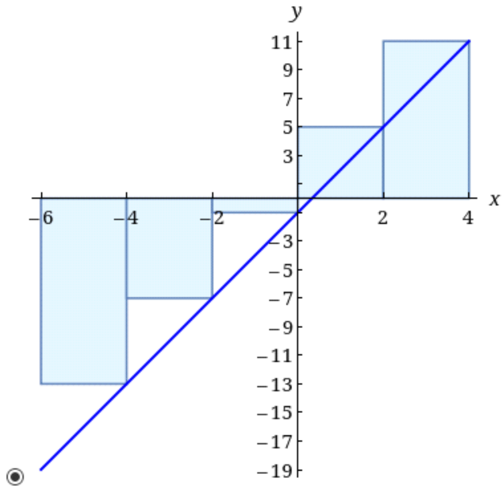
Q1

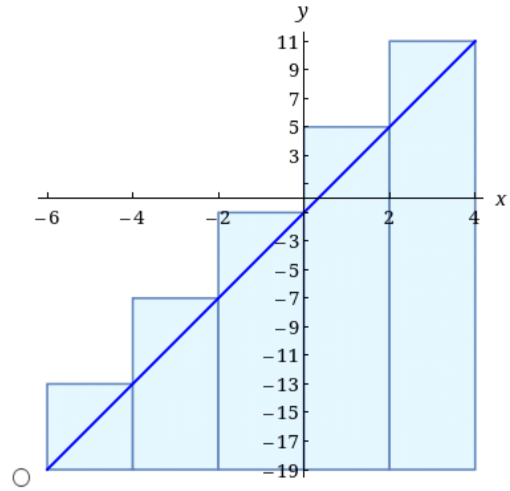
Wednesday, November 11, 2020 10:36 PM

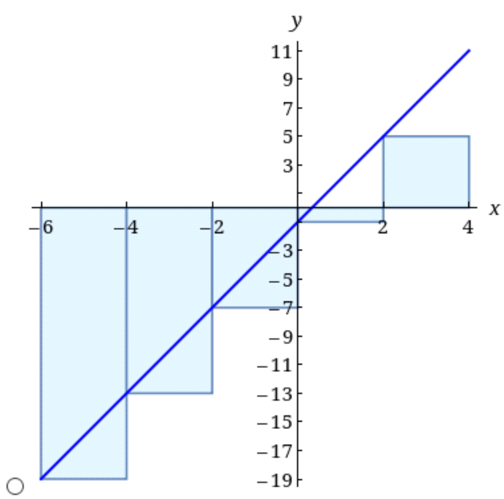
Evaluate the Riemann sum for $f(x) = 3x - 1$, $-6 \le x \le 4$, with five subintervals, taking the sample points to be right endpoints.

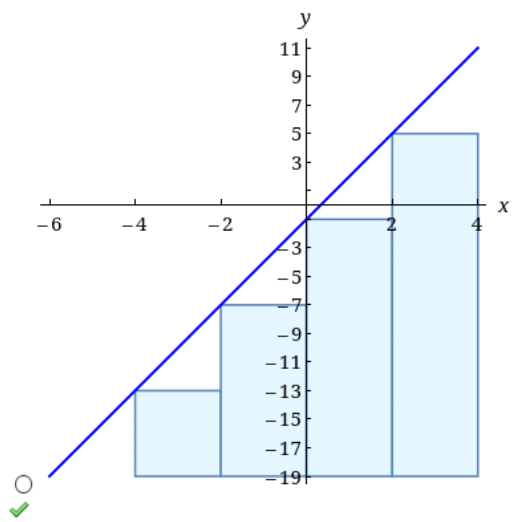
-10 ✓

Explain, with the aid of a diagram, what the Riemann sum represents.









✓

Explain.

The Riemann sum represents the net area of the rectangles with respect to the x-axis ✓ .

$f(x) = 3x - 1, \quad -6 \le x \le 4,$ five subintervals

$$\Delta x = \frac{4-(-6)}{5} = \frac{10}{5} = 2$$

$$R_1 = \sum_{i=1}^{5} f(x_i)\Delta x = \sum_{i=1}^{5} f(2i-6)\,2$$

$$f(-6+2(1))\,2 = [3(-4)-1]\,2 = (-13)\,2 = -26$$

$$f(-6+2(2))\,2 = [3(-2)-1]\,2 = (-7)\,2 = -14$$

$$f(-6+2(3))\,2 = [3(0)-1]\,2 = (-1)\,2 = -2$$

$$f(-6+2(4))\,2 = [3(2)-1]\,2 = (5)\,2 = 10$$

$$f(-6+2(5))\,2 = [3(4)-1]\,2 = (11)\,2 = 22$$

$$= \boxed{-10}$$

Q2

Thursday, November 12, 2020 11:05 AM

A table of values of an increasing function f is shown. Use the table to find lower and upper estimates for $\int_{10}^{30} f(x)\,dx$.

lower estimate -56

upper estimate 12

x	10	14	18	22	26	30
$f(x)$	−10	−9	−1	2	4	7

$$\Delta x = \frac{b-a}{n} = \frac{30-10}{5} = \frac{20}{5} = 4$$

Lower:

$$L_5 = \sum_{i=1}^{5} f(x_{i-1})\Delta x$$

$$= [f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4)]4$$

$$= (-10 - 9 - 1 + 2 + 4)4$$

$$= -56$$

Upper:

$$R_5 = \sum_{i=1}^{5} f(x_i)\Delta x$$

$$= [f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)]4$$

$$= (-9 - 1 + 2 + 4 + 7)4$$

$$= 12$$

Q3

Thursday, November 12, 2020 11:24 AM

Use the Midpoint Rule with the given value of n to approximate the integral. Round the answer to four decimal places.

$$\int_0^{96} \sin(\sqrt{x})\, dx, \quad n = 4$$

15.7087

Midpoint Rule

$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} f(\bar{x}_i)\,\Delta x = \Delta x\,[f(\bar{x}_1) + \cdots + f(\bar{x}_n)]$$

where $\Delta x = \dfrac{b-a}{n}$

and $\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i) = \text{midpoint of } [x_{i-1}, x_i]$

$$\int_0^{96} \sin(\sqrt{x})\,dx, \quad n = 4$$

$$\Delta x = \frac{b-a}{n} = \frac{96-0}{4} = 24$$

endpoints = 0, 24, 48, 72, 96

mid points = 12, 36, 60, 84

$$\int_0^{96} \sin(\sqrt{x})\,dx = \sum_{i=1}^{4} f(\bar{x}_i)\Delta x$$

$$= \left[\sin(\sqrt{12}) + \sin(\sqrt{36}) + \sin(\sqrt{60}) + \sin(\sqrt{84})\right] 24$$

$$\approx 15.7087$$

Q4

Thursday, November 12, 2020 11:39 AM

Use the form of the definition of the integral given in the theorem to evaluate the integral.

$$\int_4^6 (32 - 8x)\,dx$$

-16 ✓

Theorem If f is integrable on $[a, b]$, then

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\,\Delta x$

$$\int_4^6 (32-8x)\,dx$$

$$\Delta x = \frac{b-a}{n} = \frac{6-4}{n} = \frac{2}{n}$$

$$x_i = a + i\Delta x = 4 + i\frac{2}{n} = 4 + \frac{2i}{n}$$

$$\int_4^6 (32-8x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(4+\frac{2i}{n}\right)\frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[32-8\left(4+\frac{2i}{n}\right)\right]\frac{2}{n}$$

$$\left[32-8\left(4+\frac{2i}{n}\right)\right]\frac{2}{n}$$

$$= \left(32-32-\frac{16i}{n}\right)\frac{2}{n}$$

$$= \left(-\frac{16i}{n}\right)\frac{2}{n}$$

$$= \left(-\frac{32}{n^2}\right)i$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(-\frac{32}{n^2}\right)i$$

$$= \lim_{n\to\infty} \left(-\frac{32}{n^2}\right)\sum_{i=1}^{n} i$$

$$= \lim_{n\to\infty} \left(-\frac{32}{n^2}\right)\left[\frac{n(n+1)}{2}\right]$$

$$\left(-\frac{32}{n^2}\right)\left[\frac{n(n+1)}{2}\right]$$

$$= -\frac{n(n+1)32}{2n^2}$$

$$= -\frac{(n+1)16}{n}$$

$$= -16\left(1+\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} -16\left(1+\frac{1}{n}\right)$$

$$= -16\lim_{n\to\infty}\left(1+\frac{1}{n}\right)$$

Becomes zero as n approaches infinity

$$= -16(1)$$

$$= \boxed{-16}$$

Q5

Thursday, November 12, 2020 12:04 PM

Use the form of the definition of the integral given in the theorem to evaluate the integral.

$$\int_{-2}^{0} (8x^2 + 8x)\, dx$$

16/3

Theorem If f is integrable on $[a, b]$, then

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\,\Delta x$

$$\int_{-2}^{0} (8x^2 + 8x)\, dx$$

$$\Delta x = \frac{b-a}{n} = \frac{0-(-2)}{n} = \frac{2}{n}$$

$$x_i = a + i\Delta x = -2 + i\frac{2}{n} = -2 + \frac{2i}{n}$$

$$\int_{-2}^{0} (8x^2 + 8x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(-2 + \frac{2i}{n}\right)\frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[8\left(-2+\frac{2i}{n}\right)^2 + 8\left(-2+\frac{2i}{n}\right)\right]\frac{2}{n}$$

$$8\left(-2+\frac{2i}{n}\right)^2 = 8\left(-2+\frac{2i}{n}\right)\left(-2+\frac{2i}{n}\right) = 8\left(4 - \frac{4i}{n} - \frac{4i}{n} + \frac{4i^2}{n^2}\right) = 8\left(4 - \frac{8i}{n} + \frac{4i^2}{n^2}\right) = 32 - \frac{64i}{n} + \frac{32i^2}{n^2}$$

$$8\left(-2+\frac{2i}{n}\right) = -16 + \frac{16i}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(32 - \frac{64i}{n} + \frac{32i^2}{n^2} - 16 + \frac{16i}{n}\right)\frac{2}{n}$$

$$= \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n} \left(16 - \frac{48i}{n} + \frac{32i^2}{n^2}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n}\left(16\sum_{i=1}^{n} 1 - \frac{48}{n}\sum_{i=1}^{n} i + \frac{32}{n^2}\sum_{i=1}^{n} i^2\right)$$

$$= \lim_{n\to\infty} \frac{2}{\cancel{n}}\left[16(\cancel{n}) - \frac{48}{n}\left(\frac{\cancel{n}(n+1)}{\cancel{2}}\right) + \frac{32}{n^2}\left(\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}}\right)\right]$$

$$= \lim_{n\to\infty} \left[32 - \frac{48}{n}(n+1) + \frac{32}{n^2}\left(\frac{(n+1)(2n+1)}{3}\right)\right]$$

$$\frac{32}{n^2}\left(\frac{(n+1)(2n+1)}{3}\right) = \frac{32}{3}\left[\frac{1}{n}(n+1)\cdot\frac{1}{n}(2n+1)\right] = \frac{32}{3}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} \left[32 - 48 + \underbrace{\frac{\cancel{48}}{\cancel{n}} + \frac{32}{3}\left(1+\cancel{\frac{1}{n}}\right)\left(2+\cancel{\frac{1}{n}}\right)}\right]$$

cancel out all $\frac{k}{n}$ since they become 0 as n approaches ∞

$$= \lim_{n\to\infty} \left[32 - 48 + \frac{32}{3}(1)(2)\right] = \left(-16 + \frac{64}{3}\right) = \boxed{\frac{16}{3}}$$

Q6

Thursday, November 12, 2020 1:35 PM

Express the integral as a limit of Riemann sums. Do not evaluate the limit.

$$\int_4^6 \sqrt{3 + x^2}\, dx$$

$$\lim_{n \to \infty} \sum_{i=1}^{n} \left(\boxed{\left(\sqrt{3 + \left(4 + \frac{2i}{n}\right)^2}\right)\frac{2}{n}} \right)$$

$$\int_4^6 \sqrt{3+x^2}\, dx = \lim_{n\to\infty} \sum_{i=1}^{n}$$

Theorem If f is integrable on $[a, b]$, then

$$\int_a^b f(x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

where $\Delta x = \dfrac{b-a}{n}$ and $x_i = a + i\,\Delta x$

$f(x) = \sqrt{3+x^2}$ $\quad a = 4,\ b = 6$

$\Delta x = \dfrac{b-a}{n} = \dfrac{6-4}{n} = \dfrac{2}{n}$

$x_i = a + i\Delta x = 4 + i\dfrac{2}{n} = 4 + \dfrac{2i}{n}$

$$\int_4^6 \sqrt{3+x^2}\, dx = \boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \left[\sqrt{3+\left(4+\frac{2i}{n}\right)^2}\right]\frac{2}{n}}$$

Q7

Thursday, November 12, 2020 1:47 PM

The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.

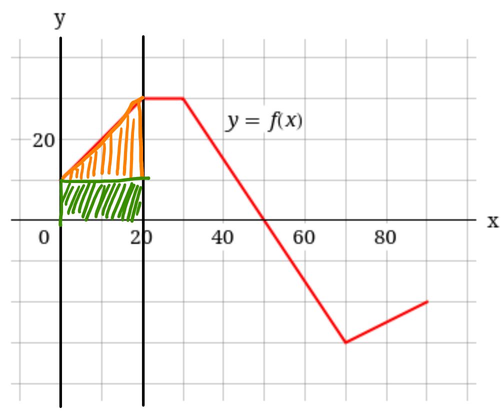
a) $\int_0^{20} f(x)\,dx$

$= \text{Area}(\square) + \text{Area}(\triangle)$

$= (20)(10) + \frac{1}{2}(20)(20)$

$= 200 + 200$

$= \boxed{400}$



b) $\int_0^{50} f(x)\,dx$

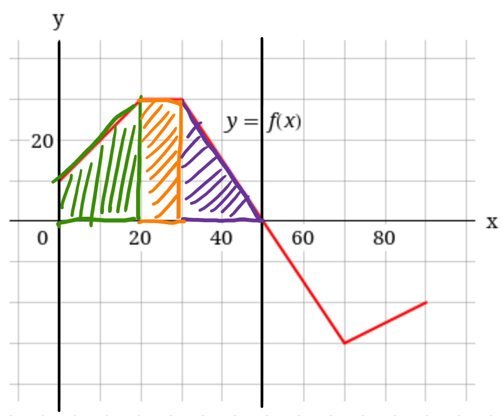
$= \int_0^{20} f(x)\,dx + \int_{20}^{30} f(x)\,dx + \int_{30}^{50} f(x)\,dx$

$= \text{Area}(\triangle) + \text{Area}(\square) + \text{Area}(\triangle)$

$= 400 + (10)(30) + \frac{1}{2}(20)(30)$

$= 400 + 300 + 300$

$= \boxed{1000}$

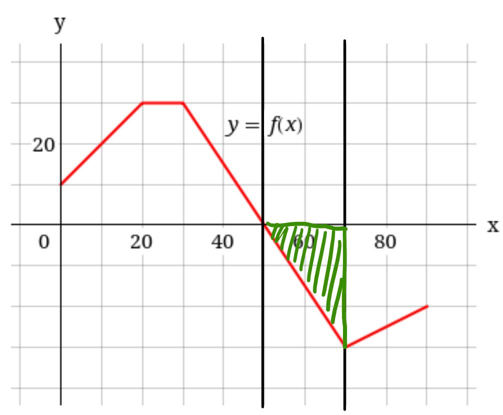


c) $\int_{50}^{70} f(x)\,dx$

$= -\text{Area}(\triangle)$

$= -\frac{1}{2}(20)(30)$

$= \boxed{-300}$



d) $\int_0^{90} f(x)\,dx$

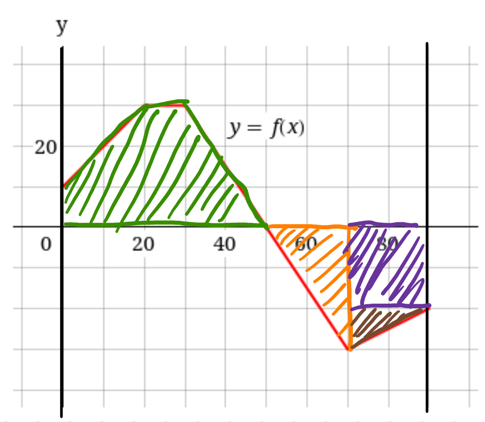
$= \int_0^{50} f(x)\,dx + \int_{50}^{70} f(x)\,dx + \int_{70}^{90} f(x)\,dx$

$= 1000 - 300 - \text{Area}(\square) - \text{Area}(\triangle)$

$= 700 - (20)(20) - \frac{1}{2}(20)(10)$

$= 700 - 400 - 100$

$= \boxed{200}$



(a) $\int_0^{20} f(x)\,dx$

400

(b) $\int_0^{50} f(x)\,dx$

1000

(c) $\int_{50}^{70} f(x)\,dx$

-300

(d) $\int_0^{90} f(x)\,dx$

200

Q8

Thursday, November 12, 2020 2:17 PM

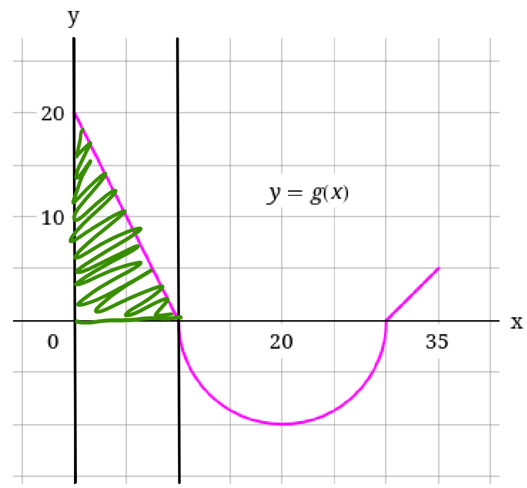
The graph of g consists of two straight lines and a semicircle. Use it to evaluate each integral.

a) $\int_0^{10} g(x)\,dx$

$= \text{Area}(\triangle)$

$= \frac{1}{2}(10)(20)$

$= \boxed{100}$



(a) $\int_0^{10} g(x)\,dx$

100

(b) $\int_{10}^{30} g(x)\,dx$

-50π

(c) $\int_0^{35} g(x)\,dx$

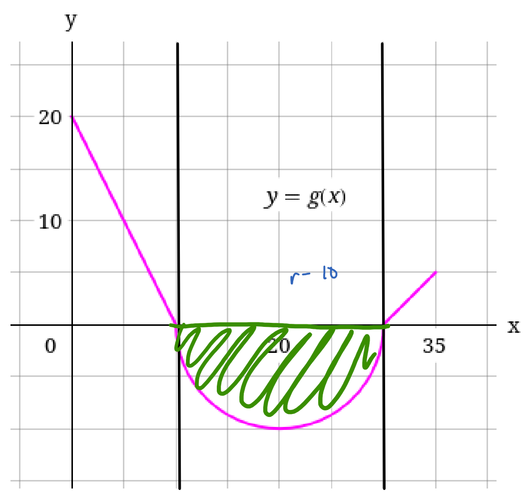
$-50\pi + \frac{225}{2}$

b) $\int_{10}^{30} g(x)\,dx$

$= -\text{Area}(\ominus)$

$= -\frac{\pi r^2}{2} = -\frac{\pi(10)^2}{2} = -\frac{10^2}{2}\pi$

$= \boxed{-50\pi}$



c) $\int_0^{35} g(x)\,dx$

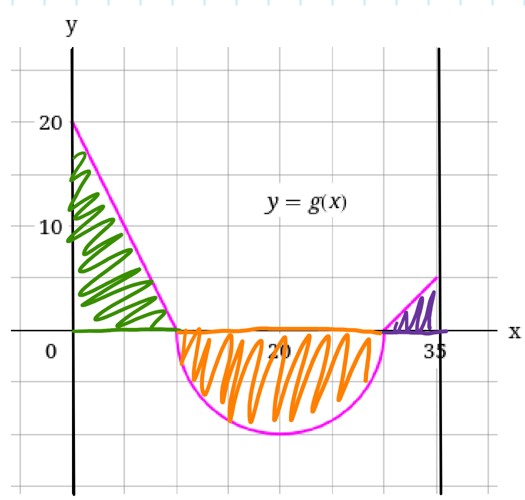
$= \int_0^{10} g(x)\,dx + \int_{10}^{30} g(x)\,dx + \int_{30}^{35} g(x)\,dx$

$= 100 - 50\pi + \text{Area}(\triangle)$

$= 100 - 50\pi + \frac{1}{2}(5)(5)$

$= 100 - 50\pi + \frac{25}{2}$

$= \boxed{-50\pi + \frac{225}{2}}$



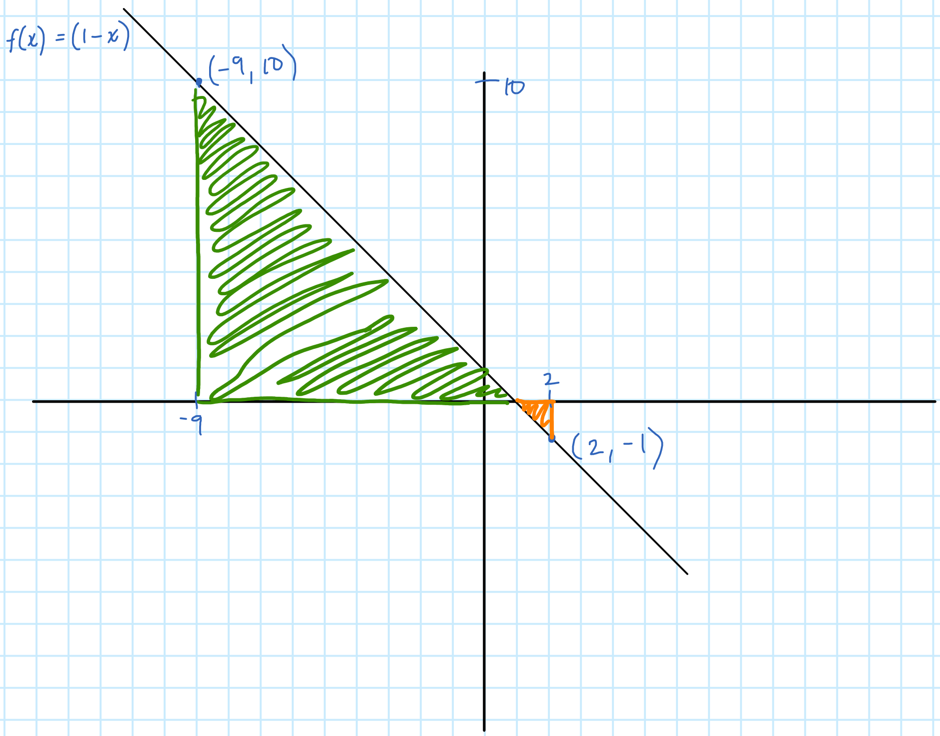
Q9

Thursday, November 12, 2020 2:43 PM

Evaluate the integral by interpreting it in terms of areas.

$$\int_{-9}^{2} (1 - x)\, dx$$

$\frac{99}{2}$



$$\int_{-9}^{2} (1 - x)\, dx$$

$$= \int_{-9}^{1} (1-x)\, dx + \int_{1}^{2} (1-x)\, dx$$

$= \text{Area}(\triangle) - \text{Area}(\triangledown)$

$= \frac{1}{2}(10)(10) - \frac{1}{2}(1)(1)$

$= 50 - \frac{1}{2}$

$= \boxed{\frac{99}{2}}$

Q10

Thursday, November 12, 2020 2:56 PM

Find $\int_0^{10} f(x)\,dx$ if

$$f(x) = \begin{cases} 6 & \text{for } x < 6 \\ x & \text{for } x \geq 6 \end{cases}.$$

68

Find $\int_0^{10} f(x)\,dx$ if

$$f(x) = \begin{cases} 6 & \text{for } x < 6 \\ x & \text{for } x \geq 6 \end{cases}$$

$$= \int_0^5 6\Delta x + \int_5^{10} x\Delta x$$

$$= \text{Area}(\square) + \text{Area}(\square) + \text{Area}(\triangle)$$

$$= (6)(6) + (4)(6) + \tfrac{1}{2}(4)(4)$$

$$= 36 + 24 + 8$$

$$= \boxed{68}$$