Q1

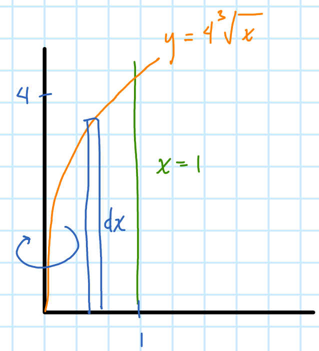
Thursday, December 3, 2020 9:21 AM

Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y-axis.

$y = 4\sqrt[3]{x}, \quad y = 0, \quad x = 1$

$\boxed{\frac{24\pi}{7}}$

$y = 4\sqrt[3]{x}, \quad y = 0, \quad x = 1$



$$V = \int_a^b 2\pi(\text{radius})(\text{height})\,dx$$

$$= 2\pi \int_a^b x(f(x))\,dx$$

$$V = 2\pi \int_0^1 x\,4\sqrt[3]{x} = 2\pi \int_0^1 4x^{4/3}\,dx$$

$$\frac{4x^{4/3+1}}{\frac{4}{3}+1} = \frac{4x^{7/3}}{\frac{7}{3}} = \frac{12x^{7/3}}{7}$$

$$= 2\pi \left[\frac{12x^{7/3}}{7}\right]_0^1$$

$$= 2\pi\left(\frac{12(1^{7/3})}{7} - \cancel{\frac{12(0^{7/3})}{7}}\right) \quad = 0$$

$$= 2\pi\left(\frac{12}{7}\right)$$

$$= \frac{24\pi}{7}$$

Q2

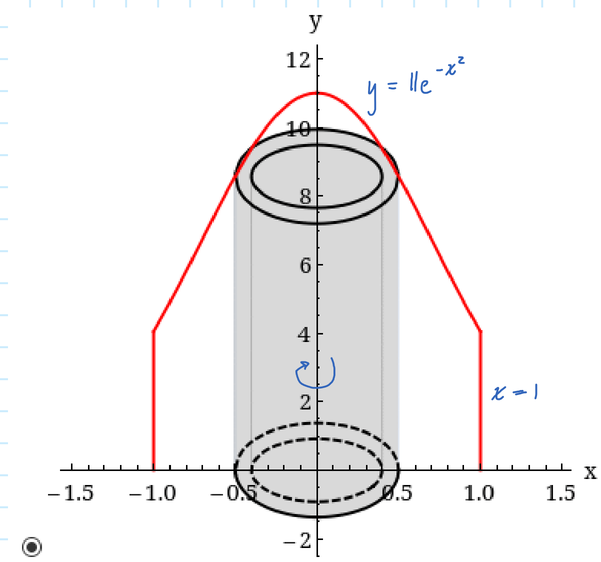
Friday, December 4, 2020 2:20 PM

Use the method of cylindrical shells to find the volume V generated by rotating the region bounded by the given curves about the y-axis.

$y = 11e^{-x^2}, \quad y = 0, \quad x = 0, \quad x = 1$

$V = -11\pi\left(\frac{1}{e} - 1\right)$

$y = 11e^{-x^2}, \; y = 0, \; x = 0, \; x = 1$



$$V = \int_a^b 2\pi(\text{radius})(\text{height})\,dx$$

$$= 2\pi \int_a^b x(f(x))\,dx$$

$u = -x^2$

$\frac{du}{dx} = -2x$

$du = -2x\,dx$

$\frac{du}{-2} = x\,dx$

$$V = 2\pi \int_0^1 x(11e^{-x^2})\,dx$$

$$= 2\pi \int_0^1 11e^u \frac{du}{-2} = -11\pi \int_0^1 e^u\,du$$

$$= -11\pi\left[e^u\right]_0^1 = \pi\left[e^{-x^2}\right]_0^1$$

$$= -11\pi\left(e^{-(1^2)} - e^{-(0^2)}\right)$$

$$= \boxed{-11\pi\left(\frac{1}{e} - 1\right)}$$

Q3

Friday, December 4, 2020 2:38 PM

Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y-axis.

$y = 4x^2, \quad y = 24x - 8x^2$

32π

$y = 4x^2, \quad y = 24x - 8x^2$

$4x^2 = 24x - 8x^2$

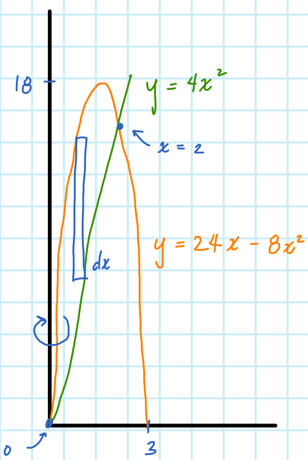
$4x^2 + 8x^2 - 24x = 0$

$12x^2 - 24x = 0$

$12x(x-2) = 0$

$x = 0 \qquad x - 2 = 0$

$x = 2$



$$V = \int_a^b 2\pi(\text{radius})(\text{height})\,dx$$

$$= 2\pi \int_a^b x(f(x))\,dx$$

$$V = 2\pi \int_0^2 x\left[(24x - 8x^2) - 4x^2\right] dx = 2\pi \int_0^2 (-12x^3 + 24x^2)\,dx$$

$$= 2\pi \int_0^2 12(-x^3 + 2x^2)\,dx = 24\pi \int_0^2 (-x^3 + 2x^2)\,dx$$

$$= 24\pi \left[-\frac{x^4}{4} + \frac{2x^3}{3}\right]_0^2$$

$$= 24\pi \left[\left(-\frac{2^4}{4} + \frac{2(2^3)}{3}\right) - \underbrace{\cancel{\left(-\frac{0^4}{4} + \frac{2(0^3)}{3}\right)}}_{=0}\right]$$

$$= 24\pi\left(-4 + \frac{16}{3}\right)$$

$$= 24\pi\left(\frac{4}{3}\right)$$

$$= \boxed{32\pi}$$

Q4

Friday, December 4, 2020 7:07 PM

Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the x-axis.

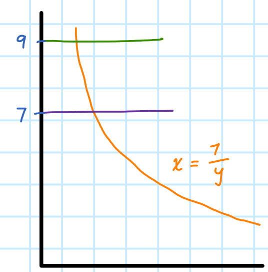
$xy = 7, \quad x = 0, \quad y = 7, \quad y = 9$

28π

$xy = 7, \; x = 0, \; y = 7, \; y = 9$

$$\downarrow$$

$$x = \frac{7}{y}$$



$$V = \int_a^b 2\pi(\text{radius})(\text{height})\,dy$$

$$= 2\pi \int_a^b y(f(y))\,dy$$

$$V = 2\pi \int_7^9 y\,\frac{7}{y}\,dy = 2\pi \int_7^9 7\,dy$$

$$= 2\pi \Big[7y\Big]_7^9$$

$$= 2\pi\left[7(9) - 7(7)\right]$$

$$= 2\pi(63 - 49) = 2\pi(14)$$

$$= \boxed{28\pi}$$

Q5

Friday, December 4, 2020 7:30 PM

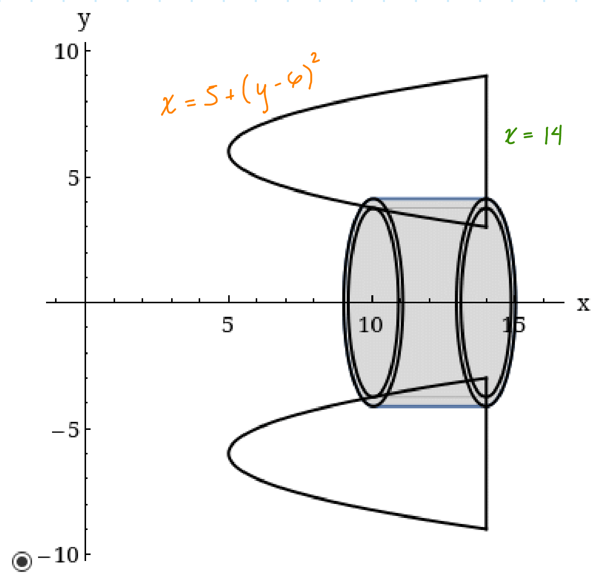
Use the method of cylindrical shells to find the volume V of the solid obtained by rotating the region bounded by the given curves about the x-axis.

$x = 5 + (y - 6)^2, \quad x = 14$

$V = 432\pi$

$$x = 5 + (y-6)^2, \quad x = 14$$

$$5 + (y-6)^2 = 14$$
$$5 - 14 + (y^2 - 12y + 36) = 0$$
$$y^2 - 12y + 27 = 0$$
$$(y-9)(y-3) = 0$$
$$y = 9 \qquad y = 3$$



$$V = \int_a^b 2\pi(\text{radius})(\text{height})\, dy$$
$$= 2\pi \int_a^b y(f(y))\, dy$$

$$V = 2\pi \int_3^9 y\left[14 - (5 + (y-6)^2)\right] dy = 2\pi \int_3^9 y(-y^2 + 12y - 27)\, dy$$

$$= 2\pi \int_3^9 (-y^3 + 12y^2 - 27y)\, dy = 2\pi \left[-\frac{y^4}{4} + 4y^3 - \frac{27y^2}{2} \right]_3^9$$

$$= 2\pi \left[\left(-\frac{9^4}{4} + 4(9^3) - \frac{27(9^2)}{2} \right) - \left(-\frac{3^4}{4} + 4(3^3) - \frac{27(3^2)}{2} \right) \right]$$

$$= 2\pi \left(\frac{729}{4} - \left(-\frac{135}{4} \right) \right) = 2\pi(216)$$

$$= \boxed{432\pi}$$

Q6

Friday, December 4, 2020 8:03 PM

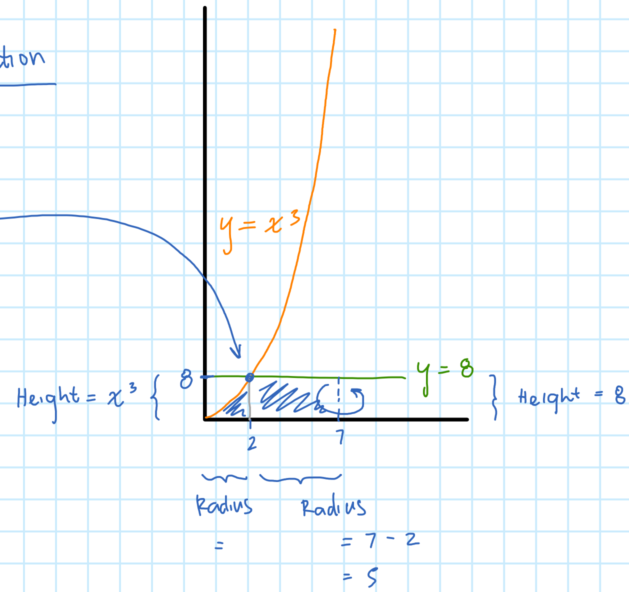
$y = x^3$, $y = 8$, $x = 0$; about $x = 7$

<u>$y = x^3$ and $y = 8$ intersection</u>

$x^3 = 8$

$x = \sqrt[3]{8}$

$x = 2$



$V = \int_a^b 2\pi(\text{radius})(\text{height})\,dx$

Q7

Sunday, December 6, 2020 8:46 PM

Use the method of cylindrical shells to find the volume V generated by rotating the region bounded by the given curves about the specified axis.

$y = 9x - x^2$, $y = 18$; about $x = 3$

$V = \boxed{\frac{27\pi}{2}}$

$y = 9x - x^2$, $y = 18$; about $x = 3$

Parabola

$y = 9x - x^2 = -x^2 + 9x$ (a, b)

x-vertex: $\frac{-b}{2a} = \frac{-9}{2(-1)} = \frac{9}{2}$

Plug in x

y-vertex $= 9\left(\frac{9}{2}\right) - \left(\frac{9}{2}\right)^2$

$= \frac{81}{2} - \frac{81}{4} = \frac{81}{4} \approx 20.25$

x-intercept: $9x - x^2 = 0$

$x(9-x) = 0$

$x = 0$ $x = 9$

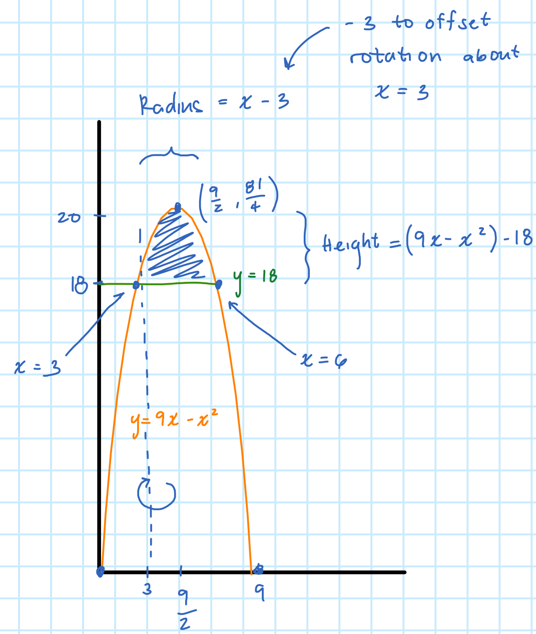
$y = 9x - x^2$ and $y = 18$ intersection points

$9x - x^2 = 18$

$0 = x^2 - 9x + 18$

$0 = (x-3)(x-6)$

$x = 3$ $x = 6$



$$V = \int_a^b 2\pi(\text{radius})(\text{height})\, dx$$

$$= 2\pi \int_3^6 (x-3)\left[(9x - x^2) - 18\right] dx = 2\pi \int_3^6 (-x^3 + 12x^2 - 45x + 54)\, dx$$

$$= 2\pi \left[-\frac{x^4}{4} + 4x^3 - \frac{45x^2}{2} + 54x\right]_3^6$$

$$= 2\pi \left[\left(-\frac{6^4}{4} + 4(6^3) - \frac{45(6^2)}{2} + 54(6)\right) - \left(-\frac{3^4}{4} + 4(3^3) - \frac{45(3^2)}{2} + 54(3)\right)\right]$$

$$= 2\pi\left(\frac{27}{4}\right) = \boxed{\frac{27\pi}{2}}$$

Q8

Sunday, December 6, 2020 10:04 PM

$x = 5y^2,\ y \geq 0,\ x = 5$; about $y = 2$

$x = \overset{a}{5}y^2$

y-vertex : $\frac{-b}{2(a)} = \frac{-0}{2(5)} = 0$

x-vertex : $5(0^2) = 0$

$x = 5y^2,\ y = 2$ intersection

$x = 5y^2$

$\frac{x}{5} = y^2$

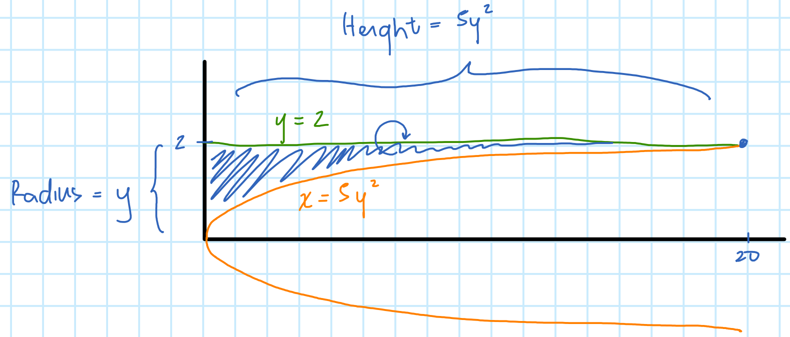
$\sqrt{\frac{x}{5}} = y$

$\sqrt{\frac{x}{5}} = 2$

$\frac{x}{5} = 2^2$

$x = 4(5)$

$\underline{x = 20}$



$V = \int_a^b 2\pi(\text{radius})(\text{height})\,dy$

$= 2\pi \int_0^2 y(5y^2)\,dy$