# Q1

Tuesday, October 27, 2020 12:21 PM

Find two numbers whose difference is 136 and whose product is a minimum.

-68 (smaller number)

68 (larger number)

$$x - y = 136 \qquad xy = f'(x) = 0$$

$$-y = 136 - x$$

$$y = x - 136$$

$$f'(x) = \frac{d}{dx}\left[x(x-136)\right]$$

$$= \frac{d}{dx}\left(x^2 - 136x\right)$$

$$f'(x) = 2x - 136$$

$$2x - 136 = 0$$

$$2x = 136$$

$$x = \frac{136}{2}$$

$$\boxed{x = 68}$$

$$y = x - 136$$

$$= 68 - 136$$

$$\boxed{y = -68}$$

# Q2

Wednesday, October 28, 2020 9:00 PM

What is the maximum vertical distance between the line $y = x + 30$ and the parabola $y = x^2$ for $-5 \le x \le 6$?

121/4

$y = x + 30$ , $y = x^2$ for $-5 \le x \le 6$

$V(x) = (x + 30) - x^2$

$V'(x) = \frac{d}{dx}\left[(x + 30) - x^2\right]$

$= 1 + 0 - 2x$

$= 1 - 2x$

$1 - 2x = 0$

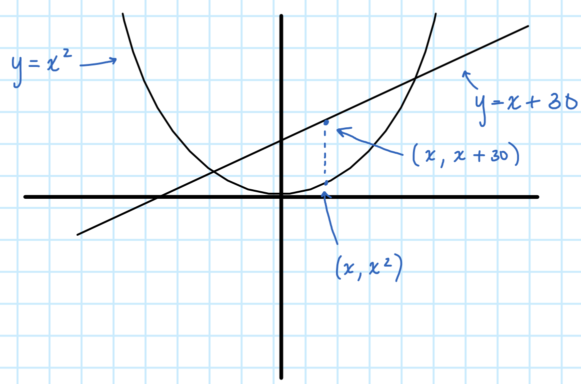
$-2x = -1$

$x = \frac{1}{2}$

$V\left(\frac{1}{2}\right) = (x + 30) - x^2$

$= \left(\frac{1}{2} + 30\right) - \left(\frac{1}{2}\right)^2$

$= \boxed{\frac{121}{4}}$




Vertical Distance $= |y_1 - y_2|$

# Q3

Wednesday, October 28, 2020 9:27 PM

A model used for the yield $Y$ of an agricultural crop as a function of the nitrogen level $N$ in the soil (measured in appropriate units) is

$$Y = \frac{kN}{25 + N^2}$$

where $k$ is a positive constant. What nitrogen level gives the best yield?

$N =$ 5

$$Y = \frac{KN}{25+N^2} \quad \text{for } K \text{ is constant} > 0$$

$$Y' = \frac{d}{dx}\left(\frac{KN}{25+N^2}\right)$$

$$= \frac{(25+N^2)\frac{d}{dx}(KN) - KN\frac{d}{dx}(25+N^2)}{(25+N^2)^2}$$

$$= \frac{(25+N^2)K - KN(2N)}{(25+N^2)^2}$$

$$= \frac{25K + KN^2 - 2KN^2}{(25+N^2)^2} = \frac{25K - KN^2}{(25+N^2)^2}$$

$$Y' = \frac{K(5+N)(5-N)}{(25+N^2)^2}$$

$$\frac{K(5+N)(5-N)}{(25+N^2)^2} = 0$$

$$5+N = 0 \qquad 5-N=0$$

$$N = -5 \qquad N = 5$$

check if Local max

Assume K is always positive

$$Y'(4) = \frac{K(5+N)(5-N)}{(25+N^2)^2} > 0$$

$$Y'(6) = \frac{K(5+N)(5-N)}{(25+N^2)^2} < 0$$

$\boxed{N=5}$ is the Local max

$$Y(5) = \frac{KN}{25+N^2}$$

$$= \frac{K(5)}{25+(5)^2}$$

$$= \frac{K5}{50}$$

$$= \frac{K}{10}$$

# Q4.1

Thursday, October 29, 2020 11:14 AM

Consider the following problem: A farmer with 850 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?

(a) Draw several diagrams illustrating the situation, some with shallow, wide pens and some with deep, narrow pens. Find the total areas of these configurations. Does it appear that there is a maximum area? If so, estimate it.

(b) Draw a diagram illustrating the general situation. Let $x$ denote the length of each of two sides and three dividers. Let $y$ denote the length of the other two sides.

(c) Write an expression for the total area A in terms of both $x$ and $y$.

$A = x \cdot y$

(d) Use the given information to write an equation that relates the variables.

$850 = 5x + 2y$

(e) Use part (d) to write the total area as a function of one variable.

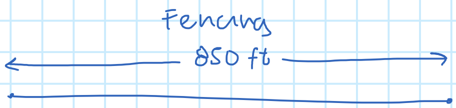
$A(x) = 425x - \frac{5}{2}x^2$

(f) Finish solving the problem by finding the largest area.

18062.5 ft$^2$

# Q4.2

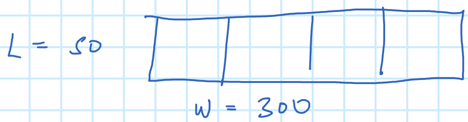
Wednesday, October 28, 2020 11:10 PM




Area = Length · Width

$\boxed{A = x \cdot y}$

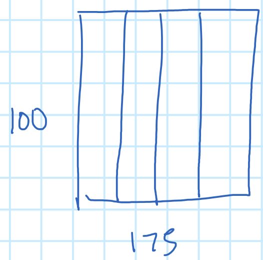
shallow wide




$A = L \cdot W$

$= 15\,000 \text{ ft}^2$

Narrow deep




$A = L\ W$

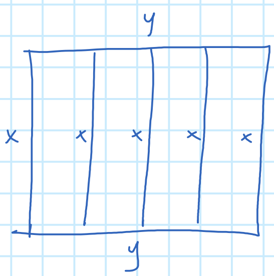
$= 17500 \text{ ft}^2$

max Area

Narrower deeper


$A = L \cdot W$

$= 15\,000 \text{ ft}^2$




$\boxed{850 = 5x + 2y}$

$850 = 5x + 2y$

$850 - 5x = 2y$

$\frac{850 - 5x}{2} = y$

$425 - \frac{5x}{2} = y$

Area $= x \cdot y$

$= x\left(425 - \frac{5x}{2}\right)$

$\boxed{A(x) = 425x - \frac{5}{2}x^2}$

$A'(x) = \frac{d}{dx}\left(425x - \frac{5}{2}x^2\right)$

$= 425 - \frac{5}{2}(2)x$

$A'(x) = 425 - 5x$

$425 - 5x = 0$

$425 = 5x$

$\frac{425}{5} = x$

$x = 85$

$A''(x) = -5$

$A''(85) = -5 < 0$ concave downwards

$\therefore$ $x = 85$ critical point is the max value

$850 = 5x + 2y$

$850 = 5(85) + 2y$

$850 = 425 + 2y$

$850 - 425 = 2y$

$425 = 2y$

$y = 212.5$

$A = x \cdot y$

$= (85)(212.5)$

$\boxed{A = 18062.5 \text{ ft}^2}$

# Q5.1

Thursday, October 29, 2020 12:17 PM

Consider the following problem: A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

(a) Draw several diagrams to illustrate the situation, some short boxes with large bases and some tall boxes with small bases. Find the volumes of several such boxes.

(b) Draw a diagram illustrating the general situation. Let $x$ denote the length of the side of the square being cut out. Let $y$ denote the length of the base.

(c) Write an expression for the volume $V$ in terms of both $x$ and $y$.

$V = xy^2$ ✓

(d) Use the given information to write an equation that relates the variables $x$ and $y$.

$y = 3 - 2x$ ✓

(e) Use part (d) to write the volume as a function of only $x$.

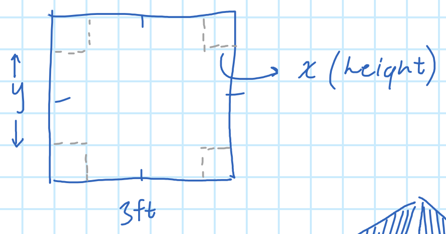
$V(x) = x(3 - 2x)^2$ ✓

(f) Finish solving the problem by finding the largest volume that such a box can have.

$V = 2$ ✓ $\text{ft}^3$

# Q5.2

Thursday, October 29, 2020 11:15 AM



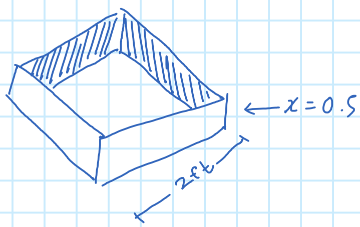

Volume = L · W · H

$V = x \cdot y \cdot h$

$V = x \cdot y \; y$

$\boxed{V = x y^2}$

$y = 3 - 2x$

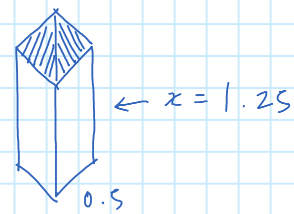



$V = xy^2$

$= (0.5)(2)^2$

$V = 2 ft^3$

max volume




$V = xy^2$

$= (1.25)(0.5)^2$

$V = 0.3125 \; ft^3$

$V(x) = x y^2$

$= x(3-2x)^2$

$V'(x) = \frac{d}{dx}\left[x(3-2x)^2\right]$

$= x \frac{d}{dx}\left[(3-2x)^2\right] + (3-2x)^2 \frac{d}{dx}(x)$

$= x\left[2(3-2x)\right] \frac{d}{dx}(3-2x) + (3-2x)^2(1)$

$= x(6-4x)(-2) + (3-2x)^2$

$= (6x - 4x^2)(-2) + (3-2x)^2$

$= -12x + 8x^2 + (3-2x)^2$

$(-2x+3)(-2x+3)$

$$\begin{array}{r} 4x^2 - 6x \\ -6x + 9 \\ \hline 4x^2 - 12x + 9 \end{array}$$

$= -12x + 8x^2 + (4x^2 - 12x + 9)$

$V'(x) = 12x^2 - 24x + 9$

$V''(x) = \frac{d}{dx}\left(12x^2 - 24x + 9\right)$

$= (2)12x - 24 + 0$

$V''(x) = 24x - 24$

$12x^2 - 24x + 9 = 0$

$3(4x^2 - 8x + 3) = 0$

$4x^2 - 8x + 3 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(3)}}{2(4)}$

$x = \frac{3}{2}, \quad x = \frac{1}{2}$

$V''\left(\frac{3}{2}\right) = 24x - 24 > 0$ concave upward

$V''\left(\frac{1}{2}\right) = 24x - 24 < 0$ concave downward

$\therefore x = \frac{1}{2}$ is the max value

$V(x) = x(3-2x)^2$

$= \frac{1}{2}\left(3 - 2\left(\frac{1}{2}\right)\right)^2$

$= 2$

$\boxed{V = 2 ft^3}$

# Q6

Thursday, October 29, 2020 12:18 PM

A box with a square base and open top must have a volume of 32,000 cm$^3$. Find the dimensions of the box that minimize the amount of material used.

| | | |
|---|---|---|
| sides of base | 40 ✓ | cm |
| height | 20 ✓ | cm |

The Volume of a box with a square base $x$ by $x$ cm and height $h$ cm is $V = x^2 h$

The amount of material used is directly proportional to the surface area, so we will minimize the amount of material by minimizing the surface area.

The surface area of the box described is $A = x^2 + 4xh$

We need $A$ as a function of $x$ alone, so we'll use the fact that $V = x^2 h = 32,000$ cm^3

which gives us $h = \frac{32,000}{x^2}$, so the area becomes:

$$A = x^2 + 4x\left(\frac{32,000}{x^2}\right) = x^2 + \frac{128,000}{x}$$

We want to minimize $A$, so

$$A' = 2x - \frac{128,000}{x^2} = 0 \text{ when } \frac{2x^3 - 128,000}{x^2} = 0$$

Which occurs when $x^3 - 64,000 = 0$ or $x = 40$

The only critical number is $x = 40$ cm.

The second derivative test verifies that $A$ has a minimum at this critical number:

$A'' = 2 + \frac{256,000}{x^3}$ which is positive at $x = 40$.

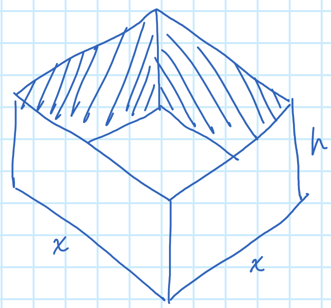
**The box should have base 40 cm by 40 cm and height 20 cm.**

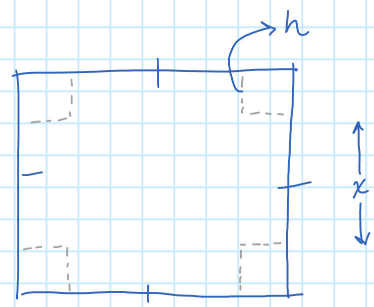
(use $h = \frac{32,000}{x^2}$ and $x = 40$)

Ref: https://socratic.org/answers/137984

$V = l \cdot w \cdot h$

$V = 32,000 \text{ cm}^3$




$$\boxed{x = 40}$$

$$\boxed{h = \frac{32000}{40^2} = 20}$$

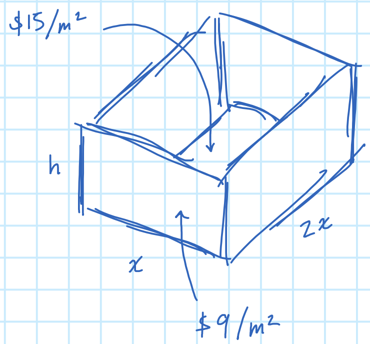
# Q7

Thursday, October 29, 2020 2:47 PM

A rectangular storage container with an open top is to have a volume of 10 $m^3$. The length of this base is twice the width. Material for the base costs $15 per square meter. Material for the sides costs $9 per square meter. Find the cost of materials for the cheapest such container. (Round your answer to the nearest cent.)

$ 245.3

$V = 10\,m^3$




$V = x \cdot 2x \cdot h$

$10 = x \cdot 2x \cdot h$

$10 = 2x^2 h$

$\frac{5}{x^2} = h$

$$\text{Total cost} = \underbrace{\$15(x)(2x)}_{\text{Base}} + \underbrace{\$9(x)(h)(2) + \$9(2x)(h)(2)}_{\text{Sides}}$$

$$= 30x^2 + 18xh + 36xh$$

$$= 30x^2 + 54xh$$

$$= 30x^2 + 54x\left(\frac{5}{x^2}\right)$$

$$\text{Total cost} = 30x^2 + \frac{270}{x}$$

$$C(x) = 30x^2 + 270x^{-1}$$

$$C'(x) = \frac{d}{dx}\left(30x^2 + 270x^{-1}\right)$$

$$= 60x - 270x^{-2}$$

$$C'(x) = 60x - \frac{270}{x^2}$$

$$60x - \frac{270}{x^2} = 0$$

$$60x = \frac{270}{x^2}$$

$$60x^3 = 270$$

$$x^3 = \frac{270}{60}$$

$$x^3 = \frac{9}{2}$$

$$x = \sqrt[3]{\frac{9}{2}}$$

$$C''(x) = \frac{d}{dx}\left(60x - 270x^{-2}\right)$$

$$= 60 + 540x^{-3}$$

$$C''(x) = 60 + \frac{540}{x^3}$$

$$C''\left(\sqrt[3]{\frac{9}{2}}\right) = 60 + \frac{540}{x^3} > 0 \quad \text{concave upwards}$$

$$\therefore x = \sqrt[3]{\frac{9}{2}} \text{ is the local min}$$

$$C(x) = 30x^2 + \frac{270}{x}$$

$$C\left(\sqrt[3]{\frac{9}{2}}\right) = 30\left(\sqrt[3]{\frac{9}{2}}\right)^2 + \frac{270}{\sqrt[3]{\frac{9}{2}}}$$

$$\approx 245.3$$

$$\boxed{\$245.3}$$

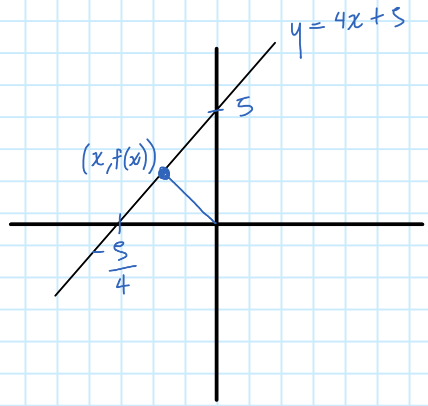
# Q8

Thursday, October 29, 2020 1:34 PM

Find the point on the line $y = 4x + 5$ that is closest to the origin.

$(x, y) = \left( -\frac{20}{17}, \frac{5}{17} \right)$

$y = 4x + 5$




Distance formula

$d = \sqrt{(x_1 - x^2)^2 + (y_1 - y_2)^2}$

$d = \sqrt{(x-0)^2 + (y-0)^2}$

$d = \sqrt{x^2 + y^2}$

or

$D = d^2 = x^2 + y^2$

$D(x) = x^2 + y^2$

$= x^2 + (4x+5)^2$

$D'(x) = \frac{d}{dx}\left[x^2 + (4x+5)^2\right]$

$= 2x + 2(4x+5)\frac{d}{dx}(4x+5)$

$= 2x + (8x+10)(4)$

$= 2x + (32x + 40)$

$D'(x) = 34x + 40$

$34x + 40 = 0$

$34x = -40$

$x = -\frac{40}{34}$

$x = -\frac{20}{17}$

$D''(x) = \frac{d}{dx}(34x + 40)$

$D''(x) = 34$

$D''\left(-\frac{20}{17}\right) = 34 > 0$

concave upwards

$\therefore\ x = -\frac{20}{17}$ is max value

$y = 4x + 5$

$= 4\left(-\frac{20}{17}\right) + 5$

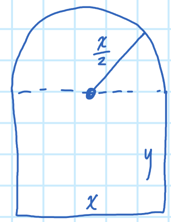
$y = \frac{5}{17}$

$\boxed{\left(-\frac{20}{17}, \frac{5}{17}\right)}$

# Q9

Thursday, October 29, 2020 2:48 PM

Let $x$ = width and
$y$ = height




Perimeter of the window is $24 = 2y + x + \pi\left(\frac{x}{2}\right)$

$$24 = 2y + x + \pi\left(\frac{x}{2}\right)$$

$$24 - x - \pi\frac{x}{2} = 2y$$

$$\frac{24 - x - \pi\frac{x}{2}}{2} = y$$

$$12 - \frac{x}{2} - \frac{\pi x}{4} = y$$

Area of the window $= xy + \frac{\pi}{2}\left(\frac{x}{2}\right)^2$

$$A(x) = x\left(12 - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{\pi}{2}\left(\frac{x}{2}\right)^2$$

$$= 12x - \frac{1}{2}x^2 - \frac{\pi}{4}x^2 + \frac{\pi}{8}x^2$$

$$A(x) = 12x - \frac{1}{2}x^2 - \frac{\pi}{8}x^2$$

$$A'(x) = \frac{d}{dx}\left(12x - \frac{1}{2}x^2 - \frac{\pi}{8}x^2\right)$$

$$= 12 - x - \frac{\pi}{4}x$$

$$A'(x) = 12 - x\left(1 + \frac{\pi}{4}\right)$$

$$12 - x\left(1 + \frac{\pi}{4}\right) = 0$$

$$12 = x\left(1 + \frac{\pi}{4}\right)$$

$$\frac{12}{1 + \frac{\pi}{4}} = x$$

$$\frac{12}{\frac{4+\pi}{4}} = x$$

$$\frac{48}{4+\pi} = x$$

$$A''(x) = \frac{d}{dx}\left(12 - x - \frac{\pi}{4}x\right)$$

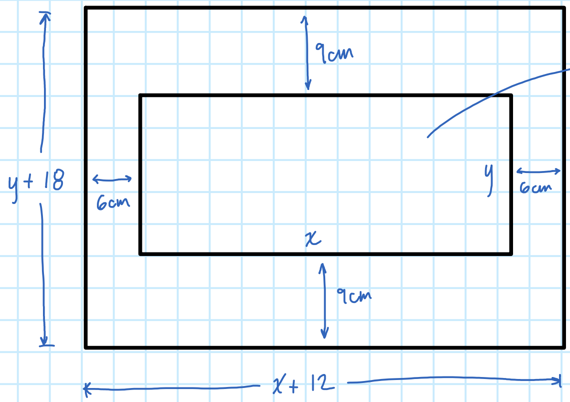
$$A''(x) = -1 - \frac{\pi}{4}$$

$$A''\left(\frac{48}{4+\pi}\right) = -1 - \frac{\pi}{4} < 0$$ concave downwards

$\therefore$ $\boxed{x = \frac{48}{4+\pi}}$ is the local max

# Q10

Thursday, October 29, 2020 3:00 PM




$A_1 = xy$

$A_1 = 864\ cm^2$

$xy = 864$

$y = \frac{864}{x}$

$A_2 = (x+12)(y+18)$

$A(x) = (x+12)\left(\frac{864}{x} + 18\right)$

$= 864 + 18x + \frac{10368}{x} + 216$

$= 18x + 10368x^{-1} + 1080$

$A'(x) = \frac{d}{dx}\left(18x + 10368x^{-1} + 1080\right)$

$A'(x) = 18 - \frac{10368}{x^2}$

$A''(x) = \frac{d}{dx}\left(18 - 10368x^{-2}\right)$

$A''(x) = 10368x^{-3}$

$18 - \frac{10368}{x^2} = 0$

$18 = \frac{10368}{x^2}$

$18x^2 = 10368$

$x^2 = \frac{10368}{18}$

$x = \sqrt{576}$

$x = 24$

$y = \frac{864}{x}$

$= \frac{864}{24}$

$y = 36$

$A''(24) = 10368x^{-3} > 0$, concave upwards

$\therefore x = 24$ is the min

Width $= x + 12$

$= 24 + 12$

$= \boxed{36\ cm}$

Height $= y + 18$

$= 36 + 18$

$= \boxed{54\ cm}$

# Q11

Thursday, October 29, 2020 3:27 PM

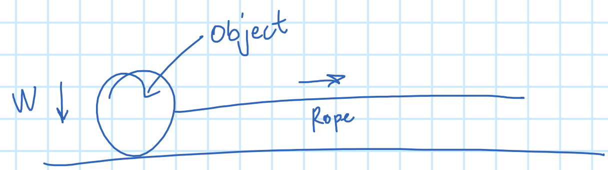
An object with weight $W$ is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle $\theta$ with a plane, then the magnitude of the force is

$$F = \frac{\mu W}{\mu \sin(\theta) + \cos(\theta)}$$

where $\mu$ is a constant called the coefficient of friction. For what value of $\theta$ is $F$ smallest?

$\theta = \boxed{\tan^{-1}\mu}$ ✓

Let $x = \theta$




$$F(x) = \frac{\mu W}{\mu \sin(x) + \cos(x)}$$

$$F'(x) = \frac{d}{dx}\left(\frac{\mu W}{\mu \sin(x) + \cos(x)}\right)$$

$$= \frac{\mu \sin(x) + \cos(x)\,\frac{d}{dx}(\mu W) - \mu W \frac{d}{dx}[\mu \sin(x) + \cos(x)]}{[\mu \sin(x) + \cos(x)]^2}$$

$$= \frac{\cancel{[\mu \sin(x) + \cos(x)](0)} - \mu W[\mu \cos(x) - \sin(x)]}{[\mu \sin(x) + \cos(x)]^2}$$

$$= \frac{-\mu^2 W \cos(x) + \mu W \sin(x)}{[\mu \sin(x) + \cos(x)]^2}$$

$$F'(x) = \frac{\mu W[\sin(x) - \mu \cos(x)]}{[\mu \sin(x) + \cos(x)]^2}$$

$$\frac{\mu W[\sin(x) - \mu \cos(x)]}{[\mu \sin(x) + \cos(x)]^2} = 0$$

$$\sin(x) - \mu \cos(x) = 0$$

$$\sin(x) = \mu \cos(x)$$

$$\frac{\sin(x)}{\cos(x)} = \mu$$

$$\mu = \tan(x)$$

$$\boxed{x = \tan^{-1}\mu}$$

Ref:

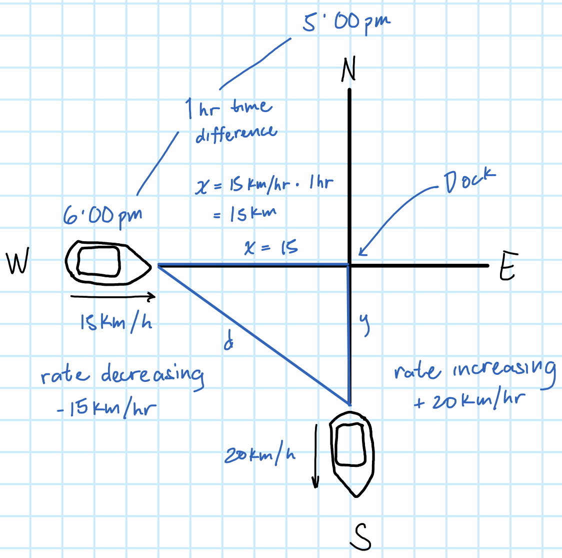
- https://sccollege.edu/Departments/MATH/Documents/Math%20180/04-01-072_Maximum_and_Minimum_Values.pdf
- https://www.slader.com/discussion/question/an-object-with-weight-w-is-dragged-along-a-horizontal-plane-by-a-force-acting-along-a-rope-attache-4/#

# Q12

Thursday, October 29, 2020 3:27 PM

A boat leaves a dock at 5:00 PM and travels due south at a speed of 20 km/h. Another boat has been heading due east at 15 km/h and reaches the same dock at 6:00 PM. How many minutes after 5:00 PM were the two boats closest together? (Round your answer to the nearest minute.)

21.6 min




Let $t$ = time (hrs)

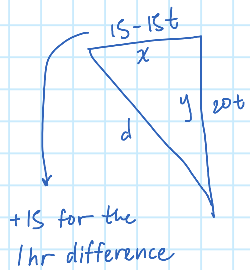
Distance = rate · time

| | | |
|---|---|---|
| east | $-15$ | $\cdot\ t$ |
| south | $20$ | $\cdot\ t$ |

$$\text{Distance} = \sqrt{x^2 + y^2}$$

$$d = \sqrt{x^2 + y^2}$$

$$S = d^2 = x^2 + y^2$$




$$S = (15 - 15t)^2 (20t)^2$$

$$S = 225 - 450t + 225t^2 + 400t^2$$

$$S = 225 - 450t + 625t^2$$

$$S' = \frac{d}{dx}\left(225 - 450t + 625t^2\right)$$

$$S' = -450 + 1250t$$

$$-450 + 1250t = 0$$

$$1250t = 450$$

$$t = \frac{450}{1250}$$

$$t = 0.36$$

$$0.36\,hrs\,(60\,min) \longrightarrow \text{convert to minutes}$$

$$= 21.6$$

$$\boxed{\text{minutes after } 5{:}00 = 21.6}$$