

Q1

Wednesday, August 26, 2020

9:05 PM

Given that

$$\lim_{x \rightarrow 2} f(x) = 4 \quad \lim_{x \rightarrow 2} g(x) = -2 \quad \lim_{x \rightarrow 2} h(x) = 0,$$

find the limits, if they exist. (If an answer does not exist, enter DNE.)

(a) $\lim_{x \rightarrow 2} [f(x) + 4g(x)]$

(b) $\lim_{x \rightarrow 2} [g(x)]^3$

(c) $\lim_{x \rightarrow 2} \sqrt{f(x)}$

(d) $\lim_{x \rightarrow 2} \frac{5f(x)}{g(x)}$

(e) $\lim_{x \rightarrow 2} \frac{g(x)}{h(x)}$

(f) $\lim_{x \rightarrow 2} \frac{g(x)h(x)}{f(x)}$

$$\begin{aligned} (a) \lim_{x \rightarrow 2} [f(x) + 4g(x)] &= \lim_{x \rightarrow 2} f(x) + \lim_{x \rightarrow 2} 4g(x) \\ &= 4 + 4(-2) \\ &= -4 \end{aligned}$$

$$\begin{aligned} (b) \lim_{x \rightarrow 2} [g(x)]^3 &= (-2)^3 \\ &= -8 \end{aligned}$$

$$(c) \lim_{x \rightarrow 2} \sqrt{f(x)}$$

$$= \sqrt{4}$$

$$= 2$$

$$(d) \lim_{x \rightarrow 2} \frac{5f(x)}{g(x)}$$

$$= \frac{\lim_{x \rightarrow 2} 5f(x)}{\lim_{x \rightarrow 2} g(x)}$$

$$= \frac{5(4)}{-2}$$

$$= -10$$

$$(e) \lim_{x \rightarrow 2} \frac{g(x)}{h(x)}$$

$$= \frac{\lim_{x \rightarrow 2} g(x)}{\lim_{x \rightarrow 2} h(x)}$$

$$= \frac{-2}{0}$$

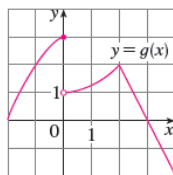
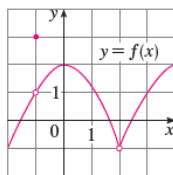
$$= \text{DNE}$$

$$(f) \lim_{x \rightarrow 2} \frac{g(x)h(x)}{f(x)}$$

$$= \frac{\lim_{x \rightarrow 2} g(x) \cdot \lim_{x \rightarrow 2} h(x)}{\lim_{x \rightarrow 2} f(x)}$$

$$= \frac{(-2)(0)}{(4)} = 0$$

The graphs of f and g are given. Use them to evaluate each limit, if it exists. (If an answer does not exist, enter DNE.)



(a) $\lim_{x \rightarrow 2} [f(x) + g(x)]$

(b) $\lim_{x \rightarrow 0} [f(x) - g(x)]$

(c) $\lim_{x \rightarrow -1} [f(x)g(x)]$

(d) $\lim_{x \rightarrow 3} \frac{f(x)}{g(x)}$

(e) $\lim_{x \rightarrow 2} [x^2 f(x)]$

(f) $f(-1) + \lim_{x \rightarrow -1} g(x)$

$$\begin{aligned} (a) \lim_{x \rightarrow 2} [f(x) + g(x)] &= \lim_{x \rightarrow 2} f(x) + \lim_{x \rightarrow 2} g(x) \\ &= -1 + 2 \\ &= 1 \end{aligned}$$

$$\begin{aligned} (b) \lim_{x \rightarrow 0} [f(x) - g(x)] &= \lim_{x \rightarrow 0} f(x) - \lim_{x \rightarrow 0} g(x) \\ &= 2 - \text{DNE} \\ &= \text{DNE} \end{aligned}$$

$$\begin{aligned} (c) \lim_{x \rightarrow -1} [f(x)g(x)] &= \lim_{x \rightarrow -1} f(x) \cdot \lim_{x \rightarrow -1} g(x) \\ &= (1)(2) \\ &= 2 \end{aligned}$$

$$\begin{aligned} (d) \lim_{x \rightarrow 3} \frac{f(x)}{g(x)} &= \frac{\lim_{x \rightarrow 3} f(x)}{\lim_{x \rightarrow 3} g(x)} \\ &= \frac{1}{0} \\ &= \text{DNE} \end{aligned}$$

$$\begin{aligned} (e) \lim_{x \rightarrow 2} [x^2 f(x)] &= \lim_{x \rightarrow 2} x^2 \cdot \lim_{x \rightarrow 2} f(x) \\ &= (2)^2 \cdot -1 \\ &= -4 \end{aligned}$$

$$\begin{aligned} (f) f(-1) + \lim_{x \rightarrow -1} g(x) &= 3 + 2 \\ &= 5 \end{aligned}$$

Q3

Wednesday, August 26, 2020 9:59 PM

Evaluate the limit using the appropriate Limit Law(s). (If an answer does not exist, enter DNE.)

$$\lim_{x \rightarrow 3} (4x^3 - 3x^2 + x - 2)$$

$$\begin{aligned} & \lim_{x \rightarrow 3} (4x^3 - 3x^2 + x - 2) \\ &= \lim_{x \rightarrow 3} 4x^3 - \lim_{x \rightarrow 3} 3x^2 + \lim_{x \rightarrow 3} x - 2 \\ &= 4(3)^3 - 3(3)^2 + 3 - 2 \\ &= 36 - 27 + 3 - 2 \\ &= 10 \end{aligned}$$

Q4

Thursday, August 27, 2020 8:18 AM

Evaluate the limit using the appropriate Limit Law(s). (If an answer does not exist, enter DNE.)

$$\lim_{t \rightarrow 2} \frac{t^4 - 7}{2t^2 - 3t + 6}$$

$$\begin{aligned} & \lim_{t \rightarrow 2} \frac{t^4 - 7}{2t^2 - 3t + 6} \\ &= \frac{\lim_{t \rightarrow 2} t^4 - 7}{\lim_{t \rightarrow 2} 2t^2 - \lim_{t \rightarrow 2} 3t + 6} \\ &= \frac{(2)^4 - 7}{2(2)^2 - 3(2) + 6} \\ &= 1.125 \end{aligned}$$

Q5

Thursday, August 27, 2020 11:01 AM

Evaluate the limit using the appropriate Limit Law(s). (If an answer does not exist, enter DNE.)

$$\lim_{x \rightarrow 2} \sqrt{\frac{5x^2 + 5}{5x - 1}}$$

$$\begin{aligned} & \lim_{x \rightarrow 2} \sqrt{\frac{5x^2 + 5}{5x - 1}} \\ &= \sqrt{\frac{\lim_{x \rightarrow 2} 5x^2 + 5}{\lim_{x \rightarrow 2} 5x - 1}} \\ &= \sqrt{\frac{5(2)^2 + 5}{5(2) - 1}} \\ &= \frac{5}{3} \end{aligned}$$

Q6

Thursday, August 27, 2020 12:47 PM

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \rightarrow 3} \frac{x^2 - 8x + 15}{x - 3}$$

$$\begin{aligned} & \lim_{x \rightarrow 3} \frac{x^2 - 8x + 15}{x - 3} \\ &= \frac{x^2 - 8x + 15}{x - 3} = \frac{\cancel{(x-3)}(x-5)}{\cancel{(x-3)}} \\ &= \lim_{x \rightarrow 3} (x - 5) \\ &= (3) - 5 \\ &= -2 \end{aligned}$$

Q7

Thursday, August 27, 2020 1:02 PM

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{t \rightarrow -6} \frac{t^2 - 36}{2t^2 + 13t + 6}$$

$$\lim_{t \rightarrow -6} \frac{t^2 - 36}{2t^2 + 13t + 6}$$

$$= \frac{t^2 - 36}{2t^2 + 13t + 6} = \frac{(t - 6)(t + 6)}{(2t + 1)(t + 6)}$$

$$= \lim_{t \rightarrow -6} \frac{t - 6}{2t + 1}$$

$$= \frac{(-6) - 6}{2(-6) + 1}$$

$$= \frac{12}{11}$$

Q8

Thursday, August 27, 2020 1:14 PM

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{h \rightarrow 0} \frac{(-6 + h)^2 - 36}{h}$$

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{(-6 + h)^2 - 36}{h} \\ &= \frac{(-6 + h)(-6 + h) - 36}{h} = \frac{(\cancel{36} - 12h + 2h^2) - \cancel{36}}{h} \\ &= \frac{(-12h + 2h^2)}{h} = \frac{\cancel{h}(-12 + 2h)}{\cancel{h}} = -12 + 2h \\ &= \lim_{h \rightarrow 0} -12 + 2h \\ &= -12 + 2(0) \\ &= -12 \end{aligned}$$

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \rightarrow -4} \frac{x+4}{x^3+64}$$

$$x^3 + a^3 = (x+a)(x^2 - ax + a^2)$$

$$\lim_{x \rightarrow -4} \frac{x+4}{x^3+64}$$

$$= \frac{x+4}{x^3+64} = \frac{\cancel{x+4}}{(\cancel{x+4})(x^2-4x+16)} = \frac{1}{x^2-4x+16}$$

$$= \lim_{x \rightarrow -4} \frac{1}{x^2-4x+16}$$

$$= \frac{1}{(-4)^2 - 4(-4) + 16} = \frac{1}{48}$$

Q10

Thursday, August 27, 2020 1:35 PM

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{h \rightarrow 0} \frac{\sqrt{36+h} - 6}{h}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{36+h} - 6}{h}$$

$$= \frac{\sqrt{36+h} - 6}{h} \cdot \frac{(\sqrt{36+h} + 6)}{(\sqrt{36+h} + 6)} = \frac{(\sqrt{36+h})^2 - (6)^2}{h(\sqrt{36+h} + 6)}$$

$$= \frac{36+h - 36}{h(\sqrt{36+h} + 6)} = \frac{\cancel{h}}{\cancel{h}(\sqrt{36+h} + 6)} = \frac{1}{\sqrt{36+h} + 6}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{36+h} + 6}$$

$$= \frac{1}{\sqrt{36+0} + 6}$$

$$= \boxed{\frac{1}{12}}$$

Q11

Thursday, August 27, 2020 1:44 PM

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{t \rightarrow 0} \left(\frac{6}{t} - \frac{6}{t^2 + t} \right)$$

$$\lim_{t \rightarrow 0} \left(\frac{6}{t} - \frac{6}{t^2 + t} \right)$$

$$= \frac{6}{t} - \frac{6}{t^2 + t} = \frac{6}{t} - \frac{6}{t(t+1)} = \frac{6}{t} \cdot \frac{(t+1)}{(t+1)} - \frac{6}{t(t+1)}$$

$$= \frac{6(t+1)}{t(t+1)} - \frac{6}{t(t+1)} = \frac{6(t+1) - 6}{t(t+1)} = \frac{6t + 6 - 6}{t(t+1)}$$

$$= \frac{6\cancel{t}}{\cancel{t}(t+1)} = \frac{6}{t+1}$$

$$= \lim_{t \rightarrow 0} \frac{6}{t+1} = \frac{6}{0+1} = 6$$

Q12

Thursday, August 27, 2020 2:27 PM

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{h \rightarrow 0} \frac{(6+h)^{-1} - 6^{-1}}{h}$$

$$\lim_{h \rightarrow 0} \frac{(6+h)^{-1} - 6^{-1}}{h}$$

$$= \frac{(6+h)^{-1} - 6^{-1}}{h} = \frac{\frac{1}{6+h} - \frac{1}{6}}{h} = \frac{\frac{1}{6+h} \cdot (-6) - \frac{1}{6} \cdot \frac{(6+h)}{6+h}}{h}$$

$$= \frac{-\frac{6}{6(6+h)} - \frac{6+h}{6(6+h)}}{h} = -\frac{6 - 6 + h}{6(6+h)} = -\frac{\cancel{6} - \cancel{6} + h}{6(6+h)} \cdot \frac{1}{\cancel{h}} = -\frac{1}{6(6+h)}$$

$$= \lim_{h \rightarrow 0} -\frac{1}{6(6+h)} = -\frac{1}{6(6+0)} = -\frac{1}{36}$$

Q13

Thursday, August 27, 2020 2:49 PM

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$$

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \\ &= \frac{(x+h)(x^2 + 2hx + h^2) - x^3}{h} = \frac{\cancel{x^3} + 3hx^2 + 3h^2x + h^3 - \cancel{x^3}}{h} \\ &= \frac{3hx^2 + 3h^2x + h^3}{h} = \frac{3hx^2}{h} + \frac{3h^2x}{h} + \frac{h^3}{h} = 3x^2 + 3hx + h^2 \\ &= \lim_{h \rightarrow 0} 3x^2 + 3hx + h^2 = 3x^2 + 3(0)x + (0)^2 = 3x^2 \end{aligned}$$

Q14

Thursday, August 27, 2020 3:14 PM

Find the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \rightarrow 8} (7x + |x - 8|)$$

$$\lim_{x \rightarrow 8} (7x + |x - 8|)$$

$$\begin{aligned} &= \lim_{x \rightarrow 8^+} (7x + x - 8) = \lim_{x \rightarrow 8^+} (8x - 8) \\ &= 8(8) - 8 \\ &= \underline{56} \end{aligned}$$

$$|x - 8| = \begin{cases} x - 8 & \text{if } x - 8 \geq 0 \\ -(x - 8) & \text{if } x - 8 < 0 \end{cases}$$

$$\text{or} \quad = \begin{cases} x - 8 & \text{if } x \geq 8 \\ -(x - 8) & \text{if } x < 8 \end{cases}$$

$$\begin{aligned} &= \lim_{x \rightarrow 8^-} (7x - (x - 8)) = \lim_{x \rightarrow 8^-} (7x - x + 8) = \lim_{x \rightarrow 8^-} (6x + 8) \\ &= 6(8) + 8 \\ &= \underline{56} \end{aligned}$$

Q15

Thursday, August 27, 2020 3:59 PM

Find the limit, if it exists. (If an answer does not exist, enter DNE.)

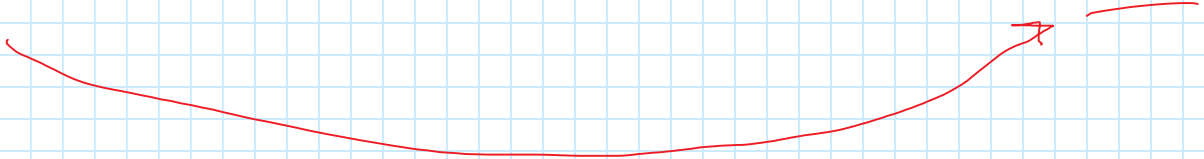
$$\lim_{x \rightarrow -7} \frac{6x + 42}{|x + 7|}$$

$$\lim_{x \rightarrow -7} \frac{6x + 42}{|x + 7|}$$

$$= \lim_{x \rightarrow -7}$$

$$x + 7 = \begin{cases} x + 7 & \text{if } x + 7 \geq 0 \\ -(x + 7) & \text{if } x + 7 < 0 \end{cases}$$

$$= \begin{cases} x + 7 & \text{if } x \geq -7 \\ -(x + 7) & \text{if } x < -7 \end{cases}$$


DNE

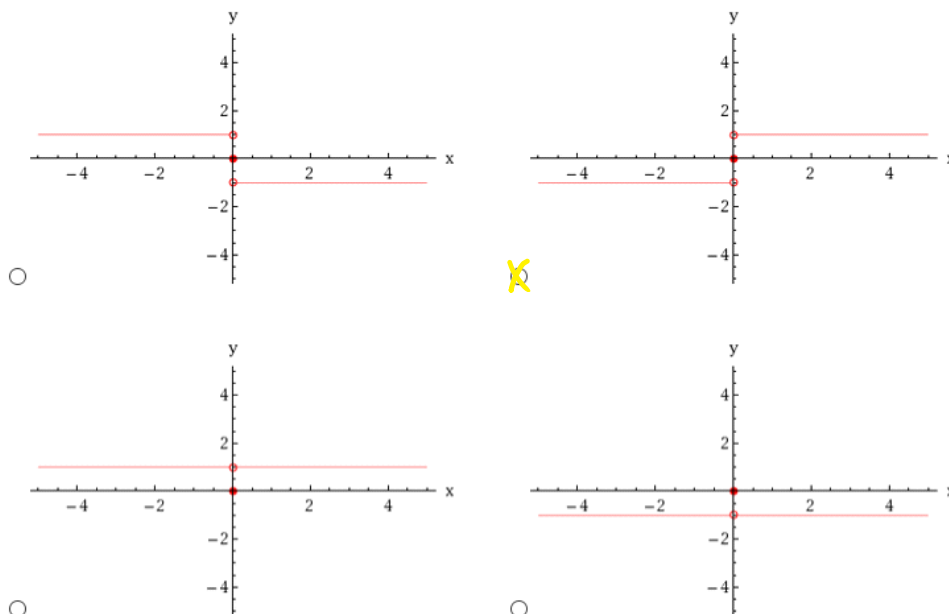
Q16

Thursday, August 27, 2020 4:06 PM

The *signum* (or *sign*) function, denoted by sgn , is defined by

$$\text{sgn } x = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

(a) Sketch the graph of this function.



(b) Find each of the following limits. (If an answer does not exist, enter DNE.)

(i) $\lim_{x \rightarrow 0^+} \text{sgn } x$

(ii) $\lim_{x \rightarrow 0^-} \text{sgn } x$

(iii) $\lim_{x \rightarrow 0} \text{sgn } x$

(iv) $\lim_{x \rightarrow 0} |\text{sgn } x|$

(i) $\lim_{x \rightarrow 0^+} \text{sgn } x = 1$ for $x > 0$

$\lim_{x \rightarrow 0^+} \text{sgn } x = \lim_{x \rightarrow 0^+} 1 = 1$

(ii) $\lim_{x \rightarrow 0^-} \text{sgn } x = -1$ for $x < 0$

$\lim_{x \rightarrow 0^-} \text{sgn } x = -1 = \lim_{x \rightarrow 0^-} -1 = -1$

(iii) $\lim_{x \rightarrow 0^-} \text{sgn } x \neq \lim_{x \rightarrow 0^+} \text{sgn } x$
 $= \text{DNE}$

(iv) $\lim_{x \rightarrow 0} |\text{sgn } x|$

$|\text{sgn } x| = 1$ for $x \neq 0$

$\lim_{x \rightarrow 0} |\text{sgn } x| = \lim_{x \rightarrow 0} 1 = 1$

Q17

Thursday, August 27, 2020 4:45 PM

Let $g(x) = \frac{x^2 + x - 12}{|x - 3|}$.

(a) Find the following limits. (If an answer does not exist, enter DNE.)

(i) $\lim_{x \rightarrow 3^+} g(x)$

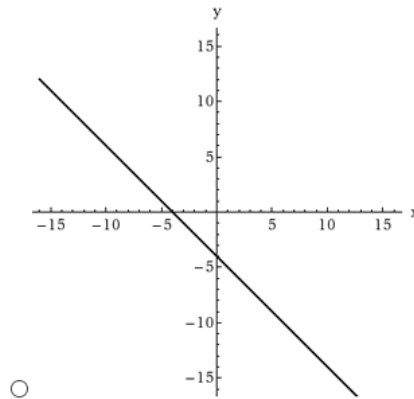
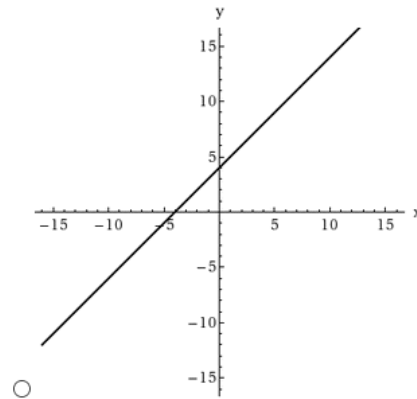
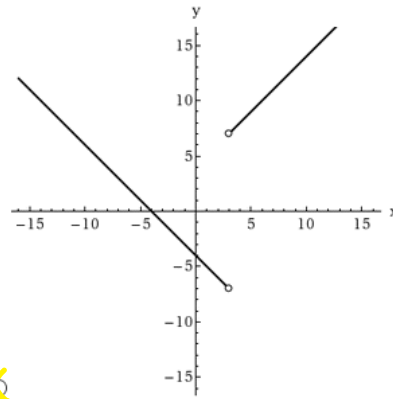
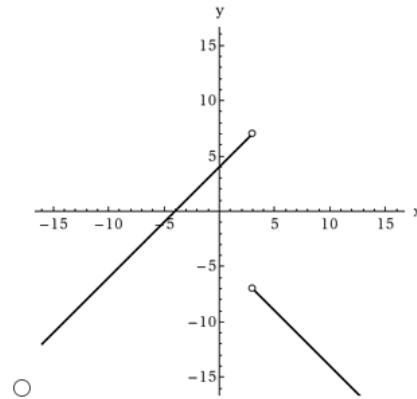
(ii) $\lim_{x \rightarrow 3^-} g(x)$

(b) Does $\lim_{x \rightarrow 3} g(x)$ exist?

☐ Yes

☒ No

(c) Sketch the graph of g .



$$g(x) = \frac{x^2 + x - 12}{|x - 3|}$$

$$|x - 3| = \begin{cases} x - 3 & \text{if } x - 3 > 0 \\ -(x - 3) & \text{if } x - 3 < 0 \end{cases}$$

or

$$\begin{cases} x - 3 & \text{if } x > 3 \\ -(x - 3) & \text{if } x < 3 \end{cases}$$

$$(i) \lim_{x \rightarrow 3^+} \frac{x^2 + x - 12}{x - 3} = \frac{(x - 3)(x + 4)}{(x - 3)}$$

$$= \lim_{x \rightarrow 3^+} x + 4 = 3 + 4 = 7$$

$$(ii) \lim_{x \rightarrow 3^-} \frac{x^2 + x - 12}{-(x - 3)} = \frac{x^2 + x - 12}{-x + 3} = \frac{(x - 3)(x + 4)}{-(x - 3)} = -x - 4$$

$$\lim_{x \rightarrow 3^-} -x - 4 = -(3) - 4 = -7$$

Q18

Thursday, August 27, 2020 5:03 PM

Let

$$f(x) = \begin{cases} x^2 + 2 & \text{if } x < 2 \\ (x-3)^2 & \text{if } x \geq 2 \end{cases}$$

(a) Find the following limits. (If an answer does not exist, enter DNE.)

$$\lim_{x \rightarrow 2^-} f(x) = \boxed{6}$$

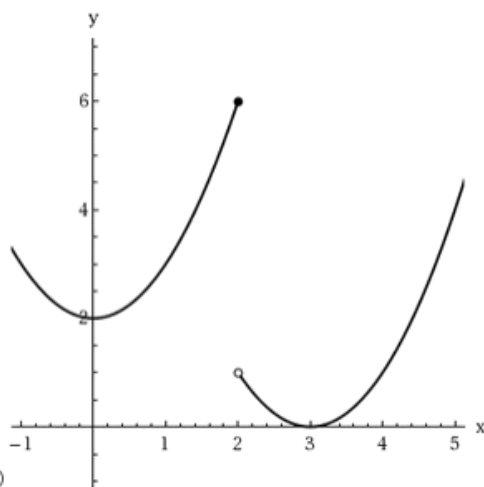
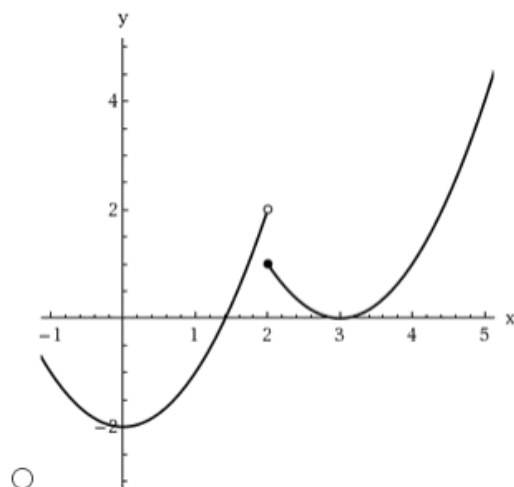
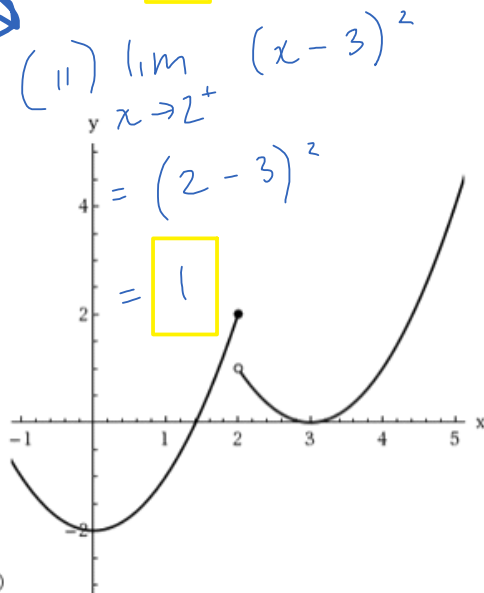
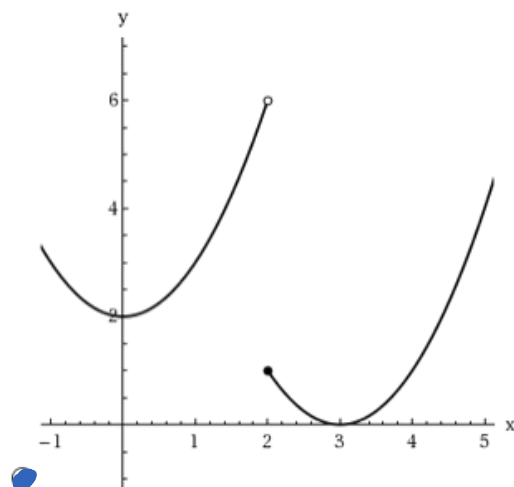
$$\lim_{x \rightarrow 2^+} f(x) = \boxed{1}$$

(b) Does $\lim_{x \rightarrow 2} f(x)$ exist?

☐ Yes

☒ No

(c) Sketch the graph of f .



$$(i) \lim_{x \rightarrow 2^-} x^2 + 2$$

$$= (2)^2 + 2$$

$$= \boxed{6}$$

$$(ii) \lim_{x \rightarrow 2^+} (x-3)^2$$

$$= (2-3)^2$$

$$= \boxed{1}$$

Q19

Thursday, August 27, 2020 5:12 PM

In the theory of relativity, the Lorentz contraction formula

$$L = L_0 \sqrt{1 - v^2/c^2}$$

expresses the length L of an object as a function of its velocity v with respect to an observer, where L_0 is the length of the object at rest and c is the speed of light. Find $\lim_{v \rightarrow c^-} L$.

Why is a left-hand limit necessary?

- ☐ L is not defined for $v < 0$.
- ☐ L is not defined for $v > c$.
- ☐ L is not defined for $v < c$.
- ☐ L is not defined for $v > 0$.

Q20

Thursday, August 27, 2020 5:24 PM

If $\lim_{x \rightarrow 1} \frac{f(x) - 6}{x - 1} = 8$, evaluate $\lim_{x \rightarrow 1} f(x)$.

$$\lim_{x \rightarrow 1} \frac{f(x) - 6}{x - 1} = 8$$

$$= \frac{\lim_{x \rightarrow 1} [f(x) - 6]}{\lim_{x \rightarrow 1} (x - 1)} = 8$$

$$= \lim_{x \rightarrow 1} [f(x) - 6] = 8 \lim_{x \rightarrow 1} (x - 1)$$

$$= \lim_{x \rightarrow 1} f(x) - \lim_{x \rightarrow 1} 6 = 8 \lim_{x \rightarrow 1} (1 - 1)$$

$$= \lim_{x \rightarrow 1} f(x) - (6) = 8(0)$$

$$= \lim_{x \rightarrow 1} f(x) - 6 = 0$$

$$= \lim_{x \rightarrow 1} f(x) = 6$$