Q1

Wednesday, August 26, 2020 9:05 PM

Given that

$$\lim_{x \to 2} f(x) = 4 \qquad \lim_{x \to 2} g(x) = -2 \qquad \lim_{x \to 2} h(x) = 0,$$

find the limits, if they exist. (If an answer does not exist, enter DNE.)

(a) $\lim_{x \to 2} [f(x) + 4g(x)]$

(b) $\lim_{x \to 2} [g(x)]^3$

(c) $\lim_{x \to 2} \sqrt{f(x)}$

(d) $\lim_{x \to 2} \frac{5f(x)}{g(x)}$

(e) $\lim_{x \to 2} \frac{g(x)}{h(x)}$

(f) $\lim_{x \to 2} \frac{g(x)h(x)}{f(x)}$

(a) $\lim_{x \to 2} [f(x) + 4g(x)]$

$= \lim_{x \to 2} f(x) + \lim_{x \to 2} 4g(x)$

$= 4 + 4(-2)$

$= \boxed{-4}$

(b) $\lim_{x \to 2} [g(x)]^3$

$= (-2)^3$

$= \boxed{-8}$

(c) $\lim_{x \to 2} \sqrt{f(x)}$

$= \sqrt{4}$

$= \boxed{2}$

(d) $\lim_{x \to 2} \frac{5f(x)}{g(x)}$

$= \frac{\lim_{x \to 2} 5f(x)}{\lim_{x \to 2} g(x)}$

$= \frac{5(4)}{-2}$

$= \boxed{-10}$

(e) $\lim_{x \to 2} \frac{g(x)}{h(x)}$

$= \frac{\lim_{x \to 2} g(x)}{\lim_{x \to 2} h(x)}$

$= \frac{-2}{0}$

$= \boxed{DNE}$

(f) $\lim_{x \to 2} \frac{g(x)h(x)}{f(x)}$

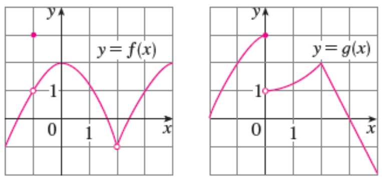
$= \frac{\lim_{x \to 2} g(x) \cdot \lim_{x \to 2} h(x)}{\lim_{x \to 2} f(x)}$

$= \frac{(-2)(0)}{(4)} = \boxed{0}$

Q2

Wednesday, August 26, 2020 9:34 PM

The graphs of f and g are given. Use them to evaluate each limit, if it exists. (If an answer does not exist, enter DNE.)



(a) $\lim\limits_{x \to 2} [f(x) + g(x)]$

(b) $\lim\limits_{x \to 0} [f(x) - g(x)]$

(c) $\lim\limits_{x \to -1} [f(x)g(x)]$

(d) $\lim\limits_{x \to 3} \dfrac{f(x)}{g(x)}$

(e) $\lim\limits_{x \to 2} [x^2 f(x)]$

(f) $f(-1) + \lim\limits_{x \to -1} g(x)$

(a) $\lim\limits_{x \to 2} [f(x) + g(x)]$

$= \lim\limits_{x \to 2} f(x) + \lim\limits_{x \to 2} g(x)$

$= -1 + 2$

$= \boxed{1}$

(b) $\lim\limits_{x \to 0} [f(x) - g(x)]$

$= \lim\limits_{x \to 0} f(x) - \lim\limits_{x \to 0} g(x)$

$= 2 - \text{DNE}$

$= \boxed{\text{DNE}}$

(c) $\lim\limits_{x \to -1} [f(x)\, g(x)]$

$= \lim\limits_{x \to -1} f(x) \cdot \lim\limits_{x \to -1} g(x)$

$= (1)(2)$

$= \boxed{2}$

(d) $\lim\limits_{x \to 3} \dfrac{f(x)}{g(x)}$

$= \dfrac{\lim\limits_{x \to 3} f(x)}{\lim\limits_{x \to 3} g(x)}$

$= \dfrac{1}{0}$

$= \boxed{\text{DNE}}$

(e) $\lim\limits_{x \to 2} [x^2 f(x)]$

$= \lim\limits_{x \to 2} x^2 \cdot \lim\limits_{x \to 2} f(x)$

$= (2)^2 \cdot -1$

$= \boxed{-4}$

(f) $f(-1) + \lim\limits_{x \to -1} g(x)$

$= 3 + 2$

$= \boxed{5}$

Q3

Wednesday, August 26, 2020 9:59 PM

Evaluate the limit using the appropriate Limit Law(s). (If an answer does not exist, enter DNE.)

$$\lim_{x \to 3} (4x^3 - 3x^2 + x - 2)$$

$$\lim_{x \to 3} (4x^3 - 3x^2 + x - 2)$$

$$= \lim_{x \to 3} 4x^3 - \lim_{x \to 3} 3x^2 + \lim_{x \to 3} x - 2$$

$$= 4(3)^3 - 3(3)^2 + 3 - 2$$

$$= 36 - 27 + 3 - 2$$

$$= \boxed{10}$$

Q4

Thursday, August 27, 2020 8:18 AM

Evaluate the limit using the appropriate Limit Law(s). (If an answer does not exist, enter DNE.)

$$\lim_{t \to 2} \frac{t^4 - 7}{2t^2 - 3t + 6}$$

$$\lim_{t \to 2} \frac{t^4 - 7}{2t^2 - 3t + 6}$$

$$= \frac{\lim_{t \to 2} t^4 - 7}{\lim_{t \to 2} t^2 - \lim_{t \to 2} 3t + 6}$$

$$= \frac{(2)^4 - 7}{2(2)^2 - 3(2) + 6}$$

$$= \boxed{1.125}$$

Q5

Thursday, August 27, 2020 11:01 AM

Evaluate the limit using the appropriate Limit Law(s). (If an answer does not exist, enter DNE.)

$$\lim_{x \to 2} \sqrt{\frac{5x^2 + 5}{5x - 1}}$$

$$\lim_{x \to 2} \sqrt{\frac{5x^2+5}{5x-1}}$$

$$= \sqrt{\frac{\lim_{x \to 2} 5x^2 + 5}{\lim_{x \to 2} 5x - 1}}$$

$$= \sqrt{\frac{5(2)^2 + 5}{5(2) - 1}}$$

$$= \boxed{\frac{5}{3}}$$

Q6

Thursday, August 27, 2020 12:47 PM

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \to 3} \frac{x^2 - 8x + 15}{x - 3}$$

$$\lim_{x \to 3} \frac{x^2 - 8x + 15}{x - 3}$$

$$= \frac{x^2 - 8x + 15}{x - 3} = \frac{\cancel{(x-3)}(x-5)}{\cancel{(x-3)}}$$

$$= \lim_{x \to 3} (x - 5)$$

$$= (3) - 5$$

$$= -2$$

Q7

Thursday, August 27, 2020 1:02 PM

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{t \to -6} \frac{t^2 - 36}{2t^2 + 13t + 6}$$

$$\lim_{t \to -6} \frac{t^2 - 36}{2t^2 + 13t + 6}$$

$$= \frac{t^2 - 36}{2t^2 + 13t + 6} = \frac{(t-6)\cancel{(t+6)}}{(2t+1)\cancel{(t+6)}}$$

$$= \lim_{t \to -6} \frac{t-6}{2t+1}$$

$$= \frac{(-6)-6}{2(-6)+1}$$

$$= \frac{12}{11}$$

Q8

Thursday, August 27, 2020 1:14 PM

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{h \to 0} \frac{(-6 + h)^2 - 36}{h}$$

$$\lim_{h \to 0} \frac{(-6+h)^2 - 36}{h}$$

$$= \frac{(-6+h)(-6+h) - 36}{h} = \frac{(\cancel{36} - 12h + 2h^2) - \cancel{36}}{h}$$

$$= \frac{(-12h + 2h^2)}{h} = \frac{\cancel{h}(-12 + 2h)}{\cancel{h}} = -12 + 2h$$

$$= \lim_{h \to 0} -12 + 2h$$

$$= -12 + 2(0)$$

$$= \boxed{-12}$$

Q9

Thursday, August 27, 2020 1:23 PM

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \to -4} \frac{x + 4}{x^3 + 64}$$

$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$

$$\lim_{x \to -4} \frac{x+4}{x^3+64}$$

$$= \frac{x+4}{x^3+64} = \frac{\cancel{x+4}}{\cancel{(x+4)}(x^2 - 4x + 16)} = \frac{1}{x^2 - 4x + 16}$$

$$= \lim_{x \to -4} \frac{1}{x^2 - 4x + 16}$$

$$= \frac{1}{(-4)^2 - 4(-4) + 16} = \frac{1}{48}$$

Q10

Thursday, August 27, 2020 1:35 PM

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{h \to 0} \frac{\sqrt{36 + h} - 6}{h}$$

$$\lim_{h \to 0} \frac{\sqrt{36+h} - 6}{h}$$

$$= \frac{\sqrt{36+h} - 6}{h} \cdot \frac{\left(\sqrt{36+h} + 6\right)}{\left(\sqrt{36+h} + 6\right)} = \frac{\left(\sqrt{36+h}\right)^2 - (6)^2}{h\left(\sqrt{36+h} + 6\right)}$$

$$= \frac{36 + h - 36}{h\left(\sqrt{36+h} + 6\right)} = \frac{\cancel{h}}{\cancel{h}\left(\sqrt{36+h} + 6\right)} = \frac{1}{\sqrt{36+h} + 6}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{36+h} + 6}$$

$$= \frac{1}{\sqrt{36+0} + 6}$$

$$= \boxed{\frac{1}{12}}$$

Q11

Thursday, August 27, 2020 1:44 PM

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{t \to 0} \left(\frac{6}{t} - \frac{6}{t^2 + t} \right)$$

$$\lim_{t \to 0} \left(\frac{6}{t} - \frac{6}{t^2+t} \right)$$

$$= \frac{6}{t} - \frac{6}{t^2+t} = \frac{6}{t} - \frac{6}{t(t+1)} = \frac{6}{t} \frac{(t+1)}{(t+1)} - \frac{6}{t(t+1)}$$

$$= \frac{6(t+1)}{t(t+1)} - \frac{6}{t(t+1)} = \frac{6(t+1) - 6}{t(t+1)} = \frac{6t + 6 - 6}{t(t+1)}$$

$$= \frac{6\cancel{t}}{\cancel{t}(t+1)} = \frac{6}{t+1}$$

$$= \lim_{t \to 0} \frac{6}{t+1} = \frac{6}{0+1} = \boxed{6}$$

Q12

Thursday, August 27, 2020 2:27 PM

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{h \to 0} \frac{(6+h)^{-1} - 6^{-1}}{h}$$

$$\lim_{h \to 0} \frac{(6+h)^{-1} - 6^{-1}}{h}$$

$$= \frac{(6+h)^{-1} - 6^{-1}}{h} = \frac{\frac{1}{6+h} - \frac{1}{6}}{h} = \frac{\frac{1}{6+h} \cdot \frac{(-6)}{(-6)} - \frac{1}{6} \cdot \frac{(6+h)}{6+h}}{h}$$

$$= \frac{-\frac{6}{6(6+h)} - \frac{6+h}{6(6+h)}}{h} = -\frac{\frac{6-6+h}{6(6+h)}}{h} = -\frac{\cancel{h}}{6(6+h)} \cdot \frac{1}{\cancel{h}} = -\frac{1}{6(6+h)}$$

$$= \lim_{h \to 0} -\frac{1}{6(6+h)} = -\frac{1}{6(6+0)} = \boxed{-\frac{1}{36}}$$

Q13

Thursday, August 27, 2020 2:49 PM

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \frac{(x+h)(x^2+2hx+h^2) - x^3}{h} = \frac{\cancel{x^3} + 3hx^2 + 3h^2x + h^3 - \cancel{x^3}}{h}$$

$$= \frac{3hx^2 + 3h^2x + h^3}{h} = \frac{3\cancel{h}x^2}{\cancel{h}} + \frac{3h^{\cancel{2}\,1}x}{\cancel{h}} + \frac{h^{\cancel{3}\,2}}{\cancel{h}} = 3x^2 + 3hx + h^2$$

$$= \lim_{h \to 0} 3x^2 + 3x + h^2 = 3x^2 + 3(0)x + (0)^2 = \boxed{3x^2}$$

Q14

Thursday, August 27, 2020 3:14 PM

Find the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \to 8} (7x + |x - 8|)$$

$$\lim_{x \to 8} (7x + |x-8|)$$

$$|x-8| = \begin{cases} x-8 & \text{if } x-8 \geq 0 \\ -(x-8) & \text{if } x-8 < 0 \end{cases}$$

or

$$= \begin{cases} x-8 & \text{if } x \geq 8 \\ -(x-8) & \text{if } x < 8 \end{cases}$$

$$= \lim_{x \to 8^+} (7x + x - 8) = \lim_{x \to 8^+} (8x - 8)$$

$$= 8(8) - 8$$

$$= \boxed{56}$$

$$= \lim_{x \to 8^-} (7x - (x-8)) = \lim_{x \to 8^-} (7x - x + 8) = \lim_{x \to 8^-} (6x + 8)$$

$$= 6(8) + 8$$

$$= \boxed{56}$$

Q15

Thursday, August 27, 2020 3:59 PM

Find the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \to -7} \frac{6x + 42}{|x + 7|}$$

$$\lim_{x \to -7} \frac{6x + 42}{|x + 7|}$$

$$= \lim_{x \to -7}$$

$$|x + 7| = \begin{cases} x + 7 & \text{if } x + 7 \geq 0 \\ -(x + 7) & \text{if } x + 7 < 0 \end{cases}$$

$$= \begin{cases} x + 7 & \text{if } x \geq -7 \\ -(x + 7) & \text{if } x < -7 \end{cases}$$

$\boxed{\text{DNE}}$

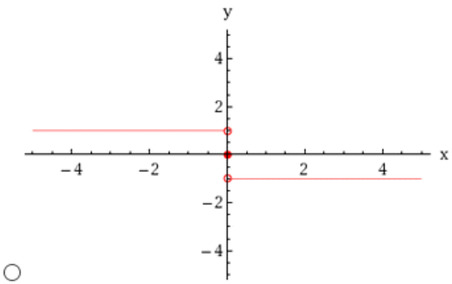
Q16

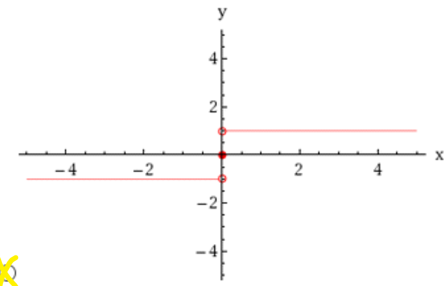
Thursday, August 27, 2020 4:06 PM

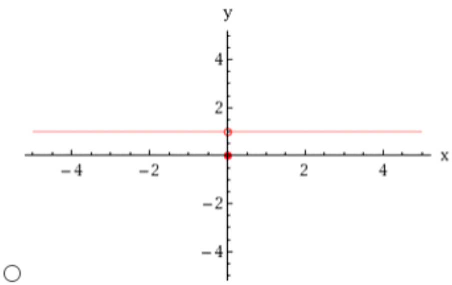
The *signum* (or sign) *function*, denoted by sgn, is defined by

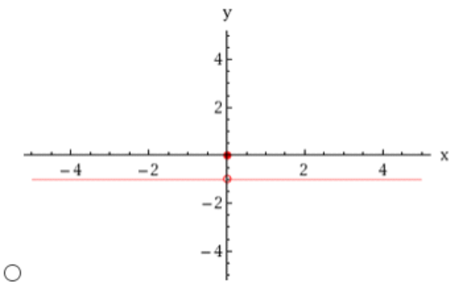
$$\operatorname{sgn} x = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}.$$

(a) Sketch the graph of this function.









(b) Find each of the following limits. (If an answer does not exist, enter DNE.)

(i) $\lim_{x \to 0^+} \operatorname{sgn} x$

(ii) $\lim_{x \to 0^-} \operatorname{sgn} x$

(iii) $\lim_{x \to 0} \operatorname{sgn} x$

(iv) $\lim_{x \to 0} |\operatorname{sgn} x|$

(i) $\lim_{x \to 0^+} \operatorname{sgn} x = 1$ for $x > 0$

$\lim_{x \to 0^+} \operatorname{sgn} x = \lim_{x \to 0^+} 1 = \boxed{1}$

(ii) $\lim_{x \to 0^-} \operatorname{sgn} x = -1$ for $x < 0$

$\lim_{x \to 0^-} \operatorname{sgn} x = -1 = \lim_{x \to 0^-} = -1 = \boxed{-1}$

(iii) $\lim_{x \to 0^-} \operatorname{sgn} x \neq \lim_{x \to 0^+} \operatorname{sgn} x$

$= \boxed{DNE}$

(iv) $\lim_{x \to 0} |\operatorname{sgn} x|$

$|\operatorname{sgn} x| = 1$ for $x \neq 0$

$\lim_{x \to 0} |\operatorname{sgn} x| = \lim_{x \to 0} 1 = \boxed{1}$

Q17

Thursday, August 27, 2020 4:45 PM

Let $g(x) = \dfrac{x^2 + x - 12}{|x - 3|}$.

(a) Find the following limits. (If an answer does not exist, enter DNE.)

(i) $\lim_{x \to 3^+} g(x)$

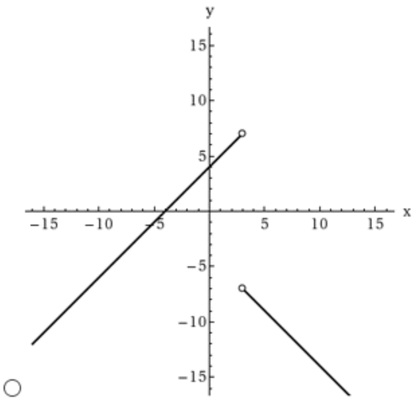
(ii) $\lim_{x \to 3^-} g(x)$

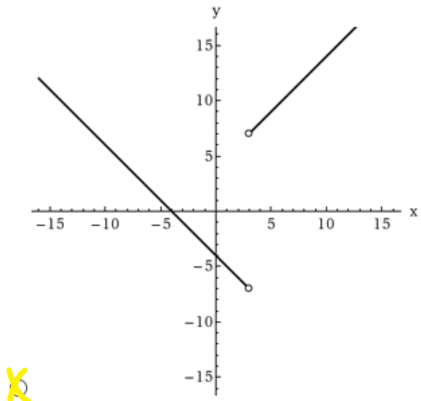
(b) Does $\lim_{x \to 3} g(x)$ exist?

○ Yes

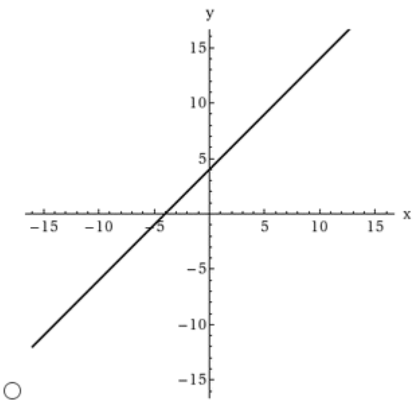
X No

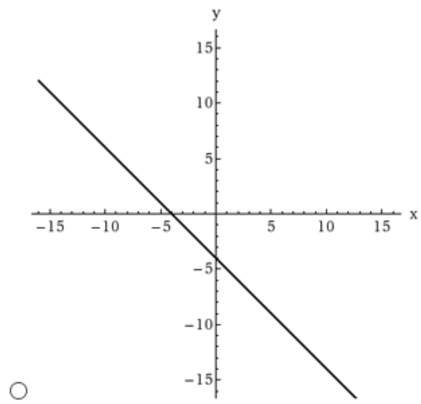
(c) Sketch the graph of g.





X





$$g(x) = \frac{x^2 + x - 12}{|x - 3|}$$

$$|x-3| = \begin{cases} x-3 & \text{if } x-3 > 0 \\ -(x-3) & \text{if } x-3 < 0 \end{cases}$$

or

$$\begin{cases} x-3 & \text{if } x > 3 \\ -(x-3) & \text{if } x < 3 \end{cases}$$

(i) $\lim_{x \to 3^+} = \dfrac{x^2 + x - 12}{x - 3} = \dfrac{\cancel{(x-3)}(x+4)}{\cancel{(x-3)}} = \lim_{x \to 3^+} x + 4 = \lim_{x \to 3^+} 3 + 4 = \boxed{7}$

(ii) $\lim_{x \to 3^-} \dfrac{x^2 + x - 12}{-(x-3)} = \dfrac{x^2 + x - 12}{-x + 3} = \dfrac{\cancel{(-x+3)}(-x-4)}{\cancel{-x+3}} = -x - 4$

$\lim_{x \to 3^-} -x - 4 = -(3) - 4 = \boxed{-7}$

Q18

Thursday, August 27, 2020 5:03 PM

Let

$$f(x) = \begin{cases} x^2 + 2 & \text{if } x < 2 \\ (x-3)^2 & \text{if } x \geq 2 \end{cases}.$$

(a) Find the following limits. (If an answer does not exist, enter DNE.)

$\lim_{x \to 2^-} f(x) =$ 6

$\lim_{x \to 2^+} f(x) =$ 1

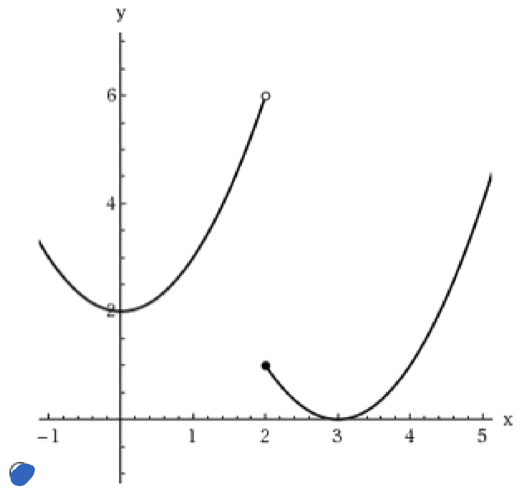
(b) Does $\lim_{x \to 2} f(x)$ exist?

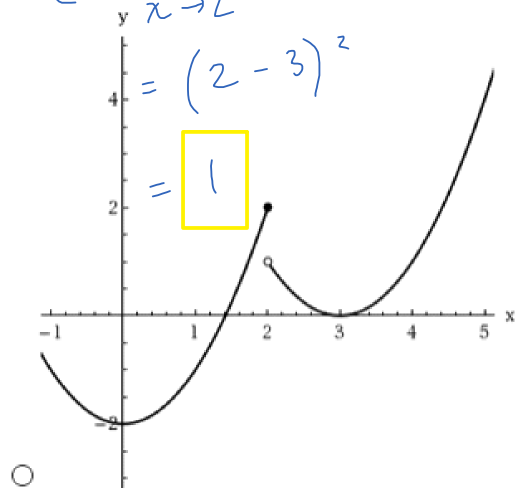
- Yes
- No (selected)

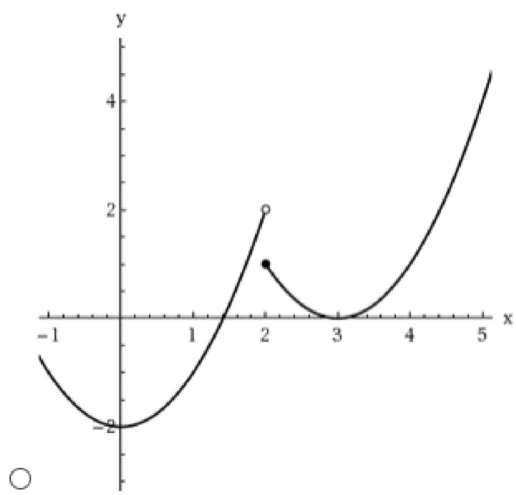
(c) Sketch the graph of f.

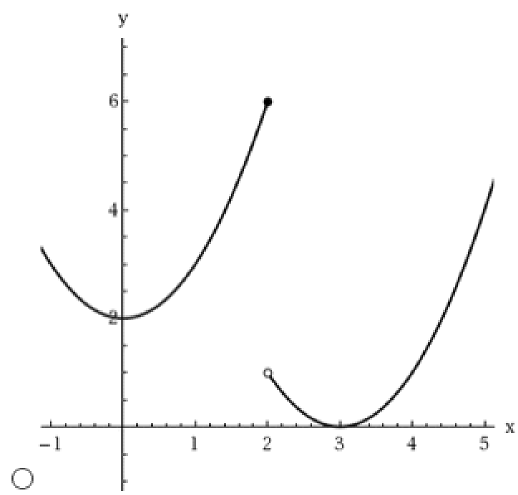
$$(i)\ \lim_{x \to 2^-} x^2 + 2 = (2)^2 + 2 = \boxed{6}$$

$$(ii)\ \lim_{x \to 2^+} (x-3)^2 = (2-3)^2 = \boxed{1}$$









Q19

Thursday, August 27, 2020 5:12 PM

In the theory of relativity, the Lorentz contraction formula

$$L = L_0\sqrt{1 - v^2/c^2}$$

expresses the length L of an object as a function of its velocity v with respect to an observer, where L_0 is the length of the object at rest and c is the speed of light. Find $\lim\limits_{v \to c^-} L$.

0 ✓

Why is a left-hand limit necessary?

- ○ L is not defined for $v < 0$.
- ◉ L is not defined for $v > c$.
- ○ L is not defined for $v < c$.
- ○ L is not defined for $v > 0$.

✓

$L = L_0\sqrt{1 - v^2/c^2}$, find $\lim\limits_{v \to c^-} L$

$$\lim_{v \to c^-} L_0\sqrt{1 - v^2/(v)^2} = L_0\sqrt{1 - 1} = L_0\sqrt{0} = \boxed{0}$$

Q20

Thursday, August 27, 2020 5:24 PM

If $\lim_{x \to 1} \frac{f(x) - 6}{x - 1} = 8$, evaluate $\lim_{x \to 1} f(x)$.

$$\lim_{x \to 1} \frac{f(x) - 6}{x - 1} = 8$$

$$= \frac{\lim_{x \to 1} [f(x) - 6]}{\lim_{x \to 1} (x - 1)} = 8$$

$$= \lim_{x \to 1} [f(x) - 6] = 8 \lim_{x \to 1} (x - 1)$$

$$= \lim_{x \to 1} f(x) - \lim_{x \to 1} 6 = 8 \lim_{x \to 1} (1 - 1)$$

$$= \lim_{x \to 1} f(x) - (6) = 8(0)$$

$$= \lim_{x \to 1} f(x) - 6 = 0$$

$$= \lim_{x \to 1} f(x) = \boxed{6}$$