Markov Chain Monte Carlo II

PSYC 573

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The original Metropolis (random walk) algorithm allows posterior sampling, without the need to solving the integral

However, it is inefficient, especially in high dimension problems (i.e., many parameters)

Data Example

Taking notes with a pen or a keyboard?

Mueller & Oppenheimer (2014, Psych Science)

R code | Data | Data

```r
# Use haven::read_sav() to import SPSS data
nt_dat <- haven::read_sav("https://osf.io/qrs5y/download")
```

> Do people write more or less words when asked to use longhand?

Normal model

Consider only the laptop group first

$$\textbf{wc\_laptop}_i \sim N(\mu, \sigma^2)$$

Two parameters: μ (mean), σ^2 (variance)

Gibbs Sampling

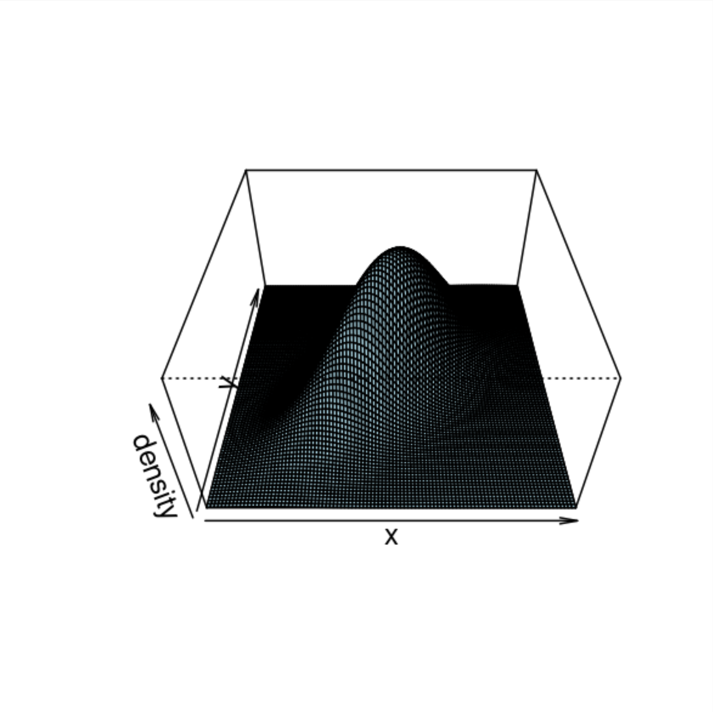
Gibbs sampling is efficient by generating smart proposed values, using **conjugate** or **semiconjugate** priors

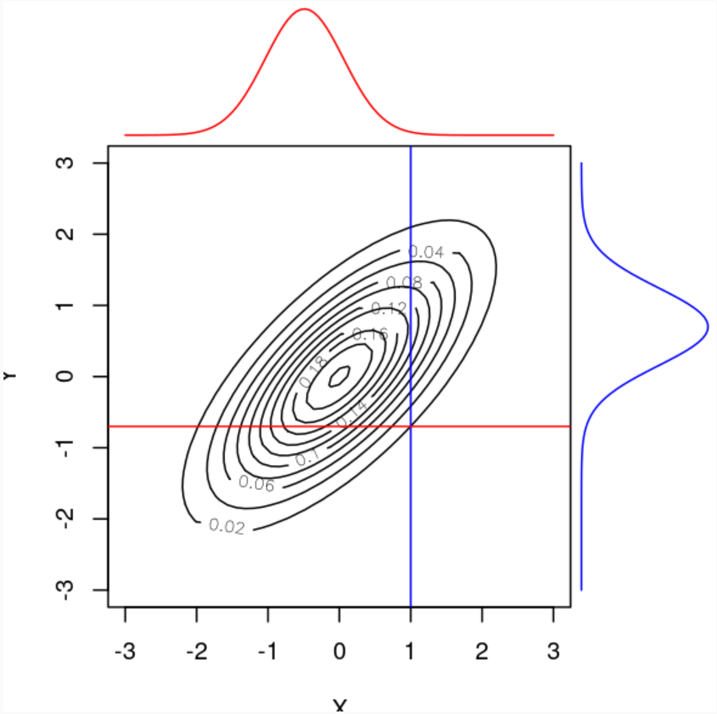
Implemented in software like BUGS and JAGS

Useful when:

- Joint posterior is intractable, but the **conditional distributions** are known

Another example





Conjugate priors for conditional distributions

$$\mu \sim N(\mu_0, \tau_0^2)$$
$$\sigma^2 \sim \text{Inv-Gamma}(\nu_0/2, \nu_0\sigma_0^2/2)$$

- μ_0: Prior mean, τ_0^2: Prior variance (i.e., uncertainty) of the mean
- ν_0: Prior sample size for the variance; σ_0^2: Prior expectation of the variance

Posterior

$$\mu \mid \sigma^2, y \sim N(\mu_n, \tau_n^2)$$
$$\sigma^2 \mid \mu^2 \sim \text{Inv-Gamma}(\nu_n/2, \nu_n\sigma_n^2[\mu]/2)$$

- $\tau_n^2 = \left(\frac{1}{\tau_0^2} + \frac{n}{\sigma^2}\right)^{-1}$; $\mu_n = \tau_n^2\left(\frac{\mu_0}{\tau_0^2} + \frac{n\bar{y}}{\sigma^2}\right)$
- $\nu_n = \nu_0 + n$; $\sigma_n^2(\mu) = \frac{1}{\nu_n}\left[\nu_0\sigma_0^2 + (n-1)s_y^2 + \sum(\bar{y} - \mu)^2\right]$

No need for a separate proposal distribution; directly sample the conditional posterior

- Thus, all draws are accepted

Posterior Summary

2 chains, 10,000 draws each, half warm-ups

$\mu_0 = 5$, $\sigma_0^2 = 1$, $\tau_0^2 = 100$, $\nu_0 = 1$

variable	mean	median	sd	mad	q5	q95	rhat	ess_bulk	ess_tail
mu	3.1	3.10	0.213	0.211	2.753	3.45	1	9928	9936
sigma2	1.4	1.33	0.378	0.337	0.904	2.11	1	10189	10136

The ESS is almost as large as # of draws