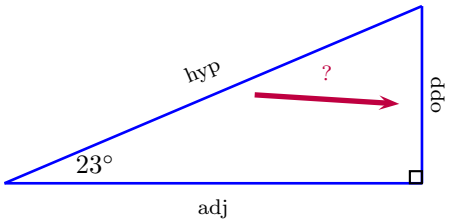
The Idea

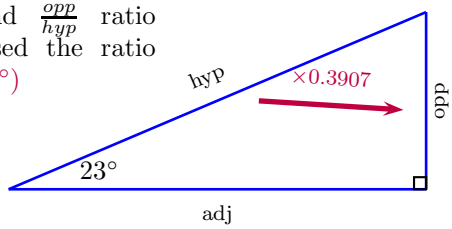
Now we turn our attention to solving for the angles on a right triangle. We also take the opportunity to emphasize what it means for us to completely solve a triangle. We see triangles as 6 pieces of information, 3 sides and 3 corresponding angles. To *solve the triangle* will mean for us to determine all 6 items, the 3 sides and the 3 angles. The last section, using $\sin(\theta)$, $\cos(\theta)$ and $\tan(\theta)$, provided for us all the ratios for all the triangles, and using the ratios and just one side, we can easily solve all sides. That was a a major milestone, and we are now ready to complete the solving of right triangles challenge by learning to solve for angles.

The Inverse Functions

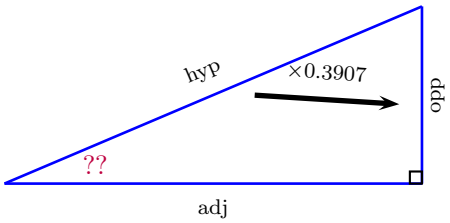
To solve for angles, we introduce the idea of inverse trig functions. Let us take an example from the past. For a 23-67 triangle we used the $\sin(23°)$ to determine the opp/hyp ratio, approximately .3907.



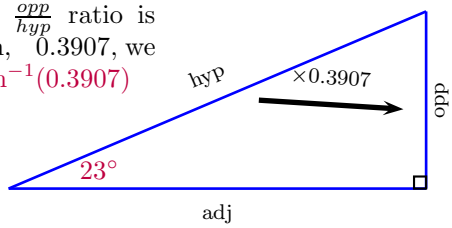
to find $\frac{opp}{hyp}$ ratio we used the ratio $\sin(23°)$



The question we now contemplate is *what if we knew the ratio, but not the angle?*. How then could we determine the angles. One primitive approach would be to use a protractor, assuming we can draw such a triangle to correct scale, a protractor would help us get a working approximation of the angle. But there is more. A more accurate answer is to use the sine inverse function. The sine inverse function take ratios as input values and it gives angles as output values. The sine inverse function is often written as $\sin^{-1}(k)$, where k is the ratio for which we want to find the angle. The sine inverse function[1] is sometimes also known as the arc-sine function and written as $\arcsin(k)$ both names and notations are common and most scientific calculators have these functions built-in. You should become familiar with such functions on your calculator [also with the radians-degree settings on the calculator]. While we are on the calculator topic, while you use the calculator to compute these values, your self-conscious may feel a bit uneasy, it should feel as though you are not really learning how or where these values come from. You should embrace such feelings as these will foster an enduring anticipation for the day when you really learn where and how these values are computed. For now we make do with our calculators and at least begin to understand what these inverse functions do for us.

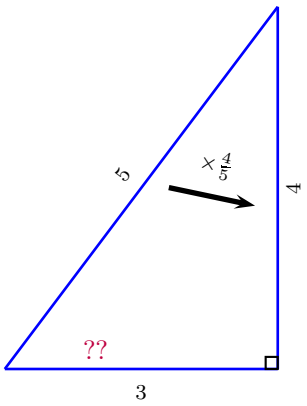


If the $\frac{opp}{hyp}$ ratio is known, 0.3907, we use $\sin^{-1}(0.3907)$

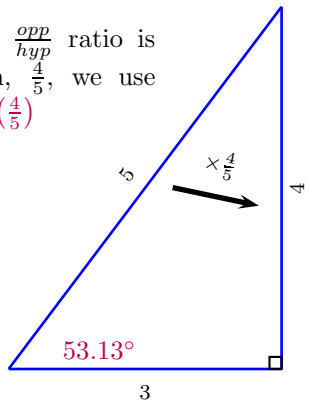


Here is another look at the a similar problem. There is a famous triangle going around, the 3, 4, 5 triangle. It happens to be a right triangle, and the sides are famous with $3^2 + 4^2 = 5^2$. The sides are famous, but here we ask, what are the angles? Have you ever wondered?

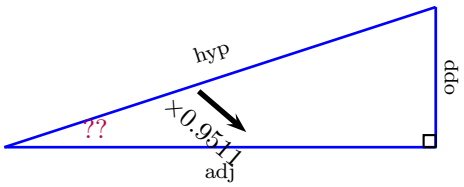
[1] this definition will be extended to a broader sense later



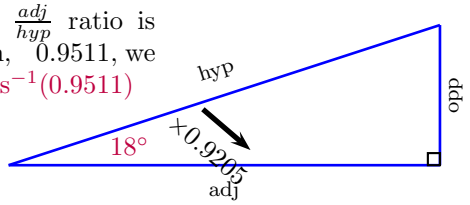
If the $\frac{opp}{hyp}$ ratio is known, $\frac{4}{5}$, we use $\sin^{-1}\left(\frac{4}{5}\right)$



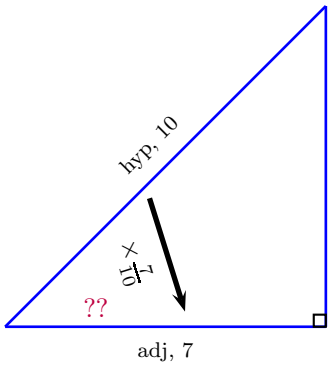
But wait, there is more. Now we can extend these ideas to other ratios. For example if we know the $\frac{adj}{hyp}$ ratio, we can use the arc-cosine function also known as the inverse cosine to compute the angle. For example:



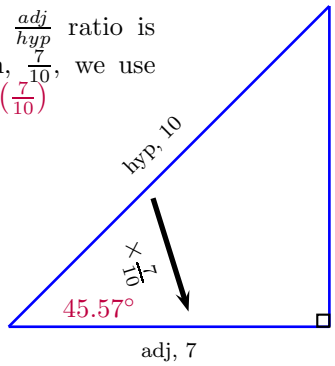
If the $\frac{adj}{hyp}$ ratio is known, 0.9511, we use $\cos^{-1}(0.9511)$



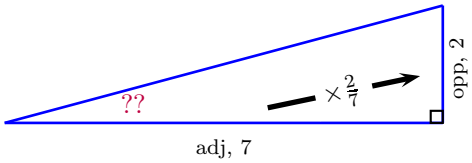
Suppose we knew the sides, adjacent and hypothenuse, then we compute the ratio, then the angle using the cosine inverse function.



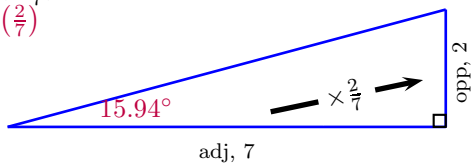
If the $\frac{adj}{hyp}$ ratio is known, $\frac{7}{10}$, we use $\cos^{-1}\left(\frac{7}{10}\right)$



Similarly for arctangent, if we know the ratio, either by knowing the ratio or by knowing the sides, opp & adj, we can find the respective angle.

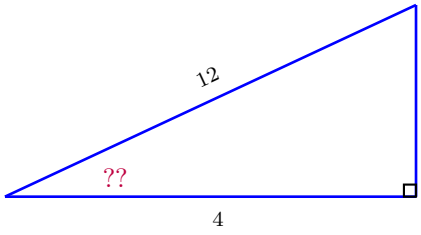


If the $\frac{opp}{adj}$ ratio is known, $\frac{2}{7}$, we use $\tan^{-1}\left(\frac{2}{7}\right)$

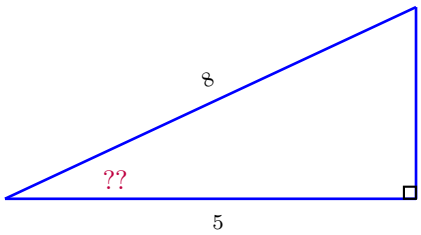


1.

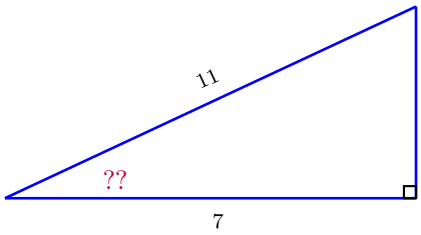
2. Find the angle in question.



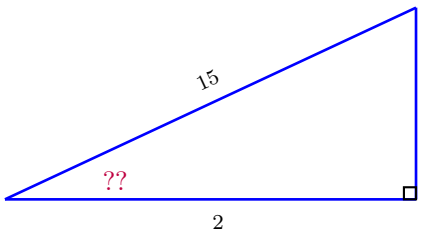
3. Find the angle in question.



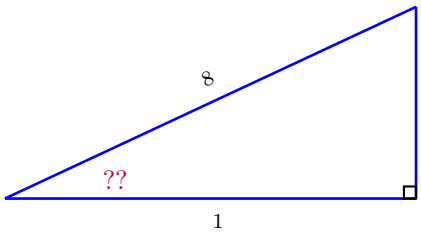
4. Find the angle in question.



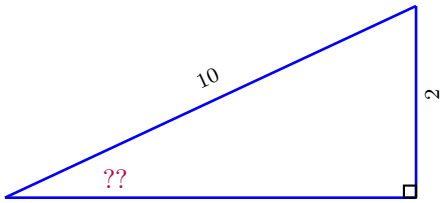
5. Find the angle in question.



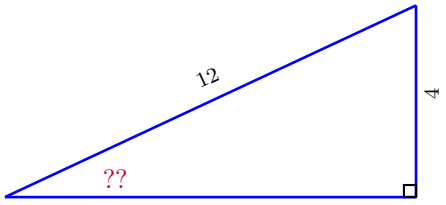
6. Find the angle in question.



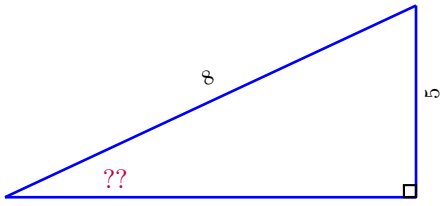
7. Find the angle in question.



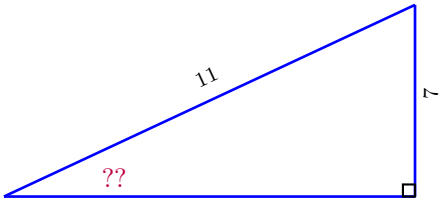
8. Find the angle in question.



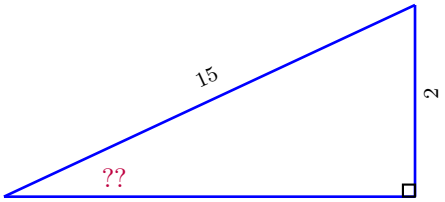
9. Find the angle in question.



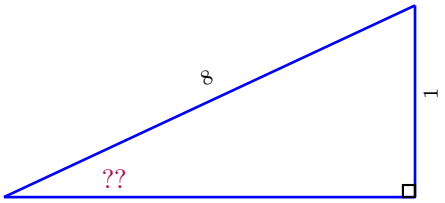
10. Find the angle in question.



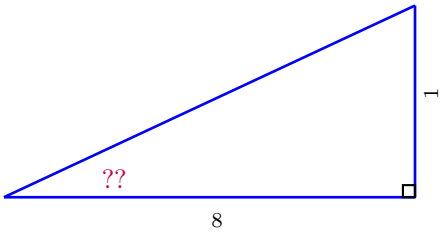
11. Find the angle in question.



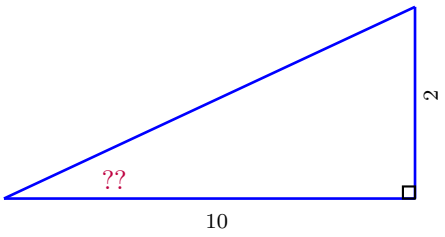
12. Find the angle in question.



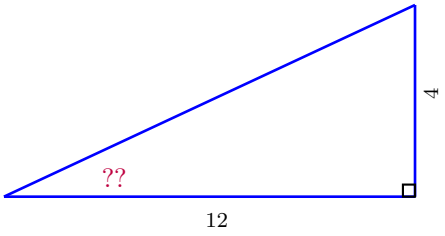
13. Find the angle in question.



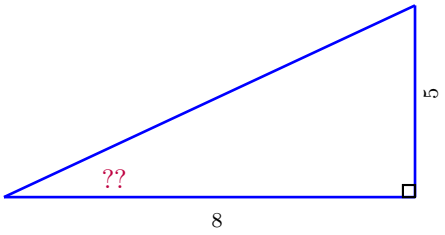
14. Find the angle in question.



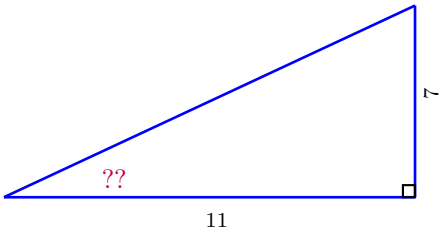
15. Find the angle in question.



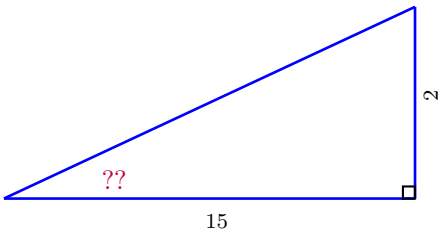
16. Find the angle in question.



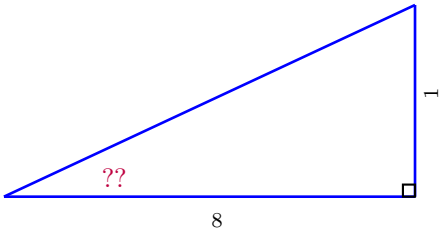
17. Find the angle in question.



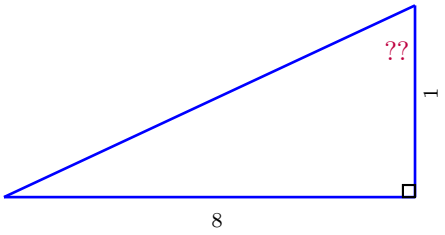
18. Find the angle in question.



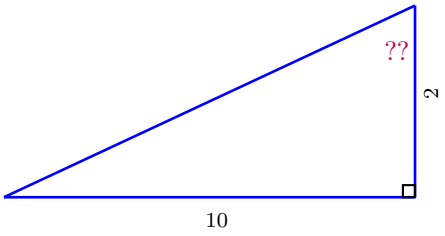
19. Find the angle in question.



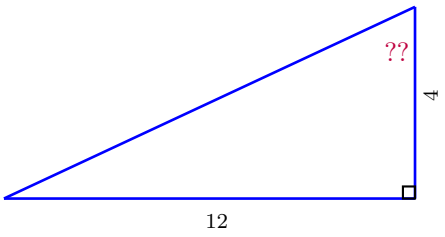
20. Find the angle in question.



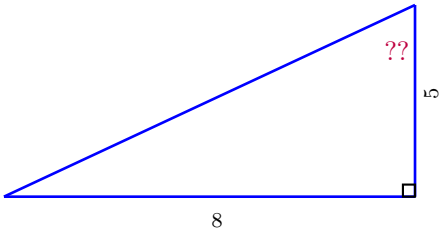
21. Find the angle in question.



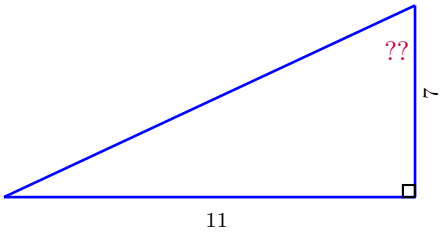
22. Find the angle in question.



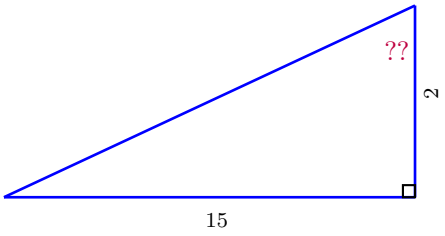
23. Find the angle in question.



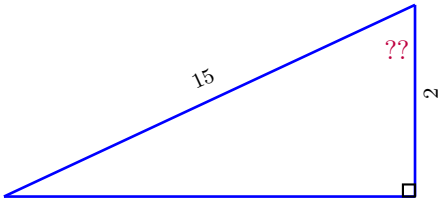
24. Find the angle in question.



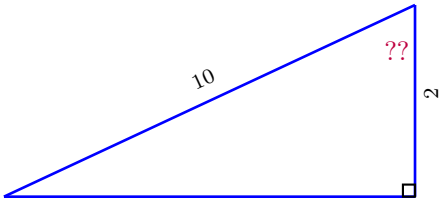
25. Find the angle in question.



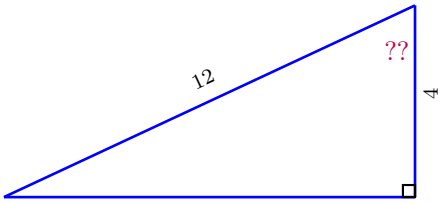
26. Find the angle in question.



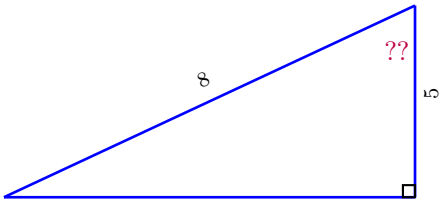
27. Find the angle in question.



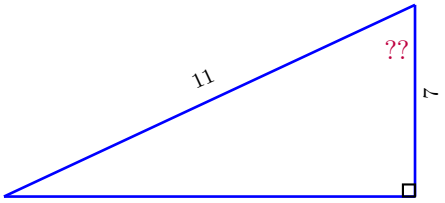
28. Find the angle in question.



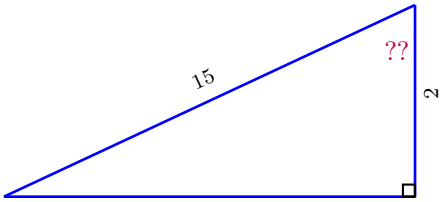
29. Find the angle in question.



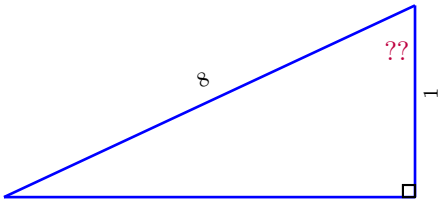
30. Find the angle in question.



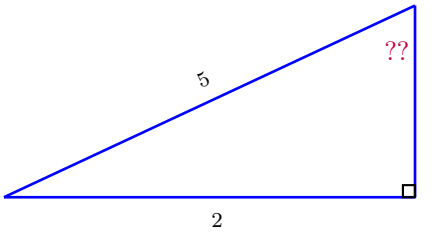
31. Find the angle in question.



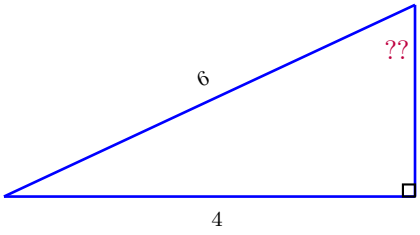
32. Find the angle in question.



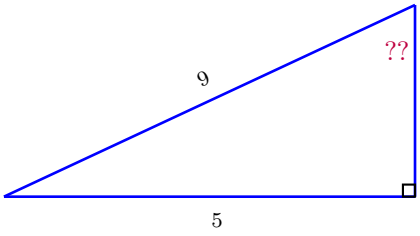
33. Find the angle in question.



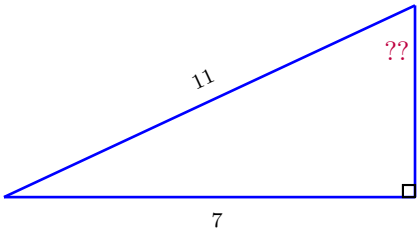
34. Find the angle in question.



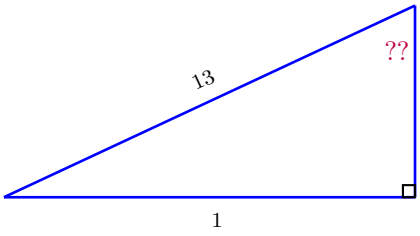
35. Find the angle in question.



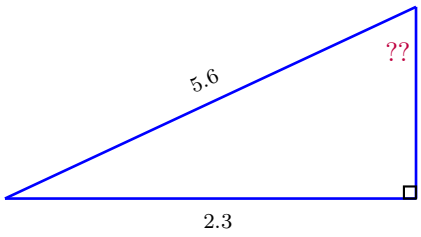
36. Find the angle in question.



37. Find the angle in question.



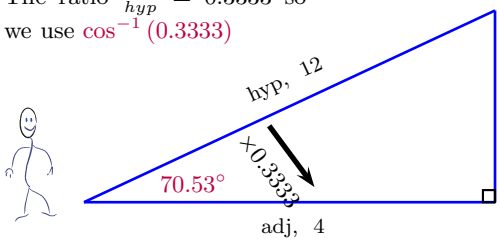
38. Find the angle in question.



1.

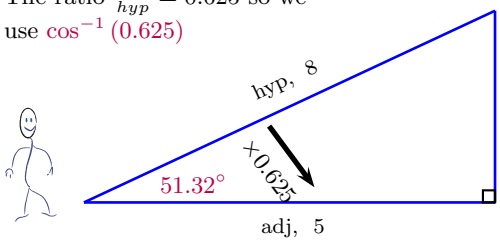
2. To find the angle in question:

The ratio $\frac{adj}{hyp} = 0.3333$ so we use $\cos^{-1}(0.3333)$



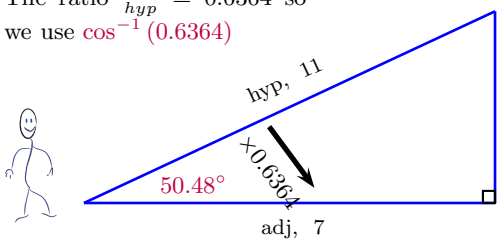
3. To find the angle in question:

The ratio $\frac{adj}{hyp} = 0.625$ so we use $\cos^{-1}(0.625)$



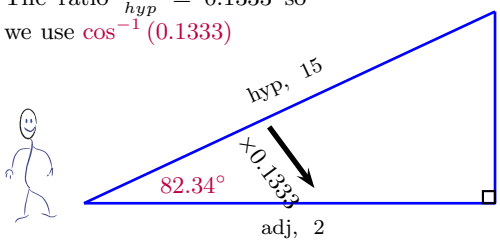
4. To find the angle in question:

The ratio $\frac{adj}{hyp} = 0.6364$ so we use $\cos^{-1}(0.6364)$



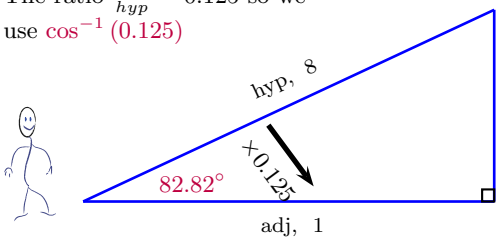
5. To find the angle in question:

The ratio $\frac{adj}{hyp} = 0.1333$ so we use $\cos^{-1}(0.1333)$



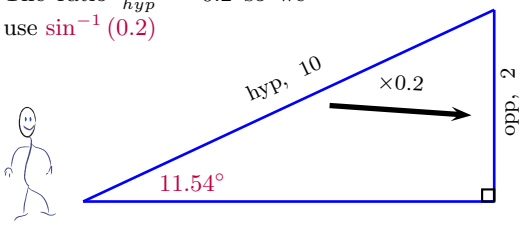
6. To find the angle in question:

The ratio $\frac{adj}{hyp} = 0.125$ so we use $\cos^{-1}(0.125)$



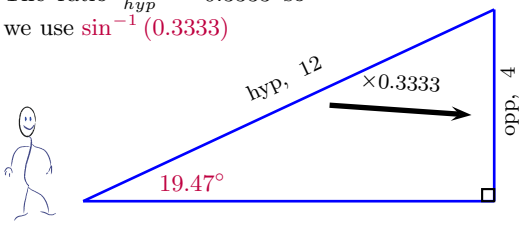
7. To find the angle in question:

The ratio $\frac{opp}{hyp} = 0.2$ so we use $\sin^{-1}(0.2)$



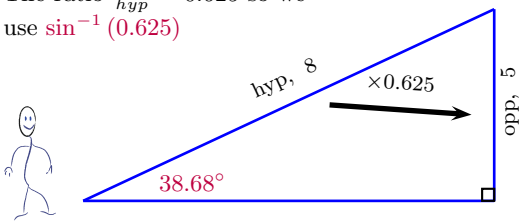
8. To find the angle in question:

The ratio $\frac{opp}{hyp} = 0.3333$ so we use $\sin^{-1}(0.3333)$



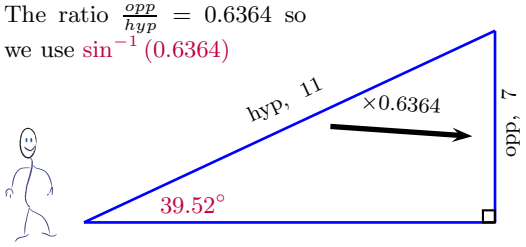
9. To find the angle in question:

The ratio $\frac{opp}{hyp} = 0.625$ so we use $\sin^{-1}(0.625)$

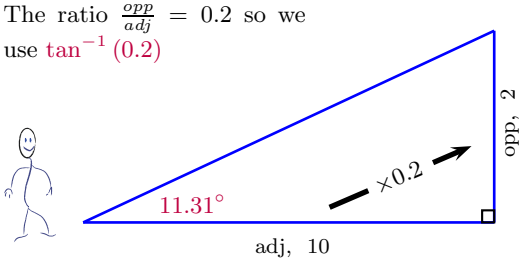


10. To find the angle in question:

The ratio $\frac{opp}{hyp} = 0.6364$ so we use $\sin^{-1}(0.6364)$

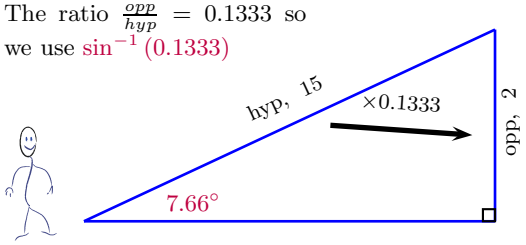


The ratio $\frac{opp}{adj} = 0.2$ so we use $\tan^{-1}(0.2)$



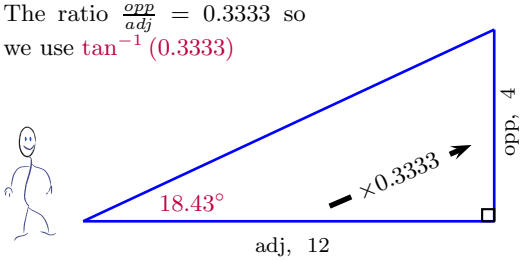
11. To find the angle in question:

The ratio $\frac{opp}{hyp} = 0.1333$ so we use $\sin^{-1}(0.1333)$



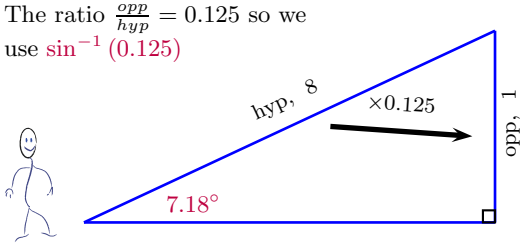
15. To find the angle in question:

The ratio $\frac{opp}{adj} = 0.3333$ so we use $\tan^{-1}(0.3333)$



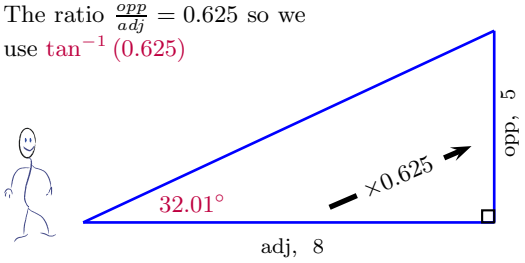
12. To find the angle in question:

The ratio $\frac{opp}{hyp} = 0.125$ so we use $\sin^{-1}(0.125)$



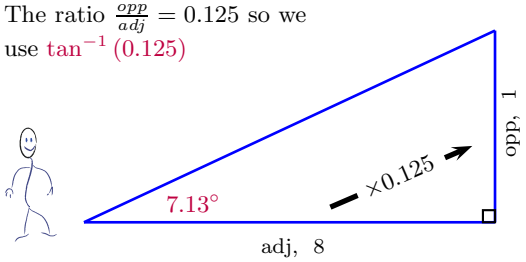
16. To find the angle in question:

The ratio $\frac{opp}{adj} = 0.625$ so we use $\tan^{-1}(0.625)$



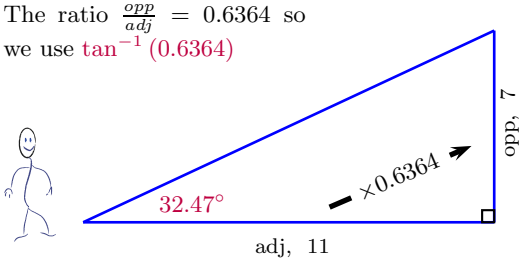
13. To find the angle in question:

The ratio $\frac{opp}{adj} = 0.125$ so we use $\tan^{-1}(0.125)$



17. To find the angle in question:

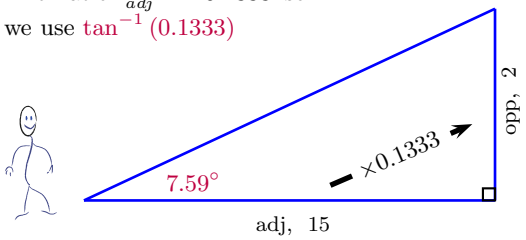
The ratio $\frac{opp}{adj} = 0.6364$ so we use $\tan^{-1}(0.6364)$



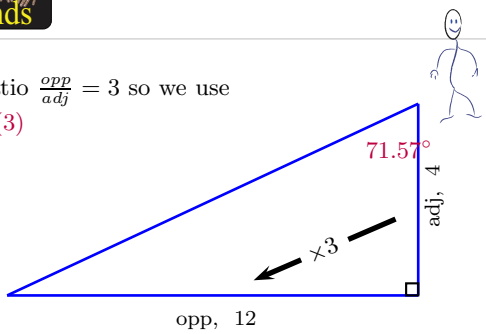
14. To find the angle in question:

18. To find the angle in question:

The ratio $\frac{opp}{adj} = 0.1333$ so we use $\tan^{-1}(0.1333)$

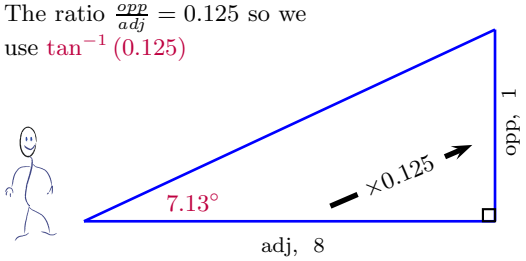


The ratio $\frac{opp}{adj} = 3$ so we use $\tan^{-1}(3)$



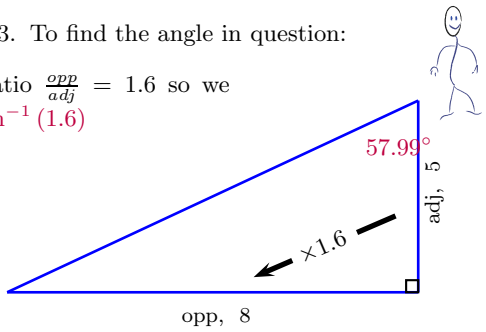
19. To find the angle in question:

The ratio $\frac{opp}{adj} = 0.125$ so we use $\tan^{-1}(0.125)$



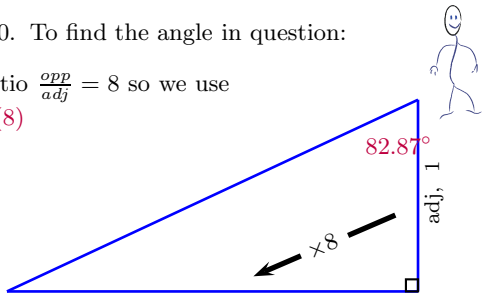
23. To find the angle in question:

The ratio $\frac{opp}{adj} = 1.6$ so we use $\tan^{-1}(1.6)$



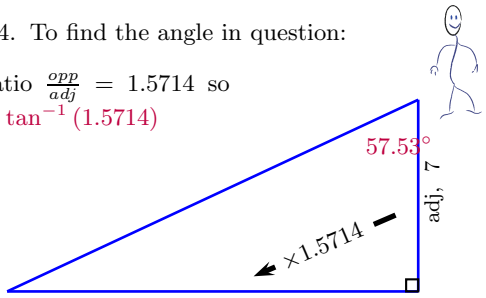
20. To find the angle in question:

The ratio $\frac{opp}{adj} = 8$ so we use $\tan^{-1}(8)$



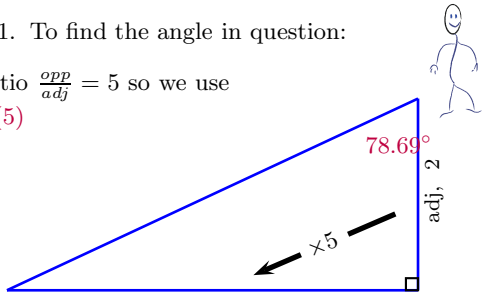
24. To find the angle in question:

The ratio $\frac{opp}{adj} = 1.5714$ so we use $\tan^{-1}(1.5714)$



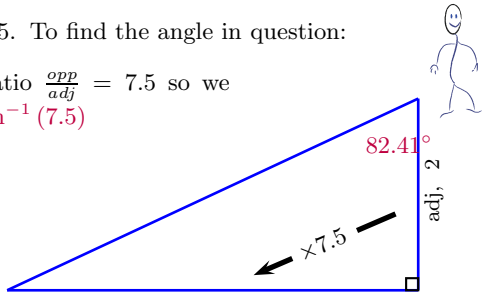
21. To find the angle in question:

The ratio $\frac{opp}{adj} = 5$ so we use $\tan^{-1}(5)$



25. To find the angle in question:

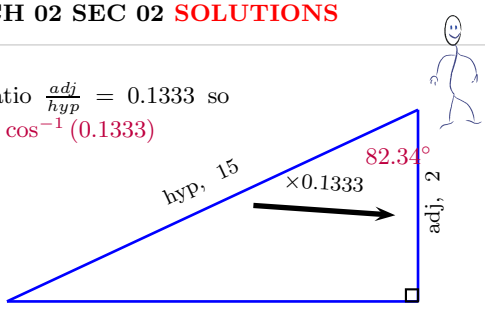
The ratio $\frac{opp}{adj} = 7.5$ so we use $\tan^{-1}(7.5)$



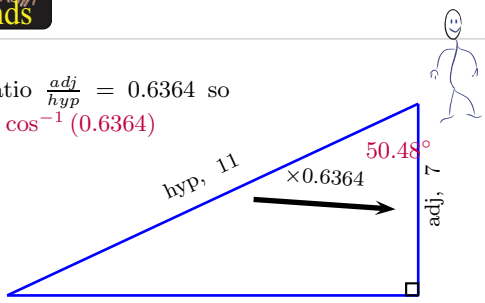
22. To find the angle in question:

26. To find the angle in question:

The ratio $\frac{adj}{hyp} = 0.1333$ so we use $\cos^{-1}(0.1333)$

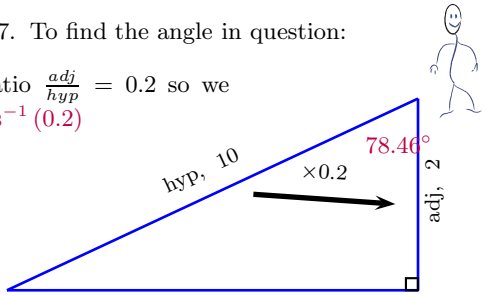


The ratio $\frac{adj}{hyp} = 0.6364$ so we use $\cos^{-1}(0.6364)$



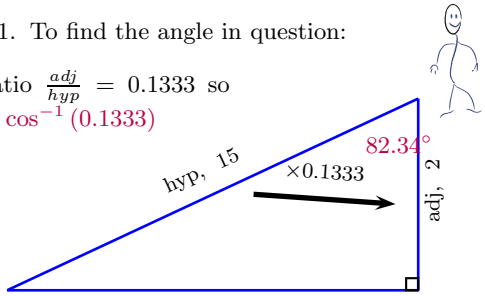
27. To find the angle in question:

The ratio $\frac{adj}{hyp} = 0.2$ so we use $\cos^{-1}(0.2)$



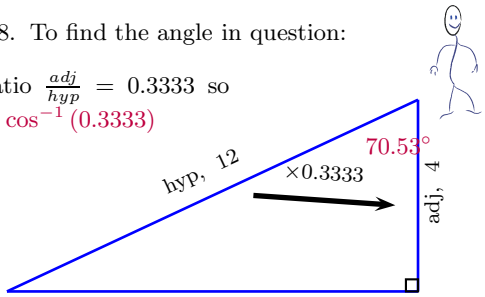
31. To find the angle in question:

The ratio $\frac{adj}{hyp} = 0.1333$ so we use $\cos^{-1}(0.1333)$



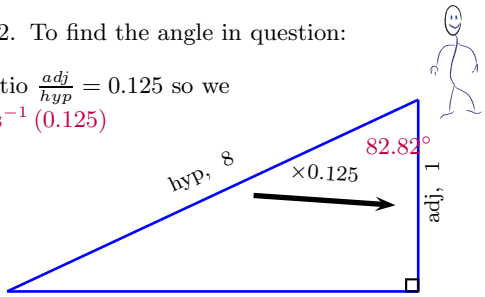
28. To find the angle in question:

The ratio $\frac{adj}{hyp} = 0.3333$ so we use $\cos^{-1}(0.3333)$



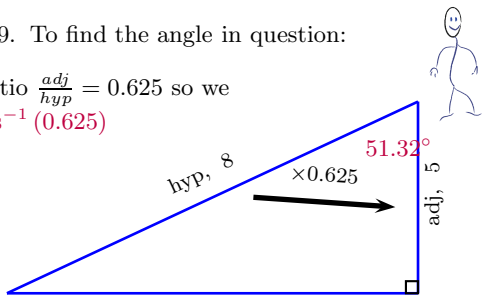
32. To find the angle in question:

The ratio $\frac{adj}{hyp} = 0.125$ so we use $\cos^{-1}(0.125)$



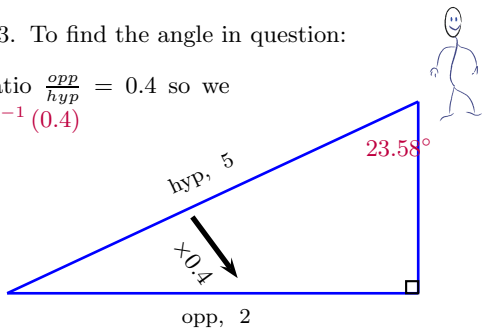
29. To find the angle in question:

The ratio $\frac{adj}{hyp} = 0.625$ so we use $\cos^{-1}(0.625)$



33. To find the angle in question:

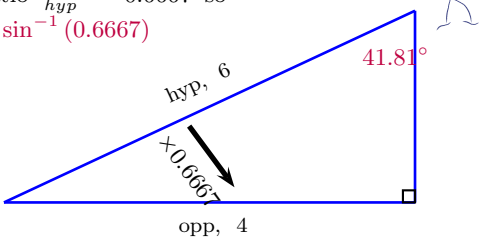
The ratio $\frac{opp}{hyp} = 0.4$ so we use $\sin^{-1}(0.4)$



30. To find the angle in question:

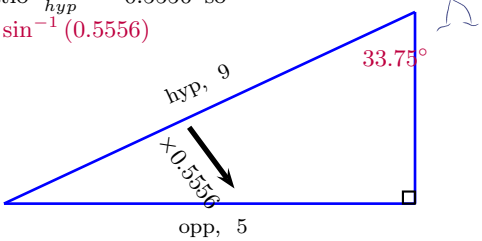
34. To find the angle in question:

The ratio $\frac{opp}{hyp} = 0.6667$ so
we use $\sin^{-1}(0.6667)$



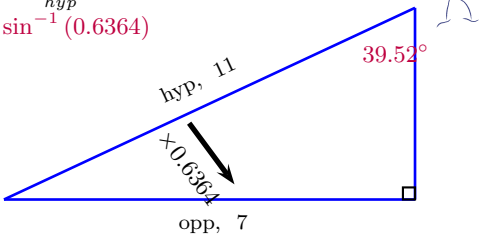
35. To find the angle in question:

The ratio $\frac{opp}{hyp} = 0.5556$ so
we use $\sin^{-1}(0.5556)$



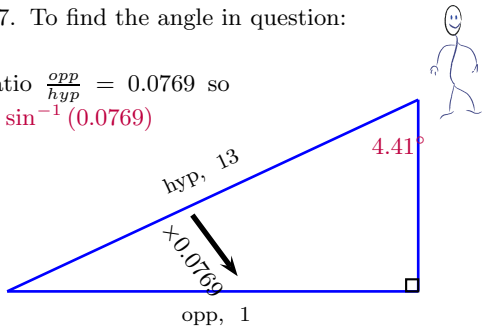
36. To find the angle in question:

The ratio $\frac{opp}{hyp} = 0.6364$ so
we use $\sin^{-1}(0.6364)$



37. To find the angle in question:

The ratio $\frac{opp}{hyp} = 0.0769$ so
we use $\sin^{-1}(0.0769)$



38. To find the angle in question:

The ratio $\frac{opp}{hyp} = 0.4107$ so
we use $\sin^{-1}(0.4107)$