1 & 2

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1) Let f(x)= $x^2\sec(2x)$. Find f'(x).
2) Let f(x) = tan(xsin(x)). Find f'(x).
3) Use implicit differentiation to find y' when $e^y + y^2 = xlnx$
4) Find the derivative of $f(x) = x^2 tan^{-1}(e^x)$

The Product Rule

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)\frac{d}{dx}\left[g(x)\right] + g(x)\frac{d}{dx}\left[f(x)\right]$$

1) $f(x) = x^2\sec(2x)$. Find $f'(x)$

$= x^2\frac{d}{dx}\left[\sec(2x)\right] + \sec(2x)\frac{d}{dx}(x^2)$

Chain Rule

$u = 2x$

$f(u) = \sec(u) = y$

$\frac{dy}{du}\cdot\frac{du}{dx}$

$= x^2\frac{dy}{du}\left[\sec(u)\right]\frac{du}{dx}(u) + \sec(2x)(2x)$

$= x^2\left[\sec(u)\tan(u)\right]\frac{d}{dx}(2x) + 2x\sec(2x)$

$= x^2\sec(2x)\tan(2x)(2) + 2x\sec(2x)$

$$f'(x) = 2x\sec(2x)\left[x\tan(2x) + 1\right]$$

2) $f(x) = \tan(x\sin(x))$. Find $f'(x)$

Chain Rule

$u = x\sin(x)$

$f(u) = \tan(u) = y$

$\frac{dy}{du}\quad\frac{du}{dx}$

$= \frac{dy}{du}\left[\tan(u)\right]\frac{du}{dx}(u)$

$= \sec^2(u)\frac{d}{dx}\left[x\sin(x)\right]$

$= \sec^2(x\sin(x))\left[x\frac{d}{dx}(\sin(x)) + \sin(x)\frac{d}{dx}(x)\right]$

$= \sec^2(x\sin(x))\left[x\cos(x) + \sin(x)(1)\right]$

$$f'(x) = \sec^2(x\sin(x))\left[x\cos(x) + \sin(x)\right]$$

3 & 4

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1) Let f(x)= $x^2\sec(2x)$. Find f'(x).
2) Let f(x) = tan(xsin(x)). Find f'(x).
3) Use implicit differentiation to find y' when $e^y + y^2 = xlnx$
4) Find the derivative of $f(x) = x^2 tan^{-1}(e^x)$

The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

3) $e^y + y^2 = x\ln x$, Find y'

$e^y + y^2 = x\ln x$

$y' = \frac{d}{dx}(e^y + y^2) = \frac{d}{dx}(x\ln x)$

$\frac{d}{dy}(e^y)\frac{dy}{dx} + \frac{d}{dy}(y^2)\frac{dy}{dx} = x\frac{d}{dx}(\ln(x)) + \ln x\frac{d}{dx}(x)$

$e^y\frac{dy}{dx} + 2y\frac{dy}{dx} = x\left(\frac{1}{x}\right)\frac{d}{dx}(x) + \ln x(1)$

$e^y y' + 2yy' = \frac{x}{x}(1) + \ln x$

$y'(e^y + 2y) = 1 + \ln x$

$\frac{y'(e^y+2y)}{(e^y+2y)} = \frac{1+\ln x}{(e^y+2y)} = y' = \frac{1+\ln x}{e^y+2y}$

4) $f(x) = x^2\tan^{-1}(e^x)$, Find $f'(x)$

$= x^2\frac{d}{dx}[\tan^{-1}(e^x)] + \tan^{-1}(e^x)\frac{d}{dx}(x^2)$

Chain rule

$u = e^x$

$f(x) = \tan^{-1}(u) = y$

$\frac{dy}{du}$ $\frac{du}{dx}$

$= x^2\frac{dy}{du}[\tan^{-1}(u)]\frac{du}{dx}(u) + \tan^{-1}(e^x)(2x)$

$= x^2\left(\frac{1}{1+u^2}\right)\frac{d}{dx}(e^x) + 2x\tan^{-1}(e^x)$

$= x^2\left(\frac{1}{1+(e^x)^2}\right)(e^x) + 2x\tan^{-1}(e^x)$

$$f'(x) = \frac{x^2e^x}{1+e^{2x}} + 2x\tan^{-1}(e^x)$$