Chapter 4 (draft): Data and Pattern Matching

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Part I
Introduction

clean up!

The encoded data presented in Chapter 2 is unrealistically inconvenient and is especially implausible for a dependently typed programming language intended to be easy to use.

User defined data is an essential feature of a realistic programming language. Simple data types like `Nat` and `Bool` are essential for organizing readable programs. Dependent data like `Id` can represent the mathematical predicate of equality. Dependent data can also be used to preserve invariants like the length in `Vec`.

We have opted for data definitions like those found in systems like Agda and Coq. A data definition is formed by a type constructor indexed by value arguments, and a set of constructors that tag data and characterize their arguments. See Figure 1 for the definitions of several standard data types. Data is easy to build and reason about, since data can only be created from its constructors. Unfortunately data elimination is more murky.

How should data be used? One option is a direct eliminator scheme, like Coq uses in its core language. It is worth formalizing that example.

Part II
Data and elimination

A minimal accounting of data can be given by extending the surface language syntax in ...

The slightly awkward eliminator syntax is designed to be forward compatible with the pattern matching system defined in the rest of this section. Mutual patterns. Recursion does the work of induction in the MLTT style

abbreviate away dumb arrows, unstated separator is a space, also usual syntax (:*)? also shorthands for telescopes

ebnf? if r
underline

pat or p?

alternativ

1 Incomplete Eliminations

2 (non) Strict Positivity

3 Specification

4 Bidirectional extension

a convenient bidirectional interpretation

$$\frac{\mathsf{data}\ D\ \Delta \in \Gamma}{\Gamma \vdash D \overrightarrow{?} \Delta \to *} \cdots$$

$$\frac{d\ :\ \Theta \to D\overline{m} \in \Gamma}{\Gamma \vdash d \overrightarrow{?} \Theta \to D\overline{m}} \cdots$$

```
data Bool : * {
| True : Bool
| False : Bool
};

data Nat : * {
| Z : Nat
| S : Nat -> Nat
};
-- Syntactic sugar expands decimal numbers
-- into their unary representation.

data Vec : (A : *) -> Nat -> * {
| Nil  : (A : *) -> Vec A Z
| Cons : (A : *) -> A -> (x : Nat)
         -> Vec A x -> Vec A (S x)
};

data Id : (A : *) -> A -> A -> * {
| refl  : (A : *) -> (a : A) -> Id A a a
};
```

Figure 1: Definitions of Common Data Types

bidirectional non dependent elimination

$$\frac{\begin{array}{c}\mathsf{data}\, D\, \Delta \in \Gamma \\ \Gamma \vdash n \overrightarrow{:} D\overline{N} \\ \Gamma, \overline{x} : \Delta, z : D\,\overline{x} \vdash M \overleftarrow{:} \star \\ \forall\, d\, :\, \Theta \to D\overline{m} \in \Gamma. \quad \Gamma, \overline{x} : \Delta, \overline{y}_d : \Theta \vdash m_d \overleftarrow{:} M\end{array}}{\Gamma \vdash \mathsf{case}\, n\, \left\{\overline{\mid (d\,\overline{y}_d) \Rightarrow m_d}\right\} \overleftarrow{:} M}\ \ldots$$

bidirectional dependent elimination

$$\frac{\begin{array}{c}\mathsf{data}\, D\, \Delta \in \Gamma \\ \Gamma \vdash \overline{N} \overleftarrow{:} \Delta \quad \Gamma \vdash n \overleftarrow{:} D\overline{N} \\ \Gamma, \overline{x} : \Delta, z : D\,\overline{x} \vdash M \overleftarrow{:} \star \\ \forall\, d\, :\, \Theta \to D\overline{m} \in \Gamma. \quad \Gamma, \overline{x} : \Delta, \overline{y}_d : \Theta \vdash m_d \overleftarrow{:} M\end{array}}{\begin{array}{c}\Gamma \vdash \mathsf{case}\, \overline{N}, n\ \langle \overline{x} \Rightarrow z : D\,\overline{x} \Rightarrow M\rangle \left\{\overline{\mid \overline{x} \Rightarrow (d\,\overline{y}_d) \Rightarrow m_d}\right\} \\ : M\left[\overline{x} \coloneqq \overline{N}, z \coloneqq n\right]\end{array}}\ \ldots$$

may not need scrut wf check? oh but what about the empty types!

$$\frac{\Gamma \vdash \Delta \overleftarrow{wf}}{\Gamma \vdash \mathsf{data}\, D\, \Delta \overleftarrow{wf}}\ \ldots$$

abuse of notation...

ok?$\Gamma \vdash \Delta \overleftarrow{:} \star$, perhaps $\Gamma \vdash \Delta\, wf$ and $\Gamma \vdash \Delta \overleftarrow{wf}$. or $\Gamma \vdash \Delta\, ok$ ...or $\Gamma \vdash \Delta \vdash \ldots$

abuse of notation...

$$\frac{\Gamma \vdash \mathsf{data}\, D\, \Delta \quad \forall d. \Gamma, \mathsf{data}\, D\, \Delta \vdash \Theta_d \quad \forall d.\, \Gamma, \mathsf{data}\, D\, \Delta, \Theta_d \vdash \overline{m}_d : \Delta}{\Gamma \vdash \mathsf{data}\, D\, :\, \Delta\, \left\{\overline{\mid d\, :\, \Theta_d \to D\overline{m}_d}\right\}}\ \ldots$$

telescope,

Δ, Θ	$::=$	$.$	empty telescope
	$\mid$	$x : M,\, \Delta$	extend telescope

list of O, separated with s

$\overline{sO}$,$\overline{Os}$	$::=$	s	empty list
	$\mid$	$sO\overline{sO}$	extend list

data type identifier,

D	$::=$	...	

data constructor identifier,

d	$::=$	...	

contexts,

Γ	$::=$	...	
	$\mid$	$\Gamma, \mathsf{data}\, D\, :\, \Delta \to * \left\{\overline{\mid d\, :\, \Theta \to D\overline{m}}\right\}$	data def
	$\mid$	$\Gamma, \mathsf{data}\, D\, :\, \Delta \to *$	abstract data
m, n, M, N	$::=$	...	
	$\mid$	$D\,\overline{m}$	type cons.
	$\mid$	$d\,\overline{m}$	data cons.
	$\mid$	$\mathsf{case}\,\overline{N}$,$m\, \left\{\overline{\mid pat \Rightarrow m}\right\}$	data elim. without motive
	$\mid$	$\mathsf{case}\,\overline{N}$,$m\, \langle \overline{x \Rightarrow} y : D\,\overline{x} \Rightarrow M\rangle \left\{\overline{\mid pat \Rightarrow m}\right\}$	data elim. with motive

(minimal) patterns,

pat	$::=$	$\overline{\Rightarrow x} \Rightarrow (d\,\overline{y})$	
pat	$::=$	$x \Rightarrow pat$	match a variable
	$\mid$	$(d\,\overline{x})$	match a constructor

values,

v	$::=$	...	
	$\mid$	$D\,\overline{v}$	
	$\mid$	$d\overline{v}$	

differentiate identifiers with font

motive should not need to insist on the type info of the binder?

Grey out things that are surface syntax but not needed for theory

Make identifiers consistent with chapter 2, and locations in chapter 3

Make a module syntax?

Figure 2: Surface Language Data

$$\frac{}{\Gamma \vdash .}\ \ldots$$

$$\frac{\Gamma \vdash M : \star \quad \Gamma, x : M \vdash \Delta}{\Gamma \vdash x : M, \Delta}\ \ldots$$

suspect this also hinges on regularity

$$\frac{\Gamma \vdash}{\Gamma \vdash \Diamond : .}\ \ldots$$

$$\frac{\Gamma, x : M \vdash \Delta \quad \Gamma \vdash m : M \quad \Gamma \vdash \overline{n}\,[x := m] : \Delta\,[x := m]}{\Gamma \vdash m, \overline{n}\ :\ x : M, \Delta}\ \ldots$$

Figure 3: Surface Language Type Assignment System

Part III

Pattern Matching

Unfortunately, eliminators are cumbersome for programmers to deal with directly. For instance, in figure 7 we show how `Vec` data can be directly eliminated in the definition of `head′`. The `head′` function needs to redirect impossible inputs to a dummy type and requires several copies of the same variable that cannot be identified automatically. Pattern matching is much more ergonomic than a direct eliminator, where variables will be assigned their definitions as needed, and unreachable branches can be omitted from code. For this reason, pattern matching has been considered an "essential" feature for dependently typed languages since [Coq92] and is implemented in Agda and the user facing language of Coq.

Figure 8 shows the extensions to the surface language for data and pattern matching. The syntax of data constructors and data type constructors is standard. Our case eliminators match a tuple of expressions, allowing us to be very precise about the typing of branches.

While pattern matching is an extremely practical feature, theoretical accounts tend to be messy. To implement standard pattern matching, a unification procedure is needed to resolve the equational constraints that arise. There are many different strategies to handle these equational constraints. Several options are explored in [CD18]. Worse, the constraints are undecidable in general, since arbitrary computation can be embedded in the type of a constructor.

there is a lot of jenkyness about unification in general, but I think the additional points lose focus?

Parentheses are used to distinguish between matching a single variable and a constructor that takes no arguments

Part IV

Cast Language Data

Surprisingly, the cast language can be extended with a pattern matching construct without unification. This becomes clear when considering the normal forms of data terms. While in standard dependent type theory a normal form of data must have the data constructor in head position (justifying the pattern syntax), in the cast language the normal form of data will have a stack of casts applied to it. Casts will be wrapped around constructors during the elaboration procedure, and will accumulate during evaluation. If the cast language is extended with a path variable that can represent the stack of equalities then that stack can be matched and used in the body of the pattern. Since the type constructor is known, it is possible to check the coverage of the patterns. If every constructor is accounted for, only blameable data remains. Quantifying over casts allows blame to be redirected, so if the program gets stuck in a pattern branch it can blame the malformed input. We conjecture that this extension preserves cast soundness.

It makes sense to further extend the cast syntax so that blame can be invoked directly by a path. For example, in **??**. Branches set up in this way prove their "unreachability" by using their input to generate blame. Once direct blame is added it makes sense to allow operations on paths, such as concatenation, and reverse so that more proofs of inequality are possible. Proofs of inequality can be further helped by inspecting the arguments of type constructors and data constructors. See **??** for the extended path syntax.

Extending the syntax in this way complicates the type system and the reduction rules. We include a sample of our reduction rules in **??**. We conjecture that these extensions support all of our expected properties.

s are al-

Part V

Elaborating Eliminations

To make the overall system behave as expected we do not want to expose users to equality patterns, or force them to manually do the blame bookkeeping. To work around this we extend a standard unification algorithm to cast patterns with instrumentation to remember paths that were required for the solution. Then if pattern matching is satisfiable, compile additional casts into the branch based on its assignments. Unlisted patterns can be checked to confirm they are unsatisfiable. If the pattern is unsatisfiable then elaboration can use the proof of unsatisfiability to construct explicit blame. If an unlisted pattern cannot be proven "unreachable" then we could warn the user, and like most functional programming languages, blame the incomplete match if that pattern ever occurs.

$$\frac{\mathsf{data}\ D\ \Delta \in \Gamma}{\Gamma \vdash D\ :\ \Delta \to *}\ \ldots$$

$$\frac{d\ :\ \Theta \to D\overline{m} \in \Gamma}{\Gamma \vdash d\ :\ \Theta \to D\overline{m}}\ \ldots$$

define these $\in$

$$\frac{\begin{array}{c}\mathsf{data}\ D\ \Delta \in \Gamma \\ \Gamma \vdash \overline{N} : \Delta \quad \Gamma \vdash n : D\overline{N} \\ \Gamma, \overline{x} : \Delta, z : D\,\overline{x} \vdash M : \star \\ \forall\, d\ :\ \Theta \to D\overline{m} \in \Gamma.\quad \Gamma, \overline{x} : \Delta, \overline{y}_d : \Theta \vdash m_d : M\end{array}}{\begin{array}{c}\Gamma \vdash \mathsf{case}\ \overline{N}, n\ \left\{\overline{\mid \overline{x} \Rightarrow (d\,\overline{y}_d) \Rightarrow m_d}\right\} \\ : M\left[\overline{x} \coloneqq \overline{N}, z \coloneqq n\right]\end{array}}\ \ldots$$

don't need $\Gamma \vdash \overline{N} : \Delta$?

$$\frac{\begin{array}{c}\mathsf{data}\ D\ \Delta \in \Gamma \\ \Gamma \vdash \overline{N} : \Delta \quad \Gamma \vdash n : D\overline{N} \\ \Gamma, \overline{x} : \Delta, z : D\,\overline{x} \vdash M : \star \\ \forall\, d\ :\ \Theta \to D\overline{m} \in \Gamma.\quad \Gamma, \overline{x} : \Delta, \overline{y}_d : \Theta \vdash m_d : M\end{array}}{\begin{array}{c}\Gamma \vdash \mathsf{case}\ \overline{N}, , n\ \langle \overline{x} \Rightarrow z : D\,\overline{x} \Rightarrow M\rangle\ \left\{\overline{\mid \overline{x} \Rightarrow (d\,\overline{y}_d) \Rightarrow m_d}\right\} \\ : M\left[\overline{x} \coloneqq \overline{N}, z \coloneqq n\right]\end{array}}\ \ldots$$

may not need scrut wf check? oh but what about the empty types!

$$\frac{\Gamma \vdash \Delta}{\Gamma \vdash \mathsf{data}\ D\ \Delta}\ \ldots$$

$$\frac{\Gamma \vdash \mathsf{data}\ D\ \Delta \quad \forall d.\Gamma, \mathsf{data}\ D\ \Delta \vdash \Theta_d \quad \forall d.\ \Gamma, \mathsf{data}\ D\ \Delta, \Theta_d \vdash \overline{m}_d : \Delta}{\Gamma \vdash \mathsf{data}\ D\ :\ \Delta\left\{\overline{\mid d\ :\ \Theta_d \to D\overline{m}_d}\right\}}\ \ldots$$

to ensure regularity

$$\frac{\Gamma \vdash \mathsf{data}\ D\ \Delta}{\Gamma, \mathsf{data}\ D\ \Delta \vdash}\ \ldots$$

$$\frac{\Gamma \vdash \mathsf{data}\ D\ :\ \Delta\left\{\overline{\mid d\ :\ \Theta \to D\overline{m}}\right\}}{\Gamma, \mathsf{data}\ D\ :\ \Delta\left\{\overline{\mid d\ :\ \Theta \to D\overline{m}}\right\} \vdash}\ \ldots$$

Figure 4: Surface Language Type Assignment System + Minimal Data

$$\frac{\begin{array}{c}\overline{N} \Rrightarrow \overline{N'} \quad \overline{m} \Rrightarrow \overline{m'} \\ \forall \overline{x} \Rrightarrow (d\,\overline{y}_d) \Rightarrow m_d \in \left\{\overline{\mid \Rrightarrow \overline{x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\}.\, m_d \Rrightarrow m'_d \\ m_d \Rrightarrow m'_d \end{array}}{\mathsf{case}\,\overline{N}, d\overline{m}\ \langle ... \rangle \left\{\overline{\mid \Rrightarrow \overline{x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\} \Rrightarrow \mathsf{case}\,\overline{N'}, d\overline{m'} \left\{\overline{\mid \Rrightarrow \overline{x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m'_{d'}}\right\}} \Rrightarrow\text{-}\mathsf{case}\text{<>-red}$$

it's actually kind of fine discriminating between non converting motives?

$$\frac{\begin{array}{c}\overline{N} \Rrightarrow \overline{N'} \quad \overline{m} \Rrightarrow \overline{m'} \\ \exists \overline{x} \Rrightarrow (d\,\overline{y}_d) \Rightarrow m_d \in \left\{\overline{\mid \overline{x} \Rrightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\} \\ m_d \Rrightarrow m'_d \end{array}}{\mathsf{case}\,\overline{N}, d\overline{m}\ \left\{\overline{\mid \overline{x} \Rrightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\} \Rrightarrow m'_d\left[\overline{x} \coloneqq \overline{N'}, \overline{y}_d \coloneqq \overline{m'}\right]} \Rrightarrow\text{-}\mathsf{case}\text{-red}$$

structural reductions

$$\frac{\begin{array}{c}\overline{N} \Rrightarrow \overline{N'} \quad m \Rrightarrow m' \\ M \Rrightarrow M' \\ \forall d.\, \Rrightarrow \overline{x} \Rightarrow (d\,\overline{y}_d) \Rightarrow m_d \in \left\{\overline{\mid \Rrightarrow \overline{x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\} \\ m_d \Rrightarrow m'_d \end{array}}{\begin{array}{c}\mathsf{case}\,\overline{N}, m\ \langle \overline{x} \Rrightarrow z : D\,\overline{x} \Rightarrow M \rangle \left\{\overline{\mid \Rrightarrow \overline{x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\} \Rrightarrow \\ \mathsf{case}\,\overline{N}, m'\ \langle \overline{x} \Rrightarrow z : D\,\overline{x} \Rightarrow M' \rangle \left\{\overline{\mid \Rrightarrow \overline{x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\}\end{array}} \Rrightarrow\text{-}\mathsf{case}\text{<>}$$

$$\frac{\begin{array}{c}\overline{N} \Rrightarrow \overline{N'} \quad m \Rrightarrow m' \\ M \Rrightarrow M' \\ \forall d.\, \Rrightarrow \overline{x} \Rightarrow (d\,\overline{y}_d) \Rightarrow m_d \in \left\{\overline{\mid \Rrightarrow \overline{x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\} \\ m_d \Rrightarrow m'_d \end{array}}{\begin{array}{c}\mathsf{case}\,\overline{N}, m\ \left\{\overline{\mid \Rrightarrow \overline{x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\} \Rrightarrow \\ \mathsf{case}\,\overline{N}, m'\ \left\{\overline{\mid \Rrightarrow \overline{x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\}\end{array}} \Rrightarrow\text{-}\mathsf{case}\text{<>}$$

$$\frac{\overline{m} \Rrightarrow \overline{m'}}{D\overline{m} \Rrightarrow D\overline{m'}} \cdots$$

$$\frac{\overline{m} \Rrightarrow \overline{m'}}{d\overline{m} \Rrightarrow d\overline{m'}} \cdots$$

extend reductions over lists

Figure 5: Surface Language Parallel Reductions

$$\frac{}{\mathsf{case}\,\overline{N}, n\ \langle ...\rangle \left\{\overline{|\Rightarrow x \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\} \rightsquigarrow \mathsf{case}\,\overline{N}, n \left\{\overline{|\Rightarrow x \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\}} ...$$

$$\frac{\exists \overline{x} \Rightarrow (d\,\overline{y}_d) \Rightarrow m_d \in \left\{\overline{|\,\overline{x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\}}{\mathsf{case}\,\overline{V}, d\overline{v} \left\{\overline{|\,\overline{x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\} \rightsquigarrow m_d\,[\overline{x} := \overline{V}, \overline{y}_d := \overline{v}]} \Rightarrow\text{-case-red}$$

$$\frac{\overline{n} \rightsquigarrow \overline{n'}}{\mathsf{case}\,\overline{V}, d\overline{n} \left\{\overline{|\,\overline{x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\} \rightsquigarrow \mathsf{case}\,\overline{V}, d\overline{n'} \left\{\overline{|\,\overline{x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\}}$$

$$\frac{\overline{N} \rightsquigarrow \overline{N'}}{\mathsf{case}\,\overline{N}, d\overline{n} \left\{\overline{|\,\overline{x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\} \rightsquigarrow \mathsf{case}\,\overline{N'}, d\overline{n} \left\{\overline{|\,\overline{x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\}}$$

structural reductions

$$\frac{\begin{array}{c}\overline{N} \Rrightarrow \overline{N'} \quad m \Rrightarrow m' \\ M \Rrightarrow M' \\ \forall d.\, \overline{\Rightarrow x} \Rightarrow (d\,\overline{y}_d) \Rightarrow m_d \in \left\{\overline{|\,\overline{\Rightarrow x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\} \\ m_d \Rrightarrow m'_d \end{array}}{\begin{array}{c}\mathsf{case}\,\overline{N}, m\ \langle \overline{x} \Rightarrow z : D\,\overline{x} \Rightarrow M\rangle \left\{\overline{|\,\overline{\Rightarrow x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\} \Rrightarrow \\ \mathsf{case}\,\overline{N}, m'\ \langle \overline{x} \Rightarrow z : D\,\overline{x} \Rightarrow M'\rangle \left\{\overline{|\,\overline{\Rightarrow x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\}\end{array}} \Rrightarrow\text{-case<>}$$

$$\frac{\begin{array}{c}\overline{N} \Rrightarrow \overline{N'} \quad m \Rrightarrow m' \\ M \Rrightarrow M' \\ \forall d.\, \overline{\Rightarrow x} \Rightarrow (d\,\overline{y}_d) \Rightarrow m_d \in \left\{\overline{|\,\overline{\Rightarrow x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\} \\ m_d \Rrightarrow m'_d \end{array}}{\begin{array}{c}\mathsf{case}\,\overline{N}, m \left\{\overline{|\,\overline{\Rightarrow x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\} \Rrightarrow \\ \mathsf{case}\,\overline{N}, m' \left\{\overline{|\,\overline{\Rightarrow x} \Rightarrow (d'\,\overline{y}_{d'}) \Rightarrow m_{d'}}\right\}\end{array}} \Rrightarrow\text{-case<>}$$

$$\frac{\overline{m} \rightsquigarrow \overline{m'}}{D\overline{m} \rightsquigarrow D\overline{m'}} ...$$

what about D? how much of a value should it be?

$$\frac{\overline{m} \rightsquigarrow \overline{m'}}{D\overline{m} \rightsquigarrow D\overline{m'}} ...$$

extend step over lists

Figure 6: Surface Language cbv

```
-- eliminator style
head' : (A : *) -> (n : Nat) ->
  Vec A (S n) ->
  A ;
head' A n v =
  case A, (S n), v <
    A' => n' => _ : Vec A' n' =>
      case n' < _ => *> {
        | (Z  ) => Unit
        | (S _) => A'
      }
  >{
  | _ => (Z)   => (Nil _        ) => tt
  | _ => (S _) => (Cons _ a _ _) => a
  } ;

 -- pattern match style
head : (A : *) -> (n : Nat) ->
  Vec A (S n) ->
  A ;
head A n v =
  case v < _ => A > {
  | (Cons _ a _ _) => a
  } ;
```

clean when I get motive inference working

syntax highlighting would be bomb

Figure 7: Eliminators vs. Pattern Matching

"can", "could", weasel word until implemented

add rules of unification? the rules as implemented are standard, needing extended normalization is weird

We found that our normalization procedure (Figure **??**) unexpectedly blocked unification. This is because the normalization is conservative about throwing away casts that could later lead to errors. For instance, with normalization, unification will get stuck on terms like $x ::_{\mathbb{N}} \mathbb{N} = 5$. Thus we employ a more optimistic normalization procedure that will collapse casts that are equivalent Thus $x ::_{\mathbb{N}} \mathbb{N} \overset{\cdot\cdot}{\rightsquigarrow} x$ allows us to solve the unification problem and assign $x := 5$. This approach has worked well so far.

In order to make the additional casts generated from unification we further extend the cast construct so it supports congruence. For instance, if x has type $Vec\, y\, z$ and the unification procedure established $y := Bool$ and $z := 5$ then we should cast $x :: Vec\, Bool\, 5$. Unlike in Section 3 this requires that we use evidence to inject a cast within a type. This can be done inherently by adding an "untyped" component to the cast annotation. The syntax that achieves this can be seen in **??**.

Part VI
Related work

5 Systems with Data

Minimal data with Sigma and Unit

ML W types

UTT[Luo90, Luo94]

I am unaware of any clear, complete account of CIC in English. A bidirectional account of CIC is given in [LB21], though it uses a different style of biderectionality then discussed here to maintain compatibility with the

$$
\begin{array}{lcll}
m... & ::= & ... & \\
 & | & \mathsf{case}\,\overline{n},\ \left\{\overline{|\,\overline{pat} \Rightarrow m}\right\} & \text{data elim. without motive} \\
 & | & \mathsf{case}\,\overline{n},\ \langle \overline{x} \Rightarrow M\rangle\ \left\{\overline{|\,\overline{pat} \Rightarrow m}\right\} & \text{data elim. with motive}
\end{array}
$$

(minimal) patterns,

$$
\begin{array}{lcll}
pat & ::= & x & \text{match a variable} \\
 & | & (d\,\overline{pat}) & \text{match a constructor}
\end{array}
$$

Figure 8: Surface Language Data

existing Galina grammar.

6 Pattern matching

Early work by Coq92 [Coq92]
with a lot of follow up from McBride [MM04]
reiterated in [Nor07]

A tutorial implementation of dynamic pattern unification Adam Gundry and Conor McBride (2012) http://adam.gundry.co.uk/pub/pattern-unify/ (this links give you the choice to read a more detailed chapter of Adam Gundry's thesis instead)

with substantial follow up in [CD18]

https://research.chalmers.se/en/publication/519011 ?

https://sozeau.gitlabpages.inria.fr/www/research/publications/Equations:_A_Dependent_Pattern-Matching_Compiler.pdf ?

http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.99.1405&rep=rep1&type=pdf ?

talk about normalization

References

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greyed out test

TODO (inline) test

Part VII
TODO

- review chapter 9 of file:///Users/stephaniesavir/Downloads/Computation-and-Reasoning.dependently