Intro to Polar Coordinates

Main Idea

The cartesian coordinates system uses x and y coordinates to describe each point on a plane. For example, once a point $(0,0)$ is designated as 'the orgigin' all other points on the plane have coordinates that identify them. For example, $(3,4)$ refers to the point 3 to the right and 4 up from the origin. Polar coordinates serve a similar function. They are also ordered pairs, (r, θ), and each ordered pair describes a unique point on the $\mathbf{R}^2$ plane. The principal difference is that rather than describing a point by how much to go left or right or up or down, polar coordinates describe the point by describing in which direction the point is located, and how far from the origin. "4 miles 3 O'Clock' might be a statement of the type most resembling polar coordinates, and most familiar from everyday language. The angle, θ, is measured, as usual, from the positive side of the x-axis, clockwise for negative angles, and counter-clockwise for positive angles. While r represents the distance from the origin in the direction θ, OR backwards for negatives values of r^1.

Examples: Plotting Points

The following is just a random sequence of point in polar coordinates. Practice by plotting and labeling each of the points:

$$p_1 = (2, 30°), \quad p_2 = (-4, 30°), \quad p_3 = (-4, -30°), \quad p_4 = (4, -210°), \quad p_5 = (-4, 135°)$$

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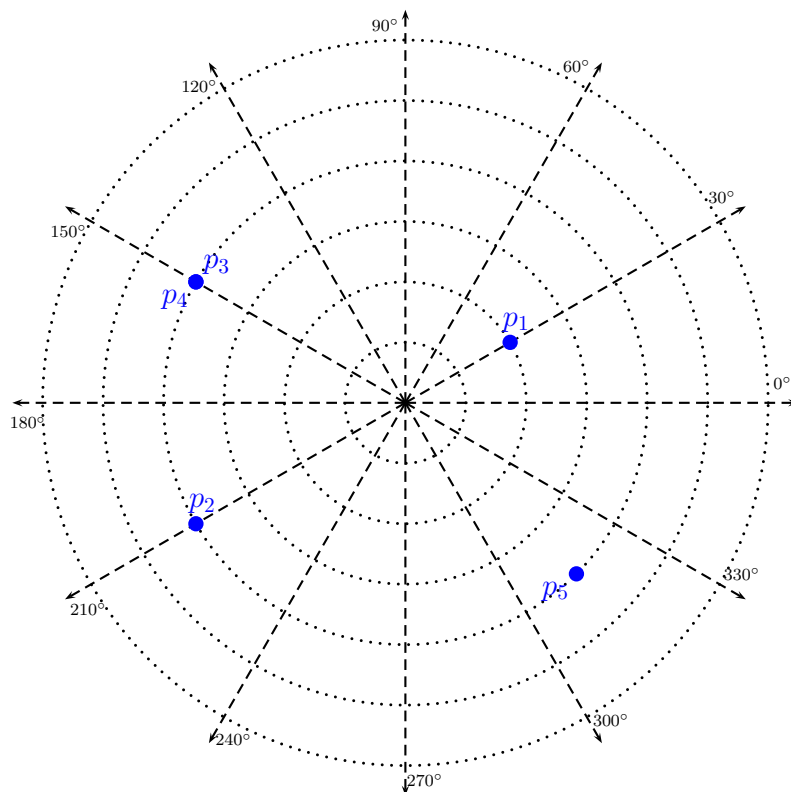
[1]Note: in polar coordinates, a point may have many (infinite many) different coordinates, i.e. $(4, 0°)$ and $(4, 360°)$ take on the same point in polar coordinates. When translating from cartesian to polar there will always be infinite many choices

Converting: Polar Coordinates to Cartesian Coordinates

The translating 'dictionary' to go from polar-to-cartesian is as follows:

From the defining features of the polar and cartesian cordinates, we conclude that generally:
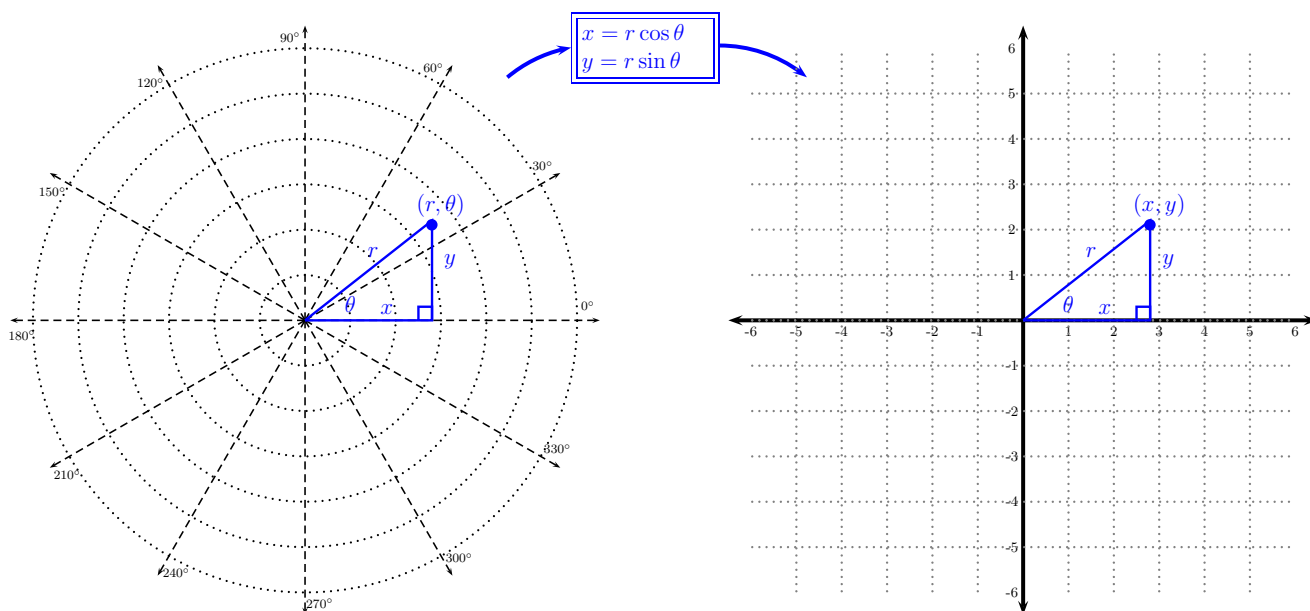
$$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$$
$$r\cos\theta = x$$

(.. then..)

and similarly

$$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$$
$$r\sin\theta = y$$

(.. then..)

Therefore, we have the translating 'dictionary' to translate from polar-to-cartesian

$$x = r\cos\theta$$
$$y = r\sin\theta$$

Converting: Cartesian Coordinates to Polar Coordinates

The translating 'dictionary' to go from cartesian-to-polar is as follows:

From the defining features of the polar and cartesian cordinates, we conclude that generally:

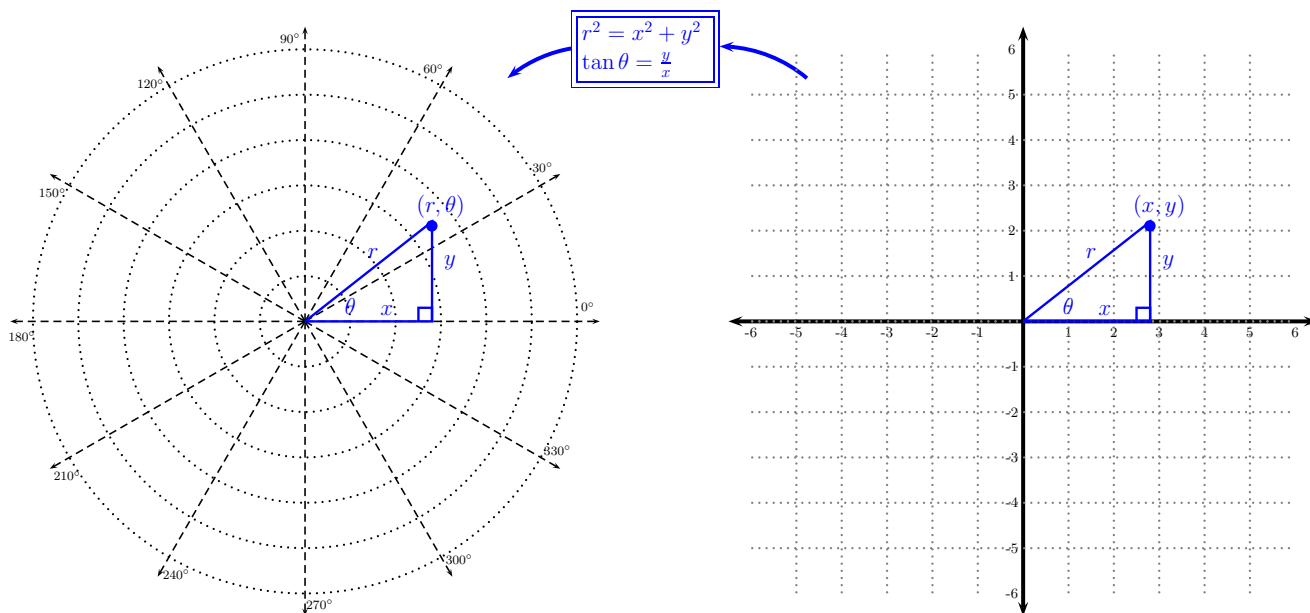$$x^2 + y^2 = r^2$$

(by pythagoras)

It is important to note that each single point may be referenced by many [infinite] different polar coordinates. For example, coordinates $(3, 180°)$, $(3, -180°)$ and $(-3, 0°)$ all refer to the same point on the polar plane. Therefore, translating from cartesian to polar coordinates can get ambiguous, since for each cartesian coordinate there are infinite many possible polar coordinates. The equation above shows already two choices for r, namely, $r = \sqrt{x^2 + y^2}$ or $r = -\sqrt{x^2 + y^2}$. The equation below shows that there are generally, infinite more possibilities for θ
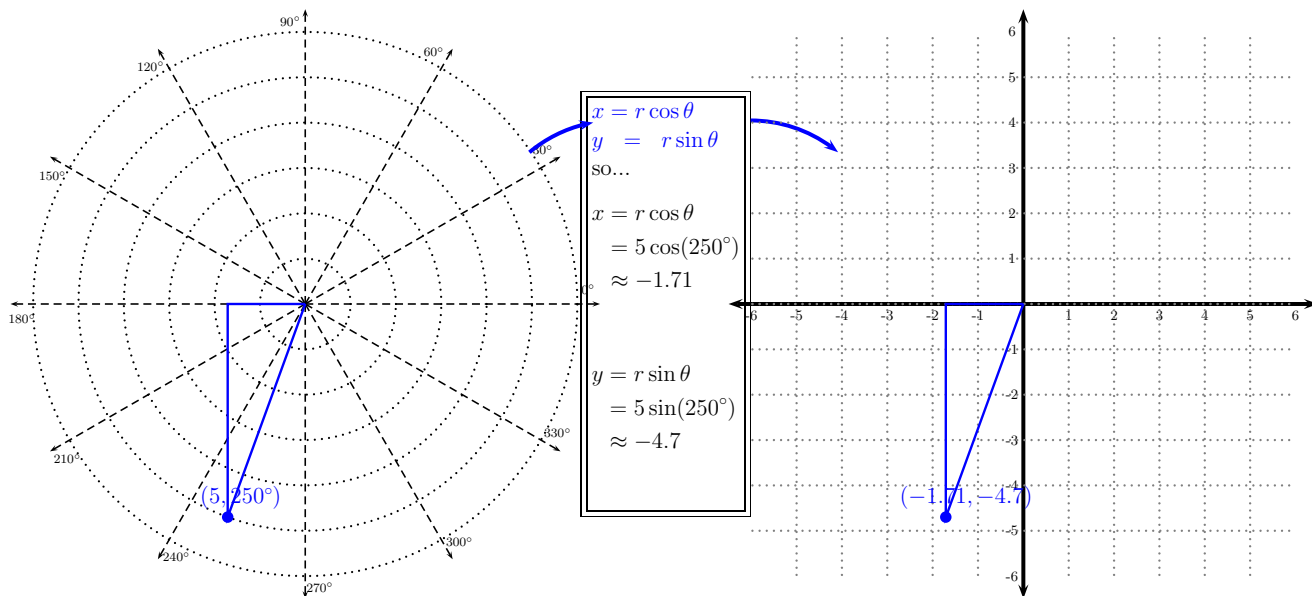
$$\tan \theta = \frac{opp}{adj} = \frac{y}{x}$$

Note this equation, $\tan \theta = y/x$ generally yields infinite solutions, some appropriate some not so much, depending on the above choice of r. We no summarize, and follow with some examples.

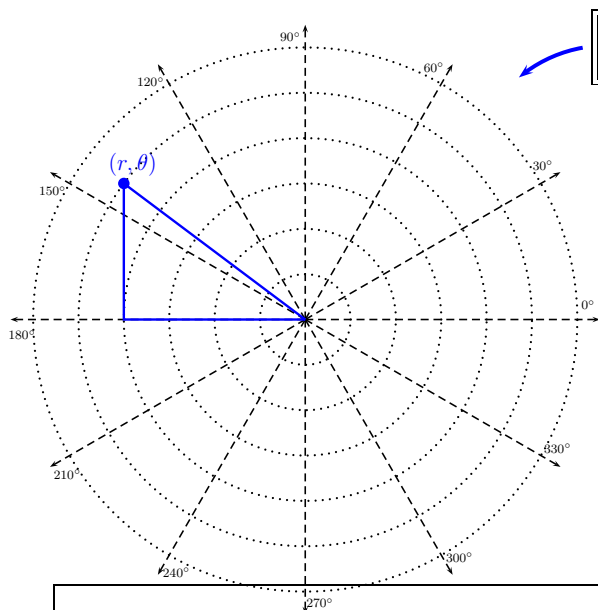Therefore, we have the translating 'dictionary' to translate from cartesian-to-polar

EXAMPLEs: Polar Coordinates $< -- >$ Cartesian Coordinates

Convert the polar coordinates, $(5, 250°)$ to cartisian coordinates

$$x = r\cos\theta$$
$$y = r\sin\theta$$
so...

$$x = r\cos\theta$$
$$= 5\cos(250°)$$
$$\approx -1.71$$

$$y = r\sin\theta$$
$$= 5\sin(250°)$$
$$\approx -4.7$$

$(5, 250°)$

$(-1.71, -4.7)$
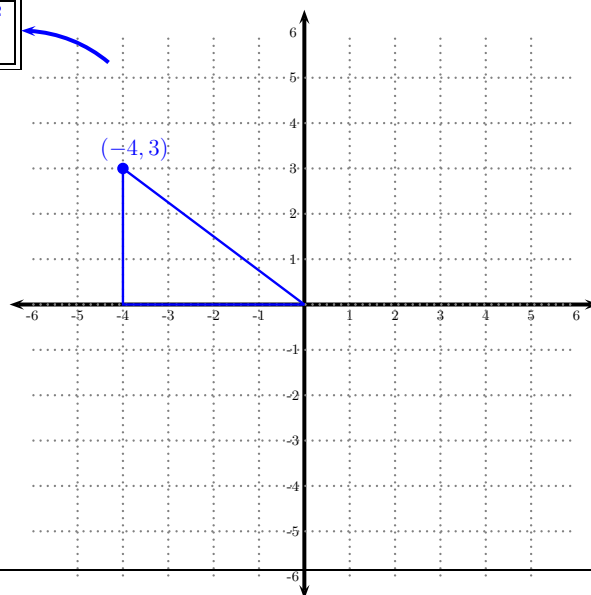
Convert the cartesian coordinates, (-4, 3), to polar coordinates.

$$r^2 = x^2 + y^2$$
$$\tan\theta = y/x$$

(r, θ)

$(-4, 3)$

$$\tan\theta = 3/-4 \quad \text{(note: eq has infinite solutions)}$$
$$\theta \approx -36.87°$$

(note: this is just one [acceptable] value for θ)

$$r^2 = x^2 + y^2$$
$$r^2 = (-4)^2 + (3)^2$$
$$r^2 = 25$$
$$r = \pm\sqrt{25} \approx \pm 5$$

note: if we use the angle $-36.87°$, then we must choose the negative value of r, $r \approx -5$, thus the polar cordinates $(-5, -36.87°)$ is *one* way to represent the point $(-4, 3)$

Intro to Polar Coordinates

1. The following is just a random sequence of point in polar coordinates. Practice by plotting and labeling each of the points:
$$p_1 = (3, 120°), \quad p_2 = (-6, 120°), \quad p_3 = (-3, -120°), \quad p_4 = (3, -300°), \quad p_5 = (-3, 135°)$$

2. The following is just a random sequence of point in polar coordinates. Practice by plotting and labeling each of the points:
$$p_1 = (2, -120°), \quad p_2 = (-4, -120°), \quad p_3 = (-4, 120°), \quad p_4 = (4, -60°), \quad p_5 = (-4, 135°)$$

3. The following is just a random sequence of point in polar coordinates. Practice by plotting and labeling each of the points:
$$p_1 = (1, 45°), \quad p_2 = (-2, 45°), \quad p_3 = (-5, -45°), \quad p_4 = (5, -225°), \quad p_5 = (-5, 135°)$$

4. Convert the polar coordinates, $(-4, 210°)$ to cartisian coordinates

5. Convert the polar coordinates, $(4, 210°)$ to cartisian coordinates

6. Convert the polar coordinates, $(5, 150°)$ to cartisian coordinates

7. Convert the polar coordinates, $(6, -240°)$ to cartisian coordinates

8. Convert the polar coordinates, $(-5, 0°)$ to cartisian coordinates

9. Convert the polar coordinates, $(6, -180°)$ to cartisian coordinates

10. Convert the polar coordinates, $(6, 180°)$ to cartisian coordinates

11. Convert the cartesian coordinates, (-2, 3), to polar coordinates.

12. Convert the cartesian coordinates, (2, -3), to polar coordinates.

13. Convert the cartesian coordinates, (-5, -3), to polar coordinates.

14. Convert the cartesian coordinates, (-4, -1), to polar coordinates.

15. Convert the cartesian coordinates, (-6, 3), to polar coordinates.

16. Convert the cartesian coordinates, (-8, 4), to polar coordinates.

17. **Plot Points, for a Famous Graph** Plot the following points and connect the points.
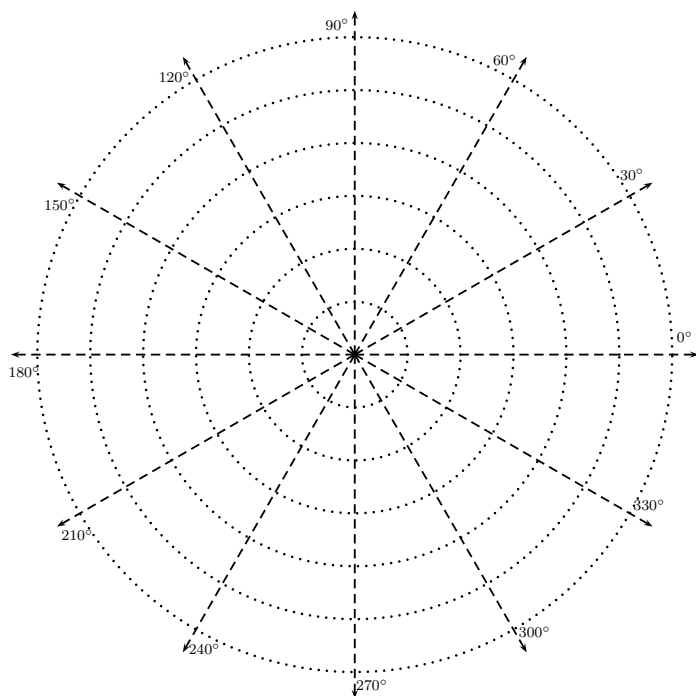
angle theta	r
0	5.00
15	4.33
30	2.50
45	0.00
60	-2.50
75	-4.33
90	-5.00
105	-4.33
120	-2.50
135	-0.00
150	2.50
165	4.33
180	5.00
195	4.33
210	2.50
225	0.00
240	-2.50
255	-4.33
270	-5.00
285	-4.33
300	-2.50
315	-0.00
330	2.50
345	4.33
360	5.00

18. **Plot Points, for a Famous Graph** Plot the following points and connect the points.
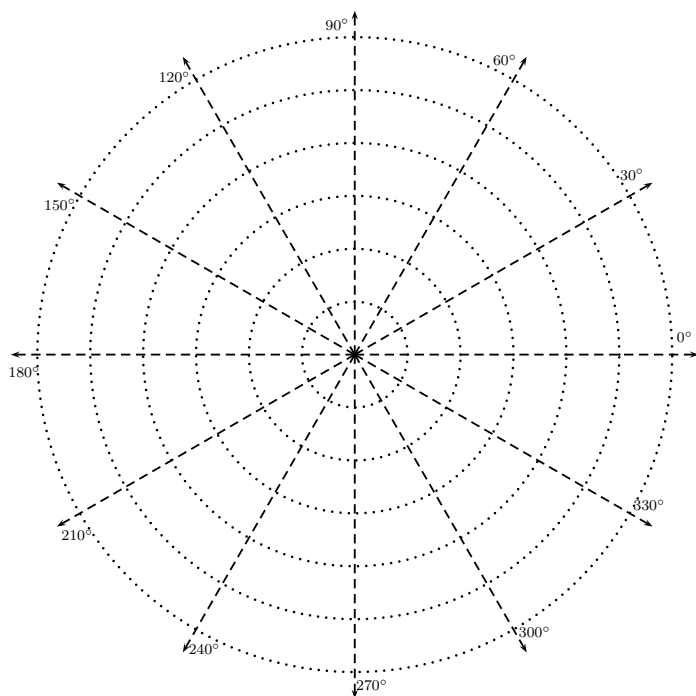
angle theta	r	
0	0.00	
15	1.29	
30	2.50	
45	3.54	
60	4.33	
75	4.83	
90	5.00	
105	4.83	
120	4.33	
135	3.54	
150	2.50	
165	1.29	
180	0.00	
195	-1.29	
210	-2.50	
225	-3.54	
240	-4.33	
255	-4.83	
270	-5.00	
285	-4.83	
300	-4.33	
315	-3.54	
330	-2.50	
345	-1.29	
360	-0.00	
	5sin th	