Q1

Sunday, October 25, 2020 1:58 PM

Let $f(x) = \frac{x}{x^3-1}$. Use algebra and calculus to find the following. You must show your work.

a) Intervals of increasing or decreasing

b) local max's and min's

c) concavity and inflection points

$$f(x) = \frac{x}{x^3-1}$$

$$f'(x) = \frac{(x^3-1)\frac{d}{dx}(x) - x\frac{d}{dx}(x^3-1)}{(x^3-1)^2}$$

$$= \frac{(x^3-1)(1) - x(3x^2)}{(x^3-1)^2}$$

$$= \frac{x^3-1-3x^3}{(x^3-1)^2}$$

$$f'(x) = \frac{-2x^3-1}{(x^3-1)^2}$$

$$\frac{-2x^3-1}{(x^3-1)^2} = 0$$

$-2x^3-1=0$
$-2x^3=1$
$x^3=-\frac{1}{2}$
$x=\sqrt[3]{-\frac{1}{2}}$

$x^3-1=0$
$x^3=1$
$x=1$

$$f\left(\sqrt[3]{-\tfrac{1}{2}}\right) = \frac{x}{x^3-1}$$

$$= \frac{\sqrt[3]{-\frac{1}{2}}}{\left(\sqrt[3]{-\frac{1}{2}}\right)^3-1}$$

$$= \frac{\sqrt[3]{-\frac{1}{2}}}{-\frac{1}{2}-1}$$

$$= \frac{\sqrt[3]{-\frac{1}{2}}}{-\frac{3}{2}}$$

$$= -\frac{2\sqrt[3]{-\frac{1}{2}}}{3}$$

$$f(1) = \frac{x}{x^3-1}$$

$$= \frac{1}{(1)^3-1}$$

$$= \frac{1}{0}$$

undefined

$+ \quad - \quad -$

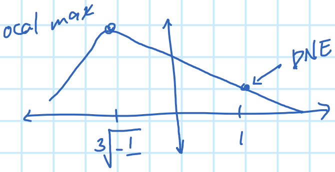
$\sqrt[3]{-\frac{1}{2}} \qquad 1$

$x<\sqrt[3]{-\frac{1}{2}} \quad \sqrt[3]{-\frac{1}{2}}<x<1 \quad x>1$

Intervals	$f'(x)$
$x<\sqrt[3]{-\frac{1}{2}}$	$f(-1) = +$
$\sqrt[3]{-\frac{1}{2}}<x<1$	$f(0) = -$
$x>1$	$f(2) = -$

Local max

DNE



f' is increasing @ $\left(-\infty, \sqrt[3]{-\frac{1}{2}}\right)$

f' is decreasing @ $\left(\sqrt[3]{-\frac{1}{2}}, 1\right) \cup (1, \infty)$

Local max value $= -\frac{2\sqrt[3]{-\frac{1}{2}}}{3}$

Local min value = DNE

$$f''(x) = \frac{d}{dx}\left(\frac{-2x^3-1}{(x^3-1)^2}\right)$$

$$= \frac{(x^3-1)^2\frac{d}{dx}(-2x^3-1) - (-2x^3-1)\frac{d}{dx}\left[(x^3-1)^2\right]}{(x^3-1)^2}$$

$$= \frac{(x^3-1)^2(-6x^2) - (-2x^3-1)2(x^3-1)\frac{d}{dx}(x^3-1)}{(x^3-1)^2}$$

$$= \frac{(x^3-1)^2(-6x^2) - (-2x^3-1)2(x^3-1)(3x^2)}{(x^3-1)^2}$$

$$= \frac{\cancel{(x^3-1)}\left[(x^3-1)(-6x^2) - (-2x^3-1)2(3x^2)\right]}{(x^3-1)^{\cancel{2}}}$$

$$= \frac{-6x^5+6x^2-\left[2(-6x^2-3x^2)\right]}{x^3-1}$$

$$= \frac{-6x^5+6x^2-12x^2-6x^2}{x^3-1}$$

Out of time

concavity upwards $f''(x)>0$

concavity downwards $f''(x)<0$