Q1

Sunday, October 18, 2020 2:55 PM

1) Find the absolute maximum and minimum of $f(x) = x + \frac{1}{x}$ on $\left[\frac{1}{2}, 5\right]$ by using algebra and calculus. You must show your work.

$f(x) = x + \frac{1}{x}$, $\left[\frac{1}{2}, 5\right]$

$f'(x) = \frac{d}{dx}\left(x + \frac{1}{x}\right)$

$= 1 + \frac{d}{dx}\left(x^{-1}\right)$

$= 1 + \left(-x^{-2}\right)$

$f'(x) = 1 - \frac{1}{x^2}$

Get critical points

$1 - \frac{1}{x^2} = 0$

$\cancel{(x^2)}\ \frac{1}{\cancel{x^2}} = 1\ (x^2)$

$1 = x^2$

$x = 1$

$f(c) = 0$

$f(5) = x + \frac{1}{x}$

$= 5 + \frac{1}{5}$

$= \frac{25}{5} + \frac{1}{5}$

$\boxed{f(5) = \frac{26}{5} \longrightarrow \text{Absolute max}}$

$f\left(\frac{1}{2}\right) = x + \frac{1}{x}$

$= \frac{1}{2} + \frac{1}{\frac{1}{2}}$

$= \frac{1}{2} + 2$

$= \frac{1}{2} + \frac{4}{2}$

$f\left(\frac{1}{2}\right) = \frac{5}{2}$

$f(1) = x + \frac{1}{x}$

$= 1 + \frac{1}{1}$

$= 1 + 1$

$\boxed{f(1) = 2 \longrightarrow \text{Absolute min}}$

Q2

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2) Find the "c" that satisfies the conclusion of the Mean Value Theorem for $f(x) = \sqrt{x}$ On the interval [0,4].

$f(x) = \sqrt{x}$, $[0,4]$

$f(0) = \sqrt{x}$

$= \sqrt{0}$

$f(0) = 0$

$f(4) = \sqrt{x}$

$= \sqrt{4}$

$= 2$

$f'(x) = \frac{d}{dx}\left(\sqrt{x}\right)$

$= \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$

$f'(x) = \frac{1}{2}x^{-1/2} \longrightarrow f'(c) = \frac{1}{2}c^{-1/2}$

$f'(c) = \frac{f(b) - f(a)}{b - a}$

$= \frac{f(4) - f(0)}{4 - 0}$

$= \frac{2 - 0}{4 - 0}$

$= \frac{1}{2} \longrightarrow f'(c) = \frac{1}{2}$

$\frac{1}{2}c^{-1/2} = \frac{1}{2}$

$\cancel{(2)}\ \cancel{\frac{1}{2}}\left(\frac{1}{\sqrt{c}}\right) = \cancel{\frac{1}{2}}\ \cancel{(2)}$

$\cancel{(\sqrt{c})}\ \frac{1}{\cancel{\sqrt{c}}} = 1\ (\sqrt{c})$

$1 = \sqrt{c}$

$\boxed{c = 1}$