1. Consider the famous identity:

$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$

as well as the also famous identity:

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

Select true statements:

A $\cos(a-b) + \cos(a+b) = \cos(2a)$

B (**) $\cos(a-b) + \cos(a+b) = 2\cos(a)\cos(b)$

C If true, (**) leads to the very famous identity

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$

2.

$$\cos\left(\frac{\pi}{7}\right) - \cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{3\pi}{7}\right) = \cos\left(\frac{\pi}{3}\right)$$

A false B true

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3. Algebraic manipulations and/or substitutions on the famous identity:

$$\sin(2x) = 2\sin(x)\cos(x)$$

would correctly yield the following identity/ies:

A $\sin(50x) = 2\sin(25x)\cos(25x)$

B $\sin(4x) = 2\sin(2x)\cos(2x)$

C $\sin\left(\frac{x}{5}\right) = 2\sin\left(\frac{x}{10}\right)\cos\left(\frac{x}{10}\right)$

D $\sin(10x) = 2\sin(5x)\cos(5x)$

E $\sin(x) = 2\sin(.5x)\cos(.5x)$

F $\sin(x) = 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)$

G none of these

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4. Consider the famous identity:

$$\frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right] = \sin(a)\sin(b)$$

Select true statements.

A Substituting $u = \frac{x}{2}$ and $v = \frac{y}{2}$ on (B) would yield the (also very famous) identity:

$$\cos(y) - \cos(x) = 2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

B This identity can be re-written as

$$\cos(a-b) - \cos(a+b) = 2\sin(a)\sin(b)$$

C Substituting $a = u+v$ and $b = u-v$ on (A) would yield :

$$\cos(2v) - \cos(2u) = 2\sin(u+v)\sin(u-v)$$

D none of these

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5. The expression:

$$\cos\left(\frac{x+\pi}{2}\right)\cos\left(\frac{x-\pi}{2}\right) = \frac{1}{2}\cos(x) - \frac{1}{2}$$

is an identity.

A False B True

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6. Select expressions equivalent to:

$$\cos(15°)$$

A $\frac{1}{2} \cdot \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$

B $\cos(60°)\cos(45°) + \sin(60°)\sin(45°)$

C $\frac{1}{3}\cos(45°)$

D $\cos(60° - 45°)$

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7. The expression:

$$\sin(x + 90°) + \sin(x - 30°) = \cos(60° - x)$$

is an identity.

A True B False

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8. The expression:

$$\cos\left(x + \frac{\pi}{2}\right)\cos\left(x - \frac{\pi}{2}\right) = \frac{1}{2}\left[\cos(x) + \cos\left(\frac{\pi}{2}\right)\right]$$

is an identity.

A True B False

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9. Consider the famous identity:

$$\sin(a - b) = \sin(a)\cos(b) - \cos(a)\sin(b)$$

as well as the also famous identity:

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

Select true statements:

A $\sin(a - b) + \sin(a + b) = 2\sin(a)\cos(b)$

B $\sin(a - b) + \sin(a + b) = \sin(2a)$

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10. The expression:

$$\sin(15x) + \sin(3x) = 2\sin(9x)\cos(6x)$$

is an identity.

A False B True

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11. Consider the famous identity:

$$\frac{1}{2}[\sin(a + b) + \sin(a - b)] = \sin(a)\cos(b)$$

Select true statements.

A Substituting $a = \frac{x+y}{2}$ and $b = \frac{x-y}{2}$ on (***) would yield the (also very famous) identity:

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

B (***)This identity can be re-written as

$$\sin(a + b) + \sin(a - b) = 2\sin(a)\cos(b)$$

C none of these

---

12. The very famous famous identity:

$$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

can be proven from the MOTA identity:

$$\cos(a - b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$

If we start with the substitution:

A $a = x$ and $b = y$ B $a = x$ and $b = -y$ C $a = x$ and $b = x$

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13. Algebraic manipulations on the famous identity:

$$\cos(2x) = 1 - 2\sin^2(x)$$

would correctly yield the following identity/ies:

A $4\sin^4(x) = 1 - \cos(4x)$

B $\sin^2(x) = \frac{1-\cos(2x)}{2}$

C $2\sin^2(x) = 1 - \cos(2x)$

14. Select the true identities

A
$$\cos(\pi/7) - \cos(2\pi/7) + \cos(3\pi/7) = \cos(\pi/7)$$

B
$$\sin^2(x) - \cos^2(x) - \tan^2(x) = \frac{2\sin^2(x) - 2\sin^4(x) - 1}{1 - \sin^2(x)}$$

C
$$\cos(\pi/7) - \cos(2\pi/7) + \cos(3\pi/7) = \cos(\pi/3)$$

D
$$\arcsin(x) + \arccos(x) = \frac{\pi}{2}$$

E
$$\cos(x) + \cos(3x) + \cos(5x) = \frac{\sin(6x)}{2\sin(x)}$$

F
$$\frac{\csc x}{\csc x - \sin x} = \sec^2 x$$

G
$$\cos(20^o) \cdot \cos(40^o) \cdot \cos(80^o) = \frac{1}{8}$$

15. Select expressions equivalent to:
$$\tan(x + y)$$

A
$$\frac{\sin(x + y)}{\cos(x + y)}$$

B
$$\frac{\sin(x)\cos(y) + \cos(x)\sin(y)}{\cos(x)\cos(y) - \sin(x)\sin(y)}$$

C
$$\frac{\frac{\sin(x)\cos(y)}{\cos(x)\cos(y)} + \frac{\cos(x)\sin(y)}{\cos(x)\cos(y)}}{\frac{\cos(x)\cos(y)}{\cos(x)\cos(y)} + \frac{\sin(x)\sin(y)}{\cos(x)\cos(y)}}$$

16. Select expressions equivalent to:
$$\tan(x + y)$$

A
$$\frac{\sin(x)\cos(y) + \cos(x)\sin(y)}{\cos(x)\cos(y) - \sin(x)\sin(y)}$$

B
$$\frac{\sin(x + y)}{\cos(x + y)}$$

C
$$\frac{\frac{\sin(x)\cos(y)}{\cos(x)\cos(y)} + \frac{\cos(x)\sin(y)}{\cos(x)\cos(y)}}{\frac{\cos(x)\cos(y)}{\cos(x)\cos(y)} - \frac{\sin(x)\sin(y)}{\cos(x)\cos(y)}}$$

D none of these

17.

Algebraic manipulations on the famous identity:

$$\cos(2x) = 1 - 2\sin^2(x)$$

would correctly yield the following identity/ies:

A $4\sin^4(x) = 1 - \cos(4x)$

B $\sin^2(x) = \frac{1-\cos(2x)}{2}$

C $2\sin^2(x) = 1 - \cos(2x)$

---

18. Algebraic manipulations and/or substitutions on the famous identity:

$$\cos(2x) = 2\cos^2(x) - 1$$

would correctly yield the following identity/ies:

A $\cos(3x) = 3\cos^3(x) - 1$

B $\cos(60°) = 2\cos^2(30°) - 1$

C $\cos(\theta) = 2\cos^2\left(\frac{\theta}{2}\right) - 1$

D $\cos(90°) = 2\cos^2(45°) - 1$

E $\cos(blah) = 2\cos^2(blah/2) - 1$

F $\cos(18t) = 2\cos^2(9t) - 1$

G $\cos(6t) = 2\cos^2(3t) - 1$

H $\frac{1+\cos(\theta)}{2} = \cos^2\left(\frac{\theta}{2}\right)$

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19. The very famous famous identity:

$$\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

can be proven from the MOTA identity:

$$\cos(a - b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$

If we start with the substitution:

A $a = 90° - x$ and $b = -y$

B $a = 90° - x$ and $b = 90° - y$

C $a = 90° - x$ and $b = y$

---

20. The expression:

$$\cos(10x)\cos(4x) = \frac{1}{2}[\cos(14x) + \cos(6x)]$$

is an identity.

A True B False

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21. Consider the famous identity:

$$\frac{1}{2}[\sin(a+b) + \sin(a-b)] = \sin(a)\cos(b)$$

Select true statements.

A Substituting $a = \frac{x+y}{2}$ and $b = \frac{x-y}{2}$ on (**) would yield the (also very famous) identity:

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

B (**) This identity can be re-written as

$$\sin(a+b) + \sin(a-b) = 2\sin(a)\cos(b)$$

C none of these

---

22. Algebraic manipulations and/or substitutions on the famous identity:

$$\cos(2x) = 2\cos^2(x) - 1$$

would correctly yield the following identity:

$$\cos^2(15°) = \frac{1}{2} + \frac{1}{2}\cos(30°)$$

A True B False

23.

The expression:

$$\cos(10x)\cos(4x) = \cos(40x^2)$$

is an identity.

A False B True

24.

$$\cos(x) + \cos(3x) + \cos(5x) = \frac{\sin(6x)}{2\sin(x)}$$

A identity B not an identity

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25. Algebraic manipulations on the famous identity:

$$\cos(2x) = 2\cos^2(x) - 1$$

would correctly yield the following identity/ies:

A $3\sin^3(x) = 1 - \cos(3x)$

B $\cos^2(x) = \frac{1+\cos(2x)}{2}$

C $2\cos^2(x) = 1 + \cos(2x)$

---

26. Algebraic manipulations and/or substitutions on the famous identity:

$$\cos(2x) = 1 - 2\sin^2(x)$$

would correctly yield the following identity/ies:

A $2\sin^2(x) = \cos^2(x) + \sin^2(x) - \cos(2x)$

B $4\sin^4(x) = 1 - \cos(4x)$

C $2\sin^2(x) = 1 - \cos(2x)$

D $\sin^2(x) = \frac{1-\cos(2x)}{2}$

E $\sin^2(x) = \cos^2(x) - \cos(2x)$

---

27. Select expressions equivalent to:

$$\cos(3x - x)$$

A $\cos(7x)\cos(5x) + \sin(7x)\sin(5x)$

B $\cos(3)$

C $\cos(7x - 5x)$

D $\cos(3x)\cos(x) + \sin(3x)\sin(x)$

E $\cos(2x)$

---

28. Consider the famous identity:

$$\cos(a - b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$

as well as the also famous identity:

$$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

Select true statements:

A $\cos(a - b) - \cos(a + b) = \cos(-2b)$

B $\cos(a - b) - \cos(a + b) = 2\sin(a)\sin(b)$

C If true, (B) leads to the very famous identity

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a - b) - \cos(a + b)\right]$$

---

29. Substituting $a = x$ and $b = -x$ on the famous identity:

$$\cos(a - b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$

would yield the also famous identity:

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

A True B False

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30. Substituting $a = x$ and $b = x$ on the famous identity:

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

would yield the also famous identity:

$$\sin(2x) = 2\sin(x)\cos(x)$$

A True B False