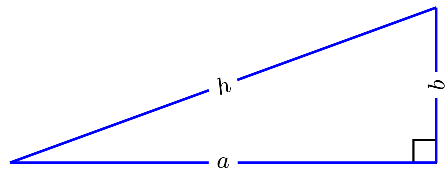
Pythagoras Theorem [PT] Says:

For any right triangle in Euclidean space, having sides a, and b with hypothenuse[1], h, the sides of such triangle are related by:

$$\boxed{a^2 + b^2 = h^2}$$

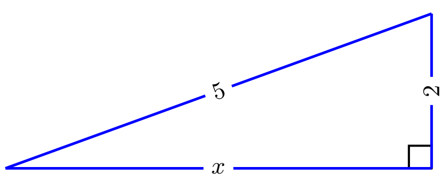


What is useful for: Delivering the *third* side

The Pythagoras Theorem is useful for nearly a billion different occasions. Anything from proving other theorems to checking to make sure walls are built straight up or to determine the radius of the earth. *Most notably and most generally, it is useful in solving the third side of a right triangle when any two of the sides are known.*

Example:

Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



$$\begin{aligned} x^2 + 2^2 &= 5^2 && \text{(PT)} \\ x^2 + 4 &= 25 && \text{(BI)} \\ x^2 &= 21 && \text{(CLA, Bi)} \\ x &= \pm\sqrt{21} && \text{(SRP)} \\ x &= \sqrt{21} && \text{(assume positive real sides)} \end{aligned}$$

Example:

Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.

[1]hypothenuse is defined to be the side across from the 90° angle

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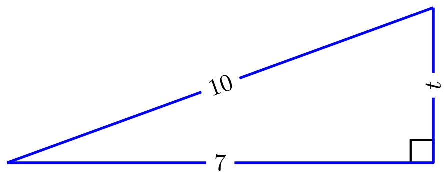
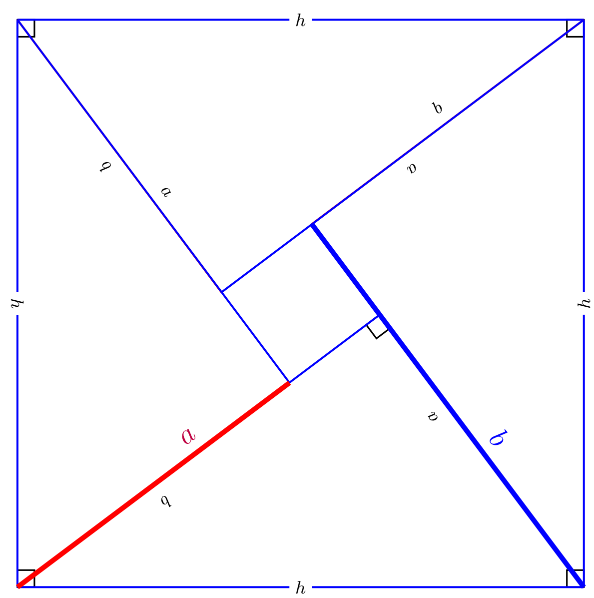


$$7^2 + t^2 = 10^2 \quad \text{(PT)}$$
$$t^2 + 49 = 100 \quad \text{(BI)}$$
$$t^2 = 51 \quad \text{(CLA, Bi)}$$
$$t = \pm\sqrt{51} \quad \text{(SRP)}$$
$$t = \sqrt{51} \quad \text{(assume positive real sides)}$$

Other uses for the Pythagoras Theorem [or Pythagorean Theorem] are explored in the exercises and on the self quiz as well as subsequent sections. In particular, we will emphasize one very important use of the Pythagoras Theorem. That is the idea that *knowing one side and one ratio* is enough to figure out the entire right triangle. We devote the next section specifically to elaborate on this point. For now, we turn our attention to a timeless question, *why?* Why is it that the on a right triangle the square of the hypothenuse is equal to the sum of the squares of the each of the other two sides? It is the natural question any curious human being would ask, and we now contemplate and savor such question.

Make it Yours!

There are many many ways to prove or explain why the pythagoras theorem is true. Here we provide one way to look at it, and in the exercises we provide two more ways of seeing it. The idea below is a classic one and it lies on the simple but creative idea of making a large square composed of 4 triangles and a small square, then the brilliant breakthrough, to note that the area of the entire square is equal to the sum of the areas of the little pieces!!



Key, Essential, & Brilliant Idea

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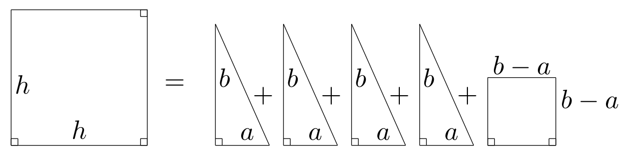
Look at this square and compute the area two different ways:

Area of Large Square = Sum of Areas of small pieces



The idea is brilliant, the execution is easy. Note the area of the big square is h^2 while each triangle has $area = base \cdot height/2$. Thus, for each triangle $A = \frac{ab}{2}$. Then, note the area of the smaller square on the right side is given by $(b-a)(b-a) = b^2 - 2ab + a^2$. Fill in the details, clean up the algebra and you will own the world famous, perpetually celebrated, always useful *Pythagoras Theorem*...

$$h^2 = \frac{ba}{2} + \frac{ba}{2} + \frac{ba}{2} + \frac{ba}{2} + (b-a)(b-a)$$

$$h^2 = 4\left(\frac{ba}{2}\right) + (b-a)(b-a)$$

$$h^2 = 2ba + (b-a)(b-a)$$

$$h^2 = 2ba + b^2 - 2ba + a^2$$

$$h^2 = b^2 + a^2$$

$$\boxed{a^2 + b^2 = h^2}$$

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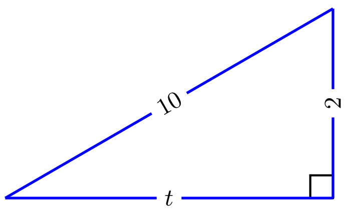
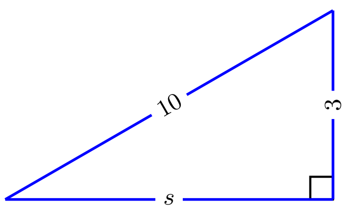
1. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



2. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



3. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.

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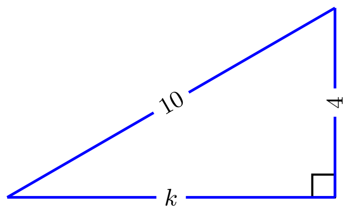
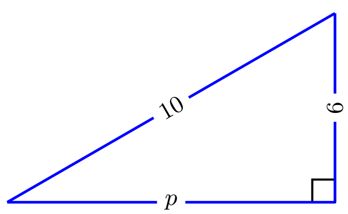
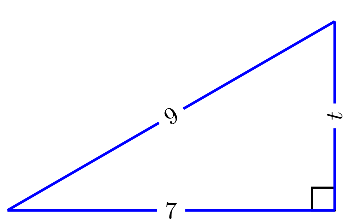


4. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



5. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



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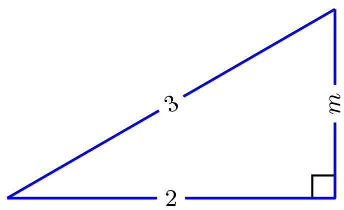
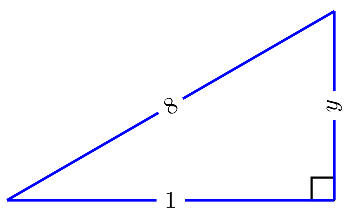
6. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



7. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



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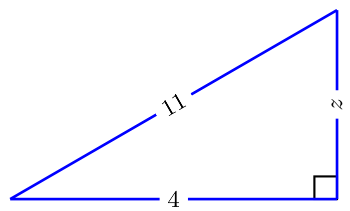
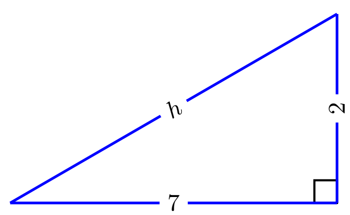
8. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



9. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



10. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.

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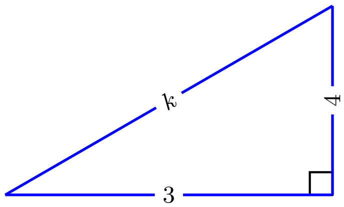
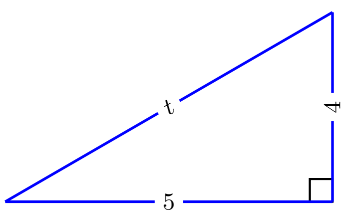
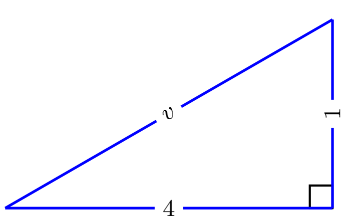


11. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



12. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



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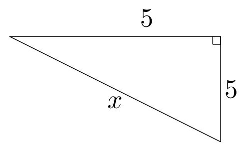
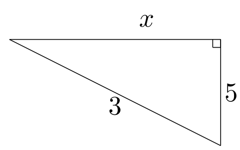
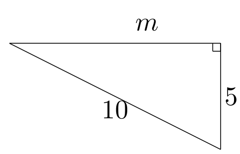
13. **Solve for** x



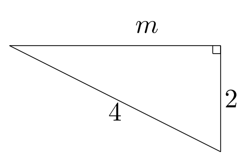
14. **Solve for** x or What is wrong with this picture?



15. **A Very Important Exercise** Solve for the m



16. **A Very Important Exercise** Solve for the m



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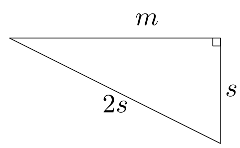
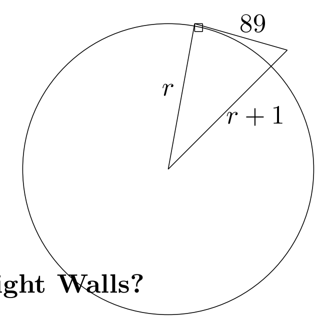
17. **A Very Important Exercise** Solve for the m

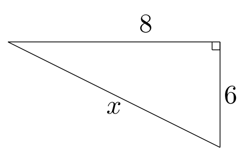


18. **Small World, NOT!** Assume the earth is perfectly round. Suppose when climbing to an altitude of 1 mile, the furthest visible point on the horizon is 89 miles away. *Find the radius of the earth.*

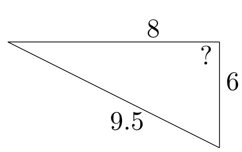


19. **Got Straight Walls?**

(a) Suppose you are planning to build a rectangular room 8ft. by 6ft., In theory, how long should the diagonal measurement be? solve for x



(b) Suppose when you are done building the first two walls shown below, you check the diagonal and it measures 9.5 ft. What can you say about the angle between the walls. A. Corner angle is $< 90°$ B. Corner angle is $> 90°$ C. Corner angle is $= 90°$ D. Impossible to know if larger or smaller than $90°$ E. None of These



(c) Do you think this theorem is 'reversible' that is, if $a^2 + b^2 = h^2$ does that mean we have a right triangle for sure?

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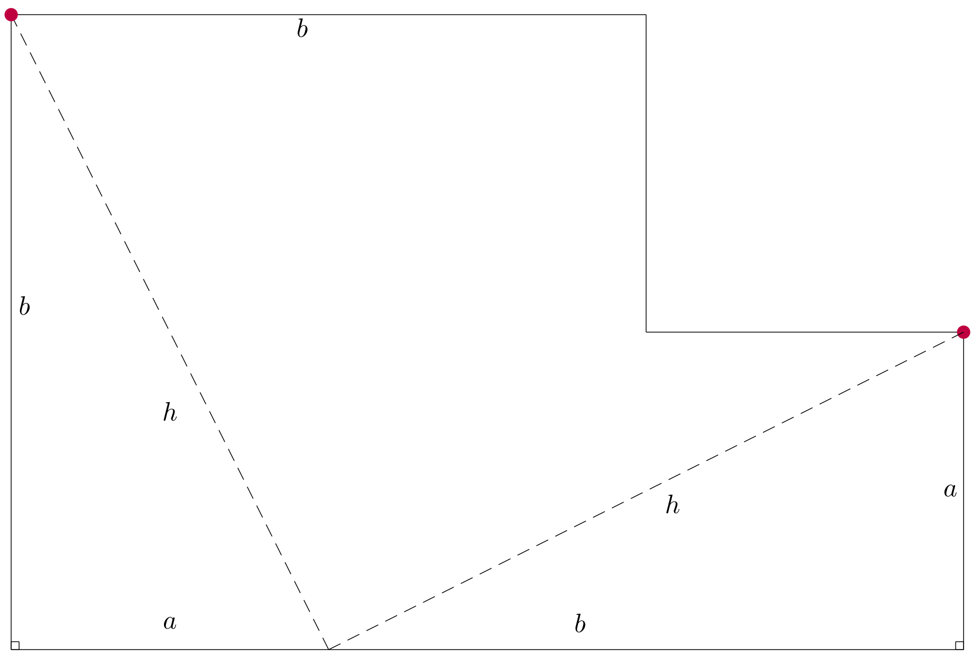
20. **Another Proof of Pythagoras Thm.** Find the area of the figure below. Note its is made up of two squares, one b by b, the other measures, a by a. Then, use scissors to cut out the figure below. After cutting the figure, cut the dashed lines. Can you make a perfect square out of the 3 cut figures? What is the area of the square? What does this show?



21. **Proof by US President, James Garfield (1876)** You know what to do (compare the area of the trapezoid to the sum of the little areas). Just keep in mind: area of a trapezoid is width, $a + b$ in this case, times the average of the heights, $(a + b)/2$.

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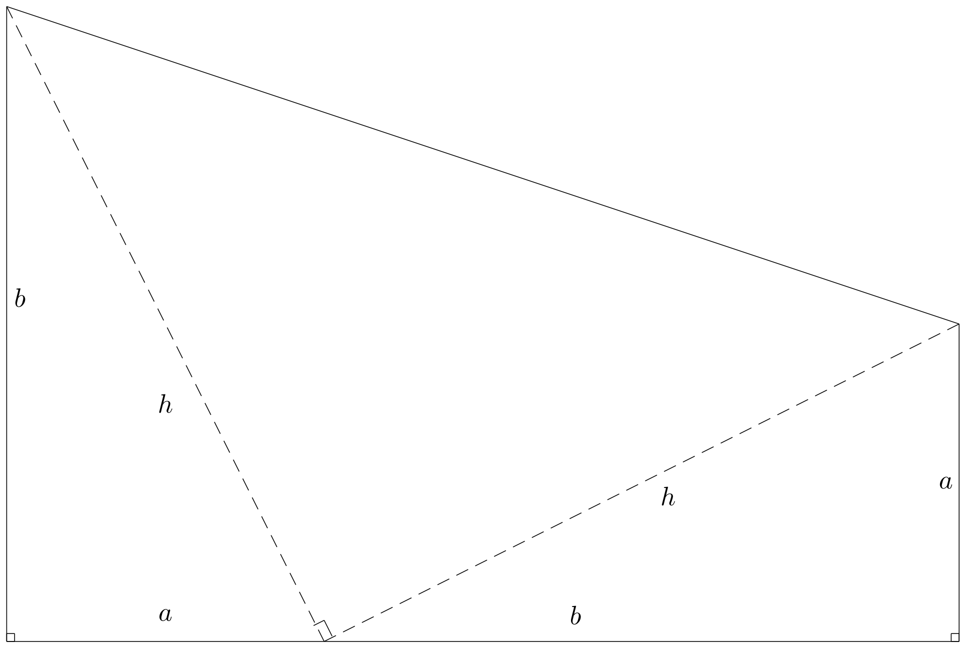

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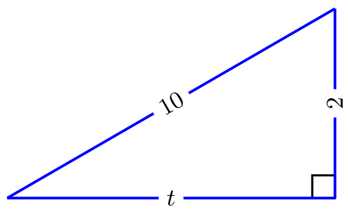
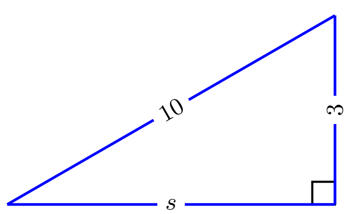
1. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



Solution:

$$t^2 + 2^2 = 10^2 \quad \text{(PT)}$$
$$t^2 + 4 = 100 \quad \text{(BI)}$$
$$t^2 = 96 \quad \text{(CLA, Bi)}$$
$$t = \pm\sqrt{96} \quad \text{(SRP)}$$
$$t = \sqrt{96} \quad \text{(assume positive real sides)}$$

2. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



Solution:

$$s^2 + 3^2 = 10^2 \quad \text{(PT)}$$
$$s^2 + 9 = 100 \quad \text{(BI)}$$
$$s^2 = 91 \quad \text{(CLA, Bi)}$$
$$s = \pm\sqrt{91} \quad \text{(SRP)}$$
$$s = \sqrt{91} \quad \text{(assume positive real sides)}$$

3. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.

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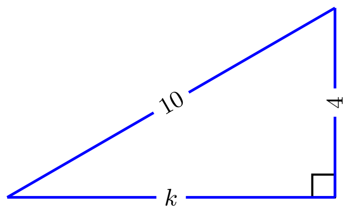
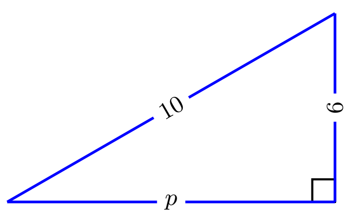


Solution:

$$\begin{aligned} k^2 + 4^2 &= 10^2 && \text{(PT)} \\ k^2 + 16 &= 100 && \text{(BI)} \\ k^2 &= 84 && \text{(CLA, Bi)} \\ k &= \pm\sqrt{84} && \text{(SRP)} \\ k &= \sqrt{84} && \text{(assume positive real sides)} \end{aligned}$$

---

4. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.

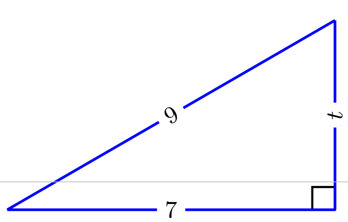


Solution:

$$\begin{aligned} p^2 + 6^2 &= 10^2 && \text{(PT)} \\ p^2 + 36 &= 100 && \text{(BI)} \\ p^2 &= 64 && \text{(CLA, Bi)} \\ p &= \pm\sqrt{64} && \text{(SRP)} \\ p &= \sqrt{64} && \text{(assume positive real sides)} \end{aligned}$$

---

5. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



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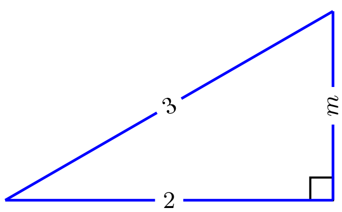
Solution:

$$
\begin{aligned}
7^2 + t^2 &= 9^2 && \text{(PT)}\\
t^2 + 49 &= 81 && \text{(BI)}\\
t^2 &= 32 && \text{(CLA, Bi)}\\
t &= \pm\sqrt{32} && \text{(SRP)}\\
t &= \sqrt{32} && \text{(assume positive real sides)}
\end{aligned}
$$

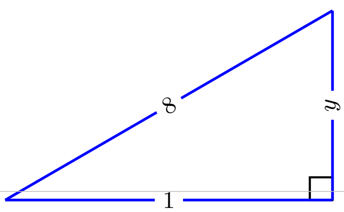
6. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



Solution:

$$
\begin{aligned}
2^2 + m^2 &= 3^2 && \text{(PT)}\\
m^2 + 4 &= 9 && \text{(BI)}\\
m^2 &= 5 && \text{(CLA, Bi)}\\
m &= \pm\sqrt{5} && \text{(SRP)}\\
m &= \sqrt{5} && \text{(assume positive real sides)}
\end{aligned}
$$

7. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



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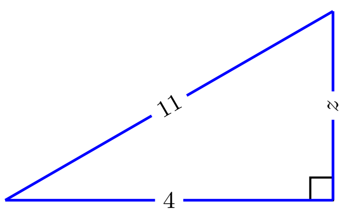
Solution:

$$
\begin{aligned}
1^2 + y^2 &= 8^2 && \text{(PT)}\\
y^2 + 1 &= 64 && \text{(BI)}\\
y^2 &= 63 && \text{(CLA, Bi)}\\
y &= \pm\sqrt{63} && \text{(SRP)}\\
y &= \sqrt{63} && \text{(assume positive real sides)}
\end{aligned}
$$

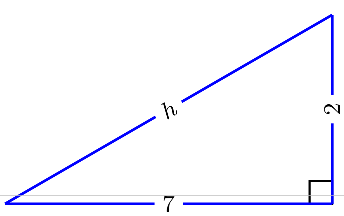
8. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



Solution:

$$
\begin{aligned}
4^2 + z^2 &= 11^2 && \text{(PT)}\\
z^2 + 16 &= 121 && \text{(BI)}\\
z^2 &= 105 && \text{(CLA, Bi)}\\
z &= \pm\sqrt{105} && \text{(SRP)}\\
z &= \sqrt{105} && \text{(assume positive real sides)}
\end{aligned}
$$

9. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



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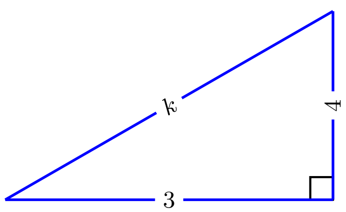
Solution:

$$
\begin{aligned}
7^2 + 2^2 &= h^2 && \text{(PT)}\\
49 + 4 &= h^2 && \text{(BI)}\\
53 &= h^2 && \text{(CLA, Bi)}\\
h &= \pm\sqrt{53} && \text{(SRP)}\\
h &= \sqrt{53} && \text{(assume positive real sides)}
\end{aligned}
$$

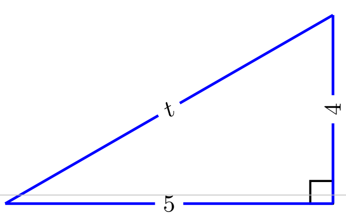
10. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



Solution:

$$
\begin{aligned}
3^2 + 4^2 &= k^2 && \text{(PT)}\\
9 + 16 &= k^2 && \text{(BI)}\\
25 &= k^2 && \text{(CLA, Bi)}\\
k &= \pm\sqrt{25} && \text{(SRP)}\\
k &= \sqrt{25} && \text{(assume positive real sides)}
\end{aligned}
$$

11. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



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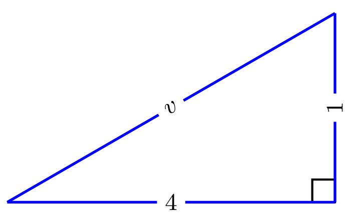
Solution:

$$
\begin{aligned}
5^2 + 4^2 &= t^2 && \text{(PT)} \\
25 + 16 &= t^2 && \text{(BI)} \\
41 &= t^2 && \text{(CLA, Bi)} \\
t &= \pm\sqrt{41} && \text{(SRP)} \\
t &= \sqrt{41} && \text{(assume positive real sides)}
\end{aligned}
$$

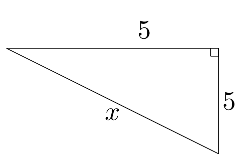
12. Two sides are given, use the *pythagoras theorem* to solve for the third side. For now, assume sides are of real positive length.



Solution:

$$
\begin{aligned}
4^2 + 1^2 &= v^2 && \text{(PT)} \\
16 + 1 &= v^2 && \text{(BI)} \\
17 &= v^2 && \text{(CLA, Bi)} \\
v &= \pm\sqrt{17} && \text{(SRP)} \\
v &= \sqrt{17} && \text{(assume positive real sides)}
\end{aligned}
$$

13. **Solve for** x



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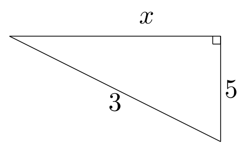
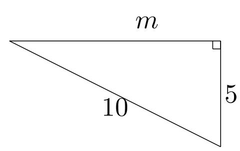
Solution:
$x = 5\sqrt{2}$ .... very famous..

14. **Solve for** x or What is wrong with this picture?



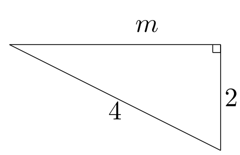
Solution:
impossible.. hypothenuse must be bigger..

15. **A Very Important Exercise** Solve for the m



Solution:
$x = 5\sqrt{3}$ very famous...

16. **A Very Important Exercise** Solve for the m



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Solution:
$x = 2\sqrt{3}$ very famous...

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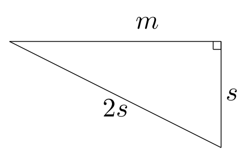
17. **A Very Important Exercise** Solve for the m



Solution:
$x = s\sqrt{3}$ very famous...

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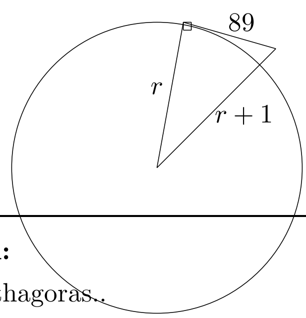
18. **Small World, NOT!** Assume the earth is perfectly round. Suppose when climbing to an altitude of 1 mile, the furthest visible point on the horizon is 89 miles away. *Find the radius of the earth.*



Solution:
apply pythagoras..

$$r^2 + 89^2 = (r+1)^2$$

$r \approx 3961$ clean up..

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19. **Got Straight Walls?**

(a) Suppose you are planning to build a rectangular room 8ft. by 6ft., In theory, how long should the diagonal measurement be? solve for x

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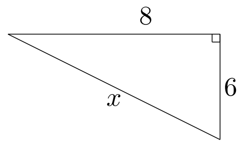
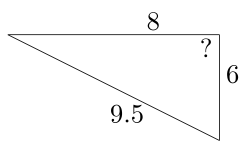


Solution:
$x = 10$

(b) Suppose when you are done building the first two walls shown below, you check the diagonal and it measures 9.5 ft. What can you say about the angle between the walls. A. Corner angle is $< 90°$ B. Corner angle is $> 90°$ C. Corner angle is $= 90°$ D. Impossible to know if larger or smaller than $90°$ E. None of These

Solution:
if the 'hypothenuse' is shorter than 10, then across from it must be an angles smaller than $90°$



(c) Do you think this theorem is 'reversible' that is, if $a^2 + b^2 = h^2$ does that mean we have a right triangle for sure?

Solution:
yes, the above suggests so....

20. **Another Proof of Pythagoras Thm.** Find the area of the figure below. Note its is made up of two squares, one b by b, the other measures, a by a. Then, use scissors to cut out the figure below. After cutting the figure, cut the dashed lines. Can you make a perfect square out of the 3 cut figures? What is the area of the square? What does this show?

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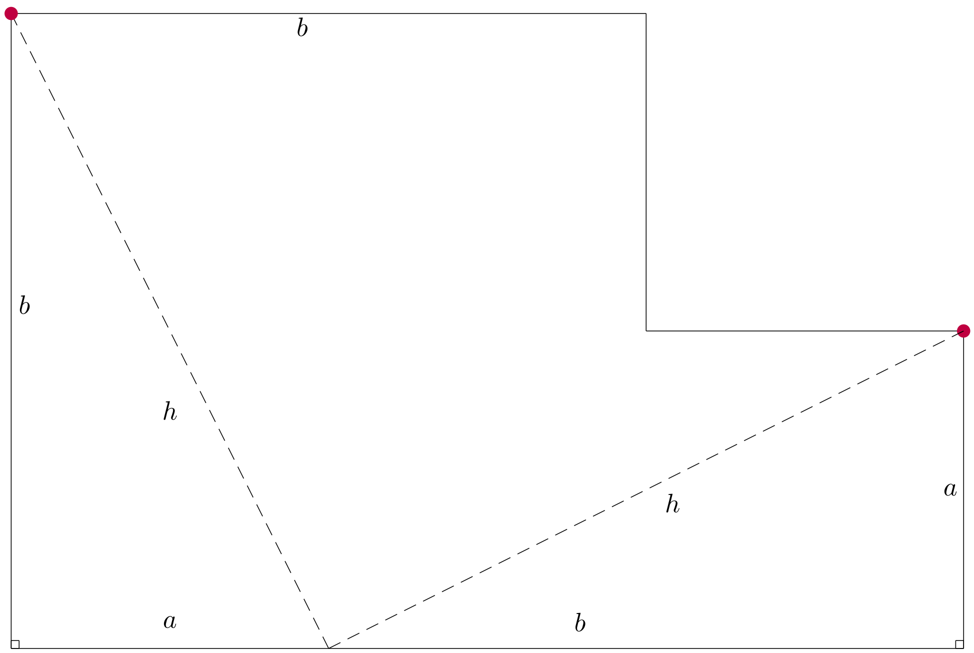

Solution:

cut the dotted lines, then use the noted dots, as pivot points to rotate the cut triangles, this should turn the $a^2 + b^2$ shape into a c^2 shape..

21. **Proof by US President, James Garfield (1876)** You know what to do (compare the area of the trapezoid to the sum of the little areas). Just keep in mind: area of a trapezoid is width, $a + b$ in this case, times the average of the heights, $(a + b)/2$.

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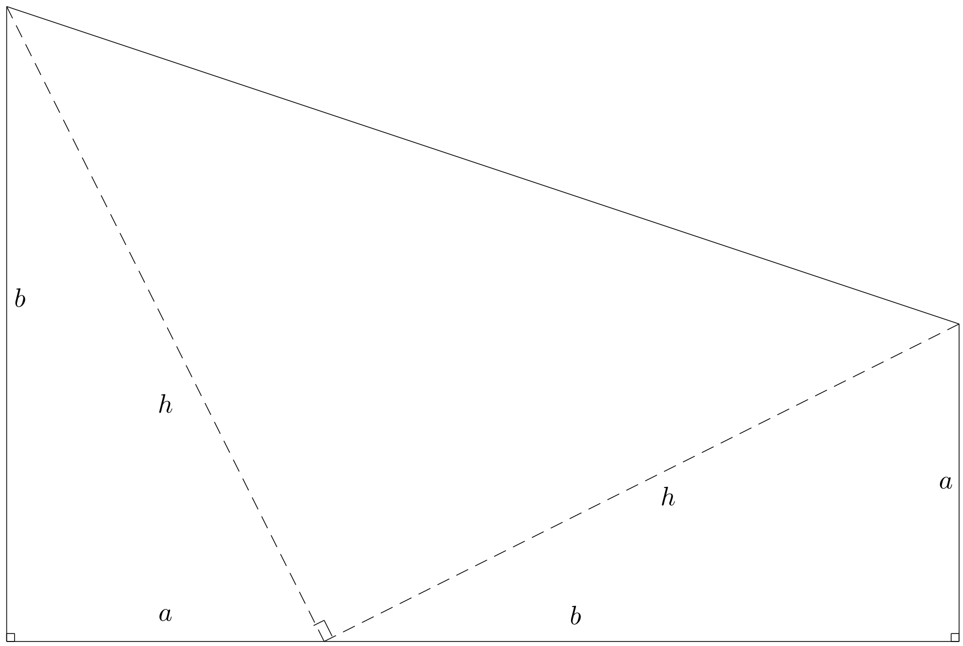

Solution:

a very nice problem ... think about it...

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