



## Famous Trig Identities

### Definition 1: Common Limits

$$\begin{aligned}\sin \theta &= \frac{\text{opp}}{\text{hyp}} & \cos \theta &= \frac{\text{adj}}{\text{hyp}} & \tan \theta &= \frac{\text{opp}}{\text{adj}} \\ \cot \theta &= \frac{\text{adj}}{\text{opp}} & \tan \theta &= \frac{\sin \theta}{\cos \theta} & \cot \theta &= \frac{\cos \theta}{\sin \theta} \\ \csc \theta &= \frac{1}{\sin \theta} & \sec \theta &= \frac{1}{\cos \theta}\end{aligned}$$

### Co-Function 2: Identities

$$\begin{aligned}\sin \theta &= \cos(90^\circ - \theta) & \tan \theta &= \cot(90^\circ - \theta) \\ \sec \theta &= \csc(90^\circ - \theta) & \sin \theta &= \cos(\theta - 90^\circ) \\ -\cos \theta &= \sin(\theta - 90^\circ) & \cos \theta &= \sin(90^\circ - \theta)\end{aligned}$$

### Even/Odd 3: Identities

$$\begin{aligned}\sin(-\theta) &= -\sin \theta & \cos(-\theta) &= \cos \theta \\ \csc(-\theta) &= -\csc \theta & \sec(-\theta) &= \sec \theta \\ \tan(-\theta) &= -\tan \theta & \cot(-\theta) &= -\cot \theta\end{aligned}$$

### Pythagoras 4: Identities

$$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 & \tan^2 \theta + 1 &= \sec^2 \theta \\ \sin^2 \theta &= 1 - \cos^2 \theta & \cos^2 \theta &= 1 - \sin^2 \theta \\ \tan^2 \theta &= \sec^2 \theta - 1 & \cot^2 \theta + 1 &= \csc^2 \theta \\ \cot^2 \theta &= \csc^2 \theta - 1 & \sin \theta &= \pm \sqrt{1 - \cos^2 \theta} \\ \tan \theta &= \pm \sqrt{\sec^2 \theta - 1} & \sec \theta &= \pm \sqrt{\tan^2 \theta + 1}\end{aligned}$$

### Guidelines 5: To PROVE Identities

1. Work on sides independently.
2. Tweak a known Identity.
3. Look at Graphs.
4. Start with something amazingly creative and brilliant!

### Sum Angle 6: Identities

**the M.O.T.A.**  $\mapsto$

$$\begin{aligned}\cos(a - b) &= \cos a \cos b + \sin a \sin b \\ \cos(a + b) &= \cos a \cos b - \sin a \sin b \\ \sin(a + b) &= \sin a \cos b + \cos a \sin b \\ \sin(a - b) &= \sin a \cos b - \cos a \sin b \\ \tan(a + b) &= \frac{\tan a + \tan b}{1 - \tan a \tan b} & \tan(a - b) &= \frac{\tan a - \tan b}{1 + \tan a \tan b}\end{aligned}$$

### Double Angle 7: Identities

$$\begin{aligned}\cos(2a) &= \cos^2 a - \sin^2 a & \cos(2a) &= 2 \cos^2 a - 1 \\ \cos(2a) &= 1 - 2 \sin^2 a & \sin(2a) &= 2 \sin a \cos a \\ \cos^2 \theta &= \frac{1 + \cos 2\theta}{2} & \sin^2 \theta &= \frac{1 - \cos 2\theta}{2} \\ \tan(2a) &= \frac{2 \tan a}{1 - \tan^2 a} & \tan^2 \theta &= \frac{1 - \cos 2\theta}{1 + \cos 2\theta}\end{aligned}$$

### Product-to-Sums 8: Identities

$$\begin{aligned}\sin a \sin b &= \frac{1}{2} [\cos(a - b) - \cos(a + b)] \\ \cos a \cos b &= \frac{1}{2} [\cos(a - b) + \cos(a + b)] \\ \sin a \cos b &= \frac{1}{2} [\sin(a + b) + \sin(a - b)]\end{aligned}$$

### Sum-to-Products 9: Identities

$$\begin{aligned}\sin a + \sin b &= 2 \sin \left( \frac{a+b}{2} \right) \cos \left( \frac{a-b}{2} \right) \\ \sin a - \sin b &= 2 \sin \left( \frac{a-b}{2} \right) \cos \left( \frac{a+b}{2} \right) \\ \cos a + \cos b &= 2 \cos \left( \frac{a+b}{2} \right) \cos \left( \frac{a-b}{2} \right) \\ \cos a - \cos b &= -2 \sin \left( \frac{a+b}{2} \right) \sin \left( \frac{a-b}{2} \right)\end{aligned}$$

### EULER 10: The Master of Us All

$$\begin{aligned}e^{i\theta} &= \cos \theta + i \sin \theta \\ \cos(\theta) &= \frac{e^{i\theta} + e^{-i\theta}}{2} & \sin(\theta) &= \frac{e^{i\theta} - e^{-i\theta}}{2i}\end{aligned}$$