

Q1

Sunday, October 25, 2020 1:58 PM

Let $f(x) = \frac{x}{x^3-1}$. Use algebra and calculus to find the following. You must show your work.

a) Intervals of increasing or decreasing

b) local max's and min's

c) concavity and inflection points

$$f(x) = \frac{x}{x^3-1}$$

$$f'(x) = \frac{(x^3-1) \frac{d}{dx}(x) - x \frac{d}{dx}(x^3-1)}{(x^3-1)^2}$$

$$= \frac{(x^3-1)(1) - x(3x^2)}{(x^3-1)^2}$$

$$= \frac{x^3-1-3x^3}{(x^3-1)^2}$$

$$= \frac{-2x^3-1}{(x^3-1)^2}$$

$$f'(x) = \frac{-2x^3-1}{(x^3-1)^2}$$

$$\frac{-2x^3-1}{(x^3-1)^2} = 0$$

$$\begin{aligned} -2x^3-1 &= 0 \\ -2x^3 &= 1 \\ x^3 &= -\frac{1}{2} \\ x &= \sqrt[3]{-\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} x^3-1 &= 0 \\ x^3 &= 1 \\ x &= 1 \end{aligned}$$

f' is increasing @ $(-\infty, \sqrt[3]{-\frac{1}{2}})$
 f' is decreasing @ $(\sqrt[3]{-\frac{1}{2}}, 1) \cup (1, \infty)$

$$\text{Local max value} = -\frac{2\sqrt[3]{-\frac{1}{2}}}{3}$$

$$\text{Local min value} = \text{DNE}$$

$$f''(x) = \frac{d}{dx} \left(\frac{-2x^3-1}{(x^3-1)^2} \right)$$

$$= \frac{(x^3-1)^2 \frac{d}{dx}(-2x^3-1) - (-2x^3-1) \frac{d}{dx}[(x^3-1)^2]}{(x^3-1)^4}$$

$$= \frac{(x^3-1)^2(-6x^2) - (-2x^3-1)2(x^3-1)\frac{d}{dx}(x^3-1)}{(x^3-1)^4}$$

$$= \frac{(x^3-1)^2(-6x^2) - (-2x^3-1)2(x^3-1)(3x^2)}{(x^3-1)^4}$$

$$= \frac{(x^3-1)[(x^3-1)(-6x^2) - (-2x^3-1)2(3x^2)]}{(x^3-1)^4}$$

$$= \frac{-6x^5+6x^2-2(-6x^2-3x^2)}{x^3-1}$$

$$= \frac{-6x^5+6x^2-12x^2-6x^2}{x^3-1}$$

Out of time

concavity upwards $f''(x) > 0$
 concavity downwards $f''(x) < 0$

$$f\left(\sqrt[3]{-\frac{1}{2}}\right) = \frac{x}{x^3-1}$$

$$= \frac{\sqrt[3]{-\frac{1}{2}}}{\left(\sqrt[3]{-\frac{1}{2}}\right)^3-1}$$

$$= \frac{\sqrt[3]{-\frac{1}{2}}}{-\frac{1}{2}-1}$$

$$= \frac{\sqrt[3]{-\frac{1}{2}}}{-\frac{3}{2}}$$

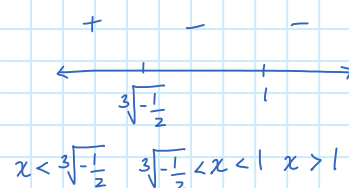
$$= -\frac{2\sqrt[3]{-\frac{1}{2}}}{3}$$

$$f(1) = \frac{x}{x^3-1}$$

$$= \frac{1}{(1)^3-1}$$

$$= \frac{1}{0}$$

undefined



Intervals	$f'(x)$
$x < \sqrt[3]{-\frac{1}{2}}$	$f'(-1) = +$
$\sqrt[3]{-\frac{1}{2}} < x < 1$	$f'(0) = -$
$x > 1$	$f'(2) = -$

