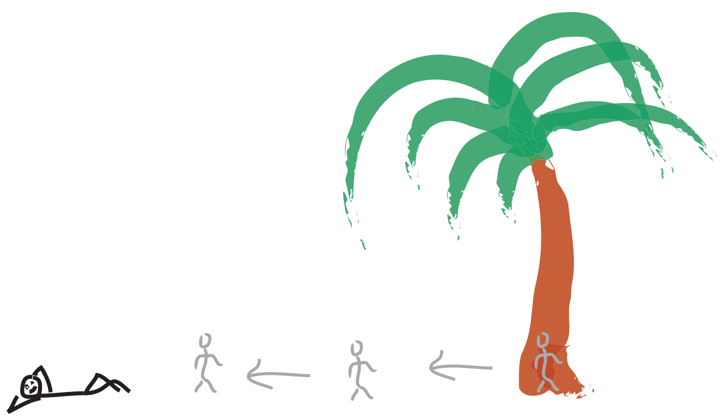
The Idea

We have done it! At this point, we have developed enough ideas to fulfill one of the fundamental promises of trigonometry, *to be able to measure things by simply looking at them*. That is, roughly speaking, we now have enough tools to be able to measure distances by looking at a couple points, measuring one or a couple angles and a side or a piece of a side. This is one of the essential goals of trigonometry, and we are ready to declare this mission accomplished.

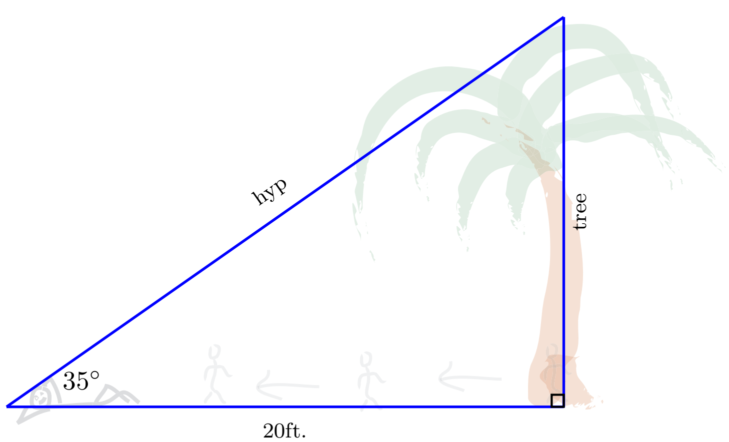
We introduce no new ideas here, rather, we simply show a few examples of how to use the ideas we have developed.

Look at a tree and measure it: Example A:

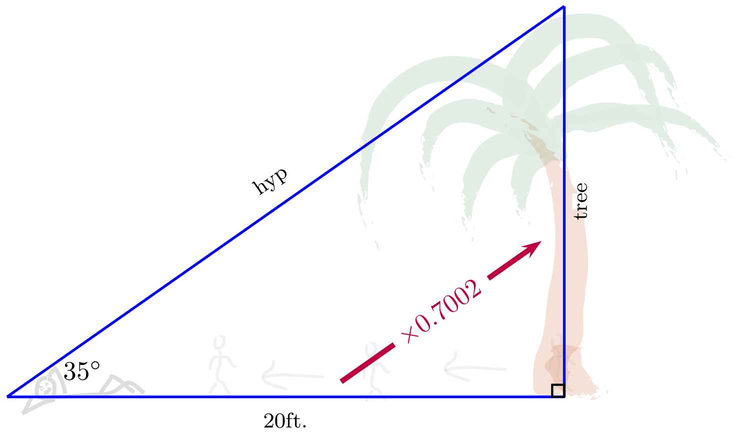
In this example, suppose we want to measure the height of a palm tree. Furthermore suppose we complete access to the area around the palm tree. We start at the trunk and walk away from the palm tree 20 ft. We lay down look up, and there it is, we determine the hight of the palm-tree.



To solve this, we need to do more than just look at the tree. We need to measure the angle from the ground to the top of the tree. Moreover, we make some extraordinary assumptions. We assume, for example that the ground is perfectly flat and the tip of the palm-tree lies exactly over the trunk, etc, etc. ... We then turn the unknown quantity, the hight of the palm-tree, into the side of a right triangle. Suppose upon laying down and looking at the tree we measure and angle of 35°. Then we note:



We then invoke our ratio functions, in this case, we *know* the side adjacent to 35° and *want* the side opposite. We recall the tangent function is taylor made to describe such ratio, opp/adj. We make note of it on the diagram to obtain:



first, we note

$\tan(35°) \approx 0.7002$

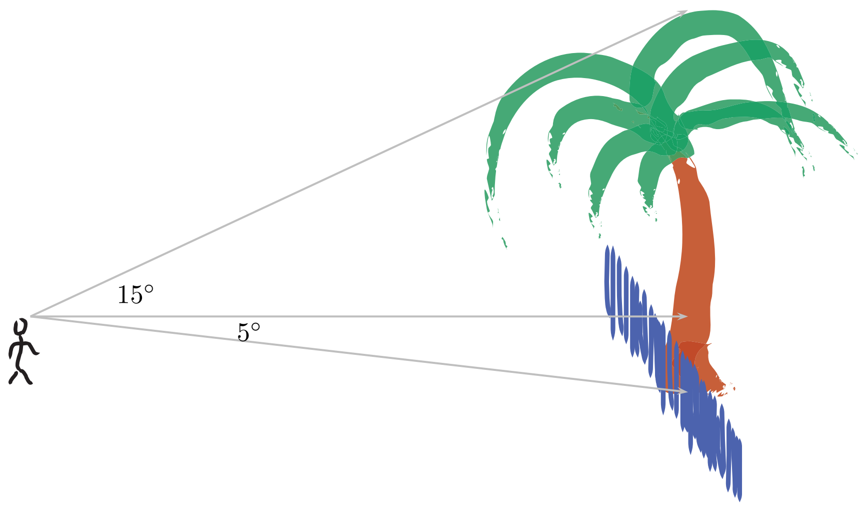
then,

$tree \approx (20ft.)(0.7002)$

$\approx 14ft.$

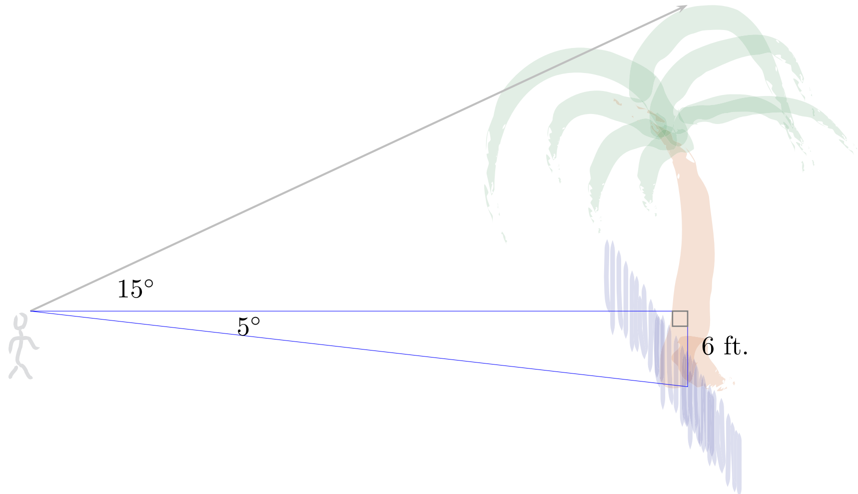
Look at a tree and measure it: Example B:

Suppose we change things just a little. This time we don't even come near the trunk of the palm-tree. This time we simply glance from far away measure the angle and then determine the height of the palm-tree. This time we make use of a fence directly next to the tree. It just so happens that we know this type of fence is exactly 6 ft tall and at exactly eye level. We measure the following angles, from that we can determine the heigh of the tree.

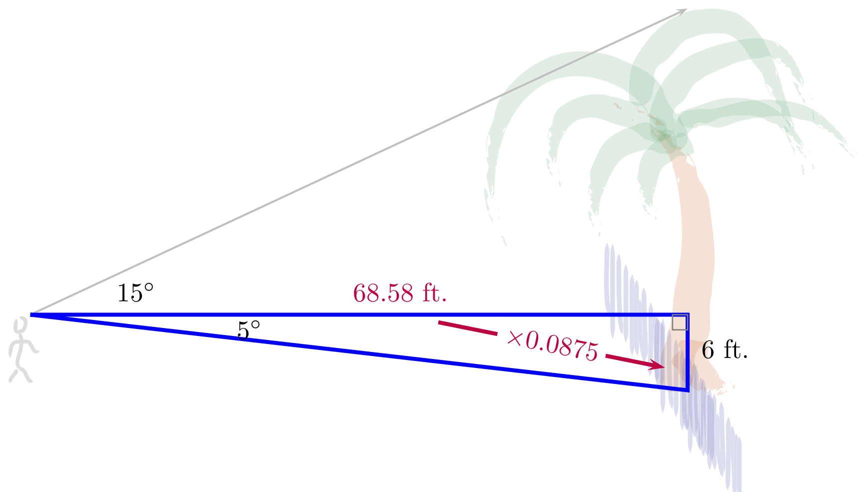


Solutions:

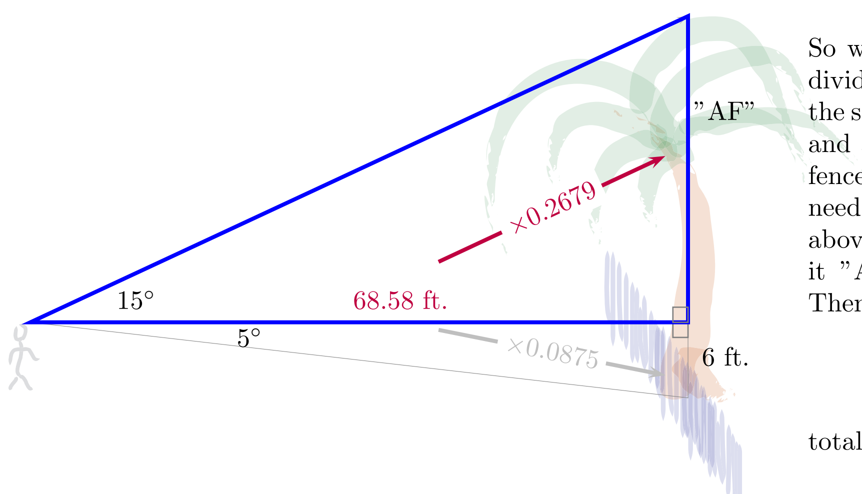
We will solve this by looking at the diagram, assuming we have two right triangles, and solving these two triangles. Let us look at the bottom triangles first.



We use the tangent function to determine the opp/adj ratio, then determine the adj side. In this step we effectively figure out how far away from the palm-tree we are standing. Note $\tan(5^\circ) \approx 0.0875$



We now turn our attention to the other right triangle. We find have the adjacent side, want the opposite side, thus we use the tangent function $\tan(15^\circ) \approx 0.2679$ Thus...



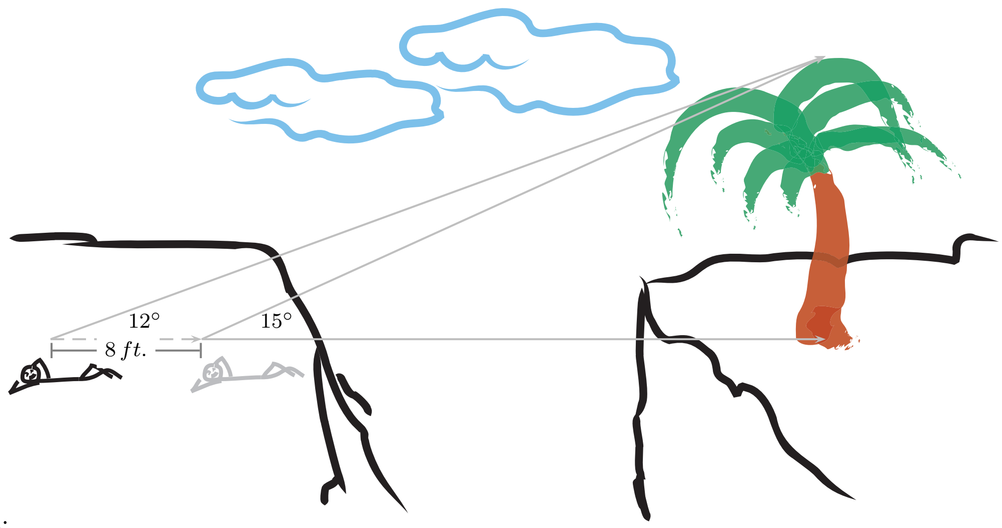
So we have the palm-tree divided into two sections the section below the fence and above the fence. The fence is 6ft. tall so we need only find the section above the fence, let us call it "AF" for above fence. Then...

$$\begin{aligned} \text{AF} &\approx (68.58)(0.2679) \\ &\approx 18.38 ft. \end{aligned}$$

total tree $\approx 24.38\ ft.$

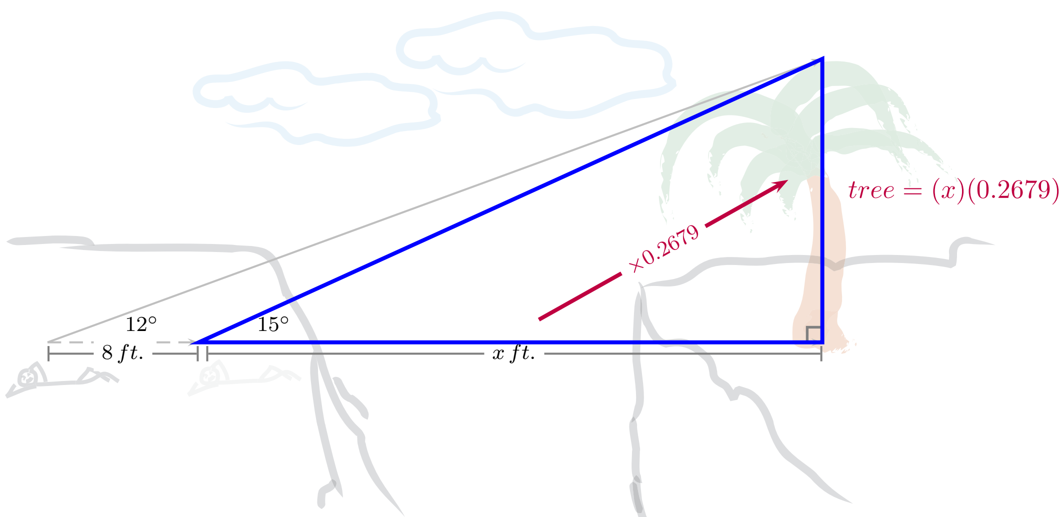
Look at a tree and measure it: Example C:

That as a nice challenge for about 2 minutes. We now make it a bit more interesting. What if there was not fence and we could not come close to the palm-tree? It turns out we can again overcome this challenge. We simply measure an angle from far away, as illustrated below, then we move away some fixed distance, such as 8 ft., we measure the angle again, and voila! done! we can then tell the hight of the tree.

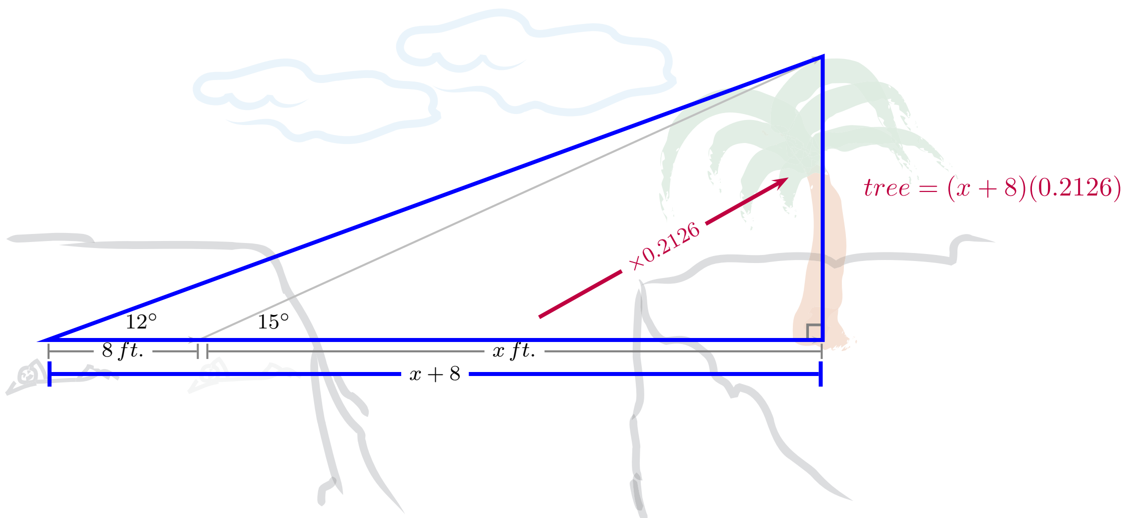


solutions:

To solve this we will look for two right triangles. Note we denote the hight of the palm-tree as *tree*, and the initial distance to the tree as x. Then we analyze the inner triangle.



Then the outer triangle.


$tree = (x + 8)(0.2126)$
$\times 0.2126$
12°
15°
$8\,ft.$
$x\,ft.$
$x + 8$

We now have two expression for "tree". On the one hand

$$tree = (x)(0.2679)$$

while on the other hand,

$$tree = (x + 8)(0.2126)$$

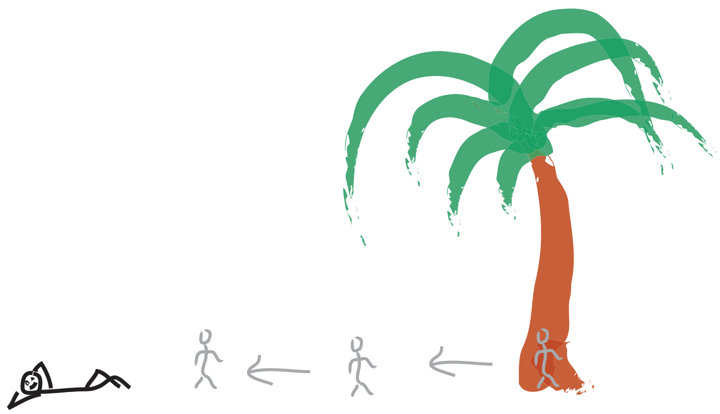
. We now set these equal to each other and solve for x, the first distance to the tree, then ultimately solve for the height of the tree.

$$\begin{aligned}
(x + 8)(0.2126) &= (x)(0.2679) \\
x(0.2126) + (8)(0.2126) &= x(0.2679) \\
(8)(0.2126) &= x(0.2679) - x(0.2126) \\
(8)(0.2126) &= x(0.2679 - 0.2126) \\
1.7005 &= x(0.0554) \\
\frac{1.7005}{0.0554} &= x \\
x &\approx 30.6949\, ft.
\end{aligned}$$

and finally,

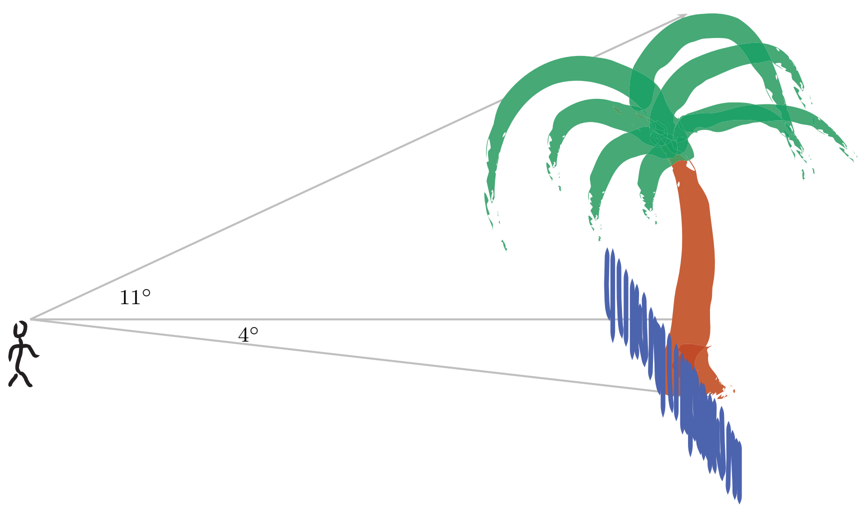
$$tree = (x)(0.2679) = (30.6949)(0.2679) \approx 8.2247\, ft.$$

1. Assume upon walking 20 ft. you lay down and measure the angle to the top of the tree to be 35°. Find the height of the tree.

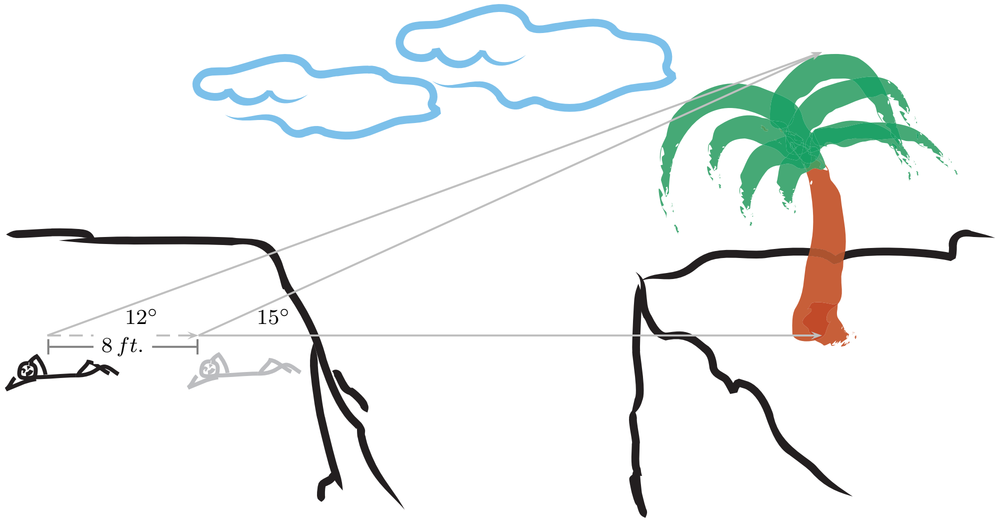


(You may also assume the line of sight, bottom of the tree and top of the tree form a right triangle.)

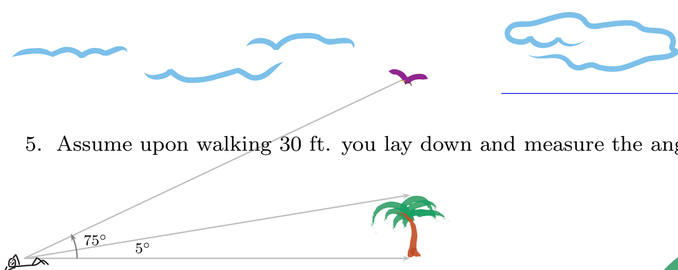
2. Suppose the fence as well as eye-level are 6 ft. from the ground. Suppose the following angles are given, then find the height of the palm-tree.



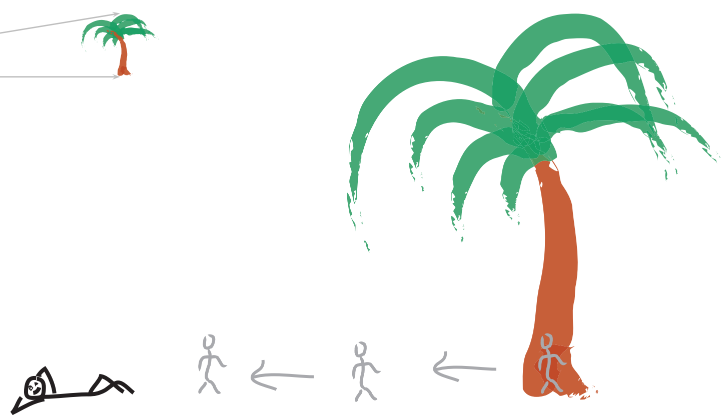
3. Solve for the height of the tree. Use the diagram below, and make the usual assumptions.



4. Suppose you know the height of your neighbor's palm treed is 17 ft. Suppose you spot a bird flying directly above the palm-tree. The following angles are given. Find the height at which the bird is flying.

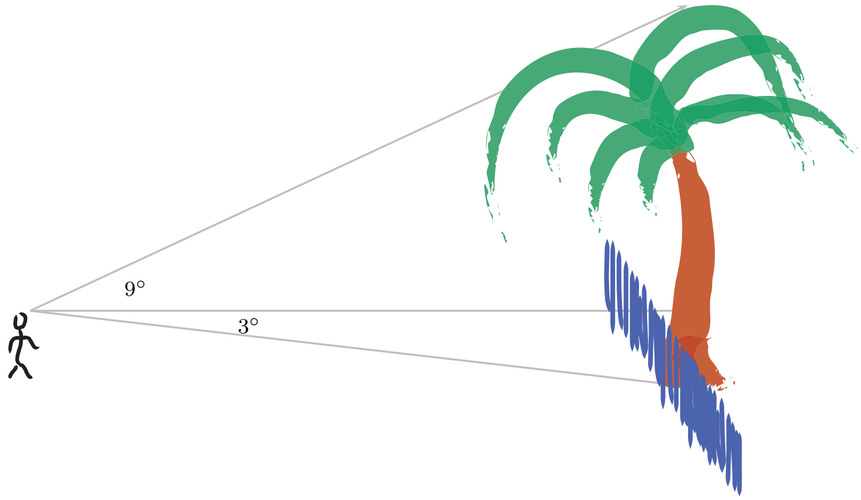


5. Assume upon walking 30 ft. you lay down and measure the angle to the top of the tree to be 19°. Find the height of the tree.

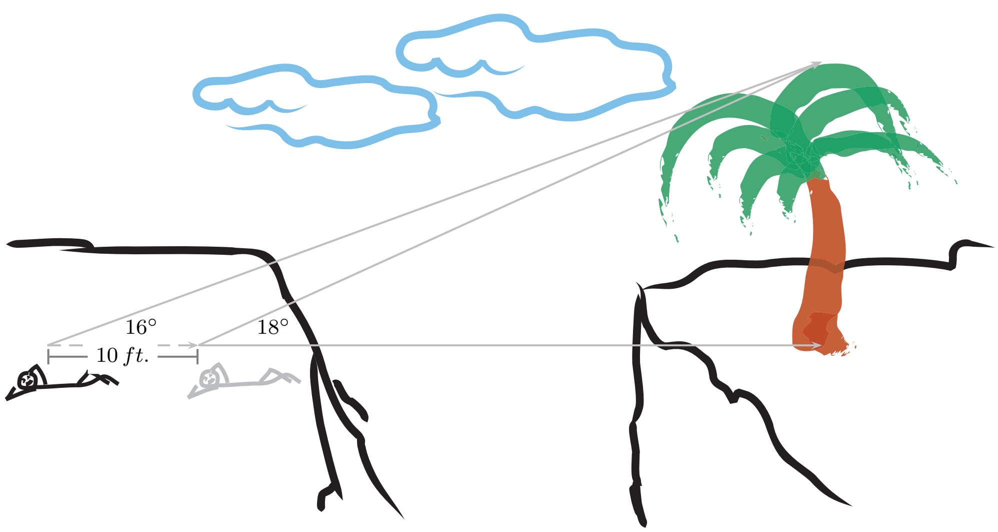


(You may also assume the line of sight, bottom of the tree and top of the tree form a right triangle.)

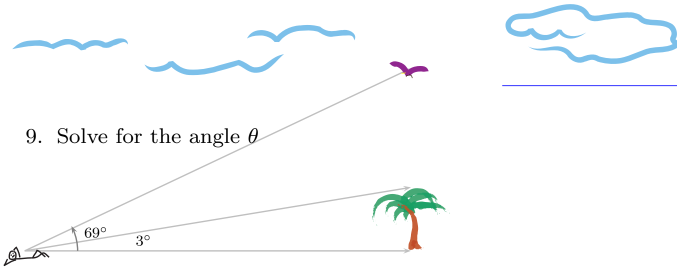
6. Suppose the fence as well as eye-level are 5.5 ft. from the ground. Suppose the following angles are given, then find the height of the palm-tree.



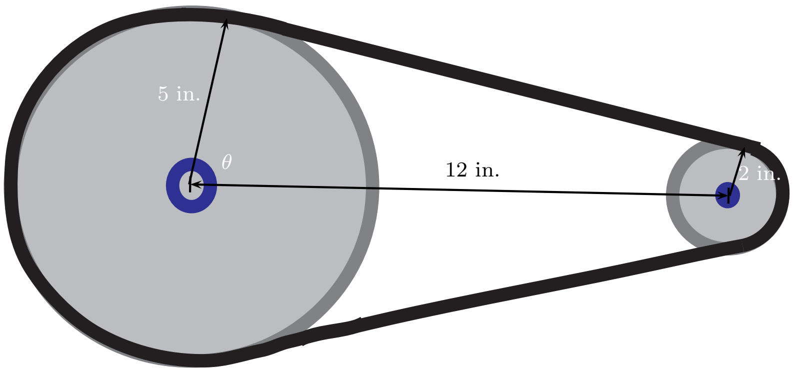
7. Solve for the height of the tree. Use the diagram below, and make the usual assumptions.



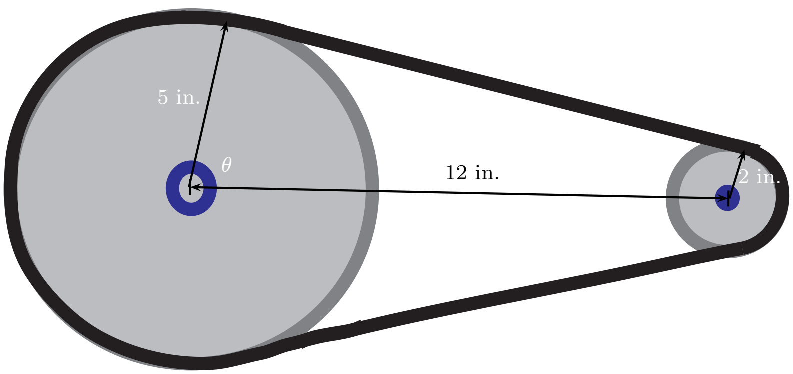
8. Suppose you know the height of your neighbor's palm treed is 20 ft. Suppose you spot a bird flying directly above the palm-tree. The following angles are given. Find the height at which the bird is flying.

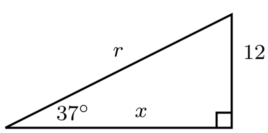


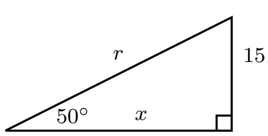
9. Solve for the angle θ

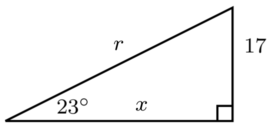


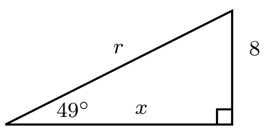
10. How long is the belt?



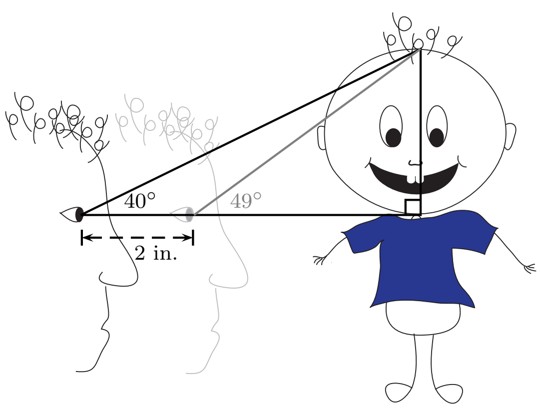
11.


12.


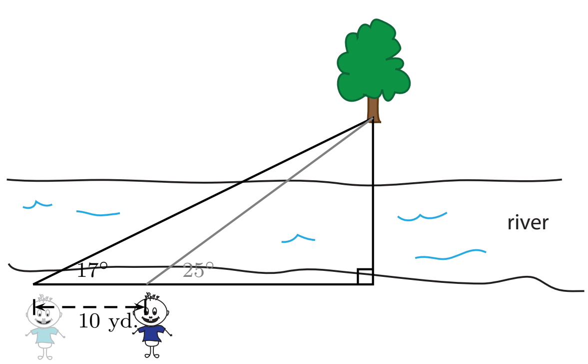
13. 

14. 

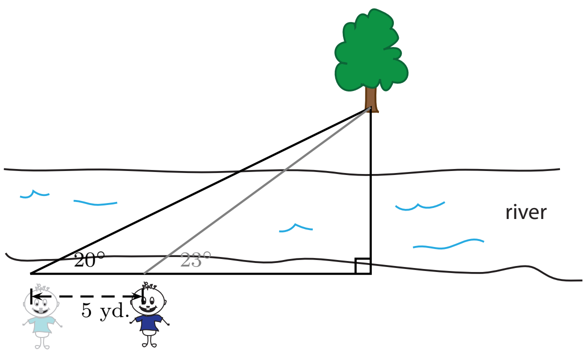
15. Just by looking at him, can you figure out how big Diego's head is? See picture for angles & measurements.



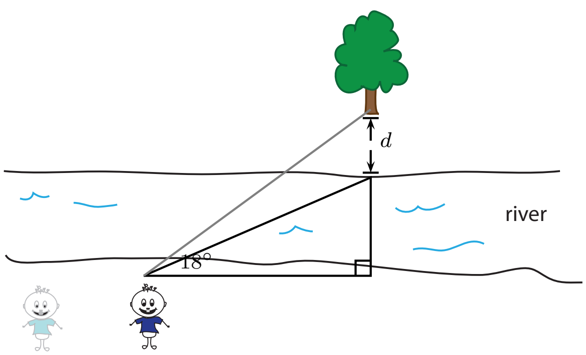
16. Determine the distance from the tree trunk to the opposite side of the river as shown.



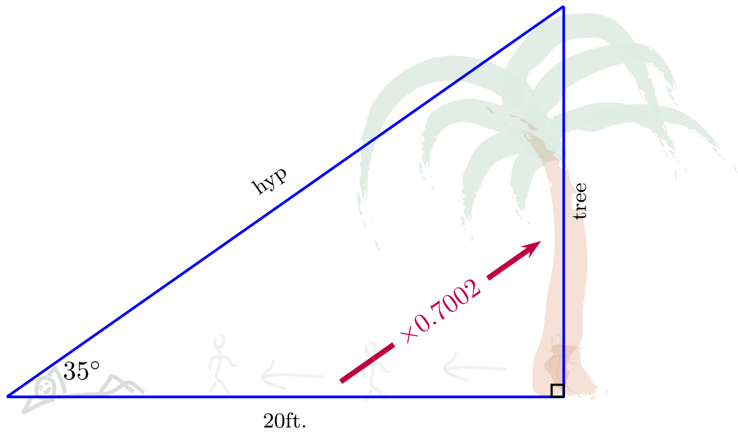
17. (a) Determine the distance from the tree trunk to the opposite side of the river as shown.



(b) (con't from previous exercise) How far away is the tree from the river? Use the diagram below.



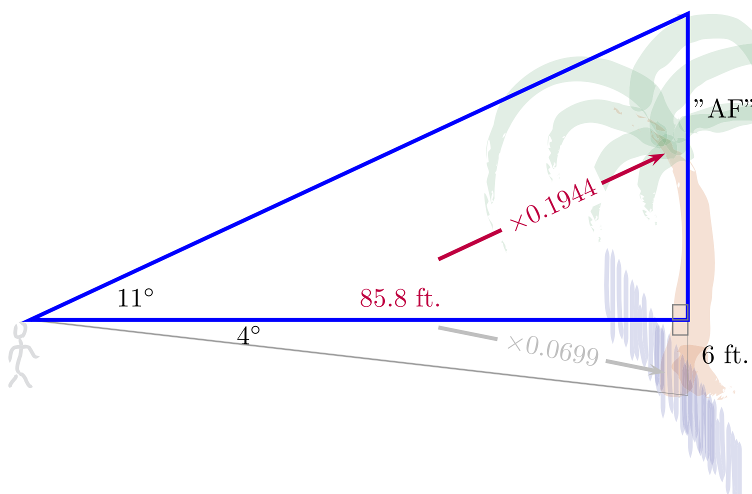
1. Assume upon walking 20 ft. you lay down and measure the angle to the top of the tree to be 35°. Find the height of the tree.



first, we note
$\tan(35°) \approx 0.7002$
then,
$$tree \approx (20ft.)(0.7002)$$
$$\approx 14ft.$$

We then invoke our ratio functions, in this case, we *know* the side adjacent to 35° and *want* the side opposite. We recall the tangent function is taylor made to describe such ratio, opp/adj. We make note of it on the diagram to obtain:

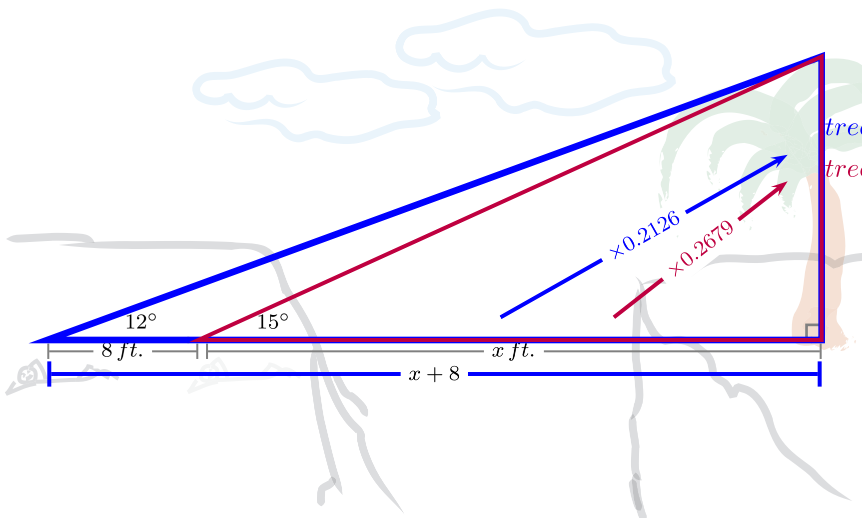
2. Suppose the fence as well as eye-level are 6 ft. from the ground. Suppose the following angles are given, then find the height of the palm-tree.



First we find $\tan(4°) \approx 0.0699$ and we use the heigh of the fence 6 ft. to find the distance to the tree. Then we use the $\tan(11°) \approx 0.1944$ ration to go from distance to the tree to distance above the fence "AF"... thus..

$$\text{AF} \approx (85.8\,ft.)(0.1944)$$
$$\approx 16.68ft.$$
total tree $\approx 22.68\,ft.$

3. Solve for the height of the tree. Use the diagram below, and make the usual assumptions.



$tree = (x+8)(0.2126)$

$tree = (x)(0.2679)$

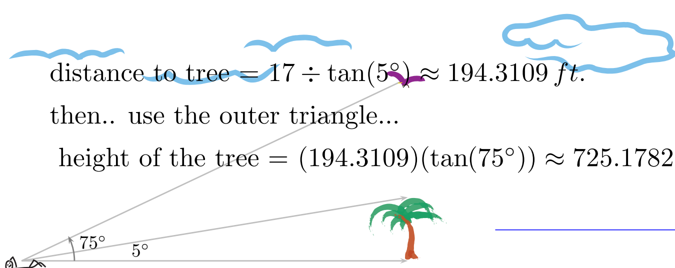
then set equal and solve $(x)(0.2679) = (x+8)(0.2126)$ $x \approx 30.6949\,ft.$ and $tree \approx 8.2247\,ft.$

4. Suppose you know the height of your neighbor's palm treed is 17 ft. Suppose you spot a bird flying directly above the palm-tree. The following angles are given. Find the height at which the bird is flying.

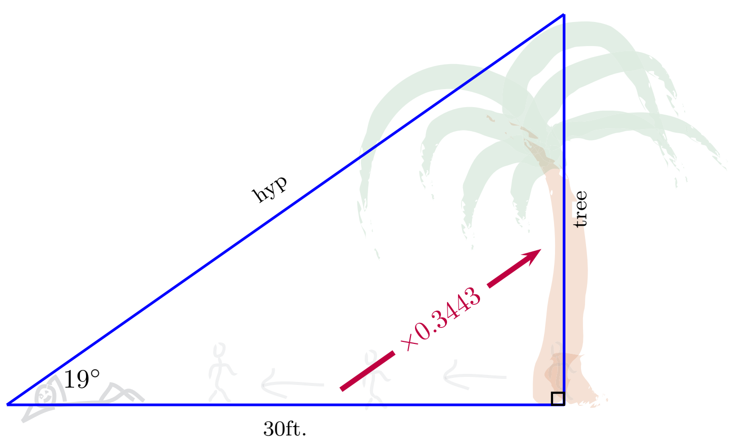
distance to tree $= 17 \div \tan(5°) \approx 194.3109\,ft.$

then.. use the outer triangle...

height of the tree $= (194.3109)(\tan(75°)) \approx 725.1782\,ft.$



5. Assume upon walking 30 ft. you lay down and measure the angle to the top of the tree to be 19°. Find the height of the tree.



first, we note

$\tan(19°) \approx 0.3443$

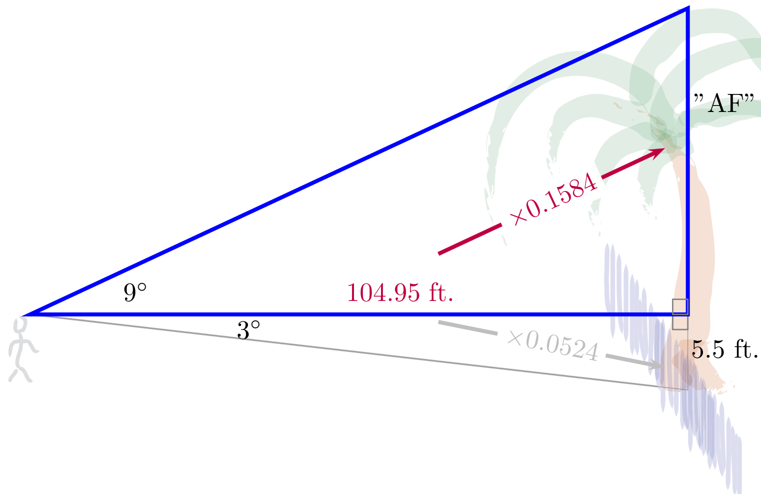
then,

$tree \approx (30\,ft.)(0.3443)$

$\approx 10.33\,ft.$

We then invoke our ratio functions, in this case, we *know* the side adjacent to 19° and *want* the side opposite. We recall the tangent function is taylor made to describe such ratio, opp/adj. We make note of it on the diagram to obtain:

6. Suppose the fence as well as eye-level are 5.5 ft. from the ground. Suppose the following angles are given, then find the height of the palm-tree.

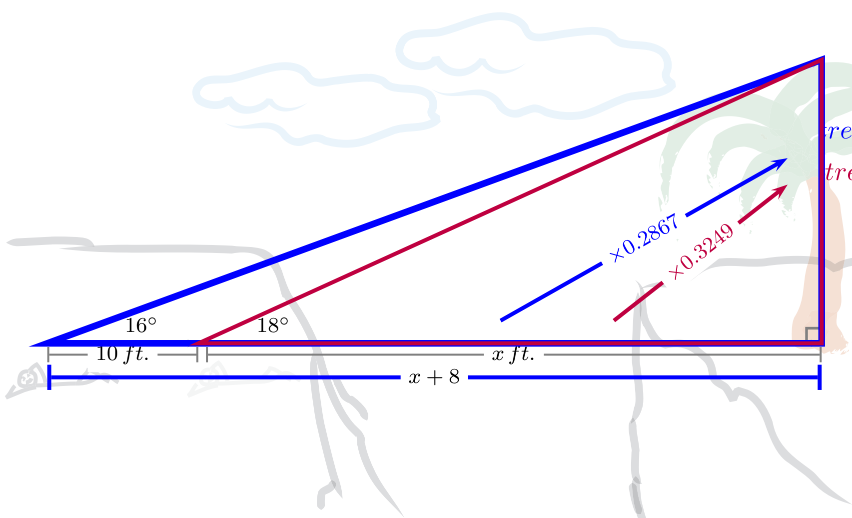


First we find $\tan(3°) \approx 0.0524$ and we use the heigh of the fence 5.5 ft. to find the distance to the tree. Then we use the $\tan(9°) \approx 0.1584$ ration to go from distance to the tree to distance above the fence "AF"... thus..

$$\text{AF} \approx (104.95\,ft.)(0.1584)$$
$$\approx 16.62\,ft.$$

total tree $\approx 22.12\,ft.$

7. Solve for the height of the tree. Use the diagram below, and make the usual assumptions.



$tree = (x + 10)(0.2867)$

$tree = (x)(0.3249)$

then set equal and solve $(x)(0.3249) = (x + 10)(0.2867)$ $x \approx 75.0654\,ft.$ and $tree \approx 24.3902\,ft.$

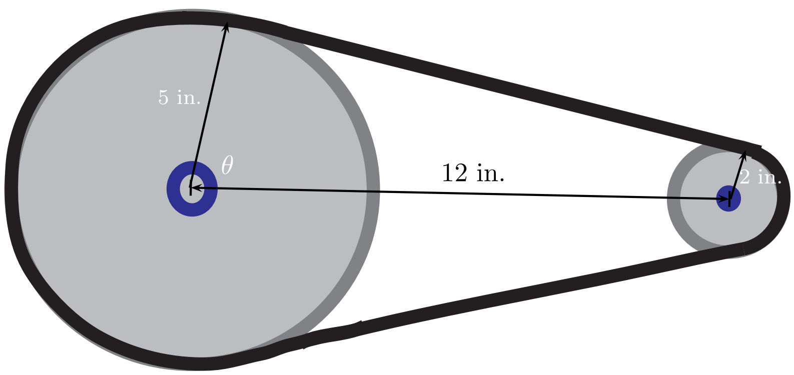
8. Suppose you know the height of your neighbor's palm treed is 20 ft. Suppose you spot a bird flying directly above the palm-tree. The following angles are given. Find the height at which the bird is flying.

distance to tree $= 17 \div \tan(3°) \approx 381.6227\,ft.$

then.. use the outer triangle...

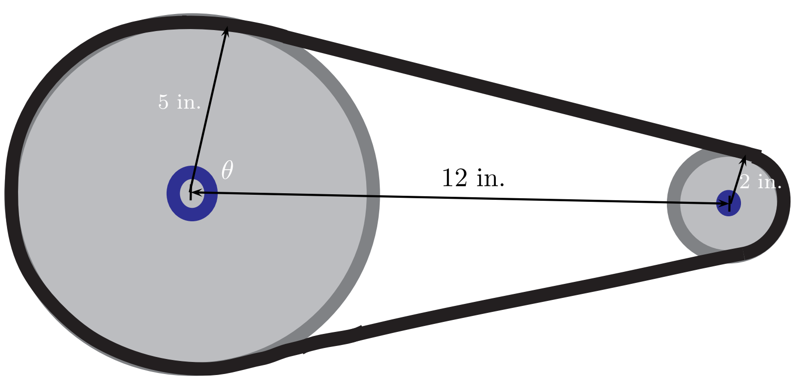
height of the tree $= (381.6227)(\tan(69°)) \approx 994.1611\,ft.$

9. Solve for the angle θ



> **Solution:**
>
> to solve for θ we could look at it as one of the angles on a right triangle, by drawing a line parallel to the top portion of the belt between the pulleys.
>
> the difference in radii is 3, this is one of the sides, the hypothenuse is 12, thus
>
> $$\cos\theta = \frac{3}{12}$$
>
> , from here we get
>
> $$\theta = \cos^{-1}(3/12) \approx 75.5°$$

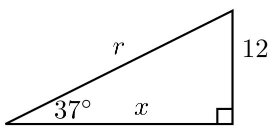
10. How long is the belt?



Solution:

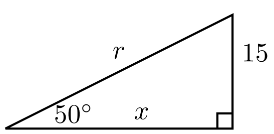
once the angle is found.. you should proceed to find the belt one piece at a time.. to find the portions around the pulleys you may need that $s = \pi\theta$ (in radians) or $s = \frac{d}{360} \cdot 2\pi r$ in degrees.. where s is the arc-length...

---

11. 

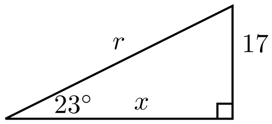
Solution: $\tan 37° = \frac{12}{x}$ Therefore..
$x = \frac{12}{\tan 37°} \approx 15.92$
also..
$\sin 37° = \frac{12}{r}$ Therefore.. $r = \frac{12}{\sin 37°} \approx 19.94$

---

12. 

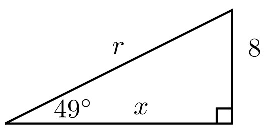
Solution: $\tan 50° = \frac{15}{x}$ Therefore..
$x = \frac{15}{\tan 50°} \approx 12.59$
also..
$\sin 50° = \frac{15}{r}$ Therefore.. $r = \frac{15}{\sin 50°} \approx 19.58$

---

13. 

Solution: $\tan 23° = \frac{17}{x}$ Therefore..
$x = \frac{17}{\tan 23°} \approx 40.05$
also..
$\sin 23° = \frac{17}{r}$ Therefore.. $r = \frac{17}{\sin 23°} \approx 43.51$

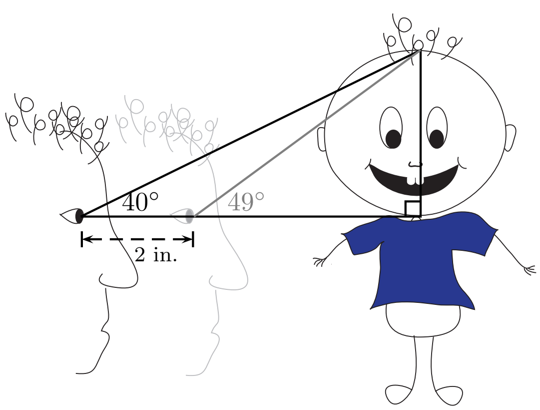
---

14. 

Solution: $\tan 49° = \frac{8}{x}$ Therefore..
$x = \frac{8}{\tan 49°} \approx 6.95$
also..
$\sin 49° = \frac{8}{r}$ Therefore.. $r = \frac{8}{\sin 49°} \approx 10.60$

---

15. Just by looking at him, can you figure out how big Diego's head is? See picture for angles & measurements.



Solution: We first call the size of his head, h and the lower side of the inner triangle, k. Then, we look at the inner triangle to conclude

$$\tan 49^\circ = \frac{h}{k}$$

, this means that $k = \frac{h}{\tan 49^\circ}$. Similarly, we look at the outer triangle to conclude

$$\tan 40^\circ = \frac{h}{2+k}$$

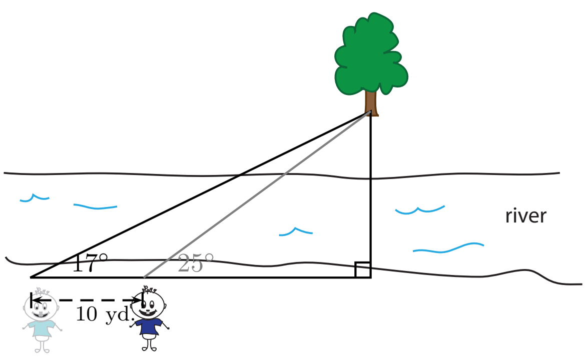
this means $2 + k = \frac{h}{\tan 40^\circ}$ Or $k = -2 + \frac{h}{\tan 40^\circ}$. Having solved for k in two different ways, we set the two equal, then solve for h, the sought quantity.

$$\frac{h}{\tan 49^\circ} = -2 + \frac{h}{\tan 40^\circ}$$

then... we move all h terms to one side factor the h and solve......

$$h \approx 6.202 \text{ in.}$$

16. Determine the distance from the tree trunk to the opposite side of the river as shown.



Solution: We first call the sought distance, h and the lower side of the inner triangle, k. Then, we look at the inner triangle to conclude

$$\tan 25^\circ = \frac{h}{k}$$

, this means that $k = \frac{h}{\tan 25^\circ}$. Similarly, we look at the outer triangle to conclude

$$\tan 17^\circ = \frac{h}{10+k}$$

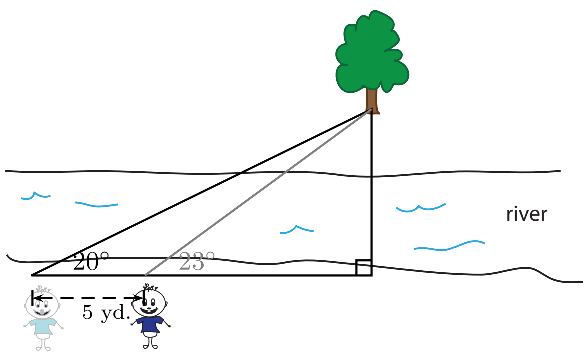
this means $10 + k = \frac{h}{\tan 17^\circ}$ Or $k = -10 + \frac{h}{\tan 17^\circ}$. Having solved for k in two different ways, we set the two equal, then solve for h, the sought quantity.

$$\frac{h}{\tan 25^\circ} = -10 + \frac{h}{\tan 17^\circ}$$

then... we move all h terms to one side factor the h and solve......

$$h \approx 8.878 \text{ yd.}$$

17. (a) Determine the distance from the tree trunk to the opposite side of the river as shown.



Solution: We first call the sought distance, h and the lower side of the inner triangle, k. Then, we look at the inner triangle to conclude

$$\tan 23° = \frac{h}{k}$$

, this means that $k = \frac{h}{\tan 23°}$. Similarly, we look at the outer triangle to conclude

$$\tan 20° = \frac{h}{5+k}$$

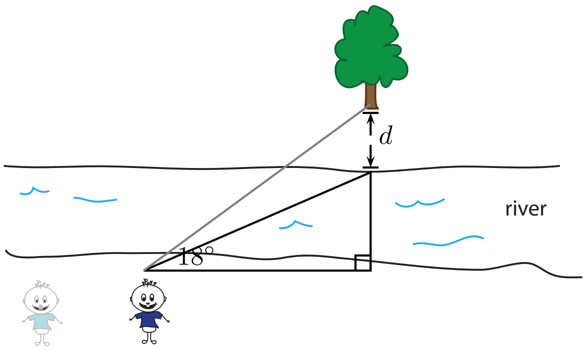
this means $5 + k = \frac{h}{\tan 20°}$ Or $k = -5 + \frac{h}{\tan 20°}$. Having solved for k in two different ways, we set the two equal, then solve for h, the sought quantity.

$$\frac{h}{\tan 23°} = -5 + \frac{h}{\tan 20°}$$

then... we move all h terms to one side factor the h and solve......

$$h \approx 12.767 \text{ yd.}$$

(b) (con't from previous exercise) How far away is the tree from the river? Use the diagram below.



Solution: from the above we can determine h, the distance from the trunk to the opposite side of the river.

$$h \approx 12.767 \text{ yd.}$$

If we can determine the width of the river we can subtract the quantities and obtain the distance, d. To figure out the width of the river, we can calculate k, the lower side of the triangle, from part a)...

$$\tan 23° = \frac{h}{k}$$

thus... $k = \frac{h}{\tan 23°} \approx \frac{12.767}{\tan 23°} \approx 30.111$
Then, we can use $\tan 18° \approx \frac{'river'}{k}$, so..

$$\begin{aligned} \tan 18° &= \frac{'river'}{k} \\ river &= k * \tan 18° \\ river &\approx 30.111 * 0.325 \\ river &\approx 9.786 \end{aligned}$$

..etc...etc.. solve for 'river' and subtract from h to get

$$"trunk - to - river" = h - river \approx 12.767 - 9.786 \approx 2.981$$