Chapter 5 (draft): Data Rules

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temporary
$\mathbb{N}, \mathbb{B} : \star$

$$\frac{}{\Gamma \vdash \mathbb{N} \sqsupseteq \mathbb{N} : \star}$$

$$\frac{}{\Gamma \vdash \mathbb{B} \sqsupseteq \mathbb{B} : \star}$$

$$\frac{}{\mathbb{N}\ \mathbf{Val}}$$

$$\frac{}{\mathbb{B}\ \mathbf{Val}}$$

1 Endpoint Rules

some of these "ok" checks may be redundant.

$$\frac{}{\Gamma \vdash \star \sqsupseteq \star : \star}$$

$$\frac{\begin{array}{c}\Gamma \vdash L\ \mathbf{ok} \\ \Gamma \vdash L \sqsupseteq A : \star \\ \Gamma \vdash L \sqsupseteq B : \star \\ \Gamma \vdash a \sqsupseteq a' : A\end{array}}{\Gamma \vdash a :: L \sqsupseteq a' :: L : B}$$

$$\frac{\begin{array}{c}\Gamma \vdash L\ \mathbf{ok} \\ \Gamma \vdash L \sqsupseteq A : \star \\ \Gamma \vdash L \sqsupseteq B : \star \\ \Gamma \vdash a \sqsupseteq a' : A\end{array}}{\Gamma \vdash a :: L \sqsupseteq a' :: L : B}$$

$$\frac{\begin{array}{c}\Gamma \vdash \{\overline{a},\}_{L'}\ \ \mathbf{ok} \\ a \in \overline{a} \\ \Gamma \vdash a \sqsupseteq a' : A \\ \Gamma \vdash L' \sqsupseteq B : \star\end{array}}{\Gamma \vdash \{\overline{a},\}_{L'} \sqsupseteq a' :: L' \ \ : \ \ B}$$

$$\frac{\begin{array}{c}c \in \{a, b\} \\ \Gamma \vdash c \sqsupseteq c' : C' \\ \Gamma \vdash L \sqsupseteq c' : C' \\ \Gamma \vdash L \sqsupseteq D : \star \\ \Gamma \vdash L\ \mathbf{ok}\end{array}}{\Gamma \vdash a \sim^{L}_{\ell,o} b \sqsupseteq a' :: L \ \ : \ \ B}$$

1.1 Endpoint Conversions

$$\frac{\Gamma \vdash a \sqsupseteq a' : A' \quad \Gamma \vdash A' \equiv B : \star}{\Gamma \vdash a \sqsupseteq a' : B}$$

$$\frac{\Gamma \vdash a \sqsupseteq a' : A' \quad \Gamma \vdash a' \equiv b : A'}{\Gamma \vdash a \sqsupseteq b : A'}$$

2 Empty

$$\frac{}{.\ \textbf{Empty}}$$

3 Step

the exact order doesn't really matter for most of these rules. But it will be helpful if it is made deterministic

$$\frac{L \rightsquigarrow L'}{\{\overline{a,}\}_L \rightsquigarrow \{\overline{a,}\}_{L'}}$$

$$\frac{L\ \textbf{Val} \quad \overline{a}\ \textbf{Val} \quad \star \in \overline{a}}{\{\overline{a,}\star, \overline{b,}\}_L \rightsquigarrow \{\overline{a,}\overline{b,}\}_L}$$

not determinisic...

$$\frac{L\ \textbf{Val} \quad \overline{a} \rightsquigarrow \overline{a'}}{\{\overline{a,}\}_L \rightsquigarrow \{\overline{a',}\}_L}$$

$$\frac{L\ \textbf{Val} \quad \overline{a}\ \textbf{Val} \quad b\ \textbf{Val} \quad b :: L'}{\{\overline{a,}b :: L', \overline{c,}\}_L \rightsquigarrow \{\overline{a,}b, \overline{c,}\}_{L' \cup L}}$$

perhaps it is easier to include $\cup$ as a sintax, instead of an operator? but needs to deal with the type of the type,

$$\frac{}{\{\star\}_\star \rightsquigarrow \star}$$

$$\frac{}{\star \sim_{\ell,o}^{\star} \star \rightsquigarrow \star}$$

$$\frac{}{(a :: L) :: L' \rightsquigarrow a' :: (L \cup L')}$$

$$\frac{}{a :: L' \sim_{\ell,o}^{L} b \rightsquigarrow a \sim_{\ell,o}^{L \cup L'} b}$$

$$\frac{}{a \sim_{\ell,o}^{L} b :: L' \rightsquigarrow a \sim_{\ell,o}^{L \cup L'} b}$$

structrural rules, perhaps invest in evaluation contexts

$$\frac{L \rightsquigarrow L'}{a :: L \rightsquigarrow a :: L'}$$

$$\frac{a \rightsquigarrow a' \quad L\ \textbf{Val}}{a :: L \rightsquigarrow a' :: L}$$

$$\frac{L \rightsquigarrow L'}{a \sim_{\ell,o}^{L} b \rightsquigarrow a \sim_{\ell,o}^{L'} b}$$

$$\frac{a \rightsquigarrow a'}{a \sim_{\ell,o}^{L} b \rightsquigarrow a' \sim_{\ell,o}^{L} b}$$

$$\frac{b \rightsquigarrow b'}{a \sim_{\ell,o}^{L} b \rightsquigarrow a \sim_{\ell,o}^{L} b'}$$

4 Values

$$\frac{}{\star\ \mathbf{Val}}$$

5 Blame

$$\frac{a\ \mathbf{Blame}_{\ell,o} \quad a \in \overline{a,}}{\{\overline{a,}\}_{\star}\ \mathbf{Blame}_{\ell,o}}$$

$$\frac{L\ \mathbf{Blame}_{\ell,o}}{a :: L\ \mathbf{Blame}_{\ell,o}}$$

$$\frac{\mathbf{head}\ a \neq \mathbf{head}\ b}{a \sim^{L}_{\ell,o} b\ \mathbf{Blame}_{\ell,o}}$$

$$\frac{L\ \mathbf{Blame}_{\ell,o}}{a \sim^{L}_{\ell,o} b\ \mathbf{Blame}_{\ell,o}}$$

$$\frac{c \in \{a, b\} \quad c\ \mathbf{Blame}_{\ell,o}}{a \sim^{L}_{\ell,o} b\ \mathbf{Blame}_{\ell,o}}$$

6 Definitional equality

(consider untyped varient)

equality has these properties, the exact definition may differ, and some of these properties would be hard to build directly

respects evaluation

$$\frac{a \rightsquigarrow a' \quad \Gamma \vdash a : A}{\Gamma \vdash a \equiv a' : A}$$

Assume a congruent equivalence that

$$\frac{\Gamma \vdash a \equiv a' : A \quad \Gamma, x : A \vdash b : B}{\Gamma \vdash b\,[x ::= a] \equiv b\,[x ::= a'] : B\,[x ::= a]}$$

$$\frac{\Gamma \vdash a : A}{\Gamma \vdash a \equiv a : A}$$

$$\frac{\Gamma \vdash a \equiv b : A}{\Gamma \vdash b \equiv a : A}$$

$$\frac{\Gamma \vdash a \equiv b : A \quad \Gamma \vdash b \equiv c : A}{\Gamma \vdash b \equiv c : A}$$

associates trivial casts with uncast terms

$$\frac{\Gamma \vdash a : A}{\Gamma \vdash a \equiv a :: \{A\}_{\star} : A}$$

associates casts at the same endpoints

$$\frac{\Gamma \vdash a \equiv a' : A \quad \Gamma \vdash a :: L\ : B \quad \Gamma \vdash a' :: L'\ : B}{\Gamma \vdash a :: L\ \equiv\ a' :: L'\ :\ B}$$

7 ok

just going to alow okness to imply connectedness, seems easier then 2 seperate judgments

$$\frac{}{\Gamma \vdash \star \ \mathbf{ok}}$$

$$\frac{\begin{array}{c}\Gamma \vdash a \ \mathbf{ok} \\ \Gamma \vdash A \ \mathbf{ok} \\ \Gamma \vdash a \sqsupseteq a' : A' \\ \Gamma \vdash A \sqsupseteq A' : \star\end{array}}{\Gamma \vdash \{a\}_A \ \ \mathbf{ok}}$$

$$\frac{\begin{array}{c}\Gamma \vdash L \ \mathbf{ok} \\ L = \{\overline{a,}\}_{L'} \\ \Gamma \vdash b \ \mathbf{ok} \\ \Gamma \vdash b \sqsupseteq a' : A' \\ \Gamma \vdash L \sqsupseteq a' : A' \\ \Gamma \vdash L' \ \mathbf{ok} \\ \Gamma \vdash L' \sqsupseteq A' : \star\end{array}}{\Gamma \vdash \{b, \overline{a,}\}_{L'} \ \ \mathbf{ok}}$$

$$\frac{\begin{array}{c}\Gamma \vdash A \ \mathbf{ok} \\ \Gamma \vdash L \ \mathbf{ok} \\ \Gamma \vdash a \sqsupseteq a' : A' \\ \Gamma \vdash L \sqsupseteq A' : \star\end{array}}{\Gamma \vdash a :: L \ \mathbf{ok}}$$

$$\frac{\begin{array}{c}\Gamma \vdash L \ \mathbf{ok} \\ \Gamma \vdash a \sqsupseteq a' : A' \\ \Gamma \vdash L \sqsupseteq A' : \star \\ \Gamma \vdash b \sqsupseteq b' : A'\end{array}}{\Gamma \vdash a \sim^{L}_{\ell,o} b \ \mathbf{ok}}$$