Q1 ✗

1) Sketch a graph of a function with all of the following properties:

$\lim\limits_{x\to3+} f(x) = 7$ $\lim\limits_{x\to3-} f(x) = -1$ $f(3)=2$

$\lim\limits_{x\to\infty} f(x) = 5$ $\lim\limits_{x\to-\infty} f(x) = 3$ $f(6)=0$

11 points

$\lim\limits_{x\to3^+} f(x) = 7$

$\lim\limits_{x\to\infty} f(x) = 5$

$\lim\limits_{x\to-\infty} f(x) = 3$

$f(3) = 2$

$f(6) = 0$

$\lim\limits_{x\to3^-} f(x) = -1$

open dot @ 6

Q2 ✗
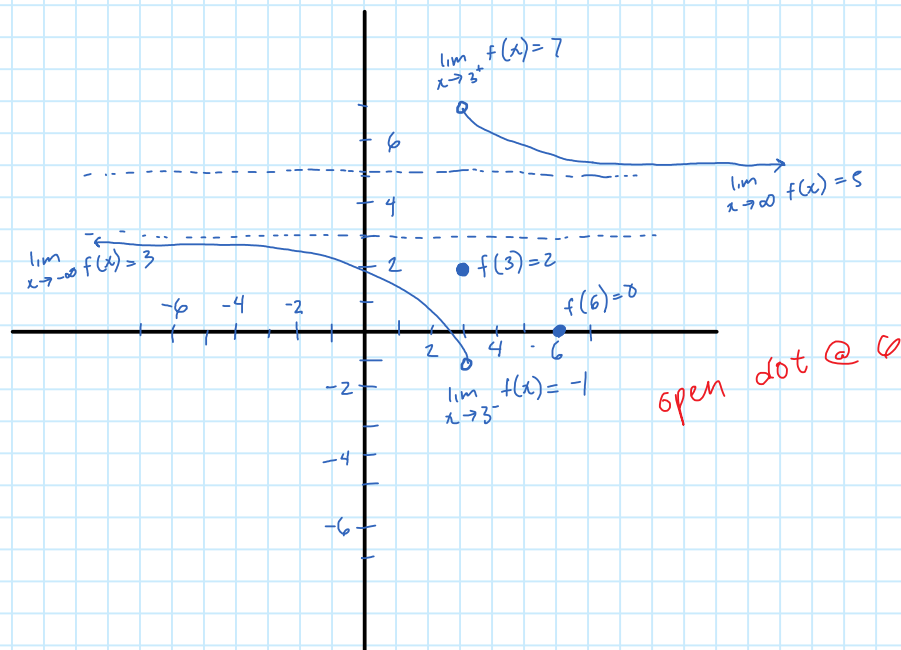
2) Evaluate $\lim\limits_{x \to 9} \dfrac{9-x}{3-\sqrt{x}}$. You must show all of your algebra.   11 points

multiply top and bottom
by the conjugate

$$\lim\limits_{x \to 9} \frac{9-x}{3-\sqrt{x}} = \frac{9-(9)}{3-\sqrt{(9)}} = \frac{0}{0} = 0$$

The limit as x
approaches 9, DOES NOT EXIST

Q3

3) Let $f(x) = \begin{cases} x^2 + 1 \ for \ x \geq 0 \\ \cos(x) \ for \ x < 0 \end{cases}$ — Right — Left

Is f(x) continuous at x=0? Why or why not? Your explanation must include an <u>algebraic</u> argument.   11 points

$$\lim_{x \to 0^-} f(x) = \cos(x)$$

$$= \cos(0)$$

$$= 1$$

$$\boxed{\lim_{x \to 0^-} f(x) = 1}$$

$$\lim_{x \to 0^+} f(x) = x^2 + 1$$

$$= (0)^2 + 1$$

$$= 1$$

$$\boxed{\lim_{x \to 0^+} f(x) = 1}$$

$$\lim_{x \to 0^-} = \lim_{x \to 0^+}$$

Therefore, f(x) is continuous at x=0

Q4

4) Evaluate $\lim\limits_{x \to \infty} \dfrac{\sqrt{4x^2 + x}}{x - 3}$. You must show all of your algebra.

11 points

$$\lim_{x \to \infty} \frac{\sqrt{4x^2 + x}}{x - 3}$$

Take the highest term from the numerator and denominator

$$\frac{\sqrt{4x^2}}{x} = \frac{2x}{x} = \boxed{2}$$

Q5 ✗

5) Find $\lim_{x \to b} \frac{(x-b)^{5000}-x+b}{x-b}$ 11 points

$\lim_{x \to b} \dfrac{(x-b)^{5000} - x + b}{x-b}$

$\dfrac{(x-b)(x-b)^{4999} - x + b}{x-b}$ → $\dfrac{(x-5)^{5000}}{x-b} \cdot \cdot -1 \dfrac{x-b}{x-b}$

$= (x-b)^{4999} - \dfrac{x+b}{x-b}$ $= \dfrac{(x-5)^{5000}}{x-6} \cdot \cdot -1$

$= (x-5)^{4999} - \dfrac{x+b}{-1(x+b)}$ $= -\dfrac{(x-5)^{5000}}{x-6} \cdot \cdot \cdot$

$= (x-b)^{4999} + 1$

$\lim_{x \to b} (x-b)^{4999} + 1$

$= (b-b)^{4999} + 1 = (0)^{4999} + 1$

$= 1$

$\boxed{\lim_{x \to b} = 1}$

6) Let f(x)= x² + 1   12 points

Find $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ You must show all of your algebra. No using more advanced short cuts from later chapters.

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \qquad f(x) = x^2 + 1$$

$$\lim_{h \to 0} \frac{(x+h)^2 + 1 - (x^2+1)}{h}$$

$$= \frac{x^2 + 2xh + h^2 + 1 - x^2 - 1}{h}$$

$$= \frac{2xh + h^2}{h} = \frac{h(2x+h)}{h}$$

$$\lim_{h \to 0} = 2x + h$$

$$= 2x + (0)$$

$$\boxed{\lim_{h \to 0} = 2x}$$

Q7

7) Evaluate $\lim\limits_{x \to 0}\left(\dfrac{1}{x} - \dfrac{1}{x^2+x}\right)$

11 points

$\lim\limits_{x \to 0} \quad \dfrac{1}{x} - \dfrac{1}{x^2+x}$

$$= \dfrac{1}{x}\dfrac{(x^2+x)}{(x^2+x)} - \dfrac{1}{x^2+x}\dfrac{(x)}{(x)}$$

$$= \dfrac{x^2+x}{x^3+x^2} - \dfrac{x}{x^3+x^2}$$

$$= \dfrac{x^2}{x^3+x^2}$$

$$= \dfrac{x^2}{x^2(x+1)}$$

$$\lim\limits_{x \to 0} = \dfrac{1}{x+1} = \dfrac{1}{0+1} = \dfrac{1}{1} = \boxed{1}$$

8) Let $f(x) = \frac{x^2-x-6}{x^2-16}$ 11 points

Find all horizontal and vertical asymptotes.

$f(x) = \dfrac{x^2 - x - 6}{x^2 - 16}$
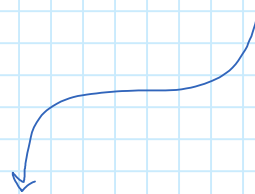
Take the highest term from both the numerator and the denominator

$\dfrac{x^2}{x^2} = 1$

$\boxed{y = 1}$

$\dfrac{x^2 - x - 6}{x^2 - 16}$

$= \dfrac{x^2 - x - 6}{(x-4)(x+4)}$

$\boxed{x = 4 , \quad x = -4}$

Q9

9) Let $f(x) = \frac{1}{x}$ 12 points

Find $\lim_{x \to 2} \frac{f(x) - f(a)}{x - a}$ You must show all of your algebra. No using more advanced short cuts from later chapters.

$$\lim_{x \to 2} \frac{f(x) - f(a)}{x - a} \quad , \quad f(x) = \frac{1}{x}$$

$$\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{a}}{x - a} = \frac{\frac{1}{x}\frac{(a)}{(a)} - \frac{1}{a}\frac{(x)}{(x)}}{x - a}$$

$$= \frac{\frac{a}{xa} - \frac{x}{xa}}{x - a} = \frac{a - x}{xa} \cdot \frac{1}{x - a}$$

$$= \frac{a - x}{xa(x - a)} = \frac{-1(-a + x)}{xa(x - a)}$$

$$= \frac{-1}{xa}$$

$$\lim_{x \to 2} \frac{-1}{xa} = \frac{-1}{(2)a} = \boxed{-\frac{1}{2}a}$$