Dissertation Prospectus

A Full-Spectrum Dependently typed language for testing with dynamic equality

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Abstract

Dependent type systems offer a powerful tool to eliminate bugs from programs. Interest in dependent types is often driven by the inherent usability of such systems: Dependent types systems can re-use that methodology and syntax that functional programmers are familiar with for formal proofs. This insight has lead to several Full-Spectrum languages that try and present programmers with a consistent and unrestricted view of proofs and programs. However these languages still have substantial usability issues: missing features like general recursion, confusingly conservative equality, an inability to prototype, and no straight forward way to test specifications that have not yet been proven.

I attempt to solve these problems by building a new language that contains standard functional programming features such as general recursion, with a gradualized equality, runtime proof search and a testing system.

Contents

1 Introduction

The promise of dependent types in a practical programming language has been the goal of research projects for decades. There have been many formalization and prototypes that make different compromises in the design space. One popular direction to explore is "Full-spectrum" dependently types languages, these languages tend to have a minimalist approach: computation can appear anywhere in a term or type. Such a design purposely exposes the Curry-Howard correspondence, as opposed to trying to hide it as a technical foundation: a proof has the exact same syntax and behavior as a program. This direct approach tries to make clear to the programmer the subtleties of the proof system that are often obscured by other formal method systems. Even though this style makes writing efficient programs hard, and drastically complicates the ability to encode effects, it can be seen in some of the most popular dependently typed languages (notably Agda and Idris).

However there are several inconveniences with languages in this style:

1. Restrictions on standard programming features, such as general recursion
2. A subtle and weak notion of type equality
3. Difficulties in prototyping proofs and programs

4. Difficulties in testing programs that make use of dependent types

While each problem will be treated as separately as possible, the nature of dependent types requires that equality is modified before testing and prototyping can be handled. The notion of equality itself is also very sensitive to which programmatic features are included. My thesis will solve these problems by

- Defining a full-spectrum dependently typed base language, with a few of the most essential programming features like general recursion and user defined data types
- A generalization of that base language that supports dynamic equality checking with blame tracking
- Syntax that supports runtime proof search
- A symbolic testing system that will exercise terms with uncertain equalities and runtime proof search

2 A Dependently Typed Base Language

The base language contains the features:

- Full-Spectrum Dependent types
- Unrestricted user defined dependent data types (no requirement of strict positivity)
- Unrestricted recursion (no required termination checking)
- Type-in-type (no predictive hierarchy of universes)

Any one of these features can result in logical unsoundness[1], but they are widely used in mainstream functional programming. In spite of the logical unsoundness, the resulting language is still has type soundness[2]. This seems ideal for a programming language since logically sound proofs can still be defined and logical unsoundness can be discovered through traditional testing. Importantly no desirable computation is prevented.

Though this language is not logically sound it supports a partial correctness property for first order data types when run with CBV, for instance:

$$\vdash M : \sum x : \mathbb{N}.\mathtt{IsEven}\, x$$

[1]Every type is inhabited by an infinite loop.

[2]No term with a reduct that applies an argument to a non-function in the empty context will type.

$\mathtt{fst}\,M$ may not terminate, but if it does, $\mathtt{fst}\,M$ will be an even $\mathbb{N}$. However, this property does not extend to functions

$$\vdash M : \sum x : \mathbb{N}.\, (y : \mathbb{N}) \rightarrow x \leq y$$

it is possible that $\mathtt{fst}\,M \equiv 7$ if

$$M \equiv \langle 7, \lambda y.\mathtt{loopForever} \rangle$$

The hope would be that the type is sufficient to communicate intent, in the same way unproductive non-termination is typeable in mainstream programming languages but still considered a bug.

Since arbitrary computation can appear in types, the type systems need to characterize what computations are equivalent. The base language associates all terms that are $\alpha\beta$ equivalent, a conventional choice for intensional type theories. $\alpha\beta$ equivalence is undecidable for general recursion so type checking for this language is undecidable, however this has not been a problem in practice[3].

The implementation additionally supports top level functions, unrestricted mutual recursion for functions and data and is written in a bidirectional style allowing some annotations to be inferred.

2.1 Prior work for the Base Language

While many of these features have been explored in theory and implemented in practice, I am unaware of any development with exactly this formulation.

Unsound logical systems go back to at least to Church's lambda calculus which was originally intended to be a logical foundation. Martin Lof proposed a system with Type-in-type that was shown logically unsound by Girard. The first proof of type soundness for general recursive functions that I am aware of came form the Trellys Project [27], it contains many similar features, but base language uses a simpler notion of equality and dependent data resulting in an arguably simpler proof of type soundness. Further work in the Trellys Project[6, 5] used modalities

[3]While languages like Coq and Agda claim decidable typechecking, it is easy to construct terms who's type verification would exceed the computational resources of the universe.

to separate the terminating and non terminating fragments of the language, thought the annotations burden seemed too high in practice.

Many implementations support this combination of features without proofs of type soundness. Cayenne [4] was an early Haskell like language combined dependent types with and non-termination. Agda supports general recursion and Type-in-type with compiler flags, and can simulate some non-positive data types using coinduction. Idris supports similar "unsafe" features.

The base language has been deeply informed by the Trellys Project[15][27][6, 5] [28] [26] and the Zombie Language[4] it produced.

[14] claims a similar "partial correctness" criterion.

3 A Language with Dynamic Type Equality

A key issue with full-spectrum dependent type theories is the characterization of definitional equality. Since computation can appear at the type level, and types must be checked for equality, traditional dependent type theories pick a subset of equivalences to support. For instance, the base language follows the common choice of $\alpha\beta$ equivalence of terms. However this causes many obvious programs to not type-check:

$$\begin{aligned} \texttt{Vec} &: \mathbb{N} \to * \to * \\ \texttt{rep} &: (x : \mathbb{N}) \to \texttt{Vec}\, x\, \mathbb{B} \\ \texttt{head} &: (x : \mathbb{N}) \to \texttt{Vec}\ (1 + x)\ \mathbb{B} \to \mathbb{B} \\ &\nvdash \lambda x.\texttt{head}\, x\ (\texttt{rep}\ (x+1))\ :\ \mathbb{N} \to \mathbb{B} \end{aligned}$$

Since $1 + x$ does not have the same definition as $x + 1$.

Overly fine definitional equalities directly results in the poor error messages that are common for dependently typed languages [10]. For instance, the above will give the error message "x + 1 != suc x of type $\mathbb{N}$ when checking that the expression rep (x + 1) has type Vec Bool (1 + x)" in Agda. The error is confusing since it objects to an intended property of addition, and if addition were buggy no hints would be given to fix the problem. Ideally the

[4] https://github.com/sweirich/trellys

error messages would give a specific instance of x where $x+1 \neq 1+x$ or remain silent. There is some evidence that specific examples can help clarify the error messages in OCaml[24] and there has been an effort to make refinement type error messages more concrete in Liquid Haskell[13].

Strengthening the equality relation in dependently typed languages is used to motivate many research projects (to name a few [7, 28]). It is unlikely those impressive efforts are suitable for non exerts, since programmers expect the data types and functions they define to have the properties the expect to have. Asking a programmer to build a custom confluent rewrite system on top of their functions is unrealistic[7]. Asking programmers to translate their datatype into SAT input, likewise requires too much perquisite knowledge. Even asking programmers to prove the equational properties that can then be used automatically[28] is off putting. Needing to know the details of definitional equality makes choices that would be irreverent in Haskell, subtle and complicated.

Additionally , every attempt to extend the definitional equalities of dependent type theory I am aware of intends to preserve decidable type checking and/or logical soundness, so equality will never be complete[5]. Since dependently typed languages with the practical features outlined in base language are already incompatible with logical soundness and decidable type checking, perhaps equality can also be made more convenient.

Building off the base language I purpose a dynamic cast language, and a cast type system. Many programs that do not type in the base language can be elaborated into the cast language. The cast language has a weaker notion of type soundness such that

1. $\vdash_c e' : M'$ then
   (a) $e' \Downarrow v'$ and $\vdash_c v' : M'$
   (b) or $e' \Uparrow$
   (c) or $e' \Downarrow blame$

Type soundness is preserved, or inequality can be proven at a specific source location. In the example above $\lambda x.\mathtt{head}\, x\ (\mathtt{rep}\ (x+1))\ :\ \mathbb{N} \to \mathbb{B}$ will not emit any errors at compile time or runtime (though a static warning may be given).

If the example is changed to

[5] I am also unaware of any suitable notion of complete extensional equality for dependent type theory though it is considered in [26] .

$$\lambda x.\mathtt{head}\,x\ (\mathtt{rep}\,x)\ :\ \mathbb{N} \to \mathbb{B}$$

at runtime the blame tracking system will blame the exact static location that uses unequal types with a direct proof of inequality, allowing an error like "failed at application $\left(\mathtt{head}\,x : \mathtt{Vec}\,\underline{(1+x)}\,\mathbb{B} \to ...\right)(rep\,x : \mathtt{Vec}\,\underline{x}\,\mathbb{B})$ since when $x = 3$, $1 + x = 4 \neq 3 = x$", regardless of where in the program the discrepancy was discovered.

Just as standard type theories allow many possible characterizations of equality that support logical soundness, there are many choices of runtime checking. The minimal choice that supports type soundness is likely too permissive in practice. Alternatively, I conjecture that checking that matches the partial correctness criteria above would be reasonably intuitive[6].

Taking inspiration from the "gradual guarantee" of gradual typing, there are several basic properties in addition to type soundness that this cast language hopes to fulfill:

1. $\vdash e : M$, $elab\,(M, *) = M'$, and $elab\,(e, M') = e'$ then $\vdash_c e' : M'$.
2. $\vdash_c e' : M'$ and $e' \downarrow blame$ then there is no $\vdash e : M$ such that $elab\,(M, *) = M'$, $elab\,(e, M') = e'$
3. $\vdash_c e' : *$ and $elab\,(e, *) = e'$ then
   (a) if $e' \downarrow *$ then $e \downarrow *$
   (b) if $e' \downarrow (x : M') \to N'$ then $e \downarrow (x : M) \to N$
   (c) if $e' \downarrow TCon\triangle'$ then $e \downarrow TCon\triangle$

The first condition states that every typed term in the base language can be embedded in the cast language. The second condition shows that errors are not spurious. The third condition shows that except for error, observations are consistent (with large eliminations, term constructors can also be observed).

3.1 Prior work

It is unsurprising that dynamic quality is shares many of the same concerns as the large amount of work for contracts, hybrid types, gradual types, and blame. In fact, this work could be seen as gradualizing the Reflection Rule in Extensional Type Theory.

[6] Extending checks into non dependent function types also seems reasonable, and would allow simple static type checking

Blame has been strongly advocated for in [30, 29]. Blame tracking can establish the reasonableness of monitoring systems systems by linking a dynamic failure directly to the broken static invariant. As many authors have noticed, proving blame correctness is tedious and error prone, it is often only conjectured.

The basic correctness conditions are inspired by the Gradual Guarantee [25]. The implementation also takes inspiration from "Abstracting gradual typing"[12], where static evidence annotations become runtime checks. Unlike some impressive attempts to gradualize the polymorphic lambda calculus [3], dynamic equality does not attempt to preserve any parametric properties of the base language. It is unclear how useful such a restriction would be in practice.

A direct attempt has been made to gradualize a full spectrum dependently typed language to an untyped lambda calculus using the AGT philosophy in [11]. However that system retains the definitional style of equality and user defined data types are not supported. The paper is largely concerned with establishing decidable type checking via an approximate term normalization.

A refinement type system with higher order features is gradualized in [32] though it does not appear powerful enough to be characterized a a full-spectrum dependent type theory. [32] builds on earlier refinement type system work, which described itself as "dynamic". A notable example is [22] which describes a refinement system that limit's predicates to base types.

4 Prototyping proofs and programs

Just as "obvious" equalities are missing from the definitional relation, "obvious" proofs and programs are not always conveniently available to the programmer. For instance, in Agda it is possible to write a sorting sorting function quickly using simple types. With effort is it possible to prove that sorting procedure correct by rewriting it with the necessarily invariants. However very little is offered in between. The problem is magnified if module boundaries hide the implementation details of a function, since the details are exactly what is needed to make a proof! This is especially important for larger scale software where a library may require proof terms that while "correct" are not constructable from the exports of other libraries.

The solution proposed here is some additional syntax that will search for a term of the type when resolved at runtime. Given the sorting function

$$\texttt{sort} : \texttt{List}\,\mathbb{N} \to \texttt{List}\,\mathbb{N}$$

and given the first order predicate that

$$\texttt{IsSorted} : \texttt{List}\,\mathbb{N} \to *$$

then it is possible to assert that `sort` behaves as expected with

$$\lambda x.? : (x : \texttt{List}\,\mathbb{N}) \to \texttt{IsSorted}\,(\texttt{sort} x)$$

this term will act like any other term at runtime, given a list input it will verify that the `sort` function correctly handles that input, give an error, or non-terminate.

Additionally this would allow simple prototyping form first order specification. For instance,

$$\begin{aligned}
&data\,\texttt{Mult} : \mathbb{N} \to \mathbb{N} \to \mathbb{N} \to *\,where \\
&\quad \texttt{base} : (x : \mathbb{N}) \to \texttt{Mult} 0\,x\,0 \\
&\quad\; \texttt{suc} : (x\,y\,z : \mathbb{N}) \to \texttt{Mult}\,x\,y\,z \to \texttt{Mult}\ (1+x)\ y\,(y+z)
\end{aligned}$$

can be used to prototype

$$\texttt{div} = \lambda x.\lambda y.\texttt{fst}\left(? : \sum z : \mathbb{N}.\texttt{Mult} x\,y\,z\right)$$

The term search can be surprisingly subtle, for instance

$$? : \sum f : \mathbb{N} \to \mathbb{N}.\texttt{Id}\,(f, \lambda x.x+1)\,\&\texttt{Id}\,(f, \lambda x.1+x)$$

depends on the definitional properties of functions. To avoid this subtly I plan to only support term search over first order data.

Though the proof search is currently primitive, better search methods could be incorporated in future work.

4.1 Prior work

Proof search is often used for static term generation in dependently typed languages (for instance Coq tactics). A first order theorem prover is attached to Agda in [21].

Twelf made use of runtime proof search but the underling theory cannot be considered full spectrum.

5 Testing dependent programs

Both dynamic equalities and dynamic proof search vastly weaken the guarantees of normal dependent type systems. Programmers still would like a evidence of correctness, even while they intend to provide full proofs of properties in the future. However, there are few options available in full spectrum dependently typed languages aside from costly and sometimes unconstructable proofs.

The mainstream software industry has similar needs for evidence of correctness, and has made use of testing done in a separate execution phase. Given the rich specifications that dependent types provide it is possible to improve on the hand crafted tests used by most of the industry. Instead we can use a type directed symbolic execution, to run questionable equalities over concrete values and engage and precompute the searched proof terms. Precomputed proof terms can be cached, so that exploration is not too inefficient in the common case of repeating tests at regular intervals of code that is mostly the same. Precomputed terms can be made available at runtime, covering for the inefficient search procedure.

Interestingly dynamic equality is necessary for testing like this, since otherwise, definitional properties of functions would need to be accounted for. Using dynamic equality it is possible only consider the extensional behavior of functions.

Finally future work can add more advanced methods of testing and proof generation. This architecture should make it easier to add more advanced exploration and search without changing the underlining definitional behavior.

5.1 Prior work

5.1.1 Symbolic Execution

Most research for Symbolic Execution targets popular languages (like C) and uses SMT solvers to efficiently explore conditional branches that depend on base types. Most work does not support higher order functions or makes simplifying assumptions about the type system. There are however some relevant papers:

- [13] presents a symbolic execution engine supporting Haskell's lazy execution and type system. Higher order functions are not handled
- The draft work[31], handles higher order functions as and inputs provides a proof of completeness
- Symbolic execution for higher order functions for a limited untyped variant of PCF is described in [20]

5.1.2 Testing dependent types

There has been a long recognized need for testing in addition to proving in dependent type systems

- In [9] a QuickCheck style framework was added to an earlier version of Agda
- QuickChick[7] [8][18, 17, 16] is a research project to add testing to Coq. However testing requires building types classes that establish the properties needed by the testing framework such as decidable equality. This is presumably out of reach of novice Coq users.

6 Reservations

6.1 Purity

The base language is pure in the Haskell sense[8], an thus has many of the usability issues of languages in that style. Handling effects in a principled convenient way in mainstream functional programming languages is an ongoing research effort. Notable areas of research include Algebraic Effects and Handlers, and Linear Type systems.

Effects have been combined with dependent types in a limited fashion with Hoare Type Theory (HTT)[19] and ATS.

[7] https://github.com/QuickChick/QuickChick

[8] the only effects are non termination and unhandled exceptions.

Combing effects with Full-Spectrum dependent types is substantially more difficult because effectful equality is hard to characterize for individual effects or effects in combination. Several attempts have been made, [23] [2, 1][23] but there is still a lot of work to be done.

6.2 Implicit arguments

The base language currently has no mechanism for marking arguments implicit. Implicit arguments drastically improve the usability of dependent type systems, often taking the place of type inference in ML style languages. They have been left out of the base language so that alternatives can be explored.

6.3 Dependent data

The current presentation of data in the base language is powerful but there are inconveniences:

- The base language is based on eliminators with an explicit motive. Nested pattern matching is not supported which is surprisingly subtle [21] with dependent types.
- Data types in the base language support dependent indices, but not "Parameters".

There is no reduction in expressiveness, but these features should be added to make the language more convenient.

7 Status

- I have a prototype of the cast language supporting all features. However the casting of data types will need to be reworked. I believe I can prove all the properties on the sublanguage without data. I expect it is possible to prove the properties on the language with data as well, though I need to work through those details.
- Substantial work was completed on the proof-search for the extended abstract, though it will need to be revisited.

References

[1] Danel Ahman. Fibred computational effects. *arXiv preprint arXiv:1710.02594*, 2017.

[2] Danel Ahman. Handling fibred algebraic effects. *Proceedings of the ACM on Programming Languages*, 2(POPL):1–29, 2017.

[3] Amal Ahmed, Dustin Jamner, Jeremy G. Siek, and Philip Wadler. Theorems for free for free: Parametricity, with and without types. *Proc. ACM Program. Lang.*, 1(ICFP), August 2017.

[4] Lennart Augustsson. Cayenne a language with dependent types. In *Proceedings of the Third ACM SIGPLAN International Conference on Functional Programming*, ICFP '98, pages 239–250, New York, NY, USA, 1998. Association for Computing Machinery.

[5] Chris Casinghino. Combining proofs and programs. 2014.

[6] Chris Casinghino, Vilhelm Sjöberg, and Stephanie Weirich. Combining proofs and programs in a dependently typed language. *ACM SIGPLAN Notices*, 49(1):33–45, 2014.

[7] Jesper Cockx, Nicolas Tabareau, and Théo Winterhalter. The taming of the rew: A type theory with computational assumptions. *Proceedings of the ACM on Programming Languages*, 2021.

[8] Maxime Dénès, Catalin Hritcu, Leonidas Lampropoulos, Zoe Paraskevopoulou, and Benjamin C Pierce. Quickchick: Property-based testing for coq. In *The Coq Workshop*, 2014.

[9] Peter Dybjer, Qiao Haiyan, and Makoto Takeyama. Combining testing and proving in dependent type theory. In *International Conference on Theorem Proving in Higher Order Logics*, pages 188–203. Springer, 2003.

[10] Joseph Eremondi, Wouter Swierstra, and Jurriaan Hage. A framework for improving error messages in dependently-typed languages. *Open Computer Science*, 9(1):1–32, 2019.

[11] Joseph Eremondi, Éric Tanter, and Ronald Garcia. Approximate normalization for gradual dependent types. *Proc. ACM Program. Lang.*, 3(ICFP), July 2019.

[12] Ronald Garcia, Alison M. Clark, and Éric Tanter. Abstracting gradual typing. In *Proceedings of the 43rd Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages*, POPL '16, pages 429–442, New York, NY, USA, 2016. Association for Computing Machinery.

[13] William T. Hallahan, Anton Xue, Maxwell Troy Bland, Ranjit Jhala, and Ruzica Piskac. Lazy counterfactual symbolic execution. In *Proceedings of the 40th ACM SIGPLAN Conference on Programming Language Design and Implementation*, PLDI 2019, pages 411–424, New York, NY, USA, 2019. Association for Computing Machinery.

[14] Limin Jia, Jianzhou Zhao, Vilhelm Sjöberg, and Stephanie Weirich. Dependent types and program equivalence. In *Proceedings of the 37th annual ACM SIGPLAN-SIGACT symposium on Principles of programming languages*, pages 275–286, 2010.

[15] Garrin Kimmell, Aaron Stump, Harley D Eades III, Peng Fu, Tim Sheard, Stephanie Weirich, Chris Casinghino, Vilhelm Sjöberg, Nathan Collins, and Ki Yung Ahn. Equational reasoning about programs with general recursion and call-by-value semantics. In *Proceedings of the sixth workshop on Programming languages meets program verification*, pages 15–26, 2012.

[16] Leonidas Lampropoulos. Random testing for language design. 2018.

[17] Leonidas Lampropoulos, Diane Gallois-Wong, Cătălin Hriţcu, John Hughes, Benjamin C Pierce, and Li-yao Xia. Beginner's luck: a language for property-based generators. In *Proceedings of the 44th ACM SIGPLAN Symposium on Principles of Programming Languages*, pages 114–129, 2017.

[18] Leonidas Lampropoulos, Zoe Paraskevopoulou, and Benjamin C Pierce. Generating good generators for inductive relations. *Proceedings of the ACM on Programming Languages*, 2(POPL):1–30, 2017.

[19] Aleksandar Nanevski and Greg Gregory Morrisett. Dependent type theory of stateful higher-order functions. 2005.

[20] Phuc C Nguyen, Sam Tobin-Hochstadt, and David Van Horn. Higher order symbolic execution for contract verification and refutation. *Journal of Functional Programming*, 27, 2017.

[21] Ulf Norell. *Towards a practical programming language based on dependent type theory*, volume 32. Citeseer, 2007.

[22] Xinming Ou, Gang Tan, Yitzhak Mandelbaum, and David Walker. Dynamic typing with dependent types. In Jean-Jacques Levy, Ernst W. Mayr, and John C. Mitchell, editors, *Exploring New Frontiers of Theoretical Informatics*, pages 437–450, Boston, MA, 2004. Springer US.

[23] Pierre-Marie Pédrot and Nicolas Tabareau. The fire triangle how to mix substitution, dependent elimination, and effects. 2020.

[24] Eric L. Seidel, Ranjit Jhala, and Westley Weimer. Dynamic witnesses for static type errors (or, ill-typed programs usually go wrong). In *Proceedings of the 21st ACM SIGPLAN International Conference on Func-*

tional Programming, ICFP 2016, pages 228–242, New York, NY, USA, 2016. Association for Computing Machinery.

[25] Jeremy G. Siek, Michael M. Vitousek, Matteo Cimini, and John Tang Boyland. Refined Criteria for Gradual Typing. In Thomas Ball, Rastislav Bodik, Shriram Krishnamurthi, Benjamin S. Lerner, and Greg Morrisett, editors, *1st Summit on Advances in Programming Languages (SNAPL 2015)*, volume 32 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 274–293, Dagstuhl, Germany, 2015. Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik.

[26] Vilhelm Sjöberg. A dependently typed language with nontermination. 2015.

[27] Vilhelm Sjöberg, Chris Casinghino, Ki Yung Ahn, Nathan Collins, Harley D Eades III, Peng Fu, Garrin Kimmell, Tim Sheard, Aaron Stump, and Stephanie Weirich. Irrelevance, heterogeneous equality, and call-by-value dependent type systems. *Mathematically Structured Functional Programming*, 76:112–162, 2012.

[28] Vilhelm Sjöberg and Stephanie Weirich. Programming up to congruence. In *Proceedings of the 42nd Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages*, pages 369–382, 2015.

[29] Philip Wadler. A Complement to Blame. In Thomas Ball, Rastislav Bodik, Shriram Krishnamurthi, Benjamin S. Lerner, and Greg Morrisett, editors, *1st Summit on Advances in Programming Languages (SNAPL 2015)*, volume 32 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 309–320, Dagstuhl, Germany, 2015. Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik.

[30] Philip Wadler and Robert Bruce Findler. Well-typed programs can't be blamed. In Giuseppe Castagna, editor, *Programming Languages and Systems*, pages 1–16, Berlin, Heidelberg, 2009. Springer Berlin Heidelberg.

[31] Shu-Hung You, Robert Bruce Findler, and Christos Dimoulas. Dynamic symbolic execution of higher-order functions, 2020.

[32] Jakub Zalewski, James McKinna, J. Garrett Morris, and Philip Wadler. Î»db: Blame tracking at higher fidelity. 2020. First ACM SIGPLAN Workshop on Gradual Typing 2020, WGT 2020 ; Conference date: 19-01-2020 Through 25-01-2020.