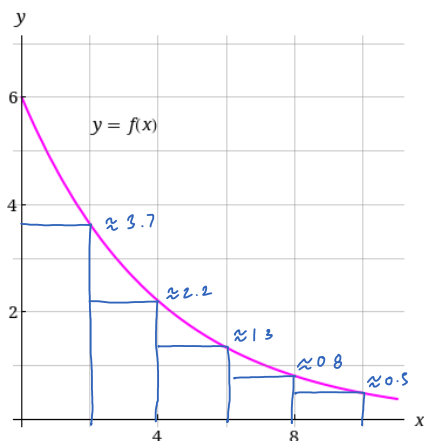


Q1.1

Friday, November 6, 2020 7:58 PM

Consider the following.



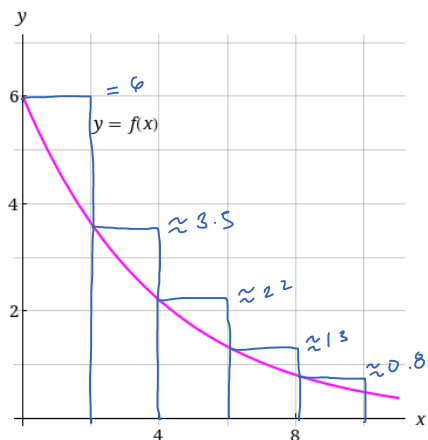
$$\Delta x = \frac{b-a}{n} = \frac{10-0}{5} = 2$$

$$\begin{aligned} R_5 &= \sum_{i=1}^5 f(x_i) \Delta x \\ &= f(x_1)(2) + f(x_2)(2) + f(x_3)(2) + f(x_4)(2) + f(x_5)(2) \\ &= 2[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)] \\ &= 2(3.7 + 2.2 + 1.3 + 0.8 + 0.5) \\ &= 17 \end{aligned}$$

(a) By reading values from the given graph of f , use five rectangles to find a lower estimate for the area under the given graph of f from $x = 0$ to $x = 10$. (Round your answer to one decimal place.)

17 ✓

Consider the following.



$$\Delta x = \frac{b-a}{n} = \frac{10-0}{5} = 2$$

$$\begin{aligned} L_5 &= \sum_{i=1}^5 f(x_i) \Delta x \\ &= f(x_1)(2) + f(x_2)(2) + f(x_3)(2) + f(x_4)(2) + f(x_5)(2) \\ &= 2[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)] \\ &= 2(6 + 3.5 + 2.2 + 1.3 + 0.8) \\ &= 27.6 \end{aligned}$$

By reading values from the given graph of f , use five rectangles to find an upper estimate for the area under the given graph of f from $x = 0$ to $x = 10$. (Round your answer to one decimal place.)

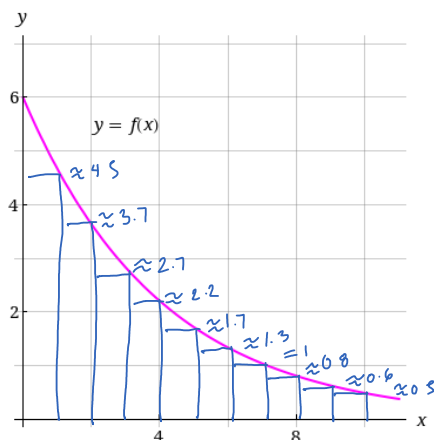
27.6 ✓

Q1.2

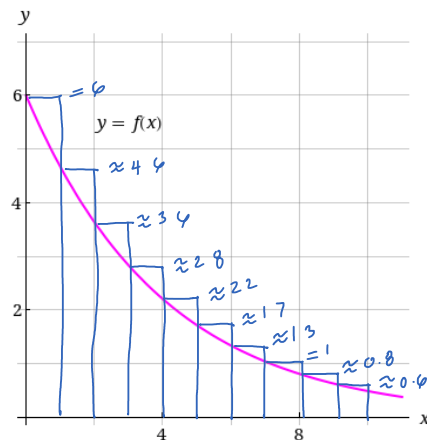
Friday, November 6, 2020

8:45 PM

Consider the following.



Consider the following.



$$\Delta x = \frac{b-a}{n} = \frac{10-0}{10} = 1$$

$$\begin{aligned} R_{10} &= \sum_{i=1}^{10} f(x_i) \Delta x \\ &= f(x_1)(1) + f(x_2)(1) + f(x_3)(1) + f(x_4)(1) + f(x_5)(1) + f(x_6)(1) + f(x_7)(1) + f(x_8)(1) + f(x_9)(1) + f(x_{10})(1) \\ &= 1 [f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6) + f(x_7) + f(x_8) + f(x_9) + f(x_{10})] \\ &= 1 (4.5 + 3.7 + 2.7 + 2.2 + 1.7 + 1.3 + 0.8 + 0.6 + 0.5) \\ &= 19 \end{aligned}$$

$$\Delta x = \frac{b-a}{n} = \frac{10-0}{10} = 1$$

$$\begin{aligned} L_{10} &= \sum_{i=1}^{10} f(x_i) \Delta x \\ &= f(x_1)(1) + f(x_2)(1) + f(x_3)(1) + f(x_4)(1) + f(x_5)(1) + f(x_6)(1) + f(x_7)(1) + f(x_8)(1) + f(x_9)(1) + f(x_{10})(1) \\ &= 1 [f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6) + f(x_7) + f(x_8) + f(x_9) + f(x_{10})] \\ &= 1 (6 + 4.6 + 3.4 + 2.8 + 2.2 + 1.7 + 1.3 + 0.8 + 0.6) \\ &= 24.6 \end{aligned}$$

(b) Find new estimates using ten rectangles in each case. (Round your answers to one decimal place.)

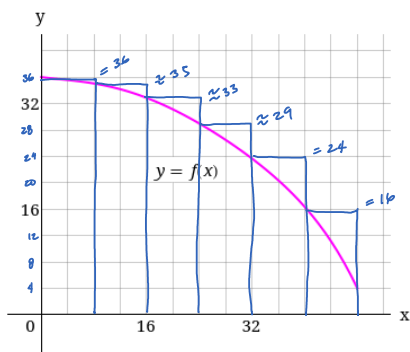
19 ✓ (lower estimate)

24.6 ✓ (upper estimate)

Q2

Friday, November 6, 2020 9:03 PM

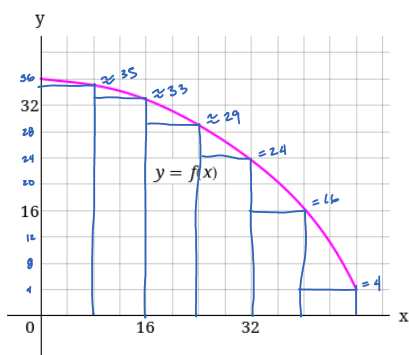
Consider the following.



$$\Delta x = \frac{b-a}{n} = \frac{48-0}{6} = 8$$

$$\begin{aligned} L_6 &= \sum_{i=1}^6 f(x_i) \Delta x \\ &= f(x_1)(8) + f(x_2)(8) + f(x_3)(8) + f(x_4)(8) + f(x_5)(8) + f(x_6)(8) \\ &= 8[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6)] \\ &= 8(36 + 35 + 33 + 29 + 24 + 16) \\ &= 1384 \end{aligned}$$

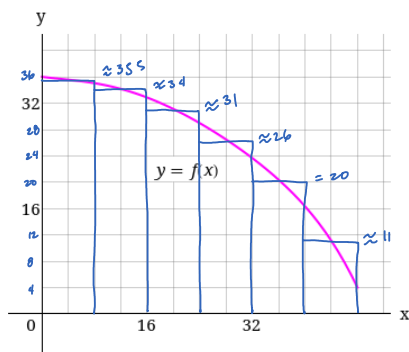
Consider the following.



$$\Delta x = \frac{b-a}{n} = \frac{48-0}{6} = 8$$

$$\begin{aligned} R_6 &= \sum_{i=1}^6 f(x_i) \Delta x \\ &= f(x_1)(8) + f(x_2)(8) + f(x_3)(8) + f(x_4)(8) + f(x_5)(8) + f(x_6)(8) \\ &= 8[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6)] \\ &= 8(35 + 33 + 29 + 24 + 16 + 4) \\ &= 1128 \end{aligned}$$

Consider the following.



$$\Delta x = \frac{b-a}{n} = \frac{48-0}{6} = 8$$

$$\begin{aligned} M_6 &= \sum_{i=1}^6 f(x_i) \Delta x \\ &= f(x_1)(8) + f(x_2)(8) + f(x_3)(8) + f(x_4)(8) + f(x_5)(8) + f(x_6)(8) \\ &= 8[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6)] \\ &= 8(35.5 + 34 + 31 + 26 + 20 + 11) \\ &= 1260 \end{aligned}$$

(a) Use six rectangles to find estimates of each type for the area under the given graph of f from $x = 0$ to $x = 48$.

(i) Sample points are left endpoints.

$$L_6 = \boxed{1380} \quad \checkmark$$

(ii) Sample points are right endpoints.

$$R_6 = \boxed{1128} \quad \checkmark$$

(iii) Sample points are midpoints.

$$M_6 = \boxed{1260} \quad \checkmark$$

(b) Is L_6 an underestimate or overestimate of the true area?

- ☒ overestimate
☐ underestimate

(c) Is R_6 an underestimate or overestimate of the true area?

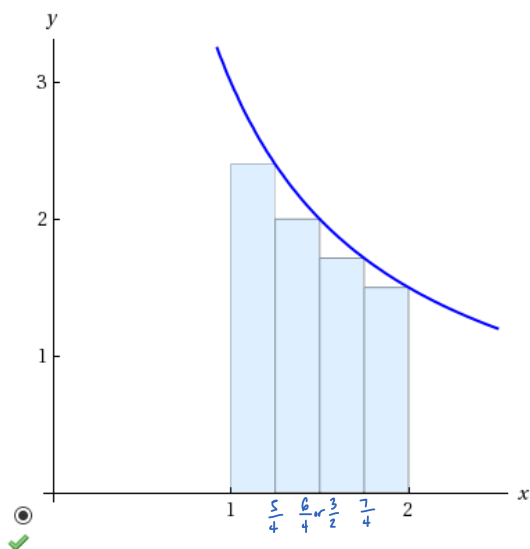
- ☐ overestimate
☒ underestimate

(d) Which of the numbers gives the best estimate?

- ☒ M_6
☐ L_6
☐ R_6

Q3

Friday, November 6, 2020 9:35 PM



$$f(x) = \frac{3}{x}$$

$$\Delta x = \frac{b-a}{n} = \frac{2-1}{4} = \frac{1}{4}$$

$$R_4 = \sum_{i=1}^4 f(x_i) \Delta x$$

$$= \frac{1}{4} [f(x_1) + f(x_2) + f(x_3) + f(x_4)]$$

$$= \frac{1}{4} \left(\frac{3}{5/4} + \frac{3}{6/4} + \frac{3}{7/4} + \frac{3}{2} \right)$$

$$\approx 1.9036$$

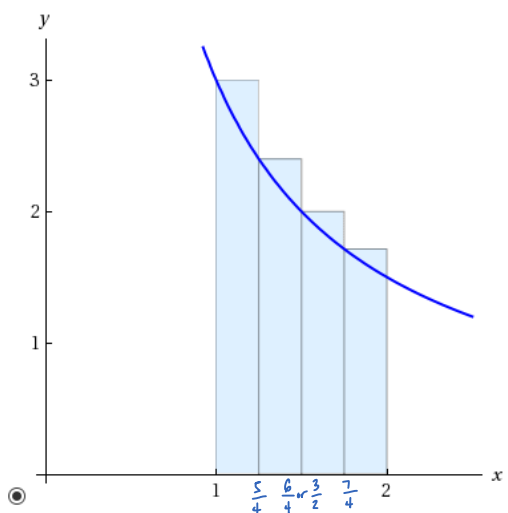
(a) Estimate the area under the graph of $f(x) = 3/x$ from $x = 1$ to $x = 2$ using four approximating rectangles and right endpoints. (Round your answer to four decimal places.)

1.9036

Is your estimate an underestimate or an overestimate?

☒ underestimate

☐ overestimate



$$f(x) = \frac{3}{x}$$

$$\Delta x = \frac{b-a}{n} = \frac{2-1}{4} = \frac{1}{4}$$

$$L_4 = \sum_{i=1}^4 f(x_i) \Delta x$$

$$= \frac{1}{4} [f(x_1) + f(x_2) + f(x_3) + f(x_4)]$$

$$= \frac{1}{4} \left(\frac{3}{1} + \frac{3}{5/4} + \frac{3}{6/4} + \frac{3}{7/4} \right)$$

$$\approx 2.2786$$

(b) Repeat part (a) using left endpoints. (Round your answer to four decimal places.)

2.2786

Is your estimate an underestimate or an overestimate?

☐ underestimate

☒ overestimate

Q4.1

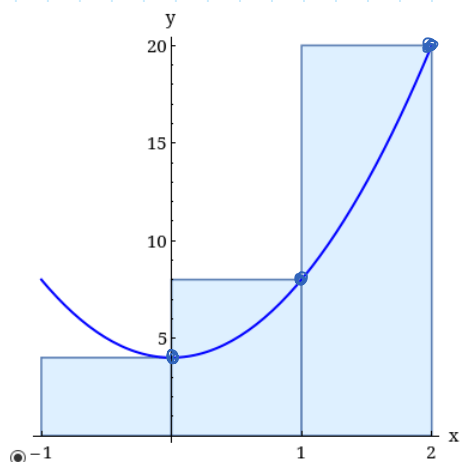
Friday, November 6, 2020 10:00 PM

(a) Estimate the area under the graph of $f(x) = 4 + 4x^2$ from $x = -1$ to $x = 2$ using three rectangles and right endpoints.

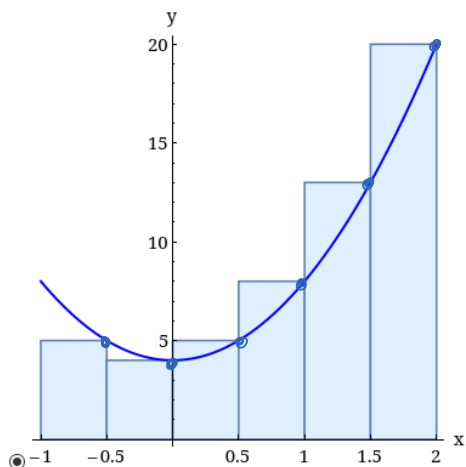
$R_3 =$ ✓

Then improve your estimate by using six rectangles.

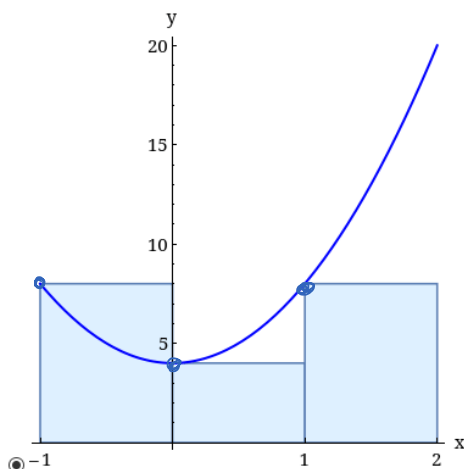
$R_6 =$ ✓



$$\begin{aligned} f(x) &= 4 + 4x^2 \\ \Delta x &= 1 \\ R_3 &= \sum_{i=1}^3 f(x_i) \Delta x \\ &= 1 [f(0) + f(1) + f(2)] \\ &= 1 (4 + 8 + 20) \\ &= 32 \end{aligned}$$



$$\begin{aligned} f(x) &= 4 + 4x^2 \\ \Delta x &= \frac{1}{2} \\ R_6 &= \sum_{i=1}^6 f(x_i) \Delta x \\ &= \frac{1}{2} [f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5) + f(2)] \\ &= \frac{1}{2} (5 + 4 + 5 + 8 + 13 + 20) \\ &= 27.5 \end{aligned}$$



$$\begin{aligned} f(x) &= 4 + 4x^2 \\ \Delta x &= 1 \\ L_3 &= \sum_{i=1}^3 f(x_i) \Delta x \\ &= 1 [f(-1) + f(0) + f(1)] \\ &= 1 (8 + 4 + 8) \\ &= 20 \end{aligned}$$

(b) Repeat part (a) using left endpoints.

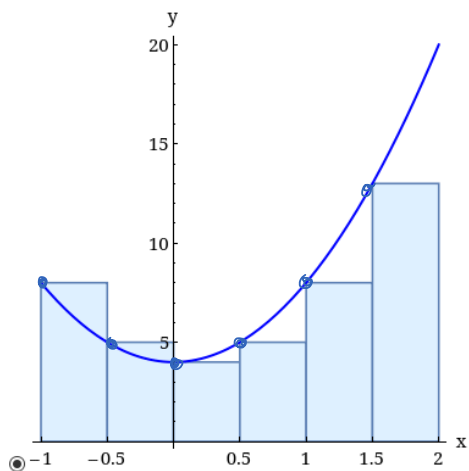
$L_3 =$ ✓

$L_6 =$ ✓

Q4.2

Friday, November 6, 2020

10:19 PM



$$f(x) = 4 + 4x^2$$

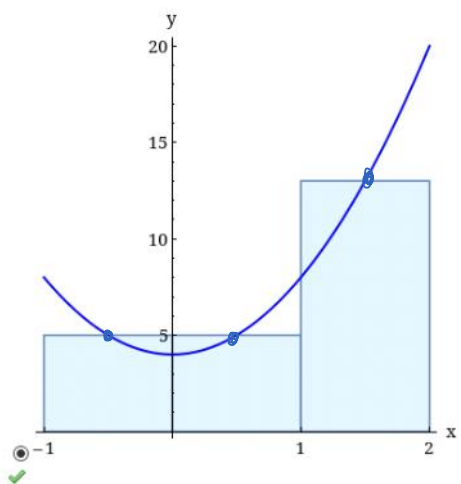
$$\Delta x = \frac{1}{2}$$

$$R_6 = \sum_{i=1}^6 f(x_i) \Delta x$$

$$= \frac{1}{2} [f(-1) + f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5)]$$

$$= \frac{1}{2} (8 + 5 + 4 + 5 + 8 + 13)$$

$$= 21.5$$



$$f(x) = 4 + 4x^2$$

$$\Delta x = 1$$

$$M_3 = \sum_{i=1}^3 f(x_i) \Delta x$$

$$= 1 [f(-0.5) + f(0.5) + f(1.5)]$$

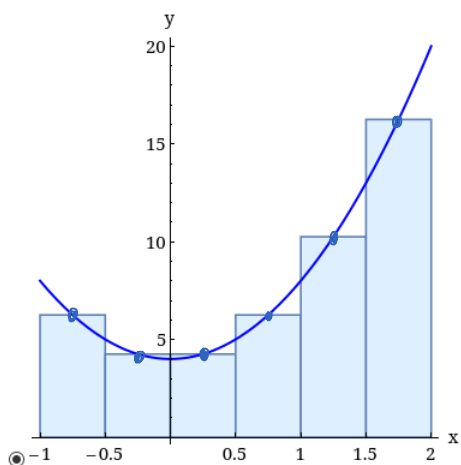
$$= 1 (5 + 5 + 13)$$

$$= 23$$

(c) Repeat part (a) using midpoints.

$$M_3 = 23$$

$$M_6 = 23.75$$



$$f(x) = 4 + 4x^2$$

$$\Delta x = \frac{1}{2}$$

$$R_6 = \sum_{i=1}^6 f(x_i) \Delta x$$

$$= \frac{1}{2} [f(-0.75) + f(-0.25) + f(0.25) + f(0.75) + f(1.25) + f(1.75)]$$

$$= \frac{1}{2} (6.25 + 4.25 + 4.25 + 6.25 + 10.25 + 16.25)$$

$$= 23.75$$

(d) From your sketches in parts (a)-(c), which appears to be the best estimate?

☐ L_6

☒ M_6

☐ R_6



Q5

Friday, November 6, 2020 10:33 PM

The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find lower and upper estimates for the distance that she traveled during these three seconds.

34.1 ✓ ft (smaller value)

44.1 ✓ ft (larger value)

t (s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v (ft/s)	0	6.7	9.2	14.1	18.8	19.4	20

lower estimate

upper estimate

$$\text{Distance} = \text{rate} \cdot \text{time}$$

Let Δd = change in distance

$$\Delta d = 0.5$$

Lower estimate

$$= 0.5(0 + 6.7 + 9.2 + 14.1 + 18.8 + 19.4)$$

$$= 34.1 \text{ ft}$$

Upper estimate

$$= 0.5(6.7 + 9.2 + 14.1 + 18.8 + 19.4 + 20)$$

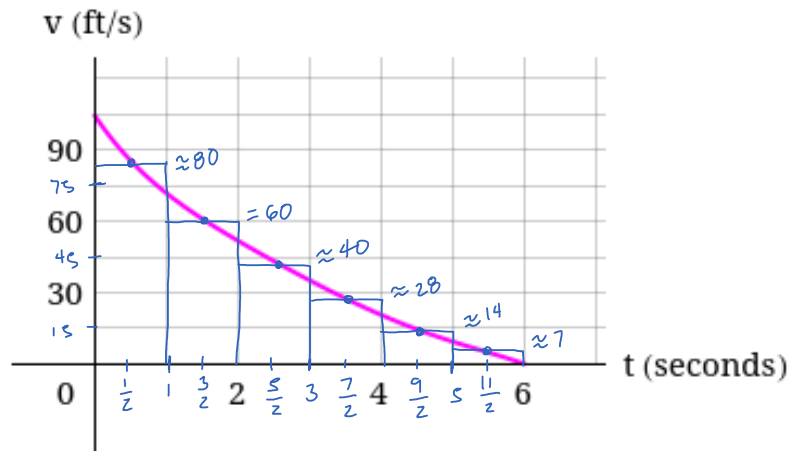
$$= 44.1 \text{ ft}$$

Q6

Friday, November 6, 2020 10:49 PM

The velocity graph of a braking car is shown. Use it to estimate the distance traveled by the car while the brakes are applied. (Use M_6 to get the most precise estimate.)

229 ft



Let Δt = change in time

$$\Delta t = 1$$

$$M_6 = \sum_{i=1}^6 f(v_i) \Delta t$$

$$= 1(80 + 60 + 40 + 28 + 14 + 7)$$

$$= 229 \text{ ft}$$