Q1

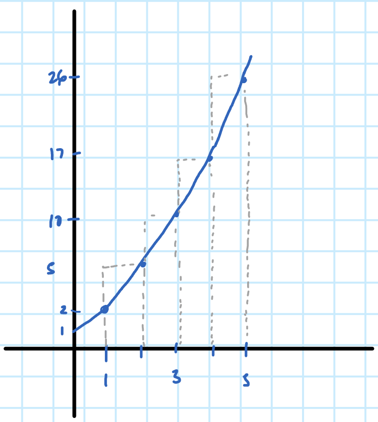
Sunday, November 15, 2020 11:48 AM

Let $f(x)=x^2+1$

Estimate the area under the graph of f(x) from 1 to 5 using 4 rectangles and right endpoints. Sketch the graph with the rectangles.

Now set up the Riemann sum and evaluate the limit to find the exact area.

DO NOT USE THE FUNDIMENIL THEORM OF CALCULUS. Though you could use it check your answer if you want.



$$\Delta x = \frac{b-a}{4} = \frac{5-1}{4} = 1$$

$$R_4 = \sum_{i=1}^{4} f(x_i)\Delta x$$

$$= f(x_1)(1) + f(x_2)(1) + f(x_3)(1) + f(x_4)(1)$$

$$= 1(2+5+10+26)$$

$$= \boxed{43}$$

$$\int_1^5 (x^2+1)\,dx$$

$$\Delta x = \frac{b-a}{n} = \frac{5-1}{n} = \frac{4}{n}$$

$$x_i = a + i\Delta x = 1 + i\frac{4}{n} = 1 + \frac{4i}{n}$$

$$\int_1^5 (x^2+1)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(1+\frac{4i}{n}\right)\frac{4}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(1+\frac{4i}{n}\right)^2 + 1\right]\frac{4}{n}$$

$$\left(1+\frac{4i}{n}\right)\left(1+\frac{4i}{n}\right)$$

$$= 1 + \frac{4i}{n} + \frac{4i}{n} + \frac{16i^2}{n^2}$$

$$= \frac{16i^2}{n^2} + \frac{8i}{n} + 1$$

$$= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} \left(\frac{16i^2}{n^2} + \frac{8i}{n} + 2\right)$$

$$= \lim_{n\to\infty} \frac{4}{\cancel{n}} \left[\frac{\overset{8}{\cancel{16}}}{n^2}\left(\frac{\cancel{n}(n+1)(2n+1)}{\underset{3}{\cancel{6}}}\right) + \frac{\overset{4}{\cancel{8}}}{n}\left(\frac{\cancel{n}(n+1)}{\cancel{2}}\right) + 2(\cancel{n})\right]$$

$$\frac{32}{n^2}\left[\frac{(n+1)(2n+1)}{3}\right]$$

$$= \frac{32}{3}\left[\frac{1}{n}(n+1)\cdot\frac{1}{n}(2n+1)\right]$$

$$= \frac{32}{3}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} \left[\frac{32(n+1)(2n+1)}{n^2 \cdot 3} + \frac{16(n+1)}{n} + 8\right]$$

$$= \lim_{n\to\infty} \left[\frac{32}{3}\left(1+\cancel{\frac{1}{n}}\right)\left(2+\cancel{\frac{1}{n}}\right) + 16 + \cancel{\frac{16}{n}} + 8\right] = \frac{32}{3}(1)(2) + 16 + 8 = \boxed{\frac{136}{3} \approx 45.3}$$

cancel out $\frac{1}{n}$s since they equals to zero as $n \to \infty$