Q1

Tuesday, November 17, 2020 1:02 PM

Evaluate the integral by making the given substitution. (Use C for the constant of integration.)

$$\int \cos(8x)\,dx, \quad u = 8x$$

$$\boxed{\frac{1}{8}\sin(8x) + C}$$

$$\int \cos(8x)\,dx, \quad u = 8x$$

$$\frac{du}{dx} = 8$$

$$\cancel{dx}\,\frac{du}{\cancel{dx}} = 8\,dx$$

$$du = 8\,dx$$

$$= \int \frac{1}{8}\cdot 8\cos(u)\,dx = \frac{1}{8}\int \cos(u)\,8dx = \frac{1}{8}\int \cos(u)\,du$$

$$= \frac{1}{8}\left[\sin(u) + C_1\right] = \boxed{\frac{1}{8}\sin(8x) + C}$$

Q2

Tuesday, November 17, 2020 1:15 PM

Evaluate the integral by making the given substitution. (Use C for the constant of integration.)

$$\int x^2\sqrt{x^3+35}\,dx, \quad u = x^3 + 35$$

$$\boxed{\frac{2}{9}\left(x^3+35\right)^{\left(\frac{3}{2}\right)} + C}$$

$$\frac{du}{dx} = 3x^2$$

$$\cancel{dx}\,\frac{du}{\cancel{dx}} = 3x^2\,dx$$

$$\underline{\underline{du = 3x^2\,dx}}$$

$$\int x^2\sqrt{x^3+35}\,dx, \quad u = x^3 + 35$$

$$= \int \frac{1}{3}\cdot 3x^2\sqrt{u}\,dx = \frac{1}{3}\int \sqrt{u}\,3x^2\,dx = \frac{1}{3}\int\sqrt{u}\,du = \frac{1}{3}\int u^{1/2}\,du$$

$$= \frac{1}{3}\left(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C_1\right) = \frac{1}{3}\left(\frac{2}{3}u^{3/2} + C_1\right) = \frac{2}{9}u^{3/2} + C$$

$$= \boxed{\frac{2}{9}\left(x^3+35\right)^{3/2} + C}$$

Q3

Tuesday, November 17, 2020 1:51 PM

Tutorial Exercise

Evaluate the integral by making the given substitution.

$$\int x^2\sqrt{x^3+20}\,dx, \quad u = x^3+20$$

Step 1

We know that if $u = f(x)$, then $du = f'(x)\,dx$. Therefore, if $u = x^3 + 20$, then

$du = \boxed{3x^2}$ ✓ $\boxed{3x^2}\,dx$.

Step 2

If $u = x^3 + 20$ is substituted into $\int x^2\sqrt{x^3+20}\,dx$, then we have $\int x^2\,(u)^{1/2}\,dx = \int u^{1/2}\,x^2\,dx$.

We must also convert $x^2\,dx$ into an expression involving u.

We know that $du = 3x^2\,dx$, and so $x^2\,dx = \boxed{\frac{1}{3}}$ ✓ $\boxed{\frac{1}{3}}\,du$.

Step 3

Now, if $u = x^3 + 20$, then $\int x^2\sqrt{x^3+20}\,dx = \int u^{1/2}\left(\frac{1}{3}\,du\right) = \frac{1}{3}\int u^{1/2}\,du$.

This evaluates as

$$\frac{1}{3}\int u^{1/2}\,du = \boxed{\frac{2}{9}(u)^{\left(\frac{3}{2}\right)}} ✓ \boxed{\frac{2u^{3/2}}{9}} + C.$$

Step 4

Since $u = x^3 + 20$, then converting back to an expression in x we get

$$\frac{2}{9}u^{3/2} + C = \boxed{\frac{2}{9}(x^3+20)^{\left(\frac{3}{2}\right)} + C} ✓ \boxed{C + \frac{2}{9}(x^3+20)^{3/2}}.$$

You have now completed the Master It.

Q4

Tuesday, November 17, 2020 1:56 PM

Evaluate the integral by making the given substitution. (Use C for the constant of integration.)

$$\int \sin^5(\theta)\cos(\theta)\,d\theta, \quad u = \sin(\theta)$$

$$\boxed{\frac{\sin^6(\theta)}{6} + C}$$

$\frac{du}{d\theta} = \cos(\theta)$

$du = \cos(\theta)\,d\theta$

$\int \sin^5(\theta)\cos(\theta)\,d\theta, \quad u = \sin(\theta)$

$= \int [\sin(\theta)]^5 \cos(\theta)\,d\theta = \int u^5\,du = \frac{u^6}{6} + C$

$= \boxed{\frac{\sin^6(\theta)}{6} + C}$

Q5

Tuesday, November 17, 2020 2:16 PM

Evaluate the integral by making the given substitution. (Use C for the constant of integration. Remember to use absolute values where appropriate.)

$$\int \frac{x^4}{x^5 - 8}\,dx, \quad u = x^5 - 8$$

$$\frac{\ln\left(\left|x^5 - 8\right|\right)}{5} + C$$

$$\int \frac{x^4}{x^5 - 8}\,dx, \quad u = x^5 - 8 \qquad \frac{du}{dx} = 5x^4$$

$$du = 5x^4\,dx$$

$$= \int x^4 \frac{1}{x^5 - 8}\,dx = \int \frac{1}{5} \cdot 5x^4 \frac{1}{x^5 - 8}\,dx = \frac{1}{5}\int \frac{1}{u} 5x^4\,dx = \frac{1}{5}\int \frac{1}{u}\,du$$

$$= \frac{1}{5}\ln(|u|) + C = \frac{1}{5}\ln\left(\left|x^5 - 8\right|\right) + C$$

$$= \boxed{\frac{\ln\left(\left|x^5 - 8\right|\right)}{5} + C}$$

Q6

Tuesday, November 17, 2020 2:37 PM

Evaluate the indefinite integral. (Use C for the constant of integration.)

$$\int x\sqrt{8 - x^2}\, dx$$

$$\boxed{-\frac{2}{6}\left(8 - x^2\right)^{\left(\frac{3}{2}\right)} + C}$$

$$\frac{du}{dx} = -2x$$

$$du = -2x\, dx$$

$$\int x\sqrt{8 - x^2}\, dx \quad , \quad u = 8 - 2x$$

$$= \int -\frac{1}{2} \cdot -2x\sqrt{8 - x^2}\, dx = -\frac{1}{2}\int u^{1/2}\, 2x\, dx$$

$$= -\frac{1}{2}\left(\frac{2}{3}u^{3/2}\right) + C = \boxed{-\frac{2}{6}(8 - x^2)^{3/2} + C}$$

Q7

Tuesday, November 17, 2020 4:10 PM

Evaluate the indefinite integral. (Use C for the constant of integration.)

$$\int (8 - 3x)^8 \, dx$$

$$\boxed{-\frac{(8-3x)^9}{27} + C}$$

$$\int (8-3x)^8 \, dx \quad , \quad u = 8 - 3x$$

$$\frac{du}{dx} = -3$$

$$du = -3\,dx$$

$$= \int -\frac{1}{3} \cdot -3u^8 \, dx = -\frac{1}{3} \int u^8 \, -3\,dx$$

$$= -\frac{1}{3}\left(\frac{u^{8+1}}{8+1}\right) + C = -\frac{u^9}{27} + C = \boxed{-\frac{(8-3x)^9}{27} + C}$$

Q8

Tuesday, November 17, 2020 4:19 PM

Evaluate the indefinite integral. (Remember to use absolute values where appropriate. Use C for the constant of integration.)

$$\int \frac{dx}{9 - 5x}$$

$$\boxed{-\frac{\ln(|9-5x|)}{5} + C}$$

$$\int \frac{dx}{9-5x}, \quad u = 9-5x \qquad \frac{du}{dx} = -5$$

$$du = -5\,dx$$

$$= \int -\frac{1}{5} \cdot -5 \, \frac{1}{9-5x}\, dx = -\frac{1}{5}\int \frac{1}{u} -5\,dx$$

$$= -\frac{1}{5}\ln(|u|) + C = \boxed{-\frac{\ln(|9-5x|)}{5} + C}$$

Q9

Tuesday, November 17, 2020 4:25 PM

Evaluate the indefinite integral. (Use C for the constant of integration.)

$$\int \frac{e^u}{(8 - e^u)^2}\, du$$

$$\boxed{\frac{1}{8 - e^u} + C}$$

Let x- = u-substitution

$$\int \frac{e^u}{(8-e^u)^2}\, du \quad , \quad x = 8 - e^u \qquad \frac{dx}{du} = -e^u$$

$$dx = -e^u\, du$$

$$= \int -1 \cdot -e^u \frac{1}{(8-e^u)^2}\, du = -\int u^{-2} - e^u\, du$$

$$= -\frac{u^{-2+1}}{-2+1} + C = -\frac{u^{-1}}{-1} + C = \boxed{\frac{1}{8-e^u} + C}$$

Q10

Tuesday, November 17, 2020 4:42 PM

Evaluate the indefinite integral. (Use C for the constant of integration.)

$$\int \frac{a + bx^8}{\sqrt{9ax + bx^9}}\, dx$$

$$\boxed{\frac{2\sqrt{9ax + bx^9}}{9} + C}$$

$$\int \frac{a+bx^8}{\sqrt{9ax+bx^9}} \quad , \quad u = 9ax + bx^9$$

$$\frac{du}{dx} = 9a + 9bx^8$$

$$du = 9(a+bx^8)\,dx$$

$$= \int \frac{1}{9} \cdot 9(a+bx^8) \frac{1}{\sqrt{9ax+bx^9}}\, dx = \frac{1}{9} \int u^{-1/2}\, du$$

$$= \frac{1}{9}\left(2u^{1/2}\right) + C = \frac{2\sqrt{u}}{9} + C = \boxed{\frac{2\sqrt{9ax+bx^9}}{9} + C}$$

Q11

Tuesday, November 17, 2020 5:05 PM

Evaluate the indefinite integral. (Use C for the constant of integration.)

$$\int \frac{(\ln(x))^{44}}{x}\,dx$$

$$\boxed{\frac{\ln^{45}(x)}{45} + C}$$

$$\int \frac{(\ln(x))^{44}}{x}\,dx \quad , \quad u = \ln(x)$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$du = \frac{1}{x}\,dx$$

$$= \int (\ln(x))^{44}\,\frac{1}{x}\,dx = \int u^{44}\,du$$

$$= \frac{u^{45}}{45} + C = \boxed{\frac{\ln^{45}(x)}{45} + C}$$

Q12

Tuesday, November 17, 2020 5:25 PM

Evaluate the indefinite integral. (Use C for the constant of integration.)

$$\int e^x \sqrt{43 + e^x}\, dx$$

$$\boxed{\frac{2\left(43+e^x\right)^{\left(\frac{3}{2}\right)}}{3} + C}$$

$$\int e^x \sqrt{43+e^x}\, dx \quad , \quad u = 43 + e^x$$

$$\frac{du}{dx} = e^x$$

$$du = e^x\, dx$$

$$= \int \sqrt{43+e^x}\; e^x dx = \int \sqrt{u}\, du = \int u^{1/2} du$$

$$= \frac{2}{3} u^{3/2} + C = \boxed{\frac{2(43+e^x)^{3/2}}{3} + C}$$

Q13

Tuesday, November 17, 2020 5:31 PM

Evaluate the indefinite integral. (Use C for the constant of integration.)

$$\int \frac{\cos(\pi/x^{43})}{x^{44}}\,dx$$

$$\boxed{-\frac{\sin\left(\frac{\pi}{x^{43}}\right)}{43\pi} + C}$$

$$\int \frac{\cos\left(\frac{\pi}{x^{43}}\right)}{x^{44}}\,dx, \quad u = \frac{\pi}{x^{43}} \qquad \frac{du}{dx} = 43\pi x^{-44}$$

$$du = \frac{43\pi}{x^{44}}\,dx$$

$$= \int \frac{1}{43\pi} \cdot 43\pi \frac{1}{x^{44}} \cos\left(\frac{\pi}{x^{43}}\right) dx = \frac{1}{43\pi} \int \cos(u)\,du$$

$$= \frac{1}{43\pi} \cdot \sin(u) + C = \boxed{-\frac{\sin\left(\frac{\pi}{x^{43}}\right)}{43\pi} + C}$$

Q14

Tuesday, November 17, 2020 5:47 PM

Evaluate the indefinite integral. (Use C for the constant of integration.)

$$\int \frac{dt}{\cos^2(t)\sqrt[6]{8+\tan(t)}}$$

$$\frac{6(8+\tan(t))^{\left(\frac{5}{6}\right)}}{5}+C$$

$$\int \frac{dt}{\cos^2(t)\sqrt[6]{8+\tan(t)}}, \quad u = 8+\tan(t)$$

$$\frac{du}{dt} = \sec^2(t)$$

$$du = \sec^2(t)\,dt \quad \text{or} \quad \frac{1}{\cos^2(t)}\,dt$$

$$= \int \frac{1}{\sqrt[6]{8+\tan(t)}}\frac{1}{\cos^2(t)}\,dt = \int \frac{1}{u^{1/6}}\,du = \int u^{-1/6}\,du$$

$$= \frac{u^{-1/6+1}}{-\frac{1}{6}+1}+C = \frac{6u^{5/6}}{5}+C = \boxed{\frac{6(8+\tan(t))^{5/6}}{5}+C}$$

Q15

Tuesday, November 17, 2020 6:10 PM

Evaluate the indefinite integral. (Use C for the constant of integration.)

$$\int \frac{\sin(44x)}{1+\cos^2(44x)}\,dx$$

$$-\frac{\arctan(\cos(44x))}{44}+C$$

$$\int \frac{\sin(44x)}{1+\cos^2(44x)}\,dx \quad , \quad u=\cos(44x)$$

$$\frac{du}{dx} = -44\sin(44x)$$

$$du = -44\sin(44x)\,dx$$

$$-\frac{1}{44}\,du = \sin(44x)\,dx$$

$$= \int -\frac{1}{44}\cdot -44\sin(44x)\,dx\,\frac{1}{1+\cos^2(44x)} = -\frac{1}{44}\int \frac{1}{1+u^2}\,du$$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$$

$$= -\frac{1}{44}\tan^{-1}(u) + C = \boxed{-\frac{\tan^{-1}(\cos(44x))}{44} + C \text{ or } -\frac{\arctan(\cos(44x))}{44} + C}$$

https://www.symbolab.com/solver/step-by-step/%5Cint%5Cfrac%7Bsin%5Cleft(44x%5Cright)%7D%7B1%2Bcos%5E%7B2%7D%5Cleft(44x%5Cright)%7Ddx

Q16

Tuesday, November 17, 2020 10:01 PM

Evaluate the indefinite integral. (Remember to use absolute values where appropriate. Use C for the constant of integration.)

$$\int \cot(44x)\, dx$$

$$\frac{\ln(|\sin(44x)|)}{44} + C$$

$$\int \cot(44x)\, dx = \int \frac{\cos(44x)}{\sin(44x)}\, dx \quad , \quad u = \sin(44x)$$

$$\frac{du}{dx} = 44\cos(44x)$$

$$du = 44\cos(44x)\, dx$$

$$= \int \frac{1}{44}\, 44\cos(44x)\, \frac{1}{\sin(44x)}\, dx = \frac{1}{44}\int \frac{1}{u}\, du$$

$$= \frac{1}{44}\ln(|u|) + C = \boxed{\frac{\ln(|\sin(44x)|)}{44} + C}$$

Q17

Tuesday, November 17, 2020 10:11 PM

Evaluate the indefinite integral. (Use C for the constant of integration.)

$$\int \frac{x^9}{1 + x^{20}}\, dx$$

$$\frac{\tan^{-1}\left(x^{10}\right)}{10} + C$$

$$\int \frac{x^9}{1+x^{20}}\, dx, \quad u = x^{10} \qquad \frac{du}{dx} = 10x^9$$

$$du = 10x^9\, dx$$

$$= \int \frac{1}{10}\, 10x^9 \frac{1}{1+x^{20}}\, dx = \frac{1}{10}\int \frac{1}{1+u^2}\, du$$

$$= \frac{1}{10}\tan^{-1}(u) + C = \boxed{\frac{\tan^{-1}(x^{10})}{10} + C}$$

Q18

Tuesday, November 17, 2020 10:47 PM

Evaluate the indefinite integral. (Use C for the constant of integration.)

$$\int \frac{8 + 6x}{1 + x^2}\, dx$$

$$\boxed{8 \tan^{-1}(x) + 3 \ln\left(1 + x^2\right) + C}$$

$$\int \frac{8+6x}{1+x^2}\,dx = \int \frac{8}{1+x^2}\,dx + \int \frac{6x}{1+x^2}\,dx, \quad u = 1+x^2$$

$$\frac{du}{dx} = 2x$$

$$du = 2x\,dx$$

$$3du = 6x\,dx$$

$$= \int 8\frac{1}{1+x^2}\,dx + 3\int \frac{1}{u}\,du$$

$$= 8\tan^{-1}(x) + 3\ln(|u|) + C = \boxed{8\tan^{-1}(x) + 3\ln(|1+x^2|) + C}$$

or

$$\boxed{8\tan^{-1}(x) + 3\ln(1+x^2) + C}$$

since x^2 will always be positive, absolute value operator is not necessary

Q19

Tuesday, November 17, 2020 10:59 PM

Evaluate the definite integral.

$$\int_0^1 8\cos(\pi t/2)\,dt$$

$\frac{16}{\pi}$

$$\int_0^1 8\cos\left(\frac{\pi t}{2}\right)dt \quad , \quad u = \frac{\pi}{2}t$$

$$\frac{du}{dt} = \frac{\pi}{2}$$

$$du = \frac{\pi}{2}dt$$

$$\frac{16}{\pi}du = 8\,dt$$

$$= \frac{16}{\pi}\int_0^1 \cos(u)\,du = \frac{16}{\pi}\sin(u) + C = \frac{16}{\pi}\sin\left(\frac{\pi}{2}t\right) + C$$

$$= F(1) - F(0)$$

$$= \left[\frac{16}{\pi}\sin\left(\frac{\pi}{2}(1)\right)\right] - \left[\frac{16}{\pi}\sin\left(\frac{\pi}{2}(0)\right)\right]$$

$$= \frac{16}{\pi} - 0 = \boxed{\frac{16}{\pi}}$$

Q20

Tuesday, November 17, 2020 11:10 PM

Evaluate the definite integral.

$$\int_0^{\pi/6} \frac{\sin(t)}{\cos^2(t)}\, dt$$

$$\boxed{\sec\left(\frac{\pi}{6}\right) - \sec(0)}$$

$$\int_0^{\pi/6} \frac{\sin(t)}{\cos^2(t)}\, dt \quad , \; u = \cos(t)$$

$$\frac{du}{dt} = -\sin(t)$$

$$du = -\sin(t)\, dt$$

$$= \int_0^{\pi/6} \frac{1}{\cos^2(t)} \sin(t)\, dt = -\int_0^{\pi/6} u^{-2}\, du$$

$$= -\frac{u^{-2+1}}{-2+1} + C = \frac{1}{u} + C = \frac{1}{\cos(t)} + C = \sec(t) + C$$

$$= F\left(\frac{\pi}{6}\right) - F(0)$$

$$= \boxed{\sec\left(\frac{\pi}{6}\right) - \sec(0)}$$

Q21

Tuesday, November 17, 2020 11:25 PM

Evaluate the definite integral.

$$\int_1^2 \frac{e^{1/x^5}}{x^6}\,dx$$

$$\frac{-e^{\left(\frac{1}{32}\right)}+e}{5}$$

✓

$$\int_1^2 \frac{e^{1/x^5}}{x^6}\,dx \quad , \quad u = \frac{1}{x^5} \qquad \frac{du}{dx} = -5x^{-6}$$

$$du = -\frac{5}{x^6}\,dx$$

$$= \int_1^2 -\frac{1}{5} \cdot -\frac{5}{x^6}\,dx\; e^{1/x^5} = -\frac{1}{5}\int_1^2 e^u\,du$$

$$= -\frac{1}{5}e^u + C = -\frac{1}{5}e^{1/x^5} + C$$

$$= F(2) - F(1)$$

$$= \left[-\frac{e^{1/2^5}}{5}\right] - \left[-\frac{e^{1/1^5}}{5}\right] = \boxed{\frac{-e^{1/32}+e}{5}}$$

Q22

Wednesday, November 18, 2020 12:11 PM

Evaluate the definite integral.

$$\int_8^9 x\sqrt{x-8}\,dx$$

$\frac{86}{15}$ ✓

$$\frac{du}{dx} = 1 \qquad u = x-8$$
$$du = dx \qquad u+8 = x$$

$$\int_8^9 x\sqrt{x-8}\,dx, \quad u = x-8$$

$$= \int_8^9 x\sqrt{x-8}\,dx = \int_8^9 (u+8)u^{1/2}\,du = \int_8^9 u^{3/2} + 8u^{1/2}\,du = \int_8^9 u^{3/2}\,du + \int_8^9 8u^{1/2}\,du$$

$$= \left[\frac{2u^{5/2}}{5} + C\right] + \left[\frac{16u^{3/2}}{3} + C\right] = \left[\frac{2(x-8)^{5/2}}{5} + C\right] + \left[\frac{16(x-8)^{3/2}}{3} + C\right]$$

$$= F(9) - F(8)$$

$$= \left[\frac{2(9-8)^{5/2}}{5} + \frac{16(9-8)^{3/2}}{3}\right] - \left[\frac{2(8-8)^{5/2}}{5} + \frac{16(8-8)^{3/2}}{3}\right]$$

$$= \left[\frac{2}{5} + \frac{16}{3}\right] - 0 = \boxed{\frac{86}{15}}$$

Q23

Wednesday, November 18, 2020 12:46 PM

Evaluate the definite integral.

$$\int_{e^{64}}^{e^{81}} \frac{dx}{x\sqrt{\ln(x)}}$$

2

$$\int_{e^{64}}^{e^{81}} \frac{dx}{x\sqrt{\ln(x)}} \quad , u = \ln(x)$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$du = \frac{1}{x}\,dx = \frac{dx}{x}$$

$$= \int_{e^{64}}^{e^{81}} \frac{1}{\sqrt{\ln(x)}}\,\frac{dx}{x} = \int_{e^{64}}^{e^{81}} u^{-1/2}\,du$$

$$= F(e^{81}) - F(e^{64})$$

$$= 2u^{1/2} + C = 2\sqrt{\ln(x)} + C$$

$$= 2\sqrt{\ln(e^{81})} - 2\sqrt{\ln(e^{64})} = 2\sqrt{81} - 2\sqrt{64} = 18 - 16 = \boxed{2}$$