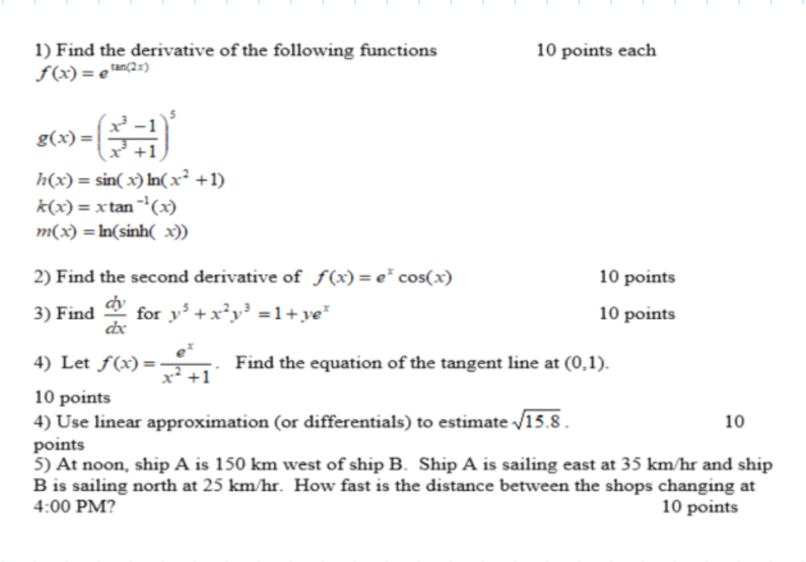
Q1

Tuesday, October 6, 2020 2:16 PM



1) Find the derivative of the following functions 10 points each

$f(x) = e^{\tan(2x)}$

$g(x) = \left(\frac{x^3-1}{x^3+1}\right)^5$

$h(x) = \sin(x)\ln(x^2+1)$

$k(x) = x\tan^{-1}(x)$

$m(x) = \ln(\sinh(x))$

2) Find the second derivative of $f(x) = e^x\cos(x)$ 10 points

3) Find $\frac{dy}{dx}$ for $y^5 + x^2y^3 = 1 + ye^x$ 10 points

4) Let $f(x) = \frac{e^x}{x^2+1}$. Find the equation of the tangent line at (0,1). 10 points

4) Use linear approximation (or differentials) to estimate $\sqrt{15.8}$. 10 points

5) At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/hr and ship B is sailing north at 25 km/hr. How fast is the distance between the shops changing at 4:00 PM? 10 points

1) Find the derivative

$$f(x) = e^{\tan(2x)}$$

$$f'(x) = \frac{d}{dx}\left[e^{\tan(2x)}\right]$$

$$= \frac{de^u}{du}\left[e^{\tan(2x)}\right]\frac{d}{dx}\left[\tan(2x)\right]$$

$$= e^{\tan(2x)}\frac{d\tan(u)}{du}\left[\tan(2x)\right]\frac{d}{dx}(2x)$$

$$= e^{\tan(2x)}\sec^2(2x)(2)$$

$$f'(x) = \boxed{2\sec^2(2x)e^{\tan(2x)}}$$

$$g(x) = \left(\frac{x^3-1}{x^3+1}\right)^5$$

$$g'(x) = \frac{d}{dx}\left[\left(\frac{x^3-1}{x^3+1}\right)^5\right]$$

$$= \frac{du^5}{du}\left[\left(\frac{x^3-1}{x^3+1}\right)^5\right]\frac{d}{dx}\left(\frac{x^3-1}{x^3+1}\right)$$

$$\begin{array}{r} 1+\frac{2}{x^3-1} \\ x^3-1\overline{)\,x^3+1} \\ -\ x^3-1 \\ \hline 0+2 \end{array}$$

$$= 5\left(\frac{x^3-1}{x^3+1}\right)^4\frac{d}{dx}\left(1+\frac{2}{x^3-1}\right)$$

$$= 5\left(\frac{x^3-1}{x^3+1}\right)^4\left[0+\frac{(x^3-1)\frac{d}{dx}(2)-2\frac{d}{dx}(x^3-1))}{(x^3-1)^2}\right]$$

$$= 5\left(\frac{x^3-1}{x^3+1}\right)^4\left[\frac{(x^3-1)-2(3x^2)}{(x^3-1)^2}\right]$$

$$g'(x) = \boxed{5\left(\frac{x^3-1}{x^3+1}\right)^4\left[\frac{x^3-6x^2-1}{(x^3-1)^2}\right]}$$

$$h(x) = \sin(x)\ln(x^2+1)$$

$$h'(x) = \frac{d}{dx}\left[\sin(x)\ln(x^2+1)\right]$$

$$= \sin(x)\frac{d}{dx}\left[\ln(x^2+1)\right] + \ln(x^2+1)\frac{d}{dx}\left[\sin(x)\right]$$

$$= \sin(x)\frac{d\ln(u)}{du}\left[\ln(x^2+1)\right]\frac{d}{dx}(x^2+1) + \ln(x^2+1)\cos(x)$$

$$= \sin(x)\frac{1}{x^2+1}(2x) + \ln(x^2+1)\cos(x)$$

$$h'(x) = \boxed{\frac{2x\sin(x)}{x^2+1} + \cos(x)\ln(x^2+1)}$$

$$k(x) = x\tan^{-1}(x)$$

$$K'(x) = \frac{d}{dx}\left[x\tan^{-1}(x)\right]$$

$$= x\frac{d}{dx}\left[\tan^{-1}(x)\right] + \tan^{-1}(x)\frac{d}{dx}(x)$$

$$= x\left(\frac{1}{1+x^2}\right) + \tan^{-1}(x)(1)$$

$$K'(x) = \boxed{\frac{x}{1+x^2} + \tan^{-1}(x)}$$

$$m(x) = \ln(\sinh(x))$$

$$= \frac{d}{dx}\left[\ln(\sinh(x))\right]$$

$$= \frac{d\ln(u)}{du}\left[\ln(\sinh(x))\right]\frac{d}{dx}\left[\sinh(x)\right]$$

$$= \frac{1}{\sinh(x)}\cosh(x)$$

$$m(x) = \boxed{\frac{\cosh(x)}{\sinh(x)} \text{ or } \coth(x)}$$

Q2

Tuesday, October 6, 2020 2:16 PM

2) Find the second derivative of

$f(x) = e^x \cos(x)$

$$f'(x) = e^x \cos(x)$$
$$= \frac{d}{dx}\left[e^x \cos(x)\right]$$
$$= e^x \frac{d}{dx}\left[\cos(x)\right] + \cos(x)\frac{d}{dx}\left(e^x\right)$$
$$= e^x(-\sin(x)) + \cos(x)(e^x)$$
$$= -e^x \sin(x) + e^x \cos(x)$$
$$f'(x) = e^x(-\sin(x) + \cos(x))$$

$$f''(x) = \frac{d}{dx}\left[e^x(-\sin(x) + \cos(x))\right]$$
$$= e^x \frac{d}{dx}\left[(-\sin(x) + \cos(x))\right] + (-\sin(x) + \cos(x))\frac{d}{dx}\left(e^x\right)$$
$$= e^x(-\cos(x) - \sin(x)) + (-\sin(x) + \cos(x))e^x$$
$$= \cancel{-e^x \cos(x)} - e^x \sin(x) - e^x \sin(x) + \cancel{e^x \cos(x)}$$
$$f''(x) = \boxed{-2e^x \sin(x)}$$

Q3

Tuesday, October 6, 2020 5:05 PM

3) Find $\frac{dy}{dx}$ for $y^5 + x^2y^2 = 1 + ye^x$

$$\frac{dy}{dx}\left(y^5 + x^2y^2\right) = \frac{dy}{dx}\left(1 + ye^x\right)$$

$$5y^4\frac{dy}{dx} + \left[x^2\frac{d}{dy}(y^2) + y^2\frac{d}{dy}(x^2)\right] = 0 + \left[y\frac{d}{dx}(e^x) + e^x\frac{d}{dy}(y)\right]$$

$$5y^4\frac{dy}{dx} + x^2\left(2y\frac{dy}{dx}\right) + y^2(2x) = y(e^x) + e^x\frac{dy}{dx}$$

$$5y\frac{dy}{dx} + 2x^2y\frac{dy}{dx} - e^x\frac{dy}{dx} = e^xy - 2xy^2$$

$$\frac{dy}{dx}\left(5y + 2x^2y - e^x\right) = y\left(e^x - 2xy\right)$$

$$\frac{\frac{dy}{dx}\cancel{\left(5y + 2x^2y - e^x\right)}}{\cancel{\left(5y + 2x^2y - e^x\right)}} = \frac{y\left(e^x - 2xy\right)}{\left(5y + 2x^2y - e^x\right)}$$

$$\boxed{\frac{dy}{dx} = \frac{y\left(e^x - 2xy\right)}{5y + 2x^2y - e^x}}$$

Q4

Tuesday, October 6, 2020 5:22 PM

4) Let $f(x) = \frac{e^x}{x^2+1}$, find tangent line @ $(0,1)$

$$f'(x) = \frac{d}{dx}\left(\frac{e^x}{x^2+1}\right)$$

$$= \frac{(x^2+1)\frac{d}{dx}(e^x) - e^x\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)e^x - e^x(2x)}{(x^2+1)^2}$$

$$= \frac{e^x x^2 + e^x - 2xe^x}{(x^2+1)^2}$$

Let $x = 0$

$$f'(0) = \frac{e^x(x^2+1-2x)}{(x^2+1)^2}$$

$$= \frac{e^0(0^2+1-2(0))}{(0^2+1)^2}$$

$$= \frac{1(0+1-0)}{(0+1)^2}$$

$$m = \frac{1}{1} = 1$$

$$y - y_1 = m(x - x_1) \quad , (0,1)$$

$$y - (1) = 1(x - 0)$$

$$y - 1 = x$$

$$\boxed{y = x - 1}$$

Q5

Tuesday, October 6, 2020 5:36 PM

5) Use linear approximation to estimate $\sqrt{15\ 8}$

$\sqrt{15.8} \approx \sqrt{16}$

$f(x) = \sqrt{x}$, $a = 16$

$f'(x) = \frac{d}{dx}(x^{1/2})$

$= \frac{1}{2} x^{-1/2}$

$L(x) = f(a) + f'(a)(x-a)$

$= f(16) + f'(16)(x-a)$

$= \sqrt{16} + \frac{1}{2}(16)^{-1/2}(x-16)$

$= 4 + \frac{1}{2}\left(\frac{1}{\sqrt{16}}\right)(x-16)$

$= 4 + \frac{1}{8}(x-16)$

$= 4 + \frac{1}{8}x - 2$

$L(x) = \frac{1}{8}x + 2$

$\sqrt{15\ 8} = L(15.8)$

$= \frac{1}{8}(15\ 8) + 2$

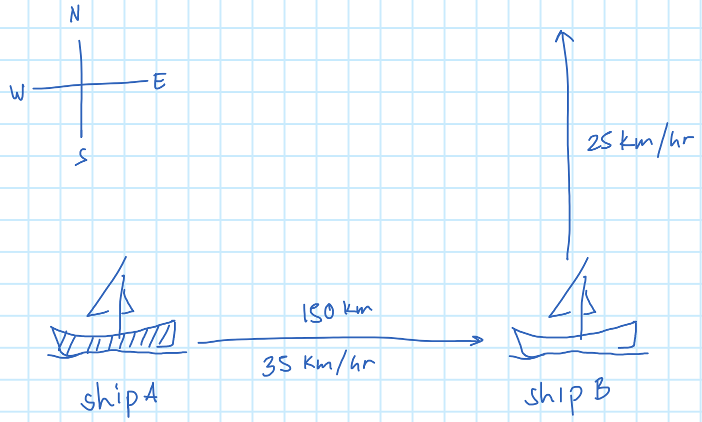
$= \boxed{3.975}$

Q6

Tuesday, October 6, 2020 5:48 PM

6)

@ 12 00 nn



@ 16'00 pm

ship A Distance traveled = 35km (16 − 12)
= 140 km

ship B Distance traveled = 25(16 − 12)
= 100 km

ship A Distance away from
initial position of ship B = 150 − 140
= 10 km

$$z^2 = x^2 + y^2$$

$$\frac{dz}{dt}(z^2) = \frac{dx}{dt}(x^2) + \frac{dy}{dt}(y^2)$$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{1}{\cancel{2z}}\left(\cancel{2}z\frac{dz}{dt}\right) = \left(\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}\right)\frac{1}{\cancel{2}z}$$

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$$

$$\frac{dz}{dt} = \frac{10(35) + 100(25)}{\sqrt{10100}}$$

$$\boxed{\frac{dz}{dt} = \frac{2850}{\sqrt{10100}}\ \text{km/hr}}$$

$$z^2 = x^2 + y^2$$

$$z^2 = 10^2 + 100^2$$

$$= 100 + 10000$$

$$z = \sqrt{10100}$$