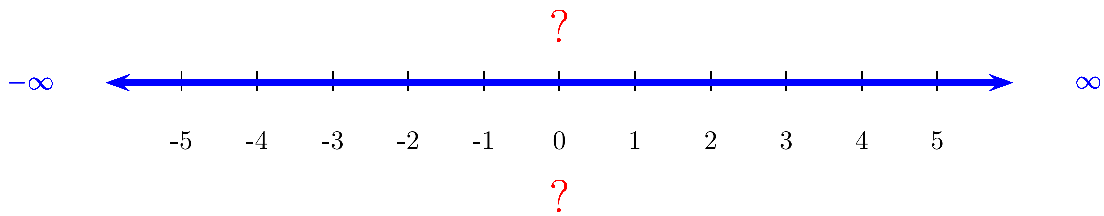
Intro to Complex Numbers

The Question

In the last section we wondered what might be far to the right and to the left of the real number line. In doing so, we entertained the existence of the very special unreal number ∞. In this section, we continue on this path and now consider *what lies above or below the real number line?*



Out of Audacity, a number is born

Here is the account, directly from the world famous, the almost immortal, mathematician Euler himself....

> "I sat in that soft comfortable chair leaned back and enjoyed the million thoughts dancing inside my head. A blank paper, a pencil and my awesome coffee was all there was on the desk. On the paper the equation
>
> $$x^2 = -1$$
>
> The equation was screaming, enticing, talking trash, challenging me, saying "you can't solve me!"
>
> Hours went by faster than I would have liked. Days past by, weeks and months...There was no real number that would solve the equation. But the forces were greater. The inspiration divine. I would not be stopped.. and one day it happened. There was no real number solution, I had looked on the positive side on the negative side and all numbers between. Resolved to avoid defeat at all costs, *I invented a number*. From my own imagination, I gathered all my might, my courage, and my audacity, and I thought...I will create a number. I will call it i, and I will solve my problem by declaring $i = \sqrt{-1}$. It's my number so I can make it behave however I please, just as the artists paints the clouds at his whim... This solves the equation
>
> $$x^2 = -1$$
>
> and marks the birth of a grand elegant family of numbers called the complex numbers, $\mathbb{C}$. With the complex numbers also came a batch of fresh new ideas. These ideas include the meaning of negative radicals, a new family of numbers to add, multiply and divide, and a whole new world that adds perspective to our previous views."

Needless, to say, I have taken some artistic liberties with this account of events. In fact, traces of complex numbers or 'imaginary' numbers can be found in 9th century's Al-Khwarizmi's *Algebra* text. During the next couple centuries these ideas made their way to Italy and France, as people were learning to solve degree 3 equations. By the turn of the 17th century Descartes coined the phrase "imaginary" numbers, referring to numbers such as $\sqrt{-1}$. At last, it was Euler, in the 18th century who named such number i, declaring $i = \sqrt{-1}$.

Thus... by definition of i;

$$i^2 = -1$$

and, i is a solution to

$$x^2 = -1$$

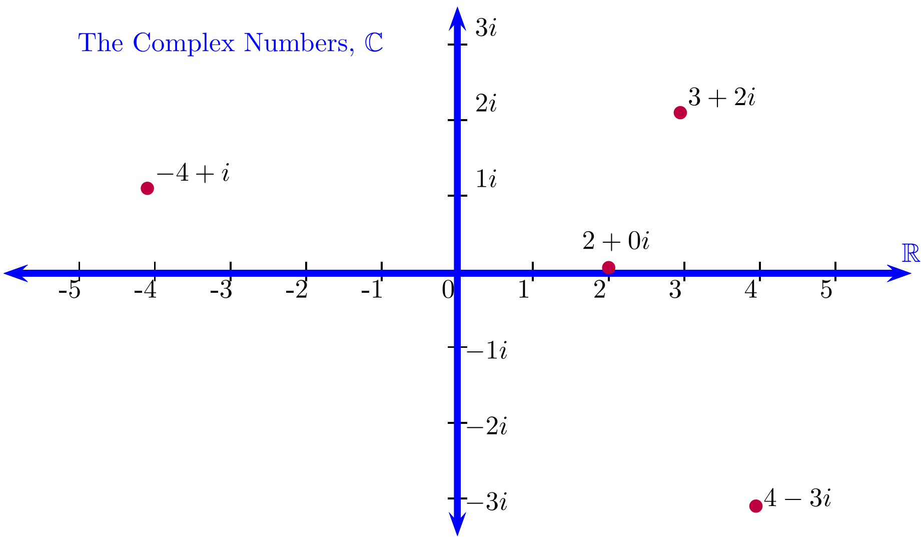
In addition to solving the equation $x^2 =^{-} 1$, and this marks the birth of a grand and elegant family of numbers called the complex numbers, $\mathbb{C}$. With the complex numbers also came a batch of fresh new ideas. These ideas include the meaning of negative radicals, a new family of numbers to add, multiply and divide, and a whole new world that adds perspective to our previous views.

Where do all the i's Live

With the invent of i came other numbers such as $2i$, $3i$, $-5i$, or $2+3i$. Generally, complex numbers are numbers that can be written in the form

$$a + bi$$

where a and b are real numbers. Now, in the previous sections we noted a visual representation of the real numbers using the real number line. Considering this, the natural question is where or how do we represent the complex numbers visually? Over the centuries, the most powerful and common way to represent the complex numbers is to place them as an extension of the real number line, extending it above and below to make what we commonly call the *complex plane*. In essence, this is done by placing an 'imaginary' axis perpendicular to the real number line. Then we position every number $a+bi$ on the plane similarly to placing the ordered point (a,b) on the cartesian $[xy]$-plane. came numbers such as $2i, 3i, 4i, \ldots$ as well as numbers such as $2+3i$. Here are some visual representations of a few complex numbers along with their position relative to the known real number line.



Negative Radicals

With the invention of i we can now make sense of radicals (i.e. square roots) of negative real numbers. Consider the radical $\sqrt{-1}$, the number whose square is -1. Recall when we first defined $\sqrt{4}$ we did so as 'the number whose number square is 4.' But there are two such numbers 2 and -2. By default, we declared to radical to mean the *positive* number whose square is 4. We follow a similar logic here, as we are confronted with the same dilemma. If we define $\sqrt{-1}$ as the number whose square is -1, we will find there are two possible choices, i and $-i$ (see examples). By convention, we will define negative radical to be, *the positive* i rather than $-i$.

Examples

1. Simplify $\sqrt{-4}$

$$\begin{aligned} \sqrt{-4} &= i\sqrt{4} && \text{(neg rad)} \\ &= i2 && \text{(def rad)} \\ &= 2i && \text{(CoLM)} \end{aligned}$$

2. Simplify $\sqrt{-10}$

$$\sqrt{-10} = i\sqrt{10} \qquad \text{(neg rad)}$$

3. Simplify $\sqrt{-15}$

$$\sqrt{-15} = i\sqrt{15} \qquad \text{(neg rad)}$$

4. Simplify $\sqrt{-x}$

Solution:

Notice, we do not know the value of x. We don't know if x is positive or negative. This means we don't know if $-x$ is positive or negative therefore we don't know if the radical $\sqrt{-x}$ is positive or negative.

5. Adding in $\mathbb{C}$

$$3 + 5i + 2 + 3i$$

Solution:

$$\begin{aligned} 3 + 5i + 2 + 3i &= && \text{(given)} \\ &= (3 + 2) + (5i + 3i) && \text{(ALA)} \\ &= 3 + 2 + (5 + 3)i && \text{(DL)} \\ &= 5 + 8i && \text{(AT)} \end{aligned}$$

6. Multiplying in $\mathbb{C}$

$$(3 + 5i)(2 + 3i)$$

Solution:

$$\begin{aligned} (3 + 5i)(2 + 3i) &= && \text{(given)} \\ &= 3 \cdot 2 + 5i \cdot 2 + 3 \cdot 3i + 5i \cdot 3i && \text{(FOIL)} \\ &= 6 + 10i + 9i + 15i^2 && \text{(BI)} \\ &= 6 + (10 + 9)i + 15i^2 && \text{(DL)} \\ &= 6 + 19i + 15i^2 && \text{(AT)} \\ &= 6 + 19i + 15 \cdot -1 && \text{(Def of } i) \\ &= 6 + 19i + -15 && \text{(BI)} \\ &= -9 + 19i && \text{(BI)} \end{aligned}$$

7. Multiplying in $\mathbb{C}$

$$(4 +^{-} 5i)(2 + 3i)$$

Solution:

$$\begin{aligned}
(4+^{-}5i)(2+3i) &= && \text{(given)}\\
&= 4\cdot 2+^{-}5i\cdot 2+4\cdot 3i+^{-}5i\cdot 3i && \text{(FOIL)}\\
&= 8+^{-}10i+12i+^{-}15i^2 && \text{(BI)}\\
&= 8+(^{-}10+12)i+^{-}15i^2 && \text{(DL)}\\
&= 6+2i+^{-}15i^2 && \text{(BI)}\\
&= 6+2i+^{-}15\cdot -1 && \text{(Def of } i)\\
&= 6+2i+15 && \text{(NNT)}\\
&= 21+2i && \text{(BI)}
\end{aligned}$$

8. Multiplying in $\mathbb{C}$

$$(4+3i)(2+3i)$$

Solution:

$$\begin{aligned}
(4+3i)(2+3i) &= && \text{(given)}\\
&= 8+12i+6i+9i^2 && \text{(FOIL)}\\
&= 8+12i+6i+9\cdot ^{-}1 && \text{(def } i)\\
&= 8+12i+6i+^{-}9 && \text{(BI)}\\
&=^{-}1+18i && \text{(BI)}
\end{aligned}$$

9. Multiplying in $\mathbb{C}$

$$i^7$$

Solution:

$$\begin{aligned}
i^7 &= iiiiiii && \text{(+Expo)}\\
&= i^2i^2i^2i && \text{(+Expo)}\\
&= -1\cdot -1\cdot -1\cdot i && \text{(Def of } i)\\
&= -1\cdot i && \text{(BI)}\\
&= -i && \text{(MT)}
\end{aligned}$$

And Now, Divide

We have now introduced the imaginary number, their standard form '$a+bi$', we introduced their home, the complex plane, and we introduced some simple arithmetic operations on them such as adding/multiplying. In this section, we continue on the same theme, adding to that some division skills, we add some some famous terminology, such as 'conjugates', and we look further into the calculation of many exponential powers of i.

How to divide in the $\mathbb{C}$-world The layman way to divide.

The key lies in the observation that multiplying pairs of conjugate complex numbers always yields real numbers. In a way, it is sort of a way to smack a complex number on its head and turn it into a real number, sort of. Every complex number has a conjugate defined as follows, when written in standard form, the conjugate of $a+bi$ is $a-bi$. In other words, the conjugate of a complex number is the same number with the sign of the complex part switched. Now, observe how the product of conjugates *always* yields a real number. Take, for example, the complex number $2+3i$, its conjugate is $2-3i$:

$$\begin{aligned}(2+3i)(2-3i) &= 4+6i-6i-9\cdot i^2 && \text{(FOIL)}\\ &= 4+0-9\cdot(-1) && \text{(BI)}\\ &= 13 && \text{(as promised, a real number)}\end{aligned}$$

Now, we see how this will help us divide. Suppose we want to divide $\frac{3+5i}{2+3i}$

Divide

$$\frac{5i+3}{3i+2}$$

$$\begin{aligned}\frac{5i+3}{3i+2} &= \frac{5i+3}{3i+2}\cdot 1 && \text{(MiD)}\\ &= \frac{5i+3}{3i+2}\cdot\frac{-3i+2}{-3i+2} && \text{(JOT)}\\ &= \frac{-15i^2+i+6}{-9i^2+4} && \text{(MAT, FOIL)}\\ &= \frac{i+21}{13} && \text{(BI)}\\ &= \frac{21}{13}+\frac{i}{13} && \text{(BI)}\end{aligned}$$

Here is another example,

Divide

$$\frac{5i-3}{i+3}$$

$$\begin{aligned}\frac{5i-3}{i+3} &= \frac{5i-3}{i+3}\cdot 1 && \text{(MiD)}\\ &= \frac{5i-3}{i+3}\cdot\frac{-i+3}{-i+3} && \text{(JOT)}\\ &= \frac{-5i^2+18i-9}{-i^2+9} && \text{(MAT, FOIL)}\\ &= \frac{18i-4}{10} && \text{(BI)}\\ &= \frac{-4}{10}+\frac{18i}{10} && \text{(BI)}\end{aligned}$$

How to divide in the $\mathbb{C}$-world

As usual, to divide means to multiply by the multiplicative inverse. Thus, we need and want to address this question: for any non-zero complex number, $a + bi$ what is its multiplicative inverse? We claim the inverse is $\frac{a-bi}{a^2+b^2}$. To check this we simply check that their product is 1.

Multiplicative Inverses in **C**

$$\begin{aligned}
(a+bi)\left(\frac{a-bi}{a^2+b^2}\right) &= \frac{(a+bi)}{1}\left(\frac{a-bi}{a^2+b^2}\right) \\
&= \frac{a^2+abi-abi-bi^2}{a^2+b^2} \\
&= = \frac{a^2+b^2}{a^2+b^2} = 1
\end{aligned}$$

Example Dividing in **C** :

$$\begin{aligned}
(3-2i) \div (1+3i) &= (3-2i)\cdot\frac{1-3i}{1^2+3^2} \\
&= \frac{3-9i-2i+6i^2}{10} \\
&= \frac{-3-11i}{10} = \frac{-3}{10} - \frac{11}{10}i
\end{aligned}$$

Another way to 'divide' and in essence carry out the same computation is to multiply numerator and denominator by the conjugate of the denominator. For example, if the denominator is $a + bi$, then multiplying both numerator and denominator will annihilate the i's on the denominator. This is a very popular method of 'dividing. For example.

Compute w/ Complex Numbers Calculate and write in standard form.

$$\frac{2i+1}{i+1}$$

There are at least a couple ways to go about this.. one way, to note 'divide' means 'multiply by inverse'... so..

$$\begin{aligned}
\frac{2i+1}{i+1} &= 2i+1\cdot\frac{-i+1}{1^2+1^2} \\
&= \frac{2i+1-i+1}{2} \\
&= \frac{i+3}{2} \\
&= \frac{3}{2}+\frac{1}{2}i
\end{aligned}$$

Another way to do it.. (more popular) is to simply multiply numerator and denominator by the conjugate of the denominator.

$$\begin{aligned}\frac{2i+1}{i+1} &= \frac{2i+1}{i+1}\cdot 1\\ &= \frac{2i+1}{i+1}\cdot\frac{-i+1}{-i+1}\\ &= \frac{i+3}{2}\\ &= \frac{3}{2}+\frac{1}{2}i\end{aligned}$$

Compute w/ Complex Numbers Calculate and write in standard form.

$$\frac{2i+3}{5i-2}$$

There are at least a couple ways to go about this.. one way, to note 'divide' means 'multiply by inverse'... so..

$$\begin{aligned}\frac{2i+3}{5i-2} &= 2i+3\cdot\frac{-5i-2}{-2^2+5^2}\\ &= \frac{2i+3-5i-2}{29}\\ &= \frac{-19i+4}{29}\\ &= \frac{4}{29}+\frac{-19}{29}i\end{aligned}$$

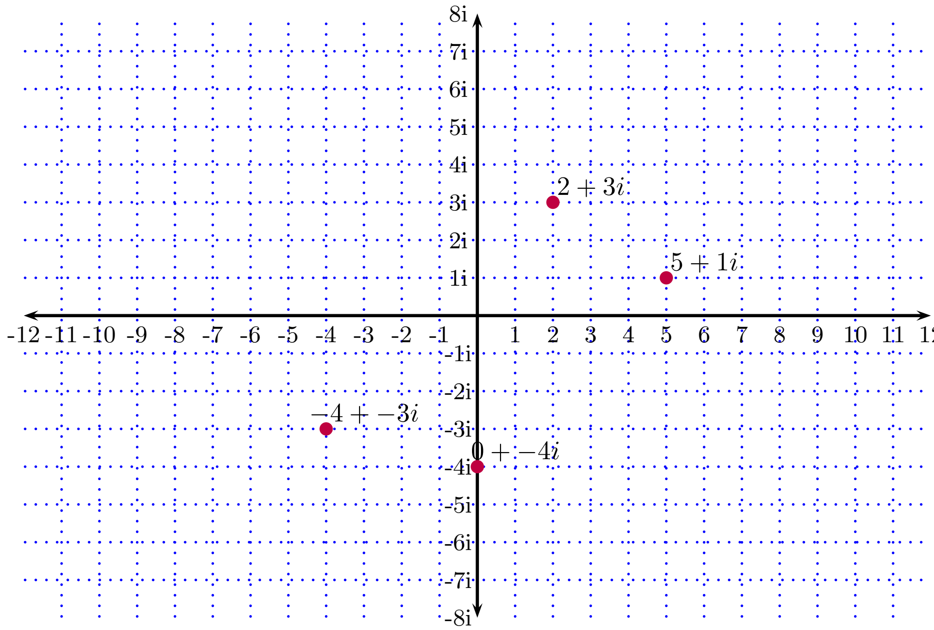
Another way to do it.. (more popular) is to simply multiply numerator and denominator by the conjugate of the denominator.

$$\begin{aligned}\frac{2i+3}{5i-2} &= \frac{2i+3}{5i-2}\cdot 1\\ &= \frac{2i+3}{5i-2}\cdot\frac{-5i-2}{-5i-2}\\ &= \frac{-19i+4}{29}\\ &= \frac{4}{29}+\frac{-19}{29}i\end{aligned}$$

Intro to Complex Numbers

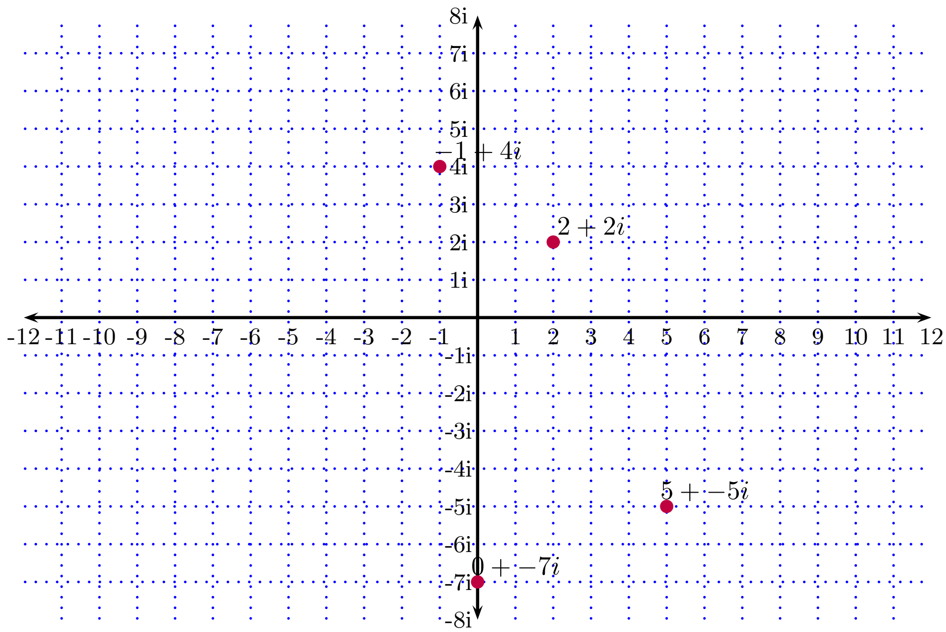
1. Plot the following points.
   (a) $2 + 3i$
   (b) $5 + 1i$
   (c) $-4 + -3i$
   (d) $-4i$

Solution:



2. Plot the following points.
   (a) $-1 + 4i$
   (b) $2 + 2i$
   (c) $5 + -5i$
   (d) $-7i$

Solution:



Simplify $\sqrt{-90}$

Solution:

$$
\begin{aligned}
&\sqrt{-90} && \text{(given)}\\
&= i\sqrt{90} && \text{(def neg rad)}\\
&= i\sqrt{9\cdot 10} && \text{(TT)}\\
&= i\sqrt{9}\sqrt{10} && \text{(RP=PR)}\\
&= i\cdot 3\sqrt{10} && \text{(def rad)}\\
&= 3i\sqrt{10} && \text{(BI)}
\end{aligned}
$$

3. Simplify $\sqrt{-150}$

Solution:

$$
\begin{aligned}
&\sqrt{-150} && \text{(given)}\\
&= i\sqrt{150} && \text{(def neg rad)}\\
&= i\sqrt{25\cdot 6} && \text{(TT)}\\
&= i\sqrt{25}\sqrt{6} && \text{(RP=PR)}\\
&= i\cdot 5\sqrt{6} && \text{(def rad)}\\
&= 5i\sqrt{6} && \text{(BI)}
\end{aligned}
$$

5. Simplify $\sqrt{-375}$

Solution:

$$
\begin{aligned}
&\sqrt{-375} && \text{(given)}\\
&= i\sqrt{375} && \text{(def neg rad)}\\
&= i\sqrt{25\cdot 15} && \text{(TT)}\\
&= i\sqrt{25}\sqrt{15} && \text{(RP=PR)}\\
&= i\cdot 5\sqrt{15} && \text{(def rad)}\\
&= 5i\sqrt{15} && \text{(BI)}
\end{aligned}
$$

6. Simplify $\sqrt{-18}$

Solution:

$$
\begin{aligned}
&\sqrt{-18} && \text{(given)}\\
&= i\sqrt{18} && \text{(def neg rad)}\\
&= i\sqrt{9\cdot 2} && \text{(TT)}\\
&= i\sqrt{9}\sqrt{2} && \text{(RP=PR)}\\
&= i\cdot 3\sqrt{2} && \text{(def rad)}\\
&= 3i\sqrt{2} && \text{(BI)}
\end{aligned}
$$

7. Simplify $\sqrt{-50}$

Solution:

$$
\begin{aligned}
&\sqrt{-50} && \text{(given)}\\
&= i\sqrt{50} && \text{(def neg rad)}\\
&= i\sqrt{25\cdot 2} && \text{(TT)}\\
&= i\sqrt{25}\sqrt{2} && \text{(RP=PR)}\\
&= i\cdot 5\sqrt{2} && \text{(def rad)}\\
&= 5i\sqrt{2} && \text{(BI)}
\end{aligned}
$$

8. Simplify $\sqrt{-8}$

Solution:

$$
\begin{aligned}
&\sqrt{-8} && \text{(given)}\\
&= i\sqrt{8} && \text{(def neg rad)}\\
&= i\sqrt{4\cdot 2} && \text{(TT)}\\
&= i\sqrt{4}\sqrt{2} && \text{(RP=PR)}\\
&= i\cdot 2\sqrt{2} && \text{(def rad)}\\
&= 2i\sqrt{2} && \text{(BI)}
\end{aligned}
$$

9. Simplify $\sqrt{-125}$

Solution:

$$
\begin{aligned}
&\sqrt{-125} && \text{(given)}\\
&= i\sqrt{125} && \text{(def neg rad)}\\
&= i\sqrt{25\cdot 5} && \text{(TT)}\\
&= i\sqrt{25}\sqrt{5} && \text{(RP=PR)}\\
&= i\cdot 5\sqrt{5} && \text{(def rad)}\\
&= 5i\sqrt{5} && \text{(BI)}
\end{aligned}
$$

10. Simplify $\sqrt[3]{-8}$

Solution: -2 since $(-2)^3 = -8$

11. Simplify $\sqrt[3]{-125}$

Solution: -5 since $(-5)^3 = -125$

12. Simplify $\sqrt[3]{-1}$

Solution: -1 since $(-1)^3 = -1$

13. Add $3i + 2 + 5i + 4$

Solution: by convention, we write in 'standard form', "$a + bi$"

$$\begin{aligned}
(2+3i)+(4+5i) &= 2+(3i+4)+5i && \text{(ALA)}\\
&= 2+(4+3i)+5i && \text{(CoLA)}\\
&= (2+4)+(3i+5i) && \text{(ALA)}\\
&= (2+4)+(3+5)i && \text{(DL)}\\
&= 6+8i && \text{(BI)}
\end{aligned}$$

14. Add $2i + 7 + 9i + 3$

Solution: by convention, we write in 'standard form', "$a + bi$"

$$\begin{aligned}
(7+2i)+(3+9i) &= 7+(2i+3)+9i && \text{(ALA)}\\
&= 7+(3+2i)+9i && \text{(CoLA)}\\
&= (7+3)+(2i+9i) && \text{(ALA)}\\
&= (7+3)+(2+9)i && \text{(DL)}\\
&= 10+11i && \text{(BI)}
\end{aligned}$$

15. Add $3i - 2 + \ -5i + 4$

Solution: by convention, we write in 'standard form', "$a + bi$"

$$\begin{aligned}
(-2+3i)+(4+-5i) &= -2+(3i+4)+-5i && \text{(ALA)}\\
&= -2+(4+3i)+-5i && \text{(CoLA)}\\
&= (-2+4)+(3i+-5i) && \text{(ALA)}\\
&= (-2+4)+(3+-5)i && \text{(DL)}\\
&= 2+-2i && \text{(BI)}
\end{aligned}$$

16. Add $30i + 11 + 5i + 5$

Solution: by convention, we write in 'standard form', "$a + bi$"

$$\begin{aligned}
(11 + 30i) + (5 + 5i) &= 11 + (30i + 5) + 5i && \text{(ALA)}\\
&= 11 + (5 + 30i) + 5i && \text{(CoLA)}\\
&= (11 + 5) + (30i + 5i) && \text{(ALA)}\\
&= (11 + 5) + (30 + 5)i && \text{(DL)}\\
&= 16 + 35i && \text{(BI)}
\end{aligned}$$

17. Add $i + 1 + i + 2$

Solution: by convention, we write in 'standard form', "$a + bi$"

$$\begin{aligned}
(1 + 1i) + (2 + 1i) &= 1 + (1i + 2) + 1i && \text{(ALA)}\\
&= 1 + (2 + 1i) + 1i && \text{(CoLA)}\\
&= (1 + 2) + (1i + 1i) && \text{(ALA)}\\
&= (1 + 2) + (1 + 1)i && \text{(DL)}\\
&= 3 + 2i && \text{(BI)}
\end{aligned}$$

18. Add $3i + 2 + 4$

Solution: by convention, we write in 'standard form', "$a + bi$"

$$\begin{aligned}
(2 + 3i) + (4 + 0i) &= 2 + (3i + 4) + 0i && \text{(ALA)}\\
&= 2 + (4 + 3i) + 0i && \text{(CoLA)}\\
&= (2 + 4) + (3i + 0i) && \text{(ALA)}\\
&= (2 + 4) + (3 + 0)i && \text{(DL)}\\
&= 6 + 3i && \text{(BI)}
\end{aligned}$$

19. Add $3i + i$

Solution: by convention, we write in 'standard form', "$a + bi$"

$$\begin{aligned}
(0 + 3i) + (0 + 1i) &= 0 + (3i + 0) + 1i && \text{(ALA)}\\
&= 0 + (0 + 3i) + 1i && \text{(CoLA)}\\
&= (0 + 0) + (3i + 1i) && \text{(ALA)}\\
&= (0 + 0) + (3 + 1)i && \text{(DL)}\\
&= 0 + 4i && \text{(BI)}
\end{aligned}$$

20. Multiply $\big(3i+2\big)\big(5i+4\big)$

Solution:
by convention, we write in 'standard form', "$a+bi$"

$$\begin{aligned}
\big(3i+2\big)\big(5i+4\big) &= (2)(4)+(2)(5i)+(3i)(4)+(3i)(5i) && \text{(FOIL)}\\
&= 8+10i+12i+15i^2 && \text{(BI)}\\
&= 8+10i+12i+15(-1) && \text{(def } i\text{)}\\
&= (8+-15)+(10i+12i) && \text{(CoLA, ALA,BI)}\\
&= (8+-15)+(10+12)i && \text{(DL)}\\
&= -7+22i && \text{(BI)}
\end{aligned}$$

21. Multiply $\big(2i+7\big)\big(9i+3\big)$

Solution:
by convention, we write in 'standard form', "$a+bi$"

$$\begin{aligned}
\big(2i+7\big)\big(9i+3\big) &= (7)(3)+(7)(9i)+(2i)(3)+(2i)(9i) && \text{(FOIL)}\\
&= 21+63i+6i+18i^2 && \text{(BI)}\\
&= 21+63i+6i+18(-1) && \text{(def } i\text{)}\\
&= (21+-18)+(63i+6i) && \text{(CoLA, ALA,BI)}\\
&= (21+-18)+(63+6)i && \text{(DL)}\\
&= 3+69i && \text{(BI)}
\end{aligned}$$

22. Multiply $\big(3i-2\big)\big(-5i+4\big)$

Solution:
by convention, we write in 'standard form', "$a+bi$"

$$\begin{aligned}
\big(3i-2\big)\big(-5i+4\big) &= (-2)(4)+(-2)(-5i)+(3i)(4)+(3i)(-5i) && \text{(FOIL)}\\
&= -8+10i+12i+-15i^2 && \text{(BI)}\\
&= -8+10i+12i+-15(-1) && \text{(def } i\text{)}\\
&= (-8+15)+(10i+12i) && \text{(CoLA, ALA,BI)}\\
&= (-8+15)+(10+12)i && \text{(DL)}\\
&= 7+22i && \text{(BI)}
\end{aligned}$$

23. Multiply $\big(30i+11\big)\big(5i+5\big)$

Solution:
by convention, we write in 'standard form', "$a + bi$"

$$\begin{aligned}
(30i + 11)\,(5i + 5) &= (11)(5) + (11)(5i) + (30i)(5) + (30i)(5i) && \text{(FOIL)}\\
&= 55 + 55i + 150i + 150i^2 && \text{(BI)}\\
&= 55 + 55i + 150i + 150(-1) && \text{(def } i\text{)}\\
&= (55 + -150) + (55i + 150i) && \text{(CoLA, ALA,BI)}\\
&= (55 + -150) + (55 + 150)i && \text{(DL)}\\
&= -95 + 205i && \text{(BI)}
\end{aligned}$$

24. Multiply $(i + 1)\,(i + 2)$

Solution:
by convention, we write in 'standard form', "$a + bi$"

$$\begin{aligned}
(i + 1)\,(i + 2) &= (1)(2) + (1)(1i) + (1i)(2) + (1i)(1i) && \text{(FOIL)}\\
&= 2 + 1i + 2i + 1i^2 && \text{(BI)}\\
&= 2 + 1i + 2i + 1(-1) && \text{(def } i\text{)}\\
&= (2 + -1) + (1i + 2i) && \text{(CoLA, ALA,BI)}\\
&= (2 + -1) + (1 + 2)i && \text{(DL)}\\
&= 1 + 3i && \text{(BI)}
\end{aligned}$$

25. Multiply $(3i + 2)\,(-4i + 4)$

Solution:
by convention, we write in 'standard form', "$a + bi$"

$$\begin{aligned}
(3i + 2)\,(-4i + 4) &= (2)(4) + (2)(-4i) + (3i)(4) + (3i)(-4i) && \text{(FOIL)}\\
&= 8 + -8i + 12i + -12i^2 && \text{(BI)}\\
&= 8 + -8i + 12i + -12(-1) && \text{(def } i\text{)}\\
&= (8 + 12) + (-8i + 12i) && \text{(CoLA, ALA,BI)}\\
&= (8 + 12) + (-8 + 12)i && \text{(DL)}\\
&= 20 + 4i && \text{(BI)}
\end{aligned}$$

26. Multiply $(3i - 1)\,(i - 1)$

Solution:

by convention, we write in 'standard form', "$a + bi$"

$$\begin{aligned}
\big(3i-1\big)\,\big(i-1\big) &= (-1)(-1) + (-1)(1i) + (3i)(-1) + (3i)(1i) && \text{(FOIL)}\\
&= 1 + -1i + -3i + 3i^2 && \text{(BI)}\\
&= 1 + -1i + -3i + 3(-1) && \text{(def } i\text{)}\\
&= (1 + -3) + (-1i + -3i) && \text{(CoLA, ALA,BI)}\\
&= (1 + -3) + (-1 + -3)i && \text{(DL)}\\
&= -2 + -4i && \text{(BI)}
\end{aligned}$$

27. Multiply $\big(i+1\big)\,\big(i+2\big)$

Solution:
by convention, we write in 'standard form', "$a + bi$"

$$\begin{aligned}
\big(i+1\big)\,\big(i+2\big) &= (1)(2) + (1)(1i) + (1i)(2) + (1i)(1i) && \text{(FOIL)}\\
&= 2 + 1i + 2i + 1i^2 && \text{(BI)}\\
&= 2 + 1i + 2i + 1(-1) && \text{(def } i\text{)}\\
&= (2 + -1) + (1i + 2i) && \text{(CoLA, ALA,BI)}\\
&= (2 + -1) + (1 + 2)i && \text{(DL)}\\
&= 1 + 3i && \text{(BI)}
\end{aligned}$$

28. Multiply $\big(3i+2\big)\,\big(-4i+4\big)$

Solution:
by convention, we write in 'standard form', "$a + bi$"

$$\begin{aligned}
\big(3i+2\big)\,\big(-4i+4\big) &= (2)(4) + (2)(-4i) + (3i)(4) + (3i)(-4i) && \text{(FOIL)}\\
&= 8 + -8i + 12i + -12i^2 && \text{(BI)}\\
&= 8 + -8i + 12i + -12(-1) && \text{(def } i\text{)}\\
&= (8 + 12) + (-8i + 12i) && \text{(CoLA, ALA,BI)}\\
&= (8 + 12) + (-8 + 12)i && \text{(DL)}\\
&= 20 + 4i && \text{(BI)}
\end{aligned}$$

29. Multiply $\big(3i-1\big)\,\big(i-1\big)$

Solution:

by convention, we write in 'standard form', "$a + bi$"

$$
\begin{aligned}
\big(3i-1\big)\big(i-1\big) &= (-1)(-1)+(-1)(1i)+(3i)(-1)+(3i)(1i) && \text{(FOIL)}\\
&= 1+-1i+-3i+3i^2 && \text{(BI)}\\
&= 1+-1i+-3i+3(-1) && \text{(def } i\text{)}\\
&= (1+-3)+(-1i+-3i) && \text{(CoLA, ALA,BI)}\\
&= (1+-3)+(-1+-3)i && \text{(DL)}\\
&= -2+-4i && \text{(BI)}
\end{aligned}
$$

30. multiply i^3

Solution:

$$
\begin{aligned}
i^3 &= i^2 \cdot i && \text{(JAE)}\\
&= -1 \cdot i && \text{(def of i)}\\
&= -i && \text{(MT)}
\end{aligned}
$$

31. multiply i^4

Solution:

$$
\begin{aligned}
i^4 &= i^2 \cdot i^2 && \text{(JAE)}\\
&= -1 \cdot -1 && \text{(def of i)}\\
&= 1 && \text{(NotNot)}
\end{aligned}
$$

32. multiply i^6

Solution:

$$
\begin{aligned}
i^6 &= i^4 \cdot i^2 && \text{(JAE)}\\
&= 1 \cdot -1 && \text{(see previous problem)}\\
&= -1 && \text{(MiD)}
\end{aligned}
$$

33. $(3i + 2)^3$

Solution:

$$
\begin{aligned}
&(3i+2)^3 && \text{(given)}\\
&= (3i)^3 \quad + \quad 3\cdot(3i)^2(2) \quad + \quad 3(3i)(2)^2 \quad + \quad (2)^3 && \text{(famous, PP3)}\\
&= 27i^3 + 54i^2 + 36i + 8 && \text{(BI)}\\
&= 27(-i) + 54(-1) + 36(i) + 8 && \text{(BI)}\\
&= -46 + 9i && \text{(BI)}
\end{aligned}
$$

34. $(2i+1)^3$

Solution:

$$
\begin{aligned}
&(2i+1)^3 && \text{(given)}\\
&= (2i)^3 \quad + \quad 3\cdot(2i)^2(1) \quad + \quad 3(2i)(1)^2 \quad + \quad (1)^3 && \text{(famous, PP3)}\\
&= 8i^3 + 12i^2 + 6i + 1 && \text{(BI)}\\
&= 8(-i) + 12(-1) + 6(i) + 1 && \text{(BI)}\\
&= -11 + -2i && \text{(BI)}
\end{aligned}
$$

35. $(-2i+1)^3$

Solution:

$$
\begin{aligned}
&(-2i+1)^3 && \text{(given)}\\
&= (-2i)^3 \quad + \quad 3\cdot(-2i)^2(1) \quad + \quad 3(-2i)(1)^2 \quad + \quad (1)^3 && \text{(famous, PP3)}\\
&= \; - 8i^3 + 12i^2 - 6i + 1 && \text{(BI)}\\
&= -8(-i) + 12(-1) + -6(i) + 1 && \text{(BI)}\\
&= -11 + 2i && \text{(BI)}
\end{aligned}
$$

36. $(-2i+3)^3$

Solution:

$$\begin{aligned}
&(-2i+3)^3 && \text{(given)}\\
&= (-2i)^3 \quad + \quad 3\cdot(-2i)^2(3) \quad + \quad 3(-2i)(3)^2 \quad + \quad (3)^3 && \text{(famous, PP3)}\\
&= \ -8i^3 + 36i^2 - 54i + 27 && \text{(BI)}\\
&= -8(-i) + 36(-1) + -54(i) + 27 && \text{(BI)}\\
&= -9 + -46i && \text{(BI)}
\end{aligned}$$

37. $(-1i+1)^3$

Solution:

$$\begin{aligned}
&(-1i+1)^3 && \text{(given)}\\
&= (-1i)^3 \quad + \quad 3\cdot(-1i)^2(1) \quad + \quad 3(-1i)(1)^2 \quad + \quad (1)^3 && \text{(famous, PP3)}\\
&= \ -i^3 + 3i^2 - 3i + 1 && \text{(BI)}\\
&= -1(-i) + 3(-1) + -3(i) + 1 && \text{(BI)}\\
&= -2 + -2i && \text{(BI)}
\end{aligned}$$

38. $(1i+1)^3$

Solution:

$$\begin{aligned}
&(1i+1)^3 && \text{(given)}\\
&= (1i)^3 \quad + \quad 3\cdot(1i)^2(1) \quad + \quad 3(1i)(1)^2 \quad + \quad (1)^3 && \text{(famous, PP3)}\\
&= i^3 + 3i^2 + 3i + 1 && \text{(BI)}\\
&= 1(-i) + 3(-1) + 3(i) + 1 && \text{(BI)}\\
&= -2 + 2i && \text{(BI)}
\end{aligned}$$

39. Divide

$$\frac{7i+1}{7i+3}$$

Solution:

$$\begin{aligned}
\frac{7i+1}{7i+3} &= \frac{7i+1}{7i+3}\cdot 1 && \text{(MiD)}\\
&= \frac{7i+1}{7i+3}\cdot\frac{-7i+3}{-7i+3} && \text{(JOT)}\\
&= \frac{-49i^2+14i+3}{-49i^2+9} && \text{(MAT, FOIL)}\\
&= \frac{14i+52}{58} && \text{(BI)}\\
&= \frac{52}{58}+\frac{14i}{58} && \text{(BI)}
\end{aligned}$$

40. Divide

$$\frac{4i+2}{-5i+2}$$

Solution:

$$\begin{aligned}
\frac{4i+2}{-5i+2} &= \frac{4i+2}{-5i+2}\cdot 1 && \text{(MiD)}\\
&= \frac{4i+2}{-5i+2}\cdot\frac{5i+2}{5i+2} && \text{(JOT)}\\
&= \frac{20i^2+18i+4}{-25i^2+4} && \text{(MAT, FOIL)}\\
&= \frac{18i-16}{29} && \text{(BI)}\\
&= \frac{-16}{29}+\frac{18i}{29} && \text{(BI)}
\end{aligned}$$

41. Divide

$$\frac{5i+2}{4i+2}$$

Solution:

$$\begin{aligned}
\frac{5i+2}{4i+2} &= \frac{5i+2}{4i+2}\cdot 1 && \text{(MiD)}\\
&= \frac{5i+2}{4i+2}\cdot\frac{-4i+2}{-4i+2} && \text{(JOT)}\\
&= \frac{-20i^2+2i+4}{-16i^2+4} && \text{(MAT, FOIL)}\\
&= \frac{2i+24}{20} && \text{(BI)}\\
&= \frac{24}{20}+\frac{2i}{20} && \text{(BI)}
\end{aligned}$$

42. Divide

$$\frac{3i+1}{2i+1}$$

Solution:

$$\begin{aligned}
\frac{3i+1}{2i+1} &= \frac{3i+1}{2i+1}\cdot 1 && \text{(MiD)}\\
&= \frac{3i+1}{2i+1}\cdot\frac{-2i+1}{-2i+1} && \text{(JOT)}\\
&= \frac{-6i^2+i+1}{-4i^2+1} && \text{(MAT, FOIL)}\\
&= \frac{i+7}{5} && \text{(BI)}\\
&= \frac{7}{5}+\frac{i}{5} && \text{(BI)}
\end{aligned}$$

43. Divide

$$\frac{5i+2}{i+2}$$

Solution:

$$\begin{aligned}
\frac{5i+2}{i+2} &= \frac{5i+2}{i+2}\cdot 1 && \text{(MiD)}\\
&= \frac{5i+2}{i+2}\cdot\frac{-i+2}{-i+2} && \text{(JOT)}\\
&= \frac{-5i^2+8i+4}{-i^2+4} && \text{(MAT, FOIL)}\\
&= \frac{8i+9}{5} && \text{(BI)}\\
&= \frac{9}{5}+\frac{8i}{5} && \text{(BI)}
\end{aligned}$$

44. Divide

$$\frac{7i+2}{3i+2}$$

Solution:

$$\begin{aligned}
\frac{7i+2}{3i+2} &= \frac{7i+2}{3i+2}\cdot 1 && \text{(MiD)}\\
&= \frac{7i+2}{3i+2}\cdot\frac{-3i+2}{-3i+2} && \text{(JOT)}\\
&= \frac{-21i^2+8i+4}{-9i^2+4} && \text{(MAT, FOIL)}\\
&= \frac{8i+25}{13} && \text{(BI)}\\
&= \frac{25}{13}+\frac{8i}{13} && \text{(BI)}
\end{aligned}$$

45. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$\frac{2i+1}{i+1}$$

Solution: There are at least a couple ways to go about this.. one way, to note 'divide' means 'multiply by inverse'... so..

$$\frac{2i+1}{i+1} = 2i+1 \cdot \frac{-i+1}{1^2+1^2}$$
$$= \frac{2i+1-i+1}{2}$$
$$= \frac{i+3}{2}$$
$$= \frac{3}{2} + \frac{1}{2}i$$

Another way to do it.. (more popular) is to simply multiply numerator and denominator by the conjugate of the denominator.

$$\frac{2i+1}{i+1} = \frac{2i+1}{i+1} \cdot 1$$
$$= \frac{2i+1}{i+1} \cdot \frac{-i+1}{-i+1}$$
$$= \frac{i+3}{2}$$
$$= \frac{3}{2} + \frac{1}{2}i$$

46. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$\frac{2i+1}{2i+3}$$

Solution: There are at least a couple ways to go about this.. one way, to note 'divide' means 'multiply by inverse'... so..

$$\frac{2i+1}{2i+3} = 2i+1 \cdot \frac{-2i+3}{3^2+2^2}$$
$$= \frac{2i+1-2i+3}{13}$$
$$= \frac{4i+7}{13}$$
$$= \frac{7}{13} + \frac{4}{13}i$$

Another way to do it.. (more popular) is to simply multiply numerator and denominator by the conjugate of the denominator.

$$\begin{aligned}
\frac{2i+1}{2i+3} &= \frac{2i+1}{2i+3}\cdot 1 \\
&= \frac{2i+1}{2i+3}\cdot\frac{-2i+3}{-2i+3} \\
&= \frac{4i+7}{13} \\
&= \frac{7}{13}+\frac{4}{13}i
\end{aligned}$$

47. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$\frac{1}{i}$$

Solution: hint: the bottom is $0+1i$

48. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$\frac{1}{-i}$$

Solution: hint: the bottom is $0-1i$

49. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^4$$

Solution: $i^4 = i^2\cdot i^2 = -1\cdot -1 = 1$

50. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{14}$$

Solution:

$$\begin{aligned}
i^{14} &= i^{12}\cdot i^2 && \text{(just add exponents)} \\
&= \left(i^4\right)^3\cdot i^2 && \text{(setting up to use fact } i^4=1\text{)} \\
&= (1)^3\cdot i^2 && \text{(yipei-kae-yeh...)} \\
&= i^2 \\
&= -1
\end{aligned}$$

51. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{25}$$

Solution:

$$\begin{aligned} i^{25} &= \left(i^{24}\right) \cdot i^1 && \text{(preparing to use } i^4 = 1\text{)} \\ &= \left(i^4\right)^6 \cdot i^1 && \text{(preparing to use } i^4 = 1\text{)} \\ &= (1)^6 \cdot i && \text{(used } i^4 = 1\text{)} \\ &= i && \text{(ez as sundays)} \end{aligned}$$

52. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{-3}$$

Solution:

$$\begin{aligned} i^{-3} &= i^{-3} \cdot 1 \\ &= i^{-3} \cdot i^4 && \text{(see previous problem why } i^4 = 1\text{)} \\ &= i^1 && \text{(just add exponents)} \\ &= i && \text{(note: there are many other ways to do this problem)} \end{aligned}$$

53. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{25}$$

Solution:

$$\begin{aligned} i^{25} &= \left(i^{24}\right) \cdot i^1 && \text{(preparing to use } i^4 = 1\text{)} \\ &= \left(i^4\right)^6 \cdot i^1 && \text{(preparing to use } i^4 = 1\text{)} \\ &= (1)^6 \cdot i && \text{(used } i^4 = 1\text{)} \\ &= i && \text{(ez as sundays)} \end{aligned}$$

54. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{-7}$$

Solution:

$$\begin{aligned} i^{-7} &= \left(i^{-8}\right)\cdot i^{1} && \text{(preparing to use } i^4=1) \\ &= \left(i^{4}\right)^{-2}\cdot i^{1} && \text{(preparing to use } i^4=1) \\ &= (1)^{-2}\cdot i && \text{(used } i^4=1) \\ &= i && \text{(ez as sundays)} \end{aligned}$$

55. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{-3}$$

Solution:

$$\begin{aligned} i^{-3} &= \left(i^{-4}\right)\cdot i^{1} && \text{(preparing to use } i^4=1) \\ &= \left(i^{4}\right)^{-1}\cdot i^{1} && \text{(preparing to use } i^4=1) \\ &= (1)^{-1}\cdot i && \text{(used } i^4=1) \\ &= i && \text{(ez as sundays)} \end{aligned}$$

56. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{-5}$$

Solution:

$$\begin{aligned} i^{-5} &= \left(i^{-8}\right)\cdot i^{1} && \text{(preparing to use } i^4=1) \\ &= \left(i^{4}\right)^{-2}\cdot i^{1} && \text{(preparing to use } i^4=1) \\ &= (1)^{-2}\cdot i && \text{(used } i^4=1) \\ &= i && \text{(ez as sundays)} \end{aligned}$$

57. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{2}$$

Solution:

$$\begin{aligned} i^{2} &= \left(i^{0}\right)\cdot i^{1} && \text{(preparing to use } i^4=1) \\ &= \left(i^{4}\right)^{0}\cdot i^{1} && \text{(preparing to use } i^4=1) \\ &= (1)^{0}\cdot i && \text{(used } i^4=1) \\ &= i && \text{(ez as sundays)} \end{aligned}$$

58. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{-3}$$

Solution:

$$\begin{aligned} i^{-3} &= \left(i^{-4}\right) \cdot i^1 && \text{(preparing to use } i^4 = 1) \\ &= \left(i^4\right)^{-1} \cdot i^1 && \text{(preparing to use } i^4 = 1) \\ &= (1)^{-1} \cdot i && \text{(used } i^4 = 1) \\ &= i && \text{(ez as sundays)} \end{aligned}$$

59. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{-3}$$

Solution:

$$\begin{aligned} i^{-3} &= \left(i^{-4}\right) \cdot i^1 && \text{(preparing to use } i^4 = 1) \\ &= \left(i^4\right)^{-1} \cdot i^1 && \text{(preparing to use } i^4 = 1) \\ &= (1)^{-1} \cdot i && \text{(used } i^4 = 1) \\ &= i && \text{(ez as sundays)} \end{aligned}$$

60. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{11}$$

Solution:

$$\begin{aligned} i^{11} &= \left(i^{8}\right) \cdot i^1 && \text{(preparing to use } i^4 = 1) \\ &= \left(i^4\right)^{2} \cdot i^1 && \text{(preparing to use } i^4 = 1) \\ &= (1)^{2} \cdot i && \text{(used } i^4 = 1) \\ &= i && \text{(ez as sundays)} \end{aligned}$$

61. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{-6}$$

Solution:

$$\begin{aligned}
i^{-6} &= \left(i^{-8}\right) \cdot i^1 && \text{(preparing to use } i^4 = 1) \\
&= \left(i^4\right)^{-2} \cdot i^1 && \text{(preparing to use } i^4 = 1) \\
&= (1)^{-2} \cdot i && \text{(used } i^4 = 1) \\
&= i && \text{(ez as sundays)}
\end{aligned}$$

62. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{33}$$

Solution:

$$\begin{aligned}
i^{33} &= \left(i^{32}\right) \cdot i^1 && \text{(preparing to use } i^4 = 1) \\
&= \left(i^4\right)^{8} \cdot i^1 && \text{(preparing to use } i^4 = 1) \\
&= (1)^{8} \cdot i && \text{(used } i^4 = 1) \\
&= i && \text{(ez as sundays)}
\end{aligned}$$

63. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{-150}$$

Solution:

$$\begin{aligned}
i^{-150} &= \left(i^{-152}\right) \cdot i^1 && \text{(preparing to use } i^4 = 1) \\
&= \left(i^4\right)^{-38} \cdot i^1 && \text{(preparing to use } i^4 = 1) \\
&= (1)^{-38} \cdot i && \text{(used } i^4 = 1) \\
&= i && \text{(ez as sundays)}
\end{aligned}$$

64. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)^2$$

Solution:

$$\begin{aligned}
\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)^2 &= \left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right) \\
&= \frac{1}{2}+\frac{1}{2}i+\frac{1}{2}i+\frac{1}{2}i^2 && \text{(FOIL)} \\
&= \frac{1}{2}+\frac{1}{2}i+\frac{1}{2}i-\frac{1}{2} && \text{(used } i^2=-1\text{)} \\
&= i
\end{aligned}$$

65. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^2$$

Solution:

$$\begin{aligned}
\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^2 &= \left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) \\
&= \frac{3}{4}+\frac{\sqrt{3}}{4}i+\frac{\sqrt{3}}{4}i+\frac{1}{4}i^2 && \text{(FOIL)} \\
&= \frac{3}{4}+\frac{\sqrt{3}}{4}i+\frac{\sqrt{3}}{4}i-\frac{1}{4} && \text{(used } i^2=-1\text{)} \\
&= \frac{2}{4}+\frac{2\sqrt{3}}{4}i \\
&= \frac{1}{2}+\frac{\sqrt{3}}{2}i
\end{aligned}$$

66. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^3$$

Solution: we will use the result from the previous problem:

$$\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^2 = \frac{1}{2}+\frac{\sqrt{3}}{2}i \qquad \text{(prev. problem)}$$

$$\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^2\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) = \left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) \qquad \text{(mult both sides by } \left(\tfrac{\sqrt{3}}{2}+\tfrac{1}{2}i\right)\text{)}$$

$$\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^3 = \left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) \qquad \text{(simplify left side)}$$

$$= \frac{\sqrt{3}}{4}+\frac{3}{4}i+\frac{1}{4}i+\frac{\sqrt{3}}{4}i^2 \qquad \text{(FOIL)}$$

$$= \frac{\sqrt{3}}{4}+\frac{3}{4}i+\frac{1}{4}i-\frac{\sqrt{3}}{4} \qquad \text{(use } i^2=-1\text{)}$$

$$= i \qquad \text{(yipi-kae-yeh)}$$

67. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^6$$

Solution: just for kicks.. and for the simplicity of writing, let us call this number $\frac{\sqrt{3}}{2}+\frac{1}{2}i = A$. On the previous problem we demonstrated that $A^3 = i$, we will use this fact freely and without inhibitions ... now..

$$A^6 = \left(A^3\right)^2 \qquad \text{(getting ready to use } A^3 = i\text{)}$$

$$= (i)^2 \qquad \text{(used } A^3 = i\text{)}$$

$$= -1 \qquad \text{(ez as Sundays...)}$$

68. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^{30}$$

Solution: just for kicks.. and for the simplicity of writing, let us call this number $\frac{\sqrt{3}}{2}+\frac{1}{2}i = A$. On the previous problem we demonstrated that $A^3 = i$, we will use this fact freely and without inhibitions ... now..

$$A^6 = \left(A^3\right)^{10} \qquad \text{(getting ready to use } A^3 = i\text{)}$$

$$= (i)^{10} \qquad \text{(used } A^3 = i\text{)}$$

$$= -1 \qquad \text{(ez as Sundays...)}$$

69. **Inventing Numbers** The natural numbers are in many ways natural. In some way, all other numbers are unnatural byproducts of human imagination. Which number was *invented* just to solve the following equation?

$$3 + x = 3$$

Solution: 0 was innvented

70. **Inventing Numbers** Which type of numbers were *invented* to solve the following equation?

$$3 + x = 0$$

Solution: negative numbers were innvented

71. **Inventing Numbers** Which type of numbers were *invented* to solve the following equation?

$$3x = 1$$

Solution: rational numbers were innvented

72. **Inventing Numbers** Which type of numbers were *invented* to solve the following type of equation?

$$x^2 = 3$$

Solution: square roots of numbers were innvented

73. **Inventing Numbers** Contemplate the idea of a world of numbers of the form $a + b\sqrt{3}$ where a, b are rational numbers.
    (a) add $\frac{2}{3} + 5\sqrt{3} + \frac{7}{3} + \frac{3}{5}\sqrt{3}$
    (b) multiply $\left(\frac{2}{3} + 5\sqrt{3}\right)\left(\frac{7}{3} + \frac{3}{5}\sqrt{3}\right)$
    (c) does $\frac{2}{3} + 5\sqrt{3}$ have a multiplicative inverse of the form $a + b\sqrt{3}$ where a, b are rational.

74. Is $\sqrt{i}$ a complex number? if so can you write it in standard form?

Solution: yes.. the answer is on prob #35

