Q1

Saturday, October 24, 2020 4:35 PM

Use the guidelines of this section to sketch the curve.

$y = x^3 + 6x^2$

$y = x^3 + 6x^2$

A.) Domain $= \mathbb{R}$

$y' = 3x^2 + 12x$

$3x^2 + 12x = 0$

$3x(x+4) = 0$

$3x = 0 \qquad x + 4 = 0$

$x = 0 \qquad x = -4$

B) y - intercept $= f(0) = 0$

x - intercept $= 0, -4$

C) $f(-x) \neq f(x)$, no symmetry

D) no asymptotes

$f(0) = x^3 + 6x^2$

$= 0 \longrightarrow$ Local min

$f(-4) = x^3 + 6x^2$

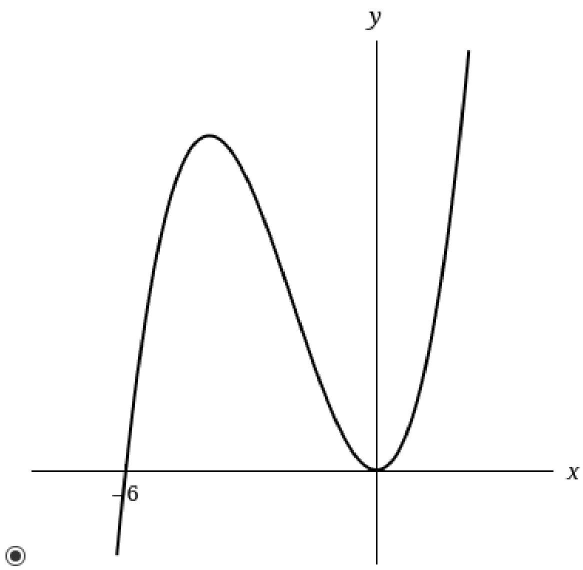
$= (-4)^3 + 6(-4)^2$

$= -64 + 96$

$= 32 \longrightarrow$ Local max

critical points

F.) $(0,0), (-4, 32)$



E) $f'(x) = 3x^2 + 12x$

$+ \quad - \quad +$

$-4 \quad 0$

$x < -4 \quad -4 < x < 0 \quad x > 0$

f' is increasing @ $(-\infty, -4] \cup [0, \infty)$

f' is decreasing @ $[-4, 0]$

G.) $f''(x) = \frac{d}{dx}(3x^2 + 12x)$

$= 6x + 12$

$6x + 12 = 0$

$6x = -12$

$x = -\frac{12}{6}$

$x = -2$

$\frown \quad \smile$

$- \quad +$

-2

$x < -2 \quad x > -2$

f'' is concave downwards @ $(-\infty, 2]$

f'' is concave upwards @ $[2, \infty)$

$f(-2) = x^3 + 6x^2$

$= (-2)^3 + 6(-2)^2$

$= 16$

Inflection point

$(-2, 16)$

Q2

Saturday, October 24, 2020 4:48 PM

Use the guidelines of this section to sketch the curve.

$$y = x^4 - 4x$$

$y = x^4 - 4x$

$y' = 4x^3 - 4$

$4x^3 - 4 = 0$

$4x^3 = 4$

$\sqrt{x^3} = \sqrt{1}$

$x = 1$

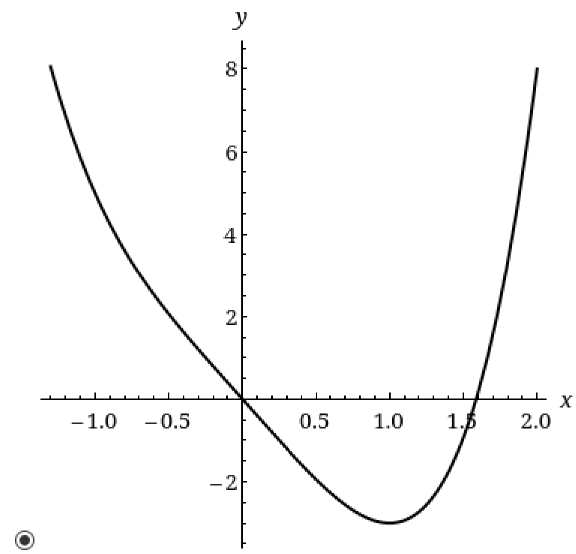
$f(1) = x^4 - 4x$

$= 1 - 4$

$= -3$

critical points

$(1, -3)$



Q3

Saturday, October 24, 2020 4:53 PM

Use the guidelines of this section to sketch the curve.

$$y = x(x-4)^3$$

$y = x(x-4)^3$

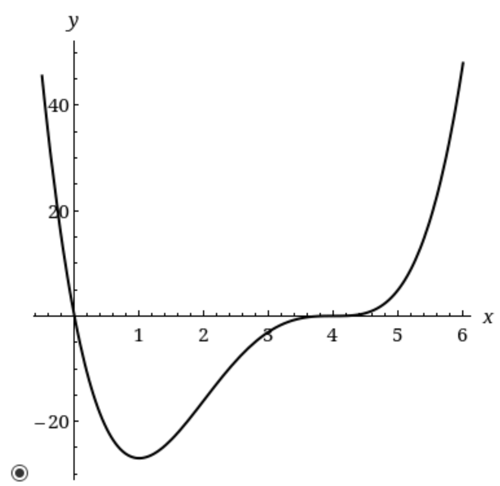
$y' = \frac{d}{dx}\left[x(x-4)^3\right]$

$= x\frac{d}{dx}\left[(x-4)^3\right] + (x-4)^3\frac{d}{dx}(x)$

$= x\,3(x-4)^2\frac{d}{dx}(x-4) + (x-4)^3(1)$

$= 3x(x-4)^2(1) + (x-4)^3$

$y' = 3x(x-4)^2 + (x-4)^3$



$3x(x-4)^2 + (x-4)^3 = 0$

$3x(x-4)^2 + (x-4)(x-4)^2 = 0$

$(x-4)^2\left[3x + (x-4)\right] = 0$

$(x-4)^2 = 0$

$x = 4$

$3x + (x-4) = 0$

$3x + x - 4 = 0$

$4x = 4$

$x = 1$

$f(4) = x(x-4)^3$

$= 4(4-4)^3$

$= 0$

critical Points

$(4,0), (1,-27)$

$f(1) = x(x-4)^3$

$= 1(1-4)^3$

$= 1(-3)^3$

$= -27$

Q4

Saturday, October 24, 2020 5:10 PM

Use the guidelines of this section to sketch the curve.

$$y = \frac{x}{x-2}$$

$$y = \frac{x}{x-2}$$

$$y' = \frac{(x-2)\frac{d}{dx}(x) - x\frac{d}{dx}(x-2)}{(x-2)^2}$$

$$= \frac{(x-2)(1) - x(1)}{(x-2)^2}$$

$$y' = \frac{x-2-x}{(x-2)^2} = \frac{\cancel{(x-2)}}{}$$

$$-\frac{2}{(x-2)} = 0 \text{ or undefined}$$

$$x - 2 = 0$$

$$x = 2$$

undefined

$$f(2) = \frac{x}{x-2}$$

$$= \frac{2}{2-2}$$

$$= \frac{2}{0} = \text{undefined}$$

Vertical Asymptote

$$\lim_{x \to 2^-} \frac{x}{x-2} = \frac{(1.9)}{(1.9)-2} = \frac{1.9}{-0.1} = -\infty$$

$$\lim_{x \to 2^+} \frac{x}{x-2} = \frac{(2.1)}{(2.1)-2} = \frac{2.1}{0.1} = \infty$$

Horizontal Asymptote

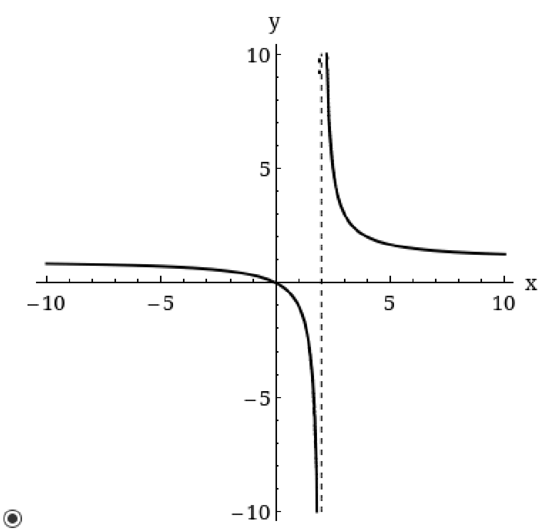
$$\lim_{x \to \pm\infty} f(x) = \frac{x}{x-2} = 1$$

As x gets bigger, f(x) approaches 1

Critical Point(s)

$(2, \infty), (2, -\infty)$

$VA = 2, \; HA = 1$



Q5

Saturday, October 24, 2020 5:31 PM

Use the guidelines of this section to sketch the curve.

$$y = \frac{x - x^2}{4 - 5x + x^2}$$

$$y = \frac{x - x^2}{4 - 5x + x^2}$$
$$= \frac{-x\cancel{(x-1)}}{(x-4)\cancel{(x-1)}}$$
$$= \frac{-x}{x-4}$$

$$y' = \frac{d}{dx}\left(\frac{-x}{x-4}\right)$$
$$= \frac{(x-4)\frac{d}{dx}(-x) - (-x)\frac{d}{dx}(x-4)}{(x-4)^2}$$
$$= \frac{(x-4)(-1) - (-x)(1)}{(x-4)^2}$$
$$= \frac{-x+4+x}{(x-4)^2}$$
$$y' = \frac{4}{(x-4)^2}$$

$$\frac{4}{(x-4)^2} = 0 \text{ or undefined}$$

$$\downarrow$$

$$x - 4 = 0$$
$$x = 4$$

undefined

$$f(4) = \frac{x - x^2}{4 - 5x + x^2}$$
$$= \frac{4 - 4^2}{4 - 5(4) + (4)^2}$$
$$= \frac{4 - 16}{4 - 20 + 16}$$
$$f(4) = \frac{-12}{0} = \text{undefined}$$

Vertical Asymptote

$$\lim_{x \to 4^-} \frac{-x}{x-4} = \frac{-(3.9)}{(3.9)-4} = + = \infty$$

$$\lim_{x \to 4^+} \frac{-x}{x-4} = \frac{-(4.1)}{(4.1)-4} = - = -\infty$$

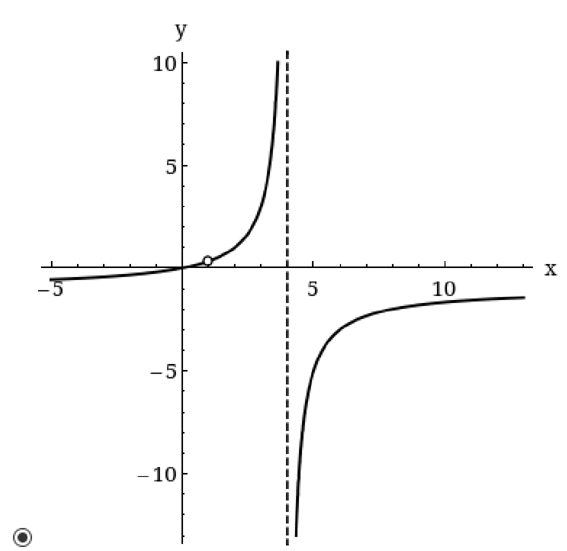
Horizontal Asymptote

$$\lim_{x \to \pm\infty} f(x) = \frac{-x}{x-4} = \frac{-\infty}{\infty - 4} = -1$$

critical point(s)

$(4, -\infty), (4, \infty)$

$VA = 4$, $HA = -1$



Q6

Saturday, October 24, 2020 9:05 PM

Use the guidelines of this section to sketch the curve.

$$y = \frac{x^2}{x^2 + 12}$$

$$\lim_{x \to \pm 0} \frac{x^2}{x^2+12} = \frac{0^2}{0^2+12} = 0$$ No Vertical Asymptote

Horizontal Asymptote

$$\lim_{x \to \pm\infty} f(x) = \frac{x^2}{x^2+12} = \frac{\infty^2}{\infty^2+12} = 1$$

$$y = \frac{x^2}{x^2+12}$$
$$= \frac{x^2}{x^2\left(1 + 12\frac{1}{x^2}\right)}$$
$$= \frac{1}{1+12x^{-2}}$$
$$= \left(1+12x^{-2}\right)^{-1}$$

$$y' = \frac{d}{dx}\left(1+12x^{-2}\right)^{-1}$$

$u = (1+12x^{-2})$

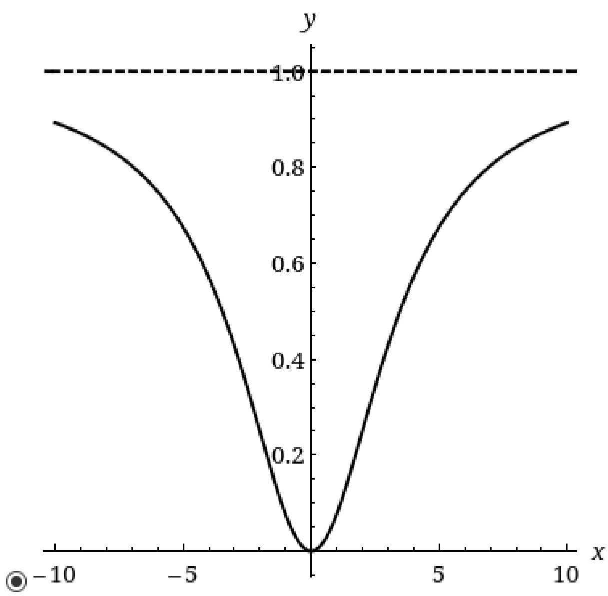
$f(u) = u^{-1}$

$\frac{df}{du} \cdot \frac{du}{dx}$

$$= \frac{d}{du}\left(u^{-1}\right)\frac{d}{dx}\left(1+12x^{-2}\right)$$
$$= -u^{-2}\left(-24x^{-3}\right)$$
$$= -\frac{1}{u^2}\left(-\frac{24}{x^3}\right)$$
$$= -\frac{1}{\left(1+12x^{-2}\right)^2}\left(-\frac{24}{x^3}\right)$$

$$1 + \frac{12}{x^2} = \frac{x^2}{x^2} + \frac{12}{x^2} = \frac{x^2+12}{x^2}$$

$$= -\frac{1}{\left(\frac{x^2+12}{x^2}\right)^2}\left(-\frac{24}{x^3}\right)$$
$$= -\frac{1}{\frac{(x^2+12)^2}{x^4}}\left(-\frac{24}{x^3}\right)$$
$$= -\frac{x^4}{(x^2+12)^2}\left(-\frac{24}{x^3}\right)$$
$$= \frac{24x^{\cancel{4}}}{(x^2+12)^2\cancel{x^3}}$$
$$y' = \frac{24x}{(x^2+12)^2}$$



$$\frac{24x}{(x^2+12)^2} = 0$$

$24x = 0$

$x = 0$

$x^2 + 12 = 0$

$x^2 = -12$

$x = \sqrt{-12}$

DNE

$$f(0) = \frac{x^2}{x^2+12}$$
$$= \frac{0}{0+12}$$
$$= 0$$

Q7

Sunday, October 25, 2020 11:38 AM

Use the guidelines of this section to sketch the curve.

$$y = (x - 6)\sqrt{x}$$

$y = (x-6)\sqrt{x}$

$y' = (x-6)\frac{d}{dx}(\sqrt{x}) + \sqrt{x}\frac{d}{dx}(x-6)$

$= (x-6)\left(\frac{1}{2}x^{-1/2}\right) + \sqrt{x}\,(1)$

$= \frac{1}{2}x\,x^{-1/2} - 3x^{-1/2} + \sqrt{x}$

$= \frac{1}{2x} - \frac{3}{x^2} + \sqrt{x}$

$= \frac{1}{2x}\frac{(x)}{(x)} - \frac{3}{x^2}\frac{(2)}{(2)} + \frac{\sqrt{x}}{1}\frac{(2x^2)}{(2x^2)}$

$= \frac{x}{2x^2} - \frac{6}{2x^2} + \frac{2x}{2x^2}$

$y' = \frac{3x-6}{2x^2}$

$\frac{3x-6}{2x^2} = 0$

$3x - 6 = 0$

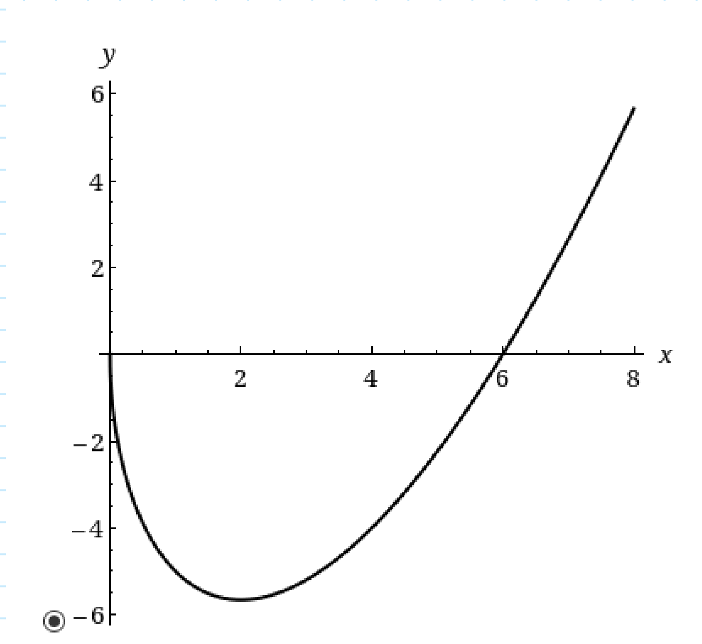
$3x = 6$

$x = \frac{6}{3}$

$x = 2$

$2x^2 = 0$

$x = 0$



$f(0) = (x-6)\sqrt{x}$

$= (0-6)\sqrt{0}$

$= 0$

critical points

$(0,0), (2, -4\sqrt{2})$

$f(2) = (x-6)\sqrt{x}$

$= (2-6)\sqrt{2}$

$= (-4)\sqrt{2}$

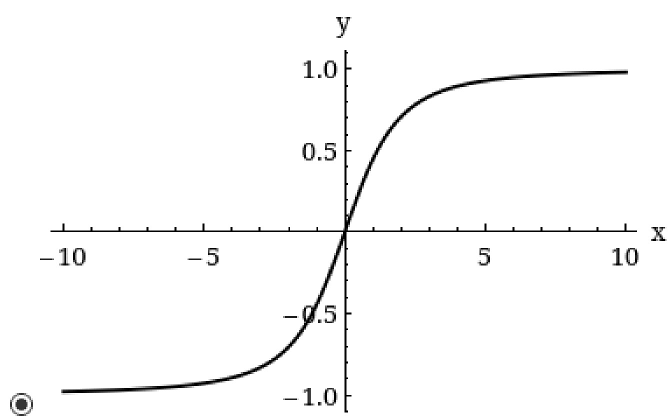
$= -4\sqrt{2}$

Q8

Sunday, October 25, 2020 12:13 PM

Use the guidelines of this section to sketch the curve.

$$y = \frac{x}{\sqrt{x^2 + 4}}$$

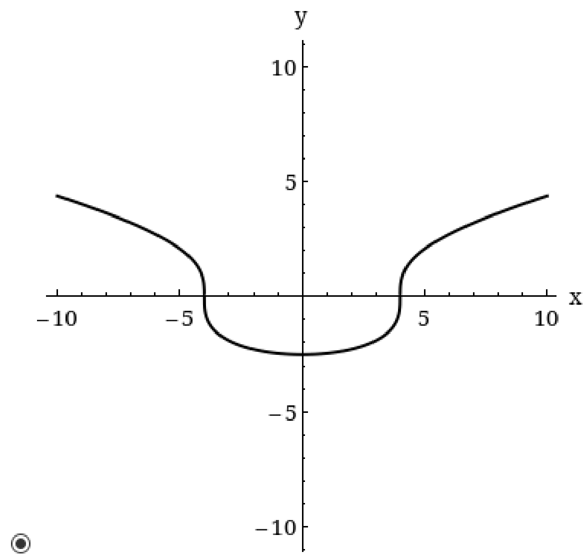


Q9

Sunday, October 25, 2020 12:17 PM

Use the guidelines of this section to sketch the curve.

$y = \sqrt[3]{x^2 - 16}$

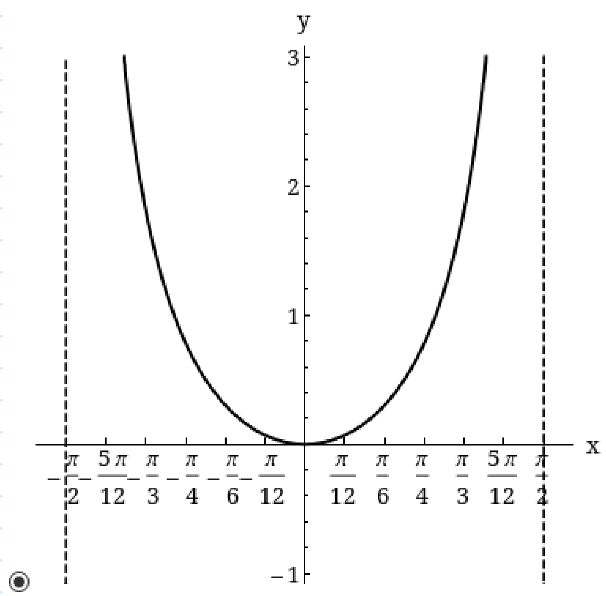


Q10

Sunday, October 25, 2020 12:18 PM

Use the guidelines of this section to sketch the curve.

$y = x \tan(x), \quad -\pi/2 < x < \pi/2$

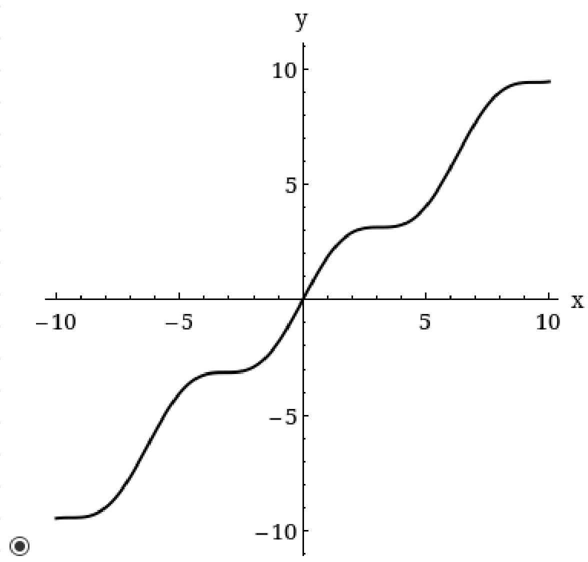


Q11

Sunday, October 25, 2020 12:18 PM

Use the guidelines of this section to sketch the curve.

$y = x + \sin(x)$

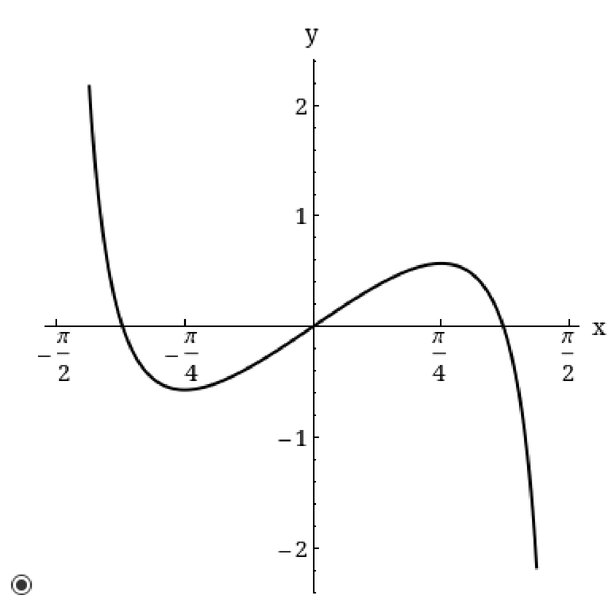


Q12

Sunday, October 25, 2020 12:19 PM

Use the guidelines of this section to sketch the curve.

$y = 2x - \tan(x), \ -\pi/2 < x < \pi/2$

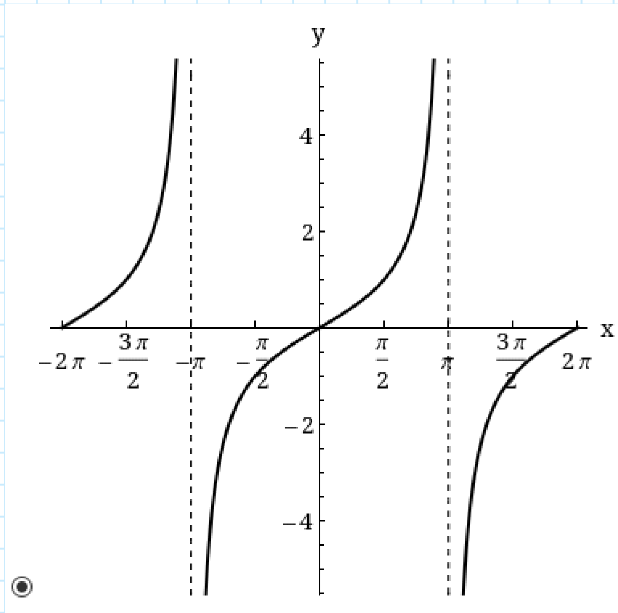


Q13

Sunday, October 25, 2020 12:20 PM

Use the guidelines of this section to sketch the curve.

$$y = \frac{\sin(x)}{1 + \cos(x)}$$

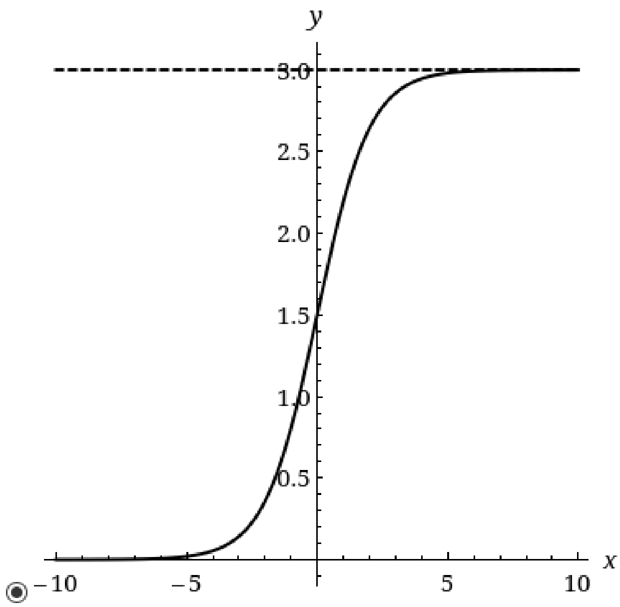


Q14

Sunday, October 25, 2020 12:21 PM

Use the guidelines of this section to sketch the curve.

$$y = \frac{3}{(1 + e^{-x})}$$

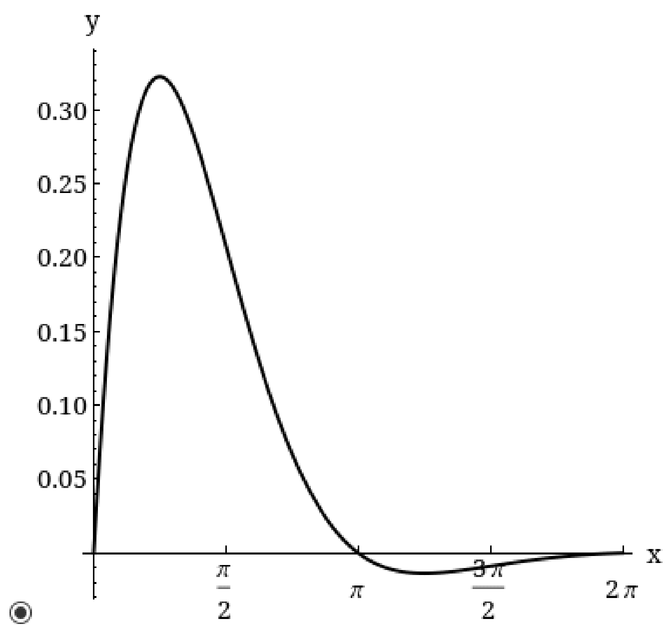


Q15

Sunday, October 25, 2020 12:22 PM

Use the guidelines of this section to sketch the curve.

$y = e^{-x}\sin(x),\ \ 0 \le x \le 2\pi$



Q16

Sunday, October 25, 2020 12:22 PM

Find an equation of the slant asymptote. Do not sketch the curve.

$$y = \frac{x^2 + 3}{x + 3}$$

$y = \boxed{x - 3}$ ✓

$$y = \frac{x^2 - 3}{x + 3}$$

has a slant asymptote because there is a rational function where the numerator has a higher degree than the denominator

Long Division

$$\begin{array}{r} x - 3 + \frac{6}{x+3} \\ x+3 \overline{) x^2 - 3} \\ -\; x^2 + 3x \\ \hline -3x - 3 \\ -\; -3x - 9 \\ \hline 6 \end{array}$$

$$f(x) = x - 3 + \frac{6}{x+3}$$

$$\lim_{x \to \pm\infty} \underbrace{x - 3}_{} + \frac{6}{x+3}$$

this becomes 0 as x becomes bigger

$\boxed{y = x - 3}$

the slant asymptote

Q17

Sunday, October 25, 2020 12:23 PM

Use the guidelines of this section to sketch the curve. In guideline D find an equation of the slant asymptote.

$$y = \frac{x^2}{x-5}$$

$y = \dfrac{x^2}{x-5}$ → has a slant asymptote because there is a rational function where the numerator has a higher degree than the denominator

Domain $= (-\infty, 5) \cup (5, \infty)$

Long Division

$$\begin{array}{r} x+5+\frac{25}{x-5} \\ x-5 \overline{\smash{)}\, x^2+0} \\ -\quad x^2-5x \\ \hline 5x+0 \\ -\quad 5x-25 \\ \hline +25 \end{array}$$

$$\lim_{x \to \pm\infty} \underbrace{x+5}_{} + \frac{25}{x-5}$$

→ this becomes 0 as x becomes bigger

$y = x+5$
slant asymptote

$$\lim_{x \to 5^-} f(x) = -\infty$$

$$\lim_{x \to 5^+} f(x) = \infty$$

Vertical Asymptote
$x = 5$

$$f(x) = x+5+\frac{25}{x-5}$$

$$f'(x) = \frac{d}{dx}\left(x+5+\frac{25}{x+5}\right)$$

$$= 1+0+\left[\frac{(x+5)\frac{d}{dx}(25) - 25\frac{d}{dx}(x+5)}{(x+5)^2}\right]$$

$$= 1+\left[\frac{0-25(1)}{(x+5)^2}\right]$$

$$f'(x) = 1-\frac{25}{(x+5)^2}$$

$$1-\frac{25}{(x+5)^2} = 0$$

$$1 = \frac{25}{(x+5)^2}$$

$$\sqrt{1} = \sqrt{\frac{25}{(x+5)^2}}$$

$$1 = \frac{5}{x+5}$$

$$x+5 = 5$$

$$x = 5-5$$

$$x = 0$$

$$f(0) = x+5+\frac{25}{x-5}$$

$$= 0+5+\frac{25}{0-5}$$

$$= 5-5$$

$$= 0$$

critical point(s)
$(0,0)$