Q1

Thursday, November 12, 2020 4:26 PM

State whether the following is true or false by differentiation.

$$\int \frac{9}{x^2\sqrt{1+x^2}}\,dx = -\frac{9\sqrt{1+x^2}}{x} + C$$

- (•) True
- ( ) False

$$\int \frac{9}{x^2\sqrt{1+x^2}}\,dx = -\frac{9\sqrt{1+x^2}}{x} + C$$

$$\frac{d}{dx}\left[-\frac{9\sqrt{1+x^2}}{x} + C\right] = \frac{d}{dx}\left[-\frac{9(1+x^2)^{1/2}}{x}\right]$$

$$= -\frac{x\,9\frac{d}{dx}\left[(1+x^2)^{1/2}\right] - 9(1+x^2)^{1/2}\frac{d}{dx}(x)}{x^2}$$

$$= -\frac{9x\left[\frac{1}{2}(1+x^2)^{-1/2}\right]\frac{d}{dx}(1+x^2) - 9(1+x^2)^{1/2}}{x^2}$$

$$= -\frac{\frac{9x}{\cancel{2}}(1+x^2)^{-1/2}(\cancel{2}x) - 9(1+x^2)^{1/2}}{x^2}$$

$$= \frac{9x^2(1+x^2)^{-1/2} - 9(1+x^2)^{1/2}}{x^2}$$

$$= \frac{9(1+x^2)^{-1/2}\left[\cancel{x^2} - (1+\cancel{x^2})\right]}{x^2}$$

$$= \frac{9\frac{1}{\sqrt{1+x^2}}(-1)}{x^2} = \frac{9}{x^2\sqrt{1+x^2}}$$

Q2

Sunday, November 15, 2020 12:23 PM

Find the general indefinite integral. (Use C for the constant of integration.)

$$\int (x^{1.9} + 9x^{3.5})\, dx$$

$$\boxed{\frac{x^{2.9}}{2.9} + 2x^{4.5} + C}$$

$$\int (x^{1.9} + 9x^{3.5})\, dx = (x^{1.9} + 9x^{3.5})$$

$$= \frac{x^{1.9+1}}{1.9+1} + \frac{9x^{3.5+1}}{3.5+1} + C$$

$$= \frac{x^{2.9}}{2.9} + \frac{9x^{4.5}}{4.5} + C$$

$$= \boxed{\frac{x^{2.9}}{2.9} + 2x^{4.5} + C}$$

Q3

Sunday, November 15, 2020 1:24 PM

Find the general indefinite integral. (Use C for the constant of integration.)

$$\int \sqrt[8]{x^9}\, dx$$

$$\frac{8}{17}x^{\left(\frac{17}{8}\right)} + C$$

$$\int \sqrt[8]{x^9}\, dx = \left[(x^9)^{1/8} \right]$$

$$= (x^9)^{1/8} = x^{9/8}$$

$$= \frac{x^{9/8+1}}{\frac{9}{8}+1} + C$$

$$= \frac{x^{17/8}}{\frac{17}{8}} + C = x^{17/8} \cdot \frac{8}{17} + C$$

$$= \boxed{\frac{8x^{17/8}}{17} + C}$$

Q4

Sunday, November 15, 2020 1:41 PM

Find the general indefinite integral. (Use C for the constant of integration.)

$$\int (u + 8)(5u + 5)\, du$$

$$\boxed{\frac{5}{3}u^3 + \frac{45}{2}u^2 + 40u + C}$$

$5u^2 + 5u + 40u + 40$

$$\int (u+8)(5u+5)\,du = \left[(u+8)(5u+5)\right] = \left[5u^2 + 45u + 40\right]$$

$$= \frac{5u^{2+1}}{2+1} + \frac{45u^{1+1}}{1+1} + 40u + C$$

$$= \boxed{\frac{5}{3}u^3 + \frac{45}{2}u^2 + 40u + C}$$

Q5

Sunday, November 15, 2020 2:24 PM

Find the general indefinite integral. (Use C for the constant of integration. Remember to use absolute values where appropriate.)

$$\int \frac{9 + \sqrt{x} + x}{x}\,dx$$

$$9\ln(|x|) + 2\sqrt{x} + x + C$$

$$\int \frac{9+\sqrt{x}+x}{x}\,dx = \left[\frac{9+\sqrt{x}+x}{x}\right] = \left[\frac{9}{x} + \frac{x^{1/2}}{x} + \frac{x}{x}\right]$$

$$= \left[9\frac{1}{x} + \frac{1}{x^{1-1/2}} + 1\right]$$

$$= 9\ln|x| + \frac{1}{x^{1/2}} + x + C$$

$$= 9\ln|x| + x^{-1/2} + x + C$$

$$= 9\ln|x| + \frac{x^{-1/2+1}}{-\frac{1}{2}+1}\; x + C \qquad \frac{x^{1/2}}{\frac{1}{2}} = 2\sqrt{x}$$

$$= \boxed{9\ln|x| + 2\sqrt{x} + x + C}$$

Q6

Sunday, November 15, 2020 2:35 PM

Find the general indefinite integral. (Use C for the constant of integration.)

$$\int (9\sin(x) + 6\sinh(x))\,dx$$

$$\boxed{-9\cos(x) + 6\cosh(x) + C}$$

$$\int (9\sin(x) + 6\sinh(x))\,dx = \left[9\sin(x) + 6\sinh(x)\right]$$

$$= \boxed{-9\cos(x) + 6\cosh(x) + C}$$

Q7

Sunday, November 15, 2020 3:52 PM

Find the general indefinite integral. (Use C for the constant of integration.)

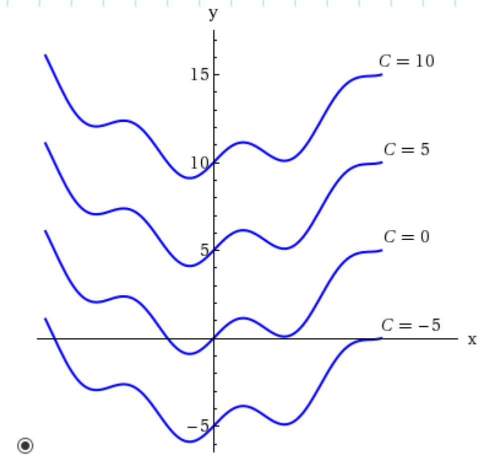
$$\int \left(\cos(x) + \frac{1}{9}x\right)dx$$

$$\boxed{\sin(x) + \frac{x^2}{18} + C}$$

$$\int \left(\cos(x) + \frac{1}{9}x\right) dx = \left[\cos(x) + \frac{1}{9}(x)\right]$$

$$= \sin(x) + \frac{x^{1+1}}{9(1+1)} + C$$

$$= \boxed{\sin(x) + \frac{x^2}{18} + C}$$



Q8

Sunday, November 15, 2020 4:01 PM

Evaluate the integral.

$$\int_{-2}^{3} (x^2 - 3)\, dx$$

$$-\frac{10}{3}$$

$$\int_{-2}^{3} (x^2 - 3)\, dx = \left[x^2 - 3\right]_{-2}^{3} = \left[\frac{x^{2+1}}{2+1} - 3x + C\right] = \left[\frac{x^3}{3} - 3x + C\right]$$

$$= F(3) - F(-2)$$

$$= \left(\frac{3^3}{3} - 3(3)\right) - \left(\frac{-2^3}{3} - 3(-2)\right)$$

$$= (9 - 9) - \left(-\frac{8}{3} + 6\right)$$

$$= \frac{8}{3} - 6 = \frac{8 - 18}{3} = \boxed{-\frac{10}{3}}$$

Q9

Sunday, November 15, 2020 4:08 PM

Evaluate the integral.

$$\int_{-2}^{0}\left(\frac{1}{4}t^5+\frac{1}{6}t^4-t\right)dt$$

$\frac{2}{5}$

$$\int_{-2}^{0}\left(\frac{1}{4}t^5+\frac{1}{6}t^4-t\right)dt=\left[\frac{1}{4}t^5+\frac{1}{6}t^4-t+C\right]_{-2}^{0}=\left[\frac{t^{5+1}}{4(5+1)}+\frac{t^{4+1}}{6(4+1)}-\frac{t^{1+1}}{1+1}+C\right]$$

$$=\left(\frac{t^6}{24}+\frac{t^5}{30}-\frac{t^2}{1+1}+C\right)_{-2}^{0}$$

$$=F(0)-F(-2)$$

$$=\left(\frac{0^6}{24}+\frac{0^5}{30}-\frac{0^2}{2}\right)-\left(\frac{(-2)^6}{24}+\frac{(-2)^5}{30}-\frac{(-2)^2}{2}\right)$$

$$=\boxed{\frac{2}{5}}$$

Q10

Sunday, November 15, 2020 5:02 PM

Evaluate the integral.

$$\int_0^{\pi} (9e^x + 8\sin(x))dx$$

$9e^{\pi} + 7$

$$\int_0^{\pi} \left(9e^x + 8\sin(x)\right) = \left[9e^x + 8\sin(x) + C\right]_0^{\pi}$$

$$= \left[9e^x - 8\cos(x) + C\right]_0^{\pi}$$

$$= F(\pi) - F(0)$$

$$= \left[9e^{\pi} - 8\cos(\pi)\right] - \left[9e^0 - 8\cos(0)\right]$$

$$= \left[9e^{\pi} - 8(-1)\right] - \left[9 - 8(1)\right]$$

$$= 9e^{\pi} + 8 - 9 + 8 = \boxed{9e^{\pi} + 7}$$

Q11

Sunday, November 15, 2020 5:13 PM

Evaluate the integral.

$$\int_0^1 x\left(8\sqrt[3]{x} + 7\sqrt[4]{x}\right)\,dx$$

$\frac{412}{63}$

$$\int_0^1 x\left(8\sqrt[3]{x}+7\sqrt[4]{x}\right)dx = \left[x\left(8x^{1/3}+7x^{1/4}\right)\right]_0^1 = \left(8x^{4/3}+7x^{5/4}\right)\Big|_0^1$$

$$= \left(\frac{8x^{4/3+1}}{\frac{4}{3}+1} + \frac{7x^{5/4+1}}{\frac{5}{4}+1} + C\right)\Bigg|_0^1$$

$$= \frac{8x^{7/3}}{\frac{7}{3}} + \frac{7x^{9/4}}{\frac{9}{4}} + C = \frac{24x^{7/3}}{7} + \frac{28x^{9/4}}{9} + C$$

$$= F(1) + F(0)$$

$$= \left(\frac{24(1)^{7/3}}{7} + \frac{28(1)^{9/4}}{9}\right) - \left(\frac{24(0)^{7/3}}{7} + \frac{28(0)^{9/4}}{9}\right)$$

$$= \boxed{\frac{412}{63}}$$

Q12

Sunday, November 15, 2020 5:28 PM

Evaluate the integral.

$$\int_1^8 \left(\frac{x}{8} - \frac{6}{x}\right) dx$$

$$\frac{63}{16} - 6\ln(8)$$

$$\int_1^8 \left(\frac{x}{8} - \frac{6}{x}\right) dx = \left[\frac{1}{8}x - 6\frac{1}{x}\right]_1^8$$

$$= \left[\frac{x^{1+1}}{8(1+1)} - 6\ln|x| + C\right]_1^8$$

$$= \left[\frac{x^2}{16} - 6\ln|x|\right]_1^8$$

$$= F(8) - F(1)$$

$$= \left[\frac{8^2}{16} - 6\ln(8)\right] - \left[\frac{1^2}{16} - 6\ln(1)\right]$$

$$= \frac{64}{16} - 6\ln(8) - \frac{1}{16} - \cancel{6\ln(1)} = 0$$

$$= \boxed{\frac{63}{16} - 6\ln(8)}$$

Q13

Sunday, November 15, 2020 5:39 PM

Evaluate the integral.

$$\int_0^{\pi/4} \frac{9 + 7\cos^2(\theta)}{\cos^2(\theta)}\, d\theta$$

$$9 + \frac{7\pi}{4}$$

Let $x = \theta$

$$\int_0^{\pi/4} \frac{9+7\cos^2(x)}{\cos^2(x)}\,dx = \left[9\frac{1}{\cos^2(x)} + \frac{7\cos^2(x)}{\cos^2(x)}\right]_0^{\pi/4} = \left[9\sec^2(x) + 7\right]$$

$$= \left[9\tan(x) + 7x\right]_0^{\pi/4}$$

$$= F\left(\frac{\pi}{4}\right) - F(0)$$

$$= \left[9\tan\left(\frac{\pi}{4}\right) + 7\left(\frac{\pi}{4}\right)\right] - \left[\underbrace{\cancel{9\tan(0) + 7(0)}}_{=0}\right]$$

$$= \boxed{9 + \frac{7\pi}{4}}$$

Q14

Sunday, November 15, 2020 5:57 PM

Evaluate the integral.

$$\int_0^{1/\sqrt{3}} 4\,\frac{t^2-1}{t^4-1}\,dt$$

$$\boxed{4\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)}$$

$$\int_0^{1/\sqrt{3}} 4\,\frac{t^2-1}{t^4-1}\,dt = \left[4\,\frac{t^2-1}{(t^2-1)(t^2+1)}\right]_0^{1/\sqrt{3}}$$

$$= \left[4\,\frac{1}{t^2+1}\right] = \left[4\arctan(t)+C\right]$$

$$= F\left(\tfrac{1}{\sqrt{3}}\right) - F(0)$$

$$= 4\tan^{-1}\left(\tfrac{1}{\sqrt{3}}\right) - \underbrace{\cancel{4\tan^{-1}(0)}}_{=0}$$

$$= \boxed{4\tan^{-1}\left(\tfrac{1}{\sqrt{3}}\right)}$$

Q15

Sunday, November 15, 2020 6:05 PM

The velocity function (in meters per second) is given for a particle moving along a line.

$v(t) = 3t - 7, \quad 0 \le t \le 4$

(a) Find the displacement.

-4 m

(b) Find the distance traveled by the particle during the given time interval.

37/3 m

$v(t) = 3t - 7, \quad 0 \le t \le 4$

$v(t) =$ Rate of change of position $= S'(t)$, where S is the position function

$$\text{Displacement} = S(t) = \int_0^4 v(t)\,dt = \int_0^4 (3t-7)\,dt$$

$$= \frac{3t^2}{2} - 7t + C \Big|_0^4$$

$$= F(4) - F(0)$$

$$= \left[\frac{3(4)^2}{2} - 7(4)\right] - \underbrace{\left[\cancel{\frac{3(0)^2}{2}} - \cancel{7(0)}\right]}_{=0}$$

$$= (24 - 28)$$

$$= \boxed{-4}$$

$$\begin{array}{ccc} & S(t) & \\ \int dt \uparrow & & \downarrow \frac{d}{dt} \\ & v(t) & \\ \int dt \uparrow & & \downarrow \frac{d}{dt} \\ & a(t) & \end{array}$$

$$\text{distance} = (\text{rate})(\text{time}) = \int_a^b |v(t)|\,dt = \lim_{n \to \infty} \sum_{i=1}^{n} \underbrace{|v(t_i^*)|}_{\text{Rate}} \underbrace{\Delta t}_{\text{Time}}$$

Absolute value to calculate distance traveled

$$\text{Distance} = \int_0^4 |3t - 7|\,dt$$

$$|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$|3t - 7| = \begin{cases} 3t - 7 & \text{if } 3t - 5 \ge 0 \\ 7 - 3t & \text{if } 3t - 5 < 0 \end{cases}$$

$$= \int_0^{7/3} (7 - 3t)\,dt + \int_{7/3}^4 (3t - 7)\,dt$$

$3t - 7 \ge 0 \qquad 3t - 7 < 0$

$t \ge \frac{7}{3} \qquad t < \frac{7}{3}$

$$= \left[7t - \frac{3t^2}{2}\right]_0^{7/3} + \left[\frac{3t^2}{2} - 7t\right]_{7/3}^4$$

At $\frac{7}{3}$, the argument changes from + to − and vice-versa

$\therefore$ the integral is split into two

$$= \left[7\left(\tfrac{7}{3}\right) - \frac{3\left(\frac{7}{3}\right)^2}{2}\right] - \left[7(0) - \frac{3(0)^2}{2}\right] + \left[\frac{3(4)^2}{2} - 7(4)\right] - \left[\frac{3\left(\frac{7}{3}\right)^2}{2} - 7\left(\tfrac{7}{3}\right)\right]$$

$$= \boxed{\frac{37}{3}}$$

Q16

Sunday, November 15, 2020 7:28 PM

The acceleration function (in m/s^2) and the initial velocity are given for a particle moving along a line.

$a(t) = t + 4, \quad v(0) = 4, \quad 0 \le t \le 11$

(a) Find the velocity at time t.

$v(t) = \boxed{\frac{t^2}{2} + 4t + 4}$ m/s

(b) Find the distance traveled during the given time interval.

3047/6 m

$a(t) = t + 4, \quad v(0) = 4, \quad 0 \le t \le 11$

$a(t)$ = Rate of change of $v(t)$ $= v'(t)$

$v(t) = \int (t+4)\,dt = \frac{t^2}{2} + 4t + C$

$v(0) = 4$

$4 = \frac{0^2}{2} + 4(0) + C$

$C = 4$

$$\boxed{v(t) = \frac{t^2}{2} + 4t + 4 \text{ m/s}}$$

$$\text{distance} = (\text{rate})(\text{time}) = \int_a^b |v(t)|\,dt = \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{|v(t_i^*)|}_{\text{Rate}} \underbrace{\Delta t}_{\text{Time}}$$

Absolute value to calculate distance traveled

$$\text{Distance} = \int_0^{11} \left| \frac{t^2}{2} + 4t + 4 \right| dt$$

No need to split between + and − since the function will never be zero when $0 \le t \le 11$

$$\left|\frac{t^2}{2} + 4t + 4\right| = \begin{cases} \frac{t^2}{2} + 4t + 5 & \text{if } \frac{t^2}{2} + 4t + 5 \ge 0 \\ -\left(\frac{t^2}{2} + 4t + 5\right) & \text{if } \frac{t^2}{2} + 4t + 5 < 0 \end{cases}$$

$$= \int_0^{11} \left(\frac{t^2}{2} + 4t + 4\right) dt$$

$$= \frac{t^{2+1}}{2(2+1)} + \frac{4t^2}{2} + 4t + C \Big|_0^{11} = \frac{t^3}{6} + 2t^2 + 4t + C \Big|_0^{11}$$

$$= F(11) - F(0)$$

$$= \left(\frac{11^3}{6} + 2(11)^2 + 4(11)\right) - \left(\frac{0^3}{6} + 2(0)^2 + 4(0)\right)$$

$$= \boxed{\frac{3047}{6}}$$