Q1

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1) Consider $f(x)=2\sqrt{x}-x$ 15 points
   a) Find all intervals of increase and decreasing.
   b) Find all local maximums and minimums.
   c) Find the intervals of concavity.
   d) Find all inflection points.
   e) Using the information from parts (a)-(d), sketch the graph of $f(x)$.

* 2) Using the definition of the derivative (the limit definition), find $f'(x)$ for $f(x)=x^3-x$. 10 points

3) Find the derivative of $f(x)=e^{\tan(2x)}$ 5 points

4) Sketch the graph of a function with the following properties: 8 points

$\lim_{x\to 3+} f(x)=3$ $\lim_{x\to 3-} f(x)=-4$ f(3)=0

$\lim_{x\to\infty} f(x)=5$ $\lim_{x\to-\infty} f(x)=10$ f(6)=1

5) Show algebraically that the following function is continuous at $x=0$.

$$f(x)=\begin{cases}\sin x & \text{if } x<0\\ -4x & \text{if } x\ge 0\end{cases}$$ 7 points

6) Evaluate $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$ 6 points

7) Prove the following statement using the delta-epsilon definition, from section 2.4, of the limit.

$\lim_{x\to 0} x^2=0$ 10 points

8) Let f(x)=5x and g(x)=x^2. Let R be the region bounded by the graphs and 0≤x≤3. Find the volume obtained by revolving R about the x-axis. 10 points

9) Find the derivative of $f(x)=\frac{e^x+\sin x}{\tan^{-1} x}$ 7 points

10) Find $\int\frac{(\ln x)^2}{x}dx$ 7 points

11) Consider the area bounder by y=3x, the x-axis, x=1 and x = 2 15 points
a) Draw the area with a representative rectangle.
b) Find an expression for the Riemann sum
c) Evaluate the Riemann sum
d) Find a definite integral that represents the area and use the Fundamental Theorem of Calculus to find the area.

$$\sum_{i=1}^{n} i=\frac{n(n+1)}{2}\qquad \sum_{i=1}^{n} i^2=\frac{n(n+1)(2n+1)}{6}\qquad \sum_{i=1}^{n} i^3=\left[\frac{n(n+1)}{2}\right]^2$$

1) $f(x)=2\sqrt{x}-x$ cannot be negative

Domain $[0,\infty)$

$2\sqrt{x}-x=0$
$2\sqrt{x}=x$
$(2\sqrt{x})^2=x^2$
$4x=x^2$
$4x-x^2=0$
$x(4-x)=0$
$x=0$ $x=4$

$f(4)=2\sqrt{x}-x$
$=2\sqrt{4}-4$
$=2(2)-4$
$=4-4$
$0=$

$f(0)=2\sqrt{x}-x$
$=2\sqrt{0}-0$
$=0$

y-intercept = f(0) = 0
x-intercept = (0, 4)

$f'(x)=\frac{d}{dx}(2x^{1/2}-x)$
$=x^{-1/2}-1$

$x^{-1/2}-1=0$
$\frac{1}{\sqrt{x}}=1$
$1=\sqrt{x}$
$x=\pm 1$

Only use positive since Domain is $[0,\infty)$

$f(1)=2\sqrt{x}-x$
$=2\sqrt{1}-1$
$=1$

$\boxed{\text{b) Local max value } = 1}$

$f''(x)=\frac{d}{dx}(x^{1/2}-1)$
$=-\frac{1}{2}x^{-3/2}$
$-\frac{1}{2}x^{-3/2}=0$
$x=0$

x cannot be zero when looking for inflection points

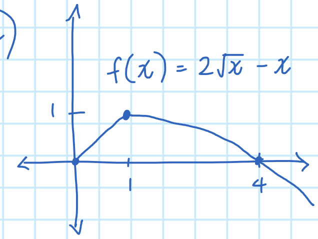
+ −
$x<1$ 1 $x>1$

Intervals	$f'(x)$
$x<1$	$f'(\frac{1}{4})=+$
$x>1$	$f'(4)=-$

$\boxed{\text{d) No inflection points}}$

a) f' is increasing @ $[0,1)$
f' is decreasing @ $[1,\infty)$

c) $f''(1)<0$
concave downwards @ $[0,\infty)$

e)


Q2

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2) $f(x) = x^3 - x$, Find $f'(x)$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$f'(x) = \lim_{h \to 0} \left[\frac{(x+h)^3 - (x+h) - (x^3 - x)}{h} \right]$$

$$= \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{-x} - h \cancel{- x^3} \cancel{+x}}{h}$$

$$= \frac{3x^2\cancel{h} + 3xh^{\cancel{2}} + h^{\cancel{3}} \cancel{-h}}{\cancel{h}} = 3x^2 + 3xh + h^2 - 1$$

$$\lim_{h \to 0} = 3x^2 + 3xh + h^2 - 1$$

$$= 3x^2 + \underbrace{\cancel{3x(0)} + \cancel{(0)^2}}_{=0} - 1$$

$$= \boxed{3x^2 - 1}$$

check

$$f'(x) = \frac{d}{dx}(x^3 - x)$$

$$= 3x^{3-1} - x^{1-1}$$

$$= 3x^2 - 1$$

Q3

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3) $f(x) = e^{\tan(2x)}$, Find derivative

$$f'(x) = \frac{d}{dx}\left[e^{\tan(2x)}\right]$$

$$= \frac{de^u}{du}\left[e^{\tan(2x)}\right]\frac{d}{dx}\left[\tan(2x)\right]$$

$$= e^{\tan(2x)}\frac{d\tan(u)}{du}\left[\tan(2x)\right]\frac{d}{dx}(2x)$$

$$= e^{\tan(2x)}\sec^2(2x)(2)$$

$$\boxed{f'(x) = 2\sec^2(2x)\,e^{\tan(2x)}}$$

Q4

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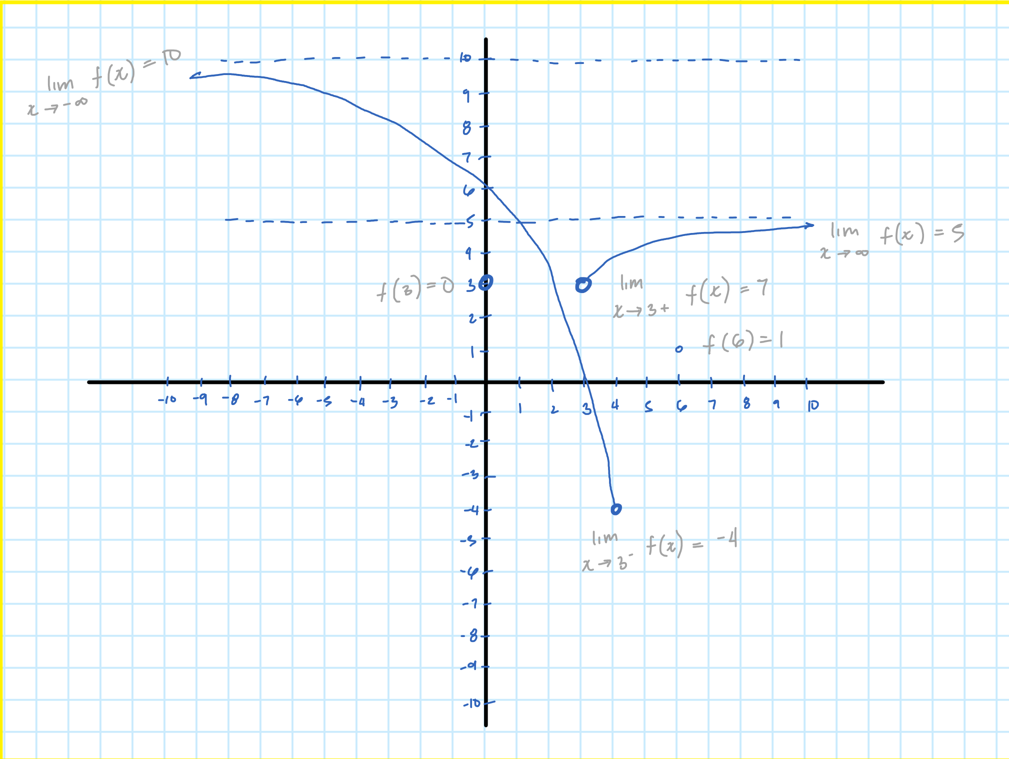
4) Sketch the graph

$\lim_{x \to 3^+} f(x) = 3$ $\lim_{x \to 3^-} f(x) = -4$ $f(3) = 0$

$\lim_{x \to \infty} f(x) = 5$ $\lim_{x \to -\infty} f(x) = 10$ $f(6) = 1$



Q5

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5) $f(x) = \begin{cases} \sin(x) & \text{if } x < 0 \\ -4x & \text{if } x \geq 0 \end{cases}$, prove $f(x)$ is continuous algebraically

$$\lim_{x \to 0^-} f(x) = \sin(x)$$
$$= \sin(0)$$
$$= 0$$

$$\lim_{x \to 0^+} f(x) = -4x$$
$$= -4(0)$$
$$= 0$$

$$\boxed{\lim_{x \to 0^-} = \lim_{x \to 0^+}}$$

$\therefore$ $f(x)$ is continuous @ $x = 0$

Q6

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6) $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$

$$\frac{\sqrt{1+x} - 1}{x} \cdot \frac{(\sqrt{1+x} + 1)}{(\sqrt{1+x} + 1)}$$

$$= \frac{1 + x - 1}{x\sqrt{1+x} + x} = \frac{\cancel{x}}{\cancel{x}\sqrt{1+x} + \cancel{x}} = \frac{1}{\sqrt{1+x} + 1}$$

$$\lim_{x \to 0} = \frac{1}{\sqrt{1+0} + 1} = \boxed{\frac{1}{2}}$$

Q7

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7) $\lim_{x \to 0} x^2 = 0$

Given $\varepsilon > 0$, $\delta > 0$ is needed

If $0 < |x - 0| < \delta$, then $|x^2 - 0| < \varepsilon$

$|x^2| < \varepsilon \iff x^2 < \varepsilon \iff |x| < \sqrt{\varepsilon}$, Then

proving that $0 |x - 0| < \delta \Rightarrow |x^2 - 0| < \varepsilon$

Q8

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8) $f(x) = 5x$, $g(x) = x^2$, $0 \le x \le 3$

Using Disk Method $V = \int \pi (R^2 - r^2)\, dx$

$$V = \int_0^3 \pi \left[(5x)^2 - (x^2)^2 \right] dx$$

$$= \pi \int_0^3 (25x^2 - x^4)\, dx = \pi \left[\frac{25x^3}{3} - \frac{x^5}{5} \right]_0^3$$

$$= F(3) - F(0)$$

$$= \pi \left[\left(\frac{25(3)^3}{3} - \frac{3^5}{5} \right) - \cancel{\left(\frac{25(0)^3}{3} - \frac{0^5}{5} \right)}_{=0} \right]$$

$$= \pi \left(225 - \frac{243}{5} \right)$$

$$= \pi \left(\frac{882}{5} \right) = \boxed{\frac{882}{5}\pi}$$

Q9

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The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

9) $$f(x) = \frac{e^x + \sin(x)}{\tan^{-1}(x)}$$

$$f'(x) = \frac{d}{dx}\left(\frac{e^x + \sin(x)}{\tan^{-1}(x)}\right)$$

$$= \frac{\tan^{-1}(x)\frac{d}{dx}\left[e^x + \sin(x)\right] - \left[e^x + \sin(x)\right]\frac{d}{dx}\left[\tan^{-1}(x)\right]}{(\tan^{-1}(x))^2}$$

$$= \frac{\tan^{-1}(x)\left[e^x + \cos(x)\right] - \left[e^x + \sin(x)\right]\left(\frac{1}{x^2+1}\right)}{(\tan^{-1}(x))^2}$$

$$\boxed{f'(x) = \frac{e^x\tan^{-1}(x) + \cos(x)\tan^{-1}(x) - \frac{e^x}{x^2+1} - \frac{\sin(x)}{x^2+1}}{(\tan^{-1}(x))^2}}$$

Q10

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10) Find $\int \frac{[\ln(x)]^2}{x}\,dx$

$u = \ln(x) \qquad \frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x}\,dx$

$$\int [\ln(x)]^2 \frac{1}{x}\,dx = \int u^2\,du$$

$$= \frac{u^3}{3} + C = \frac{[\ln(x)]^3}{3} + C = \boxed{\frac{1}{3}\ln^3(x) + C}$$

Q11

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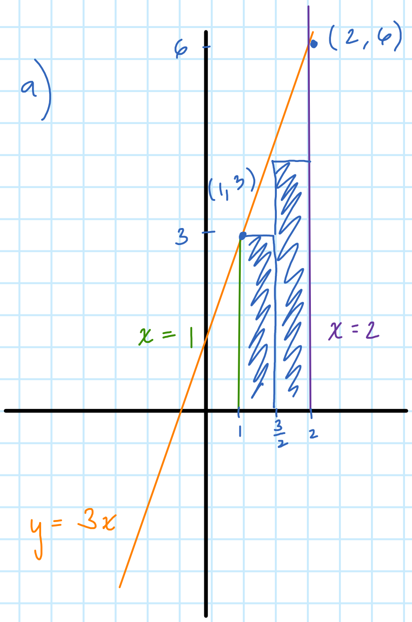
11) $y = 3x$, $x = 1$, $x = 2$

a)



b) $\Delta x = \frac{b-a}{2} = \frac{2-1}{2} = \frac{1}{2}$

$$L_2 = \sum_{i=1}^{2} f(x_{i-1}) \Delta x$$

$$= f(1)\left(\tfrac{1}{2}\right) + f\left(\tfrac{3}{2}\right)\left(\tfrac{1}{2}\right)$$

$$= \left[3(1)\left(\tfrac{1}{2}\right) + 3\left(\tfrac{3}{2}\right)\left(\tfrac{1}{2}\right)\right]$$

c) $L_2 = \frac{15}{4} = 3.75$

d) Evaluate using the Fundamental theorem of calculus

$$\int_0^2 3x \, dx = \frac{3x^2}{2} \Bigg|_0^2$$

$$= F(2) - F(0)$$

$$= \left[\frac{3(2)^2}{2} - \underbrace{\cancel{\frac{3(0)^2}{2}}}_{=0}\right]$$

$$= \frac{3(4)}{2} = 3(2) = \boxed{6}$$