Q1

Math 150   Quiz 2

Find the following derivative. You must show all of your work. Once you have taken the derivative you do not have to simplify.

1) Let $f(x) = \frac{x^3 + \cos(x) - 1}{e^x - 3}$. Find f'(x).
2) Let f(x)= x^2sin(x). Find f"(x).
3) Let f(x)= $x^5 e^x$sec(x) . Find f'(x).

The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

1.) Let $f(x) = \frac{x^3 + \cos(x) - 1}{e^x - 3}$. Find $f'(x)$

$$f(x) = x^3 + \cos(x) - 1$$
$$g(x) = e^x - 3$$

$$f'(x) = \frac{(e^x - 3)\frac{d}{dx}[x^3 + \cos(x) - 1] - [x^3 + \cos(x) - 1]\frac{d}{dx}(e^x - 3)}{(e^x - 3)^2}$$

$$= \frac{(e^x - 3)[3x^2 - \sin(x)] - [x^3 + \cos(x) - 1](e^x)}{(e^x - 3)^2}$$

$$\boxed{f'(x) = \frac{(e^x - 3)[3x^2 - \sin(x)] - e^x[x^3 + \cos(x) - 1]}{(e^x - 3)^2}}$$

Q2

The Product Rule

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)\frac{d}{dx}\left[g(x)\right] + g(x)\frac{d}{dx}\left[f(x)\right]$$

2.) Let $f(x) = x^2 \sin(x)$. Find $f''(x)$

$$f(x) = x^2$$
$$g(x) = \sin(x)$$

$$f'(x) = x^2 \sin(x)$$
$$= x^2 \frac{d}{dx}\left[\sin(x)\right] + \sin(x)\frac{d}{dx}(x^2)$$
$$= x^2 \cos(x) + \sin(x)(2x)$$

$$f'(x) = x^2 \cos(x) + 2x \sin(x)$$

$$f''(x) = \frac{d}{dx}\left[x^2 \cos(x)\right] + \frac{d}{dx}\left[2x \sin(x)\right]$$

$$= \left[x^2 \frac{d}{dx}\left[\cos(x)\right] + \cos(x)\frac{d}{dx}(x^2)\right] + \left[2x\frac{d}{dx}\left[\sin(x)\right] + \sin(x)\frac{d}{dx}(2x)\right]$$

$$= \left[-x^2 \sin(x) + \cos(x)(2x)\right] + \left[2x \cos(x) + \sin(x)(2)\right]$$

$$\boxed{f''(x) = \left[-x^2 \sin(x) + 2x\cos(x)\right] + \left[2x\cos(x) + 2\sin(x)\right]}$$

The Product Rule

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)\frac{d}{dx}\left[g(x)\right] + g(x)\frac{d}{dx}\left[f(x)\right]$$

3) Let $f(x) = x^5 e^x \sec(x)$. Find $f'(x)$

$$f(x) = x^5$$

$$g(x) = e^x \sec(x)$$

x^5 instead

$$f'(x) = x^5 \frac{d}{dx}\left[e^x \sec(x)\right] + e^x \sec(x)\frac{d}{dx}\left(e^x\right)$$

$$= x^5\left[e^x \frac{d}{dx}\left(\sec(x)\right) + \sec(x)\frac{d}{dx}\left(e^x\right)\right] + e^x \sec(x)\frac{d}{dx}\left(e^x\right)$$

$$= x^5\left[e^x\left(\sec(x)\tan(x)\right) + \sec(x)\left(e^x\right)\right] + e^x \sec(x)\left(e^x\right)$$

$$= x^5\left[e^x \sec(x)\tan(x) + e^x \sec(x)\right] + e^x \sec(x)\left(e^x\right)$$

$$= x^5\left[e^x \sec(x)\left(\tan(x) + 1\right)\right] + e^x \sec(x)\left(e^x\right)$$

$$= x^5 e^x \sec(x)\left(\tan(x) + 1\right) + e^x \sec(x)\left(e^x\right)$$

$$\boxed{f'(x) = e^x \sec(x)\left[x^2\left(\tan(x) + 1\right) + e^x\right]}$$