Q1

Tuesday, September 15, 2020 11:05 AM

Differentiate.

$f(x) = x^2 \sin(x)$

$f'(x) = \boxed{x^2\cos(x) + 2x\sin(x)}$

The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$f(x) = x^2 \sin(x)$$

$$f'(x) = x^2 \frac{d}{dx}[\sin(x)] + \sin(x)\frac{d}{dx}(x^2)$$

$$= x^2(\cos x) + \sin x\left[(2)x^{(2-1)}\right]$$

$$\boxed{f'(x) = x^2 \cos x + 2x \sin x}$$

Q2

Tuesday, September 15, 2020 12:03 PM

Differentiate.

$y = \sec(\theta)\tan(\theta)$

$y' = 2\sec^3(\theta) - \sec(\theta)$

The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$y = \sec(\theta)\tan(\theta)$$

$$y' = \sec\theta \frac{d}{dx}(\tan\theta) + \tan\theta \frac{d}{dx}(\sec\theta)$$

$$= \sec\theta(\sec^2\theta) + \tan\theta(\sec\theta\tan\theta)$$

$$= \sec^3\theta + \sec\theta\tan^2\theta$$

$$= \sec^3\theta + \sec\theta(\sec^2\theta - 1)$$

$$= \sec^3\theta + \sec^3\theta - \sec\theta$$

$$\boxed{y' = 2\sec^3\theta - \sec\theta}$$

Q3

Tuesday, September 15, 2020 12:27 PM

Differentiate with respect to t.

$$y = d\cos(t) + t^2\sin(t)$$

$$y' = \boxed{\sin(t)(2t-d) + t^2\cos(t)}$$

The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$y = d\cos(t) + t^2\sin(t)$$

Derived directly

using the product rule

$$y' = d(-\sin t) + t^2\frac{d}{dx}(\sin t) + \sin t\frac{d}{dx}[(2)t^{(2-1)}]$$

$$= -d\sin t + t^2\cos t + 2t\sin t$$

$$\boxed{y' = \sin t(2t-d) + t^2\cos t}$$

Q4

Tuesday, September 15, 2020 1:33 PM

Differentiate.

$$y = \frac{8x}{3 - \tan(x)}$$

$$y' = \boxed{\frac{8\left(3 - \tan(x) + x\sec^2(x)\right)}{\left(3 - \tan(x)\right)^2}}$$

The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$y = \frac{8x}{3 - \tan(x)}$$

$$y' = \frac{(3 - \tan x)\frac{d}{dx}(8x) - 8x\frac{d}{dx}(3 - \tan x)}{(3 - \tan x)^2}$$

$$= \frac{(3 - \tan x)\left[(1)8x^{(1-1)}\right] - 8x(0 - \sec^2 x)}{(3 - \tan x)^2}$$

$$= \frac{(3 - \tan x)8 - 8x(-\sec^2 x)}{(3 - \tan x)^2}$$

$$= \frac{24 - 8\tan x + 8x\sec^2 x}{(3 - \tan x)^2}$$

$$\boxed{y' = \frac{8(3 - \tan x + x\sec^2 x)}{(3 - \tan x)^2}}$$

Q5

Tuesday, September 15, 2020 6:46 PM

Differentiate.

$$f(\theta) = \theta \cos(\theta) \sin(\theta)$$

$$f'(\theta) = \boxed{\theta\cos(2\theta) + \frac{\sin(2\theta)}{2}}$$

The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}\cos(x)\sin(x) = \cos(2x)$$

$$\sin a \cos b = \frac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$$

$$f(x) = \theta \; , \; g(x) = \cos(\theta)\sin(\theta)$$

$$f(\theta) = \theta\cos(\theta)\sin(\theta)$$

$$f'(\theta) = \theta\frac{d}{dx}(\cos\theta\sin\theta) + \cos\theta\sin\theta\frac{d}{dx}(\theta)$$

$$= \theta(\cos 2\theta) + \cos\theta\sin\theta(1)$$

$$= \theta\cos 2\theta + \cos\theta\sin\theta$$

$$= \theta\cos 2\theta + \frac{1}{2}\left[\sin(\theta+\theta) + \sin(\theta-\theta)\right]$$

$$\boxed{f'(\theta) = \theta\cos 2\theta + \frac{\sin(2\theta)}{2}}$$

Q6

Tuesday, September 15, 2020 7:18 PM

Prove that $\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$.

$$\frac{d}{dx}(\csc(x)) = \frac{d}{dx}\left(\frac{1}{\boxed{\sin(x)}}\right)$$

$$= \frac{\left(\boxed{\sin(x)}\right)(0) - 1\left(\boxed{\cos(x)}\right)}{\sin^2(x)}$$

$$= \frac{\boxed{-\cos(x)}}{\sin^2(x)}$$

$$= -\frac{1}{\sin(x)} \cdot \frac{\boxed{\cos(x)}}{\sin(x)}$$

$$= -\csc(x)\cot(x)$$

Q7

Tuesday, September 15, 2020 8:51 PM

Find an equation of the tangent line to the curve at the given point.

$y = 9e^x \cos(x), \quad (0, 9)$

$y = 9x + 9$

The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

Get slope at (0,9)

$y = 9e^x \cos(x), \quad (0, 9)$

$$y' = 9e^x \frac{d}{dx}[\cos(x)] + \cos(x)\frac{d}{dx}(9e^x)$$

$$= 9e^x[-\sin(x)] + \cos(x)(9e^x)$$

$$= -9e^x \sin(x) + 9e^x \cos(x)$$

$$= -9e^x[\sin(x) - \cos(x)]$$

$$y'(0) = -9e^{(0)}[\sin(0) - \cos(0)]$$

$$= -9(1)(0 - 1)$$

$$= -9(-1)$$

$$y'(0) = 9 = m$$

Get equation of tangent y at (0,9)

$y - y_1 = m(x - x_1)$ point-slope form

$$y - (9) = 9(x - 0)$$

$$y - 9 = 9x$$

$$\boxed{y = 9x + 9}$$

Q8

Tuesday, September 15, 2020 9:12 PM

If $f(x) = 9e^x \cos(x)$, find $f'(x)$ and $f''(x)$.

$f'(x) =$

$f''(x) =$

The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$f(x) = 9e^x \cos(x)$$

$$f'(x) = 9e^x \frac{d}{dx}[\cos(x)] + \cos(x)\frac{d}{dx}(9e^x)$$

$$= 9e^x[-\sin(x)] + \cos(x)(9e^x)$$

$$\boxed{f'(x) = -9e^x \sin(x) + 9e^x \cos(x)}$$

$$f''(x) = \frac{d}{dx}[-9e^x \sin(x)] + \frac{d}{dx}[9e^x \cos(x)]$$

$$= -9e^x \frac{d}{dx}[\sin(x)] + \sin(x)\frac{d}{dx}(-9e^x) + 9e^x\frac{d}{dx}[\cos(x)] + \cos(x)\frac{d}{dx}(9e^x)$$

$$= -9e^x \cos(x) + \sin(x)(-9e^x) + 9e^x[-\sin(x)] + \cos(x)(9e^x)$$

$$= -9e^x \cos(x) - 9e^x \sin(x) - 9e^x \sin(x) + 9e^x \cos(x)$$

$$= -9e^x[\cos(x) + \sin(x) + \sin(x) - \cos(x)]$$

$$\boxed{f''(x) = -9e^x[2\sin(x)]}$$

Q9

Tuesday, September 15, 2020 9:34 PM

(a) Use the Quotient Rule to differentiate the function

$$f(x) = \frac{\tan(x) - 1}{\sec(x)}.$$

$f'(x) =$ $\cos(x) + \sin(x)$ ✔

(b) Simplify the expression for $f(x)$ by writing it in terms of $\sin(x)$ and $\cos(x)$, and then find $f'(x)$.

$f'(x) =$ $\cos(x) + \sin(x)$ ✔

(c) Are your answers to parts (a) and (b) equivalent?

- (●) Yes
- ( ) No

✔

The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

$$f(x) = \frac{\tan(x) - 1}{\sec(x)}$$

$$f'(x) = \frac{\sec(x)\frac{d}{dx}[\tan(x) - 1] - [\tan(x) - 1]\frac{d}{dx}[\sec(x)]}{[\sec(x)]^2}$$

$$= \frac{\sec(x)\sec^2(x) - [\tan(x) - 1][\sec(x)\tan(x)]}{[\sec(x)]^2} \rightarrow \text{Factor out } \sec(x)$$

$$= \frac{\sec(x)[\sec^2(x) - (\tan(x) - 1)(\tan(x))]}{[\sec(x)]^2}$$

$$= \frac{\sec(x)[\sec^2(x) - (\tan^2(x) - \tan(x))]}{[\sec(x)]^2}$$

$$= \frac{\cancel{\sec(x)}[\sec^2(x) - \tan^2(x) + \tan(x)]}{[\sec(x)]^{\cancel{2}}}$$

$$= \frac{\sec^2(x) - \tan^2(x) + \tan(x)}{\sec(x)}$$

$$\sec^2(x) = 1 + \tan^2(x)$$

$$= \frac{1 + \tan^2(x) - \tan^2(x) + \tan(x)}{\sec(x)}$$

$$= \frac{1 + \tan(x)}{\sec(x)}$$

$$= \frac{1 + \frac{\sin(x)}{\cos(x)}}{\frac{1}{\cos(x)}}$$

$$= \left(1 + \frac{\sin(x)}{\cos(x)}\right)\frac{\cos(x)}{1}$$

$$= \frac{\cos(x)}{1} + \frac{\sin(x)\cancel{\cos(x)}}{\cancel{\cos(x)}}$$

$$\boxed{f'(x) = \cos(x) + \sin(x)}$$

Q10

Tuesday, September 15, 2020 11:09 PM

For what values of x does the graph of f have a horizontal tangent? (Use n as your integer variable. Enter your answers as a comma-separated list.)

$f(x) = x - 2\sin(x)$

$x = \frac{\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n$

$f'(x) = 0$

$f(x) = x - 2\sin(x)$

$f'(x) = \frac{d}{dx}(x) - \frac{d}{dx}(2\sin x)$

$f'(x) = 1 - 2(\cos x)$

$1 - 2\cos x = 0$

$-2\cos x = -1$

$\cos x = \frac{1}{2}$

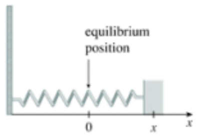
$$\boxed{x = \frac{\pi}{3} + 2\pi n \quad \text{or} \quad x = \frac{5\pi}{3} + 2\pi n}$$

see unit circle and solving trig equations

Q11

Wednesday, September 16, 2020 9:14 AM

A mass on a spring vibrates horizontally on a smooth level surface (see the figure). Its equation of motion is $x(t) = 8\sin(t)$, where t is in seconds and x is in centimeters.



(a) Find the velocity and acceleration at time t.

$v(t) = 8\cos(t)$

$a(t) = -8\sin(t)$

(b) Find the position, velocity, and acceleration of the mass at time $t = 2\pi/3$.

$x\left(\frac{2\pi}{3}\right) = 4\sqrt{3}$

$v\left(\frac{2\pi}{3}\right) = -4$

$a\left(\frac{2\pi}{3}\right) = -4\sqrt{3}$

In what direction is it moving at that time?

Since $v\left(\frac{2\pi}{3}\right)$ < 0, the particle is moving to the left .

$$x(t) = 8\sin(t)$$

$$v(t) = x'(t) = \frac{d}{dx}(8\sin t)$$

$$= 8\frac{d}{dx}(\sin t)$$

$$\boxed{v(t) = 8\cos t}$$

$$a(t) = v'(t) = x''(t) = \frac{d}{dx}(8\cos t)$$

$$= 8\frac{d}{dx}(\cos t)$$

$$\boxed{a(t) = -8\sin t}$$

$$\boxed{v\left(\frac{2\pi}{3}\right) = -4 < 0}$$

therefore, particle is moving to the $\boxed{\text{left}}$

$$x\left(\frac{2\pi}{3}\right) = 8\sin t$$

$$= 8\sin\left(\frac{2\pi}{3}\right) \quad \text{see unit circle}$$

$$= 8\left(\frac{\sqrt{3}}{2}\right)$$

$$\boxed{x\left(\frac{2\pi}{3}\right) = 4\sqrt{3}}$$

$$v\left(\frac{2\pi}{3}\right) = 8\cos t$$

$$= 8\cos\left(\frac{2\pi}{3}\right)$$

$$= 8\left(-\frac{1}{2}\right)$$

$$\boxed{v\left(\frac{2\pi}{3}\right) = -4}$$

$$a\left(\frac{2\pi}{3}\right) = -8\sin t$$

$$= -8\sin\left(\frac{2\pi}{3}\right)$$

$$= -8\left(\frac{\sqrt{3}}{2}\right)$$

$$\boxed{a\left(\frac{2\pi}{3}\right) = -4\sqrt{3}}$$