

Q1

Sunday, November 22, 2020 8:07 PM

Evaluate the following integrals. You must show all your algebra.

1) $\int_0^{\pi} \sin(x) dx$

2) $\int_1^5 \frac{2}{x} dx$

3) $\int \sin^6(x) \cos(x) dx$

$$\begin{aligned} 1) \int_0^{\pi} \sin(x) dx &= -\cos(x) + C \Big|_0^{\pi} \\ &= F(\pi) - F(0) \\ &= -\cos(\pi) - (-\cos(0)) \\ &= 1 + 1 \\ &= \boxed{2} \end{aligned}$$

$$\begin{aligned} 2) \int_1^5 \frac{2}{x} dx &= \int_1^5 2 \frac{1}{x} dx \\ &= 2 \ln(x) + C \Big|_1^5 \\ &= F(5) - F(1) \\ &= 2 \ln(5) - \cancel{2 \ln(1)}_{=0} \\ &= \boxed{2 \ln(5)} \end{aligned}$$

$$\begin{aligned} 3) \int \sin^6(x) \cos(x) dx, \quad & \begin{array}{l} u = \sin(x) \\ \frac{du}{dx} = \cos(x) \\ du = \cos(x) dx \end{array} \\ & \swarrow \quad \searrow \\ & \int (\sin(x))^6 \cos(x) dx = \int u^6 du \\ &= \frac{u^{6+1}}{6+1} + C = \frac{u^7}{7} + C = \boxed{\frac{\sin^7(x)}{7} + C} \end{aligned}$$