Q1

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Math 150 Quiz 2

Find the following derivative. You must show all of your work. Once you have taken the derivative you do not have to simplify.

1) Let $f(x) = \frac{x^3+\cos(x)-1}{e^x-3}$. Find $f'(x)$.
2) Let $f(x) = x^2\sin(x)$. Find $f''(x)$.
3) Let $f(x) = x^5e^x\sec(x)$. Find $f'(x)$.

The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

1.) Let $f(x) = \frac{x^3+\cos(x)-1}{e^x-3}$. Find $f'(x)$

$f(x) = x^3+\cos(x)-1$

$g(x) = e^x - 3$

$$f'(x) = \frac{(e^x-3)\frac{d}{dx}\left[x^3+\cos(x)-1\right] - \left[x^3+\cos(x)-1\right]\frac{d}{dx}(e^x-3)}{(e^x-3)^2}$$

$$= \frac{(e^x-3)\left[3x^2-\sin(x)\right] - \left[x^3+\cos(x)-1\right](e^x)}{(e^x-3)^2}$$

$$\boxed{f'(x) = \frac{(e^x-3)\left[3x^2-\sin(x)\right] - e^x\left[x^3+\cos(x)-1\right]}{(e^x-3)^2}}$$

Q2

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The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

2.) Let $f(x) = x^2 \sin(x)$. Find $f''(x)$

$f(x) = x^2$
$g(x) = \sin(x)$

$$f'(x) = x^2 \sin(x)$$
$$= x^2 \frac{d}{dx}[\sin(x)] + \sin(x)\frac{d}{dx}(x^2)$$
$$= x^2 \cos(x) + \sin(x)(2x)$$

$$f'(x) = x^2 \cos(x) + 2x \sin(x)$$

$$f''(x) = \frac{d}{dx}[x^2 \cos(x)] + \frac{d}{dx}[2x \sin(x)]$$
$$= \left[x^2 \frac{d}{dx}[\cos(x)] + \cos(x)\frac{d}{dx}(x^2)\right] + \left[2x \frac{d}{dx}[\sin(x)] + \sin(x)\frac{d}{dx}(2x)\right]$$
$$= \left[-x^2 \sin(x) + \cos(x)(2x)\right] + \left[2x\cos(x) + \sin(x)(2)\right]$$

$$\boxed{f''(x) = \left[-x^2 \sin(x) + 2x\cos(x)\right] + \left[2x\cos(x) + 2\sin(x)\right]}$$

Q3

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The Product Rule

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)\frac{d}{dx}\left[g(x)\right] + g(x)\frac{d}{dx}\left[f(x)\right]$$

3) Let $f(x) = x^5 e^x \sec(x)$. Find $f'(x)$

$$f(x) = x^5$$

$$g(x) = e^x \sec(x)$$

$$f'(x) = x^5 \frac{d}{dx}\left[e^x \sec(x)\right] + e^x \sec(x)\frac{d}{dx}(e^x)$$

$$= x^5\left[e^x \frac{d}{dx}(\sec(x)) + \sec(x)\frac{d}{dx}(e^x)\right] + e^x \sec(x)\frac{d}{dx}(e^x)$$

$$= x^5\left[e^x(\sec(x)\tan(x)) + \sec(x)(e^x)\right] + e^x \sec(x)(e^x)$$

$$= x^5\left[e^x \sec(x)\tan(x) + e^x \sec(x)\right] + e^x \sec(x)(e^x)$$

$$= x^5\left[e^x \sec(x)\left(\tan(x)+1\right)\right] + e^x \sec(x)(e^x)$$

$$= x^5 e^x \sec(x)\left(\tan(x)+1\right) + e^x \sec(x)(e^x)$$

$$\boxed{f'(x) = e^x \sec(x)\left[x^2\left(\tan(x)+1\right) + e^x\right]}$$