

# Q1

Sunday, September 20, 2020 5:07 PM

## Math 150

## Quiz 2

Find the following derivative. You must show all of your work. Once you have taken the derivative you do not have to simplify.

- 1) Let  $f(x) = \frac{x^3 + \cos(x) - 1}{e^x - 3}$ . Find  $f'(x)$ .
- 2) Let  $f(x) = x^2 \sin(x)$ . Find  $f''(x)$ .
- 3) Let  $f(x) = x^5 e^x \sec(x)$ . Find  $f'(x)$ .

The Quotient Rule

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2}$$

1.) Let  $f(x) = \frac{x^3 + \cos(x) - 1}{e^x - 3}$ . Find  $f'(x)$

$$f(x) = x^3 + \cos(x) - 1$$

$$g(x) = e^x - 3$$

$$f'(x) = \frac{(e^x - 3) \frac{d}{dx} [x^3 + \cos(x) - 1] - [x^3 + \cos(x) - 1] \frac{d}{dx} (e^x - 3)}{(e^x - 3)^2}$$

$$= \frac{(e^x - 3) [3x^2 - \sin(x)] - [x^3 + \cos(x) - 1] (e^x)}{(e^x - 3)^2}$$

$$f'(x) = \frac{(e^x - 3) [3x^2 - \sin(x)] - e^x [x^3 + \cos(x) - 1]}{(e^x - 3)^2}$$

## The Product Rule

$$\frac{d}{dx} [f(x)g(x)] = f(x) \frac{d}{dx} [g(x)] + g(x) \frac{d}{dx} [f(x)]$$

2.) Let  $f(x) = x^2 \sin(x)$ . Find  $f''(x)$

$$f(x) = x^2$$

$$g(x) = \sin(x)$$

$$\begin{aligned} f'(x) &= x^2 \sin(x) \\ &= x^2 \frac{d}{dx} [\sin(x)] + \sin(x) \frac{d}{dx} (x^2) \\ &= x^2 \cos(x) + \sin(x) (2x) \end{aligned}$$

$$f'(x) = x^2 \cos(x) + 2x \sin(x)$$

$$\begin{aligned} f''(x) &= \frac{d}{dx} [x^2 \cos(x)] + \frac{d}{dx} [2x \sin(x)] \\ &= \left[ x^2 \frac{d}{dx} [\cos(x)] + \cos(x) \frac{d}{dx} (x^2) \right] + \left[ 2x \frac{d}{dx} [\sin(x)] + \sin(x) \frac{d}{dx} (2x) \right] \\ &= \left[ -x^2 \sin(x) + \cos(x) (2x) \right] + \left[ 2x \cos(x) + \sin(x) (2) \right] \end{aligned}$$

$$f''(x) = [-x^2 \sin(x) + 2x \cos(x)] + [2x \cos(x) + 2 \sin(x)]$$

The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

3) Let  $f(x) = x^5 e^x \sec(x)$ . Find  $f'(x)$

$$f(x) = x^5$$

$$g(x) = e^x \sec(x)$$

$$\begin{aligned} f'(x) &= x^5 \frac{d}{dx}[e^x \sec(x)] + e^x \sec(x) \frac{d}{dx}(e^x) \\ &= x^5 \left[ e^x \frac{d}{dx}(\sec(x)) + \sec(x) \frac{d}{dx}(e^x) \right] + e^x \sec(x) \frac{d}{dx}(e^x) \\ &= x^5 \left[ e^x (\sec(x) \tan(x)) + \sec(x) (e^x) \right] + e^x \sec(x) (e^x) \\ &= x^5 \left[ e^x \sec(x) \tan(x) + e^x \sec(x) \right] + e^x \sec(x) (e^x) \\ &= x^5 \left[ e^x \sec(x) (\tan(x) + 1) \right] + e^x \sec(x) (e^x) \\ &= x^5 e^x \sec(x) (\tan(x) + 1) + e^x \sec(x) (e^x) \end{aligned}$$

$$f'(x) = e^x \sec(x) \left[ x^2 (\tan(x) + 1) + e^x \right]$$