Q1

Tuesday, September 1, 2020 2:23 PM

If f is continuous on $(-\infty, \infty)$, what can you say about its graph? (Select all that apply.)

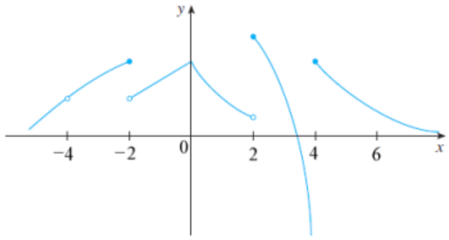
- ☐ The graph of f has a hole.
- ☐ The graph of f has a jump.
- ☐ The graph of f has a vertical asymptote.
- ☑ none of these

✓

Q2

Tuesday, September 1, 2020 2:26 PM

From the graph of f, state each x-value at which f is discontinuous. For each x-value, determine whether f is continuous from the right, or from the left, or neither. (Enter your answers from smallest to largest.)



$x =$ -4 ✔ (smallest value)

- ○ continuous from the right
- ○ continuous from the left
- ◉ neither

✔

$x =$ -2 ✔

- ○ continuous from the right
- ◉ continuous from the left
- ○ neither

✔

$x =$ 2 ✔

- ◉ continuous from the right
- ○ continuous from the left
- ○ neither

✔

$x =$ 4 ✔ (largest value)

- ◉ continuous from the right
- ○ continuous from the left
- ○ neither

✔

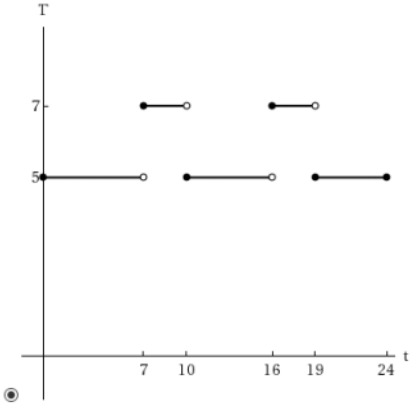
Q3

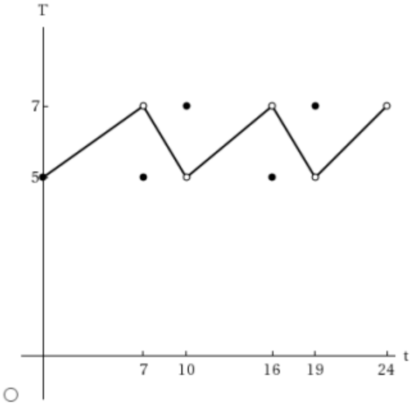
Tuesday, September 1, 2020 2:29 PM

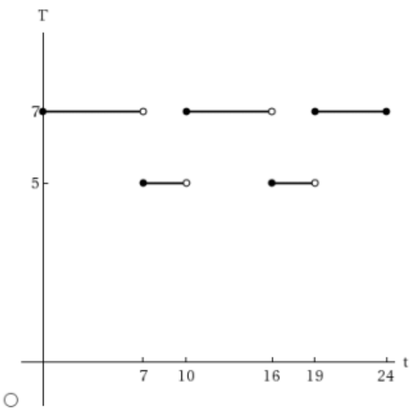
The toll T charged for driving on a certain stretch of a toll road is \$5 except during rush hours (between 7 AM and 10 AM and between 4 PM and 7 PM) when the toll is \$7.

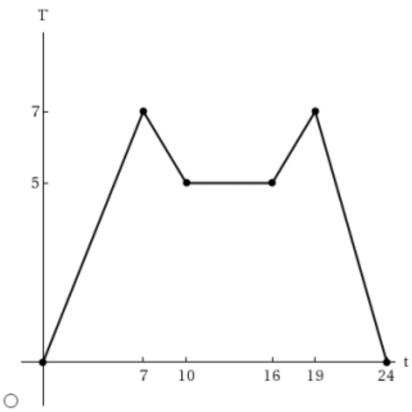
(a) Sketch a graph of T as a function of the time t, measured in hours past midnight.

Shift









(b) Locate the discontinuities of T. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

t = 7,10,16,19

Classify the discontinuities as removable, jump, or infinite.

- ○ removable
- ◉ jump
- ○ infinite
- ○ none — T is continuous

Discuss the significance of the discontinuities of T to someone who uses the road.

- ◉ Because of the sudden jumps in the toll, drivers may want to avoid the higher rates between $t = 7$ and $t = 10$ and between $t = 16$ and $t = 19$ if feasible.
- ○ The function is continuous, so there is no significance.
- ○ Because of the sudden jumps in the toll, drivers may want to avoid the higher rates between $t = 0$ and $t = 7$, between $t = 10$ and $t = 16$, and between $t = 19$ and $t = 24$ if feasible.
- ○ Because of the steady increases and decreases in the toll, drivers may want to avoid the highest rates at $t = 7$ and $t = 24$ if feasible.

Q4

Tuesday, September 1, 2020 2:32 PM

Use the definition of continuity and the properties of limits to show that the function is continuous at the given number a.

$$f(x) = \left(x + 3x^3\right)^5, \quad a = -1$$

$$\lim_{x \to -1} f(x) = \lim_{x \to -1} \left(\boxed{x + 3x^3} \right)^5$$

$$= \left(\lim_{x \to -1} x + \boxed{3} \lim_{x \to -1} x^3 \right)^5$$

$$= \left(-1 + 3\left(\boxed{-1} \right)^3 \right)^5$$

$$= \boxed{-1024}$$

$$f(-1) = \boxed{-1024}$$

Thus, by the definition of continuity, f is continuous at $a = -1$.

Q5

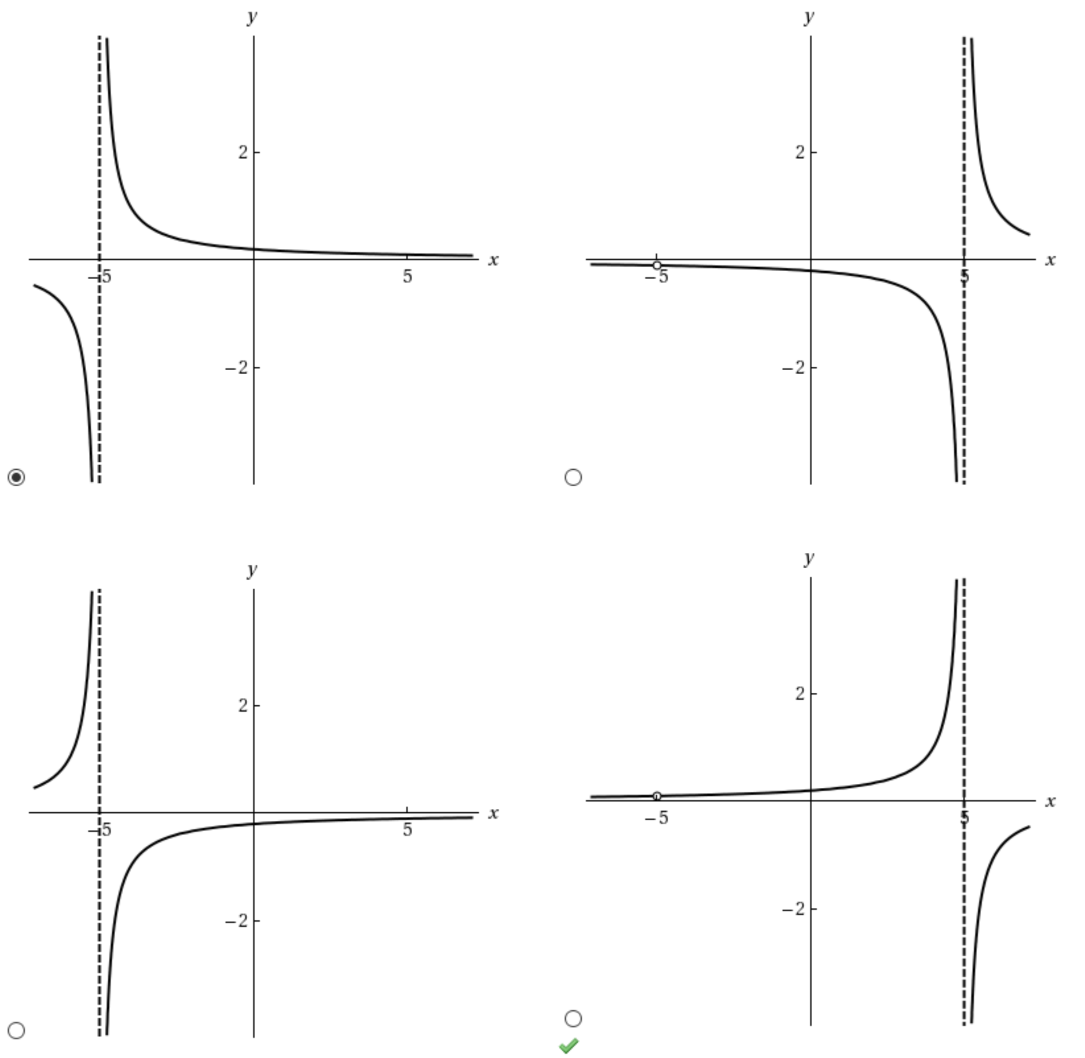
Tuesday, September 1, 2020 2:33 PM

Explain why the function is discontinuous at the given number a. (Select all that apply.)

$$f(x) = \frac{1}{x + 5} \qquad a = -5$$

- [x] $\lim_{x \to -5} f(x)$ does not exist.
- [x] $f(-5)$ is undefined.
- [ ] $\lim_{x \to -5^+} f(x)$ and $\lim_{x \to -5^-} f(x)$ exist, but are not equal.
- [ ] $f(-5)$ and $\lim_{x \to -5} f(x)$ exist, but are not equal.
- [ ] none of the above

Sketch the graph of the function.



Q6

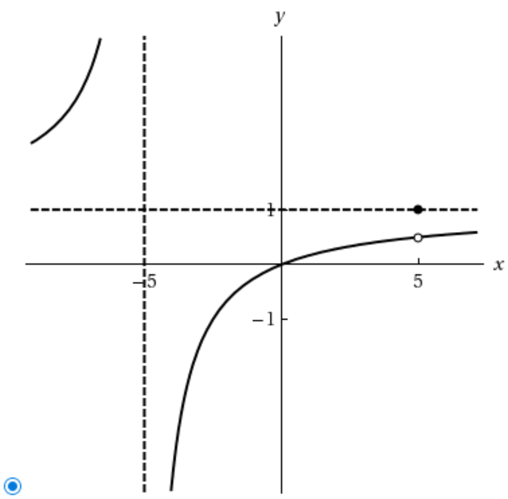
Tuesday, September 1, 2020 2:34 PM

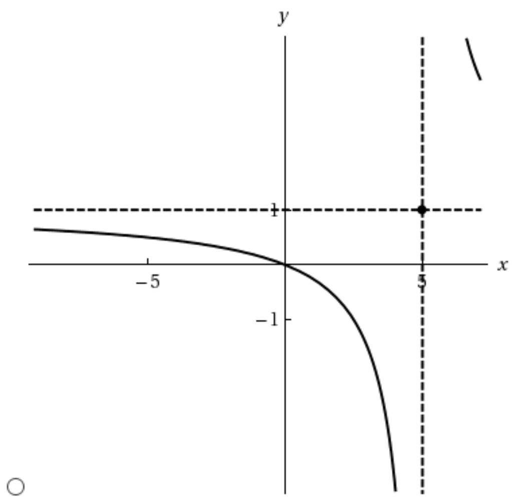
Explain why the function is discontinuous at the given number a. (Select all that apply.)

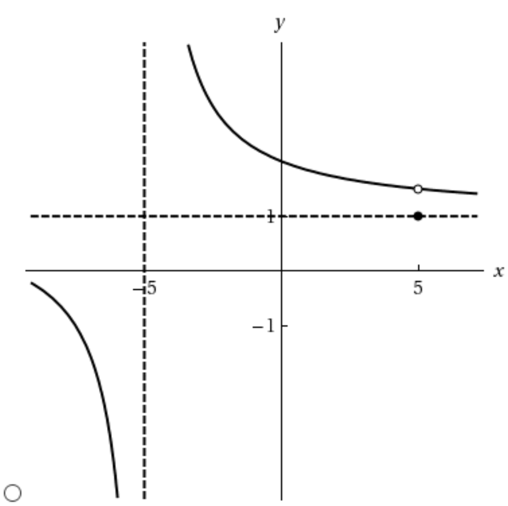
$$f(x) = \begin{cases} \dfrac{x^2 - 5x}{x^2 - 25} & \text{if } x \neq 5 \\ 1 & \text{if } x = 5 \end{cases} \qquad a = 5$$

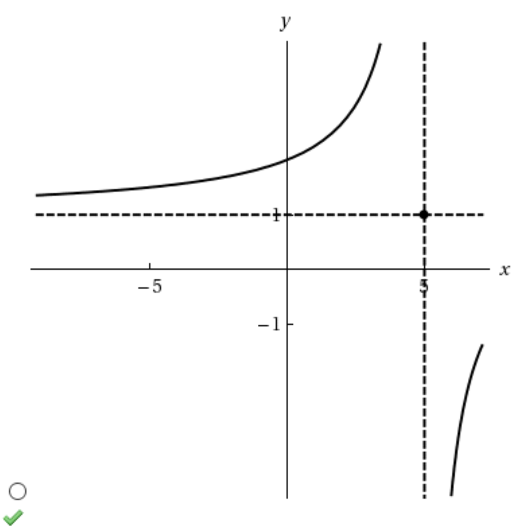
- [x] $f(5)$ is defined and $\lim_{x \to 5} f(x)$ is finite, but they are not equal.
- [ ] $\lim_{x \to 5^+} f(x)$ and $\lim_{x \to 5^-} f(x)$ are finite, but are not equal.
- [ ] $\lim_{x \to 5} f(x)$ does not exist.
- [ ] $f(5)$ is undefined.
- [ ] none of the above

Sketch the graph of the function.









Q7

Tuesday, September 1, 2020 2:35 PM

How would you "remove the discontinuity" of f? In other words, how would you define $f(5)$ in order to make f continuous at 5?

$$f(x) = \frac{x^2 - 2x - 15}{x - 5}$$

$f(5) =$ 8 ✓

$$f(x) = \frac{x^2 - 2x - 15}{x - 5}$$

$$= \frac{\cancel{(x-5)}(x+3)}{\cancel{(x-5)}}$$

$$f(5) = x + 3$$

$$= 5 + 3$$

$$= \boxed{8}$$

Q8

Tuesday, September 1, 2020 2:42 PM

Explain, using the theorems, why the function is continuous at every number in its domain.

$$F(x) = \frac{2x^2 - x - 9}{x^2 + 1}$$

- ○ $F(x)$ is a polynomial, so it is continuous at every number in its domain.
- ◉ $F(x)$ is a rational function, so it is continuous at every number in its domain.
- ○ $F(x)$ is a composition of functions that are continuous for all real numbers, so it is continuous at every number in its domain.
- ○ $F(x)$ is *not* continuous at every number in its domain.
- ○ none of these

✔

State the domain. (Enter your answer using interval notation.)

$(-\infty,\infty)$ ✔

Domain $\mathbb{R}$

Theorem If f and g are continuous at a and c is a constant, then the following functions are also continuous at a:

1. $f + g$ **2.** $f - g$ **3.** cf

4. fg **5.** $\dfrac{f}{g}$ if $g(a) \neq 0$

Theorem
(a) Any polynomial is continuous everywhere; that is, it is continuous on $\mathbb{R} = (-\infty, \infty)$.
(b) Any rational function is continuous wherever it is defined; that is, it is continuous on its domain.

Theorem The following types of functions are continuous at every number in their domains:

polynomials, rational functions, root functions

trigonometric functions, inverse trigonometric functions

exponential functions, logarithmic functions

Theorem If g is continuous at a and f is continuous at $g(a)$, then the composite function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is continuous at a.

Q9

Tuesday, September 1, 2020 2:57 PM

Explain, using the theorems, why the function is continuous at every number in its domain.

$$Q(x) = \frac{\sqrt[3]{x-9}}{x^3-9}$$

- ○ $Q(x)$ is a polynomial, so it is continuous at every number in its domain.
- ○ $Q(x)$ is a rational function, so it is continuous at every number in its domain.
- ◉ $Q(x)$ is built up from functions that are continuous for all real numbers, so it is continuous at every number in its domain.
- ○ $Q(x)$ is *not* continuous at every number in its domain.
- ○ none of these

State the domain. (Enter your answer using interval notation.)

$\left(-\infty, \sqrt[3]{9}\right) \cup \left(\sqrt[3]{9}, \infty\right)$

$Q(x)$ defined for $\mathbb{R}$, $x^3 - 2 \neq 0$

$x^3 - 9 = 0$

$x^3 = 9$

$x = \sqrt[3]{9}$

Theorem If f and g are continuous at a and c is a constant, then the following functions are also continuous at a:

1. $f + g$
2. $f - g$
3. cf
4. fg
5. $\dfrac{f}{g}$ if $g(a) \neq 0$

Theorem
(a) Any polynomial is continuous everywhere; that is, it is continuous on $\mathbb{R} = (-\infty, \infty)$.
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Theorem The following types of functions are continuous at every number in their domains:

polynomials, rational functions, root functions, trigonometric functions, inverse trigonometric functions, exponential functions, logarithmic functions

Theorem If g is continuous at a and f is continuous at $g(a)$, then the composite function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is continuous at a.

Q10

Tuesday, September 1, 2020 4:02 PM

Explain, using the theorems, why the function is continuous at every number in its domain.

$$M(x) = \sqrt{1 + \frac{9}{x}}$$

- ○ $M(x)$ is a polynomial, so it is continuous at every number in its domain.
- ○ $M(x)$ is a rational function, so it is continuous at every number in its domain.
- ◉ $M(x)$ is a composition of functions that are continuous, so it is continuous at every number in its domain.
- ○ $M(x)$ is *not* continuous at every number in its domain.
- ○ none of these

✓

State the domain. (Enter your answer using interval notation.)

$(-\infty, -9] \cup (0, \infty)$ ✓

$$M(x) \sqrt{1 + \frac{9}{x}}$$

$$= \sqrt{\frac{x+9}{x}}$$

$$\sqrt{1 + \frac{9}{x}}$$

$$= \frac{1}{1} + \frac{9}{x}$$

$$= \frac{1\,(x)}{1\,(x)} + \frac{9}{x}$$

$$= \frac{x+9}{x}$$

Defined when $\frac{x+9}{x} \geq 0$

$x + 9 \geq 0,\ x > 0 \rightarrow x > 0$

or

$x + 9 \leq 0,\ x < 0 \rightarrow x \leq 1$

$\therefore$ M has Domain

$(-\infty\ -9] \cup (0, \infty)$

Theorem If f and g are continuous at a and c is a constant, then the following functions are also continuous at a:

1. $f + g$
2. $f - g$
3. cf
4. fg
5. $\frac{f}{g}$ if $g(a) \neq 0$

Theorem
(a) Any polynomial is continuous everywhere; that is, it is continuous on $\mathbb{R} = (-\infty, \infty)$.
(b) Any rational function is continuous wherever it is defined; that is, it is continuous on its domain.

Theorem The following types of functions are continuous at every number in their domains:

polynomials, rational functions, root functions, trigonometric functions, inverse trigonometric functions, exponential functions, logarithmic functions

Theorem If g is continuous at a and f is continuous at $g(a)$, then the composite function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is continuous at a.

Q11

Tuesday, September 1, 2020 7:35 PM

Use continuity to evaluate the limit.

$$\lim_{x \to 5} x\sqrt{29 - x^2}$$

$$\lim_{x \to 5} x\sqrt{29-x^2}$$

$$\lim_{x \to 5} x\sqrt{29-x^2} = \lim_{x \to 5} 5 \cdot \sqrt{29-25}$$

$$= 5 \cdot \sqrt{4}$$

$$= 5 \cdot 2$$

$$= \boxed{10}$$

$F(x) = f(x) \cdot g(x)$

$f(x) = x$

x is continuous $(\mathbb{R})$

$g(x) = \sqrt{29-x^2}$

$\sqrt{29-x^2}$ is continuous $(29-x^2) \geq 0$

$29 \geq x^2$

$\sqrt{29} \geq x$, $x \geq -\sqrt{29}$

$F(x) = x\sqrt{29-x^2}$ is continuous for $\mathbb{R}$

$\sqrt{29} \geq x \geq -\sqrt{29}$

Q12

Tuesday, September 1, 2020 7:48 PM

Show that f is continuous on $(-\infty, \infty)$.

$$f(x) = \begin{cases} 1 - x^2 & \text{if } x \le 1 \\ \ln(x) & \text{if } x > 1 \end{cases}$$

On the interval $(-\infty, 1)$, f is a polynomial ✔ function; therefore f is continuous on $(-\infty, 1)$.

On the interval $(1, \infty)$, f is a logarithmic ✔ function; therefore f is continuous on $(1, \infty)$.

At $x = 1$,

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \left(\boxed{1 - x^2} \right) = \boxed{0} ,$$

and

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \left(\boxed{\ln(x)} \right) = \boxed{0} ,$$

so $\lim_{x \to 1} f(x) = \boxed{0}$. Also, $f(1) = \boxed{0}$. Thus, f is continuous at $x = 1$. We conclude that f is continuous on $(-\infty, \infty)$.

Q13

Tuesday, September 1, 2020 7:54 PM

Find each x-value at which f is discontinuous and for each x-value, determine whether f is continuous from the right, or from the left, or neither.

$$f(x) = \begin{cases} x + 9 & \text{if } x < 0 \\ e^x & \text{if } 0 \le x \le 1 \\ 5 - x & \text{if } x > 1 \end{cases}$$

$x =$ 0 (smaller value)

- (•) continuous from the right
- ( ) continuous from the left
- ( ) neither

$x =$ 1 (larger value)

- ( ) continuous from the right
- (•) continuous from the left
- ( ) neither

$x+9$ e^x $5-x$

cont   0   cont.   1   cont.

continuous $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (x+9) = 9$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} e^x = e^0 = 1$$

$\lim_{x \to 0} f(x)$ does not exist

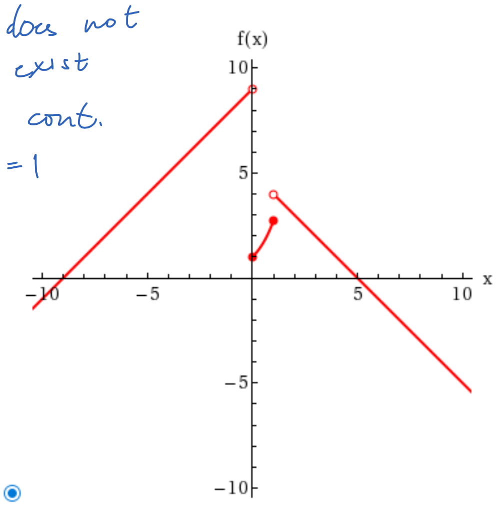
f is not cont at $x = 0$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} e^x = e^1$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (5 - x) = 4$$

$\lim f(x)$ does not exist

f is not cont. at $x = 1$



Q14

Tuesday, September 1, 2020 10:25 PM

The gravitational force exerted by the planet Earth on a unit mass at a distance r from the center of the planet is

$$F(r) = \begin{cases} \dfrac{GMr}{R^3} & \text{if } r < R \\ \dfrac{GM}{r^2} & \text{if } r \geq R \end{cases}$$

where M is the mass of Earth, R is its radius, and G is the gravitational constant. Is F a continuous function of r?

- (●) Yes, F is a continuous function of r.
- ( ) No, F is not a continuous function of r

Q15

Tuesday, September 1, 2020 10:26 PM

For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 8x & \text{if } x < 3 \\ x^3 - cx & \text{if } x \geq 3 \end{cases}$$

$c =$ 1/4

Both polynomial

cont on $(-\infty, 3) \cup (3, \infty)$

$$\lim_{x \to 3^-} (cx^2 + 8x)$$
$$= c(3)^2 + 8(3)$$
$$= 9c + 24$$

$$\lim_{x \to 3^+} (x^3 - cx)$$
$$= (3)^3 - c(3)$$
$$= 27 - 3c$$

$$9c + 24 = 27 - 3c$$
$$9c + 3c = 27 - 24$$
$$12c = 3$$
$$\frac{\cancel{12}c}{\cancel{12}} = \frac{3}{12}$$
$$\boxed{c = \frac{1}{4}}$$