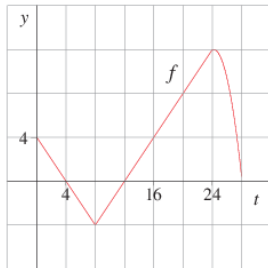


# Q1

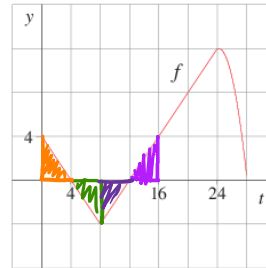
Thursday, November 12, 2020 4:25 PM

Let  $g(x) = \int_0^x f(t) dt$ , where  $f$  is the function whose graph is shown.

$$g(0) = \int_0^0 f(t) dt = 0$$

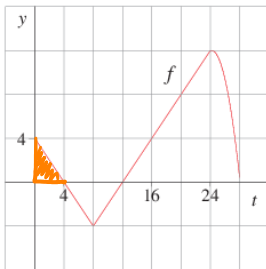


$$g(16) = \int_0^{16} f(t) dt = 0$$

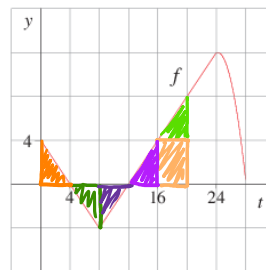


$g(0) = 0$	✓
$g(4) = 8$	✓
$g(8) = 0$	✓
$g(12) = -8$	✓
$g(16) = 0$	✓
$g(20) = 24$	✓
$g(24) = 64$	✓

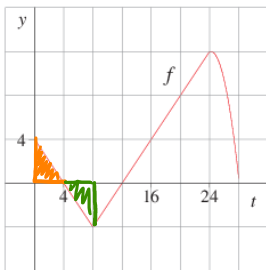
$$g(4) = \int_0^4 f(t) dt = 8$$



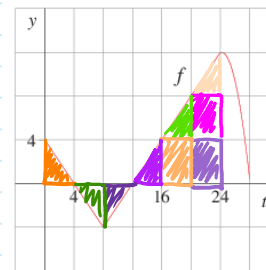
$$g(20) = \int_0^{20} f(t) dt = 24$$



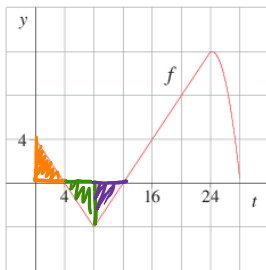
$$g(8) = \int_0^8 f(t) dt = 0$$



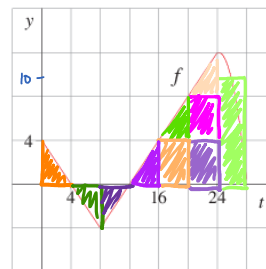
$$g(20) = \int_0^{20} f(t) dt = 64$$



$$g(12) = \int_0^{12} f(t) dt = -8$$



$$g(20) = \int_0^{20} f(t) dt = 104$$



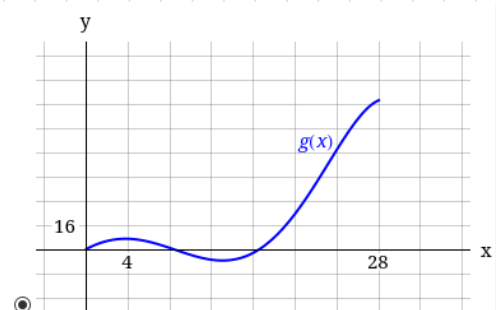
(b) Estimate  $g(28)$ . (Use the midpoint to get the most precise estimate.)

$$g(28) = 104$$

(c) Where does  $g$  have a maximum and a minimum value?

minimum  $x = 12$  ✓

maximum  $x = 28$  ✓



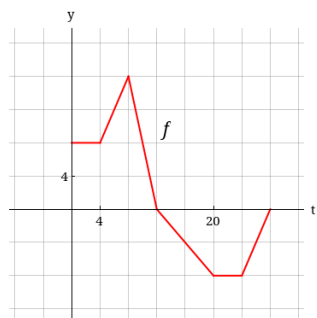
# Q2

Friday, November 13, 2020 2:39 PM

Let  $g(x) = \int_0^x f(t) dt$ , where  $f$  is the function whose graph is shown.

$$g(0) = \int_0^0 f(t) dt$$

$$= 0$$



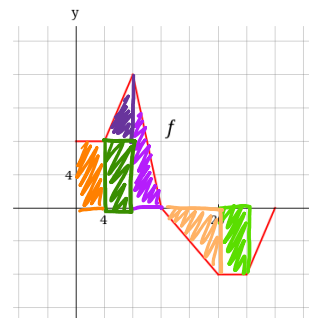
$$g(24) = \int_0^{24} f(t) dt$$

$$= \boxed{\times 2} + \triangle + \triangle - \triangle - \square$$

$$= 2[(4)(8)] + \frac{1}{2}(4)(8)$$

$$+ \frac{1}{2}(4)(16) - \frac{1}{2}(8)(8) - (4)(8)$$

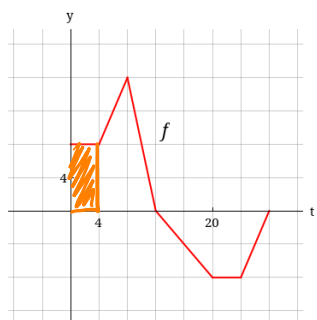
$$= 48$$



$$g(4) = \int_0^4 f(t) dt$$

$$= (4)(8)$$

$$= 32$$

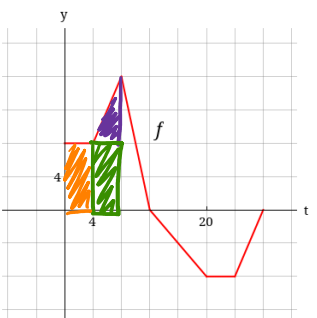


$$g(8) = \int_0^8 f(t) dt$$

$$= \boxed{\times 2} + \triangle$$

$$= 2[(4)(8)] + \frac{1}{2}(4)(8)$$

$$= 80$$



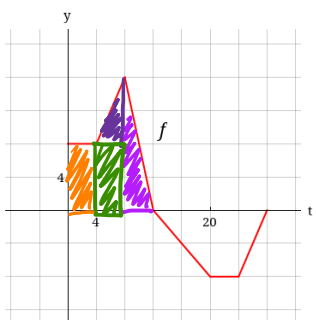
$$g(12) = \int_0^{12} f(t) dt$$

$$= \boxed{\times 2} + \triangle + \triangle$$

$$= 2[(4)(8)] + \frac{1}{2}(4)(8)$$

$$+ \frac{1}{2}(4)(16)$$

$$= 112$$



(a) Evaluate  $g(0)$ ,  $g(4)$ ,  $g(8)$ ,  $g(12)$ , and  $g(24)$ .

$$g(0) = 0 \quad \checkmark +$$

$$g(4) = 32 \quad \checkmark +$$

$$g(8) = 80 \quad \checkmark +$$

$$g(12) = 112 \quad \checkmark +$$

$$g(24) = 48 \quad \checkmark -$$

increasing

(b) On what interval is  $g$  increasing? (Enter your answer using interval notation.)

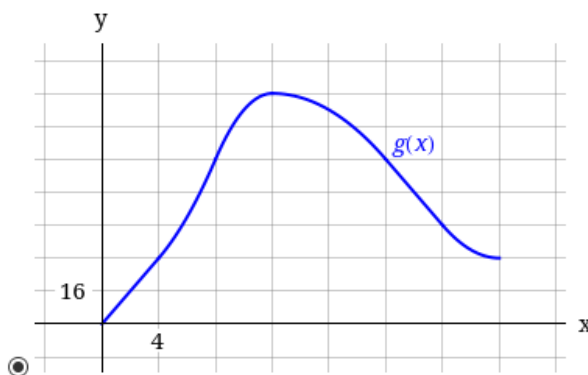
$$(0, 12)$$

✓

(c) Where does  $g$  have a maximum value?

$$x = 12 \quad \checkmark$$

(d) Sketch a rough graph of  $g$ .



## Q3

Friday, November 13, 2020 3:45 PM

Use Part 1 of the **Fundamental Theorem of Calculus** to find the derivative of the function.

$$g(x) = \int_0^x \sqrt{t^3 + t^5} dt$$

$$g'(x) = \sqrt{x^3 + x^5} \quad \checkmark$$

$$g(x) = \int_0^x \sqrt{t^3 + t^5} dt$$

$$\text{Let } f(t) = \sqrt{t^3 + t^5}$$

$$g'(x) = f(x) = \sqrt{x^3 + x^5}$$

**The Fundamental Theorem of Calculus** Suppose  $f$  is continuous on  $[a, b]$ .

1. If  $g(x) = \int_a^x f(t) dt$ , then  $g'(x) = f(x)$ .
2.  $\int_a^b f(x) dx = F(b) - F(a)$ , where  $F$  is any antiderivative of  $f$ , that is,  $F' = f$ .

# Q4

Friday, November 13, 2020 3:53 PM

Use Part 1 of the **Fundamental Theorem of Calculus** to find the derivative of the function.

$$g(s) = \int_7^s (t - t^4)^2 dt$$

$$g'(s) = \boxed{(s - s^4)^2} \quad \checkmark$$

$$g(s) = \int_7^s (t - t^4)^2 dt \quad \text{let } f(t) = (t - t^4)^2$$

$$g'(s) = f(s) = \boxed{(s - s^4)^2}$$

Evaluate the integral.

$$\int_4^6 (x^2 + 2x - 4) dx$$

$$\frac{188}{3}$$



**The Fundamental Theorem of Calculus** Suppose  $f$  is continuous on  $[a, b]$ .

1. If  $g(x) = \int_a^x f(t) dt$ , then  $g'(x) = f(x)$ .
2.  $\int_a^b f(x) dx = F(b) - F(a)$ , where  $F$  is any antiderivative of  $f$ , that is,  $F' = f$ .

$$\begin{aligned}
 \int_4^6 (x^2 + 2x - 4) dx &= \left[ \frac{x^{2+1}}{2+1} + \frac{2x^{1+1}}{1+1} - 4x \right]_4^6 \\
 &= \left[ \frac{x^3}{3} + x^2 - 4x \right]_4^6 \\
 &= F(6) - F(4) \\
 &= \left( \frac{6^3}{3} + 6^2 - 4(6) \right) - \left( \frac{4^3}{3} + 4^2 - 4(4) \right) \\
 &= \frac{188}{3}
 \end{aligned}$$

## Q6

Friday, November 13, 2020 8:10 PM

Evaluate the integral.

$$\int_0^4 \left( \frac{4}{5}t^3 - \frac{3}{4}t^2 + \frac{2}{5}t \right) dt$$

$$\frac{192}{5}$$



$$\begin{aligned} \int_0^4 \left( \frac{4}{5}t^3 - \frac{3}{4}t^2 + \frac{2}{5}t \right) dt &= \left[ \frac{4}{5} \frac{1}{3+1} t^{3+1} - \frac{3}{12} \frac{1}{2+1} t^{2+1} + \frac{2}{5} \frac{1}{1+1} t^{1+1} \right]_0^4 \\ &= \left[ \frac{4}{20} t^4 - \frac{3}{12} t^3 + \frac{2}{10} t^2 \right]_0^4 \\ &= \left[ \frac{1}{5} t^4 - \frac{1}{4} t^3 + \frac{1}{5} t^2 \right]_0^4 \\ &= F(4) - F(0) \\ &= \left( \frac{1}{5}(4)^4 - \frac{1}{4}(4)^3 + \frac{1}{5}(4)^2 \right) - \left( \frac{1}{5}(0)^4 - \frac{1}{4}(0)^3 + \frac{1}{5}(0)^2 \right) \\ &= \frac{192}{5} \end{aligned}$$

Q7

Friday, November 13, 2020 8:18 PM

Evaluate the integral.

$$\int_4^9 \sqrt{x} \, dx$$

$$\frac{38}{3}$$



$$\begin{aligned} \int_4^9 \sqrt{x} \, dx &= \int_4^9 x^{1/2} \, dx = \left[ \frac{x^{1/2+1}}{\frac{1}{2}+1} \right]_4^9 \\ &= \left[ \frac{2}{3} x^{3/2} \right]_4^9 \\ &= F(9) - F(4) \\ &= \left( \frac{2}{3} 9^{3/2} \right) - \left( \frac{2}{3} 4^{3/2} \right) \\ &= \frac{38}{3} \end{aligned}$$

## Q8

Friday, November 13, 2020 8:25 PM

Evaluate the integral.

$$\int_1^9 \frac{4 + x^2}{\sqrt{x}} dx$$

$$\frac{564}{5}$$



$$\begin{aligned}
 \int_1^9 \frac{4 + x^2}{\sqrt{x}} dx &= \int_1^9 4x^{-1/2} + x^{3/2} dx = \left[ \frac{4x^{-1/2+1}}{-\frac{1}{2}+1} + \frac{x^{3/2+1}}{\frac{3}{2}+1} \right]_1^9 \\
 &= \left[ \frac{2}{1} 4x^{1/2} + \frac{2}{5} x^{5/2} \right]_1^9 \\
 &= \left[ 8x^{1/2} + \frac{2}{5} x^{5/2} \right]_1^9 \\
 &= F(9) - F(1) \\
 &= \left[ 8(9)^{1/2} + \frac{2}{5}(9)^{5/2} \right] - \left[ 8(1)^{1/2} + \frac{2}{5}(1)^{5/2} \right] \\
 &= \frac{564}{5}
 \end{aligned}$$



Q9

Friday, November 13, 2020 8:50 PM

Evaluate the integral.

$$\int_1^2 \frac{v^5 + 4v^7}{v^4} dv$$

$$\frac{33}{2}$$



$$\begin{aligned}\int_1^2 \frac{v^5 + 4v^7}{v^4} dv &= \int_1^2 v + 4v^3 = \left[ \frac{v^{1+1}}{1+1} + \frac{4v^{3+1}}{3+1} \right]_1^2 \\&= \left[ \frac{v^2}{2} + v^4 \right]_1^2 \\&= F(2) - F(1) \\&= \left( \frac{2^2}{2} + 2^4 \right) - \left( \frac{1^2}{2} + 1^4 \right) \\&= \frac{33}{2}\end{aligned}$$

# Q10

Friday, November 13, 2020 8:55 PM

Evaluate the integral.

$$\int_0^1 (7x^e + 5e^x) dx$$

$$\frac{7}{e+1} + 5e - 5$$



$$\begin{aligned}\int_0^1 (7x^e + 5e^x) dx &= \left[ \frac{7x^{e+1}}{e+1} + 5e^x \right]_0^1 \\&= F(1) - F(0) \\&= \left( \frac{7(1)^{e+1}}{e+1} + 5e^1 \right) - \left( \frac{7(0)^{e+1}}{e+1} + 5e^0 \right) \\&= \frac{7}{e+1} + 5e - 5\end{aligned}$$

# Q11

Friday, November 13, 2020 9:03 PM

Evaluate the integral.

$$\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{7}{1+x^2} dx$$

$$\frac{7\pi}{6}$$



$$\begin{aligned} \int_{1/\sqrt{3}}^{\sqrt{3}} \frac{7}{1+x^2} dx &= \left[ 7 \tan^{-1} x \right]_{1/\sqrt{3}}^{\sqrt{3}} & \frac{d}{dx} (\tan^{-1} x) &= \frac{1}{1+x^2} \\ &= F(\sqrt{3}) - F\left(\frac{1}{\sqrt{3}}\right) \\ &= 7 \tan^{-1} \sqrt{3} - 7 \tan^{-1} \frac{1}{\sqrt{3}} \\ &= 7 \left( \frac{\pi}{3} - \frac{\pi}{6} \right) \\ &= \frac{7\pi}{6} \end{aligned}$$

## Q12

Friday, November 13, 2020 9:30 PM

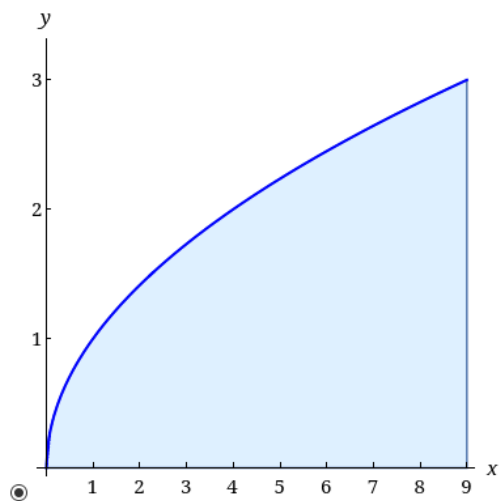
Sketch the region enclosed by the given curves. (A graphing calculator is recommended.)

$$y = \sqrt{x}, \quad y = 0, \quad x = 9$$

$$\begin{aligned}\int_0^9 \sqrt{x} &= \int_0^9 x^{1/2} = \left[ \frac{2}{3} x^{3/2} \right]_0^9 \\ &= F(9) - F(0) \\ &= \left( \frac{2}{3} 9^{3/2} \right) - \left( \frac{2}{3} 0^{3/2} \right) \\ &= 18\end{aligned}$$

Calculate its area.

18



# Q13

Friday, November 13, 2020 9:25 PM

Sketch the region enclosed by the given curves. (A graphing calculator is recommended.)

$$y = 9 - x^2, \quad y = 0$$

$$\begin{aligned} \int_{-3}^3 (9 - x^2) dx &= \left[ 9x - \frac{x^3}{3} \right]_{-3}^3 \\ &= F(3) - F(-3) \\ &= \left( 9(3) - \frac{3^3}{3} \right) - \left( 9(-3) - \frac{(-3)^3}{3} \right) \\ &= 36 \end{aligned}$$

Calculate its area.

36

