Q1.1

Consider the following.
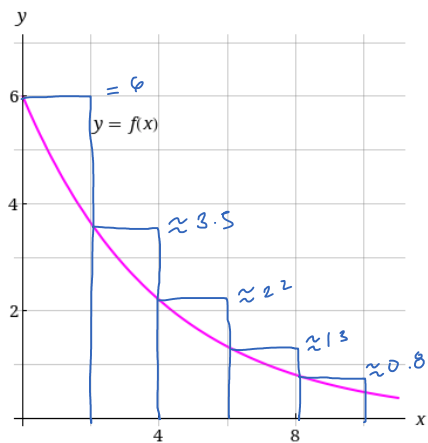

$$\Delta x = \frac{b-a}{n} = \frac{10-0}{5} = 2$$

$$R_5 = \sum_{i=1}^{5} f(x_i) \Delta x$$

$$= f(x_1)(2) + f(x_2)(2) + f(x_3)(2) + f(x_4)(2) + f(x_5)(2)$$

$$= 2\left[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) \right]$$

$$= 2(3 \cdot 7 + 2 \cdot 2 + 1 \cdot 3 + 0 \cdot 8 + 0 \cdot 5)$$

$$= \boxed{17}$$

(a) By reading values from the given graph of f, use five rectangles to find a lower estimate for the area under the given graph of f from x = 0 to x = 10. (Round your answer to one decimal place.)

17 ✔

Consider the following.

$$\Delta x = \frac{b-a}{n} = \frac{10-0}{5} = 2$$

$$L_5 = \sum_{i=1}^{5} f(x_i) \Delta x$$

$$= f(x_1)(2) + f(x_2)(2) + f(x_3)(2) + f(x_4)(2) + f(x_5)(2)$$

$$= 2\left[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) \right]$$

$$= 2(6 + 3 \cdot 5 + 2 \cdot 2 + 1 \cdot 3 + 0 \cdot 8)$$

$$27.6$$

By reading values from the given graph of f, use five rectangles to find an upper estimate for the area under the given graph of f from x = 0 to x = 10. (Round your answer to one decimal place.)
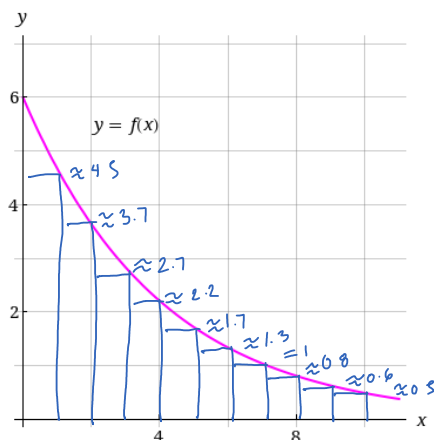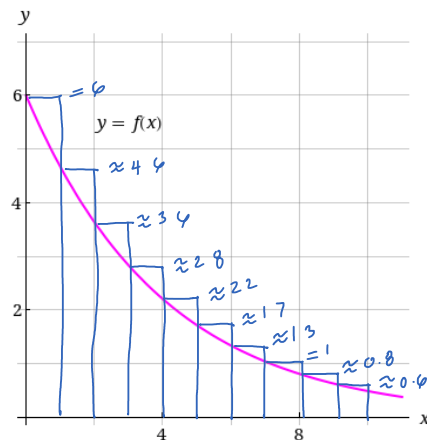
27.6 ✔

Q1.2

Consider the following.

Consider the following.

$\Delta x = \dfrac{b-a}{n} = \dfrac{10-0}{10} = 1$

$R_{10} = \sum\limits_{i=1}^{10} f(x_i)\,\Delta x$

$= f(x_1)(1) + f(x_2)(1) + f(x_3)(1) + f(x_4)(1) + f(x_5)(1) + f(x_6)(1) + f(x_7)(1) + f(x_8)(1) + f(x_9)(1) + f(x_{10})(1)$

$= 1\left[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6) + f(x_7) + f(x_8) + f(x_9) + f(x_{10}) \right]$

$= 1(4.5 + 3.7 + 2.7 + 2.2 + 1.7 + 1.3 + 1 + 0.8 + 0.6 + 0.5)$

$= \boxed{19}$

$\Delta x = \dfrac{b-a}{n} = \dfrac{10-0}{10} = 1$

$L_{10} = \sum\limits_{i=1}^{10} f(x_i)\,\Delta x$

$= f(x_1)(1) + f(x_2)(1) + f(x_3)(1) + f(x_4)(1) + f(x_5)(1) + f(x_6)(1) + f(x_7)(1) + f(x_8)(1) + f(x_9)(1) + f(x_{10})(1)$

$= 1\left[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6) + f(x_7) + f(x_8) + f(x_9) + f(x_{10}) \right]$

$= 1(6 + 4.6 + 3.6 + 2.8 + 2.2 + 1.7 + 1.3 + 1 + 0.8 + 0.6)$

$= \boxed{24.6}$

(b) Find new estimates using ten rectangles in each case. (Round your answers to one decimal place.)
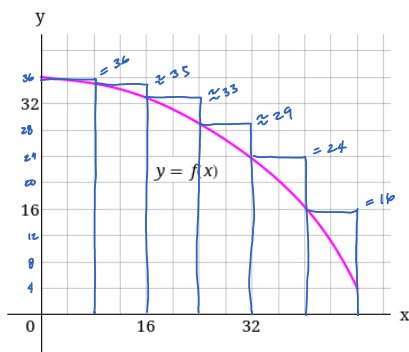
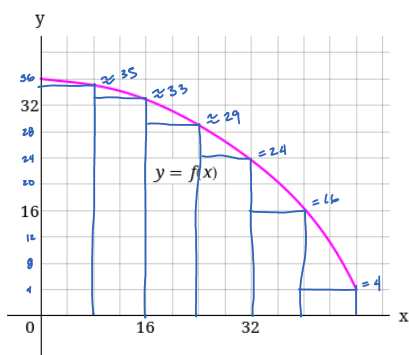| 19 | ✓ | (lower estimate) |
| 24.6 | ✓ | (upper estimate) |

Q2

Consider the following.

$y = f(x)$

$$\Delta x = \frac{b-a}{n} = \frac{48-0}{6} = 8$$

$$L_6 = \sum_{i=1}^{6} f(x_i)\Delta x$$

$$= f(x_1)(8) + f(x_2)(8) + f(x_3)(8) + f(x_4)(8) + f(x_5)(8) + f(x_6)(8)$$

$$= 8\left[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6)\right]$$

$$= 8(36 + 35 + 33 + 29 + 24 + 16)$$

$$= 1384$$

Consider the following.

$y = f(x)$

$$\Delta x = \frac{b-a}{n} = \frac{48-0}{6} = 8$$

$$R_6 = \sum_{i=1}^{6} f(x_i)\Delta x$$

$$= f(x_1)(8) + f(x_2)(8) + f(x_3)(8) + f(x_4)(8) + f(x_5)(8) + f(x_6)(8)$$

$$= 8\left[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6)\right]$$
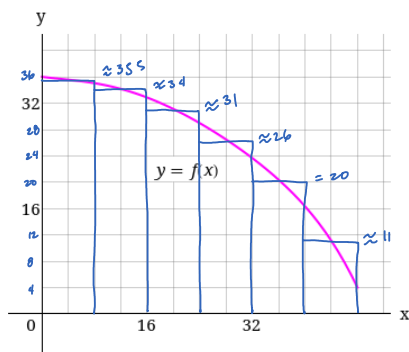
$$= 8(35 + 33 + 29 + 24 + 16 + 4)$$

$$= 1128$$

Consider the following.

$y = f(x)$

$$\Delta x = \frac{b-a}{n} = \frac{48-0}{6} = 8$$

$$M_6 = \sum_{i=1}^{6} f(x_i)\Delta x$$

$$= f(x_1)(8) + f(x_2)(8) + f(x_3)(8) + f(x_4)(8) + f(x_5)(8) + f(x_6)(8)$$

$$= 8\left[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6)\right]$$

$$= 8(35.5 + 34 + 31 + 26 + 20 + 11)$$

$$= 1260$$

(a) Use six rectangles to find estimates of each type for the area under the given graph of f from $x = 0$ to $x = 48$.

(i) Sample points are left endpoints.
$L_6 =$ 1380 ✔

(ii) Sample points are right endpoints.
$R_6 =$ 1128 ✔

(iii) Sample points are midpoints.
$M_6 =$ 1260 ✔

(b) Is L_6 an underestimate or overestimate of the true area?
- ◉ overestimate
- ○ underestimate
✔

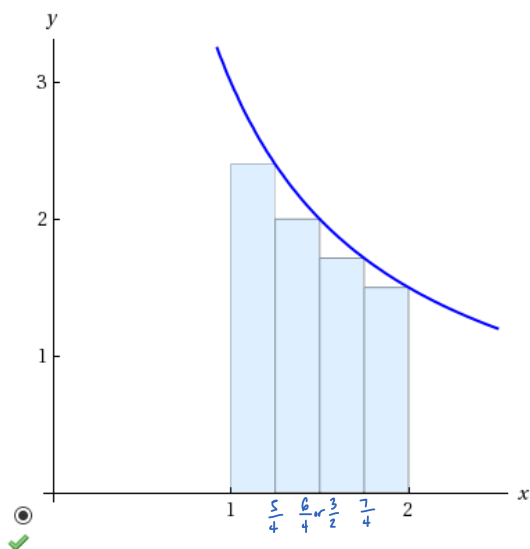(c) Is R_6 an underestimate or overestimate of the true area?
- ○ overestimate
- ◉ underestimate
✔

(d) Which of the numbers gives the best estimate?
- ◉ M_6
- ○ L_6
- ○ R_6
✔

Q3

$$f(x) = \frac{3}{x}$$

$$\Delta x = \frac{b-a}{n} = \frac{2-1}{4} = \frac{1}{4}$$

$$R_4 = \sum_{i=1}^{4} f(x_i) \Delta x$$

$$= \frac{1}{4}\left[f(x_1) + f(x_2) + f(x_3) + f(x_4) \right]$$

$$= \frac{1}{4}\left(\frac{3}{\frac{5}{4}} + \frac{3}{\frac{3}{2}} + \frac{3}{\frac{7}{4}} + \frac{3}{2} \right)$$
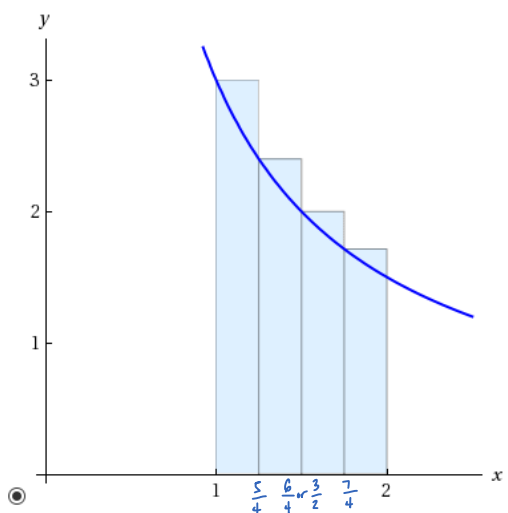
$$\approx \boxed{1.9036}$$

(a) Estimate the area under the graph of $f(x) = 3/x$ from $x = 1$ to $x = 2$ using four approximating rectangles and right endpoints. (Round your answer to four decimal places.)

1.9036 ✔

Is your estimate an underestimate or an overestimate?

◉ underestimate

○ overestimate ✔

$$f(x) = \frac{3}{x}$$

$$\Delta x = \frac{b-a}{n} = \frac{2-1}{4} = \frac{1}{4}$$

$$L_4 = \sum_{i=1}^{4} f(x_i) \Delta x$$

$$= \frac{1}{4}\left[f(x_1) + f(x_2) + f(x_3) + f(x_4) \right]$$

$$= \frac{1}{4}\left(\frac{3}{1} + \frac{3}{\frac{5}{4}} + \frac{3}{\frac{3}{2}} + \frac{3}{\frac{7}{4}} \right)$$

$$\approx \boxed{2.2786}$$

(b) Repeat part (a) using left endpoints. (Round your answer to four decimal places.)

2.2786 ✔

Is your estimate an underestimate or an overestimate?
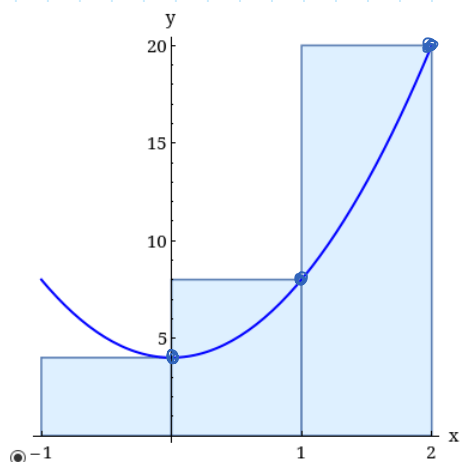
○ underestimate

◉ overestimate ✔

Q4.1

(a) Estimate the area under the graph of $f(x) = 4 + 4x^2$ from $x = -1$ to $x = 2$ using three rectangles and right endpoints.

$R_3 =$ 32 ✔

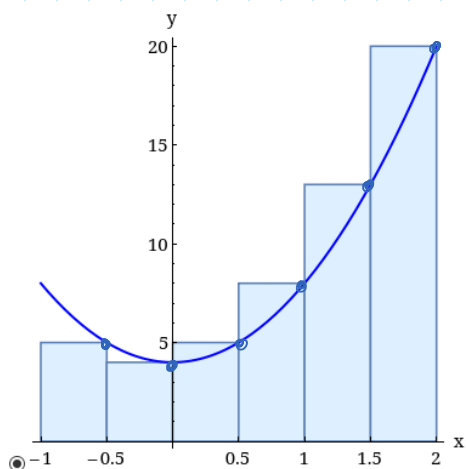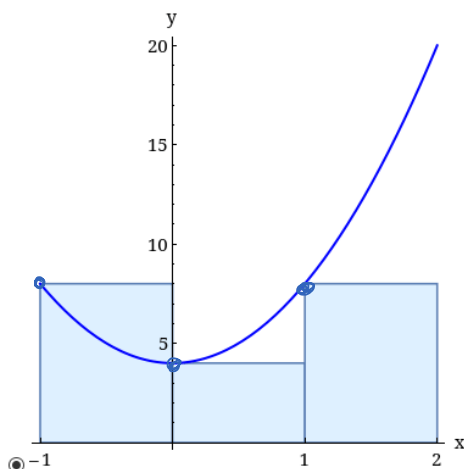Then improve your estimate by using six rectangles.

$R_6 =$ 27.5 ✔

$f(x) = 4 + 4x^2$

$\Delta x = 1$

$R_3 = \sum_{i=1}^{3} f(x_i) \Delta x$

$= 1 \left[f(0) + f(1) + f(2) \right]$

$= 1 (4 + 8 + 20)$

$= \boxed{32}$

$f(x) = 4 + 4x^2$

$\Delta x = \frac{1}{2}$

$R_6 = \sum_{i=1}^{6} f(x_i) \Delta x$

$= \frac{1}{2} \left[f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5) + f(2) \right]$

$= \frac{1}{2} (5 + 4 + 5 + 8 + 13 + 20)$

$= \boxed{27.5}$

$f(x) = 4 + 4x^2$

$\Delta x = 1$

$L_3 = \sum_{i=1}^{3} f(x_i) \Delta x$

$= 1 \left[f(-1) + f(0) + f(1) \right]$

$= 1 (8 + 4 + 8)$

$= \boxed{20}$

(b) Repeat part (a) using left endpoints.
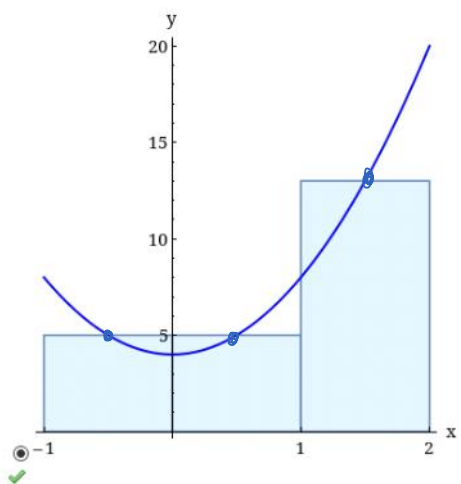
$L_3 =$ 20 ✔

$L_6 =$ 21.5 ✔

Q4.2
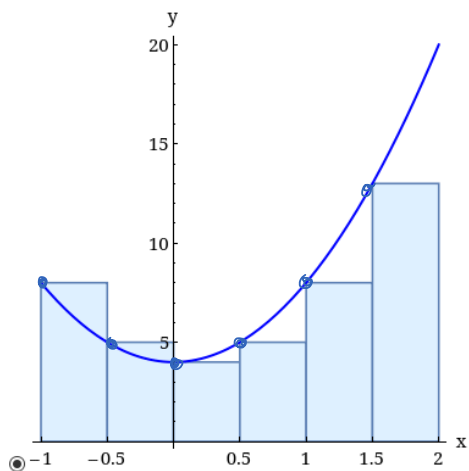
$$f(x) = 4 + 4x^2$$

$$\Delta x = \frac{1}{2}$$

$$R_6 = \sum_{i=1}^{6} f(x_i) \Delta x$$

$$= \frac{1}{2}\left[f(-1) + f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5) \right]$$

$$= \frac{1}{2}\left(8 + 5 + 4 + 5 + 8 + 13 \right)$$

$$= \boxed{21.5}$$

$$f(x) = 4 + 4x^2$$

$$\Delta x = 1$$

$$M_3 = \sum_{i=1}^{3} f(x_i) \Delta x$$

$$= 1\left[f(-0.5) + f(0.5) + f(1.5) \right]$$
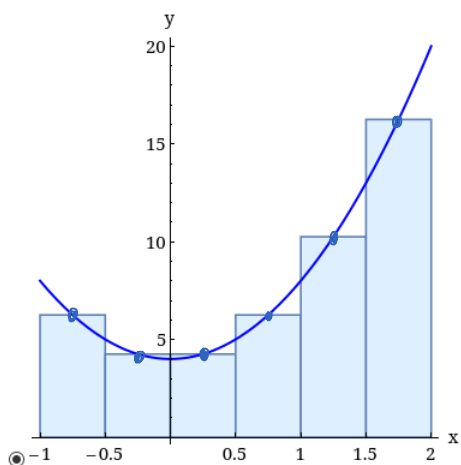
$$= 1\left(5 + 5 + 13 \right)$$

$$= \boxed{23}$$

(c) Repeat part (a) using midpoints.

$M_3 = \boxed{23}$ ✓

$M_6 = \boxed{23.75}$ ✓

$$f(x) = 4 + 4x^2$$

$$\Delta x = \frac{1}{2}$$

$$R_6 = \sum_{i=1}^{6} f(x_i) \Delta x$$

$$= \frac{1}{2}\left[f(-0.75) + f(-0.25) + f(0.25) + f(0.75) + f(1.25) + f(1.75) \right]$$

$$= \frac{1}{2}\left(6.25 + 4.25 + 4.25 + 6.25 + 10.25 + 16.25 \right)$$

$$= \boxed{23.75}$$

(d) From your sketches in parts (a)-(c), which appears to be the best estimate?

○ L_6

◉ M_6

○ R_6

✓

Q5

The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find lower and upper estimates for the distance that she traveled during these three seconds.

34.1	✔	ft (smaller value)
44.1	✔	ft (larger value)

t (s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v (ft/s)	0	6.7	9.2	14.1	18.8	19.4	20

Lower estimate   Upper estimate

Distance = rate · time

Let Δd = change in distance

$$\Delta d = 0.5$$

Lower estimate

$$= 0.5(0 + 6.7 + 9.2 + 14.1 + 18.8 + 19.4)$$

$$= \boxed{34.1 \text{ ft}}$$

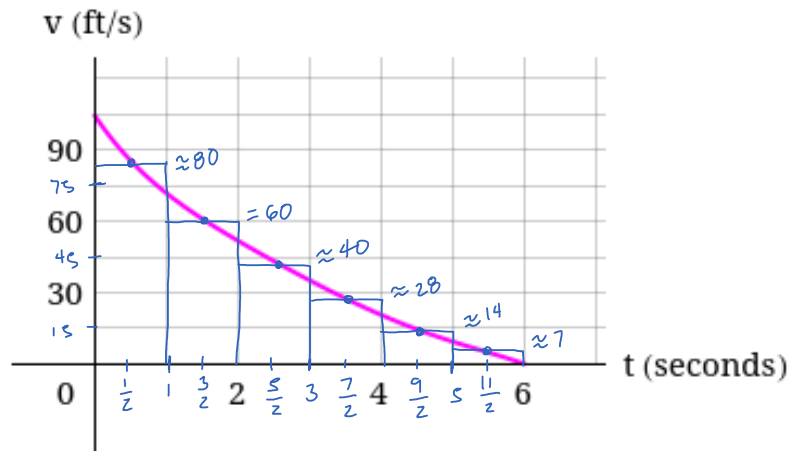Upper estimate

$$= 0.5(6.7 + 9.2 + 14.1 + 18.8 + 19.4 + 20)$$

$$= \boxed{44.1 \text{ ft}}$$

Q6

The velocity graph of a braking car is shown. Use it to estimate the distance traveled by the car while the brakes are applied. (Use M_6 to get the most precise estimate.)

229 ✔ ft

Let Δt = change in time

$$\Delta t = 1$$

$$M_6 = \sum_{i=1}^{6} f(v_i)\Delta t$$

$$= 1(80 + 60 + 40 + 28 + 14 + 7)$$

$$= \boxed{229 \text{ ft}}$$