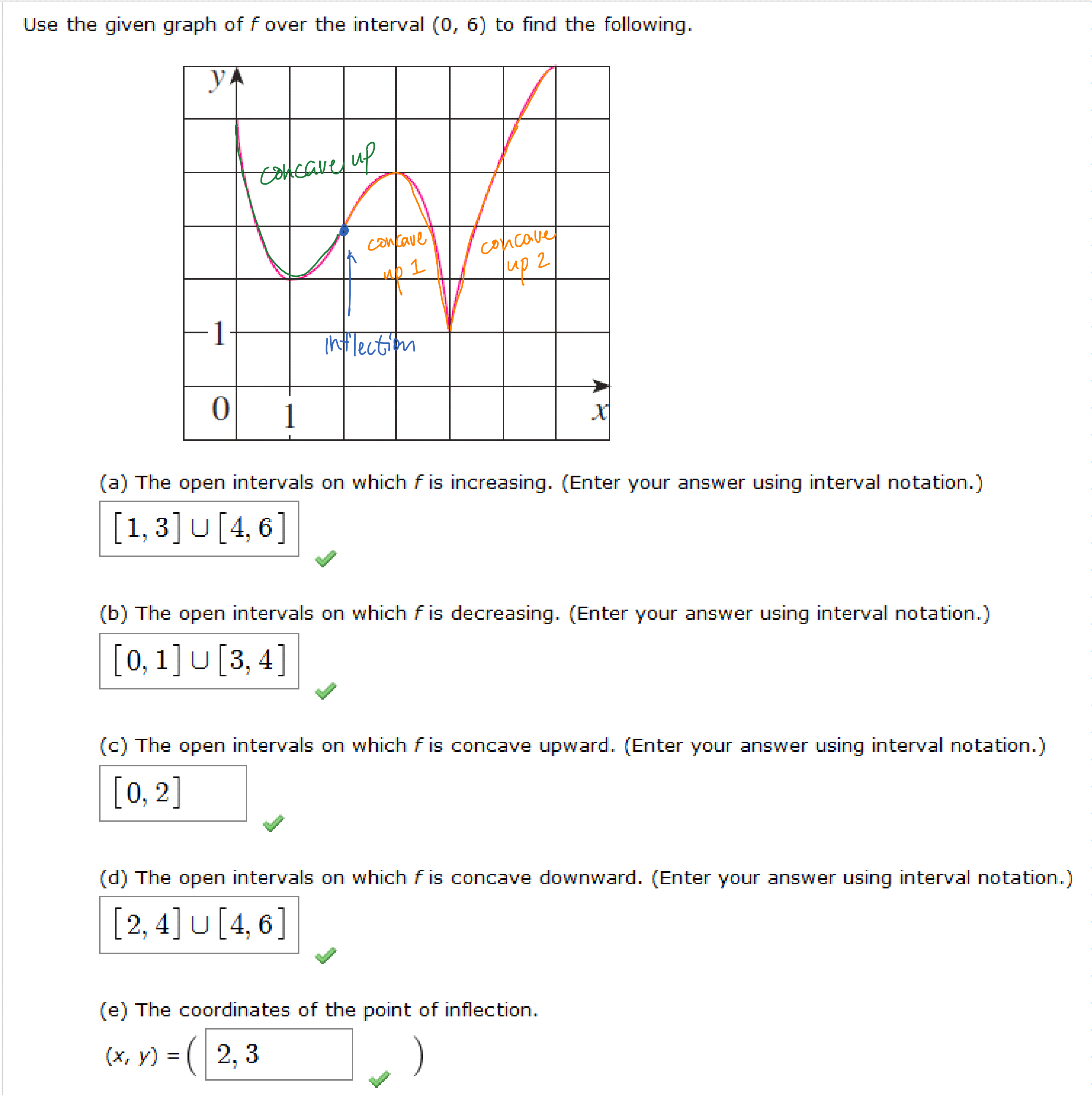
Q1

Friday, October 16, 2020 7:55 PM



Use the given graph of *f* over the interval (0, 6) to find the following.

(a) The open intervals on which *f* is increasing. (Enter your answer using interval notation.)

$[1, 3] \cup [4, 6]$ ✓

(b) The open intervals on which *f* is decreasing. (Enter your answer using interval notation.)

$[0, 1] \cup [3, 4]$ ✓

(c) The open intervals on which *f* is concave upward. (Enter your answer using interval notation.)

$[0, 2]$ ✓

(d) The open intervals on which *f* is concave downward. (Enter your answer using interval notation.)

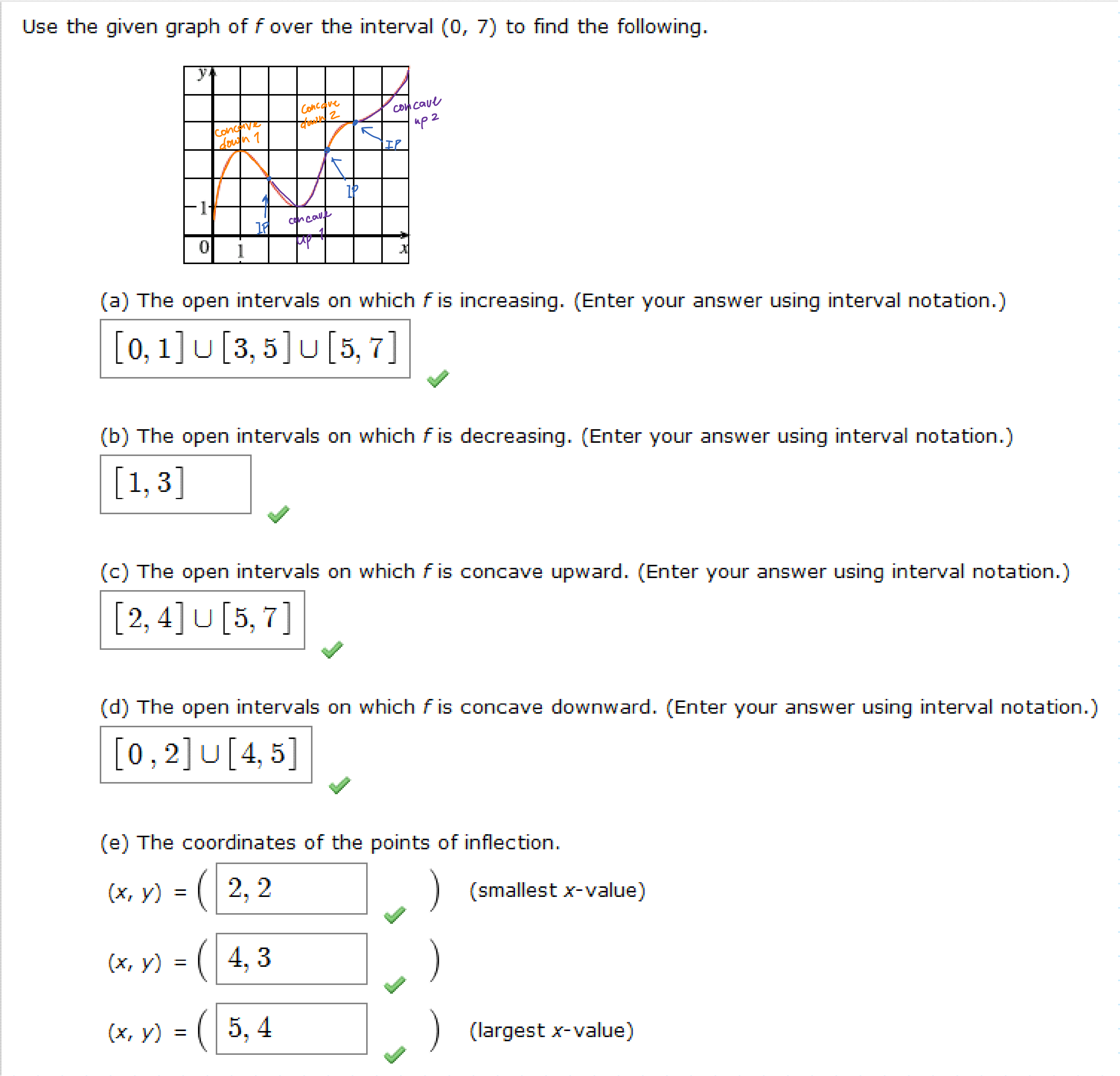
$[2, 4] \cup [4, 6]$ ✓

(e) The coordinates of the point of inflection.

$(x, y) = ($ 2, 3 ✓ $)$

Q2

Friday, October 16, 2020 8:13 PM



Use the given graph of *f* over the interval (0, 7) to find the following.

(a) The open intervals on which *f* is increasing. (Enter your answer using interval notation.)

$[0, 1] \cup [3, 5] \cup [5, 7]$ ✓

(b) The open intervals on which *f* is decreasing. (Enter your answer using interval notation.)

$[1, 3]$ ✓

(c) The open intervals on which *f* is concave upward. (Enter your answer using interval notation.)

$[2, 4] \cup [5, 7]$ ✓

(d) The open intervals on which *f* is concave downward. (Enter your answer using interval notation.)

$[0\,, 2] \cup [4, 5]$ ✓

(e) The coordinates of the points of inflection.

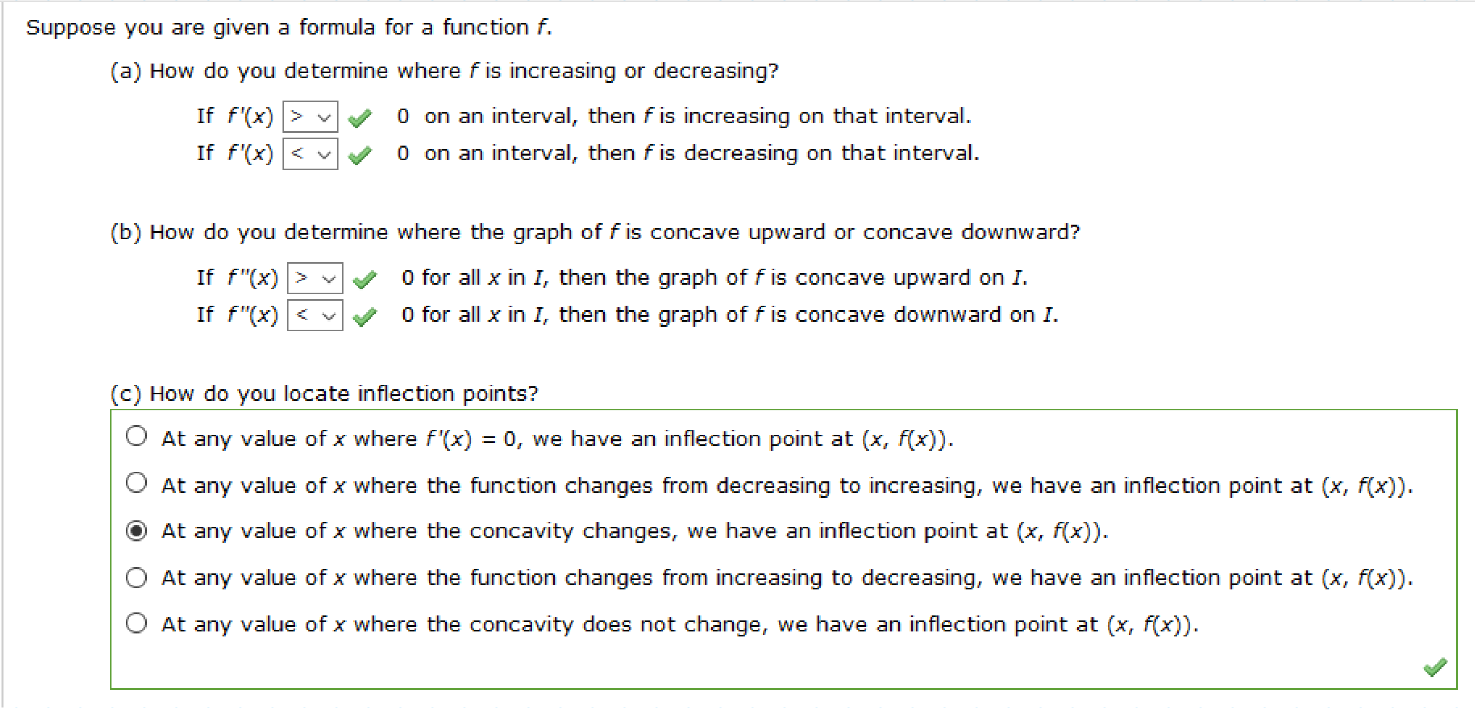
$(x, y) = (\ 2, 2\ )$ ✓ (smallest *x*-value)

$(x, y) = (\ 4, 3\ )$ ✓

$(x, y) = (\ 5, 4\ )$ ✓ (largest *x*-value)

Q3

Friday, October 16, 2020 8:22 PM



Suppose you are given a formula for a function f.

(a) How do you determine where f is increasing or decreasing?

If $f'(x)$ > ✓ 0 on an interval, then f is increasing on that interval.
If $f'(x)$ < ✓ 0 on an interval, then f is decreasing on that interval.

(b) How do you determine where the graph of f is concave upward or concave downward?

If $f''(x)$ > ✓ 0 for all x in I, then the graph of f is concave upward on I.
If $f''(x)$ < ✓ 0 for all x in I, then the graph of f is concave downward on I.

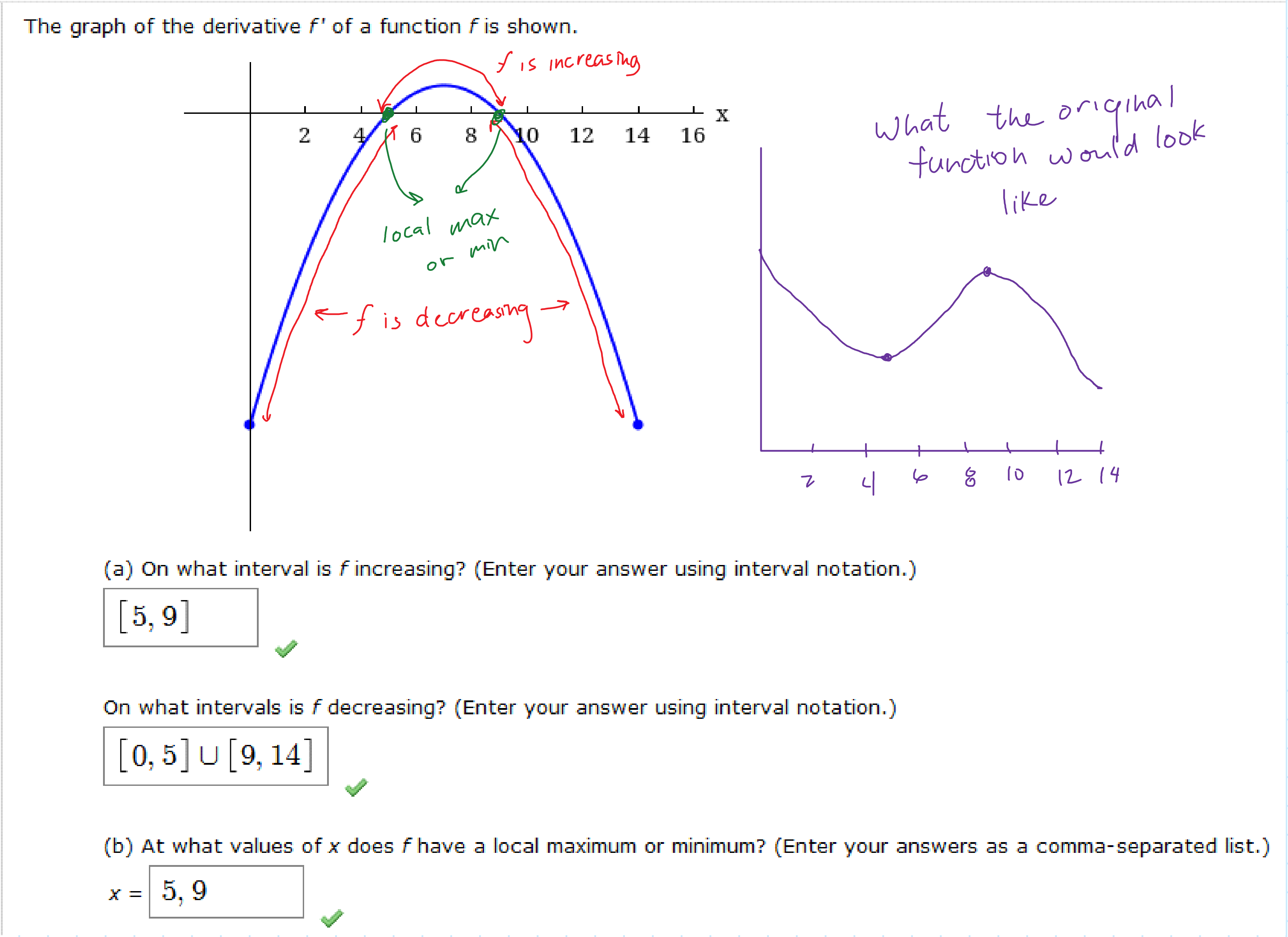
(c) How do you locate inflection points?

○ At any value of x where $f'(x) = 0$, we have an inflection point at $(x, f(x))$.
○ At any value of x where the function changes from decreasing to increasing, we have an inflection point at $(x, f(x))$.
● At any value of x where the concavity changes, we have an inflection point at $(x, f(x))$.
○ At any value of x where the function changes from increasing to decreasing, we have an inflection point at $(x, f(x))$.
○ At any value of x where the concavity does not change, we have an inflection point at $(x, f(x))$.

✓

Q4

Friday, October 16, 2020 8:26 PM



The graph of the derivative f' of a function f is shown.

(a) On what interval is f increasing? (Enter your answer using interval notation.)

$[5, 9]$ ✓

On what intervals is f decreasing? (Enter your answer using interval notation.)

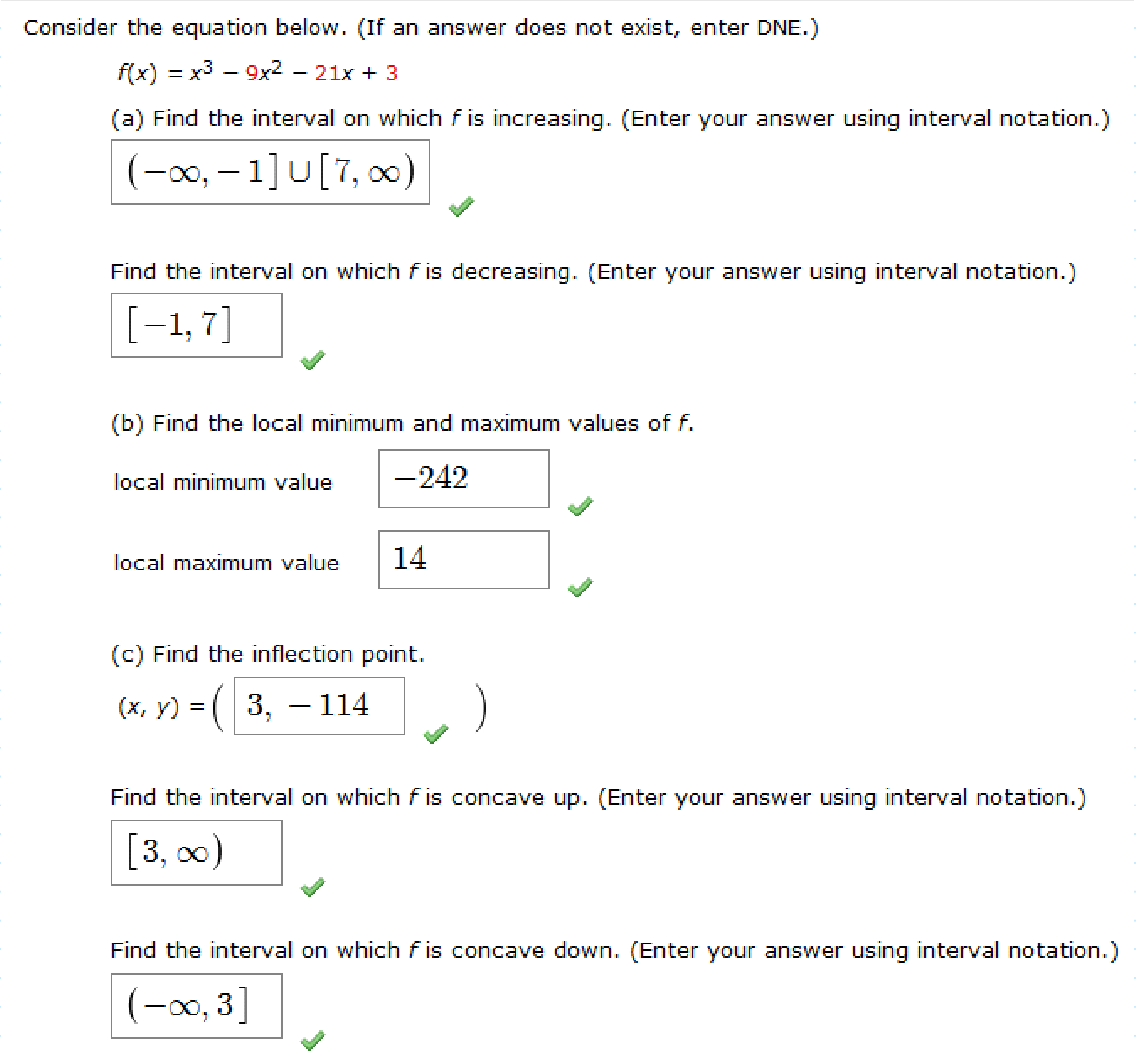
$[0, 5] \cup [9, 14]$ ✓

(b) At what values of x does f have a local maximum or minimum? (Enter your answers as a comma-separated list.)

$x =$ 5, 9 ✓

Q5.1

Saturday, October 17, 2020 6:23 PM



Consider the equation below. (If an answer does not exist, enter DNE.)

$f(x) = x^3 - 9x^2 - 21x + 3$

(a) Find the interval on which f is increasing. (Enter your answer using interval notation.)

$(-\infty, -1] \cup [7, \infty)$ ✓

Find the interval on which f is decreasing. (Enter your answer using interval notation.)

$[-1, 7]$ ✓

(b) Find the local minimum and maximum values of f.

local minimum value -242 ✓

local maximum value 14 ✓

(c) Find the inflection point.

$(x, y) = (3, -114)$ ✓

Find the interval on which f is concave up. (Enter your answer using interval notation.)

$[3, \infty)$ ✓

Find the interval on which f is concave down. (Enter your answer using interval notation.)

$(-\infty, 3]$ ✓

Q5.2

Friday, October 16, 2020 8:41 PM

$f(x) = x^3 - 9x^2 - 21x + 3$

$f'(x) = \frac{d}{dx}\left(x^3 - 9x^2 - 21x + 3\right)$

$= 3x^2 - 18x - 21$

$f'(x) = 3(x^2 - 6x - 7)$

Find Critical points

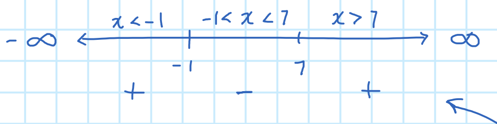
$3(x^2 - 6x - 7) = 0$

$3(x-7)(x+1) = 0$

$x - 7 = 0$ $\quad$ $x + 1 = 0$

$x = 7$ $\quad$ $x = -1$

Figure out if increasing or decreasing



Take all factors of the derivative for I/D test

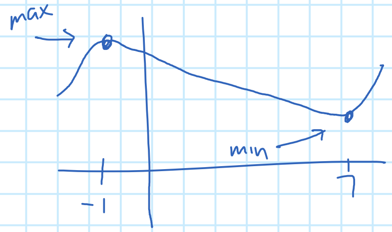
Intervals	$x-7$	$x+1$	$f'(x)$
	$(-2)-7$	$(-2)+1$	$- * - = +$
$x < -1$	$-$	$-$	$+$
$-1 < x < 7$	$-$	$+$	$-$
$x > 7$	$+$	$+$	$+$

say $x = -2$ to satisfy $x < -1$, then pass the x into the factors

decreasing @ $\boxed{[-1, 7]}$

increasing @ $\boxed{(-\infty, -1] \cup [7, \infty)}$

$f(x)$ would look like



Local max @ $x = -1$

$f(-1) = x^3 - 9x^2 - 21x + 3$

$= (-1)^3 - 9(-1)^2 - 21(-1) + 3$

$= -1 - 9 + 21 + 3$

$f(-1) = 14$

$\boxed{\text{Local max value at } 14}$

Local min $x = 7$

$f(7) = x^3 - 9x^2 - 21x + 3$

$= (7)^3 - 9(7)^2 - 21(7) + 3$

$f(7) = -242$

$\boxed{\text{Local min value at } -242}$

$f''(x) = \frac{d}{dx}\left[3x^2 - 18x - 21\right]$

$= 6x - 18$

$f''(x) = 6(x - 3)$

$x - 3 = 0$

$x = 3$

$f(3) = x^3 - 9x^2 - 21x + 3$

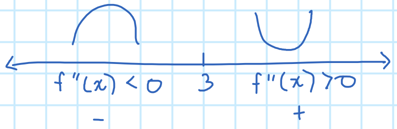
$= (3)^3 - 9(3)^2 - 21(3) + 3$

$f(3) = -114$

$\boxed{\text{Inflection point } = (3, -114)}$

$x > 3 = f''(x) > 0$ = concave upwards $[3, \infty)$

$x < 3 = f''(x) < 0$ = concave downwards $(-\infty, 3]$

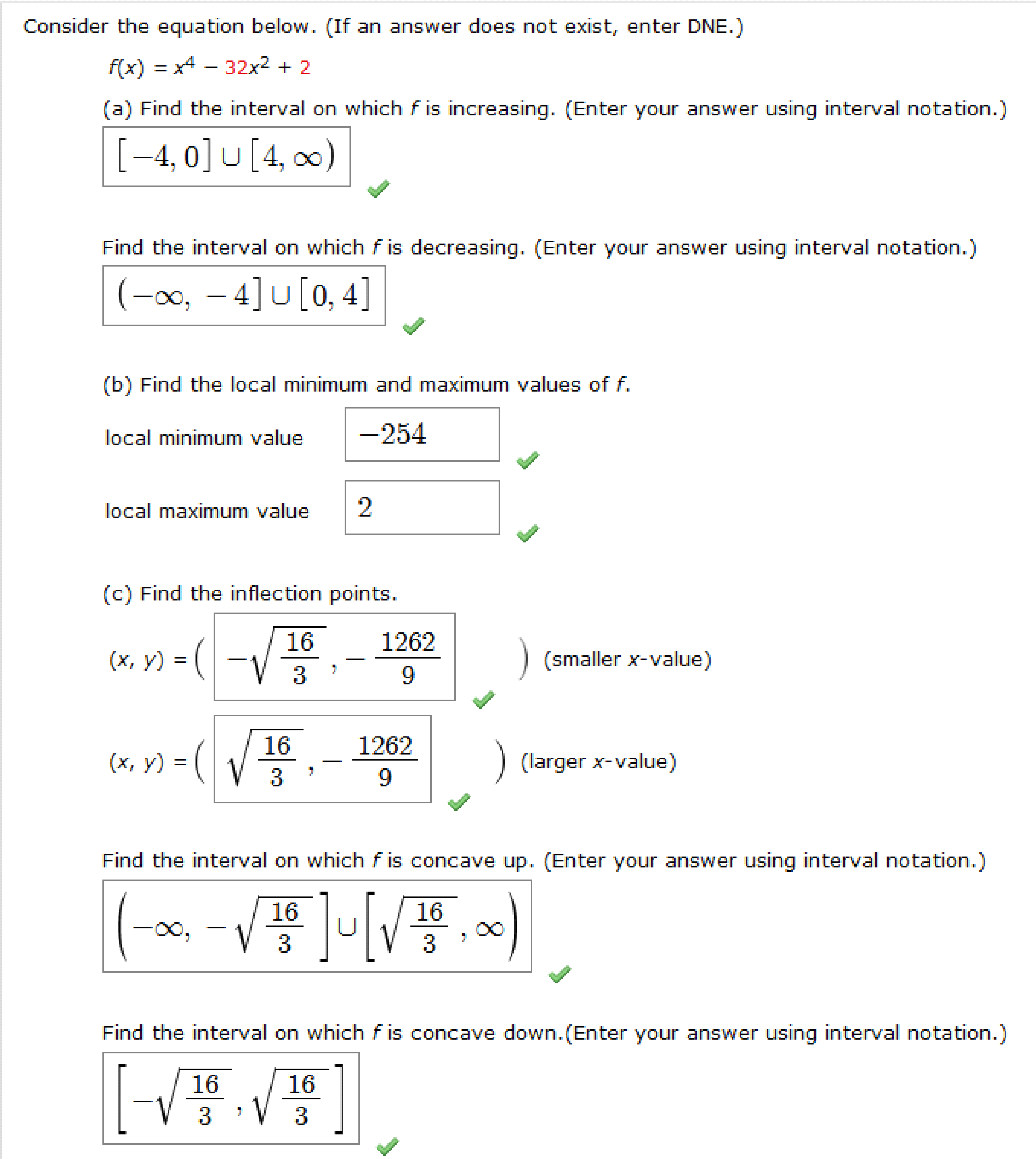


$f''(x) > 0$, concave upwards

$f''(x) < 0$, concave downwards

Q6.1

Saturday, October 17, 2020 8:14 PM



Consider the equation below. (If an answer does not exist, enter DNE.)

$f(x) = x^4 - 32x^2 + 2$

(a) Find the interval on which f is increasing. (Enter your answer using interval notation.)

$[-4, 0] \cup [4, \infty)$ ✓

Find the interval on which f is decreasing. (Enter your answer using interval notation.)

$(-\infty, -4] \cup [0, 4]$ ✓

(b) Find the local minimum and maximum values of f.

local minimum value -254 ✓

local maximum value 2 ✓

(c) Find the inflection points.

$(x, y) = \left(-\sqrt{\frac{16}{3}}, -\frac{1262}{9} \right)$ ✓ (smaller x-value)

$(x, y) = \left(\sqrt{\frac{16}{3}}, -\frac{1262}{9} \right)$ ✓ (larger x-value)

Find the interval on which f is concave up. (Enter your answer using interval notation.)

$\left(-\infty, -\sqrt{\frac{16}{3}}\right] \cup \left[\sqrt{\frac{16}{3}}, \infty\right)$ ✓

Find the interval on which f is concave down.(Enter your answer using interval notation.)

$\left[-\sqrt{\frac{16}{3}}, \sqrt{\frac{16}{3}}\right]$ ✓

Q6.2

Saturday, October 17, 2020 6:24 PM

$f(x) = x^4 - 32x^2 + 2$

$f'(x) = \frac{d}{dx}\left(x^4 - 32x^2 + 2\right)$

$= 4x^3 - 64x$

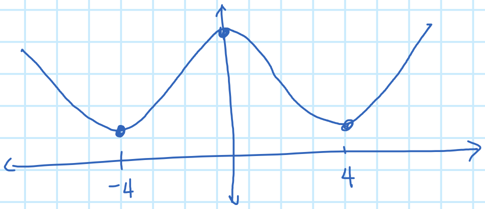
$= 4x\left(x^2 - 16\right)$

$f'(x) = 4x(x-4)(x+4)$

$4x = 0$, $x = 0$

$x - 4 = 0$, $x = 4$

$x + 4 = 0$, $x - 4$



$f(-4) = x^4 - 32x^2 + 2$

$= (-4)^4 - 32(-4)^2 + 2$

$= -254$

$f(4) = x^4 - 32x^2 + 2$

$= (4)^4 - 32(4)^2 + 2$

$= -254$

$f(0) = x^4 - 32x^2 + 2$

$= (0)^4 - 32(0)^2 + 2$

$= 2$

Local min values $= -254$

Local max values $= 2$

$x < -4$ $\quad -4 < x < 0$ $\quad 0 < x < 4$ $\quad x > 4$

$-4 \quad 0 \quad 4$

$- \quad + \quad - \quad +$

Intervals	$4x$	$x-4$	$x+4$	$f'(x)$
$x < -4$	$-$	$-$	$-$	$-$
$-4 < x < 0$	$-$	$-$	$+$	$+$
$0 < x < 4$	$+$	$-$	$+$	$-$
$x > 0$	$+$	$+$	$+$	$+$

f is increasing $[-4, 0] \cup [4, \infty)$

f is decreasing $(-\infty, -4] \cup [0, 4]$

$f''(x) = \frac{d}{dx}\left(4x^3 - 64x\right)$

$= 12x^2 - 64$

$f''(x) = 4(3x^2 - 16)$

$3x^2 - 16 = 0$

$3x^2 = 16$

$x^2 = \frac{16}{3}$

$x = \pm\sqrt{\frac{16}{3}}$

$f''(x) > 0$, concave upwards

$f''(x) < 0$, concave downwards

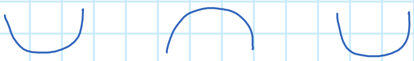
$\sqrt{\frac{16}{3}} \approx 2.31$

$+ \quad - \quad +$

$f''(x) > 0 \quad f''(x) < 0 \quad f''(x) > 0$

$-\sqrt{\frac{16}{3}} \quad \sqrt{\frac{16}{3}}$

$\left(-\infty, -\sqrt{\frac{16}{3}}\right] \quad \left[-\sqrt{\frac{16}{3}}, \sqrt{\frac{16}{3}}\right] \quad \left[\sqrt{\frac{16}{3}}, \infty\right)$



concave upward $= \left(-\infty, -\sqrt{\frac{16}{3}}\right] \cup \left[\sqrt{\frac{16}{3}}, \infty\right)$

concave downward $= \left[-\sqrt{\frac{16}{3}}, \sqrt{\frac{16}{3}}\right]$

$f\left(\sqrt{\frac{16}{3}}\right) = x^4 - 32x^2 + 2$

$= \left(\sqrt{\frac{16}{3}}\right)^4 - 32\left(\sqrt{\frac{16}{3}}\right)^2 + 2$

$= \left(\frac{16}{3}^{\frac{1}{2}}\right)^4 - 32\left(\frac{16}{3}^{\frac{1}{2}}\right)^2 + 2$

$= \frac{16^2}{3^2} - 32\left(\frac{16}{3}\right) + 2$

$= -\frac{1262}{9}$

$f\left(-\sqrt{\frac{16}{3}}\right) = x^4 - 32x^2 + 2$

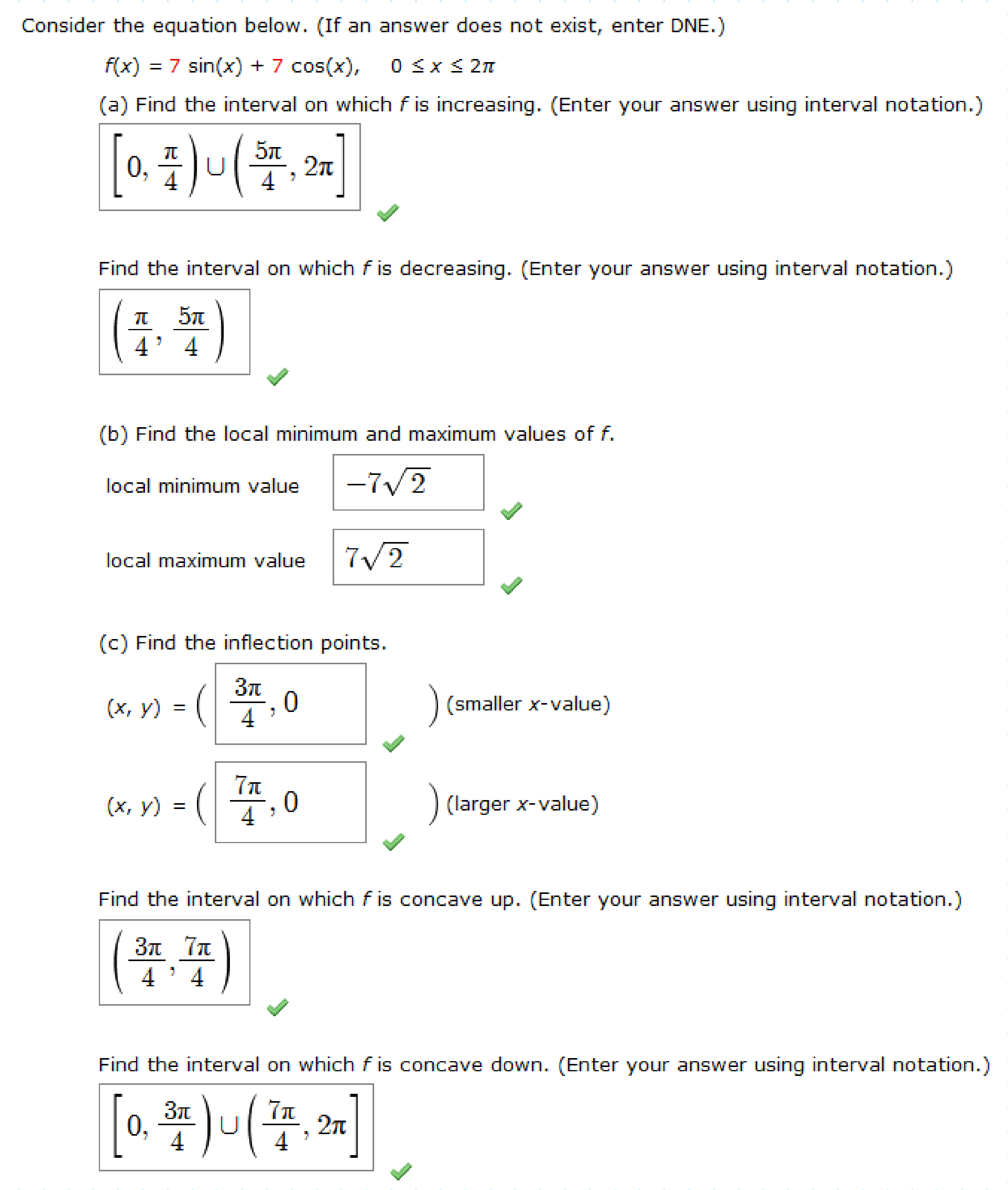
$= \left(-\sqrt{\frac{16}{3}}\right)^4 - 32\left(-\sqrt{\frac{16}{3}}\right)^2 + 2$

$= -\frac{1262}{9}$

Inflection points $= \left(\sqrt{\frac{16}{3}}, -\frac{1262}{9}\right), \left(-\sqrt{\frac{16}{3}}, -\frac{1262}{9}\right)$

Q7.1

Saturday, October 17, 2020 9:22 PM



Consider the equation below. (If an answer does not exist, enter DNE.)

$f(x) = 7\sin(x) + 7\cos(x), \quad 0 \le x \le 2\pi$

(a) Find the interval on which f is increasing. (Enter your answer using interval notation.)

$\left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$ ✓

Find the interval on which f is decreasing. (Enter your answer using interval notation.)

$\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ ✓

(b) Find the local minimum and maximum values of f.

local minimum value $-7\sqrt{2}$ ✓

local maximum value $7\sqrt{2}$ ✓

(c) Find the inflection points.

$(x, y) = \left(\frac{3\pi}{4}, 0\right)$ ✓ (smaller x-value)

$(x, y) = \left(\frac{7\pi}{4}, 0\right)$ ✓ (larger x-value)

Find the interval on which f is concave up. (Enter your answer using interval notation.)

$\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$ ✓

Find the interval on which f is concave down. (Enter your answer using interval notation.)

$\left[0, \frac{3\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right]$ ✓

Q7.2

Saturday, October 17, 2020 8:15 PM

$f(x) = 7\sin(x) + 7\cos(x)\ ,\ 0 \le x \le 2\pi$

$f'(x) = \frac{d}{dx}\left[7\sin(x) + 7\cos(x)\right]$

$= 7\cos(x) - 7\sin(x)$

$7\cos(x) - 7\sin(x) = 0$

$7\cos(x) = 7\sin(x)$

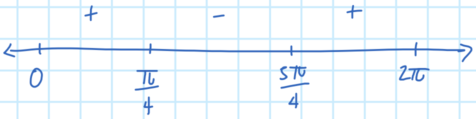
$\frac{\cancel{7\cos(x)}}{\cancel{7\cos(x)}} = \frac{\cancel{7}\sin(x)}{\cancel{7}\cos(x)}$

$1 = \frac{\sin(x)}{\cos(x)}$

$1 = \tan(x)$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$0 \le x \le 2\pi$

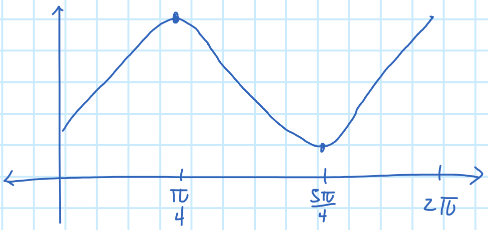


Intervals	$f'(x)$
$0 \le x < \pi/4$	$f'(\pi/8) = +$
$\pi/4 < x < 5\pi/4$	$f'(\pi) = -$
$5\pi/4 < x \le 2\pi$	$f'(\frac{3\pi}{2}) = +$

NOTE
these are random numbers
within their respective intervals

f is increasing @
$\left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$

f is decreasing @
$\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$



$f(\frac{\pi}{4}) = 7\sin(x) + 7\cos(x)$

$= 7\sin(\frac{\pi}{4}) + 7\cos(\frac{\pi}{4})$

$= 7\frac{\sqrt{2}}{2} + 7\frac{\sqrt{2}}{2}$

$= 14\frac{\sqrt{2}}{2}$

$= 7\sqrt{2}$

$f(\frac{5\pi}{4}) = 7\sin(x) + 7\cos(x)$

$= 7\sin(\frac{5\pi}{4}) + 7\cos(\frac{5\pi}{4})$

$= 7(-\frac{\sqrt{2}}{2}) + 7(-\frac{\sqrt{2}}{2})$

$= -7\frac{\sqrt{2}}{2} - 7\frac{\sqrt{2}}{2}$

$= -14\frac{\sqrt{2}}{2}$

$= -7\sqrt{2}$

Local min value $= -7\sqrt{2}$

Local max value $= 7\sqrt{2}$

$f''(x) = \frac{d}{dx}\left[7\cos(x) - 7\sin(x)\right]$

$= -7\sin(x) - 7\cos(x)$

$-7\sin(x) - 7\cos(x) = 0$

$-7\sin(x) = 7\cos(x)$

$\frac{-\cancel{7}\sin(x)}{\cancel{7}\cos(x)} = \frac{\cancel{7\cos(x)}}{\cancel{7\cos(x)}}$

$-\tan(x) = 1$

$\tan(x) = -1$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

Intervals	$f''(x)$
$0 \le x < 3\pi/4$	$f''(\frac{\pi}{2}) = -$
$3\pi/4 < x < 7\pi/4$	$f''(\pi) = +$
$7\pi/4 < x \le 2\pi$	$f''(\frac{5\pi}{8}) = -$

$f(\frac{3\pi}{4}) = 7\sin(x) + 7\cos(x)$

$= 7\sin(\frac{3\pi}{4}) + 7\cos(\frac{3\pi}{4})$

$= 7\frac{\sqrt{2}}{2} + 7(-\frac{\sqrt{2}}{2})$

$= 7\frac{\sqrt{2}}{2} - 7\frac{\sqrt{2}}{2}$

$= 0$

$f(\frac{7\pi}{4}) = 7\sin(x) + 7\cos(x)$

$= 7\sin(\frac{7\pi}{4}) + 7\cos(\frac{7\pi}{4})$

$= 7(-\frac{\sqrt{2}}{2}) + 7\frac{\sqrt{2}}{2}$

$= -7\frac{\sqrt{2}}{2} + 7\frac{\sqrt{2}}{2}$

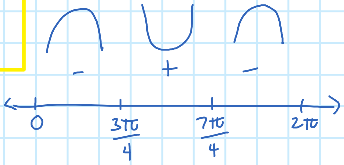
$= 0$

Inflection points
$(\frac{3\pi}{4}, 0), (\frac{7\pi}{4}, 0)$

$f''(x) > 0$, concave upwards

$f''(x) < 0$, concave downwards

$0 \le x \le 2\pi$

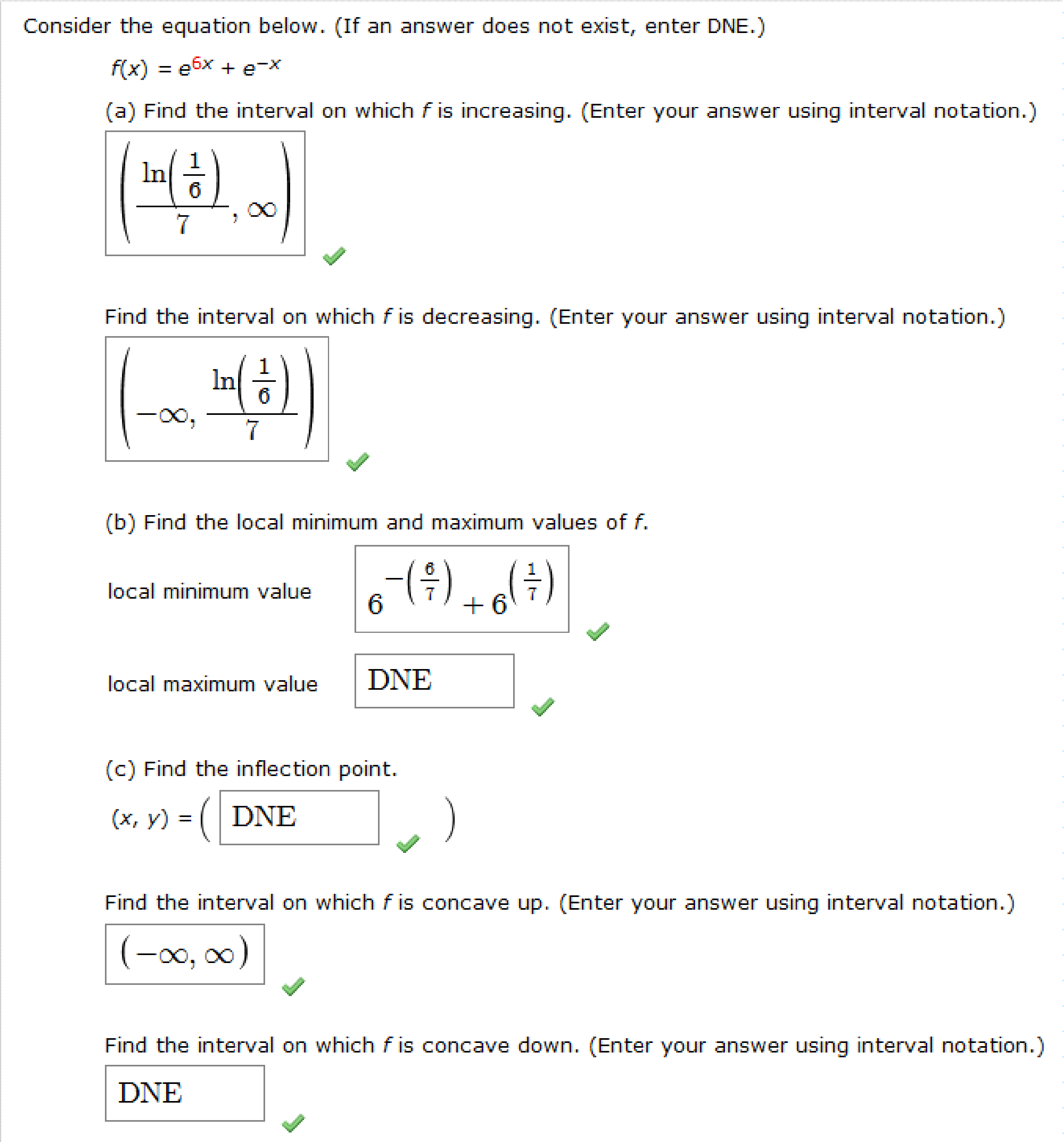


concave downwards @
$\left[0, \frac{3\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right]$

concave upwards @
$\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$

Q8.1

Monday, October 19, 2020 1:45 PM



Consider the equation below. (If an answer does not exist, enter DNE.)

$f(x) = e^{6x} + e^{-x}$

(a) Find the interval on which f is increasing. (Enter your answer using interval notation.)

$\left(\frac{\ln\left(\frac{1}{6}\right)}{7}, \infty\right)$ ✓

Find the interval on which f is decreasing. (Enter your answer using interval notation.)

$\left(-\infty, \frac{\ln\left(\frac{1}{6}\right)}{7}\right)$ ✓

(b) Find the local minimum and maximum values of f.

local minimum value $6^{-\left(\frac{6}{7}\right)} + 6^{\left(\frac{1}{7}\right)}$ ✓

local maximum value DNE ✓

(c) Find the inflection point.

$(x, y) =$ (DNE ✓)

Find the interval on which f is concave up. (Enter your answer using interval notation.)

$(-\infty, \infty)$ ✓

Find the interval on which f is concave down. (Enter your answer using interval notation.)

DNE ✓

Q8.2

Saturday, October 17, 2020 9:25 PM

$f(x) = e^{6x} + e^{-x}$

$f'(x) = \frac{d}{dx}\left(e^{6x} + e^{-x}\right)$

$= e^{6x}\frac{d}{dx}(6x) + e^{-x}\frac{d}{dx}(-x)$

$= e^{6x}(6) + e^{-x}(-1)$

$f'(x) = 6e^{6x} - e^{-x}$

$6e^{6x} - e^{-x} = 0$

$e^{-x}(6e^{7x} - 1) = 0$

will never equals to 0 for all values of x

$6e^{7x} - 1 = 0$

$6e^{7x} = 1$

$e^{7x} = \frac{1}{6}$

$7x = \ln\frac{1}{6}$

$x = \frac{\ln\frac{1}{6}}{7} \approx -0.26$

$f''(x) = \frac{d}{dx}\left(6e^{6x} - e^{-x}\right)$

$= 6(e^{6x})\frac{d}{dx}(6x) - e^{-x}\frac{d}{dx}(-x)$

$= 6e^{6x}(6) - e^{-x}(-1)$

$= 36e^{6x} + e^{-x}$

$f''(x) = e^{-x}(36e^{7x} + 1)$

$e^{-x}(36e^{7x} + 1) = 0$

$36e^{7x} + 1 = 0$

$36e^{7x} = -1$

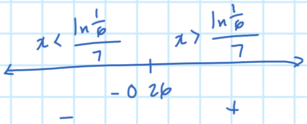
$e^{7x} = -\frac{1}{36}$

e^x where $x = \mathbb{R}$, will always return positive result

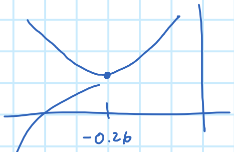
negative

$\therefore$ $e^{7x} > 0$, no inflection point

no places where $f'' = 0$ or undefined



Interval	$f'(x)$
$x < \frac{\ln\frac{1}{6}}{7}$	$f'(-1) = -$
$x > \frac{\ln\frac{1}{6}}{7}$	$f'(0) = +$



f decreasing @ $\left(-\infty, \frac{\ln\frac{1}{6}}{7}\right)$

f increasing @ $\left(\frac{\ln\frac{1}{6}}{7}, \infty\right)$

$f\left(\frac{\ln\frac{1}{6}}{7}\right) = e^{6x} + e^{-x}$

$= e^{6\left(\frac{\ln\frac{1}{6}}{7}\right)} + e^{-\left(\frac{\ln\frac{1}{6}}{7}\right)}$

$= e^{\ln\left(\frac{1}{6}\right)^{6/7}} + e^{\ln\left(\frac{1}{6}\right)^{-1/7}}$

$= \left(\frac{1}{6}\right)^{6/7} + \left(\frac{1}{6}\right)^{-1/7}$

$= (6^{-1})^{6/7} + (6^{-1})^{-1/7}$

$= 6^{-6/7} + 6^{1/7} \approx 1\,51 \rightarrow$ Local minimum

DNE → Local maximum

Check concavity

$f''(0) = 36e^{6(0)} + e^{-0}$

$= 36(1) + 1$

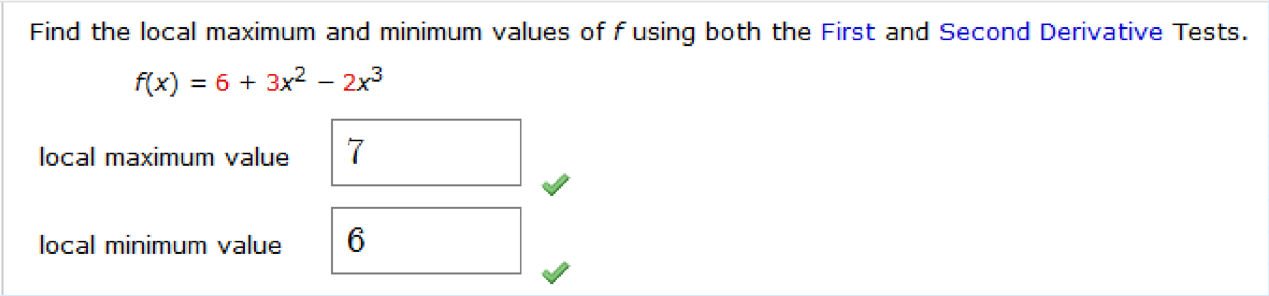
$= 37$

$37 > 0$

$\therefore$ f is concave upward on $(-\infty, \infty)$

Q9

Monday, October 19, 2020 1:45 PM



Find the local maximum and minimum values of f using both the First and Second Derivative Tests.

$f(x) = 6 + 3x^2 - 2x^3$

local maximum value 7 ✓

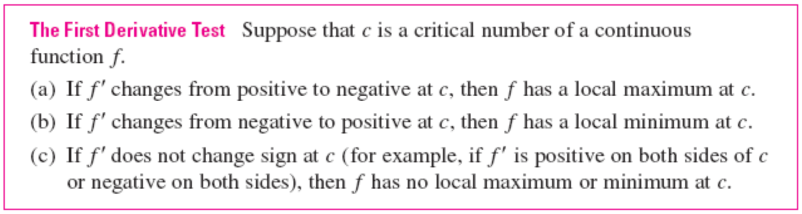
local minimum value 6 ✓

$f(x) = 6 + 3x^2 - 2x^3$

$f'(x) = \frac{d}{dx}\left(6 + 3x^2 - 2x^3\right)$

$= 0 + (2)3x - (3)2x^2$

$f'(x) = 6x - 6x^2$

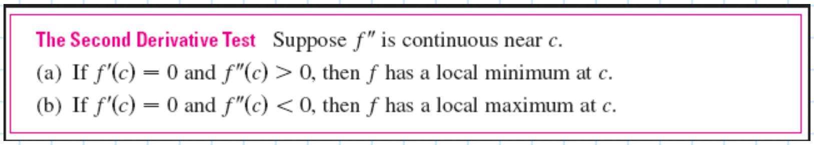


The First Derivative Test Suppose that c is a critical number of a continuous function f.

(a) If f' changes from positive to negative at c, then f has a local maximum at c.

(b) If f' changes from negative to positive at c, then f has a local minimum at c.

(c) If f' does not change sign at c (for example, if f' is positive on both sides of c or negative on both sides), then f has no local maximum or minimum at c.



The Second Derivative Test Suppose f'' is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

$6x - 6x^2 = 0$

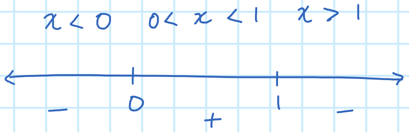
$6x(1 - x) = 0$

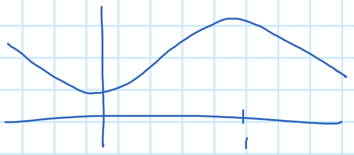
$6x = 0$ $\quad$ $1 - x = 0$

$x = 0$ $\quad$ $-x = -1$

$x = 1$

Interval	$6x$	$1-x$	$f'(x)$
$x<0$	$-$	$+$	$-$
$0<x<1$	$+$	$+$	$+$
$x>1$	$+$	$-$	$-$





$f''(x) = \frac{d}{dx}\left(6x - 6x^2\right)$

$= 6 - (2)6x$

$f''(x) = 6 - 12x$

$f''(0) = 6x - 12x$

$= 6(0) - 12(0)$

$= 0 - 0$

$f''(0) = 0$

$f''(1) = 6x - 12x$

$= 6(1) - 12(1)$

$= 6 - 12$

$f''(1) = -6$

$\downarrow$

$-6 < 0$

$x = 1$ Local max

$f(0) = 6 + 3x^2 - 2x^3$

$= 6 + 3(0)^2 - 2(0)^3$

$= 6 + 0 - 0$

$= \boxed{6 \longrightarrow \text{Local min}}$

$f(1) = 6 + 3x^2 - 2x^3$

$= 6 + 3(1)^2 - 2(1)^3$

$= 6 + 3 - 2$

$= \boxed{7 \longrightarrow \text{Local max}}$

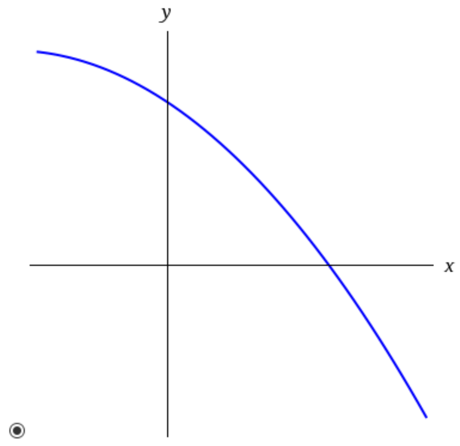
Q10

Monday, October 19, 2020 2:10 PM

Sketch the graph of a function that satisfies all of the given conditions.

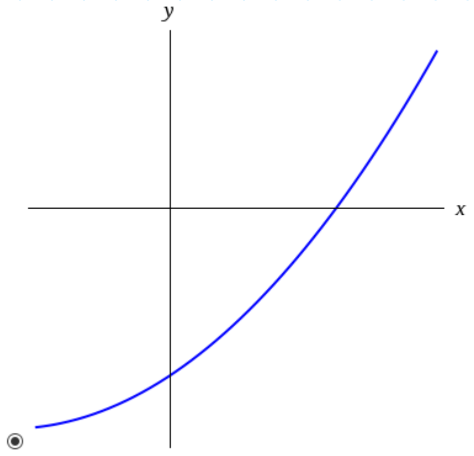
(a) $f'(x) < 0$ and $f''(x) < 0$ for all x

slope decreasing concave down



(b) $f'(x) > 0$ and $f''(x) > 0$ for all x

slope increasing concave up



Q11

Monday, October 19, 2020 2:14 PM

Sketch the graph of a function that satisfies all of the given conditions.

$f'(0) = f'(2) = f'(4) = 0,$

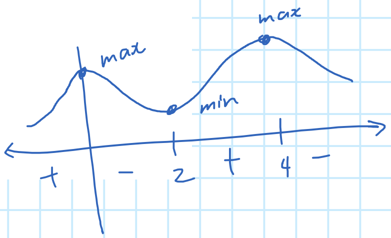
2, 4 = critical points

$f'(x) > 0$ if $x < 0$ or $2 < x < 4$, increasing

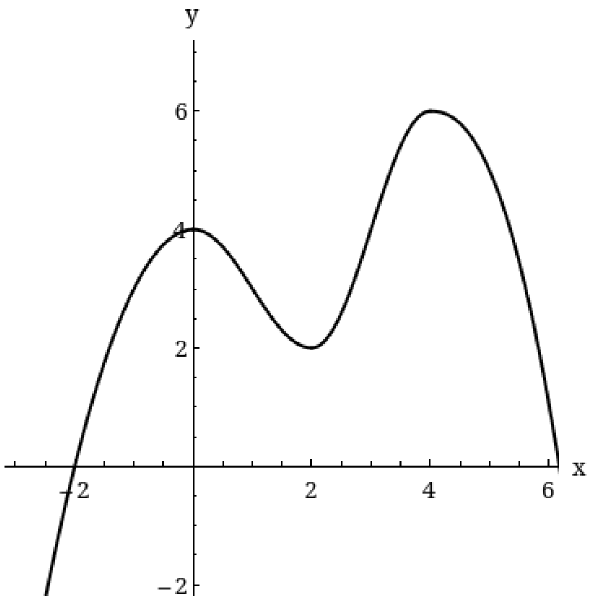
$f'(x) < 0$ if $0 < x < 2$ or $x > 4$, decreasing

$f''(x) > 0$ if $1 < x < 3$, Local min

$f''(x) < 0$ if $x < 1$ or $x > 3$ Local max



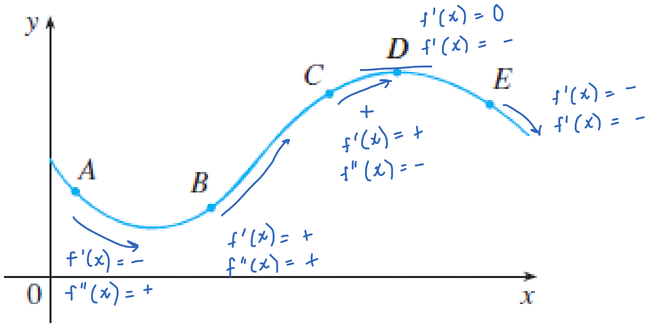
●



Q12

Monday, October 19, 2020 2:24 PM

The graph of a function $y = f(x)$ is shown. At which point(s) are the following true? (Select all that apply.)



$$f'(x) = \frac{dy}{dx} = \text{slope}$$

$$f''(x) = \frac{d^2y}{dx^2} = \text{rate of which slope is changing}$$

(a) $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ are both positive.

☐ A
☑ B
☐ C
☐ D
☐ E
✓

(b) $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ are both negative.

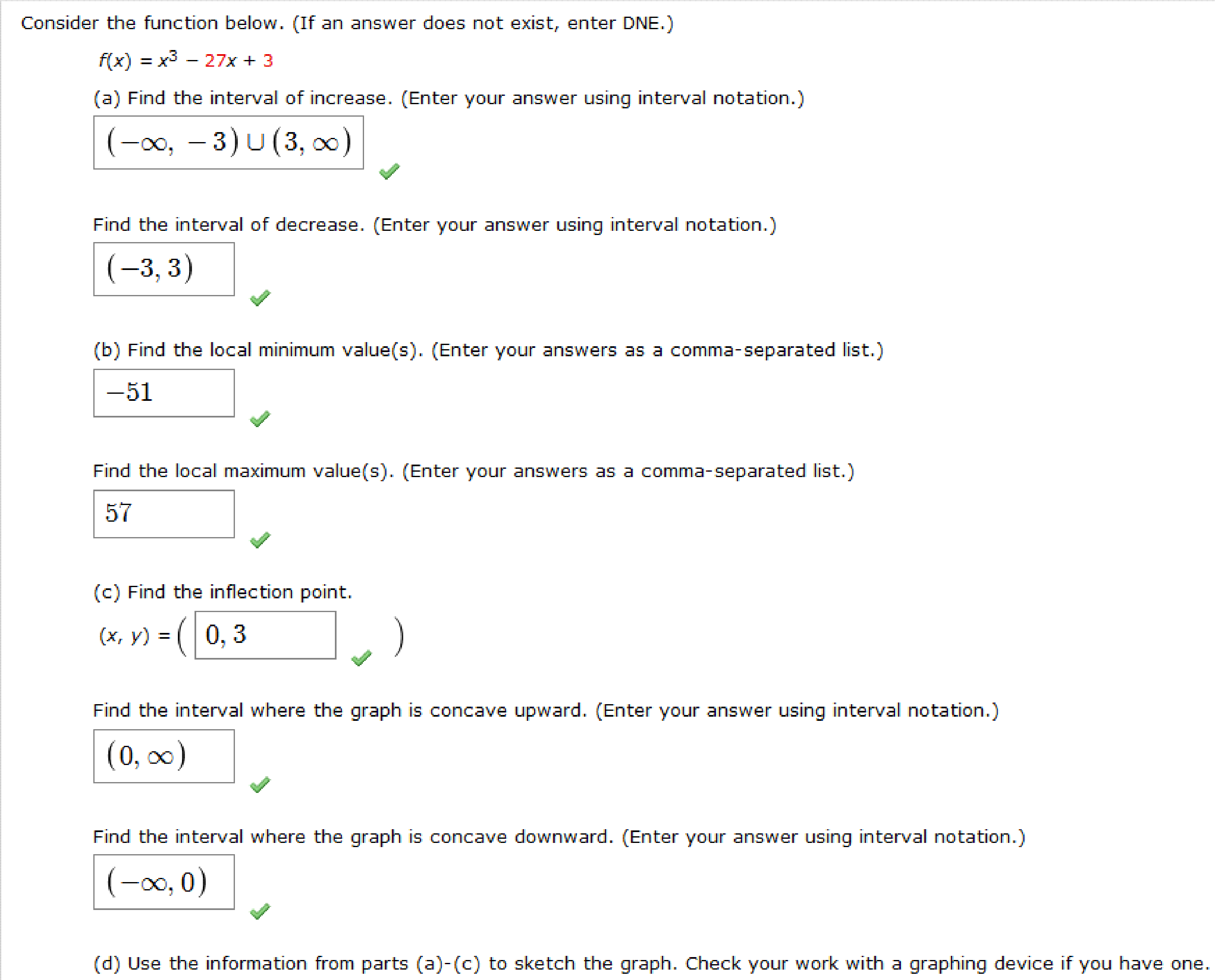
☐ A
☐ B
☐ C
☐ D
☑ E
✓

(c) $\frac{dy}{dx}$ is negative but $\frac{d^2y}{dx^2}$ is positive.

☑ A
☐ B
☐ C
☐ D
☐ E
✓

Q13.1

Monday, October 19, 2020 3:11 PM



Consider the function below. (If an answer does not exist, enter DNE.)

$f(x) = x^3 - 27x + 3$

(a) Find the interval of increase. (Enter your answer using interval notation.)

$(-\infty, -3) \cup (3, \infty)$ ✓

Find the interval of decrease. (Enter your answer using interval notation.)

$(-3, 3)$ ✓

(b) Find the local minimum value(s). (Enter your answers as a comma-separated list.)

-51 ✓

Find the local maximum value(s). (Enter your answers as a comma-separated list.)

57 ✓

(c) Find the inflection point.

$(x, y) = (0, 3)$ ✓

Find the interval where the graph is concave upward. (Enter your answer using interval notation.)

$(0, \infty)$ ✓

Find the interval where the graph is concave downward. (Enter your answer using interval notation.)

$(-\infty, 0)$ ✓

(d) Use the information from parts (a)-(c) to sketch the graph. Check your work with a graphing device if you have one.

Q13.2

Monday, October 19, 2020 2:34 PM

$f(x) = x^3 - 27x + 3$

$f'(x) = \frac{d}{dx}\left(x^3 - 27x + 3\right)$

$= 3x^2 - 27 - 0$

$f'(x) = 3x^2 - 27$

critical point(s)

$3x^2 - 27 = 0$

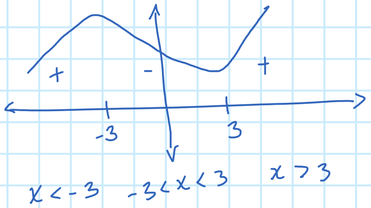
$3x^2 = 27$

$\frac{3x^2}{3} = \frac{27}{3}$

$x^2 = 9$

$\sqrt{x^2} = \sqrt{9}$

$x = \pm 3$



Intervals	$f'(x)$
$x < -3$	$f'(-4) = +$
$-3 < x < 3$	$f'(0) = -$
$x > 3$	$f'(4) = +$

f is increasing @ $(-\infty, -3) \cup (3, \infty)$

f is decreasing @ $(-3, 3)$

$f(-3) = x^3 - 27x + 3$

$= (-3)^3 - 27(-3) + 3$

$= -27 + 81 + 3$

$f(-3) = 57 \longrightarrow$ Local min

$f(3) = x^3 - 27x + 3$

$= (3)^3 - 27(3) + 3$

$= 27 - 81 + 3$

$f(3) = -51 \longrightarrow$ Local max

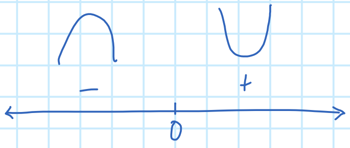
$f''(x) = \frac{d}{dx}\left(3x^2 - 27\right)$

$= (2)3x - 0$

$f''(x) = 6x$

$6x = 0$

$x = 0$



Intervals	$f''(x)$
$x < 0$	$f''(-1) = -$
$x > 0$	$f''(1) = +$

f is concave downwards @ $(-\infty, 0)$

f is concave upwords @ $(0, \infty)$

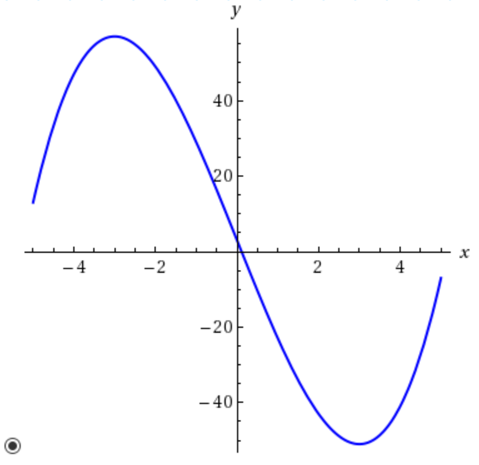
Get Inflection point(s)

$f(0) = x^3 - 27x + 3$

$= (0)^3 - 27(0) + 3$

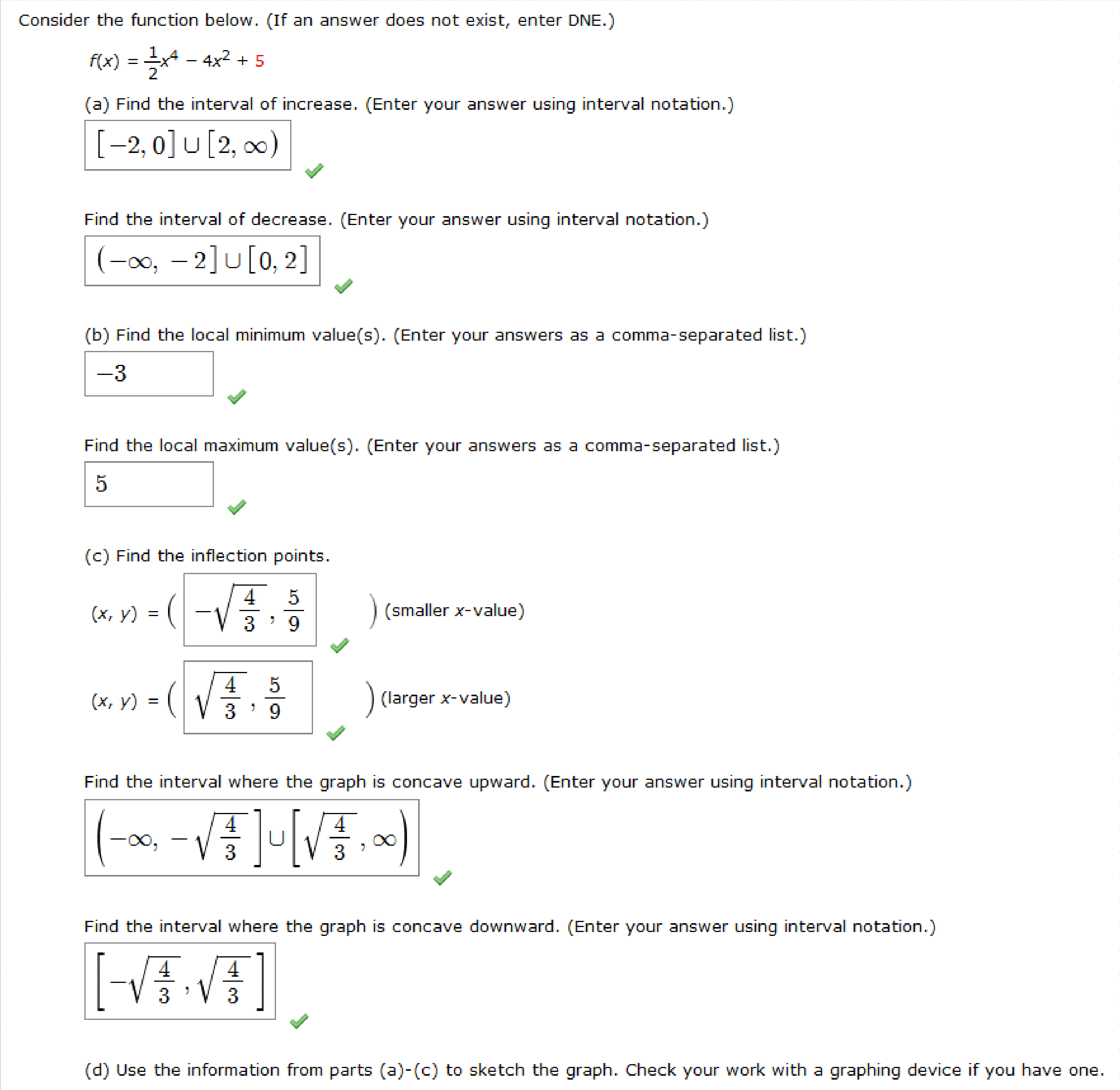
$= 3$

Inflection point @ $(0, 3)$



Q14.1

Monday, October 19, 2020 3:12 PM



Consider the function below. (If an answer does not exist, enter DNE.)

$f(x) = \frac{1}{2}x^4 - 4x^2 + 5$

(a) Find the interval of increase. (Enter your answer using interval notation.)

$[-2, 0] \cup [2, \infty)$ ✓

Find the interval of decrease. (Enter your answer using interval notation.)

$(-\infty, -2] \cup [0, 2]$ ✓

(b) Find the local minimum value(s). (Enter your answers as a comma-separated list.)

-3 ✓

Find the local maximum value(s). (Enter your answers as a comma-separated list.)

5 ✓

(c) Find the inflection points.

$(x, y) = \left(-\sqrt{\frac{4}{3}}, \frac{5}{9}\right)$ ✓ (smaller x-value)

$(x, y) = \left(\sqrt{\frac{4}{3}}, \frac{5}{9}\right)$ ✓ (larger x-value)

Find the interval where the graph is concave upward. (Enter your answer using interval notation.)

$\left(-\infty, -\sqrt{\frac{4}{3}}\right] \cup \left[\sqrt{\frac{4}{3}}, \infty\right)$ ✓

Find the interval where the graph is concave downward. (Enter your answer using interval notation.)

$\left[-\sqrt{\frac{4}{3}}, \sqrt{\frac{4}{3}}\right]$ ✓

(d) Use the information from parts (a)-(c) to sketch the graph. Check your work with a graphing device if you have one.

Q14.2

Monday, October 19, 2020 3:12 PM

$f(x) = \frac{1}{2}x^4 - 4x^2 + 5$

$f'(x) = \frac{d}{dx}\left(\frac{1}{2}x^4 - 4x^2 + 5\right)$

$= (4)\frac{1}{2}x^3 - (2)4x + 0$

$f'(x) = 2x^3 - 8x$

Get critical points

$2x^3 - 8x = 0$

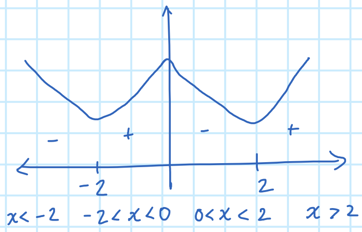
$2x(x^2 - 4) = 0$

$2x = 0$ $\qquad$ $x^2 - 4 = 0$

$x = 0$ $\qquad$ $x^2 = 4$

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$



Interval	$6x$	$x^2 - 4$	$f'(x)$
$x < -2$	−	+	−
$-2 < x < 0$	−	−	+
$0 < x < 2$	+	−	−
$x > 2$	+	+	+

$\boxed{f \text{ is increasing @ } [-2, 0] \cup [2, \infty)}$

$\boxed{f \text{ is decreasing @ } (-\infty, -2] \cup [0, 2]}$

$f(-2) = \frac{1}{2}x^4 - 4x^2 + 5$

$= \frac{1}{2}(-2)^4 - 4(-2)^2 + 5$

$= \frac{1}{2}(16) - 4(4) + 5$

$= 8 - 16 + 5$

$f(-2) = -3$

$f(0) = \frac{1}{2}x^4 - 4x^2 + 5$

$= \frac{1}{2}(0)^4 - 4(0)^2 + 5$

$= 0 - 0 + 5$

$f(0) = 5$

$f(2) = \frac{1}{2}x^4 - 4x^2 + 5$

$= \frac{1}{2}(2)^4 - 4(2)^2 + 5$

$= \frac{1}{2}(16) - 4(4) + 5$

$= 8 - 16 + 5$

$f(2) = -3$

$\boxed{\text{Local max} = 5}$

$\boxed{\text{Local min} = -3}$

$f''(x) = \frac{d}{dx}(2x^3 - 8x)$

$= (3)2x^2 - 8$

$f''(x) = 6x^2 - 8$

Get inflection points

$6x^2 - 8 = 0$

$6x^2 = 8$

$x^2 = \frac{8}{6}$

$\sqrt{x^2} = \sqrt{\frac{4}{3}}$

$x = \pm\sqrt{\frac{4}{3}} \approx \pm 1\ 15$

$f\left(-\sqrt{\frac{4}{3}}\right) = \frac{1}{2}x^4 - 4x^2 + 5$

$= \frac{1}{2}\left(-\sqrt{\frac{4}{3}}\right)^4 - 4\left(-\sqrt{\frac{4}{3}}\right)^2 + 5$

$= \frac{1}{2}\left(\frac{4}{3}^{1/2}\right)^4 - 4\left(\frac{4}{3}^{1/2}\right)^2 + 5$

$= \frac{1}{2}\left(-\frac{4}{3}\right)^2 - 4\left(\frac{4}{3}\right) + 5$

$= \frac{1}{2}\left(\frac{16}{9}\right) - \frac{16}{3} + 5$

$= \frac{8}{9} - \frac{16}{3} + \frac{5}{1}$

$= \frac{8}{9} - \frac{48}{9} + \frac{45}{9}$

$f\left(-\sqrt{\frac{4}{3}}\right) = \frac{5}{9}$

$f\left(\sqrt{\frac{4}{3}}\right) = \frac{1}{2}x^4 - 4x^2 + 5$

$= \frac{1}{2}\left(\sqrt{\frac{4}{3}}\right)^4 - 4\left(\sqrt{\frac{4}{3}}\right)^2 + 5$

$= \frac{1}{2}\left(\frac{4}{3}^{1/2}\right)^4 - 4\left(\frac{4}{3}^{1/2}\right)^2 + 5$

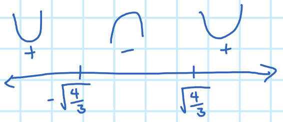
$= \frac{1}{2}\left(-\frac{4}{3}\right)^2 - 4\left(\frac{4}{3}\right) + 5$

$= \frac{1}{2}\left(\frac{16}{9}\right) - \frac{16}{3} + 5$

$= \frac{8}{9} - \frac{16}{3} + \frac{5}{1}$

$= \frac{8}{9} - \frac{48}{9} + \frac{45}{9}$

$f\left(\sqrt{\frac{4}{3}}\right) = \frac{5}{9}$



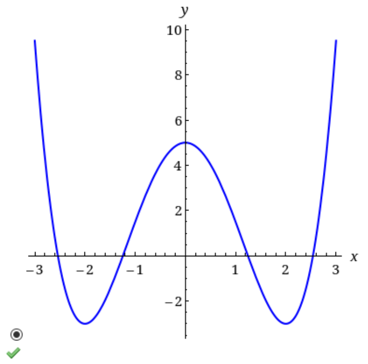
$x < -\sqrt{\frac{4}{3}}$ $\qquad$ $-\sqrt{\frac{4}{3}} < x < \sqrt{\frac{4}{3}}$ $\qquad$ $x > \sqrt{\frac{4}{3}}$

Intervals	$f''(x)$
$x < -\sqrt{\frac{4}{3}}$	$f''(-2) = +$
$-\sqrt{\frac{4}{3}} < x < \sqrt{\frac{4}{3}}$	$f''(0) = -$
$x > \sqrt{\frac{4}{3}}$	$f''(2) = +$

$\boxed{f \text{ is concave upward @ } \left(-\infty, -\sqrt{\frac{4}{3}}\right] \cup \left[\sqrt{\frac{4}{3}}, \infty\right)}$

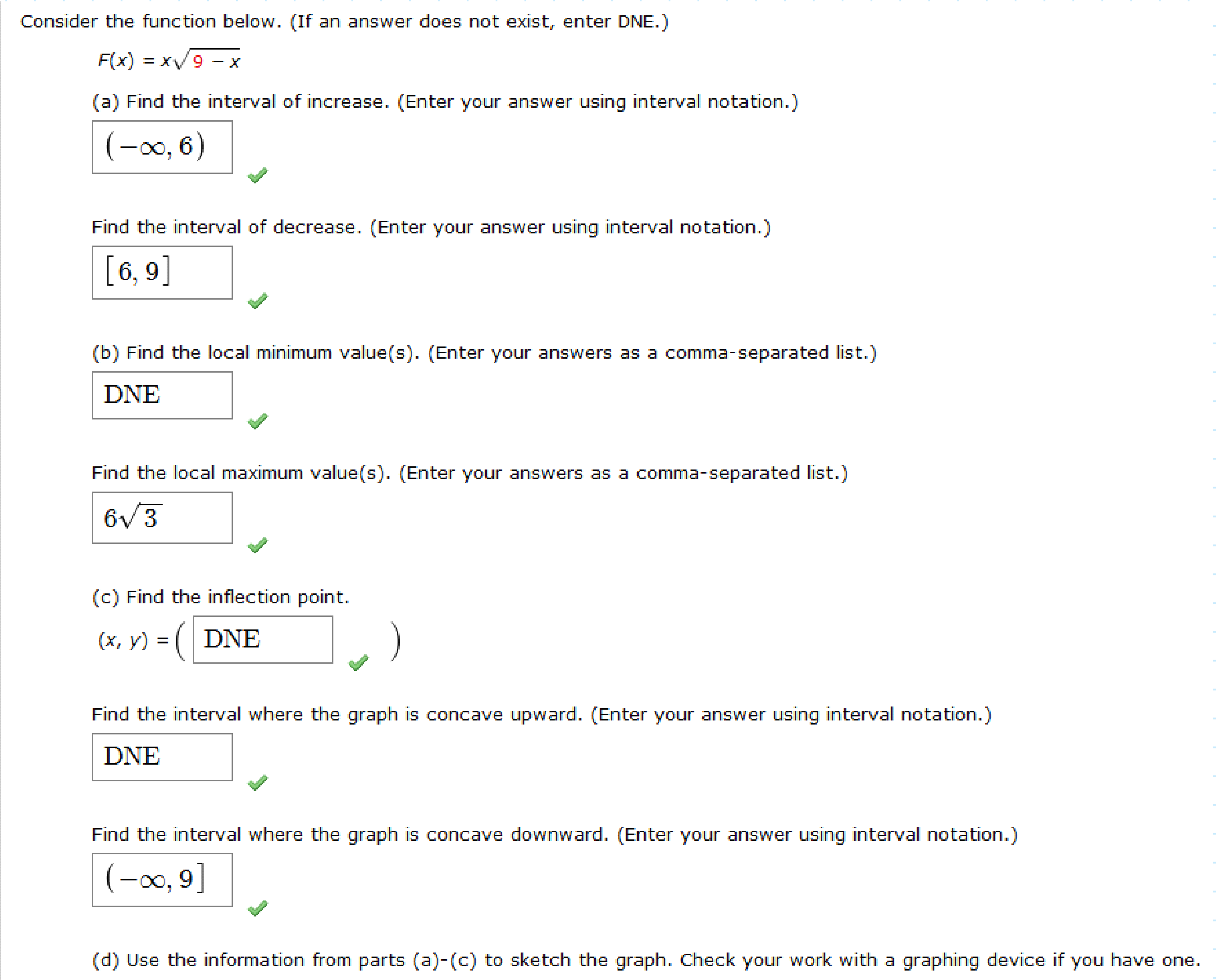
$\boxed{f \text{ is concave downward @ } \left[-\sqrt{\frac{4}{3}}, \sqrt{\frac{4}{3}}\right]}$

$\boxed{\text{Inflection points } \left(-\sqrt{\frac{4}{3}}, \frac{5}{9}\right), \left(\sqrt{\frac{4}{3}}, \frac{5}{9}\right)}$



Q15.1

Monday, October 19, 2020 8:19 PM



Consider the function below. (If an answer does not exist, enter DNE.)

$F(x) = x\sqrt{9 - x}$

(a) Find the interval of increase. (Enter your answer using interval notation.)

$(-\infty, 6)$ ✓

Find the interval of decrease. (Enter your answer using interval notation.)

$[6, 9]$ ✓

(b) Find the local minimum value(s). (Enter your answers as a comma-separated list.)

DNE ✓

Find the local maximum value(s). (Enter your answers as a comma-separated list.)

$6\sqrt{3}$ ✓

(c) Find the inflection point.

$(x, y) = ($ DNE ✓ $)$

Find the interval where the graph is concave upward. (Enter your answer using interval notation.)

DNE ✓

Find the interval where the graph is concave downward. (Enter your answer using interval notation.)

$(-\infty, 9]$ ✓

(d) Use the information from parts (a)-(c) to sketch the graph. Check your work with a graphing device if you have one.

Q15.2

Monday, October 19, 2020 8:19 PM

$$F(x) = x\sqrt{9-x}$$

$$F'(x) = \frac{d}{dx}\left(x\sqrt{9-x}\right)$$

$$= x\frac{d}{dx}\left(\sqrt{9-x}\right) + \sqrt{9-x}\,\frac{d}{dx}(x)$$

Chain Rule

$u = 9-x$

$f(u) = \sqrt{u} = y$

$\frac{dy}{du} \cdot \frac{du}{dx}$

$$= x\frac{dy}{du}\left(\sqrt{u}\right)\frac{du}{dx}(u) + \sqrt{9-x}\,(1)$$

$$= x\frac{dy}{du}\left(u^{1/2}\right)\frac{d}{dx}(9-x) + \sqrt{9-x}$$

$$= x\left(\frac{1}{2}u^{-1/2}\right)(-1) + \sqrt{9-x}$$

$$= -x\left(\frac{1}{2}[9-x]^{-1/2}\right) + \sqrt{9-x}$$

$$= -x\left(\frac{1}{2}\left[\frac{1}{\sqrt{9-x}}\right]\right) + \sqrt{9-x}$$

$$= -\frac{x}{2\sqrt{9-x}} + \sqrt{9-x}$$

$$= -\frac{x}{2\sqrt{9-x}} + \frac{\sqrt{9-x}}{1}\frac{(2\sqrt{9-x})}{(2\sqrt{9-x})}$$

$$= -\frac{x}{2\sqrt{9-x}} + \frac{2(9-x)}{2\sqrt{9-x}}$$

$$F'(x) = \frac{-x+18-2x}{2\sqrt{9-x}} = \frac{-3x+18}{2\sqrt{9-x}}$$

Get critical point where $F'(x) = 0$ or undefined

$$\frac{-3x+18}{2\sqrt{9-x}} = 0$$

$-3x + 18 = 0$

$-3x = -18$

$x = \frac{-18}{-3}$

$x = 6$

$\sqrt{9-x} = 0$

$\sqrt{9-x}^2 = 0^2$

$9 - x = 0$

$-x = -9$

$x = 9$

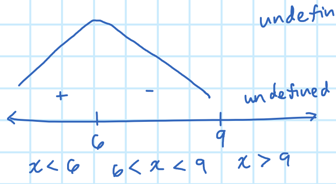
undefined

$f(6) = x\sqrt{9-x}$

$= 6\sqrt{9-6}$

$f(6) = 6\sqrt{3}$

Local max → $6\sqrt{3}$



Intervals	$-3x+18$	$2\sqrt{9-x}$	$f'(x)$
$x < 6$	+	+	+
$6 < x < 9$	−	+	−
$x > 9$	-	undefined	

f is increasing @ $(-\infty, 6]$

f is decreasing @ $[6, 9]$

$$F''(x) = \frac{d}{dx}\left(\frac{-3x+18}{2\sqrt{9-x}}\right)$$

$$= \frac{2\sqrt{9-x}\,\frac{d}{dx}(-3x+18) - (-3x+18)\frac{d}{dx}\left(2\sqrt{9-x}\right)}{(2\sqrt{9-x})^2}$$

Chain Rule

$u = 9-x$

$f(u) = \sqrt{u} = y$

$\frac{dy}{du}\ \frac{du}{dx}$

$$= \frac{2\sqrt{9-x}\,(-3) - (-3x+18)(2)\frac{dy}{du}\left(\sqrt{u}\right)\frac{du}{dx}(u)}{(2\sqrt{9-x})^2}$$

$$= \frac{-6\sqrt{9-x} - (-6x+36)\frac{dy}{du}\left(u^{1/2}\right)\frac{d}{dx}(9-x)}{(2\sqrt{9-x})^2}$$

$$= \frac{-6\sqrt{9-x} - (-6x+36)\left(\frac{1}{2}u^{-1/2}\right)(-1)}{(2\sqrt{9-x})^2} = \frac{-6\sqrt{9-x} - (-6x+36)\left(-\frac{1}{2}\left[\frac{1}{\sqrt{u}}\right]\right)}{(2\sqrt{9-x})^2}$$

$$= \frac{-6\sqrt{9-x} - (-6x+36)\left(-\frac{1}{2\sqrt{9-x}}\right)}{(2\sqrt{9-x})^2} = \frac{-6\sqrt{9-x} - \left(6x\frac{1}{2\sqrt{9-x}} - 36\frac{1}{2\sqrt{9-x}}\right)}{(2\sqrt{9-x})^2}$$

$$= \frac{-6\sqrt{9-x} - \frac{3x}{\sqrt{9-x}} + \frac{18}{\sqrt{9-x}}}{(2\sqrt{9-x})^2} = \frac{\frac{-6\sqrt{9-x}}{1}\frac{(\sqrt{9-x})}{(\sqrt{9-x})} - \frac{3x}{\sqrt{9-x}} + \frac{18}{\sqrt{9-x}}}{(2\sqrt{9-x})^2}$$

$$= \frac{\frac{-6(9-x)}{\sqrt{9-x}} - \frac{3x}{\sqrt{9-x}} + \frac{18}{\sqrt{9-x}}}{(2\sqrt{9-x})^2} = \frac{\frac{-54+6x}{\sqrt{9-x}} - \frac{3x}{\sqrt{9-x}} + \frac{18}{\sqrt{9-x}}}{(2\sqrt{9-x})^2}$$

$$= \frac{\frac{3x-36}{\sqrt{9-x}}}{(2\sqrt{9-x})^2} = \frac{\frac{3x-36}{\sqrt{9-x}}}{(2\sqrt{9-x})(2\sqrt{9-x})} = \frac{3x-36}{\sqrt{9-x}}\left[\frac{1}{(2\sqrt{9-x})(2\sqrt{9-x})}\right]$$

$$F''(x) = \frac{3(x-12)}{4(9-x)\sqrt{9-x}}$$

$$\frac{3(x-12)}{4(9-x)\sqrt{9-x}} = 0$$

$x - 12 = 0$

$x = 12$

$12 > 0$

∴ No inflection point = DNE

any number will do for concavity test without inflection point

$$F''(0) = \frac{3(x-12)}{4(9-x)\sqrt{9-x}}$$

$$= \frac{3(0-12)}{4(9-0)\sqrt{9-0}}$$

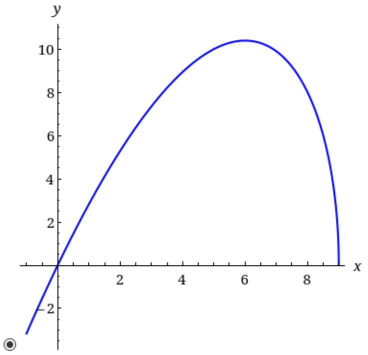
$$= \frac{-36}{36(3)}$$

$$F''(0) = \frac{-36}{108} = -\frac{1}{3}$$

$F''(x) < 0$

∴ $F(x)$ is concave downwards

$F(x)$ is concave downwards @ $(-\infty, 6]$

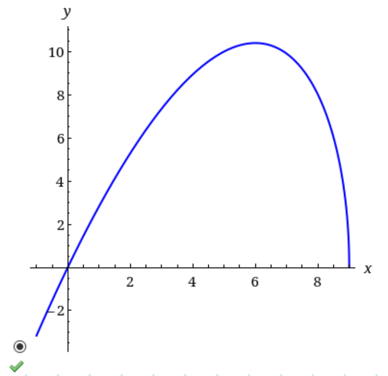


Intervals	$-3x+18$	$2\sqrt{9-x}$	$f'(x)$
$x<6$	+	+	+
$6<x<9$	−	+	−
$x>9$	−	undefined	

f is increasing @ $(-\infty, 6]$

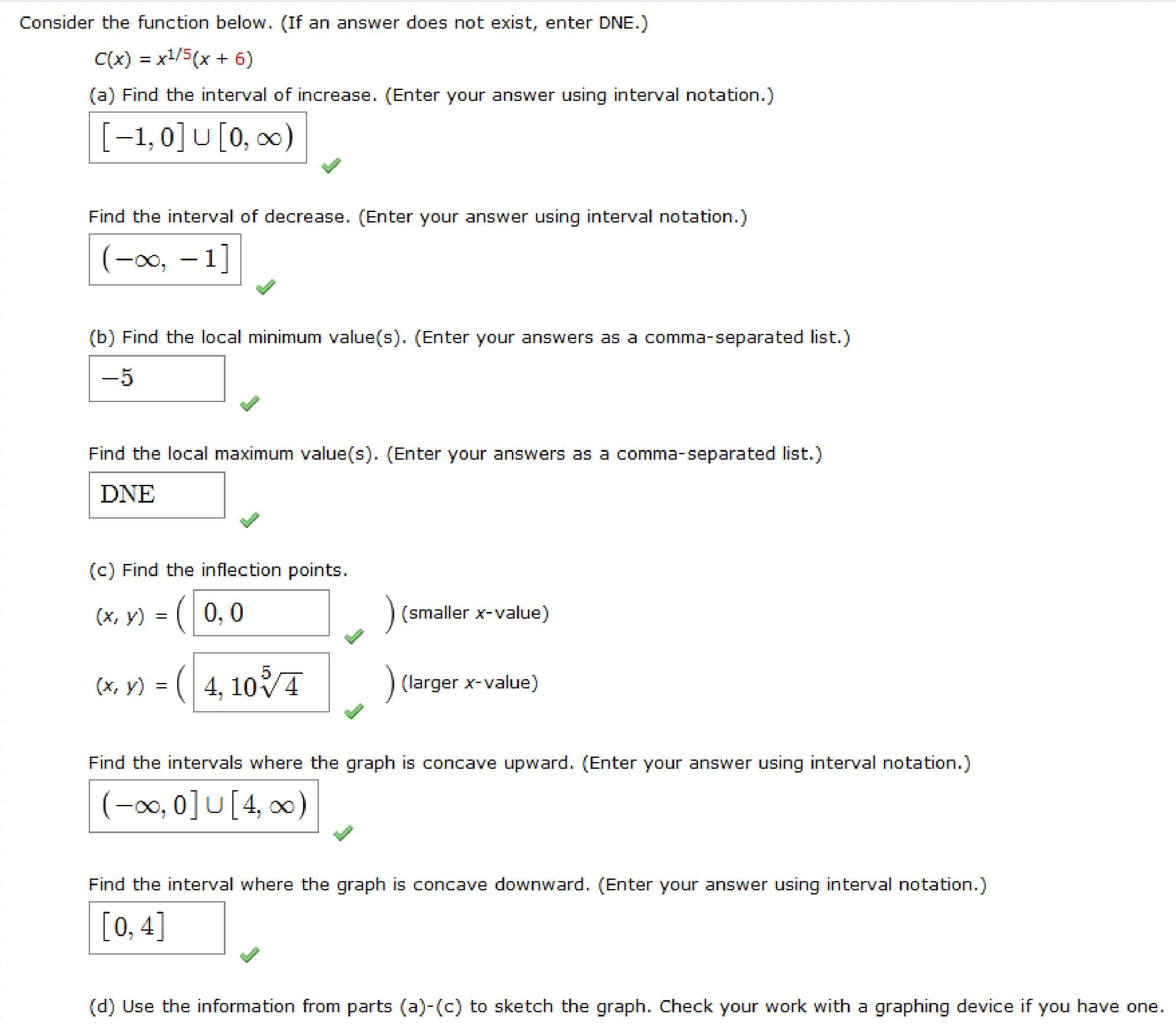
f is decreasing @ $[6, 9]$

F(x) is concave downwards @ $(-\infty, 6]$



Q16.1

Monday, October 19, 2020 11:26 PM



Consider the function below. (If an answer does not exist, enter DNE.)

$C(x) = x^{1/5}(x + 6)$

(a) Find the interval of increase. (Enter your answer using interval notation.)

$[-1, 0] \cup [0, \infty)$ ✓

Find the interval of decrease. (Enter your answer using interval notation.)

$(-\infty, -1]$ ✓

(b) Find the local minimum value(s). (Enter your answers as a comma-separated list.)

-5 ✓

Find the local maximum value(s). (Enter your answers as a comma-separated list.)

DNE ✓

(c) Find the inflection points.

$(x, y) = (0, 0)$ ✓ (smaller x-value)

$(x, y) = (4, 10\sqrt[5]{4})$ ✓ (larger x-value)

Find the intervals where the graph is concave upward. (Enter your answer using interval notation.)

$(-\infty, 0] \cup [4, \infty)$ ✓

Find the interval where the graph is concave downward. (Enter your answer using interval notation.)

$[0, 4]$ ✓

(d) Use the information from parts (a)-(c) to sketch the graph. Check your work with a graphing device if you have one.

Q16.2

Monday, October 19, 2020 11:26 PM

$C(x) = x^{1/5}(x+6)$

$C'(x) = \frac{d}{dx}\left[x^{1/5}(x+6)\right]$

$= x^{1/5}\frac{d}{dx}(x+6) + (x+6)\frac{d}{dx}(x^{1/5})$

$= x^{1/5}(1) + (x+6)\left(\frac{1}{5}x^{-4/5}\right)$

$= x^{1/5} + \frac{1}{5}x^{1/5} + \frac{6}{5}x^{-4/5}$

$C'(x) = \frac{6}{5}x^{1/5} + \frac{6}{5}x^{-4/5}$

Get Critical points where $C'(x) = 0$ or undefined

$\frac{6}{5}x^{1/5} + \frac{6}{5}x^{-4/5} = 0$

$5\left(\frac{6}{5}x^{1/5} + \frac{6}{5}x^{-4/5}\right) = (0)5$

$6x^{1/5} + 6x^{-4/5} = 0$

$6\left(x^{1/5} + x^{-4/5}\right) = 0$

$6\left(x^{1}x^{-4/5} + x^{-4/5}\right) = 0$

$6\left[x^{-4/5}(x+1)\right] = 0$

$6\left(\frac{x+1}{x^{4/5}}\right) = 0$

$\frac{6(x+1)}{x^{4/5}} = 0$

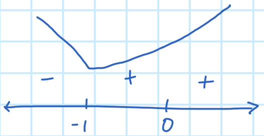
$x + 1 = 0$

$x = -1$

$x^{4/5} = 0$

$x = 0$

undefined



$x<-1$ $-1<x<0$ $x>0$

Intervals	$x+1$	$x^{4/5}$	$C'(x)$
$x<-1$	−	+	−
$-1<x<0$	+	+	+
$x>0$	+	+	+

f is increasing @ $[-1, 0] \cup [0, \infty)$

f is decreasing @ $(-\infty, -1]$

$f(-1) = x^{1/5}(x+6)$

$= -1^{1/5}(-1+6)$

$= -1(5)$

$f(-1) = -5$

Local min = −5

$x=0$, $f'(x)$ is undefined

∴ Local max = DNE

$C''(x) = \frac{d}{dx}\left(\frac{6}{5}x^{1/5} + \frac{6}{5}x^{-4/5}\right)$

$= \left(\frac{1}{5}\right)\frac{6}{5}x^{-4/5} + \left(-\frac{4}{5}\right)\frac{6}{5}x^{-9/5}$

$C''(x) = \frac{6}{25}x^{-4/5} - \frac{24}{25}x^{-9/5}$

Get inflection points where $C''(x) = 0$ or undefined

$\frac{6}{25}x^{-4/5} - \frac{24}{25}x^{-9/5} = 0$

$25\left(\frac{6}{25}x^{-4/5} - \frac{24}{25}x^{-9/5}\right) = 0(25)$

$6x^{-4/5} - 24x^{-9/5} = 0$

$6\left(x^{-4/5} - 4x^{-9/5}\right) = 0$

$6\left(x^{-4/5} - 4x^{-1}x^{-4/5}\right) = 0$

$6\left[x^{-4/5}\left(1 - 4x^{-1}\right)\right] = 0$

$6\left(\frac{1-4x^{-1}}{x^{4/5}}\right) = 0$

$6\left(\frac{1-\frac{4}{x}}{x^{4/5}}\right) = 0$

$1 - \frac{4}{x} = 0$

$(-x)\ -\frac{4}{x} = -1\ (-x)$

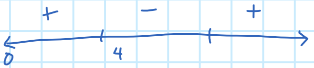
$4 = x$

$x^{4/5} = 0$

$\left(\sqrt[5]{x}\right)^4 = 0$

$x = 0$

undefined



~~x~~ $x<0$ $x < 4$ $x > 4$

Intervals	$1-\frac{4}{x}$	$x^{4/5}$	$C''(x)$
$x<0$	+	+	+
$0<x<4$	−	+	−
$x>4$	+	+	+

f is concave upward @ $(-\infty, 0] \cup [4, \infty)$

f is concave downward @ $[0, 4]$

$f(0) = x^{1/5}(x+6)$

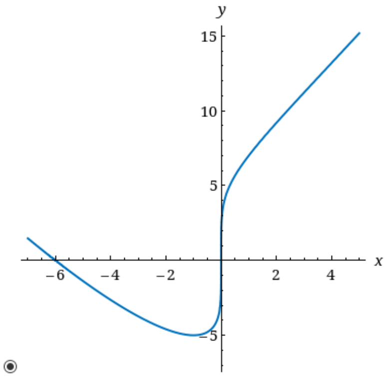
$= 0^{1/5}(0+6)$

$= 0$

$f(4) = x^{1/5}(x+6)$

$= 4^{1/5}(4+6)$

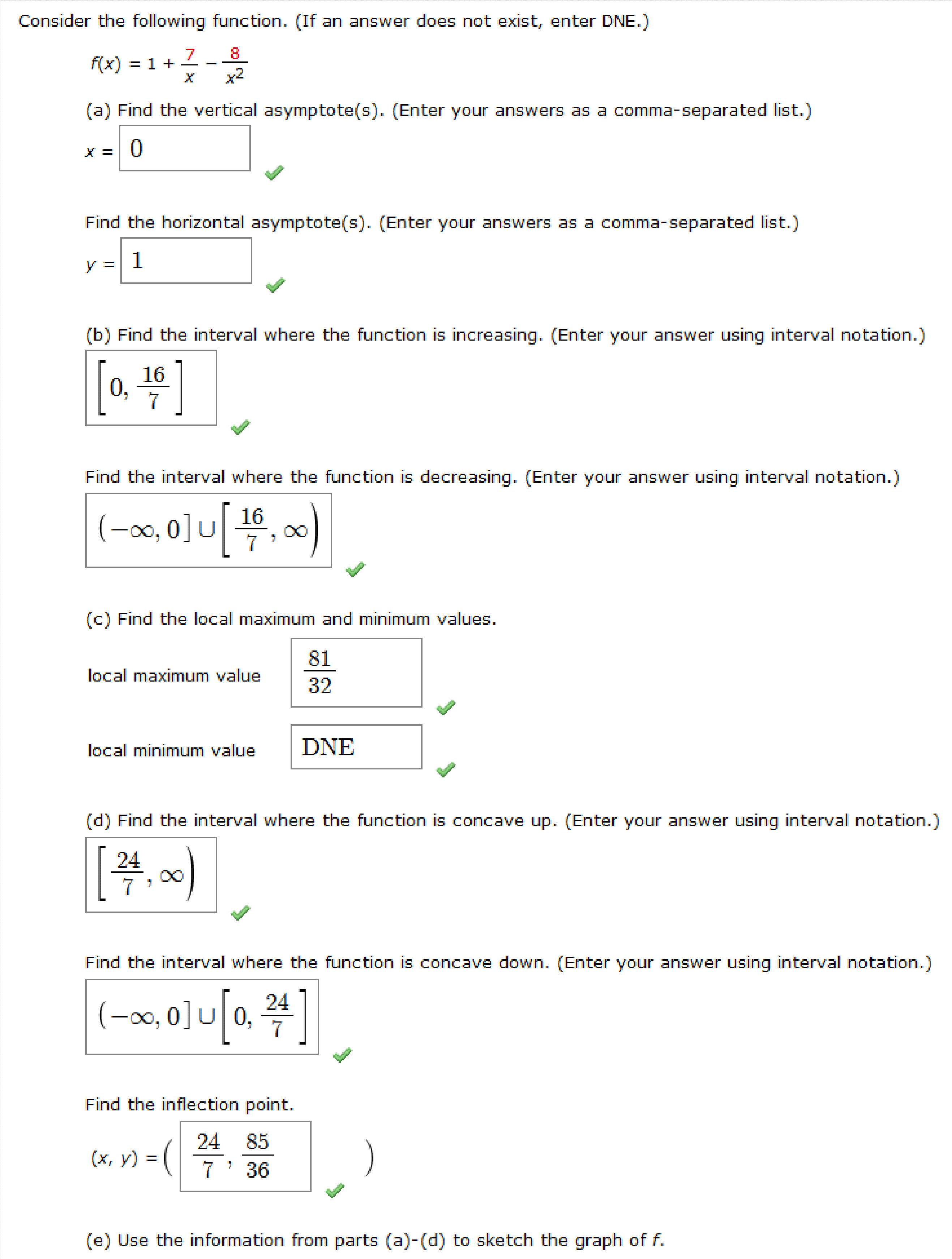
$f(4) = 4^{1/5}(10)$ or $10\sqrt[5]{4}$



Inflection points $(0,0), (4, 10\sqrt[5]{4})$

Q17.1

Tuesday, October 20, 2020 11:05 AM



Consider the following function. (If an answer does not exist, enter DNE.)

$$f(x) = 1 + \frac{7}{x} - \frac{8}{x^2}$$

(a) Find the vertical asymptote(s). (Enter your answers as a comma-separated list.)

$x =$ 0 ✓

Find the horizontal asymptote(s). (Enter your answers as a comma-separated list.)

$y =$ 1 ✓

(b) Find the interval where the function is increasing. (Enter your answer using interval notation.)

$\left[0, \frac{16}{7}\right]$ ✓

Find the interval where the function is decreasing. (Enter your answer using interval notation.)

$(-\infty, 0] \cup \left[\frac{16}{7}, \infty\right)$ ✓

(c) Find the local maximum and minimum values.

local maximum value $\frac{81}{32}$ ✓

local minimum value DNE ✓

(d) Find the interval where the function is concave up. (Enter your answer using interval notation.)

$\left[\frac{24}{7}, \infty\right)$ ✓

Find the interval where the function is concave down. (Enter your answer using interval notation.)

$(-\infty, 0] \cup \left[0, \frac{24}{7}\right]$ ✓

Find the inflection point.

$(x, y) = \left(\frac{24}{7}, \frac{85}{36}\right)$ ✓

(e) Use the information from parts (a)-(d) to sketch the graph of *f*.

Q17.2

Tuesday, October 20, 2020 11:06 AM

$$f(x) = 1 + \frac{7}{x} - \frac{8}{x^2}$$

$$f'(x) = \frac{d}{dx}\left(1 + \frac{7}{x} - \frac{8}{x^2}\right)$$

$$= 0 + \frac{d}{dx}\left(7x^{-1} - 8x^{-2}\right)$$

$$= (-1)7x^{-2} - (-2)8x^{-3}$$

$$f'(x) = -7x^{-2} + 16x^{-3}$$

Get critical points where $f'(x) = 0$ or undefined

$$-7x^{-2} + 16x^{-3} = 0$$

$$-x^{-2}(7 - 16x^{-1}) = 0$$

$$-x^{-2} = 0$$

$$-\frac{1}{x^2} = 0$$

$$x = 0$$

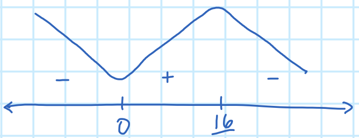
undefined

$$7 - 16x^{-1} = 0$$

$$(-x)\ -\frac{16}{x} = -7\ (-x)$$

$$16 = 7x$$

$$x = \frac{16}{7}$$



$x < 0 \quad 0 < x < \frac{16}{7} \quad x > \frac{16}{7}$

Intervals	$-x^{-2}$	$7-16x^{-1}$	$f'(x)$
$x < 0$	−	+	−
$0 < x < \frac{16}{7}$	−	−	+
$x > \frac{16}{7}$	−	+	−

f is increasing @ $\left[0, \frac{16}{7}\right]$

f is decreasing @ $(-\infty, 0] \cup \left[\frac{16}{7}, \infty\right)$

$$f\left(\frac{16}{7}\right) = 1 + \frac{7}{x} - \frac{8}{x^2}$$

$$= 1 + \frac{7}{\frac{16}{7}} - \frac{8}{\left(\frac{16}{7}\right)^2}$$

$$= 1 + \frac{49}{16} - \frac{8}{\frac{256}{49}}$$

$$= 1 + \frac{49}{16} - \frac{392}{256}$$

$$= 1 + \frac{49}{16} - \frac{49}{32}$$

$$f\left(\frac{16}{7}\right) = \frac{81}{32}$$

Local max = $\frac{81}{32}$

$x = 0$ is undefined when $f'(x) = 0$

$\therefore$ Local min = DNE

$$f''(x) = \frac{d}{dx}\left(-7x^{-2} + 16x^{-3}\right)$$

$$= (-2)-7x^{-3} + (-3)16x^{-4}$$

$$f''(x) = 14x^{-3} - 48x^{-4}$$

Get inflection points where $C''(x) = 0$ or undefined

$$14x^{-3} - 48x^{-4} = 0$$

$$2x^{-3}(7 - 24x^{-1}) = 0$$

$$2x^{-3} = 0$$

$$2\left(\frac{1}{x^3}\right) = 0$$

$$x = 0$$

undefined

$$7 - 24x^{-1} = 0$$

$$(-x)\ \frac{-24}{x} = -7\ (-x)$$

$$24 = 7x$$

$$x = \frac{24}{7}$$

$$f\left(\frac{24}{7}\right) = 1 + \frac{7}{x} - \frac{8}{x^2}$$

$$= 1 + \frac{7}{\frac{24}{7}} - \frac{8}{\left(\frac{24}{7}\right)^2}$$

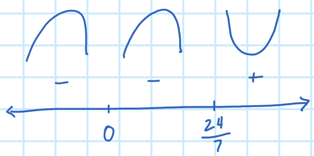
$$= 1 + \frac{49}{24} - \frac{8}{\frac{576}{49}}$$

$$= 1 + \frac{49}{24} - \frac{392}{576}$$

$$= 1 + \frac{49}{24} - \frac{49}{72}$$

$$f\left(\frac{24}{7}\right) = \frac{85}{36}$$

Inflections point = $\left(\frac{24}{7}, \frac{85}{36}\right)$



$x < 0 \quad 0 < x < \frac{24}{7} \quad x > \frac{24}{7}$

Intervals	$2x^{-3}$	$7-24x^{-1}$	$f''(x)$
$x < 0$	−	+	−
$0 < x < \frac{24}{7}$	+	−	−
$x > \frac{24}{7}$	+	+	+

f is concave upwards @ $\left[\frac{24}{7}, \infty\right)$

f is concave downwards @ $(-\infty, 0] \cup \left[0, \frac{24}{7}\right]$

Horizontal Asymptote

$$\lim_{x \to \pm\infty} f(x) = 1 + \frac{7}{x} - \frac{8}{x^2}$$

$$= 1 + \frac{7}{0} - \frac{8}{0^2}$$

$$= 1 + 0 + 0$$

$$= 1$$

Horizontal Asymptote $y = 1$

Vertical Asymptote

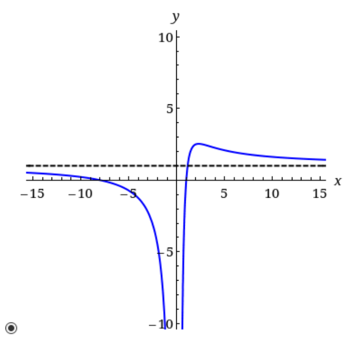
$$\lim_{x \to 0^+}\left(1 + \frac{7}{x} + \frac{8}{x^2}\right) = \left(\frac{x^2 + 7x - 8}{x^2}\right)$$

$$= -\infty$$

$$\lim_{x \to 0^-}\left(1 + \frac{7}{x} + \frac{8}{x^2}\right) = \left(\frac{x^2 + 7x - 8}{x^2}\right)$$

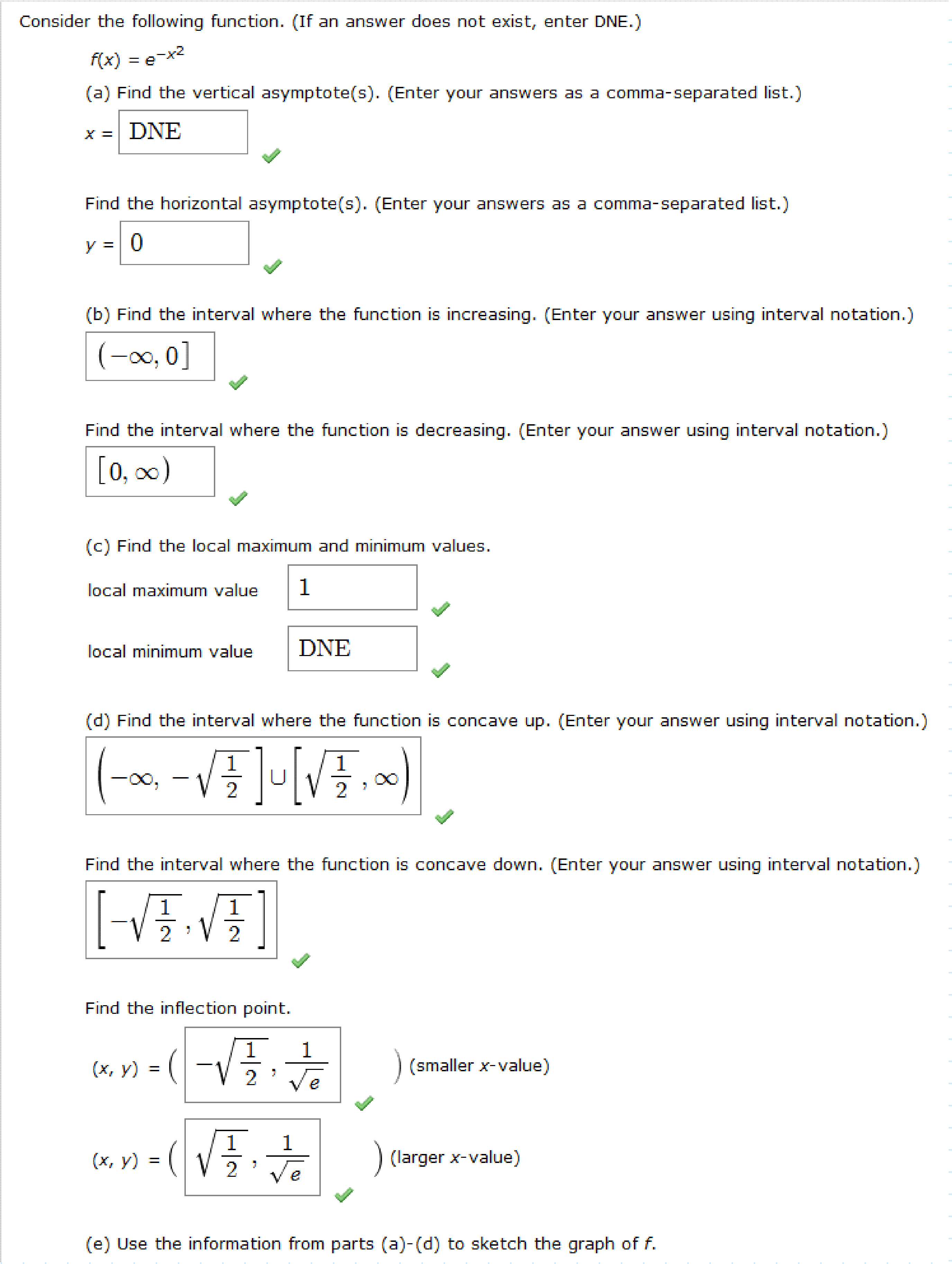
$$= -\infty$$

Vertical asymptote $x = 0$



Q18.1

Tuesday, October 20, 2020 12:47 PM



Consider the following function. (If an answer does not exist, enter DNE.)

$f(x) = e^{-x^2}$

(a) Find the vertical asymptote(s). (Enter your answers as a comma-separated list.)

$x =$ DNE ✓

Find the horizontal asymptote(s). (Enter your answers as a comma-separated list.)

$y =$ 0 ✓

(b) Find the interval where the function is increasing. (Enter your answer using interval notation.)

$(-\infty, 0]$ ✓

Find the interval where the function is decreasing. (Enter your answer using interval notation.)

$[0, \infty)$ ✓

(c) Find the local maximum and minimum values.

local maximum value 1 ✓

local minimum value DNE ✓

(d) Find the interval where the function is concave up. (Enter your answer using interval notation.)

$\left(-\infty, -\sqrt{\frac{1}{2}}\right] \cup \left[\sqrt{\frac{1}{2}}, \infty\right)$ ✓

Find the interval where the function is concave down. (Enter your answer using interval notation.)

$\left[-\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}}\right]$ ✓

Find the inflection point.

$(x, y) = \left(-\sqrt{\frac{1}{2}}, \frac{1}{\sqrt{e}} \right)$ ✓ (smaller x-value)

$(x, y) = \left(\sqrt{\frac{1}{2}}, \frac{1}{\sqrt{e}} \right)$ ✓ (larger x-value)

(e) Use the information from parts (a)-(d) to sketch the graph of f.

Q18.2

Tuesday, October 20, 2020 12:47 PM

$$f(x) = e^{-x^2}$$

$$f'(x) = \frac{d}{dx}\left(e^{-x^2}\right)$$

$$= e^{-x^2}\frac{d}{dx}\left(-x^2\right)$$

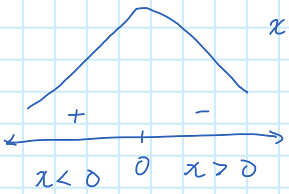
$$= e^{-x^2}(-2x)$$

$$f'(x) = -2e^{-x^2}x$$

Get critical point(s) where $f'(x) = 0$ or undefined

$$-2e^{-x^2}x = 0$$

$$x = 0$$



Intervals	$f'(x)$
$x < 0$	+
$x > 0$	−

f is increasing @ $(-\infty, 0]$

f is decreasing @ $[0, \infty)$

$$f(0) = e^{-x^2}$$

$$= e^{-0^2}$$

$$f(0) = 1$$

Local max = 1

Local min DNE

Horizontal Asymptote

$y = 0$

Vertical Asymptote

$x =$ DNE

$$f''(x) = \frac{d}{dx}\left(-2e^{-x^2}x\right)$$

$$= -2\left[e^{-x^2}\frac{d}{dx}(x) + x\frac{d}{dx}\left(e^{-x^2}\right)\right]$$

$$= -2\left[e^{-x^2}(1) + xe^{-x^2}\frac{d}{dx}\left(-x^2\right)\right]$$

$$= -2\left[e^{-x^2} + e^{-x^2}x(-2x)\right]$$

$$= -2\left(e^{-x^2} - 2e^{-x^2}x^2\right)$$

$$f''(x) = -2e^{-x^2}\left(1-2x^2\right)$$

Get inflection points where $C''(x) = 0$ or undefined

$$-2e^{-x^2}\left(1-2x^2\right) = 0$$

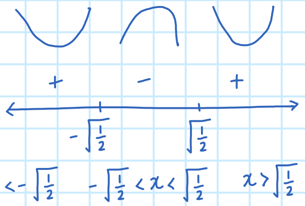
will never be zero since e^x for $x = \mathbb{R}$ is never zero

$$1 - 2x^2 = 0$$

$$-2x^2 = -1$$

$$x^2 = \frac{-1}{-2}$$

$$x = \pm\sqrt{\frac{1}{2}}$$



Internals	$f''(x)$
$x < -\sqrt{\frac{1}{2}}$	$f''(-1) = +$
$-\sqrt{\frac{1}{2}} < x < \sqrt{\frac{1}{2}}$	$f''(0) = -$
$x > \sqrt{\frac{1}{2}}$	$f''(1) = +$

f is concave upward @ $\left(-\infty, -\sqrt{\frac{1}{2}}\right] \cup \left[\sqrt{\frac{1}{2}}, \infty\right)$

f is concave downward @ $\left[-\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}}\right]$

$$f\left(-\sqrt{\tfrac{1}{2}}\right) = e^{-x^2}$$

$$= e^{-\left(-\sqrt{\frac{1}{2}}\right)^2}$$

$$= e^{-\left(-\frac{1}{2}^{1/2}\right)^2}$$

$$= e^{-\left(\frac{1}{2}\right)}$$

$$f\left(-\sqrt{\tfrac{1}{2}}\right) = \frac{1}{\sqrt{e}}$$

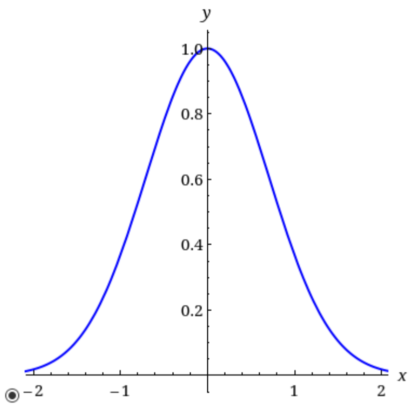
$$f\left(\sqrt{\tfrac{1}{2}}\right) = e^{-x^2}$$

$$= e^{-\left(\sqrt{\frac{1}{2}}\right)^2}$$

$$= e^{-\left(\frac{1}{2}^{1/2}\right)^2}$$

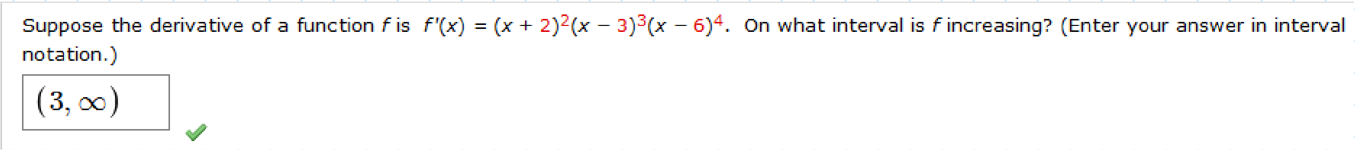
$$= e^{-\left(\frac{1}{2}\right)}$$

$$f\left(\sqrt{\tfrac{1}{2}}\right) = \frac{1}{\sqrt{e}}$$



Q19

Tuesday, October 20, 2020 2:06 PM



Suppose the derivative of a function f is $f'(x) = (x + 2)^2(x - 3)^3(x - 6)^4$. On what interval is f increasing? (Enter your answer in interval notation.)

$(3, \infty)$ ✓

$$f'(x) = (x+2)^2(x-3)^3(x-6)^4$$

$x + 2 = 0$

$x = -2$

$x - 3 = 0$

$x = 3$

$x - 6 = 0$

$x = 6$

where $f'(x) > 0$?

$$\therefore \boxed{f \text{ is increasing @ } (3, \infty)}$$

Q20

Tuesday, October 20, 2020 2:32 PM

$f(x) = ax^3 + bx^2 + cx + d$, local max $(-3, 3)$, local min $(1, 0)$

$$f'(x) = \frac{d}{dx}\left(ax^3 + bx^2 + cx + d\right) = 3ax^2 + 2bx + c$$

$f(-3) = 3$

$f(1) = 0$

These min and max are also critical points so... →

$f'(-3) = 0$ max

$f'(1) = 0$ min

$$f(-3) = ax^3 + bx^2 + cx + d = 3$$
$$= a(-3)^3 + b(-3)^2 + c(-3) + d = 3$$
$$= -27a + 9b - 3c + d = 3$$

$$f'(-3) = 3ax^2 + 2bx + c = 0$$
$$= 3a(-3)^2 + 2b(-3) + c = 0$$
$$= 27a - 6b + c = 0$$

$$f(1) = ax^3 + bx^2 + cx + d = 0$$
$$= a(1)^3 + b(1)^2 + c(1) + d = 0$$
$$= a + b + c + d = 0$$

$$f'(1) = 3ax^2 + 2bx + c = 0$$
$$= 3a(1)^2 + 2b(1) + c = 0$$
$$= 3a + 2b + c = 0$$

$$f(-3) = -27a + 9b - 3c + d = 3 \rightarrow$$
$$f(-1) = a + b + c + d = 0 \rightarrow$$

①

$$\begin{array}{rl} & -27a + 9b - 3c + d = 3 \\ - & \quad a + b + c + d = 0 \\ \hline & -28a + 8b - 4c = 3 \end{array}$$

$$f'(-3) = 27a - 6b + c = 0 \rightarrow$$
$$f'(1) = 3a + 2b + c = 0 \rightarrow$$

②

$$\begin{array}{rl} & 27a - 6b + c = 0 \\ - & 3a + 2b + c = 0 \\ \hline & 24a - 8b = 0 \end{array}$$

?

③

$$\begin{array}{rl} & -28a + 8b - 4c = 3 \\ & (3a + 2b + c = 0)\,4 \\ + & 12a + 8b + 4c = 0 \\ \hline & -16a = 3 \end{array}$$

$$-16a = 3$$
$$a = -\frac{3}{16}$$

④

$$24\left(-\frac{3}{16}\right) - 8b = 0$$
$$-\frac{72}{16} - 8b = 0$$
$$-8b = \frac{72}{16}$$
$$-8b = \frac{9}{2}$$
$$b = \frac{\frac{9}{2}}{-8}$$
$$b = -\frac{9}{16}$$

⑤

$$3\left(-\frac{3}{16}\right) + 2\left(-\frac{9}{16}\right) + c = 0$$
$$-\frac{9}{16} - \frac{18}{16} + c = 0$$
$$-\frac{27}{16} + c = 0$$
$$c = \frac{27}{16}$$

⑥

$$\left(-\frac{13}{16}\right) + \left(-\frac{9}{16}\right) + \frac{27}{16} + d = 0$$
$$\frac{-13 - 9 + 27}{16} + d = 0$$
$$\frac{5}{16} + d = 0$$
$$d = -\frac{5}{16}$$

$$f(x) = ax^3 + bx^2 + cx + d$$
$$= -\frac{13}{16}x^3 - \frac{9}{16}x^2 + \frac{27}{16}x - \frac{5}{16}$$
$$f(x) = \frac{-13x^3 - 9x^2 + 27x - 5}{16}$$