\left[(1)t^{(1-1)} - 0\right]}{(t-6)^2}$$

$$= \frac{(t-6)\left[\left(\frac{1}{3}\right)t^{(1/3-1)}\right] - \sqrt[3]{t}\,(1)}{(t-6)^2}$$

$$= \boxed{\frac{(t-6)\frac{1}{3}t^{-2/3} - t^{1/3}}{t^2-12t+36}}$$

Q7

Monday, September 14, 2020 3:24 PM

Find $f'(x)$ and $f''(x)$.

$f(x) = (x^3 + 4)e^x$

$f'(x) = e^x\left(x^3 + 3x^2 + 4\right)$ ✓

$f''(x) = e^x\left(x^3 + 6x^2 + 6x + 4\right)$ ✓

The Product Rule

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)\frac{d}{dx}\left[g(x)\right] + g(x)\frac{d}{dx}\left[f(x)\right]$$

$$f(x) = (x^3+4)e^x$$

$$f'(x) = (x^3+4)\frac{d}{dx}(e^x) + e^x\frac{d}{dx}\left[(x^3+4)\right]$$

$$= (x^3+4)e^x + e^x\left[(3)x^{(3-1)} + 0)\right]$$

$$= (x^3+4)e^x + e^x(3x^2)$$

$$\boxed{f'(x) = e^x(x^3+3x^2+4)}$$

$$f''(x) = e^x\frac{d}{dx}\left[(x^3+3x^2+4)\right] + (x^3+3x^2+4)\frac{d}{dx}(e^x)$$

$$= e^x\left[(3)x^{(3-1)} + (2)3x^{(2-1)} + 0\right] + (x^3+3x^2+4)(e^x)$$

$$= e^x(3x^2+6x) + (x^3+3x^2+4)e^x$$

$$= e^x(3x^2+6x+x^3+3x^2+4)$$

$$\boxed{f''(x) = e^x(x^3+6x^2+6x+4)}$$

Q8

Monday, September 14, 2020 4:12 PM

Find an equation of the tangent line to the given curve at the specified point.

$$y = \frac{x^2 - 1}{x^2 + x + 1}, \quad (1, 0)$$

$y = \boxed{\frac{2}{3}x - \frac{2}{3}}$ ✓

The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

$$y = \frac{x^2 - 1}{x^2 + x + 1}, \quad (1, 0)$$

Get slope at (1,0)

$$y' = \frac{(x^2 + x + 1)\frac{d}{dx}(x^2 - 1) - (x^2 - 1)\frac{d}{dx}(x^2 + x + 1)}{(x^2 + x + 1)^2}$$

$$= \frac{(x^2 + x + 1)\left[(2)x^{(2-1)} - 0\right] - (x^2 - 1)\left[(2)x^{(2-1)} + (1)x^{(1-1)} + 0\right]}{(x^2 + x + 1)^2}$$

$$= \frac{(x^2 + x + 1)\,2x - (x^2 - 1)(2x + 1)}{(x^2 + x + 1)^2}$$

$$= \frac{2x^3 + 2x^2 + 2x - (2x^2 + x^2 - 2x - 1)}{(x^2 + x + 1)^2}$$

$$= \frac{2x^3 + 2x^2 + 2x - 2x^2 - x^2 + 2x + 1}{(x^2 + x + 1)^2}$$

$$= \frac{2x^3 - x^2 + 4x + 1}{(x^2 + x + 1)^2}$$

$$y' = \frac{2x^3 - x^2 + 4x + 1}{x^4 + 2x^3 + 3x^2 + 2x + 1} = m$$

$$y'(1) = \frac{2(1)^3 - (1)^2 + 4(1) + 1}{(1)^4 + 2(1)^3 + 3(1)^2 + 2(1) + 1}$$

$$y'(1) = \frac{2 - 1 + 4 + 1}{1 + 2 + 3 + 2 + 1} = \frac{6}{9} = \frac{2}{3} = m$$

Get equation of tangent y at (1,0)

$y - y_1 = m(x - x_1)$ point-slope form

$$y - 0 = \frac{2}{3}(x - 1)$$

$$\boxed{y = \frac{2}{3}x - \frac{2}{3}}$$

Q9

Monday, September 14, 2020 4:55 PM

(a) If $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$, find $f'(x)$ and $f''(x)$.

$f'(x) = \boxed{\dfrac{4x}{x^4 + 2x^2 + 1}}$ ✓

$f''(x) = \boxed{\dfrac{-12x^4 - 8x^2 + 4}{x^8 + 4x^6 + 6x^4 + 4x^2 + 1}}$ ✓

(b) Check to see that your answers to part (a) are reasonable by comparing the graphs of f, f', and f''.

$f' = 0$ when f has a horizontal tangent ✓ .

$f'' = 0$ when f' has a horizontal tangent ✓ .

f' is negative when f is decreasing ✓ and positive when f is increasing ✓ .

f'' is negative when f' is decreasing ✓ and positive when f' is increasing ✓ .

The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

$$f(x) = \frac{x^2 - 1}{x^2 + 1}$$

$$f'(x) = \frac{(x^2+1)\frac{d}{dx}(x^2-1) - (x^2-1)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)\left[(2)x^{(2-1)} - 0\right] - (x^2-1)\left[(2)x^{(2-1)} + 0\right]}{(x^2+1)^2}$$

$$= \frac{(x^2+1)2x - (x^2-1)2x}{(x^2+1)^2}$$

$$= \frac{2x(x^2+1-x^2+1)}{(x^2+1)^2}$$

$$f'(x) = \frac{2x(2)}{(x^2+1)^2} = \boxed{\frac{4x}{x^4+2x^2+1}}$$

$$f''(x) = \frac{(x^4+2x^2+1)\frac{d}{dx}(4x) - 4x\frac{d}{dx}(x^4+2x^2+1)}{(x^4+2x^2+1)^2}$$

$$= \frac{(x^4+2x^2+1)(1)4x^{(1-1)} - 4x\left[(4)x^{(4-1)} + (2)2x^{(2-1)} + 0\right]}{(x^4+2x^2+1)^2}$$

$$= \frac{(x^4+2x^2+1)4 - 4x(4x^3+4x)}{(x^4+2x^2+1)^2}$$

$$= \frac{4x^4+8x^2+4 - (16x^4+16x^2)}{(x^4+2x^2+1)^2}$$

$$= \frac{4x^4+8x^2+4-16x^4-16x^2}{(x^4+2x^2+1)^2}$$

$$= \frac{-12x^4-8x^2+4}{(x^4+2x^2+1)^2}$$

$$f''(x) = \boxed{\frac{-12x^4-8x^2+4}{x^8+4x^6+6x^4+4x^2+1}}$$

Q10

Monday, September 14, 2020 5:27 PM

If $f(x) = x^2/(2 + x)$, find $f''(4)$.

$f''(4) =$ 1/27 ✔

The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

$$f(x) = \frac{x^2}{2+x}, \quad f''(4)$$

$$f'(x) = \frac{(2+x)\frac{d}{dx}(x^2) - x^2\frac{d}{dx}(2+x)}{(2+x)^2}$$

$$= \frac{(2+x)[(2)x^{(2-1)}] - x^2[0 + (1)x^{(1-1)}]}{(2+x)^2}$$

$$= \frac{(2+x)2x - x^2(1)}{(2+x)^2}$$

$$= \frac{4x + 2x^2 - x^2}{x^2 + 4x + 4}$$

$$f'(x) = \frac{x^2 + 4x}{x^2 + 4x + 4}$$

$$f'(x) = \frac{x^2 + 4x}{x^2 + 4x + 4}$$

$$f''(x) = \frac{(x^2+4x+4)\frac{d}{dx}(x^2+4x) - (x^2+4x)\frac{d}{dx}(x^2+4x+4)}{(x^2+4x+4)^2}$$

$$= \frac{(x^2+4x+4)[(2)x^{(2-1)} + (1)4x^{(1-1)}] - (x^2+4x)[(2)x^{(2-1)} + (1)4x^{(1-1)} + 0]}{(x^2+4x+4)^2}$$

$$= \frac{(x^2+4x+4)(2x+4) - (x^2+4x)(2x+4)}{(x^2+4x+4)^2}$$

$$= \frac{(x^2+4x+4)(2x+4) + (-x^2-4x)(2x+4)}{(x^2+4x+4)^2}$$

$$= \frac{(2x+4)(x^2+4x+4-x^2-4x)}{(x^2+4x+4)^2}$$

$$f''(x) = \frac{(2x+4)(4)}{(x^2+4x+4)^2} = \frac{8x+16}{x^4+8x^3+24x^2+32x+16}$$

$$f''(4) = \frac{8x+16}{x^4+8x^3+24x^2+32x+16}$$

$$= \frac{8(4)+16}{(4)^4+8(4)^3+24(4)^2+32(4)+16}$$

$$\boxed{f''(4) = \frac{1}{27}}$$

Q11

Monday, September 14, 2020 6:19 PM

If f is a differentiable function, find an expression for the derivative of each of the following functions.

(a) $y = x^7 f(x)$

- ○ $y' = x^6 f'(x) - 6x^7 f(x)$
- ◉ $y' = x^7 f'(x) + 7x^6 f(x)$
- ○ $y' = x^6 f'(x) + 6x^7 f(x)$
- ○ $y' = x^7 f(x) - 7x^6 f'(x)$
- ○ $y' = x^7 f(x) + 7x^6 f'(x)$

✓

(b) $y = \dfrac{f(x)}{x^3}$

- ◉ $y' = \dfrac{xf'(x) - 3f(x)}{x^4}$
- ○ $y' = \dfrac{xf'(x) + 2f(x)}{x^3}$
- ○ $y' = \dfrac{xf'(x) - 2f(x)}{x^4}$
- ○ $y' = \dfrac{xf(x) - 3f'(x)}{x^2}$
- ○ $y' = \dfrac{xf(x) + 3f'(x)}{x^3}$

✓

(c) $y = \dfrac{x^9}{f(x)}$

- ○ $y' = \dfrac{f'(x)(9x^8) - x^9 f(x)}{[f(x)]^2}$
- ○ $y' = \dfrac{f'(x)(8x^9) + x^8 f(x)}{[f(x)]^2}$
- ◉ $y' = \dfrac{f(x)(9x^8) - x^9 f'(x)}{[f(x)]^2}$
- ○ $y' = \dfrac{f(x)(8x^9) + x^9 f'(x)}{[f(x)]^2}$
- ○ $y' = \dfrac{f'(x)(8x^8) - x^9 f'(x)}{[f(x)]^2}$

✓

(d) $y = \dfrac{6 + xf(x)}{\sqrt{x}}$

- ○ $y' = \dfrac{xf'(x) + 6x^2 f(x) - 2}{2x^{3/2}}$
- ◉ $y' = \dfrac{xf(x) + 2x^2 f'(x) - 6}{2x^{3/2}}$
- ○ $y' = \dfrac{xf(x) - 6x^2 f'(x) + 6}{2x^{3/2}}$
- ○ $y' = \dfrac{xf'(x) - 2x^2 f'(x) + 6}{2x^{3/2}}$
- ○ $y' = \dfrac{xf(x) + 6x^2 f(x) - 2}{2x^{3/2}}$

✓

The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$