# Q1

Wednesday, September 30, 2020 12:57 PM

Each side of a square is increasing at a rate of 6 cm/s. At what rate is the area of the square increasing when the area of the square is 36 $cm^2$?

72 ✔ $cm^2/s$

Area of square

$A = 36\ cm^2$

$s$ = side of square

$\frac{ds}{dt} = 6\ cm/s$ = rate of change of each side with respect to time ($t$, in seconds)

$\frac{dA}{dt} = ?$ = rate of change of Area with respect to time ($t$, in seconds)

$$A = s^2$$

$$\frac{dA}{dt}(A) = \frac{ds}{dt}(s^2)$$

$$\frac{dA}{dt} = \frac{d(s^2)}{ds}(s^2)\frac{ds}{dt}$$

$$\frac{dA}{dt} = 2s\frac{ds}{dt} \longrightarrow A = 36 = s^2,\quad \frac{ds}{dt} = 6\ cm/s$$

$$\sqrt{36} = s$$

$$6 = s$$

$$\frac{dA}{dt} = 2(6)(6)$$

$$\boxed{\frac{dA}{dt} = 72\ cm^2/s}$$

# Q2

Wednesday, September 30, 2020 1:24 PM

A cylindrical tank with radius 5 m is being filled with water at a rate of 3 $m^3$/min. How fast is the height of the water increasing?

$\frac{3}{25\pi}$ m/min

Volume of a cylinder

$$V = \pi r^2 h$$

$$r = 5 \quad , \quad r^2 = 25$$

$\frac{dV}{dt} = 3 m^3/min$ = rate of change of Volume of cylinder with respect to time (minutes)

$\frac{dh}{dt} = ?$ = rate of change of height with respect to time (minutes)

$$V = \pi r^2 h$$

$$= \pi (5)^2 h$$

$$V = 25 \pi h$$

$$\frac{dV}{dt}(V) = \frac{d}{dt}(25\pi h)$$

$$\frac{dV}{dt} = 25\pi \frac{dh}{dt}(h) \longrightarrow \frac{dV}{dt} = 3 cm^3/min$$

$$3 = 25\pi \frac{dh}{dt}$$

$$\frac{3}{25\pi} = \frac{dh}{dt}$$

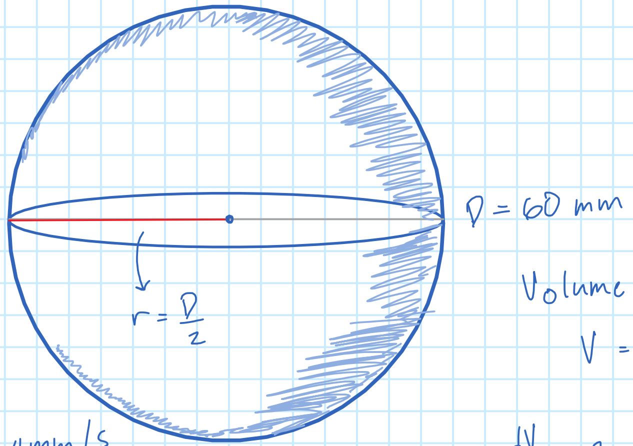
$$\boxed{\frac{dh}{dt} = \frac{3}{25\pi} \text{ m/min}}$$

# Q3

Wednesday, September 30, 2020 1:46 PM

The radius of a sphere is increasing at a rate of 4 mm/s. How fast is the volume increasing when the diameter is 60 mm?

$14400\pi$ $mm^3/s$




$\frac{dr}{dt} = 4$ mm/s

Volume of sphere

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = ?$ when $D = 60$mm

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt}(V) = \frac{dr}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\frac{dV}{dt} = \frac{4\pi}{3}\frac{dr^3}{dr}(r^3)$$

$$= \frac{4\pi}{\cancel{3}}\cancel{3}r^2\frac{dr}{dt}$$

$$= 4\pi r^2\frac{dr}{dt}$$

$r = \frac{D}{2} = \frac{60}{2} = 30$ mm

$\frac{dr}{dt} = 4$mm/s

$$= 4\pi(30)^2(4)$$

$$\boxed{\frac{dV}{dt} = 14400\pi \text{ mm}^3/\text{s}}$$

# Q4

Wednesday, September 30, 2020 2:13 PM

Suppose $y = \sqrt{2x+1}$, where $x$ and $y$ are functions of $t$.

(a) If $dx/dt = 9$, find $dy/dt$ when $x = 4$.

$\frac{dy}{dt} = 3$ ✓

(b) If $dy/dt = 3$, find $dx/dt$ when $x = 40$.

$\frac{dx}{dt} = 27$ ✓

$y = \sqrt{2x+1}$, where $x$ and $y$ are functions of $t$

a) $\frac{dx}{dt} = 9$, $\frac{dy}{dt} = ?$ when $x = 4$

$$\frac{dy}{dt}(y) = \frac{dx}{dt}\left(\sqrt{2x+1}\right)$$

$$\frac{dy}{dt} = \frac{dx}{dt}\left[(2x+1)^{\frac{1}{2}}\right]$$

Chain Rule

$u = 2x+1$

$f(u) = u^{1/2} = y$

$\frac{dy}{du} \cdot \frac{du}{dx}$

$$= \frac{dy}{du}\left(u^{1/2}\right)\frac{du}{dx}(u)$$

$$= \frac{1}{2}(2x+1)^{-1/2}\frac{dx}{dt}(2x+1)$$

$$= \frac{1}{2}\left(\frac{1}{\sqrt{2x+1}}\right)\frac{dx}{dt}(2x) + 0$$

$$= \frac{1}{2\sqrt{2x+1}} \cdot 2\frac{dx}{dt}(x)$$

$$= \frac{1}{\cancel{2}\sqrt{2x+1}} \cdot \cancel{2}\frac{dx}{dt}$$

$$= \frac{1}{\sqrt{2x+1}}\frac{dx}{dt} \to \frac{dx}{dt} = 9$$

$x = 4$

$$= \frac{9}{\sqrt{2(4)+1}}$$

$$\frac{dy}{dt} = \frac{9}{\sqrt{9}} = \frac{9}{3} = 3$$

$$\boxed{\frac{dy}{dt} = 3}$$

b) $\frac{dy}{dt} = 3$, $\frac{dx}{dt} = ?$ when $x = 40$

$$\frac{dy}{dt} = \frac{1}{\sqrt{2x+1}}\frac{dx}{dt}$$

$$3 = \frac{1}{\sqrt{2(40)+1}}\frac{dx}{dt}$$

$$3 = \frac{1}{\sqrt{81}}\frac{dx}{dt}$$

$$3 = \frac{1}{9}\frac{dx}{dt}$$

$$\frac{3}{\frac{1}{9}} = \frac{dx}{dt}$$

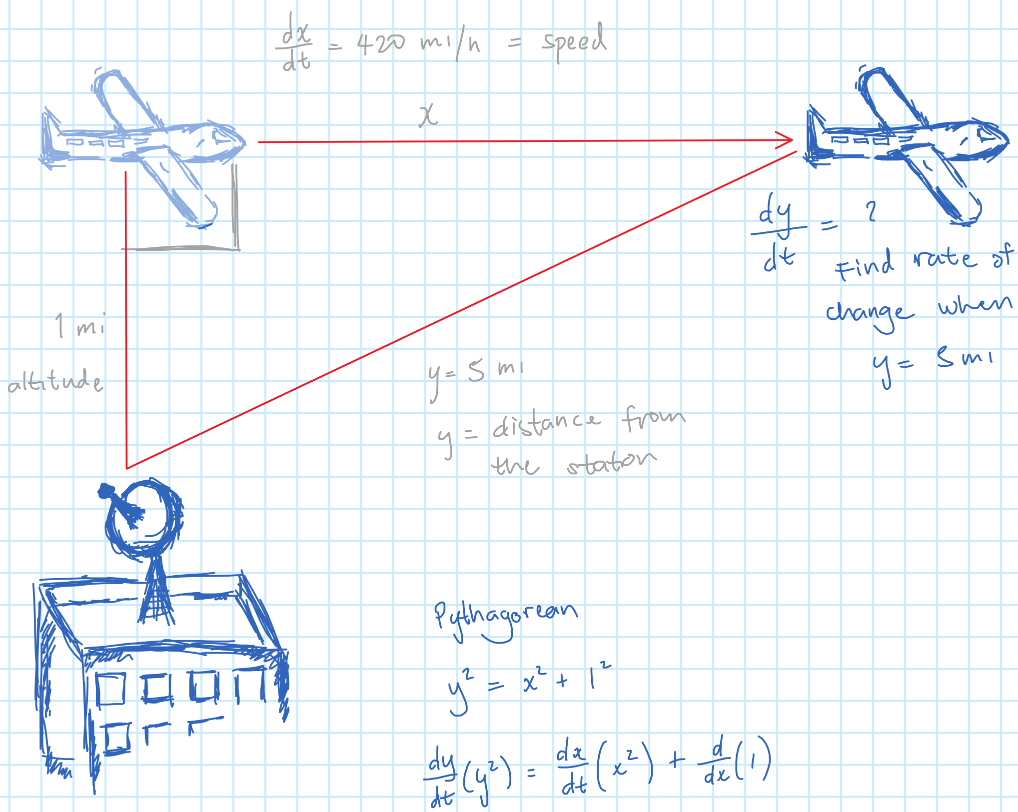
$$\boxed{\frac{dx}{dt} = 27}$$

# Q5

Wednesday, September 30, 2020 2:35 PM

A plane flying horizontally at an altitude of 1 mi and a speed of 420 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 5 mi away from the station. (Round your answer to the nearest whole number.)

412 mi/h




Pythagorean

$$y^2 = x^2 + 1^2$$

$$\frac{dy}{dt}(y^2) = \frac{dx}{dt}(x^2) + \frac{d}{dx}(1)$$

$$2y\frac{dy}{dt} = 2x\frac{dx}{dt} + 0$$

$$2y\frac{dy}{dt} = 2x\frac{dx}{dt} \longrightarrow$$

$\frac{dx}{dt} = 420$ mi/h

$y = 5$ mi

$y^2 = x^2 + 1^2$

$5^2 = x^2 + 1$

$25 - 1 = x^2$

$24 = x^2$

$x = \sqrt{24}$

$$2(5)\frac{dy}{dt} = 2(\sqrt{24})(420)$$

$$10\frac{dy}{dt} = 840\sqrt{24}$$

$$\frac{dy}{dt} = \frac{840\sqrt{24}}{10}$$

$$\frac{dy}{dt} = 84\sqrt{24}$$

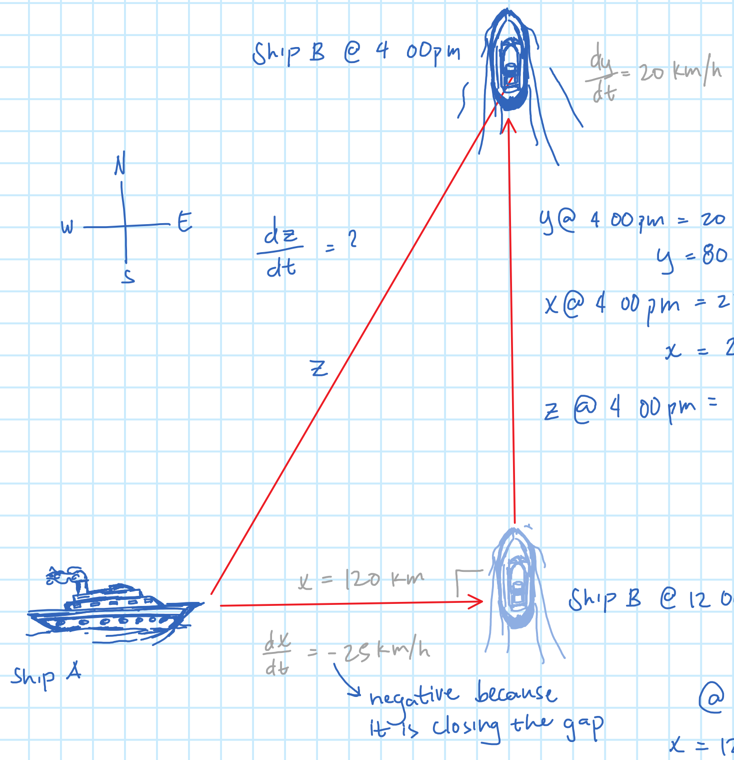
$$\boxed{\frac{dy}{dt} \approx 412 \text{ mi/h}}$$

# Q6

Wednesday, September 30, 2020 7:38 PM

At noon, ship A is 120 km west of ship B. Ship A is sailing east at 25 km/h and ship B is sailing north at 20 km/h. How fast is the distance between the ships changing at 4:00 PM?

$\dfrac{1100}{\sqrt{6800}}$ km/h ✓




$y$ @ 4 00pm = 20 km/h (4h) + 0 km
$y = 80$ km
$x$ @ 4 00 pm = 25 km/h (4h) − 120 km
$x = 20$ km
$z$ @ 4 00 pm = $\sqrt{6800}$

$z^2 = 20^2 + 80^2$
$z^2 = 6800$
$z = \sqrt{6800}$

@ 12:00 nn
$x = 120$ km
$y = 0$ km

because they are linear to each other

$$z^2 = x^2 + y^2$$

$$\frac{dz}{dt}(z^2) = \frac{dx}{dt}(x^2) + \frac{dy}{dt}(y^2)$$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\left(\frac{1}{\cancel{2}z}\right)\cancel{2}z\frac{dz}{dt} = \cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}\left(\frac{1}{\cancel{2}z}\right)$$

$$\frac{dz}{dt} = \frac{1}{z}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right) \longrightarrow$$

$x = 20$, $y = 80$, $z = \sqrt{6800}$

$\frac{dx}{dt} = -25$ km/h, $\frac{dy}{dt} = 20$ km/h

$$= \frac{1}{\sqrt{6800}}\left[20(-25) + 80(20)\right]$$

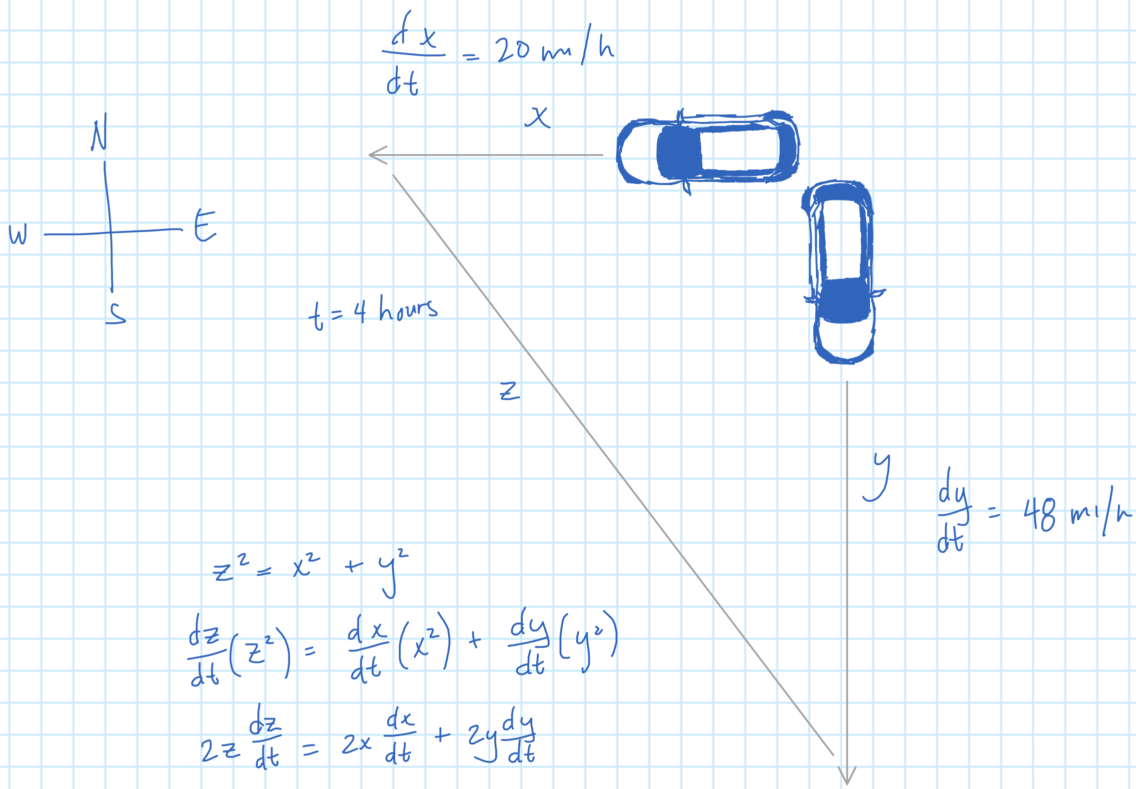
$$\boxed{\frac{dz}{dt} = \frac{1100}{\sqrt{6800}} \text{ km/h}}$$

# Q7

Wednesday, September 30, 2020    10:01 PM

Two cars start moving from the same point. One travels south at 48 mi/h and the other travels west at 20 mi/h. At what rate is the distance between the cars increasing four hours later?

52 ✓ mi/h




$$z^2 = x^2 + y^2$$

$$\frac{d z}{dt}(z^2) = \frac{dx}{dt}(x^2) + \frac{dy}{dt}(y^2)$$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{\cancel{2}z\frac{dz}{dt}}{\cancel{z}} = \frac{\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}}{\cancel{z}}$$

$$z\frac{dz}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$$

$$x = 20(4) = 80$$

$$y = 48(4) = 192$$

$$\sqrt{43264}\,\frac{dz}{dt} = 80(20) + 192(48)$$

$$208\,\frac{dz}{dt} = 10816$$

$$\frac{dz}{dt} = \frac{10816}{208}$$

$$\boxed{\frac{dz}{dt} = 52 \text{ mi/h}}$$

$$z^2 = 80^2 + 192^2$$

$$z^2 = 43264$$

$$\sqrt{z^2} = \sqrt{43264}$$

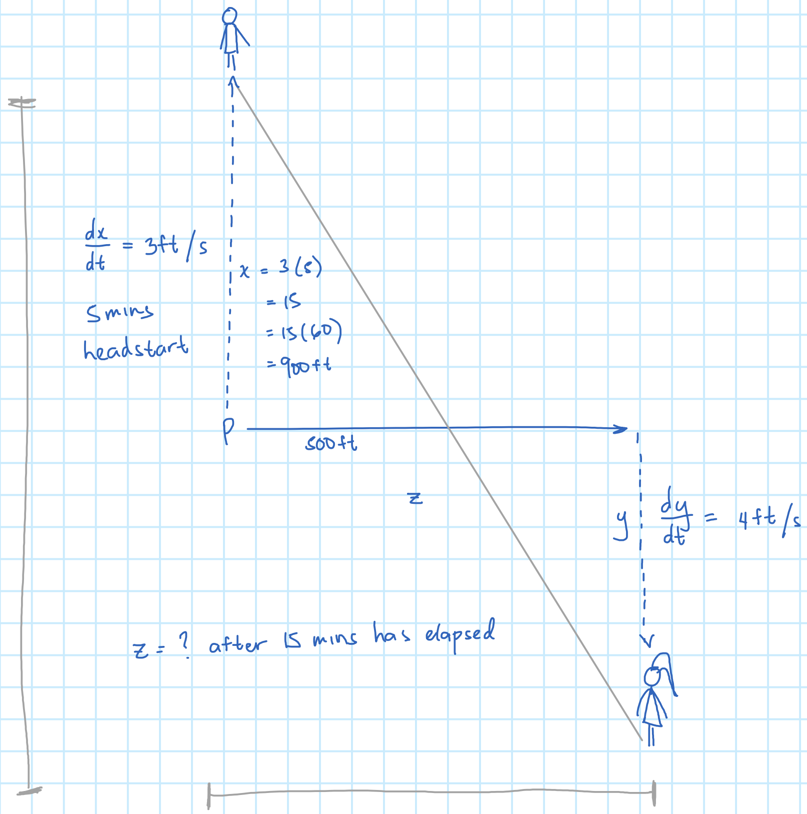
$$z = \sqrt{43264}$$

# Q8

Wednesday, September 30, 2020 10:37 PM

A man starts walking north at 3 ft/s from a point $P$. Five minutes later a woman starts walking south at 4 ft/s from a point 500 ft due east of $P$. At what rate are the people moving apart 15 min after the woman starts walking? (Round your answer to two decimal places.)

6.98 ✔ ft/s




$$z^2 = (x+y)^2 + 500^2$$

$$\frac{dz}{dt}(z^2) = \frac{d(x+y)^2}{d(x+y)}\left[(x+y)^2\right] + \frac{d}{dt}(500^2)$$

$$2z\frac{dz}{dt} = 2(x+y)\frac{d}{dt}(x+y) + 0$$

$$\frac{2z\frac{dz}{dt}}{2z} = \frac{2(x+y)\left(\frac{dx}{dt}+\frac{dy}{dt}\right)}{2z}$$

$$\frac{dz}{dt} = \frac{(x+y)\left(\frac{dx}{dt}+\frac{dy}{dt}\right)}{z}$$

$$\boxed{\frac{dz}{dt} = \frac{(3600+3600)(3+4)}{\sqrt{52{,}090{,}000}} \approx 6.98 \text{ ft/s}}$$

$y = \frac{dy}{dt} \cdot 15 \text{ min} \rightarrow \text{seconds}$

$= 4(15)(60)$

$\underline{y = 3{,}600 \text{ ft}}$

$x = \frac{dx}{dt} \cdot 15 \text{ min} \rightarrow \text{seconds} + 900 \text{ ft}$ headstart

$= 3(15)(60) + 900$

$\underline{x = 3{,}600 \text{ ft}}$

$z^2 = (3600 + 3600)^2 + 500^2$

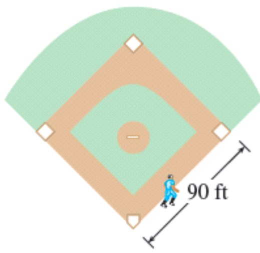
$z^2 = 52{,}090{,}000$

$z = \sqrt{52{,}090{,}000}$

# Q9

Thursday, October 1, 2020 7:13 AM

A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 26 ft/s.



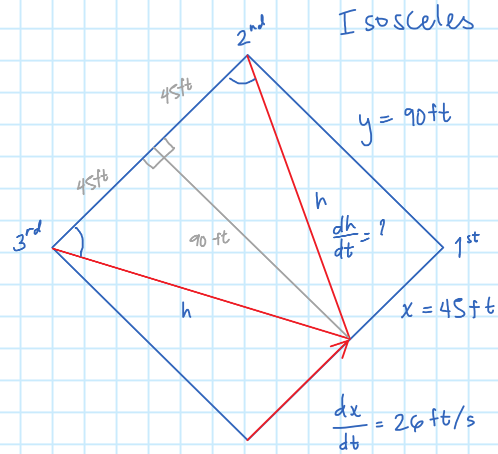

(a) At what rate is his distance from second base decreasing when he is halfway to first base? (Round your answer to one decimal place.)

11.6 ft/s

(b) At what rate is his distance from third base increasing at the same moment? (Round your answer to one decimal place.)

11.6 ft/s

Isosceles Triangle




$$h^2 = 45^2 + 90^2$$
$$h^2 = 10125$$
$$h = \sqrt{10125}$$

$$h^2 = x^2 + y^2$$

$$\frac{d}{dt}(h^2) = \frac{d}{dt}(x^2) + \frac{d}{dt}(y^2)$$

$$2h\frac{dh}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{\cancel{2}h\frac{dh}{dt}}{\cancel{2}h} = \frac{\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}}{\cancel{2}h}$$

$$\frac{dh}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{h}$$

$x = 45$, $y = 90$

$\frac{dx}{dt} = 26$ ft/s, $\frac{dy}{dt} = 0$

$h = \sqrt{10125}$

$$\frac{dh}{dt} = \frac{45(26) + 90(0)}{\sqrt{10125}}$$

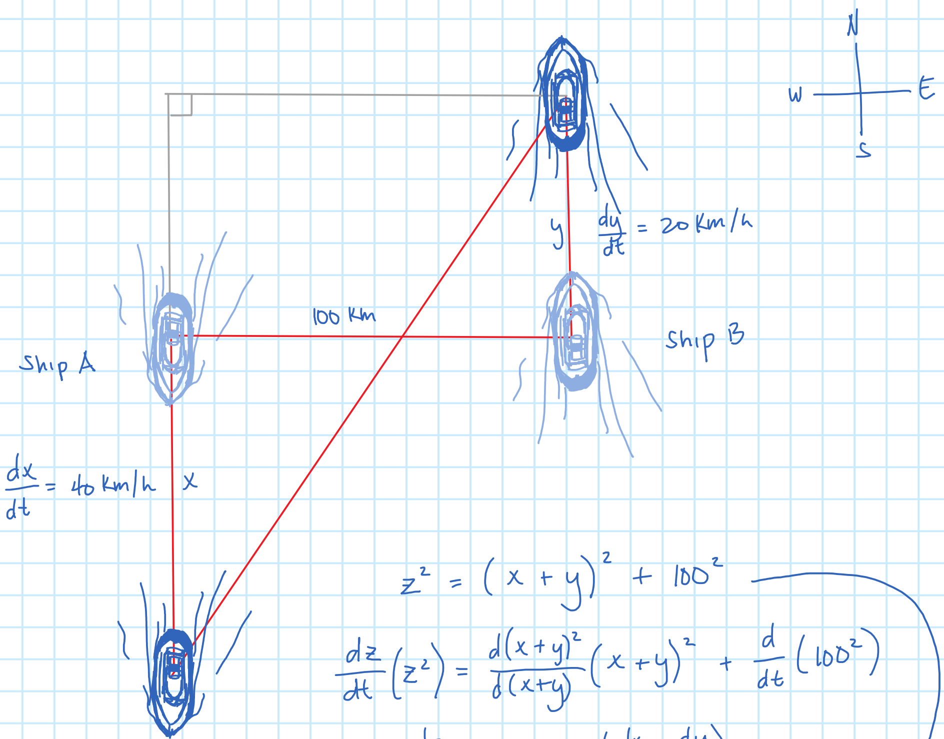
$$\boxed{\frac{dh}{dt} \approx 11.6 \text{ ft/s}}$$

# Q10

Thursday, October 1, 2020 8:02 AM

At noon, ship A is 100 km west of ship B. Ship A is sailing south at 40 km/h and ship B is sailing north at 20 km/h. How fast is the distance between the ships changing at 4:00 PM? (Round your answer to one decimal place.)

55.4 km/h




$$z^2 = (x+y)^2 + 100^2$$

$$\frac{dz}{dt}(z^2) = \frac{d(x+y)^2}{d(x+y)}(x+y)^2 + \frac{d}{dt}(100^2)$$

$$2z\frac{dz}{dt} = 2(x+y)\left(\frac{dx}{dt}+\frac{dy}{dt}\right) + 0$$

$$\frac{\cancel{2z}\frac{dz}{dt}}{\cancel{2z}} = \frac{\cancel{2}(x+y)\left(\frac{dx}{dt}+\frac{dy}{dt}\right)}{\cancel{2}z}$$

$$\frac{dz}{dt} = \frac{(x+y)\left(\frac{dx}{dt}+\frac{dy}{dt}\right)}{z}$$

$$= \frac{(160+80)(40+20)}{260}$$

$$\boxed{\frac{dz}{dt} \approx 55.4 \text{ km/h}}$$

from noon time

$x$ @ 4:00 pm $= 40$ (4 hours)

$= 160$ km

$y$ @ 4:00 pm $= 20$ (4 hours)

$= 80$ km

$z^2 = (x+y)^2 + 100^2$

$= (160+80)^2 + 100^2$

$z^2 = 67600$

$z = \sqrt{67600}$

$z = 260$

# Q11

Thursday, October 1, 2020 8:47 AM

Water is leaking out of an inverted conical tank at a rate of 10,000 $cm^3$/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank. (Round your answer to the nearest integer.)

289,253 ✔ $cm^3$/min

$$V = \frac{1}{3}\pi r^2 h$$

$\frac{r}{h} = \frac{2}{6}$ Ratio of small right triangle to big one to get r

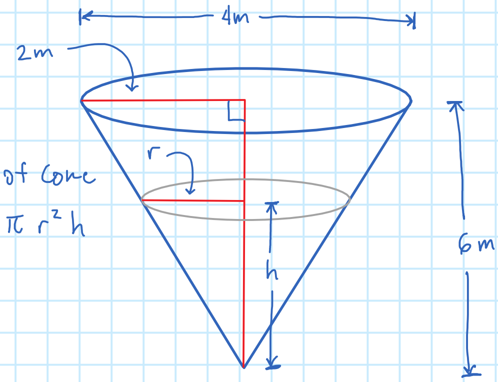
$(h)\frac{r}{h} = \frac{1}{3}(h)$

$r = \frac{1}{3}h$

$$= \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$$

$$= \frac{1}{3}\pi\left(\frac{1}{9}h^2\right)h$$

$$V = \frac{1}{27}\pi h^3$$




Volume of Cone

$V = \frac{1}{3}\pi r^2 h$

$\frac{dh}{dt} = 20\text{ cm/min}$

$$\frac{dV}{dt}(V) = \frac{d}{dt}\left(\frac{1}{27}\pi h^3\right)$$

$$\frac{dV}{dt} = \frac{1}{27}\pi\frac{dh}{dt}(h^3)$$

$$= \frac{1}{27}\pi 3h^2\frac{dh}{dt}$$

$$= \frac{1}{9}\pi h^2\frac{dh}{dt}$$

$\frac{dh}{dt} = 20\text{ cm/min}$

$$= \frac{\pi}{9}(200)^2(20)$$

$h = 2\text{ m} \to 200\text{ cm}$

$$\frac{dV}{dt} = \frac{800\,000\pi}{9}$$

C = the rate at which water is pumped into the tank

$$\frac{800\,000\pi}{9} = C - 10\,000$$

$\frac{dV}{dt} = C - \underbrace{10{,}000}_{\text{water leaking out}}$

$$C = \frac{800\,000\pi}{9} + 10\,000$$

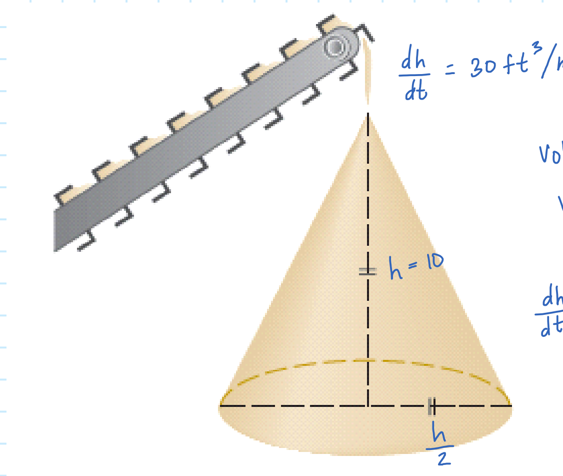
$$\boxed{C \approx 289{,}253\text{ cm}^3/\text{min}}$$

# Q12

Thursday, October 1, 2020 10:36 AM

Gravel is being dumped from a conveyor belt at a rate of 30 $ft^3$/min, and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high? (Round your answer to two decimal places.)

0.38 ft/min




$\frac{dh}{dt} = 30\ ft^3/min$

Volume of a cone

$V = \frac{1}{3}\pi r^2 h$

$\frac{dh}{dt} = ?$

$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$$
$$= \frac{1}{3}\pi \frac{h^2}{4} h$$
$$V = \frac{\pi h^3}{12}$$

$$\frac{dV}{dt}(V) = \frac{dh}{dt}\left(\frac{\pi h^3}{12}\right)$$
$$\frac{dV}{dt} = \frac{\pi}{12}\frac{dh}{dt}(h^3)$$
$$= \frac{\pi}{12} 3h^2 \frac{dh}{dt}$$
$$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$$

$$30 = \frac{\pi}{4}(10)^2 \frac{dh}{dt}$$

$\frac{dV}{dt} = 30\ cm^3/min$

$h = 10$

$$30 = \frac{100\pi}{4}\frac{dh}{dt}$$
$$30 = 25\pi \frac{dh}{dt}$$
$$\frac{30}{25\pi} = \frac{\cancel{25\pi}\frac{dh}{dt}}{\cancel{25\pi}}$$
$$\frac{6}{5\pi} = \frac{dh}{dt}$$
$$\boxed{\frac{dh}{dt} \approx 0.38\ ft/min}$$

# Q13

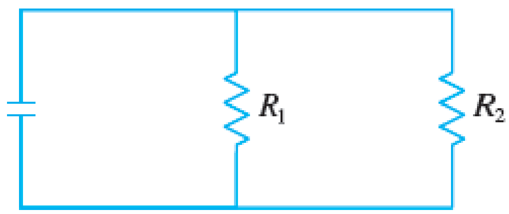
Thursday, October 1, 2020 10:54 AM

If two resistors with resistances $R_1$ and $R_2$ are connected in parallel, as in the figure below, then the total resistance $R$, measured in ohms ($\Omega$), is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

If $R_1$ and $R_2$ are increasing at rates of 0.3 $\Omega$/s and 0.2 $\Omega$/s, respectively, how fast is $R$ changing when $R_1$ = 80 $\Omega$ and $R_2$ = 90 $\Omega$? (Round your answer to three decimal places.)

0.128 ✔ $\Omega$/s




$R_1 = 80\ \Omega$, $\dfrac{dR_1}{dt} = 0.3\ \Omega/s$

$R_2 = 90\ \Omega$, $\dfrac{dR_2}{dt} = 0.2\ \Omega/s$

$\dfrac{dR}{dt} = ?$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R} = \frac{1}{80} + \frac{1}{90}$$

$$= \frac{1(90)}{80\ (90)} + \frac{1\ (80)}{90\ (80)}$$

$$\frac{1}{R} = \frac{170}{7200} = \frac{17}{720}$$

$$\frac{1}{R} = \frac{17}{720}$$

$$(R)\ \frac{1}{R} = \frac{17}{720}\ (R)$$

$$\frac{1}{\frac{17}{720}} = \frac{\cancel{\frac{17}{720}}\, R}{\cancel{\frac{17}{720}}}$$

$$R = \frac{720}{17}\ \Omega$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{dR}{dt}\left(R^{-1}\right) = \frac{dR}{dt}\left(R_1^{-1} + R_2^{-1}\right)$$

$$-R^{-2}\,\frac{dR}{dt} = -R_1^{-2}\,\frac{dR_1}{dt} - R_2^{-2}\,\frac{dR_2}{dt}$$

$$\frac{\cancel{-R^{-2}}\,\frac{dR}{dt}}{\cancel{-R^{-2}}} = \frac{\cancel{-}R_1^{-2}\,\frac{dR_1}{dt}\ \cancel{-}\ R_2^{-2}\,\frac{dR_2}{dt}}{\cancel{-}R^{-2}}$$

$$\frac{dR}{dt} = \frac{R_1^{-2}\,\frac{dR_1}{dt} + R_2^{-2}\,\frac{dR_2}{dt}}{R^{-2}}$$

$$\frac{dR}{dt} = \frac{\frac{1}{R_1^2}\,\frac{dR_1}{dt} + \frac{1}{R_2^2}\,\frac{dR_2}{dt}}{\frac{1}{R^2}}$$

$$\frac{dR}{dt} = R^2\left(\frac{1}{R_1^2}\,\frac{dR_1}{dt} + \frac{1}{R_2^2}\,\frac{dR_2}{dt}\right)$$

$R_1 = 80\Omega$, $\dfrac{dR_1}{dt} = 0.3\ \Omega/s$

$R_2 = 90\Omega$, $\dfrac{dR_2}{dt} = 0.2\ \Omega/s$

$R = \dfrac{720}{17}\ \Omega$

$$= \left(\frac{720}{17}\right)^2\left[\frac{1}{(80)^2}(0.3) + \frac{1}{(90)^2}(0.2)\right]$$

$$\boxed{\frac{dR}{dt} \approx 0.128\ \Omega/s}$$