Q1 X

Tuesday, December 8, 2020 11:20 AM

1) $y = 3x + 2$, $x = 1$, $x = 3$

You must show all of your work.

1) Consider the area bounded by y=3x +2, the x-axis, x=1 and x = 3 20 points

a) Draw the area with a representative rectangle.

b) Find an expression for the Riemann sum

c) Evaluate the Riemann sum

d) Find a definite integral that represents the area and use the Fundamental Theorem of Calculus to find the area.

These formulas might be useful:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

2) Find $\int_0^2 (x^2+1)^2\,dx$ 12 point

3) Find $\int \cot(x)dx$ 12 points

4) Find $\int e^{2x}\sqrt{1+e^{2x}}\,dx$ 12 points

5) Find the area between the graphs y=sin(x) and y=1/2 on the interval $[0,\frac{\pi}{6}]$ 12 points

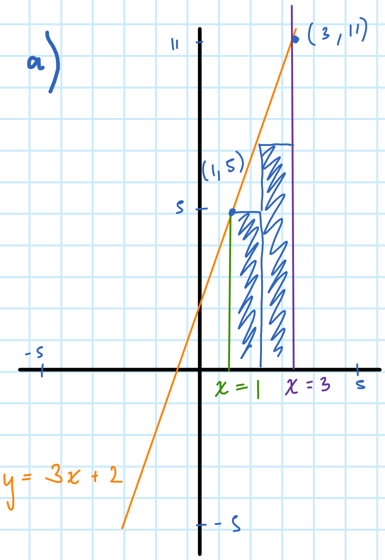
6) $\int \frac{\sin(x)}{1+\cos^2(x)}dx$ 12 points

7) **For this question, set up the integrals only. Do not evaluate.**
Use two different methods to find the volume of the solid obtained by rotating the region in the first quadrant bounded by $y=x^3$ and y=x about the x-axis. Your answer needs to include a graph of the region and a representative rectangle.

a) Use the Disk (Washer) Method 10 points

b) Use the Shell Method 10 points

a)



$\Delta x = \frac{b-a}{2} = \frac{3-1}{2} = 1$

b) $L_2 = \sum_{i=1}^{2} f(x_{i-1})\Delta x$

$= f(1)(1) + f(2)(1)$

$= (3(1)+2)(1) + (3(2)+2)(1)$

$= 5 + 8$

c) $= 13$

d) $\Delta x = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$

$x_i = a + i\Delta x = 1 + i\frac{2}{n}$

$= \frac{2i}{n} + 1$

$\int_a^b f(x_i)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

$\int_1^3 (\cancel{3x^2+2})\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left[3\left(\frac{2i}{n}+1\right)^2 + 2\right]\frac{2}{n}$

$3x + 2$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{12i^2}{n^2} + \frac{12i}{n} + 5\right)\frac{2}{n}$

$= \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left(\frac{12i^2}{n^2} + \frac{12i}{n} + 5\right)$

$= \lim_{n\to\infty} \frac{2}{n}\left(\frac{12}{n^2}\sum_{i=1}^{n} i^2 + \frac{12}{n}\sum_{i=1}^{n} i + 5\sum_{i=1}^{n} 1\right)$

$= \lim_{n\to\infty} \frac{\cancel{2}}{\cancel{n}}\left[\frac{12}{n^2}\left(\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6} = 3}\right) + \frac{12}{n}\left(\frac{\cancel{n}(n+1)}{\cancel{2}}\right) + 5(\cancel{n})\right]$

$= \lim_{n\to\infty} \left[\frac{12}{n^2}\left(\frac{(n+1)(2n+1)}{3}\right) + \frac{12}{n}(n+1) + 10\right]$

$= \lim_{n\to\infty} \left[4\left(1+\cancel{\tfrac{1}{n}}\right)\left(2+\cancel{\tfrac{1}{n}}\right) + 12 + \cancel{\tfrac{12}{n}} + 10\right]$

$= \lim_{n\to\infty} \left[4(1)(2) + 12 + 10\right] = \boxed{30}$

cancel out all $\frac{x}{n}$ since they become 0 as n approaches ∞

Did not evaluate using the Fundamental theorem of calculus

$3\left(\frac{2i}{n}+1\right)\left(\frac{2i}{n}+1\right)$

$= 3\left(\frac{4i^2}{n^2} + \frac{2i}{n} + \frac{2i}{n} + 1\right)$

$= 3\left(\frac{4i^2}{n^2} + \frac{4i}{n} + 1\right)$

$= \frac{12i^2}{n^2} + \frac{12i}{n} + 3$

$\frac{12}{n^2}\left(\frac{(n+1)(2n+1)}{3}\right)$

$= \frac{12}{3}\left(\frac{(n+1)(2n+1)}{(n)(n)}\right)$

$= \frac{12}{3}\left[\frac{1}{n}(n+1)\frac{1}{n}(2n+1)\right]$

$= 4\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$

Q2

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2) $\int_0^2 (x^2+1)^2 \, dx$

$(x^2+1)(x^2+1)$

$$\begin{array}{r} x^4 + x^2 \\ + x^2 + 1 \\ \hline x^4 + 2x^2 + 1 \end{array}$$

$$= \int_0^2 (x^4 + 2x^2 + 1) \, dx = \left[\frac{x^{4+1}}{4+1} + \frac{2x^{2+1}}{2+1} + 1x \right]_0^2$$

$$= \frac{x^5}{5} + \frac{2x^3}{3} + x \Big|_0^2$$

$$= F(2) - F(0)$$

$$= \left[\left(\frac{2^5}{5} + \frac{2(2^3)}{3} + 2 \right) - \underbrace{\cancel{\left(\frac{0^5}{5} + \frac{2(0^3)}{3} + 0 \right)}}_{=0} \right]$$

$$= \frac{32}{5} + \frac{16}{3} + 2$$

$$= \boxed{\frac{206}{15}}$$

Q3

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3) $\int \cot(x)\,dx$

$= \int \frac{\cos(x)}{\sin(x)}\,dx$, $u = \sin(x)$ $\quad \frac{du}{dx} = \cos(x)$

$du = \cos(x)\,dx$

$= \int \frac{1}{u}\cos(x)\,dx = \int \frac{1}{u}\,du$

$= \ln|u| + C$

$= \boxed{\ln|\sin(x)| + C}$

Q4

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4) $\int e^{2x}\sqrt{1+e^{2x}}\,dx$, $u = 1 + e^{2x}$

$\frac{du}{dx} = e^{2x}(2)$

$du = 2e^{2x}\,dx$

$= \int \frac{1}{2}\cdot 2e^{2x}\sqrt{u}\,dx = \frac{1}{2}\int u^{1/2}\,du$

$= \frac{1}{2}\left(\frac{u^{1/2+1}}{\frac{1}{2}+1}\right) + C = \frac{1}{2}\left(\frac{u^{3/2}}{\frac{3}{2}}\right) + C$

$= \frac{1}{2}\left[\frac{2}{3}(1+e^{2x})^{3/2}\right] + C$

$= \boxed{\frac{1}{3}(1+e^{2x})^{3/2} + C}$

Q5

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5) $y = \sin(x)$, $y = \frac{1}{2}$, $\left[0, \frac{\pi}{6}\right]$

$$\int_0^{\pi/6} \left(\frac{1}{2} - \sin(x)\right) dx = \left[\frac{x}{2} + \cos(x)\right]_0^{\pi/6}$$

$$= F\left(\frac{\pi}{6}\right) - F(0)$$

$$= \left[\left(\frac{\frac{\pi}{6}}{2} + \cos\left(\frac{\pi}{6}\right)\right) - \left(\underbrace{\cancel{\frac{0}{2}}}_{=0} + \underbrace{\cancel{\cos(0)}}_{=1}\right)\right]$$

$$= \frac{\pi}{6} \cdot \frac{1}{2} + \cos\left(\frac{\pi}{6}\right) - 1$$

$$= \boxed{\cos\left(\frac{\pi}{6}\right) + \frac{\pi}{12} - 1}$$

Q6

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6) $\int \frac{\sin(x)}{1+\cos^2(x)}\,dx$, $u = \cos(x)$ $\frac{du}{dx} = -\sin(x)$

$du = -\sin(x)\,dx$

$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$

$= \int -1 \cdot -1 \frac{\sin(x)}{1+u^2}\,dx = -\int \frac{1}{1+u^2} - \sin(x)\,dx = -\int \frac{1}{1+u^2}\,du$

$= -\left(\tan^{-1}(u)\right) + C$

$= \boxed{-\tan^{-1}\left(\cos(x)\right) + C}$

Q7

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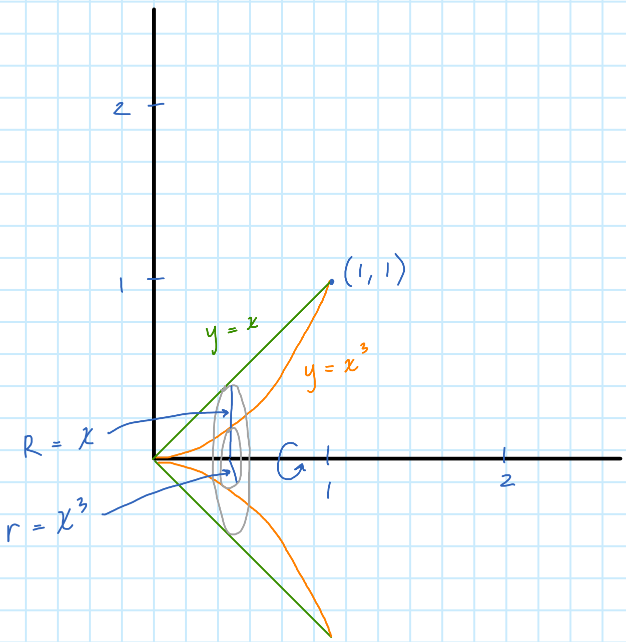
7) $y = x^3$, $y = x$; about x-axis

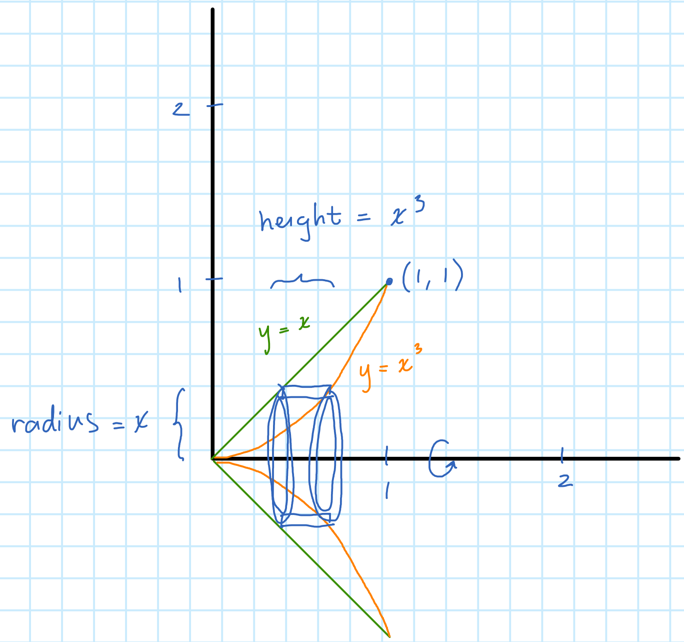
a) Disk Method



$$V = \int \pi r^2 \, dx = V = \int \pi (R^2 - r^2) \, dx$$

$$V = \int_0^1 \pi \left[x^2 - (x^3)^2 \right] dx$$

b) Shell Method



$$V = \int 2\pi (\text{radius})(\text{height}) \, dx$$

$$V = 2\pi \int_0^1 x(x^3) \, dx$$