Converting Complex numbers From and To EULER FORM

Main Idea

Perhaps the most celebrated identity of mathematics is Euler's World known timeless identity:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Among other things, it's power lies on the idea that a binomial (on the right side) can be converted to a monomial (left side). Indeed, every complex number, $a + bi$ can be written in Euler Form as

$$re^{i\theta}$$

. It will serve us well to think about Euler's identity as a sort of *bridge* between the binomials numbers which make up the entire complex plane and the monomial world expressed in Euler Form. Moreover, we have already crossed this bridge. You will see the ideas are identical to the ideas used in converting from cartesian coordinated to polar coordinates in first section of this chapter.

Converting: EULER TO & FORM Standard Form

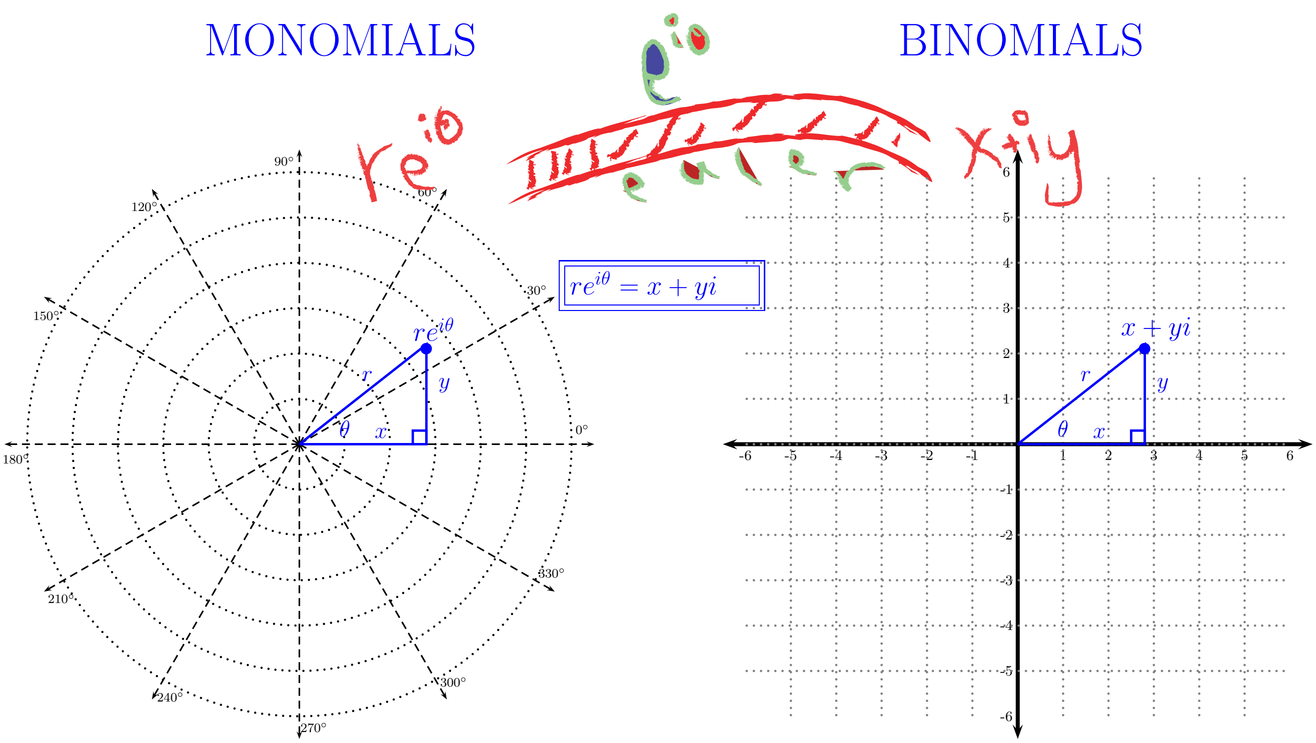
The translating 'dictionary' to go from Euler-to-Standard is as follows:

From the defining features of the polar and cartesian cordinates, we conclude that generally:

$$\begin{aligned}
re^{i\theta} &= r[e^{i\theta}] && \text{(getting ready to use Euler's ID, } e^{i\theta} = \cos(\theta) + i\sin(\theta)\text{)}\\
&= r[\cos(\theta) + i\sin(\theta)] && \text{(used } e^{i\theta} = \cos(\theta) + i\sin(\theta)\text{)}\\
&= r\cos(\theta) + ir\sin(\theta) && \text{(algebra)}\\
&= x + iy && \text{(used polar dictionary)}
\end{aligned}$$

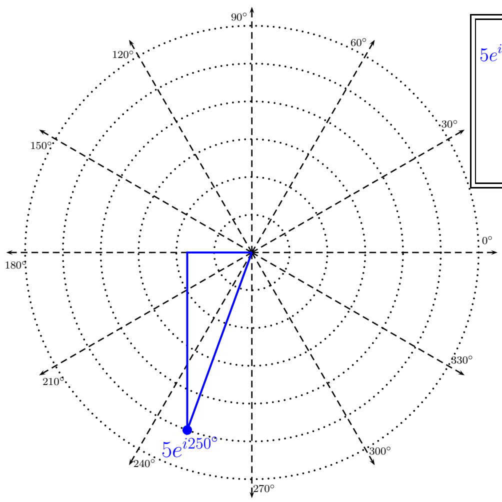
and similarly going backwards.

Therefore, we have the translating 'dictionary' to translate from EULER TO& FROM STANDARD Form

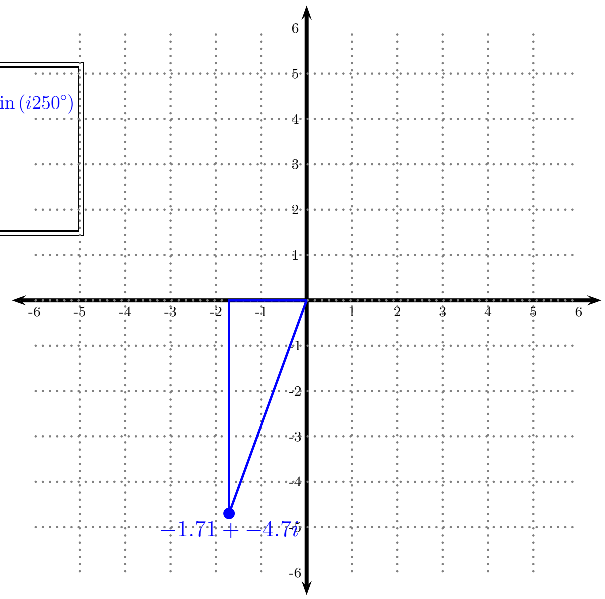


EXAMPLEs: Polar Coordinates $< -- >$ Cartesian Coordinates

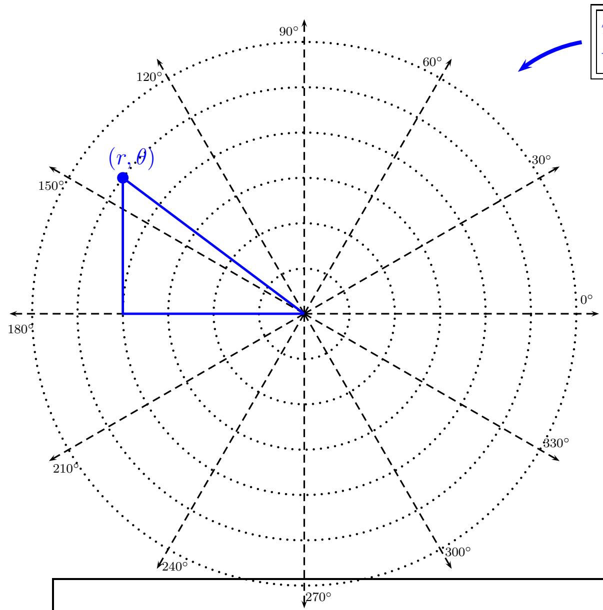
Convert the Euler Form, $5e^{i250^\circ}$ to Standard form of the complex number.



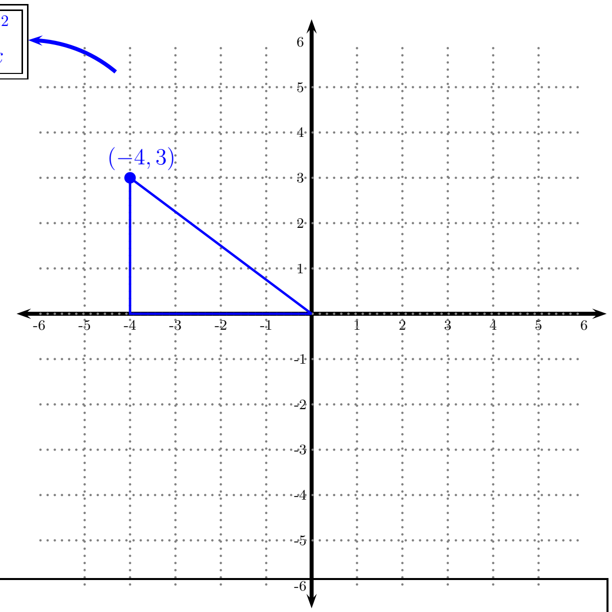
$$5e^{i250^\circ} = 5\cos(i250^\circ) + i5\sin(i250^\circ)$$
(Euler, Baby!!)
$$\approx -1.71 + -4.7i$$



Convert the cartesian coordinates, (-4, 3), to polar coordinates.



$$\boxed{\begin{array}{l} r^2 = x^2 + y^2 \\ \tan\theta = y/x \end{array}}$$



$\tan\theta = 3/-4$ (note: eq has infinite solutions)

$\theta \approx -36.87°$

(note: this is just one [acceptable] value for θ)

$$\begin{aligned} r^2 &= x^2 + y^2 \\ r^2 &= (-4)^2 + (3)^2 \\ r^2 &= 25 \\ r &= \pm\sqrt{25} \approx \pm 5 \end{aligned}$$

note: if we use the angle $-36.87°$, then we must choose the negative value of r, $r \approx -5$, thus the polar cordinates $(-5, -36.87°)$ is *one* way to represent the point $(-4, 3)$

Converting Complex numbers From and To EULER FORM

1. Convert the Euler Form, $5e^{i240^\circ}$ to Standard form of the complex number.
2. Convert the Euler Form, $7e^{i210^\circ}$ to Standard form of the complex number.
3. Convert the Euler Form, $7e^{i330^\circ}$ to Standard form of the complex number.
4. Convert the Euler Form, $5e^{i150^\circ}$ to Standard form of the complex number.
5. Convert the Euler Form, $2e^{i240^\circ}$ to Standard form of the complex number.
6. Convert the Euler Form, $7e^{i135^\circ}$ to Standard form of the complex number.
7. Convert the Euler Form, $3e^{i-30^\circ}$ to Standard form of the complex number.
8. Convert the Euler Form, $2e^{i-90^\circ}$ to Standard form of the complex number.
9. Convert the Euler Form, $4e^{i180^\circ}$ to Standard form of the complex number.
10. Convert the Euler Form, $7e^{i225^\circ}$ to Standard form of the complex number.
11. Convert the Euler Form, $5e^{i-150^\circ}$ to Standard form of the complex number.
12. Convert the Euler Form, $5e^{i-300^\circ}$ to Standard form of the complex number.
13. Convert the Euler Form, $2e^{i400^\circ}$ to Standard form of the complex number.
14. Convert the Euler Form, $3e^{i360^\circ}$ to Standard form of the complex number.
15. Convert the Euler Form, $2e^{i3600^\circ}$ to Standard form of the complex number.
16. Convert the Euler Form, $4e^{i-180^\circ}$ to Standard form of the complex number.
17. Convert the Euler Form, $5e^{i45^\circ}$ to Standard form of the complex number.
18. Convert the Euler Form, $5e^{i225^\circ}$ to Standard form of the complex number.
19. Convert the complex number $4 + 5i$ to EULER form.
20. Convert the complex number $-4 + 1i$ to EULER form.
21. Convert the complex number $-3 + -4i$ to EULER form.
22. Convert the complex number $-1 + 5i$ to EULER form.
23. Convert the complex number $1 + -5i$ to EULER form.