Q1

Math 150   Quiz 2

Find the following derivative. You must show all of your work. Once you have taken the derivative you do not have to simplify.

1) Let $f(x) = \frac{x^3 + \cos(x) - 1}{e^x - 3}$. Find $f'(x)$.
2) Let $f(x) = x^2 \sin(x)$. Find $f''(x)$.
3) Let $f(x) = x^5 e^x \sec(x)$. Find $f'(x)$.

The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

1.) Let $f(x) = \frac{x^3 + \cos(x) - 1}{e^x - 3}$. Find $f'(x)$

$$f(x) = x^3 + \cos(x) - 1$$
$$g(x) = e^x - 3$$

$$f'(x) = \frac{(e^x - 3)\frac{d}{dx}\left[x^3 + \cos(x) - 1\right] - \left[x^3 + \cos(x) - 1\right]\frac{d}{dx}\left(e^x - 3\right)}{\left(e^x - 3\right)^2}$$

$$= \frac{(e^x - 3)\left[3x^2 - \sin(x)\right] - \left[x^3 + \cos(x) - 1\right]\left(e^x\right)}{\left(e^x - 3\right)^2}$$

$$f'(x) = \frac{(e^x - 3)\left[3x^2 - \sin(x)\right] - e^x\left[x^3 + \cos(x) - 1\right]}{\left(e^x - 3\right)^2}$$

Q2

<center>The Product Rule</center>

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)\frac{d}{dx}\left[g(x)\right] + g(x)\frac{d}{dx}\left[f(x)\right]$$

2.) Let $f(x) = x^2 \sin(x)$. Find $f''(x)$

$$f(x) = x^2$$
$$g(x) = \sin(x)$$

$$f'(x) = x^2 \sin(x)$$
$$= x^2 \frac{d}{dx}\left[\sin(x)\right] + \sin(x)\frac{d}{dx}\left(x^2\right)$$
$$= x^2 \cos(x) + \sin(x)(2x)$$

$$f'(x) = x^2 \cos(x) + 2x\sin(x)$$

$$f''(x) = \frac{d}{dx}\left[x^2\cos(x)\right] + \frac{d}{dx}\left[2x\sin(x)\right]$$
$$= \left[x^2\frac{d}{dx}\left[\cos(x)\right] + \cos(x)\frac{d}{dx}\left(x^2\right)\right] + \left[2x\frac{d}{dx}\left[\sin(x)\right] + \sin(x)\frac{d}{dx}\left(2x\right)\right]$$
$$= \left[-x^2\sin(x) + \cos(x)(2x)\right] + \left[2x\cos(x) + \sin(x)(2)\right]$$

$$\boxed{f''(x) = \left[-x^2\sin(x) + 2x\cos(x)\right] + \left[2x\cos(x) + 2\sin(x)\right]}$$

The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

3) Let $f(x) = x^5 e^x \sec(x)$. Find $f'(x)$

$$f(x) = x^5$$

$$g(x) = e^x \sec(x)$$

x^5 instead

$$f'(x) = x^5 \frac{d}{dx}\left[e^x \sec(x)\right] + e^x \sec(x) \frac{d}{dx}(e^x)$$

$$= x^5\left[e^x \frac{d}{dx}(\sec(x)) + \sec(x)\frac{d}{dx}(e^x)\right] + e^x \sec(x)\frac{d}{dx}(e^x)$$

$$= x^5\left[e^x(\sec(x)\tan(x)) + \sec(x)(e^x)\right] + e^x \sec(x)(e^x)$$

$$= x^5\left[e^x \sec(x)\tan(x) + e^x \sec(x)\right] + e^x \sec(x)(e^x)$$

$$= x^5\left[e^x \sec(x)(\tan(x) + 1)\right] + e^x \sec(x)(e^x)$$

$$= x^5 e^x \sec(x)(\tan(x) + 1) + e^x \sec(x)(e^x)$$

$$\boxed{f'(x) = e^x \sec(x)\left[x^2(\tan(x) + 1) + e^x\right]}$$