

## *Dissertation Prospectus*

# **A Full-Spectrum Dependently typed language for testing with dynamic equality**

Mark Lemay

Department of Computer Science, Boston University

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### **Abstract**

Dependent type systems offer a powerful tool to eliminate bugs from programs. Interest in dependent types is often driven by the inherent usability of such systems: Dependent types systems can re-use that methodology and syntax that functional programmers are familiar with for formal proofs. This insight has lead to several Full-Spectrum languages that try and present programmers with a consistent and unrestricted view of proofs and programs. However these languages still have substantial usability issues: missing features like general recursion, confusingly conservative equality, an inability to prototype, and no straight forward way to test specifications that have not yet been proven.

I attempt to solve these problems by building a new language that contains standard functional programming features such as general recursion, with a gradualized equality, runtime proof search and a testing system.

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## 1 Introduction

The promise of dependent types in a practical programming language has been the goal of research projects for decades. There have been many formalization and prototypes that make different compromises in the design space. One popular direction to explore is “Full-spectrum” dependently types languages, these languages tend to have a minimalist approach: computation can appear anywhere in a term or type. Such a design purposely exposes the Curry-Howard correspondence, as opposed to trying to hide it as a technical foundation: a proof has the exact same syntax and behavior as a program. This direct approach tries to make clear to the programmer the subtleties of the proof system that are often obscured by other formal method systems. Even though this style makes writing efficient programs hard, and drastically complicates the ability to encode effects, it can be seen in some of the most popular dependently typed languages (notably Agda and Idris).

However there are several inconveniences with languages in this style:

1. A restriction on standard programming features, such as general recursion
2. A subtle and weak notion of equality
3. difficulties in prototyping proofs and programs
4. Difficulties in testing programs that make use of dependent types

While each problem will be treated as separately as possible, the nature of dependent types requires that equality is modified before testing and prototyping can be handled. The notion of equality itself is also very sensitive to which programmatic features are included. My thesis will solve these problems by

- Defining a full-spectrum dependently typed base language, with a few of the most essential programming features like general recursion and user defined data types

- A cast language that supports dynamic equality checking
- Syntax that supports runtime proof search
- A symbolic testing system that will exercise terms with uncertain equalities and runtime proof search

## 2 A Dependently Typed Base Language

The base language contains the features:

- Unrestricted dependent data types (no requirement of strict positivity)
- Unrestricted recursion (no required termination checking)
- Type-in-type (no predictive hierarchy of universes)

Any one of these features can result in logical unsoundness<sup>1</sup>, but they are widely used in mainstream functional programming. In spite of the logical unsoundness, the resulting language is still has type soundness<sup>2</sup>. Type checking is undecidable for this language, however this has not been a problem in practice<sup>3</sup>.

The type theory is intuitionistic, definitional equality is the  $\alpha\beta$  equivalence of terms. The implementation is written in a bidirectional style allowing some annotations to be inferred.

Though this language is non-terminating it supports a partial correctness property for first order data types when run with CBV, for instance:

$$\vdash M : \sum x : \mathbb{N}. \text{IsEven } x$$

$\text{fst } M$  may not terminate, but if it does,  $\text{fst } M$  will be an even  $\mathbb{N}$ . However, this property does not extend to functions

$$\vdash M : \sum x : \mathbb{N}. (y : \mathbb{N}) \rightarrow x \leq y$$

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<sup>1</sup>Every type is inhabited by an infinite loop.

<sup>2</sup>No term with a reduct that applies an argument to a non-function in the empty context will type.

<sup>3</sup>While languages like Coq and Agda claim decidable typechecking, it is easy to construct terms whose type verification would exceed the computational resources of the universe.

it is possible that  $\text{fst } M \equiv 7$  if

$$\langle 7, \lambda y. \text{loopForever} \rangle$$

The hope would be that the type is sufficient to communicate intent, in the same way unproductive non-termination is trappable in the vast majority of typed languages but still considered a bug.

## 2.1 Prior work for the Base Language

While the base language is in line with prior research, I am unaware of any development with exactly these features. Agda supports general recursion and Type-in-type with compiler flags, and can simulate some non-positive data types using coinduction. Idris supports similar “unsafe” features. Meta-theoretically, this base language is similar to [23] though data and equality are formulated differently. The base language has been deeply informed by the Trellys Project[12][23][3, 2] [24] [22] and the Zombie Language<sup>4</sup> it produced.

[11] claims a similar “partial correctness” criterion.

## 3 A Language with Dynamic Dependent Equality

A key issue with full-spectrum dependent type theories is the characterization of definitional equality. Since computation can appear at the type level, and types must be checked for equality, traditional dependent type theories pick a subset of equivalences to support. For instance, the base language follows the common choice of  $\alpha\beta$  equivalence of terms. However this causes many obvious programs to not type-check:

$$\begin{aligned} \text{Vec} &: \mathbb{N} \rightarrow * \rightarrow * \\ \text{rep} &: (x : \mathbb{N}) \rightarrow \text{Vec } x \, \mathbb{B} \\ \text{head} &: (x : \mathbb{N}) \rightarrow \text{Vec } (1 + x) \, \mathbb{B} \rightarrow \mathbb{B} \\ &\not\vdash \lambda x. \text{head } x \, (\text{rep } (x + 1)) : \mathbb{N} \rightarrow \mathbb{B} \end{aligned}$$

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<sup>4</sup><https://github.com/sweirich/trellys>

Since  $1 + x$  does not have the same definition as  $x + 1$ .

Overly fine definitional equalities directly results in the poor error messages that are common for dependently typed languages [7]. For instance, the above will give the error message “ $x + 1 \neq \text{succ } x$  of type  $\mathbb{N}$  when checking that the expression  $\text{rep } (x + 1)$  has type  $\text{Vec Bool } (1 + x)$ ” in Agda. The error is confusing since it objects to an obvious property of addition, and if addition were buggy no hints would be given to fix the problem. Ideally the error messages would give a specific instance of  $x$  where  $x + 1 \neq 1 + x$  or remain silent. There is some evidence that specific examples can help clarify the error messages in OCaml[20] and there has been an effort to make refinement type error messages more concrete in Liquid Haskell[10].

Strengthening the equality relation dependently typed languages is used to motivate many research projects (to name a few [4, 24, 19]). However, every formulation I am aware of intends to preserve decidable type checking or logical soundness, so equality will never be complete<sup>5</sup>. Since I intend to dispense with both decidable type checking or logical soundness I can propose a language with an equality that is more convenient in practice.

Building off the base language I purpose a dynamic cast language, a cast type system and a partial elaboration function that satisfies the following basic guarantees:

1.  $\vdash e : M$  and  $\text{elab}(e, M) = e'$  then  $\vdash e' : M'$  for some  $\vdash M' : *$ .
2.  $\vdash e' : M'$  and  $e' \downarrow \text{blame}$  then there is no  $\vdash e : M$  such that  $\text{elab}(e, M) = e'$
3.  $\vdash e' : *$  and  $\text{elab}(e, *) = e'$  then
  - (a) if  $e' \downarrow *$  then  $e \downarrow *$
  - (b) if  $e' \downarrow (x : M') \rightarrow N'$  then  $e \downarrow (x : M) \rightarrow N$
  - (c) if  $e' \downarrow TCon\Delta'$  then  $e \downarrow TCon\Delta$
4.  $\vdash e' : M'$  then
  - (a)  $e' \downarrow v'$  and  $\vdash v' : M'$
  - (b) or  $e' \uparrow$
  - (c) or  $e' \downarrow \text{blame}$

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<sup>5</sup>I am also unaware of any suitable notion of complete equality though it is considered in [22].

In the example above  $\lambda x.\text{head } x (\text{rep } (x + 1)) : \mathbb{N} \rightarrow \mathbb{B}$  will not emit any errors at compile time or runtime (though a warning may be given). If the example is changed to

$$\lambda x.\text{head } x (\text{rep } x) : \mathbb{N} \rightarrow \mathbb{B}$$

no static error will be given, but if the function is called a runtime error will be throughn. The blame tracking system will blame the exact static location that uses unequal types with a direct proof of inequality, allowing an error like “failed at application  $(\text{head } x : \text{Vec } (1 + x) \mathbb{B} \rightarrow \dots) (\text{rep } x : \text{Vec } x \mathbb{B})$  since when  $x = 3$ ,  $1 + x = 4 \neq 3 = x$ ”, regardless of where in the program the the function was called.

### 3.1 Prior work

It is unsurprising that dynamic quality is shares many of the same concerns as the large amount of work contracts, hybrid types, gradual types, and blame. In fact, this work could be seen as gradualizing the Reflection Rule in Extensional Type Theory.

Blame has been strongly advocated for in [26, 25]. Blame tracking can establish the reasonableness of gradual typing systems, though as many authors have noticed, proving blame correctness is tedious and error prone, many authors only conjecture it for their systems.

The basic correctness conditions are inspired by the Gradual Guarantee [21]. The implementation also takes inspiration from “Abstracting gradual typing”[9], where static evidence annotations become runtime checks. Unlike some impressive attempts to gradualize the polymorphic lambda calculus [1], dynamic equality does not attempt to preserve any parametric properties of the base language.

A direct attempt has been made to gradualize a full spectrum dependently typed language to an untyped lambda calculus using the AGT philosophy in [8]. However that system retains the definitional style of equality and user defined data types are not supported. The paper is largely concerned with establishing decidable type checking via an approximate term normalization.

A refinement type system with higher order features is gradualized in [28] though it does not appear powerful enough to be characterized a a full-spectrum dependent type theory. [28] builds on earlier refinement type system work, which described itself as “dynamic” . A notable example is [18] which describes a refinement system that

limit's predicates to base types.

## 4 Prototyping proofs and programs

Just as “obvious” equalities are missing from the definitional relation, “obvious” proofs and programs are not always conveniently available to the programmer. For instance, in Agda it is possible to write a sorting function quickly using simple types. With expertise and effort it is possible to prove that sorting procedure correct by rewriting it with the necessarily invariants. However very little is offered in between. The problem is magnified if module boundaries hide the implementation details of a function, since the details are exactly what is needed to make a proof! This is especially important for larger scale software where a library may expect proof terms that while “correct” are not constructible from the exports of the other library.

The solution proposed here is some additional syntax that will search for a term of the type when resolved at runtime. Given the sorting function

$$\text{sort} : \text{List } \mathbb{N} \rightarrow \text{List } \mathbb{N}$$

and given the first order predicate that

$$\text{IsSorted} : \text{List } \mathbb{N} \rightarrow *$$

then it is possible to assert that `sort` behaves as expected with

$$\lambda x. ? : (x : \text{List } \mathbb{N}) \rightarrow \text{IsSorted}(\text{sort } x)$$

this term will act like any other term at runtime, given a list input it will verify that the `sort` function correctly handles that input, give an error, or non-terminate.

Additionally this would allow simple prototyping from first order specification. For instance,

*data* **Mult** :  $\mathbb{N} \rightarrow \mathbb{N} \rightarrow \mathbb{N} \rightarrow *$  *where*

**base** :  $(x : \mathbb{N}) \rightarrow \mathbf{Mult} 0 x 0$

**suc** :  $(x y z : \mathbb{N}) \rightarrow \mathbf{Mult} x y z \rightarrow \mathbf{Mult} (1 + x) y (y + z)$

can be used to prototype

$$\mathbf{div} = \lambda x. \lambda y. \mathbf{fst} \left( ? : \sum z : \mathbb{N}. \mathbf{Mult} x y z \right)$$

The term search is made subtly easier by the the dynamic equality, otherwise examples like

$$? : \sum f : \mathbb{N} \rightarrow \mathbb{N}. \mathbf{Id} (f, \lambda x. x + 1) \& \mathbf{Id} (f, \lambda x. 1 + x)$$

which would require resolving definitional behavior. Using dynamic equality it is possible only consider the extensional behavior of functions.

Though the proof search is currently primitive, better search methods could be incorporated in future work.

#### 4.1 Prior work

Proof search is often used for static term generation in dependently typed languages (for instance Coq tactics). A first order theorem prover is attached to Agda in [17].

Twelf made use of runtime proof search but the underling theory cannot be considered full spectrum.

### 5 Testing dependent programs

Both dynamic equalities and dynamic proof search vastly weaken the guarantees of normal dependent type systems. Programmers still would like a evidence of correctness, even while they intend to provide full proofs of properties in the future. However, there are few options available in full spectrum dependently typed languages aside from costly and sometimes unconstructable proofs.



The mainstream software industry has similar needs for evidence of correctness, and has made use of testing done in a separate execution phase. Given the rich and precise specifications that dependent types provide it is possible to improve on the hand crafted tests used by most of the industry. Instead we can use a type directed symbolic execution, to run questionable equalities over concrete values and engage and precompute the searched proof terms. Precomputed proof terms can be cached, so that exploration is not too inefficient in the common case of repeating tests at regular intervals of code that is mostly the same. Precomputed terms can be made available at runtime, covering for the inefficient search procedure.

Finally future work can add more advanced methods of testing and proof generation. This architecture should make it easier to add more advanced exploration and search without changing the underlining definitional behavior.

## **5.1 Prior work**

### **5.1.1 Symbolic Execution**

Most research for Symbolic Execution targets popular languages (like C) and uses SMT solvers to efficiently explore branches that depend on base types. Most work does not support higher order functions or makes simplifying assumptions about the type system. There are however some relevant papers:

- [10] presents a symbolic execution engine supporting Haskell’s lazy execution and type system. Higher order functions are not handled
- The draft work[27], handles higher order functions as and inputs provides a proof of completeness
- Symbolic execution for higher order functions for a limited untyped variant of PCF is described in [16]

### **5.1.2 Testing dependent types**

There has been a long recognized need for testing in addition to proving in dependent type systems

- In [6] a QuickCheck style framework was added to an earlier version of Agda
- QuickChick<sup>6</sup> [5][15, 14, 13] is a research project to add testing to Coq. However testing requires building types classes that establish the properties needed by the testing framework such as decidable equality. This is presumably out of reach of novice Coq users.

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<sup>6</sup><https://github.com/QuickChick/QuickChick>

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