

# 1 & 2

Sunday, September 27, 2020

9:20 PM

- 1) Let  $f(x) = x^2 \sec(2x)$ . Find  $f'(x)$ .
- 2) Let  $f(x) = \tan(x \sin(x))$ . Find  $f'(x)$ .
- 3) Use implicit differentiation to find  $y'$  when  $e^y + y^2 = x \ln x$
- 4) Find the derivative of  $f(x) = x^2 \tan^{-1}(e^x)$

## The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

1)  $f(x) = x^2 \sec(2x)$ . Find  $f'(x)$

Chain Rule  
 $u = 2x$   
 $f(u) = \sec(u) = y$   
 $\frac{dy}{du} \cdot \frac{du}{dx}$

$$\begin{aligned} &= x^2 \frac{d}{dx}[\sec(2x)] + \sec(2x) \frac{d}{dx}(x^2) \\ &= x^2 \frac{dy}{du}[\sec(u)] \frac{du}{dx}(u) + \sec(2x)(2x) \\ &= x^2 [\sec(u) \tan(u)] \frac{d}{dx}(2x) + 2x \sec(2x) \\ &= x^2 \sec(2x) \tan(2x)(2) + 2x \sec(2x) \\ &f'(x) = 2x \sec(2x) [x \tan(2x) + 1] \end{aligned}$$

2)  $f(x) = \tan(x \sin(x))$ , Find  $f'(x)$

Chain Rule  
 $u = x \sin(x)$   
 $f(u) = \tan(u) = y$   
 $\frac{dy}{du} \cdot \frac{du}{dx}$

$$\begin{aligned} &= \frac{dy}{du}[\tan(u)] \frac{du}{dx}(u) \\ &= \sec^2(u) \frac{d}{dx}[x \sin(x)] \\ &= \sec^2(x \sin(x)) \left[ x \frac{d}{dx}(\sin(x)) + \sin(x) \frac{d}{dx}(x) \right] \\ &= \sec^2(x \sin(x)) [x \cos(x) + \sin(x)(1)] \\ &f'(x) = \sec^2(x \sin(x)) [x \cos(x) + \sin(x)] \end{aligned}$$

# 3 & 4

Sunday, September 27, 2020 9:58 PM

- 1) Let  $f(x) = x^2 \sec(2x)$ . Find  $f'(x)$ .
- 2) Let  $f(x) = \tan(x \sin(x))$ . Find  $f'(x)$ .
- 3) Use implicit differentiation to find  $y'$  when  $e^y + y^2 = x \ln x$
- 4) Find the derivative of  $f(x) = x^2 \tan^{-1}(e^x)$

The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

3)  $e^y + y^2 = x \ln x$ , Find  $y'$

$$e^y + y^2 = x \ln x$$

$$y' = \frac{d}{dx}(e^y + y^2) = \frac{d}{dx}(x \ln x)$$

$$\frac{d}{dy}(e^y) \frac{dy}{dx} + \frac{d}{dy}(y^2) \frac{dy}{dx} = x \frac{d}{dx}(\ln(x)) + \ln x \frac{d}{dx}(x)$$

$$e^y \frac{dy}{dx} + 2y \frac{dy}{dx} = x \left(\frac{1}{x}\right) \frac{d}{dx}(x) + \ln x (1)$$

$$e^y y' + 2y y' = \frac{x}{x} (1) + \ln x$$

$$y'(e^y + 2y) = 1 + \ln x$$

$$\frac{y'(e^y + 2y)}{(e^y + 2y)} = \frac{1 + \ln x}{(e^y + 2y)} = \boxed{y' = \frac{1 + \ln x}{e^y + 2y}}$$

4)  $f(x) = x^2 \tan^{-1}(e^x)$ , Find  $f'(x)$

$$= x^2 \frac{d}{dx}[\tan^{-1}(e^x)] + \tan^{-1}(e^x) \frac{d}{dx}(x^2)$$

Chain rule

$$u = e^x$$

$$f(x) = \tan^{-1}(u) = y$$

$$\frac{dy}{du} \frac{du}{dx}$$

$$= x^2 \frac{dy}{du}[\tan^{-1}(u)] \frac{du}{dx}(u) + \tan^{-1}(e^x)(2x)$$

$$= x^2 \left(\frac{1}{1+u^2}\right) \frac{d}{dx}(e^x) + 2x \tan^{-1}(e^x)$$

$$= x^2 \left(\frac{1}{1+(e^x)^2}\right) (e^x) + 2x \tan^{-1}(e^x)$$

$$\boxed{f'(x) = \frac{x^2 e^x}{1 + e^{2x}} + 2x \tan^{-1}(e^x)}$$