

Q1



Tuesday, September 8, 2020

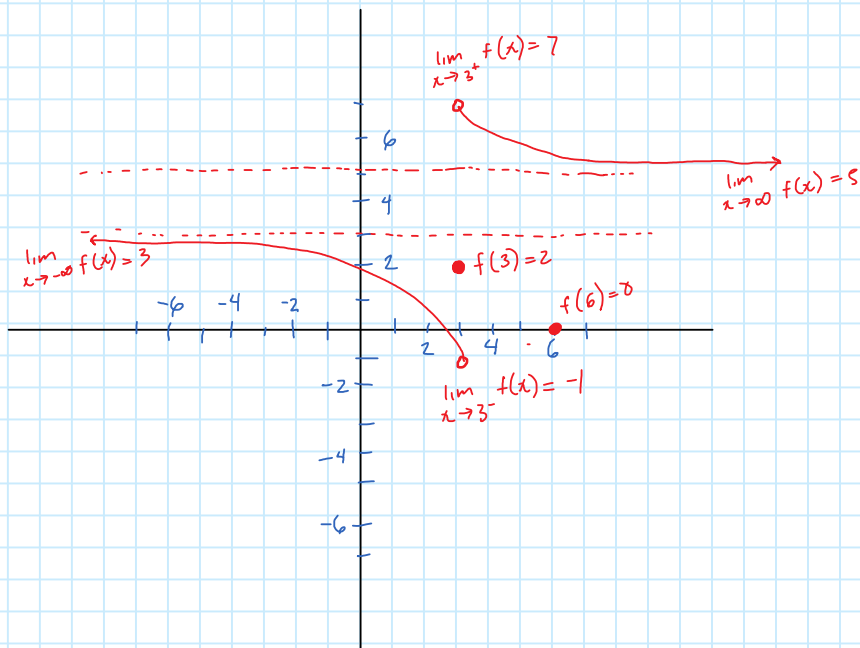
8:48 PM

1) Sketch a graph of a function with all of the following properties:

$$\lim_{x \rightarrow 3^+} f(x) = 7 \quad \lim_{x \rightarrow 3^-} f(x) = -1 \quad f(3) = 2$$

$$\lim_{x \rightarrow \infty} f(x) = 5 \quad \lim_{x \rightarrow -\infty} f(x) = 3 \quad f(6) = 0$$

11 points



Q2



Tuesday, September 8, 2020 8:56 PM

2) Evaluate $\lim_{x \rightarrow 9} \frac{9-x}{3-\sqrt{x}}$. You must show all of your algebra.

11 points

$$\lim_{x \rightarrow 9} \frac{9-x}{3-\sqrt{x}} = \frac{9-(9)}{3-\sqrt{9}} = \frac{0}{0} = 0$$

The limit as x
approaches 9, DOES NOT EXIST

Q3

Tuesday, September 8, 2020 9:03 PM

3) Let $f(x) = \begin{cases} x^2 + 1 & \text{for } x \geq 0 \quad \text{--- Right} \\ \cos(x) & \text{for } x < 0 \quad \text{--- Left} \end{cases}$

Is $f(x)$ continuous at $x=0$? Why or why not? Your explanation must include an algebraic argument. 11 points

$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= \cos(x) \\ &= \cos(0) \\ &= 1 \end{aligned}$$

$$\lim_{x \rightarrow 0^-} f(x) = 1$$

$$\begin{aligned} \lim_{x \rightarrow 0^+} f(x) &= x^2 + 1 \\ &= (0)^2 + 1 \\ &= 1 \end{aligned}$$

$$\lim_{x \rightarrow 0^+} f(x) = 1$$

$$\lim_{x \rightarrow 0^-} = \lim_{x \rightarrow 0^+}$$

Therefore, $f(x)$ is
continuous at $x=0$

Q4

Tuesday, September 8, 2020 9:13 PM

4) Evaluate $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 + x}}{x - 3}$. You must show all of your algebra.
11 points

$$\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 + x}}{x - 3}$$

↓

Take the highest
term from both
the numerator and
denominator

$$\frac{\sqrt{4x^2}}{x} = \frac{2x}{x} = 2$$

Q5



Tuesday, September 8, 2020

9:21 PM

5) Find $\lim_{x \rightarrow b} \frac{(x-b)^{5000} - x + b}{x-b}$

11 points

$$\lim_{x \rightarrow b} \frac{(x-b)^{5000} - x + b}{x-b}$$

$$\frac{\cancel{(x-b)}(x-b)^{4999} - x + b}{\cancel{x-b}}$$

$$= (x-b)^{4999} - \frac{x+b}{x-b}$$

$$= (x-b)^{4999} - \frac{\cancel{x+b}}{-1\cancel{(x+b)}}$$

$$= (x-b)^{4999} + 1$$

$$\lim_{x \rightarrow b} (x-b)^{4999} + 1$$

$$= (b-b)^{4999} + 1 = (0)^{4999} + 1$$

$$= 1$$

$$\lim_{x \rightarrow b} = 1$$

Q6

Tuesday, September 8, 2020 9:25 PM

6) Let $f(x) = x^2 + 1$

12 points

Find $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

You must show all of your algebra. No using more advanced short cuts from later chapters.

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad f(x) = x^2 + 1$$

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{(x+h)^2 + 1 - (x^2 + 1)}{h} \\ = \frac{\cancel{x^2} + 2xh + \cancel{h^2} + \cancel{1} - \cancel{x^2} - \cancel{1}}{h} \\ = \frac{2xh + h^2}{h} = \frac{\cancel{h}(2x + h)}{\cancel{h}} \end{aligned}$$

$$\begin{aligned} \lim_{h \rightarrow 0} &= 2x + h \\ &= 2x + (0) \end{aligned}$$

$$\boxed{\lim_{h \rightarrow 0} = 2x}$$

Q7

Tuesday, September 8, 2020 9:32 PM

7) Evaluate $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{x^2 + x} \right)$

11 points

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{1}{x} - \frac{1}{x^2 + x} \\ &= \frac{1}{x} \frac{(x^2 + x)}{(x^2 + x)} - \frac{1}{x^2 + x} \frac{(x)}{(x)} \\ &= \frac{x^2 + x}{x^3 + x^2} - \frac{x}{x^3 + x^2} \\ &= \frac{x^2}{x^3 + x^2} \\ &= \frac{\cancel{x^2}}{\cancel{x^2}(x+1)} \end{aligned}$$

$$\lim_{x \rightarrow 0} = \frac{1}{x+1} = \frac{1}{0+1} = \frac{1}{1} = \boxed{1}$$

8) Let $f(x) = \frac{x^2 - x - 6}{x^2 - 16}$

11 points

Find all horizontal and vertical asymptotes.

$$f(x) = \frac{x^2 - x - 6}{x^2 - 16}$$



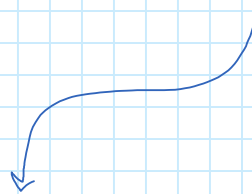
Take the highest
term from both
the numerator and
denominator

$$\frac{x^2}{x^2} = 1$$

$$y = 1$$

$$\frac{x^2 - x - 6}{x^2 - 16}$$

$$= \frac{x^2 - x - 6}{(x - 4)(x + 4)}$$



$$x = 4, x = -4$$

9) Let $f(x) = \frac{1}{x}$

12 points

Find $\lim_{x \rightarrow 2} \frac{f(x) - f(a)}{x - a}$

You must show all of your algebra. No using more advanced short cuts from later chapters.

$$\lim_{x \rightarrow 2} \frac{f(x) - f(a)}{x - a}, \quad f(x) = \frac{1}{x}$$

$$\lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{a}}{x - a} = \frac{\frac{1}{x} \frac{(a)}{(a)} - \frac{1}{a} \frac{(x)}{(x)}}{x - a}$$

$$= \frac{\frac{a}{xa} - \frac{x}{xa}}{x - a} = \frac{a - x}{xa} \cdot \frac{1}{x - a}$$

$$= \frac{a - x}{xa(x - a)} = \frac{-1 \cancel{(-a + x)}}{xa \cancel{(x - a)}}$$

$$= \frac{-1}{xa}$$

$$\lim_{x \rightarrow 2} \frac{-1}{xa} = \frac{-1}{(2)a} = \boxed{-\frac{1}{2}a}$$