Q1

Monday, October 5, 2020 7:46 AM

Find the linearization $L(x)$ of the function at a.

$$f(x) = \sin(x), \quad a = \frac{\pi}{3}$$

$$L(x) = \boxed{\frac{3\sqrt{3} - \pi}{6} + \frac{1}{2}x}$$

Tangent linear approximation

$$L(x) = f(a) + f'(a)(x - a)$$

$$f(x) = \sin(x), \quad a = \frac{\pi}{3}$$

$$\begin{aligned} f'(x) &= \sin(x) \\ &= \frac{d}{dx}\left[\sin(x)\right] \\ &= \cos(x) \end{aligned}$$

$$\begin{aligned} L\left(\tfrac{\pi}{3}\right) &= f\left(\frac{\pi}{3}\right) + f'\left(\frac{\pi}{3}\right)\left(x - \frac{\pi}{3}\right) \\ &= \frac{\sqrt{3}}{2} + \frac{1}{2}\left(x - \frac{\pi}{3}\right) \\ &= \frac{\sqrt{3}}{2} + \frac{1}{2}x - \frac{\pi}{6} \\ &= \boxed{\frac{3\sqrt{3} - \pi}{6} + \frac{1}{2}x} \end{aligned}$$

Q2

Monday, October 5, 2020 8:08 AM

Find the linearization $L(x)$ of the function at a.

$$f(x) = \sqrt{x}, \quad a = 9$$

$$L(x) = \boxed{\frac{3}{2} + \frac{1}{6}x}$$

Tangent linear approximation

$$L(x) = f(a) + f'(a)(x - a)$$

$$f(x) = \sqrt{x} \quad , \quad a = 9$$

$$f'(x) = \sqrt{x}$$

$$= \frac{d}{dx}\left(x^{1/2}\right)$$

$$= \frac{1}{2}x^{-1/2}$$

$$L(9) = f(9) + f'(9)(x-9)$$

$$= 3 + \frac{1}{2}(9)^{-1/2}(x-9)$$

$$\frac{1}{2}9^{-1/2}$$

$$\frac{1}{2}\left(\frac{1}{\sqrt{9}}\right) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

$$= 3 + \frac{1}{6}(x-9)$$

$$= 3 + \frac{1}{6}x - \frac{3}{2}$$

$$= \boxed{\frac{3}{2} + \frac{1}{6}x}$$

Q3

Monday, October 5, 2020 8:19 AM

Use a linear approximation (or differentials) to estimate the given number.

$(1.999)^4$

15.968

Tangent linear approximation

$L(x) = f(a) + f'(a)(x-a)$

$(1.999)^4$

$f(x) = x^4$, $a = 2$

$f'(x) = x^4$

$= \frac{d}{dx}(x^4)$

$= 4x^3$

$L(2) = f(2) + f'(2)(x-2)$

$= 16 + 32(x-2)$

$= 16 + 32x - 64$

$= 32x - 48$

$(1.999)^4 = f(1.999)$

$\approx 32(1.999) - 48$

$= 63.968 - 48$

$= \boxed{15.968}$

Q4

Monday, October 5, 2020 8:40 AM

Use a linear approximation (or differentials) to estimate the given number. (Round your answer to five decimal places.)

$\sqrt[3]{65}$

4.02083 ✓

Tangent linear approximation

$$L(x) = f(a) + f'(a)(x-a)$$

$$\sqrt[3]{65}$$

$f(x) = \sqrt[3]{x}$, $a = 64$

$$f'(x) = \sqrt[3]{x}$$
$$= \frac{d}{dx}\left(x^{1/3}\right)$$
$$= \frac{1}{3}x^{-2/3}$$

$$L(x) = f(64) + f'(64)(x-64)$$
$$= 4 + \frac{1}{3}(64)^{-2/3}(x-64)$$

$$\frac{1}{3}(64)^{-2/3}$$
$$\frac{1}{3}\left(\frac{1}{64^{2/3}}\right) = \frac{1}{3}\left(\frac{1}{(4^3)^{2/3}}\right) = \frac{1}{3}\left(\frac{1}{4^2}\right)$$
$$= \frac{1}{3}\left(\frac{1}{16}\right) = \frac{1}{48}$$

$$= 4 + \frac{1}{48}(x-64)$$
$$= 4 + \frac{1}{48}x - \frac{64}{48}$$
$$= \frac{x}{48} + 4 - \frac{4}{3}$$
$$= \frac{x}{48} + \frac{8}{3}$$

$$\sqrt[3]{65} = f(65)$$
$$= \frac{65}{48} + \frac{8}{3}$$
$$= \frac{193}{48}$$
$$\approx \boxed{4.02083}$$

Q5

Monday, October 5, 2020 11:34 AM

Find the linear approximation of the function $g(x) = \sqrt[3]{1 + x}$ at $a = 0$.

$g(x) \approx \frac{1}{3}x + 1$ ✓

Use it to approximate the numbers $\sqrt[3]{0.95}$ and $\sqrt[3]{1.1}$. (Round your answers to three decimal places.)

$\sqrt[3]{0.95} \approx$ 0.983 ✓

$\sqrt[3]{1.1} \approx$ 1.033 ✓

Illustrate by graphing g and the tangent line.

Tangent linear approximation

$L(x) = f(a) + f'(a)(x - a)$

$g(x) = \sqrt[3]{1 + x}$, $a = 0$

$$g'(x) = \sqrt[3]{1+x}$$
$$= \frac{d}{dx}\left[(1+x)^{1/3}\right]$$
$$= \frac{1}{3}(1+x)^{-2/3}\frac{d}{dx}(1+x)$$
$$= \frac{1}{3}\left[\frac{1}{(1+x)^{2/3}}\right](0+1)$$
$$= \frac{1}{3(1+x)^{2/3}}$$

$$L(0) = g(0) + g'(0)(x - 0)$$
$$= 1 + \frac{1}{3}(x)$$
$$\boxed{g(x) = \frac{1}{3}x + 1}$$

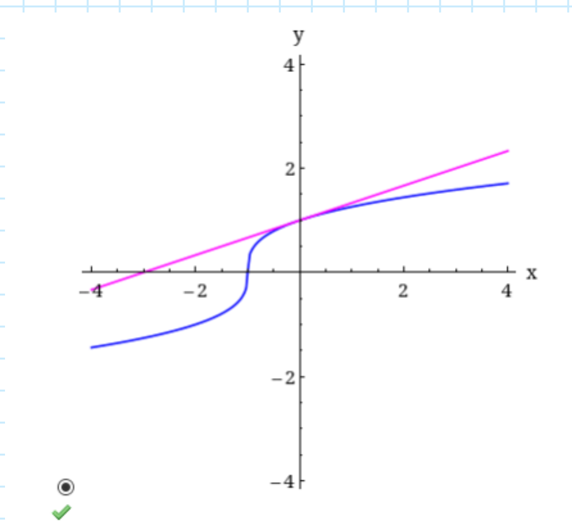
$\sqrt[3]{1+0}$
$= \sqrt[3]{1}$
$= 1$

$\frac{1}{3(1+0)^{2/3}}$
$= \frac{1}{3\left(\sqrt[3]{1+0}\right)^2}$
$= \frac{1}{3(1)}$
$= \frac{1}{3}$

$$\sqrt[3]{0.95} = L(0.95 - 1)$$
$$= \frac{1}{3}(0.95 - 1) + 1$$
$$\approx \boxed{0.983}$$

-1 to make up for the $+1$ in $g(x) = \sqrt[3]{1+x}$

$$\sqrt[3]{1.1} = L(1.1 - 1)$$
$$= \frac{1}{3}(1.1 - 1) + 1$$
$$\approx \boxed{1.033}$$



Q6

Monday, October 5, 2020 2:07 PM

Use a linear approximation (or differentials) to estimate the given number.

$e^{-0.01}$

0.99

Tangent linear approximation

$L(x) = f(a) + f'(a)(x-a)$

$e^{-0.01}$

$f(x) = e^{x}$, $a = 0$

$f'(x) = \frac{d}{dx}(e^{x})$

$= e^{x}$

$L(0) = f(0) + f'(0)(x-0)$

$= 1 + 1(x-0)$

$= 1 + x$

$e^{-0.01} = L(-0.01)$

$= 1 + (-0.01)$

$= \boxed{0.99}$