Q1

Tuesday, November 3, 2020 8:13 PM

For all of these problems you must use calculus and algebra and you must show all your work.

1)Find the absolute maximum an absolute minimum value of
$f(x) = x^3 - 6x^2 + 9x + 2$ on [-1,4]. You must show all your work. 15 points

2)Consider the function $f(x) = 2\sqrt{x} - x$. Find the following: 25 points

a) The Domain
b) All intercepts
c) Determine if f(x) is odd, even or periodic
d) All asymptotes
e) All intervals of increase or decreasing
f) All local maximums and minimums
g) Concavity and inflection points
h) Make a sketch of the graph

3) Evaluate $\lim_{x\to\infty} \frac{(\ln x)^2}{x}$ 15 points

4)A rectangular storage container with an open top is to have a volume of 10 m^3. The length of this base is twice the width. Material for the base costs $10 per square meter. Material for the side's costs $6 per square meter. Find the cost of materials for the cheapest such container. (Round your answer to the nearest cent.)
20 points

5)Let $f(x) = e^{-x^2}$ Find the following: 25 points

a) The Domain
b) All intercepts
c) Determine if f(x) is odd, even or periodic
d) All asymptotes
e) All intervals of increase or decreasing
f) All local maximums and minimums
g) Concavity and inflection points
h) Make a sketch of the graph

1)Find the absolute maximum an absolute minimum value of
$f(x) = x^3 - 6x^2 + 9x + 2$ on [-1,4]. You must show all your work.

Need to check end points as well for absolute min/max

$f(x) = x^3 - 6x^2 + 9x + 2, \; [-1, 4]$

$f'(x) = \frac{d}{dx}(x^3 - 6x^2 + 9x + 2)$

$f'(x) = 3x^2 - 12x + 9$

$3x^2 - 12x + 9 = 0$

$3(x^2 - 4x + 3) = 0$

$3(x-3)(x-1) = 0$

$x = 3 \qquad x = 1$

$f''(x) = \frac{d}{dx}(3x^2 - 12x + 9)$

$f''(x) = 6x - 12$

$f''(3) = 6x - 12 > 0$ concave upward

$f''(1) = 6x - 12 < 0$ concave downward

$f(3) = x^3 - 6x^2 + 9x + 2$
$= (3)^3 - 6(3)^2 + 9(3) + 2$
$= 2$

$f(1) = x^3 - 6x^2 + 9x + 2$
$= (1)^3 - 6(1)^2 + 9(1) + 2$
$= 6$

$\therefore$ Absolute max value = 6
Absolute min value = 2

Q2

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2)Consider the function $f(x) = 2\sqrt{x} - x$. Find the following:

a) The Domain
b) All intercepts
c) Determine if f(x) is odd, even or periodic
d) All asymptotes
e) All intervals of increase or decreasing
f) All local maximums and minimums
g) Concavity and inflection points
h) Make a sketch of the graph

$f(x) = 2\sqrt{x} - x$

x cannot be negative

a) Domain $[0, \infty)$

$2\sqrt{x} - x = 0$

$2\sqrt{x} = x$

$^2(2\sqrt{x}) - (x)^2$

$4x = x^2$

$4x - x^2 = 0$

$4x(4 - x) = 0$

$x = 0$ $x = 4$

$f(4) = 2\sqrt{x} - x$
$= 2\sqrt{4} - 4$
$= 2(2) - 4$
$= 4 - 4$
$= 0$

$f(0) = 2\sqrt{x} - x$
$= 2\sqrt{0} - 0$
$= 0$

b) y-intercept $= f(0) = 0$
x-intercept $= (0, 4)$

c) $f(-x) \neq f(x)$ can't be negative
no symmetry or odd

$f'(x) = \frac{d}{dx}(2x^{1/2} - x)$

$f'(x) = x^{-1/2} - 1$

$x^{-1/2} - 1 = 0$

$\frac{1}{\sqrt{x}} = 1$

$1 = \sqrt{x}$

$x = \pm 1$ only use positive

+	−
$x < 1$ 1	$x > 1$

Intervals	$f'(x)$
$x < 1$	$f'(\frac{1}{4}) = +$
$x > 1$	$f'(4) = -$

e) f' is increasing @ $[0, 1)$
f' is decreasing @ $(1, \infty)$

$f(1) = 2\sqrt{x} - x$
$= 2\sqrt{1} - 1$
$= 1$

f) Local max value $= 1$

$f''(x) = \frac{d}{dx}(x^{-1/2} - 1)$
$= -\frac{1}{2}x^{-3/2}$

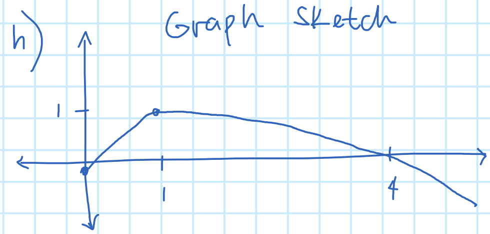
$-\frac{1}{2}x^{-3/2} = 0$

$x = 0$

x cannot be zero when looking for I.P

g) no inflection point
$f''(1) < 0$
concave downwards
@ $(0, \infty)$

h) Graph Sketch



Q3

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3) Evaluate $\lim_{x\to\infty} \frac{(\ln x)^2}{x}$

$$\lim_{x\to\infty} \frac{(\ln x)^2}{x} = \frac{\infty}{\infty}$$

l'Hospital's Rule

$$\lim_{x\to\infty} \frac{\frac{d}{dx}\left[(\ln x)^2\right]}{\frac{d}{dx}(x)} = \frac{2\ln \frac{d}{dx}(\ln x)}{1} = 2\ln(x)\frac{1}{x} = \frac{2\ln(x)}{x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{\frac{d}{dx}\left[2\ln(x)\right]}{\frac{d}{dx}(x)} = \frac{2\left(\frac{1}{x}\right)}{1} = \frac{2}{x} = \boxed{0}$$

will become zero as x approaches ∞ or gets bigger value

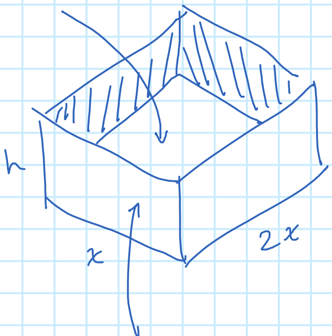
Q4

Tuesday, November 3, 2020 9:17 PM

4)A rectangular storage container with an <u>open top</u> is to have a volume of 10 m^3. The length of this base is twice the width. Material for the base costs \$10 per square meter. Material for the side's costs \$6 per square meter. Find the cost of materials for the cheapest such container. (Round your answer to the nearest cent.)
20 points

Let x = length, y = width, h = height, and V = volume (in meters cube)

Base cost = \$10/m^2



sides cost = \$6/m^2

$V = l\,w\,h$

$10 = (x)(2x)(h)$

$10 = 2x^2 h$

$\frac{10}{2x^2} = h$

$\frac{5}{x^2} = h$

$$\text{Total cost} = \underbrace{\$10(x)(2x)}_{\text{Base}} + \underbrace{\$6(x)(h)(2) + \$6(2x)(h)(2)}_{\text{sides}}$$

$$= 20x^2 + 12xh + 24xh$$

$$= 20x^2 + 36xh$$

$$= 20x^2 + 36x\left(\frac{5}{x^2}\right)$$

$$\text{Total cost} = C(x) = 20x^2 + \frac{180}{x}$$

$$C'(x) = \frac{d}{dx}\left(20x^2 + 180x^{-1}\right)$$

$$C'(x) = 40x - 180x^{-2}$$

$$40x - 180x^{-2} = 0$$

$$40x - \frac{180}{x^2} = 0$$

$$40x = \frac{180}{x^2}$$

$$40x^3 = 180$$

$$x^3 = \frac{180}{40}$$

$$x^3 = \frac{9}{2}$$

$$x = \sqrt[3]{\frac{9}{2}}$$

$$C''(x) = \frac{d}{dx}\left(40x - 180x^{-2}\right)$$

$$C''(x) = 40 + 360x^{-3}$$

make sure x = min

$$C''\left(\sqrt[3]{\frac{9}{2}}\right) = 40 + \frac{360}{x^3} > 0$$

concave upwards $\therefore x = \sqrt[3]{\frac{9}{2}}$ is the local min

$$C\left(\sqrt[3]{\frac{9}{2}}\right) = 20x^2 + \frac{180}{x}$$

$$= 20\left(\sqrt[3]{\frac{9}{2}}\right)^2 + \frac{180}{\left(\sqrt[3]{\frac{9}{2}}\right)}$$

$$\approx 163.54$$

$$\boxed{\text{Total Cheapest Cost} = \$163.5}$$

Q5

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5)Let $f(x) = e^{-x^2}$ Find the following:

a) The Domain
b) All intercepts
c) Determine if f(x) is odd, even or periodic
d) All asymptotes
e) All intervals of increase or decreasing
f) All local maximums and minimums
g) Concavity and inflection points
h) Make a sketch of the graph

$f(x) = e^{-x^2}$

a) Domain $\mathbb{R}$

$f(0) = e^{-x^2}$
$= e^{-(0)^2}$
$= 1$

$e^{-x^2} = 0$

No Solution,
x will never be zero

b) y-intercept $= f(0) = 1$
x-Intercept = DNE

c) $f(-x) = f(x)$
symmetrical or even

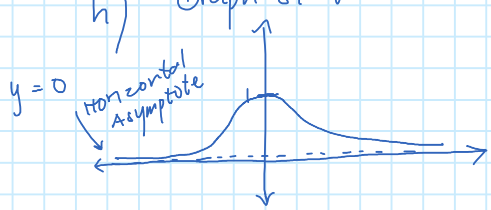
Horizontal Asymptote

$\lim_{x \to \pm\infty} f(x) = e^{-x^2} = 0$

f(x) becomes zero as x approaches ∞

d) Horizontal Asymptote
$y = 0$

h) Graph sketch



$f'(x) = \frac{d}{dx}\left(e^{-x^2}\right)$

$= e^{-x^2} \frac{d}{dx}(-x^2)$

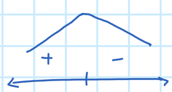
$= e^{-x^2}(-2x)$

$f'(x) = -2e^{-x^2}x$

$-2e^{-x^2}x = 0$

$\downarrow$

$x = 0$



Intervals	$f'(x)$
$x < 0$	$f(-1) = +$
$x > 0$	$f(1) = -$

e) f' is increasing @ $(\infty, 0]$
f' is decreasing @ $[0, \infty)$

$f(0) = e^{-x^2}$
$= e^{-0^2}$
$= 1$

f) Local max value $= 1$

$f''(x) = \frac{d}{dx}\left(-2e^{-x^2}x\right)$

$= -2\left[e^{-x^2}\frac{d}{dx}(x) + x\frac{d}{dx}\left(e^{-x^2}\right)\right]$

$= -2\left[e^{-x^2}(1) + x\left(e^{-x^2}\right)\frac{d}{dx}(-x^2)\right]$

$= -2\left[e^{-x^2} + e^{-x^2}x(-2x)\right]$

$= -2\left(e^{-x^2} - 2e^{-x^2}x^2\right)$

$f''(x) = -2e^{-x^2}(-1 + 2x^2)$

$-2e^{-x^2}(2x^2 - 1) = 0$

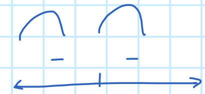
$\downarrow$

$2x^2 - 1 = 0$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \sqrt{\frac{1}{2}}$



$x < \sqrt{\frac{1}{2}}$ $\sqrt{\frac{1}{2}}$ $x > \sqrt{\frac{1}{2}}$

Intervals	$f''(x)$
$x < \sqrt{\frac{1}{2}}$	$f''(-1) = -$
$x > \sqrt{\frac{1}{2}}$	$f''(1) = -$

g) No inflection points
Concave downwards
@ $(-\infty, \infty)$