Q 1/9

Explain the steps required to find the length of a curve $y = f(x)$ between $x = a$ and $x = b$.

Choose the correct answer below.

A. Determine if f has a continuous derivative on [a,b]. If so, calculate $f'(x)$. Then evaluate the integral $\int_a^b \sqrt{1 + f'(x)}\,dx$.

B. Determine if f has a continuous derivative on [a,b]. If so, calculate $f'(x)$. Then evaluate the integral $\int_a^b (1 + f'(x))\,dx$.

C. Determine if f has a continuous derivative on [a,b]. If so, calculate $f'(x)$ and $f'(x)^2$. Then evaluate the integral $\int_a^b \sqrt{1 + f'(x)^2}\,dx$.

D. Determine if f has a continuous derivative on [a,b]. If so, calculate $f'(x)$ and $f'(x)^2$. Then evaluate the integral $\int_a^b \left(1 + f'(x)^2\right)dx$.

Write and simplify, but do not evaluate, an integral with respect to x that gives the length of the following curve on the given interval.

$y = 2\cos 3x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{4}\right]$

$f' = 2\frac{d}{dx}[\cos(3x)]$

$= 2[-\sin(3x)]\frac{d}{dx}(3x)$

$= -2\sin(3x)(3)$

$= -6\sin(3x)$

$L \int_a^b \sqrt{1+f'(x)^2}\,dx$

$$\boxed{\int_{-\frac{\pi}{2}}^{\frac{\pi}{4}} \sqrt{1+(-6\sin(3x))^2}\,dx}$$

Find the arc length of the following line.

$y = 4x + 2$; [0,4]

$$f' = \frac{d}{dx}(4x+2)$$
$$= 4$$

$$L \int_a^b \sqrt{1 + f'(x)^2}\, dx$$

$$\int_0^4 \sqrt{1+(4)^2}\, dx = \int_0^4 \sqrt{17}\, dx$$

Antiderivative

$$\frac{17^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{17^{\frac{3}{2}}}{\frac{3}{2}} = \frac{34^{\frac{3}{2}}}{3}$$

$$= \left[\sqrt{17}\, x\right]_0^4 = \left[\sqrt{17}(4) - \sqrt{17}(0)\right]$$

$$= \boxed{4\sqrt{17}}$$

Q 4/9

Find the arc length of the curve below on the given interval.

$$y = \frac{x^3}{3} + \frac{1}{4x} \text{ on } [1,3]$$

$$f' = \frac{d}{dx}\left(\frac{x^3}{3} + \frac{1}{4x}\right)$$

$$= x^2 - \frac{1}{4x^2}$$

$$L \int_a^b \sqrt{1 + f'(x)^2}\, dx$$

$$\int_1^3 \sqrt{1 + \left(x^2 - \frac{1}{4x^2}\right)^2}\, dx$$

$$\left(x^2 - \frac{1}{4x^2}\right)\left(x^2 - \frac{1}{4x^2}\right)$$

$$x^4 - \frac{1}{4}$$

$$-\frac{1}{4} + \frac{1}{16x^4}$$

$$x^4 - \frac{1}{2} + \frac{1}{16x^4}$$

$$= \int_1^3 \sqrt{1 + \left(x^4 - \frac{1}{2} + \frac{1}{16x^4}\right)}\, dx = \int_1^3 \sqrt{x^4 + \frac{1}{2} + \frac{1}{16x^4}}\, dx = \int_1^3 \sqrt{\left(x^2 + \frac{1}{4x^2}\right)^{\cancel{2}}}\, dx$$

$$= \int_1^3 \left(x^2 + \frac{1}{4x^2}\right) dx = \left[\frac{x^{2+1}}{2+1} + \frac{1}{4}\frac{x^{-2+1}}{-2+1}\right]_1^3 = \left[\frac{x^3}{3} + \frac{1}{4}\frac{x^{-1}}{-1}\right]_1^3 = \left[\frac{x^3}{3} - \frac{1}{4x}\right]_1^3$$

$$= \left[\left(\frac{(3)^3}{3} - \frac{1}{4(3)}\right) - \left(\frac{(1)^3}{3} - \frac{1}{4(1)}\right)\right]$$

$$= \left(9 - \frac{1}{12}\right) - \left(\frac{1}{3} - \frac{1}{4}\right)$$

$$= 9 - \frac{1}{12} - \frac{1}{3} + \frac{1}{4} = \boxed{\frac{53}{6}}$$

Q 5/9

a. Write and simplify the integral that gives the arc length of the following curve on the given interval.
b. If necessary, use technology to evaluate or approximate the integral.

$y = -2x^2 + 3$, for $-2 \le x \le 1$

$f' = \frac{d}{dx}\left(-2x^2 + 3\right)$

$= -4x$

$f'^2 = (-4x)^2$

$= 16x^2$

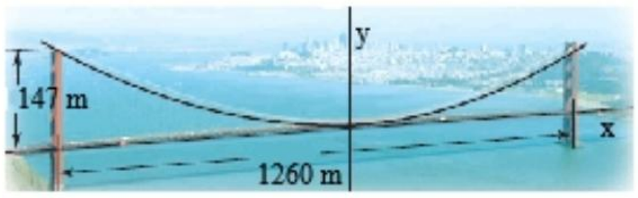
$L \int_a^b \sqrt{1 + f'(x)^2}\, dx$

$\int_{-2}^{1} \sqrt{1 + 16x^2}\, dx$

$\approx \boxed{10.733}$

Q 6/9

The profile of the cables on a suspension bridge may be modeled by a parabola. The central span of the bridge is 1260 m long and 147 m high. The parabola $y = 0.00037x^2$ gives a good fit to the shape of the cables, where $|x| \leq 630$, and x and y are measured in meters. Approximate the length of the cables that stretch between the tops of the two towers.



$$f' = \frac{d}{dx}(0.00037x^2)$$

$$= 0.00074x$$

$$f'^2 = (0.00074x)^2$$

$$= 5.476 \times 10^{-7} x^2$$

$$L \int_a^b \sqrt{1 + f'(x)^2}\, dx$$

$$\int_{-630}^{630} \sqrt{1 + 5.476 \times 10^{-7} x^2}\, dx$$

$$= \boxed{1304 \text{ meters}}$$

Q 7/9

Find a curve that passes through the point (1,5) and has an arc length on the interval [2,6] given by $\int_2^6 \sqrt{1+16x^{-6}}\,dx$?

$L \int_a^b \sqrt{1+f'(x)^2}\,dx$

Reverse Engineer $144x^{-6}$

$$f(x) = \sqrt{16x^{-6}}$$

$$= \sqrt{\frac{16}{x^6}}\,dx = \frac{\sqrt{16}}{\sqrt{x^6}}\,dx = \pm\frac{4}{x^3}\,dx$$

$$= \int \frac{4}{x^3}\,dx = 4\left[\frac{x^{-3+1}}{-3+1}\right] = 4\left[\frac{x^{-2}}{-2}\right] = -2\left(\frac{1}{x^2}\right) = -\frac{2}{x^2} + C$$

or

$$= \int -\frac{4}{x^3}\,dx = -4\left[\frac{x^{-3+1}}{-3+1}\right] = -4\left[\frac{x^{-2}}{-2}\right] = 2\left(\frac{1}{x^2}\right) = \frac{2}{x^2} + C$$

The curve $y = f(x) = \pm\frac{2}{x^2} + C$ passes through the point (1,5)

$-\frac{2}{x^2} + C$

$5 = -\frac{2}{(1)^2} + C$

$5 = -2 + C$

$5-2 = C$

$7 = C$

$\frac{2}{x^2} + C$

$5 = \frac{2}{(1)^2} + C$

$5 = 2 + C$

$5-2 = C$

$3 = C$

$$\boxed{-\frac{2}{x^2} + 7 \quad \text{or} \quad \frac{2}{x^2} + 3}$$

Q 8/9

Which curve has the greater length on the interval $[-1,1]$, $y = 1 - x^2$ or $y = \cos(\pi x / 2)$?

$$f_0' = \frac{d}{dx}(1 - x^2)$$
$$= -2x$$
$$f_0'^2 = (-2x)^2$$
$$= 4x^2$$

$$f_1' = \frac{d}{dx}\left[\cos\left(\frac{\pi}{2}x\right)\right]$$
$$= -\sin\left(\frac{\pi}{2}x\right)\frac{\pi}{2}$$
$$f_1'^2 = \left[-\sin\left(\frac{\pi}{2}x\right)\frac{\pi}{2}\right]^2$$
$$= \sin^2\left(\frac{\pi}{2}x\right)\frac{\pi^2}{4}$$
$$= \frac{\pi^2 \sin^2\left(\frac{\pi}{2}x\right)}{4}$$

$$\int_{-1}^{1} \sqrt{1 + 4x^2}\, dx$$

$$\int_{-1}^{1} \sqrt{1 + \frac{\pi^2 \sin\left(\frac{\pi}{2}x\right)}{4}}\, dx$$

≈ 2.958

≈ 2.927

curve $y = 1 - x^2$ has the greater length

Q 9/9

Suppose the graph of f on the interval [a,b] has length L, where f′ is continuous on [a,b]. Evaluate the following integrals in terms of L.

a. $\int_{a/2}^{b/2} \sqrt{1+f'(2x)^2}\,dx$

b. $\int_{a/c}^{b/c} \sqrt{1+f'(cx)^2}\,dx$ if $c \neq 0$

a. Evaluate $\int_{a/2}^{b/2} \sqrt{1+f'(2x)^2}\,dx$ in terms of L. Select the correct answer below.

- A. $\int_{a/2}^{b/2} \sqrt{1+f'(2x)^2}\,dx = 2L$
- B. $\int_{a/2}^{b/2} \sqrt{1+f'(2x)^2}\,dx = L$
- ✔ C. $\int_{a/2}^{b/2} \sqrt{1+f'(2x)^2}\,dx = \frac{L}{2}$
- D. $\int_{a/2}^{b/2} \sqrt{1+f'(2x)^2}\,dx = \frac{2L}{3}$

b. Evaluate $\int_{a/c}^{b/c} \sqrt{1+f'(cx)^2}\,dx$ in terms of L. Select the correct answer below.

- A. $\int_{a/c}^{b/c} \sqrt{1+f'(cx)^2}\,dx = Lc^2$
- ✔ B. $\int_{a/c}^{b/c} \sqrt{1+f'(cx)^2}\,dx = \frac{L}{c}$
- C. $\int_{a/c}^{b/c} \sqrt{1+f'(cx)^2}\,dx = \frac{L}{c^2}$
- D. $\int_{a/c}^{b/c} \sqrt{1+f'(cx)^2}\,dx = Lc$