Q1

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suppose A lies between 0 and 90^o and

$$\sec A + \tan A = 4$$

determine $\cos(A)$

$\sec A + \tan A = 4$

$(\sec A)^2 + \left(\sqrt{\sec^2 A - 1}\right)^2 = (4)^2$

$\sec^2 A + \sec^2 A - 1 = 16$

$2\sec^2 A = 16 - 1$

$\frac{2\sec^2 A}{2} = \frac{15}{2}$

$\sec^2 A = \frac{15}{2}$

$\sec^2 A = \frac{15}{2}$

$\cancel{\cos^2 A} \cdot \frac{1}{\cancel{\cos^2 A}} = \frac{15}{2} \cdot \cos^2 A$

$\frac{1}{\frac{15}{2}} = \frac{\frac{\cancel{15}}{\cancel{2}} \cos^2 A}{\frac{\cancel{15}}{\cancel{2}}}$

$\frac{2}{15} = \cos^2 A$

$\sqrt{\frac{2}{15}} = \cos A$

$\boxed{\text{c) none of these}}$

Q2

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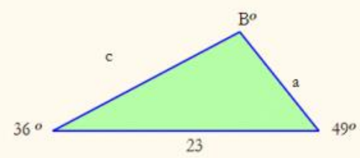
Suppose we know a triangle has $A = 36^o$, $b = 23$, $C = 49^o$, Solve the missing items, $[B^o, a, c]$.



Law of sines $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

or

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$B^\circ = 180^\circ - (36^\circ + 49^\circ)$

$\underline{\underline{B^\circ = 95^\circ}}$

$\sin(49^\circ) \dfrac{23}{\sin(95^\circ)} = \dfrac{c}{\cancel{\sin(49^\circ)}} \cancel{\sin(49^\circ)}$

$\sin(49^\circ) \cdot \dfrac{23}{\sin(95^\circ)} = c$

$\underline{\underline{\approx 17.425 = c}}$

$\sin(36^\circ) \dfrac{23}{\sin(95^\circ)} = \dfrac{a}{\cancel{\sin(36^\circ)}} \cancel{\sin(36^\circ)}$

$\sin(36^\circ) \cdot \dfrac{23}{\sin(95^\circ)} = a$

$\approx 13.571 = a$

$\boxed{B)\ [95^\circ, 13.571, 17.425]}$

Q3

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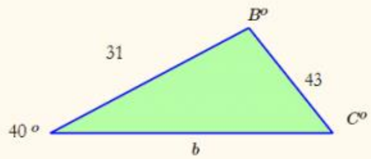
Suppose we know a triangle has $A = 40^o$, $c = 31$, $a = 43$, solve the missing items, $[b, C^o, B^o]$,



Law of sines $\quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$31 \quad \frac{\sin(40^\circ)}{43} = \frac{\sin C^\circ}{\cancel{31}} \quad \cancel{31}$$

$$31 \cdot \frac{\sin(40^\circ)}{43} = \sin C^\circ$$

$$\sin^{-1}\left(31 \cdot \frac{\sin(40^\circ)}{43}\right) = C^\circ$$

$$\approx 27.607 = C^\circ$$

$$B^\circ = 180^\circ - 40^\circ - 27.607$$

$$B^\circ \approx 112.393^\circ$$

$$\sin(112.393^\circ) \frac{43}{\sin(40^\circ)} = \frac{b}{\cancel{\sin(112.393^\circ)}} \quad \cancel{\sin(112.393^\circ)}$$

$$\sin(112.393^\circ) \cdot \frac{43}{\sin(40^\circ)} = b$$

$$\approx 61.852 = b$$

$$\boxed{B) \ [61.8517, 27.6070^\circ, 112.393^\circ]}$$

Q4

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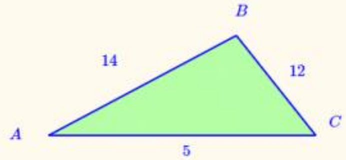
Suppose we know a triangle has $c = 14$, $b = 5$, $a = 12$, WHICH application of the LAW of COSINES



would result in an equation with only ONE unknown quantity?

Law of Cosines $a^2 = b^2 + c^2 - 2bc\cos(A)$

A) $c^2 = b^2 + a^2 - 2ba\cos(C)$

B) $a^2 = b^2 + c^2 - 2bc\cos(A)$

Q5

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Consider the following trigonometric equation

$$2\cos(3x) - 5 = 0$$

$$2\cos(3x) - 5 = 0$$

$$2\cos(3x) = 0 + 5$$

$$\frac{2\cos(3x)}{2} = \frac{5}{2}$$

$$\cos(3x) = \frac{5}{2} \rightarrow$$ cos(x) can't be greater than one for real solution

A) no real solution

Q6

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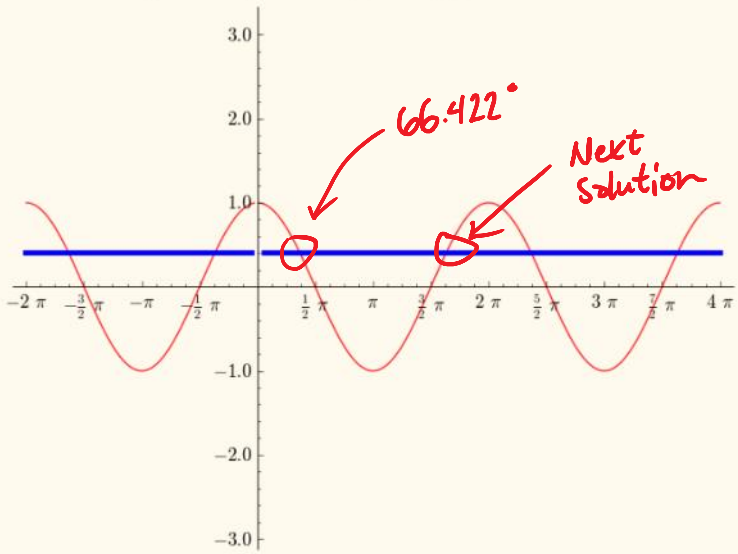
One solution to the equation $\frac{2}{5} = \cos(x)$ is approx $\mathbf{66.422^o}$



What is the NEXT real solution to the right ?

A) $x = 293.58^\circ$

B) $x = 360^\circ - 66.422$

Q7

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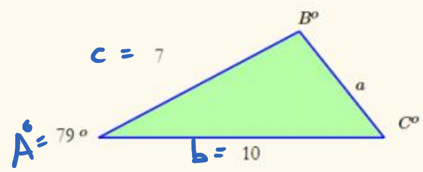
Suppose we know a triangle has $A = 79^o$, $b = 10$, $c = 7$, Solve the triangle:



Law of Cosines $a^2 = b^2 + c^2 - 2bc\cos(A)$

Law of Sines $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

or

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = 10^2 + 7^2 - 2(10)(7)\cos(79^\circ)$$

$$a^2 \approx 122.2867$$

$$a \approx \sqrt{122.2867}$$

$$a \approx 11.06$$

$$10 \cdot \frac{\sin(79^\circ)}{11.06} = \frac{\sin(B^\circ)}{\cancel{10}} \cdot \cancel{10}$$

$$10 \cdot \frac{\sin(79^\circ)}{11.06} = \sin(B^\circ)$$

$$\sin^{-1}\left(10 \cdot \frac{\sin(79^\circ)}{11.06}\right) = B^\circ$$

$$\approx 62.58 = B^\circ$$

$$7 \cdot \frac{\sin(79^\circ)}{11.06} = \frac{\sin(C^\circ)}{7} \cdot 7$$

$$7 \cdot \frac{\sin(79^\circ)}{11.06} = \sin(C^\circ)$$

$$\sin^{-1}\left(7 \cdot \frac{\sin(79^\circ)}{11.06}\right) = C^\circ$$

$$\approx 38.42 = C^\circ$$

$$\boxed{c)\ [B, a, C] = [62.58, 11.06, 38.42]}$$

Q8

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(Assume none of the quantities are zero) Suppose we know

$$\frac{A}{B} = \frac{X}{Y}$$

Select variations of the same statement as above [i.e. equivalent statements.]

$$\cancel{B} \cdot \frac{A}{\cancel{B}} = \frac{X}{Y} \cdot B \qquad \frac{A}{B} \times \frac{X}{Y} \qquad X \cdot \frac{B}{A} = \frac{Y}{\cancel{X}} \cdot \cancel{X}$$

$$A = \frac{BX}{Y} \qquad AY = BX \qquad \frac{BX}{A} = Y$$

A) $A = \frac{BX}{Y}$ B) A is to B as X is to Y

C) $\frac{B}{A} = \frac{Y}{X}$ D) X is to Y as A is to B

E) $AY = BX$ F) $\frac{Y}{X} = \frac{B}{A}$

G) $\frac{BX}{A} = Y$

Q9

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Consider the following trigonometric equation

$$\sin(x) = 4\sin^2(x)$$

$$\frac{1}{\sin(x)} \cdot \sin(x) = 4\sin^2(x) \cdot \frac{1}{\sin(x)}$$

$$\frac{\cancel{\sin(x)}}{\cancel{\sin(x)}} = \frac{4\sin^{\cancel{2}}(x)}{\cancel{\sin(x)}}$$

$$\underline{1 = 4\sin(x)}$$

$$\boxed{A)\ 1 = 4\sin(x)}$$

Q10

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Consider the following trigonometric equation

$$2\cos(3x) + \cos(2x) + 1 = 0$$

$$2(4\cos^3(x) - 3\cos(x)) + 2\cos^2(x) - 1 + 1 = 0$$

$$2(4\cos^3(x) - 3\cos(x)) + \cos(2x) + 1 = 0$$

$\cos(2x) = 2\cos^2(x) - 1$

$$\frac{2(4\cos^3(x) - 3\cos(x))}{2} = \frac{2\cos(3x)}{2}$$

$$4\cos^3(x) - 3\cos(x) = \cos(3x)$$

$$= \cos(2x + x)$$

summation identity

$$= \cos(2x)\cos(x) + \sin(2x)\sin(x)$$

double angle identity

$$= (2\cos^2(x) - 1)\cos(x) + \sin(2x)\sin(x)$$

double angle identity

$$= (2\cos^2(x) - 1)\cos(x) - 2\sin(x)\cos(x)\sin(x)$$

$$= 2\cos^3(x) - \cos(x) - 2\sin^2(x)\cos(x)$$

Pythagoras identity

$$= 2\cos^3(x) - \cos(x) - 2(1 - \cos^2(x))\cos(x)$$

$$= -\cos(x) + 2\cos^3(x) - 2\cos(x)(1 - \cos^2(x))$$ → Rearrange

$$= -\cos(x) + 2\cos^3(x) - 2\cos(x) + 2\cos^3(x)$$

$$4\cos^3(x) - 3\cos(x) = 4\cos^3(x) - 3\cos(x)$$

$$2\cos(3x) + \cos(2x) + 1 = 0$$

$$\therefore\ 2\cos(3x) = 2(4\cos^3(x) - 3\cos(x))$$

B) $2(4\cos^3(x) - 3\cos(x)) + 2\cos^2(x) - 1 + 1 = 0$

C) $\cos(2x) = 2\cos^2(x) - 1$ is helpful

Q11

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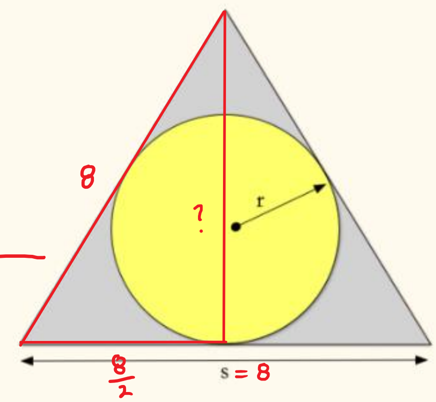
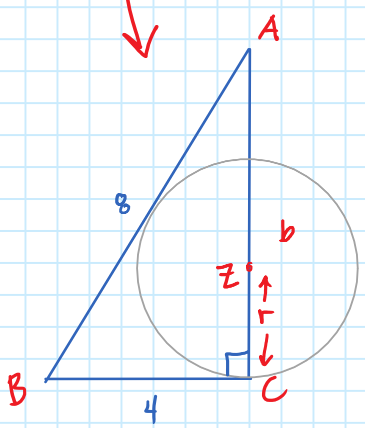
Suppose a circle is inscribed in an equilateral triangle. Find the radius r if the side is $s = 8$





$8^2 = 4^2 + b^2$

$64 = 16 + b^2$

$64 - 16 = b^2$

$\sqrt{48} = b$

$\overline{AZ} : \overline{ZC} = 2 : 1$

$AZ = 2r, \ ZC = r$

$AC = 3r$

$\sqrt{48} = 3r$

$$\frac{\sqrt{48}}{3} = r$$

c) none of these

Q12

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Consider the following trigonometric equation

$$\sin(x) = \frac{1}{\sqrt{3}}$$

$$\left(\sin(x)\right)^2 = \left(\frac{1}{\sqrt{3}}\right)^2$$

$$\sin^2(x) = \frac{1}{3}$$

$$\boxed{\text{A) } \sin^2(x) = \frac{1}{3}}$$