

Q1

Thursday, September 17, 2020 1:34 PM

Find the derivative of the function.

$$F(x) = (3x^6 + 2x^3)^4$$

$$F'(x) = 4(3x^6 + 2x^3)^3(18x^5 + 6x^2)$$



$$F(x) = (3x^6 + 2x^3)^4$$

$$u = 3x^6 + 2x^3$$

$$f(u) = y = u^4$$

$$\begin{aligned} f'(x) &= \frac{dF(x)}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \\ &= \frac{dy}{du}(u^4) \frac{du}{dx}(3x^6 + 2x^3) \\ &= 4u^3(18x^5 + 6x^2) \end{aligned}$$

$$f'(x) = 4(3x^6 + 2x^3)^3(18x^5 + 6x^2)$$

Q2

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4:18 PM

Find the derivative of the function.

$$f(x) = \sqrt{3x+2}$$

$$f'(x) = \frac{3}{2\sqrt{3x+2}}$$



$$f(x) = \sqrt{3x+2}$$

$$u = 3x+2$$

$$f(u) = \sqrt{u} = y$$

$$f'(u) = \frac{df(x)}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{dy}{du} (\sqrt{u}) \frac{du}{dx} (3x+2)$$

$$= \frac{dy}{du} (u^{1/2}) (3+0)$$

$$= \frac{1}{2} u^{-1/2} (3)$$

$$= \frac{1}{2} (3x+2)^{-1/2} (3)$$

$$= \frac{3}{2} (3x+2)^{-1/2}$$

$$= \frac{3}{2} \cdot \frac{1}{\sqrt{3x+2}}$$

$$f'(u) = \frac{3}{2\sqrt{3x+2}}$$

Q3

Thursday, September 17, 2020

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Find the derivative of the function.

$$f(\theta) = \cos(\theta^2)$$

$$f'(\theta) = -2\theta \sin(\theta^2)$$



$$f(\theta) = \cos(\theta^2)$$

$$u = \theta^2$$

$$f(u) = \cos(u) = y$$

$$f'(\theta) = \frac{df(\theta)}{d\theta} = \frac{dy}{du} \frac{du}{d\theta}$$

$$= \frac{dy}{du} [\cos(u)] \frac{du}{d\theta} (\theta^2)$$

$$= -\sin(u) (2\theta)$$

$$= -\sin(\theta^2) (2\theta)$$

$$f'(\theta) = -2\theta \sin(\theta^2)$$

Q4

Thursday, September 17, 2020

4:41 PM

Find the derivative of the function.

$$f(t) = t \sin(\pi t)$$

$$f'(t) = \pi t \cos(\pi t) + \sin(\pi t)$$



The Product Rule

$$\frac{d}{dx} [f(x)g(x)] = f(x) \frac{d}{dx} [g(x)] + g(x) \frac{d}{dx} [f(x)]$$

$$f(t) = t \sin(\pi t)$$

$$f'(t) = t \frac{d}{dx} [\sin(\pi t)] + \sin(\pi t) \frac{d}{dx} (t)$$

Apply Chain Rule

$$u = \pi t$$

$$f(u) = \sin(u) = y$$

$$\frac{d}{dx} [\sin(\pi t)] = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= t \frac{dy}{du} [\sin(u)] \frac{du}{dx} (\pi t) + \sin(\pi t) (1)$$

$$= t [\cos(u)] (\pi) + \sin(\pi t)$$

$$f'(t) = \pi t \cos(u) + \sin(\pi t)$$

Find the derivative of the function.

$$f(x) = (2x - 2)^4(x^2 + x + 1)^5$$

$$f'(x) = (2x - 2)^4 5(x^2 + x + 1)^4(2x + 1) + (x^2 + x + 1)^5 8(2x - 2)^3$$



The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

$$f(x) = (2x - 2)^4(x^2 + x + 1)^5$$

↓ product rule

$$f'(x) = (2x - 2)^4 \frac{d}{dx}(x^2 + x + 1)^5 + (x^2 + x + 1)^5 \frac{d}{dx}(2x - 2)^4$$

Chain Rule

$$u = x^2 + x + 1$$

$$f(u) = u^5 = y$$

$$\frac{dy}{du} \cdot \frac{du}{dx}$$

$$= (2x - 2)^4 \frac{dy}{du}(u^5) \frac{du}{dx}(x^2 + x + 1) + (x^2 + x + 1)^5 \frac{dy}{du}(u^4) \frac{du}{dx}(2x - 2)$$

$$= (2x - 2)^4 (5u^4) (2x + 1 + 0) + (x^2 + x + 1)^5 (4u^3) (2 - 0)$$

$$= (2x - 2)^4 [5(x^2 + x + 1)^4] (2x + 1) + (x^2 + x + 1)^5 [4(2x - 2)^3] (2)$$

Chain Rule

$$u = 2x - 2$$

$$f(u) = u^4 = y$$

$$\frac{dy}{du} \frac{du}{dx}$$

$$f'(x) = (2x - 2)^4 5(x^2 + x + 1)^4 (2x + 1) + (x^2 + x + 1)^5 8(2x - 2)^3$$

Find the derivative of the function.

$$h(t) = (t + 1)^{2/3}(2t^2 - 3)^3$$

$$h'(t) = (t + 1)^{\left(\frac{2}{3}\right)}(2t^2 - 3)^2 12t + (2t^2 - 3)^3 \left(\frac{2}{3\sqrt[3]{t+1}}\right)$$



The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

$$h(t) = \overset{f(x)}{(t+1)^{2/3}} \overset{g(x)}{(2t^2-3)^3}$$

↓ product Rule

$$h'(t) = (t+1)^{2/3} \frac{d}{dx}(2t^2-3)^3 + (2t^2-3)^3 \frac{d}{dx}(t+1)^{2/3}$$

Chain Rule

$$= (t+1)^{2/3} \frac{dy}{du}(u^3) \frac{du}{dx}(2t^2-3) + (2t^2-3)^3 \frac{dy}{du}(u^{2/3}) \frac{du}{dx}(t+1)$$

$$u = 2t^2 - 3$$

$$f(u) = u^3 = y$$

$$\frac{dy}{du} \cdot \frac{du}{dx}$$

$$= (t+1)^{2/3} (3u^2)(4t-0) + (2t^2-3)^3 \left(\frac{2}{3}u^{-1/3}\right)(1+0)$$

$$= (t+1)^{2/3} (3u^2)(4t) + (2t^2-3)^3 \left(\frac{2}{3}u^{-1/3}\right)$$

$$= (t+1)^{2/3} \left[3(2t^2-3)^2 \right] 4t + (2t^2-3)^3 \left[\frac{2}{3}(t+1)^{-1/3} \right]$$

$$= (t+1)^{2/3} (2t^2-3)^2 12t + (2t^2-3)^3 \frac{2}{3} \left(\frac{1}{\sqrt[3]{t+1}} \right)$$

$$h'(t) = (t+1)^{2/3} (2t^2-3)^2 12t + (2t^2-3)^3 \frac{2}{3\sqrt[3]{t+1}}$$

chain rule

$$u = t + 1$$

$$f(u) = u^{2/3} = y$$

$$\frac{dy}{du} \quad \frac{du}{dx}$$

Find the derivative of the function.

$$y = \sqrt{\frac{x}{x+1}}$$

$$y' = \left(\frac{1}{2\sqrt{\frac{x}{x+1}}} \right) \left(\frac{1}{x^2 + 2x + 1} \right)$$



The Quotient Rule

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2}$$

$$y = \sqrt{\frac{x}{x+1}}$$

$$\downarrow \text{Chain Rule} \quad \frac{(x+1)(1) - x(1+0)}{(x+1)^2}$$

$$u = \frac{x}{x+1}$$

$$f(u) = \sqrt{u} = y$$

$$\frac{dy}{du} \cdot \frac{du}{dx}$$

$$y' = \frac{dy}{du} \left(\sqrt{u} \right) \frac{du}{dx} \left(\frac{x}{x+1} \right)$$

Quotient rule

$$= (u^{1/2}) \left[\frac{(x+1) \frac{d}{dx}(x) - x \frac{d}{dx}(x+1)}{(x+1)^2} \right]$$

$$= \frac{1}{2} u^{-1/2} \left[\frac{x+1-x}{(x+1)^2} \right]$$

$$= \frac{1}{2} \left(\frac{x}{x+1} \right)^{-1/2} \left(\frac{x+1-x}{x^2+2x+1} \right)$$

$$= \frac{1}{2} \left(\frac{1}{\sqrt{\frac{x}{x+1}}} \right) \left(\frac{1}{x^2+2x+1} \right)$$

$$y' = \left(\frac{1}{2\sqrt{\frac{x}{x+1}}} \right) \left(\frac{1}{x^2+2x+1} \right)$$

Q8

Thursday, September 17, 2020

8:52 PM

Find the derivative of the function.

$$y = e^{\tan(\theta)}$$

$$y' = e^{\tan(\theta)} \sec^2(\theta)$$



$$y = e^{\tan(\theta)}$$

$$u = \tan(\theta)$$

$$y' = \frac{d}{dx}(e^u)$$

$$= \frac{d}{du}(e^u) \frac{du}{dx}(\tan \theta)$$

$$y' = e^{\tan \theta} \frac{d}{dx}(\tan \theta)$$

$$y' = e^{\tan \theta} \sec^2 \theta$$

Find the derivative of the function.

$$f(z) = e^{z/(z-1)}$$

$$f'(z) = e^{\left(\frac{z}{z-1}\right)} \left(-\frac{1}{z^2 - 2z + 1} \right)$$



The Quotient Rule

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2}$$

$$f(z) = e^{z/(z-1)}$$

$$u = \frac{z}{z-1}$$

$$f'(z) = \frac{d}{dx} (e^u)$$

$$= \frac{d}{du} (e^u) \frac{du}{dx} \left(\frac{z}{z-1} \right)$$

$$= e^{z/(z-1)} \frac{d}{dx} \left(\frac{z}{z-1} \right)$$

$$= e^{z/(z-1)} \left[\frac{(z-1) \frac{d}{dx} (z) - z \frac{d}{dx} (z-1)}{(z-1)^2} \right] \rightarrow \text{Quotient Rule}$$

$$= e^{z/(z-1)} \left[\frac{(z-1)(1) - z(1-0)}{(z-1)^2} \right]$$

$$= e^{z/(z-1)} \left(\frac{z-1-z}{z^2-2z+1} \right)$$

$$f'(z) = e^{z/(z-1)} \left(-\frac{1}{z^2-2z+1} \right)$$

Q10

Thursday, September 17, 2020 9:33 PM

Find the derivative of the function.

$$F(t) = e^{2t \sin(2t)}$$

$$F'(t) = e^{2t \sin(2t)} [4t \cos(2t) + 2 \sin(2t)]$$



The Product Rule

$$\frac{d}{dx} [f(x)g(x)] = f(x) \frac{d}{dx} [g(x)] + g(x) \frac{d}{dx} [f(x)]$$

$$f(t) = e^{2t \sin(2t)}$$

$$u = 2t \sin(2t)$$

$$f'(t) = \frac{d}{du} (e^u) \frac{du}{dx} (u)$$

$$= e^u \frac{du}{dx} [2t \sin(2t)]$$

$$= e^{2t \sin(2t)} \left[2 \left(t \frac{d}{dx} [\sin(2t)] + \sin(2t) \frac{d}{dx} [t] \right) \right] \rightarrow \text{product rule}$$

$$= e^{2t \sin(2t)} \left[2 \left(t \frac{dy}{du} [\sin(u)] \frac{du}{dx} [2t] + \sin(2t) \frac{d}{dx} [t] \right) \right]$$

chain rule

$$u = 2t$$

$$f(u) = \sin(u) = y$$

$$\frac{dy}{du} \quad \frac{du}{dx}$$

$$= e^{2t \sin(2t)} \left[2 \left(t [\cos(u)] [2] + \sin(2t) \frac{d}{dx} [t] \right) \right]$$

$$= e^{2t \sin(2t)} \left[2 \left(2t \cos[2t] + \sin[2t] \frac{d}{dx} [t] \right) \right]$$

$$= e^{2t \sin(2t)} \left[2 \left(2t \cos[2t] + \sin[2t] [1] \right) \right]$$

$$= e^{2t \sin(2t)} \left[2 \left(2t \cos[2t] + \sin[2t] \right) \right]$$

$$f'(t) = e^{2t \sin(2t)} [4t \cos(2t) + 2 \sin(2t)]$$

The Quotient Rule

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2}$$

$$y = \cos \left(\frac{1 - e^{2x}}{1 + e^{2x}} \right)$$

Chain Rule

$$u = \frac{1 - e^{2x}}{1 + e^{2x}}$$

$$f(u) = \cos(u) = y$$

$$\frac{dy}{du} \cdot \frac{du}{dx}$$

$$y' = \frac{dy}{du} \left[\cos(u) \right] \frac{du}{dx} \left(\frac{1 - e^{2x}}{1 + e^{2x}} \right)$$

$$= [-\sin(u)] \left[-\frac{4e^{2x}}{(1 + e^{2x})^2} \right]$$

$$= \sin(u) \left[\frac{4e^{2x}}{(1 + e^{2x})^2} \right]$$

$$y' = \sin \left(\frac{1 - e^{2x}}{1 + e^{2x}} \right) \left[\frac{4e^{2x}}{(1 + e^{2x})^2} \right]$$

Chain Rule

$$u = 2x$$

$$f(u) = e^u = y$$

$$\frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{(1 + e^{2x}) \frac{d}{dx} (1 - e^{2x}) - (1 - e^{2x}) \frac{d}{dx} (1 + e^{2x})}{(1 + e^{2x})^2}$$

$$= \frac{(1 + e^{2x}) \left[0 - \frac{dy}{du} (e^{2x}) \frac{du}{dx} (2x) \right] - (1 - e^{2x}) \left[0 + \frac{dy}{du} (e^{2x}) \frac{du}{dx} (2x) \right]}{(1 + e^{2x})^2}$$

$$= \frac{(1 + e^{2x}) (e^{2x}) (-2) - (1 - e^{2x}) (e^{2x}) (2)}{(1 + e^{2x})^2}$$

$$= \frac{-(1 + e^{2x}) 2e^{2x} - (1 - e^{2x}) 2e^{2x}}{(1 + e^{2x})^2}$$

$$= \frac{-2e^{2x} [(1 + e^{2x}) + (1 - e^{2x})]}{(1 + e^{2x})^2}$$

$$= \frac{-2e^{2x} [1 + e^{2x} + 1 - e^{2x}]}{(1 + e^{2x})^2}$$

$$= \frac{-2e^{2x} (2)}{(1 + e^{2x})^2}$$

$$= -\frac{4e^{2x}}{(1 + e^{2x})^2}$$

Find the derivative of the function.

$$f(t) = \sin^2(e^{\sin^2(t)})$$

$$f'(t) = 4 \sin(e^{\sin^2(t)}) \cos(e^{\sin^2(t)}) e^{\sin^2(t)} \sin(t) \cos(t)$$



The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

$$\begin{aligned} f(t) &= \sin^2(e^{\sin^2(t)}) \\ &= (\sin[e^{\sin^2(t)}])^2 \end{aligned}$$

Chain Rule
 $u = \sin(e^{\sin^2(t)})$
 $f'(u) = u^2 = y$
 $\frac{dy}{du} \cdot \frac{du}{dx}$

$$f'(t) = \frac{dy}{du}(u^2) \frac{du}{dx} [\sin(e^{\sin^2(t)})]$$

Chain Rule
 $v = e^{\sin^2(t)}$
 $f(v) = \sin(v)$
 $\frac{dy}{dv} \cdot \frac{dv}{dx}$

$$= 2u \left(\frac{dy}{dv} [\sin(v)] \frac{dv}{dx} [e^{\sin^2(t)}] \right)$$

Chain rule
 $w = \sin^2(t)$
 $f(w) = e^w = y$
 $\frac{dy}{dw} \cdot \frac{dw}{dx}$

$$\begin{aligned} &= 2u \left([\cos(e^{\sin^2(t)})] \frac{dy}{dw}(e^w) \frac{dw}{dx} [\sin(t)] \right) \\ &= 2[\sin(e^{\sin^2(t)})] ([\cos(e^{\sin^2(t)})]) (e^w) \frac{dy}{dw}(a^2) \frac{dw}{dx} [\sin(t)] \\ &= 2[\sin(e^{\sin^2(t)})] ([\cos(e^{\sin^2(t)})]) (e^w) (2a) [\cos(t)] \\ &= 2[\sin(e^{\sin^2(t)})] ([\cos(e^{\sin^2(t)})]) [e^{\sin^2(t)}] [2(\sin(t))] [\cos(t)] \\ &= 2[\sin(e^{\sin^2(t)})] [\cos(e^{\sin^2(t)})] [e^{\sin^2(t)}] 2[\sin(t)] [\cos(t)] \end{aligned}$$

Chain Rule
 $a = \sin^2(t)$
 $f(a) = a^2 = y$
 $\frac{dy}{da} \cdot \frac{da}{dx}$

$$f'(t) = 4 \sin(e^{\sin^2(t)}) \cos(e^{\sin^2(t)}) e^{\sin^2(t)} \sin(t) \cos(t)$$

Q13

Friday, September 18, 2020 11:30 AM

Find an equation of the tangent line to the curve at the given point.

$$y = \sin(\sin(x)), (\pi, 0)$$

$$y = -x + \pi$$



$$y = \sin(\sin[x]), (\pi, 0)$$

$$y(x) = \sin(\sin[x]) = m$$

Chain Rule

$$u = \sin(x)$$

$$f(u) = \sin(u) = y$$

$$\frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{dy}{du}(\sin[u]) \frac{du}{dx}(\sin[x])$$

$$= \cos(u) \cos(x)$$

$$y'(x) = \cos(\sin[x]) \cos(x)$$

$$y'(\pi) = \cos(\sin[\pi]) \cos(\pi)$$

$$= \cos(0)(-1)$$

$$= (1)(-1)$$

$$y'(\pi) = -1 = m$$

Point-slope form
 $y - y_1 = m(x - x_1)$

$$x_1 = \pi$$

$$y_1 =$$

$$m = -1$$

$$y - \pi = -1(x - 0)$$

$$y - \pi = -x$$

$$y = -x + \pi$$

Q14

Friday, September 18, 2020

11:51 AM

Find the derivative of the function.

$$y = \cot^2(\sin(\theta))$$

$$y' = 2 \cot(\sin(\theta)) \left(-\csc^2(\sin(\theta)) \cos(\theta) \right)$$



$$y = \cot^2(\sin[\theta])$$

$$= [\cot(\sin[\theta])]^2$$

$$y' = [\cot(\sin[\theta])]^2$$

Chain Rule

$$u = \cot(\sin[\theta])$$

$$f(u) = u^2 = y$$

$$\frac{dy}{du} \cdot \frac{du}{dx}$$

$$y' = \frac{dy}{du} (u^2) \frac{du}{dx} [\cot(\sin[\theta])]$$

$$= 2u \left[\frac{dy}{dv} (\cot[v]) \frac{dv}{dx} (\sin[\theta]) \right]$$

$$= 2 \cot(\sin[\theta]) \left[-\csc^2(v) \cos(\theta) \right]$$

$$y' = 2 \cot(\sin[\theta]) \left[-\csc^2(\sin[\theta]) \cos(\theta) \right]$$

Chain Rule

$$v = \sin(\theta)$$

$$f(v) = \cot(v) = y$$

$$\frac{dy}{dv} \cdot \frac{dv}{dx}$$

Q15

Friday, September 18, 2020 12:10 PM

Find the derivative of the function.

$$y = \cos(\sqrt{\sin(\tan(3x))})$$

$$y' = -\frac{3}{2} \sin(\sqrt{\sin(\tan(3x))}) (\sin(\tan(3x)))^{-\frac{1}{2}} \cos(\tan(3x)) \sec^2(3x)$$



$$y = \cos(\sqrt{\sin[\tan(3x)]})$$

k

$$y' = \cos(\sqrt{\sin[\tan(3x)]})$$

Chain Rule

$$u = \sqrt{\sin(\tan(3x))}$$

$$f(u) = \cos(u) = y$$

$$\frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{dy}{du} [\cos(u)] \frac{du}{dx} [\sqrt{\sin(\tan(3x))}]$$

$$= -\sin(u) \left[\frac{dy}{dv} (\sqrt{v}) \frac{dv}{dx} (\sin[\tan(3x)]) \right]$$

Chain Rule

$$v = \sin(\tan[3x])$$

$$f(v) = \sqrt{v} = y$$

$$\frac{dy}{dv} \cdot \frac{dv}{dx}$$

Chain Rule

$$w = \tan(3x)$$

$$f(w) = \sin(w) = y$$

$$\frac{dy}{dw} \cdot \frac{dw}{dx}$$

$$= -\sin(\sqrt{\sin[\tan(3x)]}) \left[\frac{dy}{dv} (\sqrt{v}) \frac{dv}{dw} (\sin[w]) \frac{dw}{dx} (\tan[3x]) \right]$$

$$= -\sin(\sqrt{\sin[\tan(3x)]}) \left[\left(\frac{1}{2} v^{-1/2} \right) \cos(w) \frac{dy}{da} (\tan[a]) \frac{da}{dx} (3x) \right]$$

$$= -\sin(\sqrt{\sin[\tan(3x)]}) \left[\left(\frac{1}{2} v^{-1/2} \right) \cos(w) \sec^2(a) (3) \right]$$

$$= -\sin(\sqrt{\sin[\tan(3x)]}) \left[\left(\frac{1}{2} [\sin(\tan[3x])]^{-1/2} \right) \cos(w) \sec^2(a) (3) \right]$$

$$= -\sin(\sqrt{\sin[\tan(3x)]}) \left[\left(\frac{1}{2} [\sin(\tan[3x])]^{-1/2} \right) \cos(\tan[3x]) \sec^2(a) (3) \right]$$

$$= -\sin(\sqrt{\sin[\tan(3x)]}) \left[\left(\frac{1}{2} [\sin(\tan[3x])]^{-1/2} \right) \cos(\tan[3x]) \sec^2(3x) (3) \right]$$

$$= -\sin(\sqrt{\sin[\tan(3x)]}) \left[\frac{3}{2} (\sin[\tan(3x)])^{-1/2} \cos(\tan[3x]) \sec^2(3x) \right]$$

$$y' = -\frac{3}{2} \sin(\sqrt{\sin[\tan(3x)]}) (\sin[\tan(3x)])^{-1/2} \cos(\tan[3x]) \sec^2(3x)$$