Q1

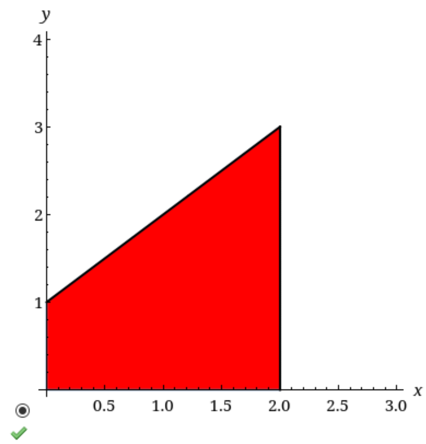
Tuesday, December 1, 2020 4:41 PM

Find the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line.

$y = x + 1$, $y = 0$, $x = 0$, $x = 2$; about the x-axis

$V = \boxed{\pi\left(\frac{8}{3} + 6\right)}$

$y = x + 1$, $y = 0$, $x = 0$, $x = 2$; about the x-axis



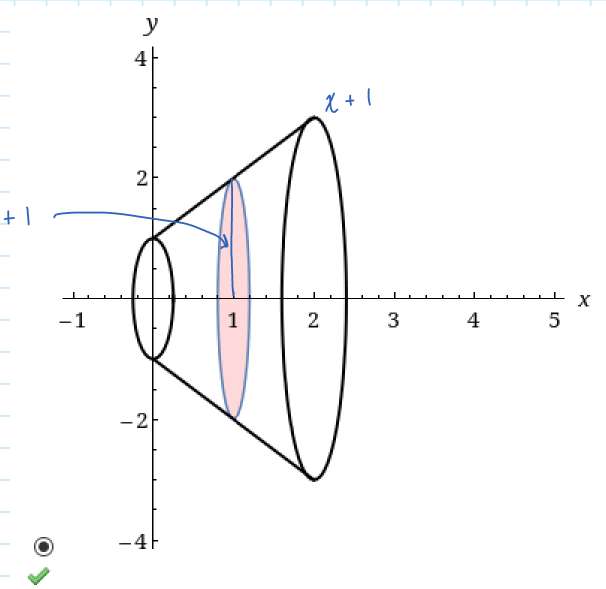
$$\int_a^b \pi f(x)^2\, dx = \pi \int_0^2 (x+1)^2\, dx = \pi \int_0^2 (x^2 + 2x + 1)\, dx$$

$$= \pi \left[\frac{x^3}{3} + x^2 + x \right]_0^2$$

$$= F(2) - F(0)$$

$$= \pi \left[\left(\frac{2^3}{3} + 2^2 + 2\right) - \underbrace{\cancel{\left(\frac{0^3}{3} + 0^2 + 0\right)}}_{=0} \right] = \pi\left(\frac{8}{3} + 4 + 2\right)$$

$$= \boxed{\pi\left(\frac{8}{3} + 6\right)}$$



Q2

Tuesday, December 1, 2020 7:43 PM

Find the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line.

$y = \sqrt{x-1}$, $y = 0$, $x = 4$; about the x-axis

$V = \boxed{\pi\left(5 - \frac{1}{2}\right)}$

$y = \sqrt{x-1}$, $y = 0$, $x = 4$

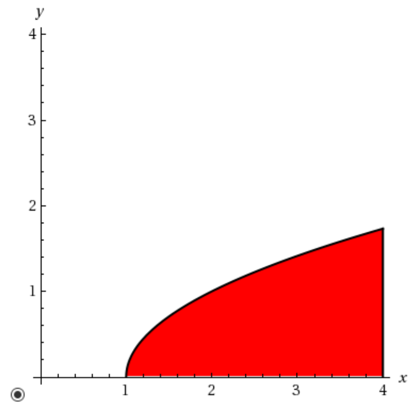
$$\int_a^b \pi f(x)^2\,dx = \pi \int_1^4 \left(\sqrt{x-1}\right)^2 dx = \pi \int_1^4 (x-1)\,dx$$

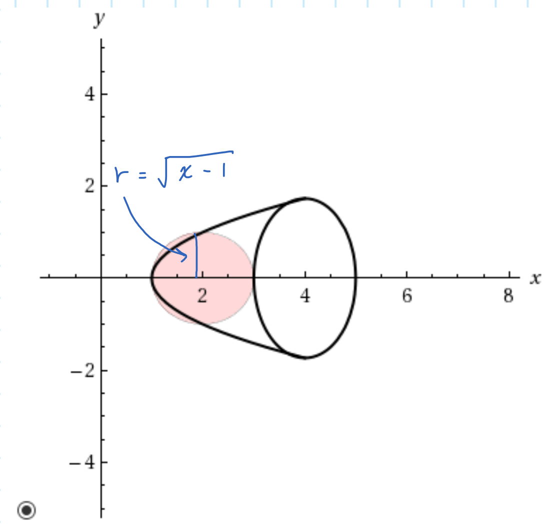
$$= \pi \left[\frac{x^2}{2} - x\right]_1^4$$

$$= F(4) - F(1)$$

$$= \pi\left[\left(\frac{4^2}{2} - 4\right) - \left(\frac{1^2}{2} - 1\right)\right] = \pi\left(8 - 4 - \frac{1}{2} + 1\right)$$

$$= \boxed{\pi\left(5 - \frac{1}{2}\right)}$$





Q3

Tuesday, December 1, 2020 7:58 PM

Find the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line.

$x = 2\sqrt{7y}$, $x = 0$, $y = 5$; about the y-axis

$V = 350\pi$

$x = 2\sqrt{7y}$, $x = 0$, $y = 5$; about y-axis

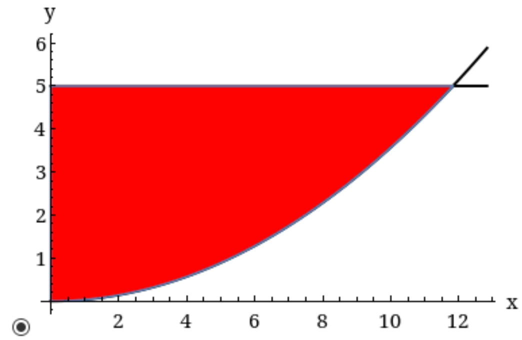
$$\int_a^b \pi f(y)^2\,dy = \pi \int_0^5 \left(2\sqrt{7y}\right)^2 dy = \pi \int_0^5 (28y)\,dy$$

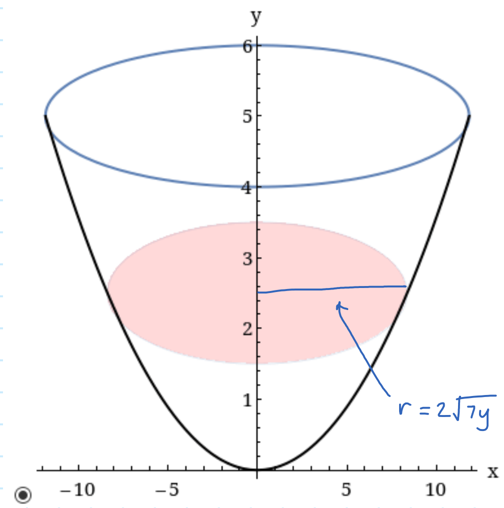
$= \pi\left(14y^2\right)$

$= F(5) - F(0)$

$= \pi\left[14(5^2) - \underbrace{14(0^2)}_{=0}\right]$

$= \boxed{350\pi}$





Q4

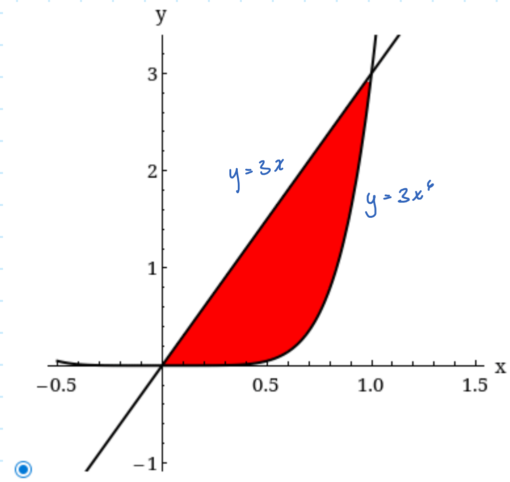
Tuesday, December 1, 2020 8:11 PM

Find the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line.

$y = 3x^6, \quad y = 3x, \quad x \geq 0;$ about the x-axis

$V = \boxed{\pi\left(3 - \frac{9}{13}\right)}$

$y = 3x^6,\ y = 3x,\ x \geq 0$; about the x-axis



Get all intersections

$$3x^6 = 3x$$

$$3x^6 - 3x = 0$$

$$3x(x^5 - 1) = 0$$

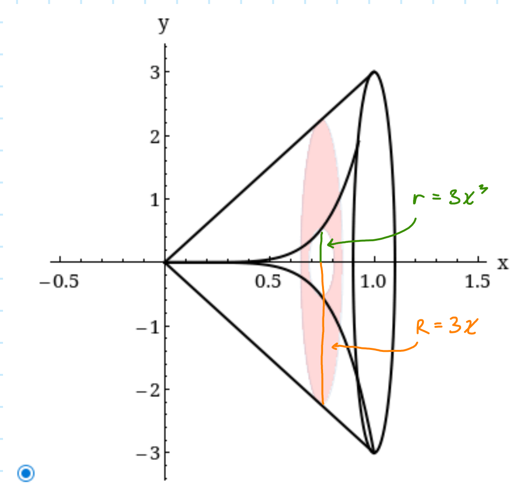
$x = 0$ $\qquad$ $x^5 - 1 = 0$

$$x^5 = 1$$

$$\underline{x = 1}$$

$$\int_a^b \pi R^2 - \pi r^2\, dx = \int_a^b \pi(R^2 - r^2)\, dx$$

where R is the radius of the outer circle and r is the radius of the inner circle



$$\int_0^1 \pi\left[(3x)^2 - (3x^6)^2\right] dx = \pi\int_0^1 \left(9x^2 - 9x^{12}\right) dx$$

$$= \pi\left[3x^3 - \frac{9x^{13}}{13}\right]_0^1$$

$$= F(1) - F(0)$$

$$= \pi\left[\left(3(1^3) - \frac{9(1^{13})}{13}\right) - \underbrace{\cancel{\left(3(0^3) - \frac{9(0^{13})}{13}\right)}}_{=0}\right]$$

$$= \boxed{\pi\left(3 - \frac{9}{13}\right)}$$

Q5

Tuesday, December 1, 2020 9:09 PM

Find the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line.

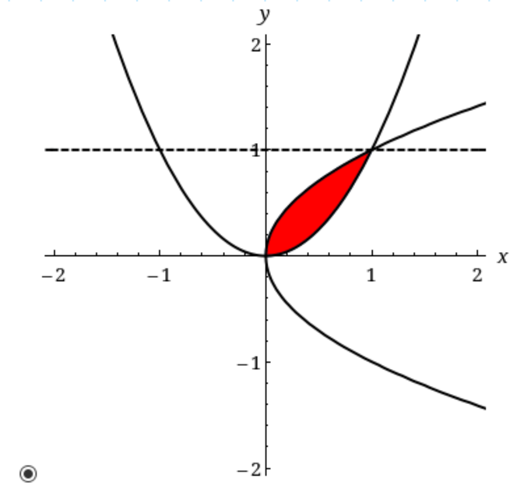
$y = x^2, \quad x = y^2; \quad$ about $y = 1$

$V = \frac{11\pi}{30}$

$y = x^2, \; x = y^2 \; ; \;$ about $y = 1$

$x = \sqrt{y}$

subtract 1 because rotating about $y = 1$



$$\int_a^b \pi(R^2 - r^2)\,dy = \pi \int_0^1 \left[(y^2-1)^2 - (\sqrt{y}-1)^2\right] dy$$

$$= \pi \int_0^1 \left[(y^4 - 2y^2 + 1) - (y - 2\sqrt{y} + 1)\right] dy = \pi \int_0^1 (y^4 - 2y^2 - y + 2\sqrt{y})\,dy$$

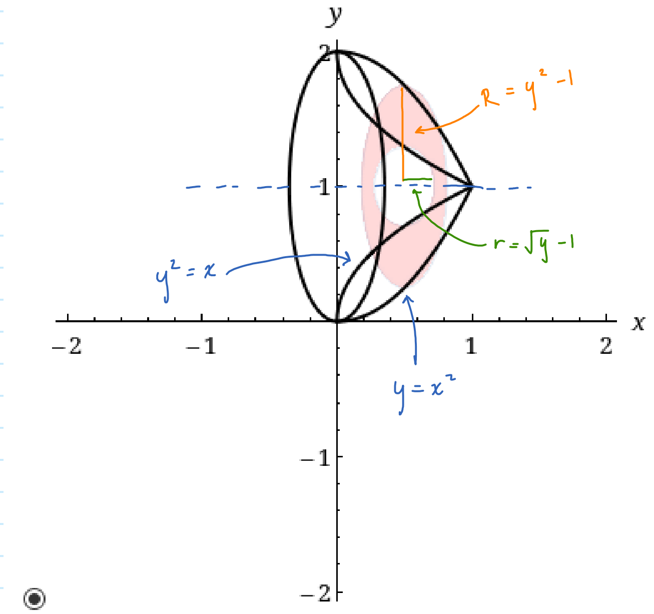
$$= \pi \left[\frac{y^5}{5} - \frac{2y^3}{3} - \frac{y^2}{2} + \frac{2y^{3/2}}{\frac{3}{2}}\right]_0^1 = \pi \left[\frac{y^5}{5} - \frac{2y^3}{3} - \frac{y^2}{2} + \frac{4y^{3/2}}{3}\right]_0^1$$

$$= F(1) - F(0)$$

$$= \pi \left[\left(\frac{1^5}{5} - \frac{2(1^3)}{3} - \frac{1^2}{2} + \frac{4(1^{3/2})}{3}\right) - \underbrace{\cancel{\left(\frac{0^5}{5} - \frac{2(0^3)}{3} - \frac{0^2}{2} + \frac{4(0^{3/2})}{3}\right)}}_{=0}\right]$$

$$= \pi\left(\frac{1}{5} - \frac{2}{3} - \frac{1}{2} + \frac{4}{3}\right)$$

$$= \boxed{\frac{11}{30}\pi}$$



Q6

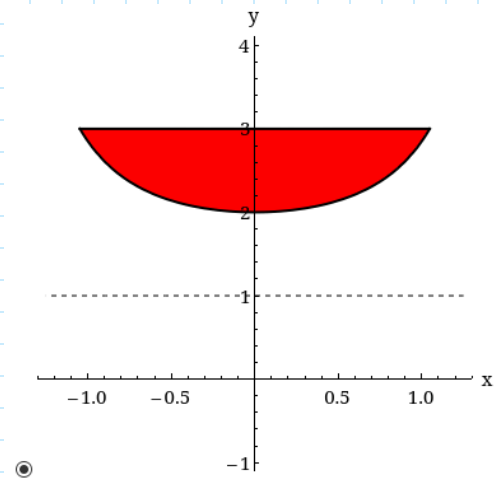
Tuesday, December 1, 2020 9:41 PM

Find the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line.

$$y = 1 + \sec(x),\ \frac{-\pi}{3} \le x \le \frac{\pi}{3},\ y = 3; \quad \text{about } y = 1$$

$$V = \boxed{2\pi\left(\frac{4\pi}{3} - \tan\left(\frac{\pi}{3}\right)\right)}$$

$y = 1 + \sec(x),\ \frac{-\pi}{3} \le x \frac{\pi}{3},\ y = 3;$ about $y = 1$



since the function is symmetrical

−1 because rotating about $y = 1$

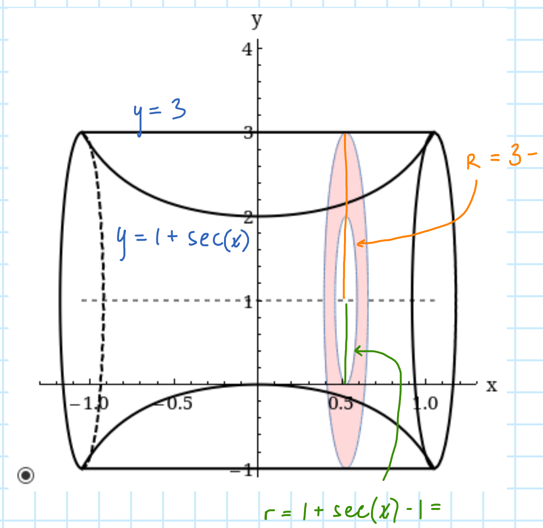
$$\int_a^b \pi(R^2 - r^2)\,dx = 2\int_0^{\pi/3} \pi\left[(3-1)^2 - (1+\sec(x)-1)^2\right] dy$$

$$= 2\pi\int_0^{\pi/3} (4 - \sec^2(x))\,dx = 2\pi\left[4x - \tan(x)\right]_0^{\pi/3}$$

$$= F\left(\frac{\pi}{3}\right) - F(0)$$

$$= 2\pi\left[\left(4\left(\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{3}\right)\right) - \underbrace{\cancel{(4(0) - \tan(0))}}_{=0}\right]$$

$$= \boxed{2\pi\left[\frac{4\pi}{3} - \tan\left(\frac{\pi}{3}\right)\right]}$$



Q7

Tuesday, December 1, 2020   10:05 PM

Find the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line.

$y = x$, $y = 0$, $x = 2$, $x = 7$;   about $x = 1$

$V = \frac{535}{3}\pi$ ✔

$y = x$, $y = 0$, $x = 2$, $x = 7$; about $x = 1$

$y = x = 2$ $y = x = 7$

$$\int_a^b \pi(R_1^2 - r_1^2)\,dy + \int_c^d \pi(R_2^2 - r_2^2)\,dy$$

$$= \int_0^2 \pi\left[(6^2 - 1^2)\right]dx + \int_2^7 \pi\left[(6^2 - (x-1)^2)\right]dx$$

$$= \pi\int_0^2 35\,dx + \pi\int_2^7 (35 - x^2 + 2x)\,dx$$

$$= F(2) - F(0) + G(7) - G(2)$$

$$= \pi(35x)\Big|_0^2 + \pi\left(35x - \frac{x^3}{3} + x^2\right)\Big|_2^7$$

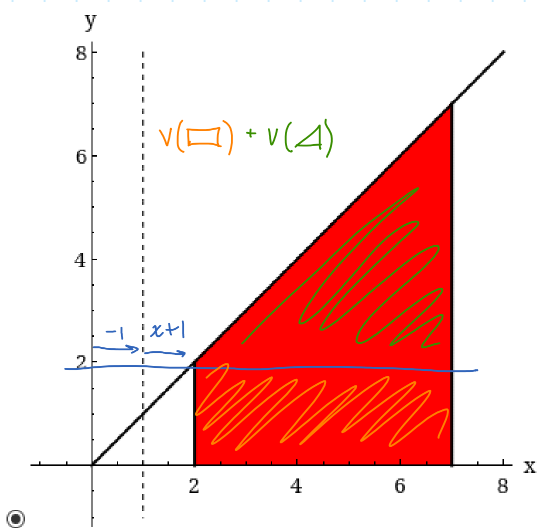
$$= \pi\left(35(2) + \underbrace{\cancel{35(0)}}_{=0}\right) + \pi\left[\left(35(7) - \left(\frac{7^3}{3}\right) + 7^2\right) - \left(35(2) - \left(\frac{2^3}{3}\right) + 2^2\right)\right]$$

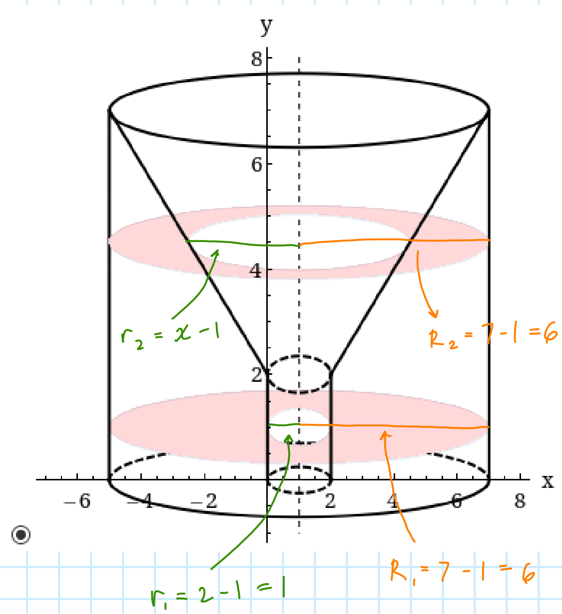
$$= 70\pi + \pi\left[\left(245 - \frac{343}{3} + 49\right) - \left(70 - \frac{8}{3} + 4\right)\right]$$

$$= 70\pi + \pi\left(\frac{539}{3} - \frac{214}{3}\right)$$

$$= 70\pi + \frac{325}{3}\pi$$

$$= \boxed{\frac{535}{3}\pi}$$





Q8

Tuesday, December 1, 2020 10:47 PM

The integral represents the volume of a solid. Describe the solid.

$$\pi \int_0^{\pi} \sin(x)\, dx$$

- (selected) The integral describes the volume of the solid obtained by rotating the region $\mathcal{R} = \left\{(x, y) \mid 0 \le x \le \pi,\ 0 \le y \le \sqrt{\sin(x)}\right\}$ of the xy-plane about the x-axis.
- The integral describes the volume of the solid obtained by rotating the region $\mathcal{R} = \left\{(x, y) \mid 0 \le x \le \pi,\ 0 \le y \le \sin(x)\right\}$ of the xy-plane about the x-axis.
- The integral describes the volume of the solid obtained by rotating the region $\mathcal{R} = \left\{(x, y) \mid 0 \le x \le \pi,\ 0 \le y \le \pi \sin(x)\right\}$ of the xy-plane about the x-axis.
- The integral describes the volume of the solid obtained by rotating the region $\mathcal{R} = \left\{(x, y) \mid 0 \le x \le \pi,\ 0 \le y \le \sqrt{\sin(x)}\right\}$ of the xy-plane about the y-axis.
- The integral describes the volume of the solid obtained by rotating the region $\mathcal{R} = \left\{(x, y) \mid 0 \le x \le \pi,\ 0 \le y \le \sin(x)\right\}$ of the xy-plane about the y-axis.

Q9

Tuesday, December 1, 2020 10:47 PM

Find the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line.

$y^2 = 2x$, $x = 2y$; about the y-axis

$V = \boxed{\dfrac{512\pi}{15}}$

$y^2 = 2x$, $x = 2y$; about the y-axis

find the intersections

$(2)\ x = 2y\ (2)$ $\qquad y^2 = 2x$

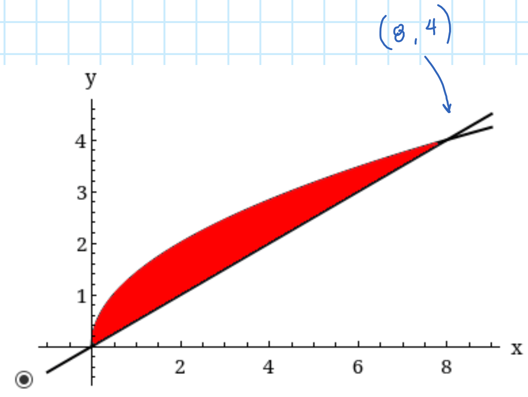
$2x = 4y$

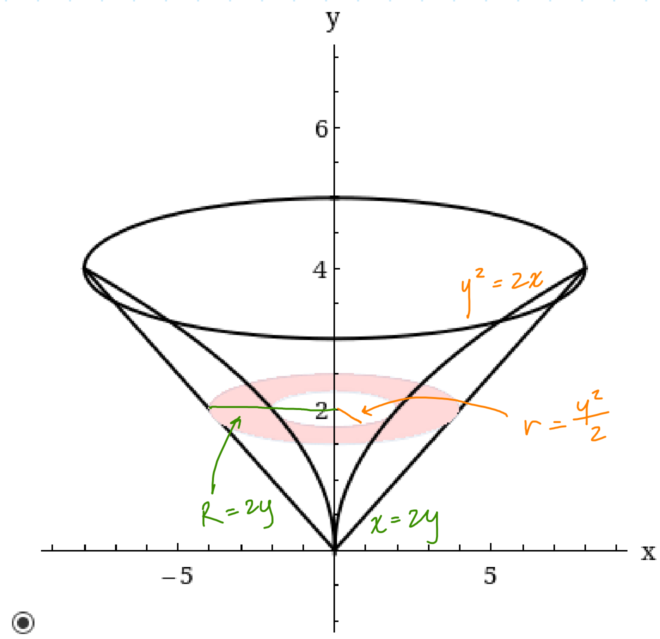
$4y = y^2$

$y^2 - 4y = 0$

$y(y-4) = 0$

$y = 0 \qquad y = 4$





$$\int_a^b \pi(R^2 - r^2)\,dy = \pi\int_0^4 \left[(2y)^2 - \left(\frac{y^2}{2}\right)^2\right] dy$$

$$= \pi\int_0^4 \left(4y^2 - \frac{y^4}{4}\right) dy = \pi\left[\frac{4y^3}{3} - \frac{y^5}{20}\right]_0^4$$

$$= F(4) - F(0)$$

$$= \pi\left[\left(\frac{4(4^3)}{3} - \frac{4^5}{20}\right) - \underbrace{\cancel{\left(\frac{4(0^3)}{3} - \frac{0^5}{20}\right)}}_{=0}\right]$$

$$= \pi\left(\frac{256}{3} - \frac{1024}{20}\right)$$

$$= \boxed{\frac{512}{15}\pi}$$

Q10

Tuesday, December 1, 2020 11:21 PM

Find the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line.

$y = 2\sin(x)$, $y = 2\cos(x)$, $0 \le x \le \pi/4$; about $y = -1$

$V = \boxed{4\sqrt{2}\,\pi - 2\pi}$

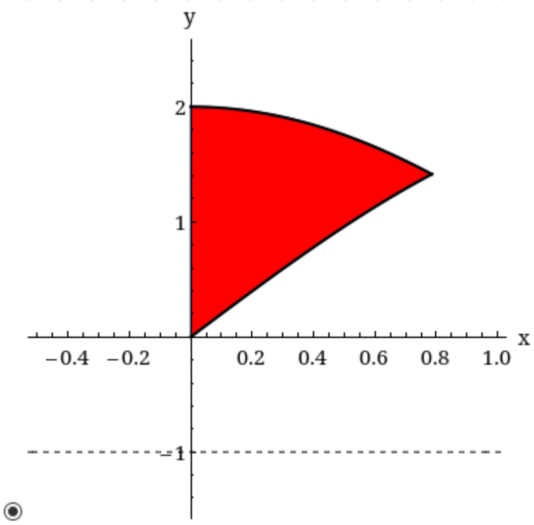
$y = 2\sin(x)$, $y = 2\cos(x)$, $0 \le x \le \frac{\pi}{4}$; about $y = -1$

$$2\sin(x) = 2\cos(x)$$

$$\frac{2\sin(x)}{2\cos(x)} = \frac{2\cos(x)}{2\cos(x)}$$

$$\tan(x) = 1$$

$$x = \underline{\underline{\frac{\pi}{4}}} + \pi n$$



$$\int_a^b \pi(R^2 - r^2)\,dx = \int_0^{\pi/4} \pi\left[(2\cos(x)+1)^2 - (2\sin(x)+1)^2\right] dx$$

$$= \pi \int_0^{\pi/4} \left[(4\cos^2(x) + 4\cos(x) + 1) - (4\sin^2(x) + 4\sin(x) + 1)\right] dx$$

$$= \pi \int_0^{\pi/4} 4\left[\cos^2(x) - \sin^2(x) + \cos(x) - \sin(x)\right] dx$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2} \qquad -\sin^2(x) = -\left(\frac{1-\cos(2x)}{2}\right)$$

$$\frac{1+\cos(2x)}{2} - \left(\frac{1-\cos(2x)}{2}\right)$$

$$= \frac{2\cos(2x)}{2} = \underline{\cos(2x)}$$

$$= 4\pi \int_0^{\pi/4} \left[\cos(2x) + \cos(x) - \sin(x)\right] dx$$

$$= 4\pi \int_0^{\pi/4} \cos(2x)\,dx + 4\pi \int_0^{\pi/4} \left[\cos(x) - \sin(x)\right] dx$$

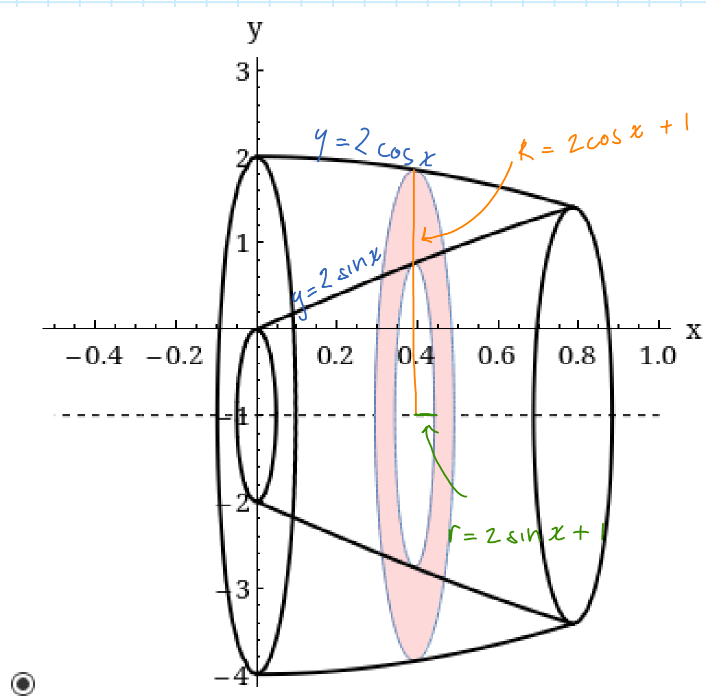
$$u = 2x \qquad \frac{du}{dx} = 2 \qquad du = 2\,dx$$

$$= 4\pi \int_0^{\pi/4} \frac{1}{2} \cdot 2\,dx\,\cos(u) = \frac{4\pi}{2}\sin(u) + C$$

$$= 2\pi\sin(2x)\Big|_0^{\pi/4} + 4\pi\left[\sin(x) + \cos(x)\right]_0^{\pi/4}$$

$$= 2\pi\left[\sin\left(2\tfrac{\pi}{4}\right) - \sin(2(0))\right] + 4\pi\left[\left(\sin\left(\tfrac{\pi}{4}\right) + \cos\left(\tfrac{\pi}{4}\right)\right) - \left(\sin(0) + \cos(0)\right)\right]$$

$$= \boxed{4\sqrt{2}\,\pi - 2\pi}$$



Q11

Wednesday, December 2, 2020 11:32 PM

Find the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line.

$y = 8x^3, \quad y = 0, \quad x = 1; \quad$ about $x = 2$

$\boxed{\dfrac{24\pi}{5}}$

$y = 8x^3$, $y = 0$, $x = 1$; about $x = 2$

Get interception(s)

$y = 8x^3$

$\dfrac{y}{8} = x^3$

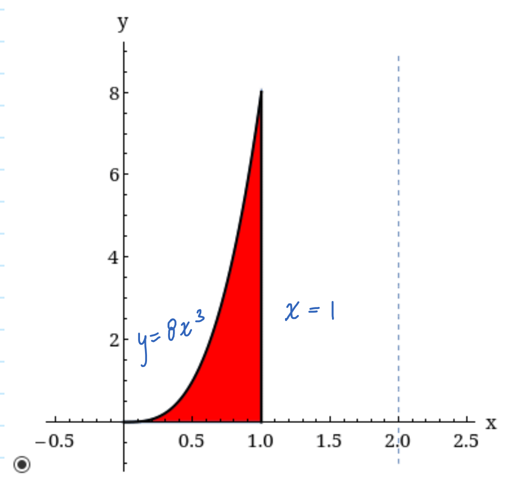
$x = \sqrt[3]{\dfrac{y}{8}}$

$x = \dfrac{\sqrt[3]{y}}{2}$

$\dfrac{\sqrt[3]{y}}{2} = 1$

$\sqrt[3]{y} = 2$

$y = 2^3 = 8$



$$\int_0^8 \pi\left[\left(\frac{\sqrt[3]{y}}{2} - 2\right)^2 - 1^2\right]dy = \pi\int_0^8 \left(\frac{y^{2/3}}{4} - 2y^{1/3} + 3\right) dy$$

$\left(\dfrac{y^{1/3}}{2} - 2\right)\left(\dfrac{y^{1/3}}{2} - 2\right)$

$= \dfrac{y^{2/3}}{4} - 4y^{1/3} + 4$

$\dfrac{y^{2/3+1}}{4\left(\frac{2}{3}+1\right)} = \dfrac{3y^{5/3}}{20}$

$-\dfrac{2y^{1/3+1}}{1/3+1} = -\dfrac{3y^{4/3}}{2}$

$$= \pi\left[\frac{3y^{5/3}}{20} - \frac{3y^{4/3}}{2} + 3y\right]_0^8$$

$$= F(8) - F(0)$$

$$= \pi\left[\left(\frac{3(8)^{5/3}}{20} - \frac{3(8)^{4/3}}{2} + 3(8)\right) - \underbrace{\left(\frac{3(0)^{5/3}}{20} - \frac{3(0)^{4/3}}{2} + 3(0)\right)}_{=0}\right]$$

$$= \pi\left(\frac{24}{5} - 24 + 24\right) = \boxed{\frac{24}{5}\pi}$$