Q1

Thursday, September 10, 2020 10:45 PM

Differentiate the function.

$f(x) = 5.8x + 2.7$

$f'(x) =$ 5.8

$f(x) = 5.8x + 2.7$

$f'(x) = 5.8x + 2.7$

$= 5.8x^1 + 0$

$= (1)5.8x^{1-1} + 0$

$= \boxed{5.8}$

Q2

Thursday, September 10, 2020 10:50 PM

Differentiate the function.

$$g(x) = x^2(1 - 8x)$$

$g'(x) = \boxed{2x - 24x^2}$

$$g(x) = x^2(1 - 8x)$$
$$= x^2 - 8x^3$$

$$g'(x) = x^2 - 8x^3$$
$$= 2x^{(2-1)} - (3)8x^{(3-1)}$$
$$= \boxed{2x - 24x^2}$$

Q3

Thursday, September 10, 2020 11:08 PM

Differentiate the function.

$$g(t) = 4t^{-3/8}$$

$$g'(t) = -\frac{12}{8}t^{-\left(\frac{11}{8}\right)}$$

$$g(t) = 4t^{-\frac{3}{8}}$$

$$g'(t) = \left(-\frac{3}{8}\right)4t^{-3/8 - 1}$$

$$= \boxed{-\frac{12}{8}t^{-11/8}}$$

Q4

Thursday, September 10, 2020 11:14 PM

Differentiate the function.

$B(y) = cy^{-8}$

$B'(y) = \boxed{-8cy^{-9}}$

$B(y) = cy^{-8}$

$B'(y) = cy^{-8}$

$= (-8)cy^{-8-1}$

$= -8cy^{-9}$

Q5

Thursday, September 10, 2020 11:15 PM

Differentiate the function.

$R(a) = (6a + 1)^2$

$R'(a) = 72a + 12$

$R(a) = (6a+1)^2$

$= 36a^2 + 12a + 1$

$R'(a) = 36a^2 + 12a + 1$

$= (2)36a^{(2-1)} + 12a^{(1-1)} + 0$

$= \boxed{72a + 12}$

Q6

Thursday, September 10, 2020 11:23 PM

Differentiate the function.

$$y = 8e^x + \frac{4}{\sqrt[3]{x}}$$

$y' = $ $8e^x + -\frac{4}{3}x^{-\left(\frac{4}{3}\right)}$ ✓

$$y = 8e^x + \frac{4}{\sqrt[3]{x}}$$

$$= 8e^x + \frac{4}{x^{1/3}}$$

$$= 8e^x + 4x^{-1/3}$$

$$y' = 8e^x + 4x^{-1/3}$$

$$= 8e^x + \left(-\frac{1}{3}\right)4x^{-1/3 - 1}$$

$$= \boxed{8e^x + -\frac{4}{3}x^{-4/3}}$$

Q7

Thursday, September 10, 2020 11:31 PM

Differentiate the function.

$$y = \frac{8x^2 + 6x + 2}{\sqrt{x}}$$

$y' = \boxed{12x^{\left(\frac{1}{2}\right)} + 3x^{-\left(\frac{1}{2}\right)} - x^{-\left(\frac{3}{2}\right)}}$ ✓

$$y = \frac{8x^2 + 6x + 2}{\sqrt{x}}$$

$$= \frac{8x^2 + 6x + 2}{x^{1/2}}$$

$$= (8x^2 + 6x + 2)\, x^{-1/2}$$

$$= 8x^{3/2} + 6x^{1/2} + 2x^{-1/2}$$

$$y' = \left(\tfrac{3}{2}\right) 8x^{(3/2 - 1)} + \left(\tfrac{1}{2}\right) 6\, x^{(1/2 - 1)} + \left(-\tfrac{1}{2}\right) 2x^{(-1/2 - 1)}$$

$$= \boxed{12x^{1/2} + 3x^{-1/2} - x^{-3/2}}$$

Q8

Friday, September 11, 2020 11:20 AM

Differentiate the function.

$$z(y) = \frac{A}{y^{13}} + Be^{y}$$

$z'(y) =$ $\boxed{-13Ay^{-14} + Be^{y}}$

$$z(y) = \frac{A}{y^{13}} + Be^{y}$$

$$= Ay^{-13} + Be^{y}$$

$$z'(y) = Ay^{-13} + Be^{y}$$

$$= (-13)Ay^{-13-1} + Be^{y}$$

$$= \boxed{-13Ay^{-14} + Be^{y}}$$

Q9

Friday, September 11, 2020 11:27 AM

Find an equation of the tangent line to the curve at the given point.

$y = 3x^3 - x^2 + 2, \quad (2, 22)$

$y = \boxed{32x - 42}$ ✔

$y = 3x^3 - x^2 + 2 \quad , \quad (2, 22)$

Get slope at (2,22)

$y'(2) = 3x^3 - x^2 + 2$

$= (3)3x^{(3-1)} - 2x^{(2-1)} + 0$

$y'(2) = 9x^2 - 2x$

$= 9(2)^2 - 2(2)$

$= 36 - 4$

$y'(2) = 32 = m \text{ at } (2, 22)$

Get equation of tangent y at (2,22)

$m = \frac{\Delta y}{\Delta x} = \frac{y - 22}{x - 2} = 32$

$\cancel{(x-2)} \frac{y - 22}{\cancel{x-2}} = 32(x-2)$

$y - 22 = 32(x-2)$

$y - 22 = 32x - 64$

$y = 32x - 64 + 22$

$y = \boxed{32x - 42}$

Q10

Friday, September 11, 2020 1:39 PM

Find an equation of the tangent line to the curve at the given point.

$y = 7x^2 - x^3$, $(1, 6)$

$y = $ $\boxed{11x - 5}$

Illustrate by graphing the curve and the tangent line on the same screen.

$$y = 7x^2 - 3x, \quad (1,6)$$

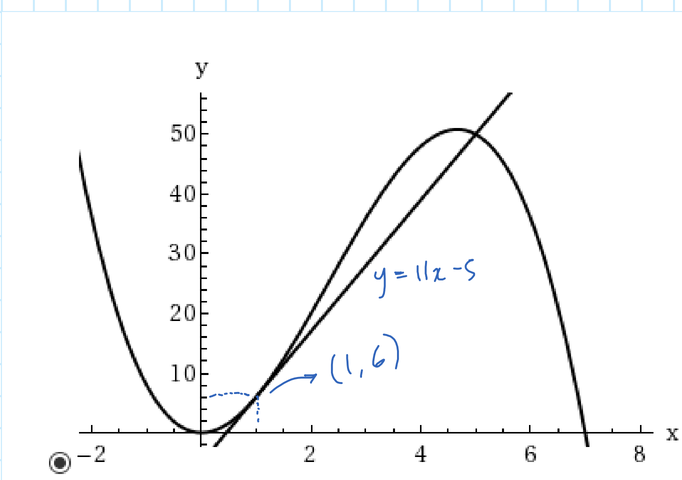
Get slope at (1,6)

$$y'(1) = 7x^2 - 3x$$
$$= (2)7x^{(2-1)} - (1)3x^{(1-1)}$$
$$y'(1) = 14x - 3$$
$$= 14(1) - 3$$
$$= 14 - 3$$
$$y'(1) = 11 = m \text{ at } (1,6)$$

Get equation of tangent at (1,6)

$y - y_1 = m(x - x_1)$ point-slope form

$$y - 6 = 11(x - 1)$$
$$y - 6 = 11x - 11$$
$$y = 11x - 11 + 6$$
$$y = \boxed{11x - 5}$$



Q11

Friday, September 11, 2020 1:54 PM

Find $f'(x)$.

$$f(x) = x^5 - 5x^3 + x - 1$$

$f'(x) = \boxed{5x^4 - 15x^2 + 1}$ ✓

Compare the graphs of f and f' and use them to explain why your answer is reasonable.

$f'(x) = 0$ when f has a horizontal tangent ✓ .

f' is positive when f is increasing ✓ .

f' is negative when f is decreasing ✓ .

$$f(x) = x^5 - 5x^3 + x - 1$$

$$f'(x) = x^5 - 5x^3 + x - 1$$

$$= (5)x^{(5-1)} - (3)5x^{(3-1)} + (1)x^{(1-1)} - 0$$

$$= \boxed{5x^4 - 15x^2 + 1}$$

Q12

Friday, September 11, 2020 3:04 PM

Find the first and second derivative of the function. Check to see that your answers are reasonable by comparing the graphs of f, f', and f''.

$f(x) = 4x - 5x^{7/8}$

$f'(x) = \boxed{4 - \left(\frac{35}{8}\right)x^{-\left(\frac{1}{8}\right)}}$ ✓

$f''(x) = \boxed{\frac{35}{64}x^{-\left(\frac{9}{8}\right)}}$ ✓

$$f(x) = 4x - 5x^{7/8}$$

$$f'(x) = 4x - 5x^{7/8}$$

$$= (1)4x^{(1-1)} - \left(\frac{7}{8}\right)5x^{7/8 - 1}$$

$$f'(x) = \boxed{4 - \frac{35}{8}x^{-1/8}}$$

$$f''(x) = 4 - \frac{35}{8}x^{-1/8}$$

$$= 0 - \left(-\frac{1}{8}\right)\frac{35}{8}x^{-1/8 - 1}$$

$$f''(x) = \boxed{\frac{35}{64}x^{-9/8}}$$

Q13

Friday, September 11, 2020 3:25 PM

The equation of motion of a particle is $s = t^3 - 27t$, where s is in meters and t is in seconds. (Assume $t \geq 0$.)

(a) Find the velocity and acceleration as functions of t.

$v(t) = 3t^2 - 27$ ✓

$a(t) = 6t$ ✓

(b) Find the acceleration after 8 s.

48 ✓ m/s^2

(c) Find the acceleration when the velocity is 0.

18 ✓ m/s^2

$s = t^3 - 27t, \quad t \geq 0$

$v(t) = s'(t) = \frac{ds}{dt} = \frac{\Delta s}{\Delta t}$

$a(t) = v'(t) = s''(t)$

$s'(t) = t^3 - 27t$

$= 3(t)^{(3-1)} - 27t^{(1-1)}$

$v(t) = \boxed{3t^2 - 27}$

(b) $t = 8$

$a(8) = 6t = 6(8) = \boxed{48 \text{ m/s}}$

$s''(t) = 3t^2 - 27$

$= (2)(3)t^{(2-1)} - 0$

$a(t) = \boxed{6t}$

(c) $v(t) = 3t^2 - 27 = 0$

$3t^2 - 27 = 0$

$3t^2 = 27$

$\frac{3t^2}{3} = \frac{27}{3}$

$t^2 = 9$

$\sqrt{t^2} = \sqrt{9}$

$t = 3$

$a(3) = 6t$

$= 6(3)$

$a(3) = 18$

$\boxed{18 \text{ m/s}^2}$

Q14

Friday, September 11, 2020 4:36 PM

Biologists have proposed a cubic polynomial to model the length L of rock bass at age A:

$$L = 0.0155A^3 - 0.372A^2 + 3.95A + 1.21$$

where L is measured in inches and A in years. Calculate

$\left.\frac{dL}{dA}\right|_{A=18}$. (Round your answer to three decimal places.)

$\left.\frac{dL}{dA}\right|_{A=18} =$ 5.624 in/yr

Interpret your answer.

- ○ A 18-year old rock fish grows at a rate of 65.893 in/yr.
- ○ A 18-year old rock fish shrinks at a rate of 65.893 in/yr.
- ◉ A 18-year old rock fish grows at a rate of 5.624 in/yr.
- ○ A 18-year old rock fish shrinks at a rate of 5.624 in/yr.
- ○ A 18-year old rock fish grows at a rate of 60.269 in/yr.

Q15

Friday, September 11, 2020 4:36 PM

Find the points on the curve $y = 2x^3 + 3x^2 - 12x + 8$ where the tangent line is horizontal.

$(x, y) = ($ $-2, 28$ $)$ (smaller x-value)

$(x, y) = ($ $1, 1$ $)$ (larger x-value)

$m = 0$

$y = 2x^3 + 3x^2 - 12x + 8$

$y' = 0$

$2x^3 + 3x^2 - 12x + 8 = 0$

$(3)2x^{(3-1)} + (2)3x^{(2-1)} - (1)12x^{(1-1)} + 0 = 0$

$6x^2 + 6x - 12 = 0$

$6(x^2 + x - 2) = 0$

$6(x+2)(x-1) = 0$

$x = -2$

or

$x = 1$

$y = 2x^3 + 3x^2 - 12x + 8$

$= 2(-2)^3 + 3(-2)^2 - 12(-2) + 8$

$= -16 + 12 + 24 + 8$

$= 28$

$y = 2x^3 + 3x^2 - 12x + 8$

$= 2(1)^3 + 3(1)^2 - 12(1) + 8$

$= 2 + 3 - 12 + 8$

$= 1$

$\boxed{(-2, 28) \text{ and } (1, 1)}$

Q16

Friday, September 11, 2020 4:48 PM

At what point on the curve $y = 8 + 2e^x - 5x$ is the tangent line parallel to the line $5x - y = 1$?

$(x, y) = (\ln(5), 18 - 5\ln(5))$

$y = 8 + 2e^x - 5x$

$y' = \frac{d}{dx}(8 + 2e^x - 5x)$

$= 0 + 2e^x - 5$

$y' = 2e^x - 5$

$5x - y = 1$

$5x - 1 = y$

$y = -1 + 5x$

slope $= \frac{5}{1} = 5 = m$

$2e^x - 5 = 5$

$2e^x = 5 + 5$

$\frac{2e^x}{2} = \frac{10}{2}$

$e^x = 5$

Apply log rule $\log_a(x^b) = b \cdot \log_a(x)$

$\ln(e^x) = \ln(5)$

Apply log rule $\log_a(a) = 1$

$x\ln(e) = \ln(5)$

NOTE: $\ln(a) = \ln_e(a)$

$x = \ln(5)$

$y = 8 + 2e^x - 5x, \quad x = \ln(5)$

$y = 8 + 2e^{\ln(5)} - 5(\ln[5])$

$y = 8 + 2(5) - 5\ln(5)$

$y = 8 + 10 - 5\ln(5)$

$y = 18 - 5\ln(5)$

$\boxed{(\ln(5), 18 - 5\ln(5))} = (x, y)$ is Parallel to the line $5x - y = 1$ or $y = 5x - 1$

$m = 5$

$y - y_1 = m(x - x_1)$

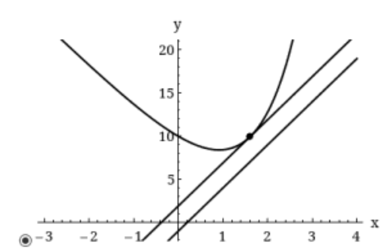
$y - [18 - 5\ln(5)] = 5[x - \ln(5)]$

$y - 18 + 5\ln(5) = 5x - 5\ln(5)$

$y = 5x - 5\ln(5) - 5\ln(5) + 18$

$y = 5x - 10\ln(5) + 18$

$\boxed{\text{CHECK Graph } 5x - 10\ln(5) + 18 \text{ and } 5x - 1}$



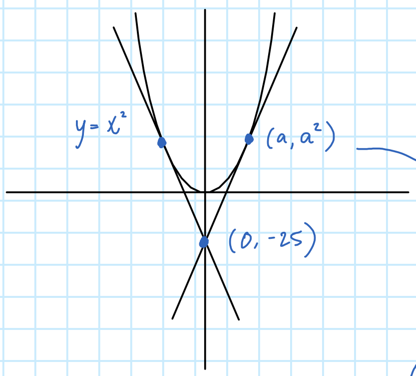
Q17

Friday, September 11, 2020 5:28 PM

Draw a diagram to show that there are two tangent lines to the parabola $y = x^2$ that pass through the point $(0, -25)$. Find the coordinates of the points where these tangent lines intersect the parabola.

$(x, y) = (\boxed{-5, 25})$ (smaller x-value)

$(x, y) = (\boxed{5, 25})$ (larger x-value)



$y - y_1 = m(x - x_1)$

$x_1 = 0 \qquad x = a$

$y_1 = -25 \qquad y = a^2$

$m = 2a$

$y - (-25) = 2a(x - 0)$

$y + 25 = 2ax$

$y = 2ax - 25$

$y = x^2$

$y' = x^2$

$= (2)x^{(2-1)}$

$y' = 2x = m$

$m = y' = 2x$

$y'(a) = 2x$

$= 2a$

$m = 2a$

Slope at point (a, a^2)

$(a^2) = 2a(a) - 25 \longrightarrow$ Plug (a, a^2)

$a^2 = 2a^2 - 25$

$25 = 2a^2 - a^2$

$\sqrt{25} = \sqrt{a^2}$

$5 = a$

$(5, (5)^2)$ and $(-5, (-5)^2)$

$\boxed{(5, 25)}$ $\boxed{(-5, 25)}$

Q18

Friday, September 11, 2020 6:06 PM

Find a second-degree polynomial P such that $P(2) = 2$, $P'(2) = 3$, and $P''(2) = 4$.

$P(x) = \boxed{2x^2 - 5x + 4}$

second - degree polynomial template $= ax^2 + bx + c \rightarrow \boxed{2x^2 - 5x + 4}$

$P(x) = ax^2 + bx + c$ $P(2) = 2$

$P(2) = ax^2 + bx + c = 2$

$(2)(2)^2 + (-5)(2) + c = 2 \rightarrow a = 2, \ b = -5$

$8 - 10 + c = 2$

$-2 + c = 2$

$c = 2 + 2 = \underline{\underline{c = 4}}$

$P'(x) = (2)ax^{(2-1)} + (1)bx^{(1-1)} + 0$ $P'(2) = 3$

$= 2ax + b$

$P'(2) = 2ax + b = 3$

$2(2)(2) + b = 3 \rightarrow a = 2$

$8 + b = 3$

$b = 3 - 8 = \underline{\underline{b = -5}}$

$P''(x) = (1)2ax^{(1-1)} + 0$ $P''(2) = 4$

$= 2a$

$P''(2) = 2a = 4$

$\dfrac{\cancel{2}a}{\cancel{2}} = \dfrac{4}{2} = \underline{\underline{a = 2}}$

Q19

Friday, September 11, 2020 6:38 PM

Find a cubic function $y = ax^3 + bx^2 + cx + d$ whose graph has horizontal tangents at the points $(-2, 11)$ and $(2, 5)$.

$y = \boxed{\frac{3}{16}x^3 - \frac{9}{4}x + 8}$

$y = ax^3 + bx^2 + cx + d = 0$

$= (3)ax^{(3-1)} + (2)bx^{(2-1)} + (1)cx^{(1-1)} + 0 = 0$

$y' = 3ax^2 + 2bx + c = 0$ at $(-2, 11)$ and $(2, 5)$

.. at $(-2, 11)$

$y'(-2) = 3ax^2 + 2bx + c = 0$

$= 3a(-2)^2 + 2b(-2) + c = 0$

$y'(-2) = 12a - 4b + c = 0$

... at $(2, 5)$

$y'(2) = 3ax^2 + 2bx + c = 0$

$= 3a(2)^2 + 2b(2) + c = 0$

$= 12a + 4b + c = 0$

Subtract to eliminate a and c to get the value of b

$\cancel{12a} - 4b + \cancel{c} = 0$

$-\ \cancel{12a} + 4b + \cancel{c} = 0$

$-8b = 0$

$\frac{-8b}{-8} = \frac{0}{-8} = \boxed{b = 0}$

Pick either one of the two to plug in $b = 0$

$12a + 4(0) + c = 0$

$12a + c = 0$

. still at $(-2, 11)$

$y(-2) = 11$

$y(-2) = ax^3 + bx^2 + cx + d = 11$

$= a(-2)^3 + b(-2)^2 + c(-2) + d = 11$

$= -8a + (0)(4) - 2c + d = 11$

$= -8a - 2c + d = 11$

. still at $(2, 5)$

$y(2) = 5$

$y(2) = ax^3 + bx^2 + cx + d = 5$

$= a(2)^3 + b(2)^2 + c(2) + d = 5$

$= 8a + (0)(4) + 2c + d = 5$

$= 8a + 2c + d = 5$

Add to eliminate a and c to get the value of d

$-\cancel{8a} - \cancel{2c} + d = 11$

$+\ \cancel{8a} + \cancel{2c} + d = 5$

$2d = 16$

$\frac{2d}{2} = \frac{16}{2} = \boxed{d = 8}$

Pick either one of the two to plug in $d = 8$

$-8a - 2c + (8) = 11$

$-8a - 2c = 11 - 8$

$-8a - 2c = 3$

or

$8a + 2c = -3$

$12a + c = 0$

$8a + 2c = -3$

$\rightarrow$

$-2(12a + c = 0)$

$8a + 2c = -3$

multiply by -2 to eliminate c to get a

$-24a - 2c = 0$

$8a + 2c = -3$

$-16a = -3$

$\rightarrow \boxed{a = \frac{3}{16}}$

Pick either one of the two to plug in $a = \frac{3}{16}$

$12a + c = 0$

$12\left(\frac{3}{16}\right) + c = 0$

$\frac{9}{4} + c = 0$

$\boxed{c = -\frac{9}{4}}$

$a = \frac{3}{16},\ b = 0,\ c = -\frac{9}{4},\ d = 8$

$y = ax^3 + bx^2 + cx + d$

$= \left(\frac{3}{16}\right)x^3 + (0)x^2 + \left(-\frac{9}{4}\right)x + (8)$

$\boxed{y = \frac{3}{16}x^3 - \frac{9}{4}x + 8}$