Q1

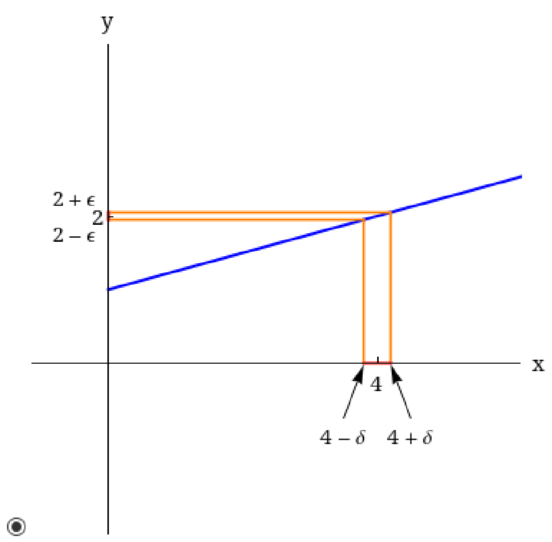
Monday, December 7, 2020 6:07 PM

Prove the statement using the ε, δ definition of a limit.

$$\lim_{x \to 4} \left(1 + \frac{1}{4}x\right) = 2$$

Given $\varepsilon > 0$, we need δ > 0 such that if $0 < |x - 4| < \delta$, then $\left|\left(1 + \frac{1}{4}x\right) - 2\right|$ $< \varepsilon$. But $\left|\left(1 + \frac{1}{4}x\right) - 2\right| < \varepsilon \Leftrightarrow \left|\frac{1}{4}x - 1\right| < \varepsilon \Leftrightarrow \left|\frac{1}{4}\right||x - 4| < \varepsilon \Leftrightarrow |x - 4| <$ 4ε . So if we choose $\delta =$ 4ε then $0 < |x - 4| < \delta \Rightarrow \left|\left(1 + \frac{1}{4}x\right) - 2\right| < \varepsilon$. Thus, $\lim_{x \to 4} \left(1 + \frac{1}{4}x\right) = 2$ by the definition of a limit.

Illustrate with a diagram.



Q2

Monday, December 7, 2020 6:20 PM

Prove the statement using the ε, δ definition of a limit.

$$\lim_{x \to 1} \frac{7 + 2x}{3} = 3$$

Given $\varepsilon > 0$, we need δ [> 0] such that if $0 < |x - 1| < \delta$, then $\left|\frac{7 + 2x}{3} - 3\right|$ [$< \varepsilon$]. But $\left|\frac{7 + 2x}{3} - 3\right| < \varepsilon \Leftrightarrow \left|\frac{2x - 2}{3}\right| < \varepsilon \Leftrightarrow \left|\frac{2}{3}\right| |x - 1| < \varepsilon \Leftrightarrow |x - 1| <$ [(3/2)ε]. So if we choose $\delta =$ [(3/2)ε], then $0 < |x - 1| < \delta \Rightarrow \left|\left(\frac{7 + 2x}{3}\right) - 3\right| < \varepsilon$. Thus, $\lim_{x \to 1} \left(\frac{7 + 2x}{3}\right) = 3$ by the definition of a limit.

Q3

Monday, December 7, 2020 6:21 PM

Prove the statement using the ε, δ definition of a limit.

$$\lim_{x \to 7} \frac{x^2 - 5x - 14}{x - 7} = 9$$

Given $\varepsilon > 0$, we need δ [> 0] such that if $0 < |x - 7| < $ [δ], then $\left|\frac{x^2 - 5x - 14}{x - 7} - 9\right|$ [$< \varepsilon$]. We have $\left|\frac{x^2 - 5x - 14}{x - 7} - 9\right| < \varepsilon \iff \left|\frac{(x - 7)(x + 2)}{x - 7} - 9\right| < \varepsilon \iff |x + 2 - 9| < \varepsilon \; [x \neq 7] \iff |x - 7| < \varepsilon$. Choose $\delta =$ [ε].

Then $0 < |x - 7| < \delta \Rightarrow |x - 7| <$ [ε] $\Rightarrow \left|\frac{x^2 - 5x - 14}{x - 7} - 9\right|$ [$< \varepsilon$]. By the definition of a limit,

$$\lim_{x \to 7} \frac{x^2 - 5x - 14}{x - 7} = 9.$$

Q4

Monday, December 7, 2020 6:24 PM

Prove the statement using the ε, δ definition of a limit.

$$\lim_{x \to a} x = a$$

Given $\varepsilon > 0$, we need δ > 0 such that if $0 < |x - a| <$ δ, then $|x - a|$ $< \varepsilon$. Choose $\delta =$ ε. Then $0 < |x - a| < \delta \Rightarrow |x - a| <$ ε. By the definition of a limit, $\lim_{x \to a} x = a$.

Q5

Monday, December 7, 2020 6:24 PM

Prove the statement using the ε, δ definition of a limit.

$$\lim_{x \to 0} x^2 = 0$$

Given $\varepsilon > 0$, we need δ > 0 such that if $0 < |x - 0| <$ δ , then $|x^2 - 0|$ $< \varepsilon$. We have $|x^2| < \varepsilon \iff x^2 < \varepsilon \iff |x| < \sqrt{\varepsilon}$. Choose $\delta = \sqrt{\varepsilon}$. Then $0 < |x - 0| < \delta \Rightarrow |x^2 - 0|$ $< \varepsilon$. By the definition of a limit, $\lim_{x \to 0} x^2 = 0$.