Q1

Tuesday, October 20, 2020 7:21 PM

Given that

$$\lim_{x\to a} f(x) = 0 \qquad \lim_{x\to a} g(x) = 0 \qquad \lim_{x\to a} h(x) = 1$$

$$\lim_{x\to a} p(x) = \infty \qquad \lim_{x\to a} q(x) = \infty,$$

evaluate the limits below where possible. (If a limit is indeterminate, enter INDETERMINATE.)

indeterminate cases are $0\cdot\infty$, $\frac{0}{0}$, $\frac{\infty}{\infty}$, 0^0, ∞^0, 1^∞, and $\infty - \infty$

a) $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{0}{0} = \boxed{\text{INDETERMINATE}}$

b) $\lim_{x\to a} \frac{f(x)}{p(x)} = \boxed{\frac{0}{\infty}}$

c) $\lim_{x\to a} \frac{h(x)}{p(x)} = \frac{1}{\infty} = \boxed{0}$

Goes infinitely bigger $\therefore \frac{h(x)}{p(x)} \to 0$

d) $\lim_{x\to a} \frac{p(x)}{q(x)} = \frac{\infty}{\infty} = \boxed{\text{INDETERMINATE}}$

(a) $\lim_{x\to a} \frac{f(x)}{g(x)}$

INDETERMINATE

(b) $\lim_{x\to a} \frac{f(x)}{p(x)}$

$\frac{0}{\infty}$

(c) $\lim_{x\to a} \frac{h(x)}{p(x)}$

0

(d) $\lim_{x\to a} \frac{p(x)}{q(x)}$

INDETERMINATE

Q2

Tuesday, October 20, 2020 10:27 PM

Given that

$$\lim_{x\to a} f(x) = 0 \quad \lim_{x\to a} g(x) = 0 \quad \lim_{x\to a} h(x) = 1$$

$$\lim_{x\to a} p(x) = \infty \quad \lim_{x\to a} q(x) = \infty,$$

evaluate the limits below where possible. (If a limit is indeterminate, enter INDETERMINATE.)

(a) $\lim_{x\to a} [f(x)p(x)]$

INDETERMINATE

(b) $\lim_{x\to a} [h(x)p(x)]$

∞

(c) $\lim_{x\to a} [p(x)q(x)]$

∞

indeterminate cases are $0 \cdot \infty$, $\frac{0}{0}$, $\frac{\infty}{\infty}$, 0^0, ∞^0, 1^∞, and $\infty - \infty$

a) $\lim_{x\to a} [f(x)p(x)] = (0)(\infty) = \boxed{\text{INDETERMINATE}}$

b) $\lim_{x\to a} [h(x)p(x)] = (1)(\infty) = \boxed{\infty}$

c) $\lim_{x\to a} [g(x)p(x)] = (\infty)(\infty) = \boxed{\infty}$

Q3

Wednesday, October 21, 2020 4:17 PM

Given that

$$\lim_{x\to a} f(x) = 0 \qquad \lim_{x\to a} g(x) = 0 \qquad \lim_{x\to a} h(x) = 1$$

$$\lim_{x\to a} p(x) = \infty \qquad \lim_{x\to a} q(x) = \infty,$$

evaluate the limits below where possible. (If a limit is indeterminate, enter INDETERMINATE.)

indeterminate cases are $0 \cdot \infty$, $\frac{0}{0}$, $\frac{\infty}{\infty}$, 0^0, ∞^0, 1^∞, and $\infty - \infty$

a) $\lim_{x\to a} [f(x) - p(x)] = 0 - \infty = \boxed{-\infty}$

b) $\lim_{x\to a} [p(x) - q(x)] = \infty - \infty = \boxed{\text{INDETERMINATE}}$

c) $\lim_{x\to a} [p(x) + q(x)] = \infty + \infty = \boxed{\infty}$

(a) $\lim_{x\to a} [f(x) - p(x)]$

$-\infty$ ✓

(b) $\lim_{x\to a} [p(x) - q(x)]$

INDETERMINATE ✓

(c) $\lim_{x\to a} [p(x) + q(x)]$

∞ ✓

Q4

Wednesday, October 21, 2020 4:22 PM

Given that

$$\lim_{x\to a} f(x) = 0 \qquad \lim_{x\to a} g(x) = 0 \qquad \lim_{x\to a} h(x) = 1$$

$$\lim_{x\to a} p(x) = \infty \qquad \lim_{x\to a} q(x) = \infty,$$

evaluate the limits below where possible. (If a limit is indeterminate, enter INDETERMINATE.)

indeterminate cases are $0 \cdot \infty$, $\frac{0}{0}$, $\frac{\infty}{\infty}$, 0^0, ∞^0, 1^∞, and $\infty - \infty$.

a) $\lim_{x\to a} [f(x)]^{g(x)} = 0^0 = \boxed{\text{INDETERMINATE}}$

b) $\lim_{x\to a} [f(x)]^{p(x)} = 0^\infty = \boxed{0}$

c) $\lim_{x\to a} [h(x)]^{p(x)} = 1^\infty = \boxed{\text{INDETERMINATE}}$

d) $\lim_{x\to a} [p(x)]^{f(x)} = \infty^0 = \boxed{\text{INDETERMINATE}}$

e) $\lim_{x\to a} [p(x)]^{q(x)} = \infty^\infty = \boxed{\infty}$

f) $\lim_{x\to a} \sqrt[q(x)]{p(x)} = \sqrt[\infty]{\infty} = \infty^{\frac{1}{\infty}} = \infty^0 = \boxed{\text{INDETERMINATE}}$

approaches zero as ∞ gets bigger

(a) $\lim_{x\to a} [f(x)]^{g(x)}$

INDETERMINATE

(b) $\lim_{x\to a} [f(x)]^{p(x)}$

0

(c) $\lim_{x\to a} [h(x)]^{p(x)}$

INDETERMINATE

(d) $\lim_{x\to a} [p(x)]^{f(x)}$

INDETERMINATE

(e) $\lim_{x\to a} [p(x)]^{q(x)}$

∞

(f) $\lim_{x\to a} \sqrt[q(x)]{p(x)}$

INDETERMINATE

Q5

Wednesday, October 21, 2020 4:37 PM

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \to 6} \frac{x - 6}{x^2 - 36}$$

$$\frac{1}{12}$$

$$\lim_{x \to 6} \frac{x-6}{x^2-36} = \frac{(6)-6}{(6)^2-36} = \frac{0}{0}$$

$$\lim_{x \to 6} \frac{\frac{d}{dx}(x-6)}{\frac{d}{dx}(x^2-36)} = \frac{1-0}{2x-0} = \frac{1}{2x} = \frac{1}{2(6)} = \boxed{\frac{1}{12}}$$

L'Hospital's Rule Suppose f and g are differentiable and $g'(x) \neq 0$ on an open interval I that contains a (except possibly at a). Suppose that

$$\lim_{x \to a} f(x) = 0 \quad \text{and} \quad \lim_{x \to a} g(x) = 0$$

or that

$$\lim_{x \to a} f(x) = \pm\infty \quad \text{and} \quad \lim_{x \to a} g(x) = \pm\infty$$

(In other words, we have an indeterminate form of type $\frac{0}{0}$ or ∞/∞.) Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if the limit on the right side exists (or is ∞ or $-\infty$).

Q6

Wednesday, October 21, 2020 4:57 PM

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it.

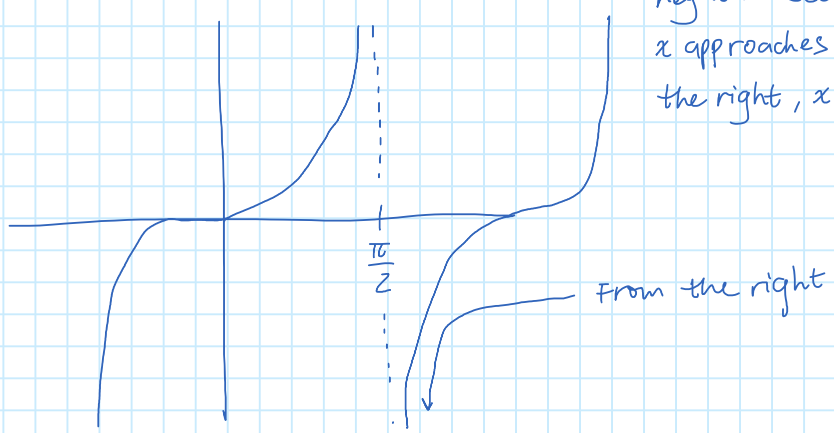
$$\lim_{x \to (\pi/2)^+} \frac{\cos(x)}{1 - \sin(x)}$$

$-\infty$

$$\lim_{x \to (\pi/2)^+} \frac{\cos(x)}{1 - \sin(x)} = \frac{\cos(\pi/2)}{1 - \sin(\pi/2)} = \frac{0}{1-1} = \frac{0}{0}$$

$$\lim_{x \to (\pi/2)^+} \frac{\frac{d}{dx}\left[\cos(x)\right]}{\frac{d}{dx}\left[1 - \sin(x)\right]} = \frac{-\sin(x)}{-\cos(x)} = \tan(x) = \tan\left(\frac{\pi}{2}\right) = \boxed{-\infty}$$

negative because x approaches $\frac{\pi}{2}$ from the right, $x \to (\pi/2)^+$



Q7

Wednesday, October 21, 2020 5:08 PM

Find the limit.

$$\lim_{t \to 0} \frac{e^{6t} - 1}{\sin(t)}$$

6

$$\lim_{t \to 0} \frac{e^{6t} - 1}{\sin(t)} = \frac{e^{6(0)} - 1}{\sin(0)} = \frac{1 - 1}{0} = \frac{0}{0}$$

$$\lim_{t \to 0} \frac{\frac{d}{dx}\left(e^{6t} - 1\right)}{\frac{d}{dx}\left[\sin(t)\right]} = \frac{e^{6t} \frac{d}{dx}(6t)}{\cos(t)} = \frac{e^{6t}(6)}{\cos(t)} = \frac{6e^{6(0)}}{\cos(0)} = \frac{6(1)}{1} = \boxed{6}$$

Q8

Wednesday, October 21, 2020 5:23 PM

$$\lim_{x \to \infty} \frac{\ln(3x)}{\sqrt{3x}} = \frac{\ln(3(\infty))}{\sqrt{3(\infty)}} = \frac{\infty}{\sqrt{\infty}} = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{\frac{d}{dx}\left[\ln(3x)\right]}{\frac{d}{dx}\left(\sqrt{3x}\right)} = \frac{\frac{1}{3x}\frac{d}{dx}(3x)}{\frac{d}{dx}\left(3x^{1/2}\right)} = \frac{\frac{1}{3x}(3)}{\left(\frac{1}{2}\right)3x^{-1/2}} = \frac{\frac{1}{x}}{\frac{3}{2\sqrt{x}}} = \frac{2\sqrt{x}}{3x} = \frac{2x^{1/2}}{3x} = \frac{2}{3x^{1/2}} = \frac{2}{3\sqrt{x}} = \boxed{0}$$

approaches zero as ∞ gets bigger

Q9

Wednesday, October 21, 2020 5:47 PM

Find the limit. Use l'Hospital's Rule if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \to 0} \frac{e^{7x} - 1 - 7x}{x^2}$$

$$\frac{49}{2}$$

$$\lim_{x \to 0} \frac{e^{7x} - 1 - 7x}{x^2} = \frac{e^{7(0)} - 1 - 7(0)}{0^2} = \frac{1 - 1 - 0}{0} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{\frac{d}{dx}\left(e^{7x} - 1 - 7x\right)}{\frac{d}{dx}\left(x^2\right)} = \frac{e^{7x}\frac{d}{dx}(7x) - 0 - 7}{2x} = \frac{e^{7x}(7) - 7}{2x} = \frac{7e^{7x} - 7}{2x} = \frac{7e^{7(0)} - 7}{2(0)} = \frac{7(1) - 7}{0} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{\frac{d}{dx}\left(7e^{7x} - 7\right)}{\frac{d}{dx}(2x)} = \frac{7e^{7x}\frac{d}{dx}(7x) - 0}{2} = \frac{7e^{7x}(7)}{2} = \frac{49e^{7x}}{2} = \frac{49e^{7(0)}}{2} = \frac{49(1)}{2} = \boxed{\frac{49}{2}}$$

Q10

Wednesday, October 21, 2020 6:16 PM

Find the limit. Use l'Hospital's Rule if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \to 0} \frac{\sin^{-1}(x)}{6x}$$

$$\frac{1}{6}$$

$$\lim_{x \to 0} \frac{\sin^{-1}(x)}{6x} = \frac{\sin^{-1}(0)}{6(0)} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{\frac{d}{dx}\left[\sin^{-1}(x)\right]}{\frac{d}{dx}(6x)} = \frac{\frac{1}{\sqrt{1-x^2}}}{6} = \frac{1}{6\sqrt{1-x^2}} = \frac{1}{6\sqrt{1-0^2}} = \frac{1}{6\sqrt{1}} = \boxed{\frac{1}{6}}$$

Q11

Wednesday, October 21, 2020 6:24 PM

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \to 0^+} \frac{\arctan(6x)}{\ln(x)}$$

0

$$\lim_{x \to 0^+} \frac{\arctan(6x)}{\ln(x)} = \frac{\tan^{-1}(6x)}{\ln(x)} = \frac{\tan^{-1}(6(0))}{\ln(0)} = \frac{0}{\infty} = \boxed{0}$$

Q12

Wednesday, October 21, 2020 6:30 PM

Find the limit. Use l'Hospital's Rule if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \to \infty} x^7 e^{-x^6}$$

0

$$\lim_{x \to \infty} x^7 e^{-x^6} = \frac{x^7}{e^{x^6}} = \frac{\infty^7}{e^{\infty^6}} = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{\frac{d}{dx}(x^7)}{\frac{d}{dx}(e^{x^6})} = \frac{7x^6}{e^{x^6}\frac{d}{dx}(x^6)} = \frac{7x^6}{e^{x^6}(6x^5)} = \frac{7x^6}{6x^5 e^{x^6}} = \frac{7\infty^6}{6\infty^5 e^{\infty^6}} = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{\frac{d}{dx}(7x^6)}{\frac{d}{dx}(6x^5 e^{x^6})} = \longrightarrow \lim_{x \to \infty} \frac{f'(x)}{g'(x)} \longrightarrow \boxed{0}$$

will keep getting $\frac{\infty}{\infty}$ until numerator and denominator becomes constant and then 0

Q13

Wednesday, October 21, 2020 7:13 PM

Find the limit. Use l'Hospital's Rule if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x\to 1}\left(\frac{7x}{x-1}-\frac{7}{\ln(x)}\right)$$

$\frac{7}{2}$

$$\lim_{x\to 1}\left(\frac{7x}{x-1}-\frac{7}{\ln(x)}\right) \longrightarrow \infty - \infty$$

Not in INDETERMINATE form

$$\frac{(\ln x)}{(\ln x)}\frac{7x}{x-1}-\frac{7}{\ln(x)}\frac{(x-1)}{(x-1)} = \frac{7x\ln(x)-(7x-7)}{x\ln(x)-\ln(x)}$$

$$\lim_{x\to 1}\frac{7x\ln(x)-(7x-7)}{x\ln(x)-\ln(x)} = \frac{7(1)\ln(1)-[7(1)-7]}{(1)\ln(1)-\ln(1)} = \frac{0-0}{0-0} = \frac{0}{0}$$

$$\lim_{x\to 1}\frac{\frac{d}{dx}[7x\ln(x)-7x+7]}{\frac{d}{dx}(x\ln(x)-\ln(x))} = \frac{7\left[x\frac{d}{dx}(\ln x)+\ln x\frac{d}{dx}(x)\right]-7+0}{x\frac{d}{dx}(\ln x)+\ln x\frac{d}{dx}(x)-\frac{d}{dx}(\ln x)} = \frac{7\left[x\frac{1}{x}+\ln(x)(1)\right]-7}{x\frac{1}{x}+\ln(x)(1)-\frac{1}{x}}$$

$$= \frac{7\ln(x)}{1+\ln(x)-\frac{1}{x}} = \frac{7\ln(1)}{1+\ln(1)-\frac{1}{1}} = \frac{0}{1+0-1} = \frac{0}{0}$$

$$\lim_{x\to 1}\frac{\frac{d}{dx}[7\ln(x)]}{\frac{d}{dx}\left[1+\ln(x)-\frac{1}{x}\right]} = \frac{\frac{7}{x}}{0+\frac{1}{x}-\frac{d}{dx}(x^{-1})} = \frac{\frac{7}{x}}{\frac{1}{x}-(-x^{-2})} = \frac{\frac{7}{x}}{\frac{1}{x}+x^{-2}} = \frac{\frac{7}{x}}{\frac{1}{x}+\frac{1}{x^2}} = \frac{\frac{7}{x}}{\frac{x+1}{x^2}} = \frac{\frac{7}{1}}{\frac{1+1}{1^2}} = \boxed{\frac{7}{2}}$$

Q14

Wednesday, October 21, 2020 8:38 PM

Find the limit. Use l'Hospital's Rule if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x\to\infty} x^{6/x}$$

1

$\lim_{x\to\infty} x^{6/x}$, Let $y = x^{6/x}$

$$y = x^{6/x}$$
$$\ln(y) = \ln\left(x^{6/x}\right) = \frac{6}{x}\ln(x) = \frac{6\ln(x)}{x}$$

$$\lim_{x\to\infty} \frac{6\ln(x)}{x} = \frac{6\ln(\infty)}{\infty} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{\frac{d}{dx}\left[6\ln(x)\right]}{\frac{d}{dx}(x)} = \frac{6\left(\frac{1}{x}\right)}{1} = \frac{\frac{6}{x}}{1} = \frac{\frac{6}{\infty}}{1} = \frac{0}{1} = 0$$

$$\lim_{x\to\infty} y = \lim_{x\to\infty} e^{\ln y} = e^{\lim_{x\to\infty} \ln y} = e^0 = \boxed{1}$$