

Supplemental Results with Improved EBM and LMC Methods

This report summarizes the results of several methods that improved upon the empirical Bayes mean (EBM) method and the latent-mean centering (LMC) method presented in the manuscript, using the same conditions as in Studies 1 and 2. The methods (and the corresponding software) here include:

- EBM-ML-FP (*lme4* in R), which uses maximum likelihood (ML) estimation, Wald confidence intervals (CIs), and finite population correction (FPC). This is also studied in the manuscript.
- EBM-REML-FP (*lme4* in R), which uses restricted maximum likelihood (REML) estimation, likelihood-based CIs, and FPC. Note that REML and likelihood-based CIs are default in the *lme4* R package.
- EBM-BA-FP (*blme* in R), which uses penalized maximum likelihood estimation with boundary-avoiding priors with the *blme* R package, likelihood-based CIs, and FPC. A $\text{gamma}(2, 0.1)$ prior is used on τ_X (with the `posterior.scale = "sd"` option).
- LMC-MLR (Mplus), which uses sandwich estimator for the standard errors with the option “ESTIMATOR=MLR” in Mplus.
- LMC-BAYES (Mplus), which uses MCMC estimation with flat priors with the option “ESTIMATOR=BAYES” in Mplus.

Study 1

Table S1, and Figures S3 and S4. The full results are stored in the file ““.

Study 2

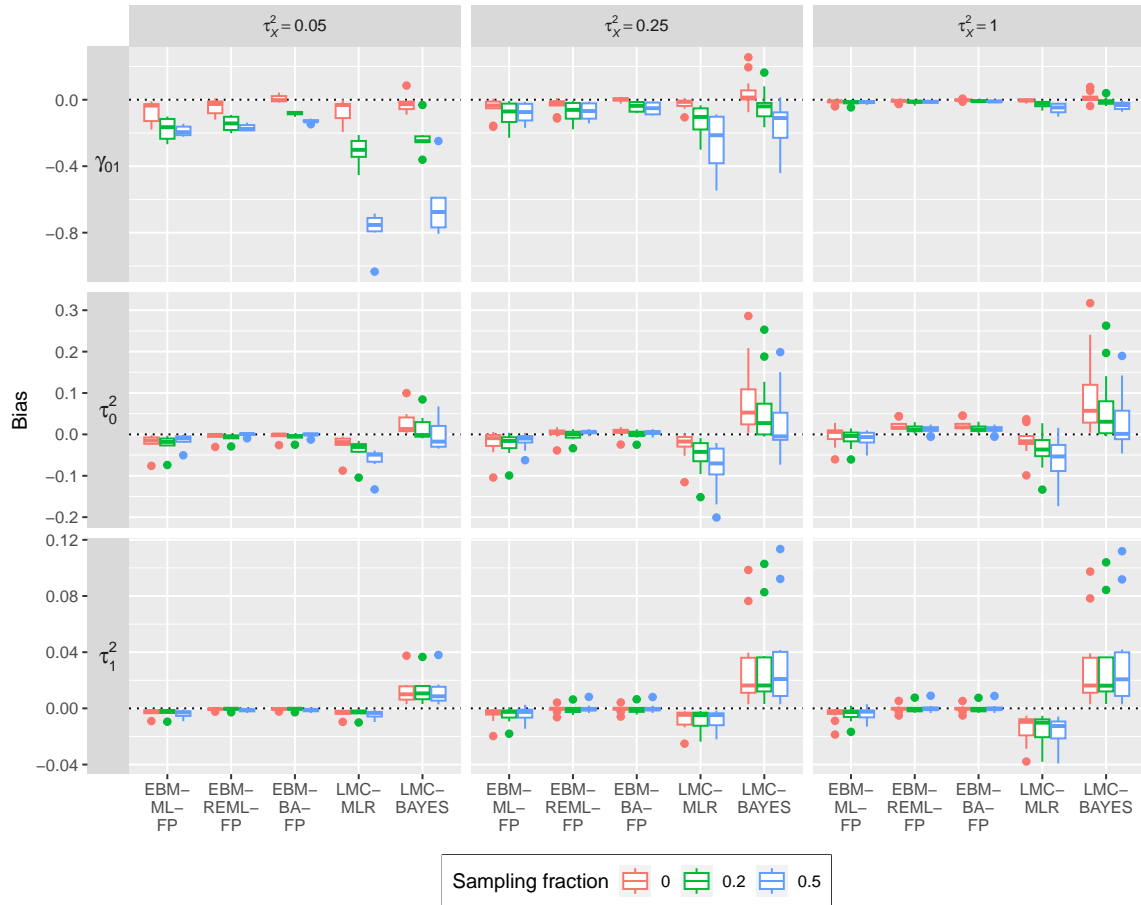
Figures S5 and S6. The full results are stored in the file ““.

Table S1

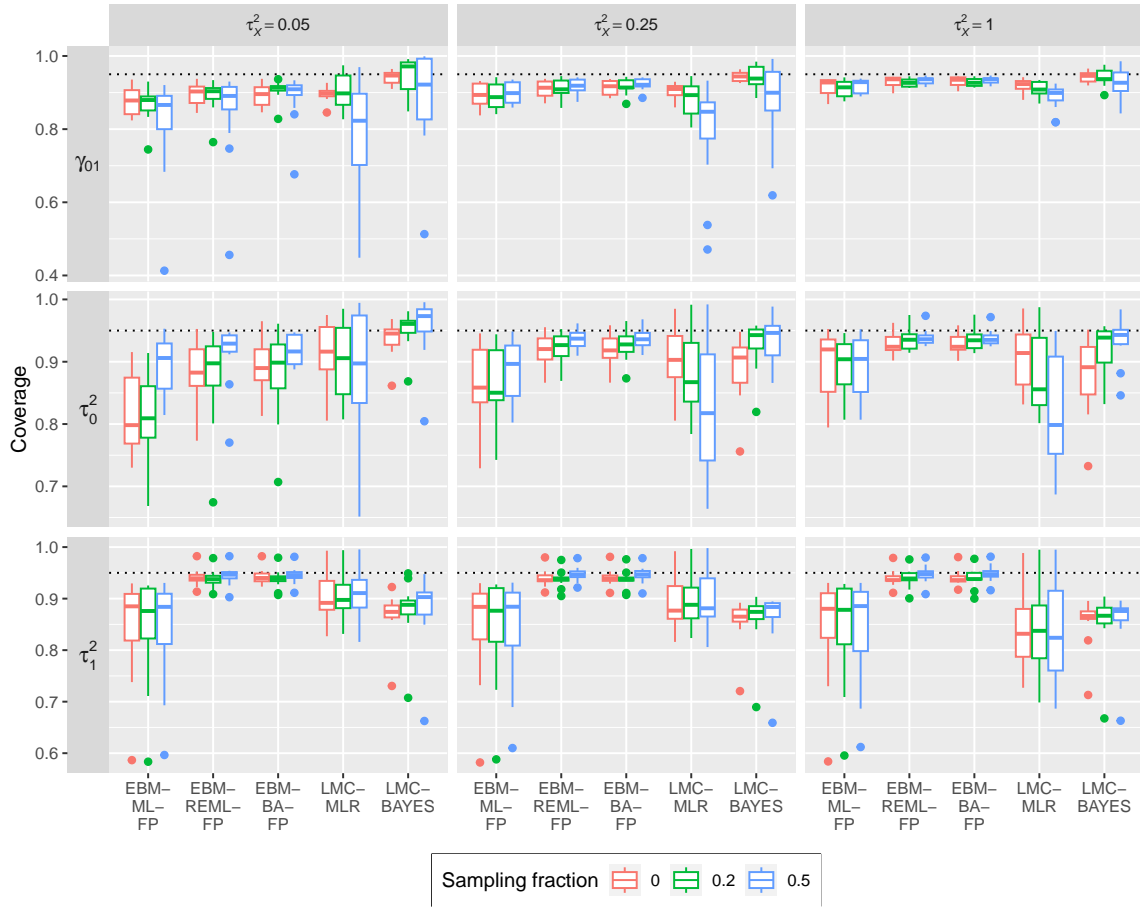
Inadmissible Solutions and Bias of Between-Level Coefficients in Low Cluster-Mean Reliability Conditions of Study 1.

\bar{n}	τ_X^2	J	SF	τ_0^2	% Inadmissible ^a		Bias for γ_{01}				
					EBMM	EBMR	EBMM	EBMR	EBMB	LMC-MLR	LMC-BAYES
5	0.05	20	0	0.1	36.40	30.30	0.02	0.08	0.32	0.20	0.67
				0.4	36.10	30.25	0.00	0.07	0.33	0.03	0.62
			0.2	0.1	54.55	48.00	-0.37	-0.29	0.10	-0.14	0.59
				0.4	53.35	46.85	-0.38	-0.35	0.12	-0.45	0.52
			0.5	0.1	86.70	81.75	-0.70	-0.63	-0.26	-1.11	0.39
				0.4	86.45	81.55	-0.73	-0.64	-0.23	-1.90	0.26
		50	0	0.1	18.55	16.55	-0.07	0.01	0.18	0.03	0.42
				0.4	18.55	16.60	-0.07	0.00	0.17	-0.22	0.32
			0.2	0.1	45.05	40.25	-0.90	-0.82	-0.33	-0.61	0.18
				0.4	45.20	40.45	-0.86	-0.76	-0.31	-1.26	-0.02
			0.5	0.1	94.00	92.10	-1.48	-1.59	-0.84	-1.72	-0.34
				0.4	93.85	92.00	-1.47	-1.51	-0.86	-3.77	-0.81
		100	0	0.1	8.60	7.25	-0.14	-0.10	0.08	-0.06	0.08
				0.4	8.60	7.25	-0.13	-0.10	0.09	-0.12	-0.12
			0.2	0.1	37.20	34.40	-1.38	-1.30	-0.70	-0.87	-0.50
				0.4	37.20	34.40	-1.38	-1.26	-0.71	-1.70	-1.14
			0.5	0.1	97.70	97.15	-2.59	-2.30	-1.08	-1.97	-1.41
				0.4	97.70	97.15	-2.66	-2.27	-1.11	-4.79	-3.01

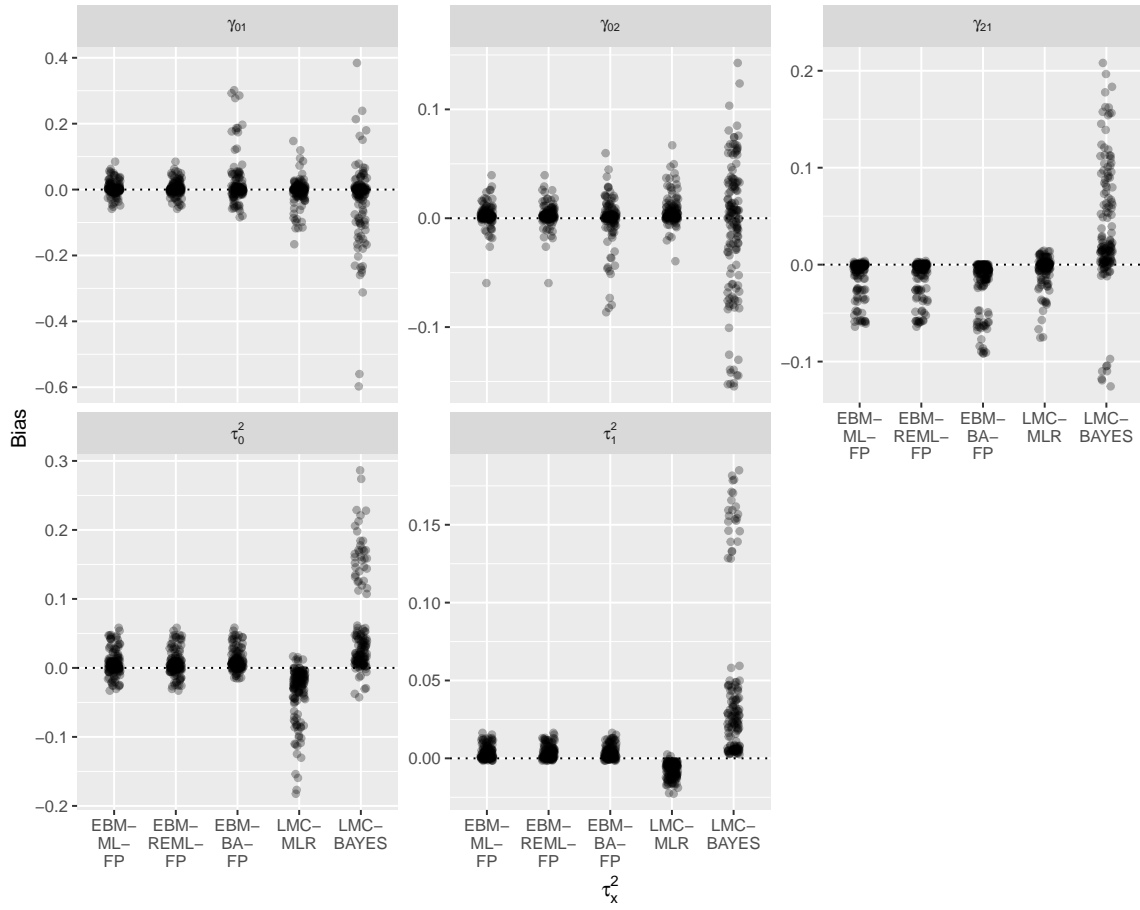
Note. ^aResults are admissible for all replications with EBM-BA-FP, LMC-MLR, and LMC-BAYES. EBMM = EBM-ML-FP. EBMR = EBM-REML-FP. EBMB = EBM-BA-FP.

**Figure S3**

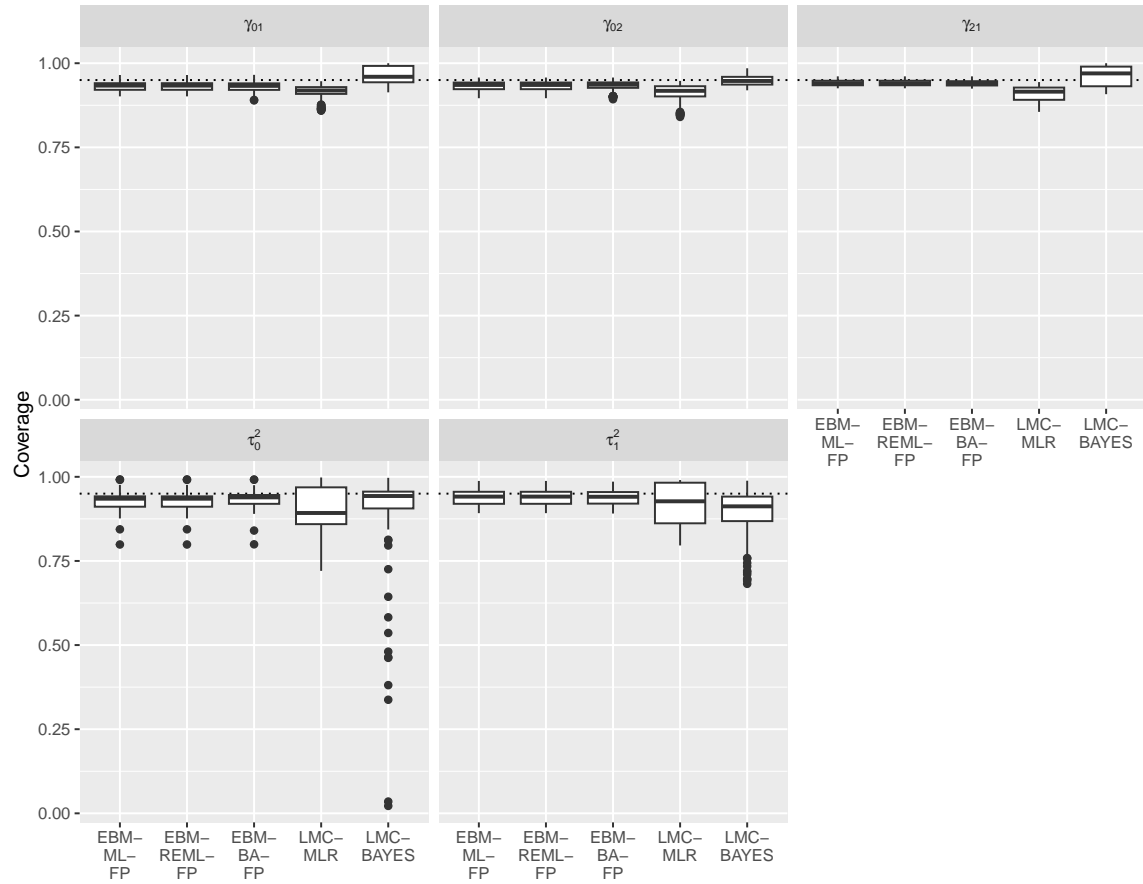
Bias for parameter estimates in Supplemental Study 1. The panels show, from top to bottom, the between-cluster effect, the conditional random intercept variance of the outcome, and the random slope variance. Conditions with average cluster size = 5 and $\tau_X^2 = 0.05$ are not shown (see Table S1).

**Figure S4**

Empirical coverage for Supplemental Study 1 (all conditions). The panels show, from top to bottom, the between-cluster effect, the conditional random intercept variance of the outcome, and the random slope variance. The dashed line represents the 95% reference rate.

**Figure S5**

Bias of parameter estimates in Supplemental Study 2. The panels show the between-cluster effect (γ_{01}), the effect of the level-2 covariate (γ_{02}), the cross-level interaction (γ_{21}), the conditional random intercept variance of the outcome (τ_0^2), and the random slope variance (τ_1^2).

**Figure S6**

Empirical coverage for Supplemental Study 2. The panels show the between-cluster effect (γ_{01}), the effect of the level-2 covariate (γ_{02}), the cross-level interaction (γ_{21}), the conditional random intercept variance of the outcome (τ_0^2), and the random slope variance (τ_1^2). The dashed line represents the 95% reference rate.