Alignment Optimization

Table of contents

Load Packages

```
library(mirt)       # for IRT analyses
```

```
Loading required package: stats4
```

```
Loading required package: lattice
```

```
# install.packages("remotes")
# remotes::install_github("marklhc/pinsearch")
library(pinsearch)  # for noninvariance effect size
library(ggplot2)    # for plotting
library(umx)        # for analyzing harmonized scores
```

```
Loading required package: OpenMx

To take full advantage of multiple cores, use:
  mxOption(key='Number of Threads', value=parallel::detectCores()) #now
  Sys.setenv(OMP_NUM_THREADS=parallel::detectCores()) #before library(OpenMx)

For an overview type '?umx'


Attaching package: 'umx'

The following object is masked from 'package:stats':

    loadings
```

Import Data

Data can be downloaded from https://github.com/jmk7cj/SEM-mnlfa.

```r
data <- read.csv(here::here("SEM-mnlfa", "data.csv"))
# Define grouping variable
data$group <- data$study_id
# Sort data by group
data <- data[order(data$group), ]
head(data)
```

```
     id study_id    sex  race T_TOCA_breakrule T_TOCA_harmpro T_TOCA_breakthg
1 FT001     FAST   male black                0             NA               0
2 FT002     FAST   male white                1             NA               0
3 FT003     FAST female white                0             NA               0
4 FT004     FAST female white                1             NA               0
5 FT005     FAST   male black                1             NA               0
6 FT006     FAST   male black                1             NA               1
  T_TOCA_takeprop T_TOCA_fight T_TOCA_lies T_TOCA_yell T_TOCA_stub T_TOCA_teas
1               0            0           0           0           0           0
2               1            1           0           1           1           1
3               0            0           0           1           0           0
4               0            1           1           0           1           0
5               0            1           0           1           1           1
6               1            1           1           1           1           1
```

```
  hs group
1  1  FAST
2  1  FAST
3  1  FAST
4  0  FAST
5  1  FAST
6  1  FAST
```

```r
# Sample sizes
# Study 1 = FAST; Study 2 = LIFT; Study 3 = PIRC1;
# Study 4 = PIRC2; Study 5 = SAFE
table(data$study_id)
```

```
 FAST  LIFT PIRC1 PIRC2  SAFE
  817   451  1884   639   157
```

```r
item_names <- names(data)[5:13]  # save item names
```

Configural Model

Run separate mirts (2PL)

```r
groups <- unique(data$group)
# Initialize output
# a. number of observations per group and item
item_n <- matrix(NA,
    nrow = length(groups),
    ncol = length(item_names),
    dimnames = list(groups, item_names))
# b. Matrix of loadings and intercepts
lambda <- nu <- item_n
# c. Loop over groups, fitting a 2PL model to each
for (g in seq_along(groups)) {
    datg <- data[data$group == groups[g], item_names]
    # Drop completely missing items
    item_n[g, ] <- colSums(!is.na(datg))
    itemsg <- which(colSums(!is.na(datg)) > 0)
    fitg <- mirt(datg[names(itemsg)], model = 1,
                 itemtype = "2PL")
```

```r
    coefg <- coef(fitg, simplify = TRUE)
    lambda[g, names(itemsg)] <- coefg$items[, "a1"]
    nu[g, names(itemsg)] <- coefg$items[, "d"]
}
```

```
Warning: data contains response patterns with only NAs
```

```
Iteration: 1, Log-Lik: -2701.839, Max-Change: 1.34190
Iteration: 2, Log-Lik: -2463.453, Max-Change: 0.72218
Iteration: 3, Log-Lik: -2412.010, Max-Change: 0.43110
Iteration: 4, Log-Lik: -2392.083, Max-Change: 0.30417
Iteration: 5, Log-Lik: -2383.941, Max-Change: 0.20276
Iteration: 6, Log-Lik: -2380.398, Max-Change: 0.13371
Iteration: 7, Log-Lik: -2377.699, Max-Change: 0.05533
Iteration: 8, Log-Lik: -2377.254, Max-Change: 0.04833
Iteration: 9, Log-Lik: -2376.981, Max-Change: 0.02805
Iteration: 10, Log-Lik: -2376.733, Max-Change: 0.02276
Iteration: 11, Log-Lik: -2376.618, Max-Change: 0.01234
Iteration: 12, Log-Lik: -2376.534, Max-Change: 0.00950
Iteration: 13, Log-Lik: -2376.384, Max-Change: 0.01048
Iteration: 14, Log-Lik: -2376.358, Max-Change: 0.00530
Iteration: 15, Log-Lik: -2376.338, Max-Change: 0.00516
Iteration: 16, Log-Lik: -2376.279, Max-Change: 0.00127
Iteration: 17, Log-Lik: -2376.278, Max-Change: 0.00099
Iteration: 18, Log-Lik: -2376.278, Max-Change: 0.00087
Iteration: 19, Log-Lik: -2376.276, Max-Change: 0.00021
Iteration: 20, Log-Lik: -2376.276, Max-Change: 0.00021
Iteration: 21, Log-Lik: -2376.276, Max-Change: 0.00021
Iteration: 22, Log-Lik: -2376.275, Max-Change: 0.00072
Iteration: 23, Log-Lik: -2376.275, Max-Change: 0.00052
Iteration: 24, Log-Lik: -2376.275, Max-Change: 0.00069
Iteration: 25, Log-Lik: -2376.275, Max-Change: 0.00021
Iteration: 26, Log-Lik: -2376.275, Max-Change: 0.00050
Iteration: 27, Log-Lik: -2376.275, Max-Change: 0.00016
Iteration: 28, Log-Lik: -2376.275, Max-Change: 0.00015
Iteration: 29, Log-Lik: -2376.275, Max-Change: 0.00049
Iteration: 30, Log-Lik: -2376.275, Max-Change: 0.00012
Iteration: 31, Log-Lik: -2376.275, Max-Change: 0.00010
Iteration: 32, Log-Lik: -2376.275, Max-Change: 0.00036
Iteration: 33, Log-Lik: -2376.275, Max-Change: 0.00048
```

```
Iteration: 34, Log-Lik: -2376.275, Max-Change: 0.00015
Iteration: 35, Log-Lik: -2376.275, Max-Change: 0.00040
Iteration: 36, Log-Lik: -2376.275, Max-Change: 0.00012
Iteration: 37, Log-Lik: -2376.275, Max-Change: 0.00011
Iteration: 38, Log-Lik: -2376.275, Max-Change: 0.00036
Iteration: 39, Log-Lik: -2376.275, Max-Change: 0.00009

Warning: data contains response patterns with only NAs


Iteration: 1, Log-Lik: -866.529, Max-Change: 1.38272
Iteration: 2, Log-Lik: -773.763, Max-Change: 0.93477
Iteration: 3, Log-Lik: -752.996, Max-Change: 0.66499
Iteration: 4, Log-Lik: -745.988, Max-Change: 0.47619
Iteration: 5, Log-Lik: -743.015, Max-Change: 0.32199
Iteration: 6, Log-Lik: -741.639, Max-Change: 0.24704
Iteration: 7, Log-Lik: -740.333, Max-Change: 0.11340
Iteration: 8, Log-Lik: -740.227, Max-Change: 0.09039
Iteration: 9, Log-Lik: -740.160, Max-Change: 0.06572
Iteration: 10, Log-Lik: -740.057, Max-Change: 0.02925
Iteration: 11, Log-Lik: -740.039, Max-Change: 0.01853
Iteration: 12, Log-Lik: -740.024, Max-Change: 0.01111
Iteration: 13, Log-Lik: -739.997, Max-Change: 0.01445
Iteration: 14, Log-Lik: -739.990, Max-Change: 0.00956
Iteration: 15, Log-Lik: -739.985, Max-Change: 0.00817
Iteration: 16, Log-Lik: -739.971, Max-Change: 0.00957
Iteration: 17, Log-Lik: -739.969, Max-Change: 0.00546
Iteration: 18, Log-Lik: -739.967, Max-Change: 0.00476
Iteration: 19, Log-Lik: -739.962, Max-Change: 0.00526
Iteration: 20, Log-Lik: -739.961, Max-Change: 0.00331
Iteration: 21, Log-Lik: -739.961, Max-Change: 0.00284
Iteration: 22, Log-Lik: -739.959, Max-Change: 0.00239
Iteration: 23, Log-Lik: -739.959, Max-Change: 0.00165
Iteration: 24, Log-Lik: -739.959, Max-Change: 0.00140
Iteration: 25, Log-Lik: -739.959, Max-Change: 0.00106
Iteration: 26, Log-Lik: -739.958, Max-Change: 0.00096
Iteration: 27, Log-Lik: -739.958, Max-Change: 0.00091
Iteration: 28, Log-Lik: -739.958, Max-Change: 0.00027
Iteration: 29, Log-Lik: -739.958, Max-Change: 0.00030
Iteration: 30, Log-Lik: -739.958, Max-Change: 0.00029
Iteration: 31, Log-Lik: -739.958, Max-Change: 0.00023
Iteration: 32, Log-Lik: -739.958, Max-Change: 0.00021
```

```
Iteration: 33, Log-Lik: -739.958, Max-Change: 0.00021
Iteration: 34, Log-Lik: -739.958, Max-Change: 0.00084
Iteration: 35, Log-Lik: -739.958, Max-Change: 0.00078
Iteration: 36, Log-Lik: -739.958, Max-Change: 0.00063
Iteration: 37, Log-Lik: -739.958, Max-Change: 0.00028
Iteration: 38, Log-Lik: -739.958, Max-Change: 0.00065
Iteration: 39, Log-Lik: -739.958, Max-Change: 0.00020
Iteration: 40, Log-Lik: -739.958, Max-Change: 0.00015
Iteration: 41, Log-Lik: -739.958, Max-Change: 0.00056
Iteration: 42, Log-Lik: -739.958, Max-Change: 0.00012
Iteration: 43, Log-Lik: -739.958, Max-Change: 0.00012
Iteration: 44, Log-Lik: -739.958, Max-Change: 0.00052
Iteration: 45, Log-Lik: -739.958, Max-Change: 0.00011
Iteration: 46, Log-Lik: -739.958, Max-Change: 0.00053
Iteration: 47, Log-Lik: -739.958, Max-Change: 0.00014
Iteration: 48, Log-Lik: -739.958, Max-Change: 0.00047
Iteration: 49, Log-Lik: -739.958, Max-Change: 0.00017
Iteration: 50, Log-Lik: -739.958, Max-Change: 0.00046
Iteration: 51, Log-Lik: -739.958, Max-Change: 0.00012
Iteration: 52, Log-Lik: -739.958, Max-Change: 0.00009

Warning: data contains response patterns with only NAs


Iteration: 1, Log-Lik: -8223.527, Max-Change: 0.97353
Iteration: 2, Log-Lik: -7401.627, Max-Change: 0.66823
Iteration: 3, Log-Lik: -7226.007, Max-Change: 0.49820
Iteration: 4, Log-Lik: -7163.834, Max-Change: 0.29288
Iteration: 5, Log-Lik: -7141.293, Max-Change: 0.19222
Iteration: 6, Log-Lik: -7131.656, Max-Change: 0.14900
Iteration: 7, Log-Lik: -7127.390, Max-Change: 0.10689
Iteration: 8, Log-Lik: -7125.368, Max-Change: 0.06951
Iteration: 9, Log-Lik: -7124.388, Max-Change: 0.05193
Iteration: 10, Log-Lik: -7123.488, Max-Change: 0.01236
Iteration: 11, Log-Lik: -7123.434, Max-Change: 0.00858
Iteration: 12, Log-Lik: -7123.407, Max-Change: 0.00632
Iteration: 13, Log-Lik: -7123.363, Max-Change: 0.00362
Iteration: 14, Log-Lik: -7123.356, Max-Change: 0.00262
Iteration: 15, Log-Lik: -7123.351, Max-Change: 0.00243
Iteration: 16, Log-Lik: -7123.332, Max-Change: 0.00084
Iteration: 17, Log-Lik: -7123.332, Max-Change: 0.00112
Iteration: 18, Log-Lik: -7123.331, Max-Change: 0.00076
```

```
Iteration: 19, Log-Lik: -7123.331, Max-Change: 0.00052
Iteration: 20, Log-Lik: -7123.331, Max-Change: 0.00039
Iteration: 21, Log-Lik: -7123.331, Max-Change: 0.00172
Iteration: 22, Log-Lik: -7123.330, Max-Change: 0.00113
Iteration: 23, Log-Lik: -7123.330, Max-Change: 0.00068
Iteration: 24, Log-Lik: -7123.330, Max-Change: 0.00061
Iteration: 25, Log-Lik: -7123.329, Max-Change: 0.00050
Iteration: 26, Log-Lik: -7123.329, Max-Change: 0.00040
Iteration: 27, Log-Lik: -7123.329, Max-Change: 0.00038
Iteration: 28, Log-Lik: -7123.329, Max-Change: 0.00030
Iteration: 29, Log-Lik: -7123.329, Max-Change: 0.00040
Iteration: 30, Log-Lik: -7123.329, Max-Change: 0.00023
Iteration: 31, Log-Lik: -7123.329, Max-Change: 0.00025
Iteration: 32, Log-Lik: -7123.329, Max-Change: 0.00018
Iteration: 33, Log-Lik: -7123.329, Max-Change: 0.00020
Iteration: 34, Log-Lik: -7123.329, Max-Change: 0.00014
Iteration: 35, Log-Lik: -7123.329, Max-Change: 0.00018
Iteration: 36, Log-Lik: -7123.329, Max-Change: 0.00011
Iteration: 37, Log-Lik: -7123.329, Max-Change: 0.00012
Iteration: 38, Log-Lik: -7123.329, Max-Change: 0.00009

Iteration: 1, Log-Lik: -2691.206, Max-Change: 1.37907
Iteration: 2, Log-Lik: -2384.546, Max-Change: 0.88232
Iteration: 3, Log-Lik: -2318.976, Max-Change: 0.53559
Iteration: 4, Log-Lik: -2296.037, Max-Change: 0.40448
Iteration: 5, Log-Lik: -2286.737, Max-Change: 0.28161
Iteration: 6, Log-Lik: -2282.220, Max-Change: 0.20677
Iteration: 7, Log-Lik: -2278.645, Max-Change: 0.07540
Iteration: 8, Log-Lik: -2278.051, Max-Change: 0.05908
Iteration: 9, Log-Lik: -2277.675, Max-Change: 0.04316
Iteration: 10, Log-Lik: -2277.059, Max-Change: 0.01590
Iteration: 11, Log-Lik: -2276.930, Max-Change: 0.01428
Iteration: 12, Log-Lik: -2276.825, Max-Change: 0.01334
Iteration: 13, Log-Lik: -2276.480, Max-Change: 0.01694
Iteration: 14, Log-Lik: -2276.450, Max-Change: 0.01265
Iteration: 15, Log-Lik: -2276.426, Max-Change: 0.01023
Iteration: 16, Log-Lik: -2276.342, Max-Change: 0.00477
Iteration: 17, Log-Lik: -2276.335, Max-Change: 0.00608
Iteration: 18, Log-Lik: -2276.330, Max-Change: 0.00386
Iteration: 19, Log-Lik: -2276.327, Max-Change: 0.00341
Iteration: 20, Log-Lik: -2276.322, Max-Change: 0.00234
Iteration: 21, Log-Lik: -2276.319, Max-Change: 0.00230
Iteration: 22, Log-Lik: -2276.306, Max-Change: 0.00122
```

```
Iteration: 23, Log-Lik: -2276.306, Max-Change: 0.00192
Iteration: 24, Log-Lik: -2276.305, Max-Change: 0.00097
Iteration: 25, Log-Lik: -2276.304, Max-Change: 0.00160
Iteration: 26, Log-Lik: -2276.304, Max-Change: 0.00066
Iteration: 27, Log-Lik: -2276.304, Max-Change: 0.00066
Iteration: 28, Log-Lik: -2276.303, Max-Change: 0.00103
Iteration: 29, Log-Lik: -2276.302, Max-Change: 0.00077
Iteration: 30, Log-Lik: -2276.302, Max-Change: 0.00085
Iteration: 31, Log-Lik: -2276.302, Max-Change: 0.00034
Iteration: 32, Log-Lik: -2276.302, Max-Change: 0.00012
Iteration: 33, Log-Lik: -2276.302, Max-Change: 0.00011
Iteration: 34, Log-Lik: -2276.302, Max-Change: 0.00013
Iteration: 35, Log-Lik: -2276.302, Max-Change: 0.00012
Iteration: 36, Log-Lik: -2276.302, Max-Change: 0.00012
Iteration: 37, Log-Lik: -2276.302, Max-Change: 0.00045
Iteration: 38, Log-Lik: -2276.302, Max-Change: 0.00046
Iteration: 39, Log-Lik: -2276.302, Max-Change: 0.00034
Iteration: 40, Log-Lik: -2276.302, Max-Change: 0.00011
Iteration: 41, Log-Lik: -2276.302, Max-Change: 0.00039
Iteration: 42, Log-Lik: -2276.302, Max-Change: 0.00008

Warning: data contains response patterns with only NAs


Iteration: 1, Log-Lik: -744.931, Max-Change: 1.23308
Iteration: 2, Log-Lik: -655.005, Max-Change: 0.95430
Iteration: 3, Log-Lik: -634.533, Max-Change: 0.80731
Iteration: 4, Log-Lik: -627.163, Max-Change: 0.58697
Iteration: 5, Log-Lik: -624.106, Max-Change: 0.43579
Iteration: 6, Log-Lik: -622.719, Max-Change: 0.33565
Iteration: 7, Log-Lik: -621.452, Max-Change: 0.10091
Iteration: 8, Log-Lik: -621.418, Max-Change: 0.07328
Iteration: 9, Log-Lik: -621.401, Max-Change: 0.05263
Iteration: 10, Log-Lik: -621.382, Max-Change: 0.01659
Iteration: 11, Log-Lik: -621.380, Max-Change: 0.01216
Iteration: 12, Log-Lik: -621.379, Max-Change: 0.00886
Iteration: 13, Log-Lik: -621.376, Max-Change: 0.00277
Iteration: 14, Log-Lik: -621.376, Max-Change: 0.00283
Iteration: 15, Log-Lik: -621.376, Max-Change: 0.00297
Iteration: 16, Log-Lik: -621.374, Max-Change: 0.00142
Iteration: 17, Log-Lik: -621.374, Max-Change: 0.00160
Iteration: 18, Log-Lik: -621.374, Max-Change: 0.00157
```

```
Iteration: 19, Log-Lik: -621.373, Max-Change: 0.00101
Iteration: 20, Log-Lik: -621.373, Max-Change: 0.00084
Iteration: 21, Log-Lik: -621.373, Max-Change: 0.00077
Iteration: 22, Log-Lik: -621.373, Max-Change: 0.00079
Iteration: 23, Log-Lik: -621.373, Max-Change: 0.00050
Iteration: 24, Log-Lik: -621.373, Max-Change: 0.00022
Iteration: 25, Log-Lik: -621.373, Max-Change: 0.00025
Iteration: 26, Log-Lik: -621.373, Max-Change: 0.00025
Iteration: 27, Log-Lik: -621.373, Max-Change: 0.00025
Iteration: 28, Log-Lik: -621.373, Max-Change: 0.00022
Iteration: 29, Log-Lik: -621.373, Max-Change: 0.00020
Iteration: 30, Log-Lik: -621.373, Max-Change: 0.00019
Iteration: 31, Log-Lik: -621.373, Max-Change: 0.00013
Iteration: 32, Log-Lik: -621.373, Max-Change: 0.00013
Iteration: 33, Log-Lik: -621.373, Max-Change: 0.00013
Iteration: 34, Log-Lik: -621.373, Max-Change: 0.00011
Iteration: 35, Log-Lik: -621.373, Max-Change: 0.00010
Iteration: 36, Log-Lik: -621.373, Max-Change: 0.00010
```

Configural Parameters

```r
knitr::kable(lambda, digits = 2)
```

Table 1: Configural Loadings

	T_TOCA_breakrule	T_TOCA_harmpra	T_TOCA_breakthg	T_TOCA_takeprop	T_TOCA_fight	T_TOCA_lies	T_TOCA_yell	T_TOCA_stub	T_TOCA_teas
FAST	3.82	NA	2.86	3.12	2.22	2.65	3.02	3.06	2.44
LIFT	2.57	5.28	3.15	2.81	4.08	2.68	5.34	2.48	2.14
PIRC1	3.27	1.91	3.77	3.86	2.86	3.20	2.71	2.50	3.01
PIRC2	2.65	4.27	4.69	4.19	4.28	3.56	2.75	1.92	2.76
SAFE	2.94	4.88	4.34	6.19	3.51	3.15	3.84	2.02	2.74

```r
knitr::kable(lambda, digits = 2)
```

Table 2: Configural Intercepts

	T_TOCA_breakrule	T_TOCA_harmpra	T_TOCA_breakthg	T_TOCA_takeprop	T_TOCA_fight	T_TOCA_lies	T_TOCA_yell	T_TOCA_stub	T_TOCA_teas
FAST	3.82	NA	2.86	3.12	2.22	2.65	3.02	3.06	2.44

Table 2: Configural Intercepts

	T_TOCA_T	breakdue_TOCA_T	harmpra_TOCA_T	breakbig_TOCA_T	takeprop_TOCA_T	figu_TOCA_T	lies_TOCA_T	yell_TOCA_T	stub_TOCA_T	teas
LIFT	2.57	5.28	3.15	2.81	4.08	2.68	5.34	2.48	2.14	
PIRC1	3.27	1.91	3.77	3.86	2.86	3.20	2.71	2.50	3.01	
PIRC2	2.65	4.27	4.69	4.19	4.28	3.56	2.75	1.92	2.76	
SAFE	2.94	4.88	4.34	6.19	3.51	3.15	3.84	2.02	2.74	

Alignment Optimization

```
aligned <- sirt::invariance.alignment(
    lambda = lambda,
    nu = nu,
    wgt = sqrt(item_n),
    optimizer = "nlminb",
    eps = .0001,
    control = list(maxit = 10000)
)
```

```
knitr::kable(aligned$lambda.aligned, digits = 2)
```

Table 3: Aligned Loadings

	T_TOCA_T	breakdue_TOCA_T	harmpra_TOCA_T	breakbig_TOCA_T	takeprop_TOCA_T	figu_TOCA_T	lies_TOCA_T	yell_TOCA_T	stub_TOCA_T	teas
FAST	3.82	NA	2.86	3.12	2.22	2.65	3.02	3.06	2.44	
LIFT	1.55	3.18	1.90	1.70	2.47	1.62	3.22	1.50	1.29	
PIRC1	2.64	1.55	3.04	3.12	2.31	2.59	2.19	2.02	2.43	
PIRC2	1.97	3.18	3.49	3.12	3.19	2.65	2.05	1.43	2.06	
SAFE	1.93	3.20	2.85	4.07	2.31	2.07	2.52	1.33	1.80	

```
knitr::kable(aligned$nu.aligned, digits = 2)
```

Table 4: Aligned Intercepts

	T_TOCA_T	breakdue_TOCA_T	harmpra_TOCA_T	breakbig_TOCA_T	takeprop_TOCA_T	figu_TOCA_T	lies_TOCA_T	yell_TOCA_T	stub_TOCA_T	teas
FAST	5.28	NA	0.33	1.53	2.53	2.01	3.13	4.08	2.75	
LIFT	2.87	0.37	-0.06	0.40	2.86	0.71	3.72	4.59	2.75	

Table 4: Aligned Intercepts

	T_TOCA_	T_TOCA_	T_TOCA_	T_TOCA_	T_TOCA_	T_TOCA_	T_TOCA_	T_TOCA_	T_TOCA_teas
PIRC1	3.15	0.37	0.39	1.53	0.61	1.45	1.62	2.50	2.26
PIRC2	3.26	1.17	0.35	1.53	2.50	1.34	1.84	1.17	2.26
SAFE	3.18	0.37	0.33	2.41	1.43	2.16	1.74	1.29	1.50

```
knitr::kable(aligned$pars, digits = 2)
```

Table 5: Aligned Latent means and variances

	alpha0	psi0
FAST	0.00	1.00
LIFT	-1.73	1.66
PIRC1	-0.79	1.24
PIRC2	-1.57	1.34
SAFE	-0.77	1.52

Quality: Effect size of noninvariance

```
# Note: we need to do this twice as `pinsearch::fmacs()`
# does not handle NAs in the input, so we need to
# treat item 2 (not in Study 1) separately
# pooled SD
vars <- apply(data[item_names],
    MARGIN = 2,
    FUN = \(x) tapply(x,
        INDEX = data$study_id,
        FUN = var, na.rm = TRUE
    )
)
item_n2 <- ifelse(item_n > 0, item_n, NA)
pooled_sd <- sqrt(
    colSums(vars * (item_n2 - 1), na.rm = TRUE) /
        colSums(item_n2 - 1, na.rm = TRUE)
)

# a. Items 1, 3-9
```

```r
thres1 <- aligned$nu.aligned[, c(1, 3:9)]
colnames(thres1) <- seq_len(ncol(thres1))
# Noninvariance effect size for items 1, 3-9
f1 <- pinsearch::fmacs_ordered(
    thresholds = thres1,
    loadings = aligned$lambda.aligned[, c(1, 3:9)],
    link = "logit",
    pooled_item_sd = pooled_sd[c(1, 3:9)]
)
# b. Item 2
thres2 <- aligned$nu.aligned[-1, 2, drop = FALSE]
colnames(thres2) <- 1
f2 <- pinsearch::fmacs_ordered(
    thresholds = thres2,
    loadings = aligned$lambda.aligned[-1, 2, drop = FALSE],
    link = "logit",
    pooled_item_sd = pooled_sd[2]
)
# Combine to form one effect size vector
f_effsize <- cbind(f1[1, 1, drop = FALSE], f2, f1[1, -1, drop = FALSE])
knitr::kable(f_effsize, digits = 2)
```

	T_TOCA_T	reakfa	TOCA_T	hampa	TOCA_T	reakfa	TOCA_T	akepa	TOCA_T	figh	TOCA_T	ies	TOCA_T	yel	TOCA_T	sno	TOCA_teas
fmacs	0.11		0.14		0.09		0.14		0.25		0.16		0.14		0.29		0.18

The magnitude of noninvariance effect size is not trivial (with fmacs $>$ .1) for most items.

Compute Harmonized Factor Scores

```r
# Refit the configural model, with latent means and variances
# from the aligned parameters
aligned_pars <- aligned$pars
# Define a syntax template to allow latent means and
# variances to be substituted by the aligned values
mod <- "
  f = 1-[I]
  START = (GROUP, MEAN_1, [mu]), (GROUP, COV_11, [sigma2])
"
```

```r
# Substitute into the aligned means and variances
fsg <- NULL  # place holder for factor scores
for (g in seq_along(groups)) {
    datg <- data[data$group == groups[g], item_names]
    # Drop completely missing items
    itemsg <- which(item_n[g, ] > 0)
    modg <- gsub("\\[I\\]", replacement = length(itemsg), mod)
    modg <- gsub("\\[mu\\]", replacement = aligned_pars[g, 1], modg)
    # Note: aligned psi is latent SD, but `mirt`
    # needs latent variance (i.e., SD^2)
    modg <- gsub("\\[sigma2\\]", replacement = aligned_pars[g, 2]^2, modg)
    fitg <- mirt(datg[names(itemsg)], model = modg,
                 itemtype = "2PL")
    fsg <- rbind(
        fsg,
        data.frame(
            fscores(fitg,
                method = "EAP", full.scores.SE = TRUE
            ),
            group = groups[g],
            mean_eta = aligned_pars[g, 1],
            sd_eta = aligned_pars[g, 2]
        )
    )
}
```

```
Warning: data contains response patterns with only NAs


Iteration: 1, Log-Lik: -2701.839, Max-Change: 1.34190
Iteration: 2, Log-Lik: -2463.453, Max-Change: 0.72218
Iteration: 3, Log-Lik: -2412.010, Max-Change: 0.43110
Iteration: 4, Log-Lik: -2392.083, Max-Change: 0.30417
Iteration: 5, Log-Lik: -2383.941, Max-Change: 0.20276
Iteration: 6, Log-Lik: -2380.398, Max-Change: 0.13371
Iteration: 7, Log-Lik: -2377.699, Max-Change: 0.05533
Iteration: 8, Log-Lik: -2377.254, Max-Change: 0.04833
Iteration: 9, Log-Lik: -2376.981, Max-Change: 0.02805
Iteration: 10, Log-Lik: -2376.733, Max-Change: 0.02276
Iteration: 11, Log-Lik: -2376.618, Max-Change: 0.01234
Iteration: 12, Log-Lik: -2376.534, Max-Change: 0.00950
Iteration: 13, Log-Lik: -2376.384, Max-Change: 0.01048
```

```
Iteration: 14, Log-Lik: -2376.358, Max-Change: 0.00530
Iteration: 15, Log-Lik: -2376.338, Max-Change: 0.00516
Iteration: 16, Log-Lik: -2376.279, Max-Change: 0.00127
Iteration: 17, Log-Lik: -2376.278, Max-Change: 0.00099
Iteration: 18, Log-Lik: -2376.278, Max-Change: 0.00087
Iteration: 19, Log-Lik: -2376.276, Max-Change: 0.00021
Iteration: 20, Log-Lik: -2376.276, Max-Change: 0.00021
Iteration: 21, Log-Lik: -2376.276, Max-Change: 0.00021
Iteration: 22, Log-Lik: -2376.275, Max-Change: 0.00072
Iteration: 23, Log-Lik: -2376.275, Max-Change: 0.00052
Iteration: 24, Log-Lik: -2376.275, Max-Change: 0.00069
Iteration: 25, Log-Lik: -2376.275, Max-Change: 0.00021
Iteration: 26, Log-Lik: -2376.275, Max-Change: 0.00050
Iteration: 27, Log-Lik: -2376.275, Max-Change: 0.00016
Iteration: 28, Log-Lik: -2376.275, Max-Change: 0.00015
Iteration: 29, Log-Lik: -2376.275, Max-Change: 0.00049
Iteration: 30, Log-Lik: -2376.275, Max-Change: 0.00012
Iteration: 31, Log-Lik: -2376.275, Max-Change: 0.00010
Iteration: 32, Log-Lik: -2376.275, Max-Change: 0.00036
Iteration: 33, Log-Lik: -2376.275, Max-Change: 0.00048
Iteration: 34, Log-Lik: -2376.275, Max-Change: 0.00015
Iteration: 35, Log-Lik: -2376.275, Max-Change: 0.00040
Iteration: 36, Log-Lik: -2376.275, Max-Change: 0.00012
Iteration: 37, Log-Lik: -2376.275, Max-Change: 0.00011
Iteration: 38, Log-Lik: -2376.275, Max-Change: 0.00036
Iteration: 39, Log-Lik: -2376.275, Max-Change: 0.00009

Warning: data contains response patterns with only NAs


Iteration: 1, Log-Lik: -866.533, Max-Change: 1.23966
Iteration: 2, Log-Lik: -793.682, Max-Change: 0.48395
Iteration: 3, Log-Lik: -774.644, Max-Change: 0.37382
Iteration: 4, Log-Lik: -764.558, Max-Change: 0.31631
Iteration: 5, Log-Lik: -758.253, Max-Change: 0.29618
Iteration: 6, Log-Lik: -753.946, Max-Change: 0.26431
Iteration: 7, Log-Lik: -743.133, Max-Change: 0.15807
Iteration: 8, Log-Lik: -742.484, Max-Change: 0.09342
Iteration: 9, Log-Lik: -742.031, Max-Change: 0.08349
Iteration: 10, Log-Lik: -740.559, Max-Change: 0.06007
Iteration: 11, Log-Lik: -740.450, Max-Change: 0.04979
Iteration: 12, Log-Lik: -740.364, Max-Change: 0.04585
```

```
Iteration: 13, Log-Lik: -740.051, Max-Change: 0.01748
Iteration: 14, Log-Lik: -740.035, Max-Change: 0.01589
Iteration: 15, Log-Lik: -740.023, Max-Change: 0.01487
Iteration: 16, Log-Lik: -739.978, Max-Change: 0.01004
Iteration: 17, Log-Lik: -739.976, Max-Change: 0.00802
Iteration: 18, Log-Lik: -739.974, Max-Change: 0.00723
Iteration: 19, Log-Lik: -739.969, Max-Change: 0.00750
Iteration: 20, Log-Lik: -739.968, Max-Change: 0.00373
Iteration: 21, Log-Lik: -739.967, Max-Change: 0.00264
Iteration: 22, Log-Lik: -739.967, Max-Change: 0.00255
Iteration: 23, Log-Lik: -739.966, Max-Change: 0.00286
Iteration: 24, Log-Lik: -739.966, Max-Change: 0.00216
Iteration: 25, Log-Lik: -739.966, Max-Change: 0.00115
Iteration: 26, Log-Lik: -739.966, Max-Change: 0.00124
Iteration: 27, Log-Lik: -739.966, Max-Change: 0.00265
Iteration: 28, Log-Lik: -739.965, Max-Change: 0.00307
Iteration: 29, Log-Lik: -739.965, Max-Change: 0.00066
Iteration: 30, Log-Lik: -739.965, Max-Change: 0.00068
Iteration: 31, Log-Lik: -739.965, Max-Change: 0.00133
Iteration: 32, Log-Lik: -739.965, Max-Change: 0.00194
Iteration: 33, Log-Lik: -739.965, Max-Change: 0.00044
Iteration: 34, Log-Lik: -739.965, Max-Change: 0.00040
Iteration: 35, Log-Lik: -739.965, Max-Change: 0.00011
Iteration: 36, Log-Lik: -739.965, Max-Change: 0.00070
Iteration: 37, Log-Lik: -739.965, Max-Change: 0.00011
Iteration: 38, Log-Lik: -739.965, Max-Change: 0.00079
Iteration: 39, Log-Lik: -739.965, Max-Change: 0.00015
Iteration: 40, Log-Lik: -739.965, Max-Change: 0.00012
Iteration: 41, Log-Lik: -739.965, Max-Change: 0.00041
Iteration: 42, Log-Lik: -739.964, Max-Change: 0.00053
Iteration: 43, Log-Lik: -739.964, Max-Change: 0.00118
Iteration: 44, Log-Lik: -739.964, Max-Change: 0.00008

Warning: data contains response patterns with only NAs


Iteration: 1, Log-Lik: -8039.425, Max-Change: 0.88834
Iteration: 2, Log-Lik: -7405.722, Max-Change: 0.64594
Iteration: 3, Log-Lik: -7251.327, Max-Change: 0.32330
Iteration: 4, Log-Lik: -7195.496, Max-Change: 0.30969
Iteration: 5, Log-Lik: -7167.166, Max-Change: 0.19066
Iteration: 6, Log-Lik: -7152.648, Max-Change: 0.12189
```

```
Iteration: 7, Log-Lik: -7144.845, Max-Change: 0.08947
Iteration: 8, Log-Lik: -7139.814, Max-Change: 0.07054
Iteration: 9, Log-Lik: -7136.303, Max-Change: 0.06529
Iteration: 10, Log-Lik: -7131.126, Max-Change: 0.05762
Iteration: 11, Log-Lik: -7129.591, Max-Change: 0.03820
Iteration: 12, Log-Lik: -7128.463, Max-Change: 0.03142
Iteration: 13, Log-Lik: -7126.882, Max-Change: 0.02838
Iteration: 14, Log-Lik: -7126.301, Max-Change: 0.02486
Iteration: 15, Log-Lik: -7125.768, Max-Change: 0.02110
Iteration: 16, Log-Lik: -7123.971, Max-Change: 0.01329
Iteration: 17, Log-Lik: -7123.853, Max-Change: 0.01148
Iteration: 18, Log-Lik: -7123.756, Max-Change: 0.00949
Iteration: 19, Log-Lik: -7123.463, Max-Change: 0.00631
Iteration: 20, Log-Lik: -7123.437, Max-Change: 0.00481
Iteration: 21, Log-Lik: -7123.418, Max-Change: 0.00424
Iteration: 22, Log-Lik: -7123.376, Max-Change: 0.00645
Iteration: 23, Log-Lik: -7123.366, Max-Change: 0.00298
Iteration: 24, Log-Lik: -7123.360, Max-Change: 0.00248
Iteration: 25, Log-Lik: -7123.339, Max-Change: 0.00146
Iteration: 26, Log-Lik: -7123.338, Max-Change: 0.00083
Iteration: 27, Log-Lik: -7123.336, Max-Change: 0.00123
Iteration: 28, Log-Lik: -7123.335, Max-Change: 0.00135
Iteration: 29, Log-Lik: -7123.334, Max-Change: 0.00114
Iteration: 30, Log-Lik: -7123.333, Max-Change: 0.00087
Iteration: 31, Log-Lik: -7123.332, Max-Change: 0.00091
Iteration: 32, Log-Lik: -7123.331, Max-Change: 0.00064
Iteration: 33, Log-Lik: -7123.331, Max-Change: 0.00024
Iteration: 34, Log-Lik: -7123.331, Max-Change: 0.00019
Iteration: 35, Log-Lik: -7123.331, Max-Change: 0.00012
Iteration: 36, Log-Lik: -7123.331, Max-Change: 0.00060
Iteration: 37, Log-Lik: -7123.330, Max-Change: 0.00060
Iteration: 38, Log-Lik: -7123.330, Max-Change: 0.00013
Iteration: 39, Log-Lik: -7123.330, Max-Change: 0.00044
Iteration: 40, Log-Lik: -7123.330, Max-Change: 0.00050
Iteration: 41, Log-Lik: -7123.330, Max-Change: 0.00023
Iteration: 42, Log-Lik: -7123.330, Max-Change: 0.00045
Iteration: 43, Log-Lik: -7123.330, Max-Change: 0.00026
Iteration: 44, Log-Lik: -7123.330, Max-Change: 0.00039
Iteration: 45, Log-Lik: -7123.330, Max-Change: 0.00017
Iteration: 46, Log-Lik: -7123.330, Max-Change: 0.00014
Iteration: 47, Log-Lik: -7123.330, Max-Change: 0.00034
Iteration: 48, Log-Lik: -7123.330, Max-Change: 0.00010
```

```
Iteration: 1, Log-Lik: -2752.068, Max-Change: 1.00488
Iteration: 2, Log-Lik: -2474.889, Max-Change: 0.48329
Iteration: 3, Log-Lik: -2397.638, Max-Change: 0.34021
Iteration: 4, Log-Lik: -2359.032, Max-Change: 0.26210
Iteration: 5, Log-Lik: -2336.196, Max-Change: 0.24034
Iteration: 6, Log-Lik: -2321.843, Max-Change: 0.23482
Iteration: 7, Log-Lik: -2311.924, Max-Change: 0.19653
Iteration: 8, Log-Lik: -2304.379, Max-Change: 0.18105
Iteration: 9, Log-Lik: -2298.886, Max-Change: 0.16132
Iteration: 10, Log-Lik: -2294.630, Max-Change: 0.14412
Iteration: 11, Log-Lik: -2291.261, Max-Change: 0.12338
Iteration: 12, Log-Lik: -2288.576, Max-Change: 0.11155
Iteration: 13, Log-Lik: -2278.957, Max-Change: 0.04534
Iteration: 14, Log-Lik: -2278.516, Max-Change: 0.04511
Iteration: 15, Log-Lik: -2278.145, Max-Change: 0.04019
Iteration: 16, Log-Lik: -2276.731, Max-Change: 0.02166
Iteration: 17, Log-Lik: -2276.661, Max-Change: 0.01864
Iteration: 18, Log-Lik: -2276.602, Max-Change: 0.01639
Iteration: 19, Log-Lik: -2276.376, Max-Change: 0.00847
Iteration: 20, Log-Lik: -2276.364, Max-Change: 0.00721
Iteration: 21, Log-Lik: -2276.354, Max-Change: 0.00619
Iteration: 22, Log-Lik: -2276.317, Max-Change: 0.00438
Iteration: 23, Log-Lik: -2276.315, Max-Change: 0.00320
Iteration: 24, Log-Lik: -2276.313, Max-Change: 0.00356
Iteration: 25, Log-Lik: -2276.310, Max-Change: 0.00266
Iteration: 26, Log-Lik: -2276.309, Max-Change: 0.00142
Iteration: 27, Log-Lik: -2276.308, Max-Change: 0.00153
Iteration: 28, Log-Lik: -2276.308, Max-Change: 0.00136
Iteration: 29, Log-Lik: -2276.307, Max-Change: 0.00171
Iteration: 30, Log-Lik: -2276.307, Max-Change: 0.00223
Iteration: 31, Log-Lik: -2276.306, Max-Change: 0.00193
Iteration: 32, Log-Lik: -2276.306, Max-Change: 0.00066
Iteration: 33, Log-Lik: -2276.306, Max-Change: 0.00053
Iteration: 34, Log-Lik: -2276.305, Max-Change: 0.00063
Iteration: 35, Log-Lik: -2276.305, Max-Change: 0.00087
Iteration: 36, Log-Lik: -2276.305, Max-Change: 0.00048
Iteration: 37, Log-Lik: -2276.305, Max-Change: 0.00020
Iteration: 38, Log-Lik: -2276.305, Max-Change: 0.00020
Iteration: 39, Log-Lik: -2276.305, Max-Change: 0.00019
Iteration: 40, Log-Lik: -2276.305, Max-Change: 0.00019
Iteration: 41, Log-Lik: -2276.305, Max-Change: 0.00018
Iteration: 42, Log-Lik: -2276.305, Max-Change: 0.00018
Iteration: 43, Log-Lik: -2276.305, Max-Change: 0.00018
```

```
Iteration: 44, Log-Lik: -2276.305, Max-Change: 0.00016
Iteration: 45, Log-Lik: -2276.305, Max-Change: 0.00016
Iteration: 46, Log-Lik: -2276.305, Max-Change: 0.00017
Iteration: 47, Log-Lik: -2276.305, Max-Change: 0.00018
Iteration: 48, Log-Lik: -2276.305, Max-Change: 0.00020
Iteration: 49, Log-Lik: -2276.305, Max-Change: 0.00020
Iteration: 50, Log-Lik: -2276.305, Max-Change: 0.00021
Iteration: 51, Log-Lik: -2276.305, Max-Change: 0.00028
Iteration: 52, Log-Lik: -2276.305, Max-Change: 0.00019
Iteration: 53, Log-Lik: -2276.305, Max-Change: 0.00024
Iteration: 54, Log-Lik: -2276.305, Max-Change: 0.00024
Iteration: 55, Log-Lik: -2276.305, Max-Change: 0.00020
Iteration: 56, Log-Lik: -2276.305, Max-Change: 0.00020
Iteration: 57, Log-Lik: -2276.305, Max-Change: 0.00028
Iteration: 58, Log-Lik: -2276.305, Max-Change: 0.00017
Iteration: 59, Log-Lik: -2276.305, Max-Change: 0.00023
Iteration: 60, Log-Lik: -2276.305, Max-Change: 0.00021
Iteration: 61, Log-Lik: -2276.305, Max-Change: 0.00018
Iteration: 62, Log-Lik: -2276.305, Max-Change: 0.00018
Iteration: 63, Log-Lik: -2276.305, Max-Change: 0.00025
Iteration: 64, Log-Lik: -2276.305, Max-Change: 0.00016
Iteration: 65, Log-Lik: -2276.305, Max-Change: 0.00021
Iteration: 66, Log-Lik: -2276.305, Max-Change: 0.00019
Iteration: 67, Log-Lik: -2276.305, Max-Change: 0.00017
Iteration: 68, Log-Lik: -2276.305, Max-Change: 0.00017
Iteration: 69, Log-Lik: -2276.305, Max-Change: 0.00023
Iteration: 70, Log-Lik: -2276.305, Max-Change: 0.00014
Iteration: 71, Log-Lik: -2276.305, Max-Change: 0.00019
Iteration: 72, Log-Lik: -2276.305, Max-Change: 0.00018
Iteration: 73, Log-Lik: -2276.305, Max-Change: 0.00015
Iteration: 74, Log-Lik: -2276.305, Max-Change: 0.00015
Iteration: 75, Log-Lik: -2276.305, Max-Change: 0.00021
Iteration: 76, Log-Lik: -2276.305, Max-Change: 0.00013
Iteration: 77, Log-Lik: -2276.305, Max-Change: 0.00017
Iteration: 78, Log-Lik: -2276.305, Max-Change: 0.00016
Iteration: 79, Log-Lik: -2276.305, Max-Change: 0.00014
Iteration: 80, Log-Lik: -2276.305, Max-Change: 0.00014
Iteration: 81, Log-Lik: -2276.305, Max-Change: 0.00020
Iteration: 82, Log-Lik: -2276.305, Max-Change: 0.00012
Iteration: 83, Log-Lik: -2276.305, Max-Change: 0.00016
Iteration: 84, Log-Lik: -2276.305, Max-Change: 0.00015
Iteration: 85, Log-Lik: -2276.305, Max-Change: 0.00013
Iteration: 86, Log-Lik: -2276.305, Max-Change: 0.00013
```

```
Iteration: 87, Log-Lik: -2276.305, Max-Change: 0.00018
Iteration: 88, Log-Lik: -2276.305, Max-Change: 0.00011
Iteration: 89, Log-Lik: -2276.305, Max-Change: 0.00015
Iteration: 90, Log-Lik: -2276.305, Max-Change: 0.00013
Iteration: 91, Log-Lik: -2276.305, Max-Change: 0.00012
Iteration: 92, Log-Lik: -2276.305, Max-Change: 0.00011
Iteration: 93, Log-Lik: -2276.305, Max-Change: 0.00017
Iteration: 94, Log-Lik: -2276.305, Max-Change: 0.00010

Warning: data contains response patterns with only NAs


Iteration: 1, Log-Lik: -699.738, Max-Change: 0.87083
Iteration: 2, Log-Lik: -648.553, Max-Change: 0.59220
Iteration: 3, Log-Lik: -635.400, Max-Change: 0.45063
Iteration: 4, Log-Lik: -629.899, Max-Change: 0.34148
Iteration: 5, Log-Lik: -627.161, Max-Change: 0.24548
Iteration: 6, Log-Lik: -625.633, Max-Change: 0.18007
Iteration: 7, Log-Lik: -623.326, Max-Change: 0.09093
Iteration: 8, Log-Lik: -623.054, Max-Change: 0.07756
Iteration: 9, Log-Lik: -622.829, Max-Change: 0.06992
Iteration: 10, Log-Lik: -621.889, Max-Change: 0.03803
Iteration: 11, Log-Lik: -621.822, Max-Change: 0.03216
Iteration: 12, Log-Lik: -621.765, Max-Change: 0.03277
Iteration: 13, Log-Lik: -621.518, Max-Change: 0.01871
Iteration: 14, Log-Lik: -621.499, Max-Change: 0.01904
Iteration: 15, Log-Lik: -621.483, Max-Change: 0.01717
Iteration: 16, Log-Lik: -621.429, Max-Change: 0.01341
Iteration: 17, Log-Lik: -621.421, Max-Change: 0.00826
Iteration: 18, Log-Lik: -621.415, Max-Change: 0.00868
Iteration: 19, Log-Lik: -621.394, Max-Change: 0.00978
Iteration: 20, Log-Lik: -621.392, Max-Change: 0.00619
Iteration: 21, Log-Lik: -621.391, Max-Change: 0.00526
Iteration: 22, Log-Lik: -621.388, Max-Change: 0.00662
Iteration: 23, Log-Lik: -621.387, Max-Change: 0.00375
Iteration: 24, Log-Lik: -621.387, Max-Change: 0.00404
Iteration: 25, Log-Lik: -621.384, Max-Change: 0.00349
Iteration: 26, Log-Lik: -621.384, Max-Change: 0.00269
Iteration: 27, Log-Lik: -621.384, Max-Change: 0.00264
Iteration: 28, Log-Lik: -621.382, Max-Change: 0.00157
Iteration: 29, Log-Lik: -621.382, Max-Change: 0.00138
Iteration: 30, Log-Lik: -621.382, Max-Change: 0.00141
```

```
Iteration: 31, Log-Lik: -621.381, Max-Change: 0.00088
Iteration: 32, Log-Lik: -621.381, Max-Change: 0.00087
Iteration: 33, Log-Lik: -621.381, Max-Change: 0.00065
Iteration: 34, Log-Lik: -621.381, Max-Change: 0.00080
Iteration: 35, Log-Lik: -621.381, Max-Change: 0.00031
Iteration: 36, Log-Lik: -621.381, Max-Change: 0.00034
Iteration: 37, Log-Lik: -621.381, Max-Change: 0.00032
Iteration: 38, Log-Lik: -621.381, Max-Change: 0.00031
Iteration: 39, Log-Lik: -621.381, Max-Change: 0.00030
Iteration: 40, Log-Lik: -621.381, Max-Change: 0.00029
Iteration: 41, Log-Lik: -621.381, Max-Change: 0.00027
Iteration: 42, Log-Lik: -621.381, Max-Change: 0.00026
Iteration: 43, Log-Lik: -621.381, Max-Change: 0.00025
Iteration: 44, Log-Lik: -621.381, Max-Change: 0.00025
Iteration: 45, Log-Lik: -621.381, Max-Change: 0.00023
Iteration: 46, Log-Lik: -621.381, Max-Change: 0.00023
Iteration: 47, Log-Lik: -621.381, Max-Change: 0.00022
Iteration: 48, Log-Lik: -621.381, Max-Change: 0.00022
Iteration: 49, Log-Lik: -621.381, Max-Change: 0.00020
Iteration: 50, Log-Lik: -621.381, Max-Change: 0.00020
Iteration: 51, Log-Lik: -621.381, Max-Change: 0.00019
Iteration: 52, Log-Lik: -621.381, Max-Change: 0.00019
Iteration: 53, Log-Lik: -621.381, Max-Change: 0.00018
Iteration: 54, Log-Lik: -621.381, Max-Change: 0.00018
Iteration: 55, Log-Lik: -621.381, Max-Change: 0.00017
Iteration: 56, Log-Lik: -621.381, Max-Change: 0.00017
Iteration: 57, Log-Lik: -621.381, Max-Change: 0.00016
Iteration: 58, Log-Lik: -621.381, Max-Change: 0.00016
Iteration: 59, Log-Lik: -621.381, Max-Change: 0.00015
Iteration: 60, Log-Lik: -621.381, Max-Change: 0.00015
Iteration: 61, Log-Lik: -621.381, Max-Change: 0.00015
Iteration: 62, Log-Lik: -621.381, Max-Change: 0.00015
Iteration: 63, Log-Lik: -621.381, Max-Change: 0.00014
Iteration: 64, Log-Lik: -621.381, Max-Change: 0.00014
Iteration: 65, Log-Lik: -621.381, Max-Change: 0.00013
Iteration: 66, Log-Lik: -621.381, Max-Change: 0.00013
Iteration: 67, Log-Lik: -621.381, Max-Change: 0.00013
Iteration: 68, Log-Lik: -621.381, Max-Change: 0.00013
Iteration: 69, Log-Lik: -621.381, Max-Change: 0.00012
Iteration: 70, Log-Lik: -621.381, Max-Change: 0.00012
Iteration: 71, Log-Lik: -621.381, Max-Change: 0.00012
Iteration: 72, Log-Lik: -621.381, Max-Change: 0.00011
Iteration: 73, Log-Lik: -621.381, Max-Change: 0.00011
```

```
Iteration: 74, Log-Lik: -621.381, Max-Change: 0.00011
Iteration: 75, Log-Lik: -621.381, Max-Change: 0.00011
Iteration: 76, Log-Lik: -621.381, Max-Change: 0.00010
Iteration: 77, Log-Lik: -621.381, Max-Change: 0.00010
Iteration: 78, Log-Lik: -621.381, Max-Change: 0.00010
Iteration: 79, Log-Lik: -621.381, Max-Change: 0.00010
```

Reliability

```
# Average reliability
rel_avg <- var(fsg[, 1], na.rm = TRUE) /
    (var(fsg[, 1], na.rm = TRUE) + mean(fsg[, 2]^2, na.rm = TRUE))
rel_avg
```

```
[1] 0.8364975
```

```
# Group-specific reliability
relg <- tapply(fsg, INDEX = fsg$group,
    FUN = \(x) var(x[, 1], na.rm = TRUE) /
        (var(x[, 1], na.rm = TRUE) + mean(x[, 2]^2, na.rm = TRUE)))
relg
```

```
     FAST      LIFT     PIRC1     PIRC2      SAFE
0.7439284 0.8468285 0.8158746 0.7938676 0.8416167
```

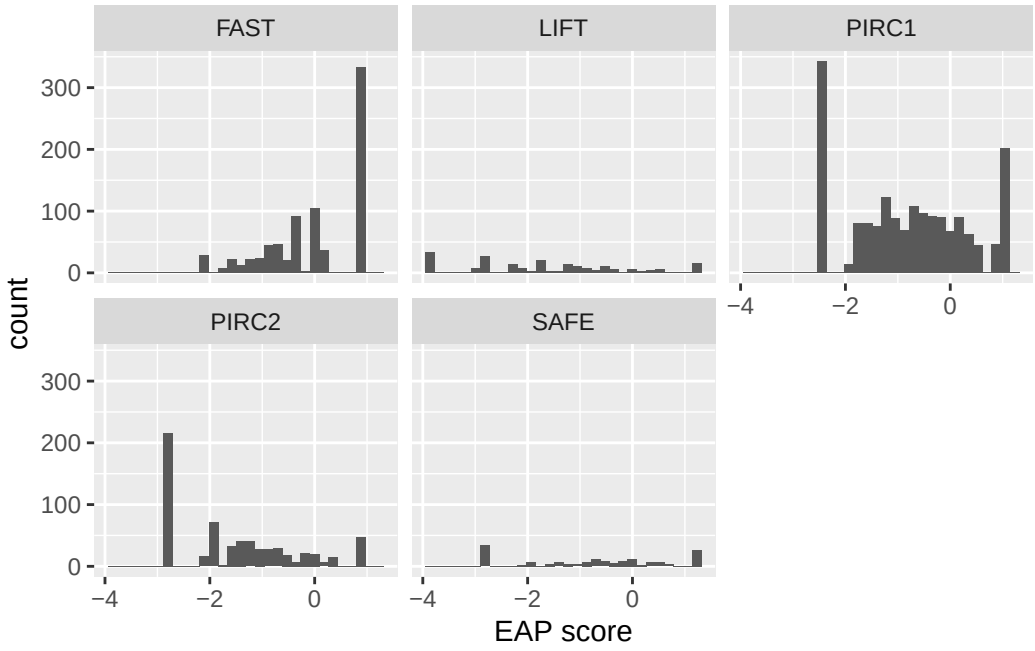
Analyses of Harmonized Scores

Descriptives

```
ggplot(fsg, aes(x = f)) +
    geom_histogram() +
    facet_wrap(~ group) +
    labs(x = "EAP score")
```

```
`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

```
Warning: Removed 386 rows containing non-finite values (`stat_bin()`).
```



Factor score regression (no reliability adjustment)

Using EAP scores to predict high school completion with probit regression

```
# Merged with other variables
# Warning: the code requires data to be sorted by study_id before analyses
data <- cbind(data, fsg)
# GLM (probit regression); interaction not significant
m_int <- glm(hs ~ f * study_id, data = data, family = binomial("probit"))
anova(m_int)
```

```
Analysis of Deviance Table

Model: binomial, link: probit

Response: hs

Terms added sequentially (first to last)

           Df Deviance Resid. Df Resid. Dev Pr(>Chi)
NULL                        3561     3809.5
f           1    95.93      3560     3713.5   <2e-16 ***
```

```
study_id    4   363.52      3556     3350.0   <2e-16 ***
f:study_id  4     3.06      3552     3347.0   0.5471
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
m_main <- glm(hs ~ f + study_id, data = data, family = binomial("probit"))
anova(m_main)
```

```
Analysis of Deviance Table

Model: binomial, link: probit

Response: hs

Terms added sequentially (first to last)

         Df Deviance Resid. Df Resid. Dev  Pr(>Chi)
NULL                      3561     3809.5
f         1    95.93      3560     3713.5 < 2.2e-16 ***
study_id  4   363.52      3556     3350.0 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
summary(m_main)
```

```
Call:
glm(formula = hs ~ f + study_id, family = binomial("probit"),
    data = data)

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept)   0.60831    0.04764  12.768  < 2e-16 ***
f            -0.15622    0.02250  -6.942 3.87e-12 ***
study_idLIFT  0.23214    0.12593   1.843   0.0653 .
study_idPIRC1 0.07322    0.06059   1.208   0.2269
study_idPIRC2 0.62058    0.09386   6.612 3.80e-11 ***
study_idSAFE -1.81915    0.13649 -13.328  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 3809.5  on 3561  degrees of freedom
Residual deviance: 3350.0  on 3556  degrees of freedom
  (386 observations deleted due to missingness)
AIC: 3362

Number of Fisher Scoring iterations: 5
```

Using `umx` (without unreliability adjustment)

```r
# Recode discrete variable to be used with OpenMx
data$hs2 <- mxFactor(data$hs, levels = c(0, 1))
# Create dummy variables for Study ID
data <- cbind(data, model.matrix(~ study_id, data = data)[, -1])
# Pooled analysis without adjusting for unreliability
# TOCA-R -> high school completion
probitreg_umx <- umxRAM("
  hs2 ~ f + study_idLIFT + study_idPIRC1 + study_idPIRC2 + study_idSAFE
  hs2 ~ 0 * 1
  hs2 ~~ 1 * hs2
",
    data = data,
    tryHard = "ordinal"  # try hard for convergence
)
```

```
Polite note: Variance of variable(s) NULL is < 0.1.
You might want to express the variable in smaller units, e.g. multiply to use cm instead of m
Alternatively umx_scale() for data already in long-format, or umx_scale_wide_twin_data for wi

object 'threshMat' created to handle: 1 binary variables:'hs2'

1 trait(s) are binary: 'hs2'
For these, you you MUST fix the mean and variance of the latent traits driving each variable
See ?umxThresholdMatrix

Using deviation-based model: Thresholds will be in'threshMat' based on deviations in 'deviati

Running m1 with 26 parameters
```

```
Beginning initial fit attempt

Running m1 with 26 parameters

 Lowest minimum so far:  21783.2747623497

Beginning fit attempt 1 of at maximum 10 extra tries

Running m1 with 26 parameters

 Fit attempt generated errors

Beginning fit attempt 2 of at maximum 10 extra tries

Running m1 with 26 parameters

 Fit attempt generated errors

Beginning fit attempt 3 of at maximum 10 extra tries

Running m1 with 26 parameters

 Fit attempt generated errors

Beginning fit attempt 4 of at maximum 10 extra tries

Running m1 with 26 parameters

 Fit attempt generated errors
```

```
Beginning fit attempt 5 of at maximum 10 extra tries

Running m1 with 26 parameters

 Fit attempt generated errors

Beginning fit attempt 6 of at maximum 10 extra tries

Running m1 with 26 parameters

 Fit attempt generated errors

Beginning fit attempt 7 of at maximum 10 extra tries

Running m1 with 26 parameters

 Fit attempt generated errors

Beginning fit attempt 8 of at maximum 10 extra tries

Running m1 with 26 parameters

 Fit attempt generated errors

Beginning fit attempt 9 of at maximum 10 extra tries

Running m1 with 26 parameters

 Fit attempt generated errors
```

```
Beginning fit attempt 10 of at maximum 10 extra tries

Running m1 with 26 parameters


 Fit attempt generated errors


Retry limit reached


Solution found

Final run, for Hessian and/or standard errors and/or confidence intervals

Running m1 with 26 parameters


 Solution found!  Final fit=21783.275 (started at 27940.692)  (11 attempt(s): 1 valid, 10 er

 Start values from best fit:

-0.151829700462419,0.312103252193123,0.12366338099553,0.597197741937646,-1.59082714570869,1.5

?umxSummary options: std=T|F', digits=, report= 'html', filter= 'NS' & more

[1] "Warning calling mxRefModels: mxRefModels can't handle all designs, including twin, and W


Table: Parameter loadings for model 'm1'

|   |name                              | Estimate|SE    |type          |
|:--|:---------------------------------|--------:|:-----|:-------------|
|7  |f_with_study_idLIFT               |   -0.098|0.009 |Manifest Cov  |
|8  |f_with_study_idPIRC1              |    0.032|0.011 |Manifest Cov  |
|9  |f_with_study_idPIRC2              |   -0.116|0.008 |Manifest Cov  |
|10 |f_with_study_idSAFE               |    0.004|0.004 |Manifest Cov  |
|13 |study_idLIFT_with_study_idPIRC1   |   -0.055|0.003 |Manifest Cov  |
```

```
|14 |study_idLIFT_with_study_idPIRC2  |   -0.018|0.002 |Manifest Cov  |
|15 |study_idLIFT_with_study_idSAFE   |   -0.005|0.001 |Manifest Cov  |
|17 |study_idPIRC1_with_study_idPIRC2 |   -0.077|0.003 |Manifest Cov  |
|18 |study_idPIRC1_with_study_idSAFE  |   -0.019|0.002 |Manifest Cov  |
|20 |study_idPIRC2_with_study_idSAFE  |   -0.006|0.001 |Manifest Cov  |
|1  |f_to_hs2                         |   -0.152|0.022 |Manifest path |
|2  |study_idLIFT_to_hs2              |    0.312|0.096 |Manifest path |
|3  |study_idPIRC1_to_hs2             |    0.124|0.064 |Manifest path |
|4  |study_idPIRC2_to_hs2             |    0.597|0.088 |Manifest path |
|5  |study_idSAFE_to_hs2              |   -1.591|0.115 |Manifest path |
|22 |one_to_f                         |   -0.853|0.021 |Mean          |
|23 |one_to_study_idLIFT              |    0.114|0.005 |Mean          |
|24 |one_to_study_idPIRC1             |    0.477|0.008 |Mean          |
|25 |one_to_study_idPIRC2             |    0.161|0.006 |Mean          |
|26 |one_to_study_idSAFE              |    0.041|0.003 |Mean          |
|6  |f_with_f                         |    1.575|0.038 |Residual      |
|11 |hs2_with_hs2                     |    1.000|0     |Residual      |
|12 |study_idLIFT_with_study_idLIFT   |    0.101|0.002 |Residual      |
|16 |study_idPIRC1_with_study_idPIRC1 |    0.249|0.006 |Residual      |
|19 |study_idPIRC2_with_study_idPIRC2 |    0.136|0.003 |Residual      |
|21 |study_idSAFE_with_study_idSAFE   |    0.038|0.001 |Residual      |
```

```
Model Fit:  ²(1) = NA, p = NA; CFI = NA; TLI = NA; RMSEA = NA
```

```
Algebra'threshMat' = -0.589CI95[-0.686, -0.492]. p-value < 0.001
```

```
# plot(probitreg_umx)  # path diagram
```

Two-Stage Path Analysis

Adjusting for measurement error

```
# Define loading and error variances
# Loading = Latent variance - SE^2
data$rel_f <- data$sd_eta^2 - data$SE_f^2
# Error variance = SE^2 * reliability
data$ev_f <- data$SE_f^2 * data$rel_f
# Drop rows with missing rel_f
data2 <- data[!is.na(data$rel_f), ]
# Incorporate measurement model for factor scores
```

```r
probitreg_2spa <- umxRAM(
    "2spa",
    # Main effects of EAP scores and study indicators
    umxPath(c("eta", "study_idLIFT", "study_idPIRC1",
              "study_idPIRC2", "study_idSAFE"),
            to = "hs2"),
    # Loading = reliability (as definition variable)
    umxPath("eta", to = "f", labels = "data.rel_f", free = FALSE),
    # Error variance (as definition variable)
    umxPath(var = "f", labels = "data.ev_f", free = FALSE),
    # Covariances of predictors
    umxPath(unique.pairs = c("eta", "study_idLIFT", "study_idPIRC1",
                             "study_idPIRC2", "study_idSAFE")),
    # Means of predictors
    umxPath(means = c("eta", "study_idLIFT", "study_idPIRC1",
                      "study_idPIRC2", "study_idSAFE")),
    # For model identification: Fix latent variate
    # to be standard normal
    umxPath(v1m0 = "hs2"),
    data = data2,
    tryHard = "ordinal"
)
```

```
A latent variable 'eta' was created.

Polite note: Variance of variable(s) NULL is < 0.1.
You might want to express the variable in smaller units, e.g. multiply to use cm instead of m
Alternatively umx_scale() for data already in long-format, or umx_scale_wide_twin_data for wi

object 'threshMat' created to handle: 1 binary variables:'hs2'

1 trait(s) are binary: 'hs2'
For these, you you MUST fix the mean and variance of the latent traits driving each variable
See ?umxThresholdMatrix

Using deviation-based model: Thresholds will be in'threshMat' based on deviations in 'deviati

Running 2spa with 26 parameters


Beginning initial fit attempt
```

```
Running 2spa with 26 parameters

 Lowest minimum so far:  19797.1146080977

Beginning fit attempt 1 of at maximum 10 extra tries

Running 2spa with 26 parameters

 Fit attempt generated errors

Beginning fit attempt 2 of at maximum 10 extra tries

Running 2spa with 26 parameters

 Fit attempt generated errors

Beginning fit attempt 3 of at maximum 10 extra tries

Running 2spa with 26 parameters

 Fit attempt generated errors

Beginning fit attempt 4 of at maximum 10 extra tries

Running 2spa with 26 parameters

 Fit attempt generated errors

Beginning fit attempt 5 of at maximum 10 extra tries
```

```
Running 2spa with 26 parameters

 Fit attempt generated errors

Beginning fit attempt 6 of at maximum 10 extra tries

Running 2spa with 26 parameters

 Fit attempt generated errors

Beginning fit attempt 7 of at maximum 10 extra tries

Running 2spa with 26 parameters

 Fit attempt generated errors

Beginning fit attempt 8 of at maximum 10 extra tries

Running 2spa with 26 parameters

 Fit attempt generated errors

Beginning fit attempt 9 of at maximum 10 extra tries

Running 2spa with 26 parameters

 Fit attempt generated errors

Beginning fit attempt 10 of at maximum 10 extra tries
```

```
Running 2spa with 26 parameters


 Fit attempt generated errors


Retry limit reached


Solution found

Final run, for Hessian and/or standard errors and/or confidence intervals

Running 2spa with 26 parameters


 Solution found!  Final fit=19797.115 (started at 24817.453)  (11 attempt(s): 1 valid, 10 er

 Start values from best fit:

0.31777416400859,0.0548483164817531,0.617036120917892,-1.77660797926986,-0.260545169656601,0

?umxSummary options: std=T|F', digits=, report= 'html', filter= 'NS' & more

[1] "Warning calling mxRefModels: mxRefModels can't handle all designs, including twin, and

Warning in mxStandardizeRAMpaths(model, SE = SE): 'model' (or one of its
submodels) contains definition variables; interpret results of
mxStandardizeRAMpaths() cautiously
```

Table: Parameter loadings for model '2spa'

	name	Estimate	SE	type
5	data.rel_f	0.782	0	Factor loading
6	eta_to_hs2	-0.261	0.04	Factor loading
23	eta_with_eta	0.682	0.027	Factor Variance

```
|13 |eta_with_study_idLIFT            |   -0.008|0.004 |Latent-Manifest Cov |
|17 |eta_with_study_idPIRC1           |   -0.023|0.008 |Latent-Manifest Cov |
|20 |eta_with_study_idPIRC2           |   -0.084|0.006 |Latent-Manifest Cov |
|22 |eta_with_study_idSAFE            |    0.008|0.003 |Latent-Manifest Cov |
|10 |study_idLIFT_with_study_idPIRC1  |   -0.028|0.002 |Manifest Cov        |
|11 |study_idLIFT_with_study_idPIRC2  |   -0.010|0.001 |Manifest Cov        |
|12 |study_idLIFT_with_study_idSAFE   |   -0.002|0.001 |Manifest Cov        |
|15 |study_idPIRC1_with_study_idPIRC2 |   -0.089|0.004 |Manifest Cov        |
|16 |study_idPIRC1_with_study_idSAFE  |   -0.022|0.002 |Manifest Cov        |
|19 |study_idPIRC2_with_study_idSAFE  |   -0.008|0.001 |Manifest Cov        |
|1  |study_idLIFT_to_hs2              |    0.318|0.124 |Manifest path       |
|2  |study_idPIRC1_to_hs2             |    0.055|0.062 |Manifest path       |
|3  |study_idPIRC2_to_hs2             |    0.617|0.095 |Manifest path       |
|4  |study_idSAFE_to_hs2              |   -1.777|0.135 |Manifest path       |
|24 |one_to_study_idLIFT              |    0.056|0.004 |Mean                |
|25 |one_to_study_idPIRC1             |    0.497|0.008 |Mean                |
|26 |one_to_study_idPIRC2             |    0.179|0.006 |Mean                |
|27 |one_to_study_idSAFE              |    0.044|0.003 |Mean                |
|28 |one_to_eta                       |   -0.563|0.016 |Mean                |
|7  |data.ev_f                        |    0.170|0     |Residual            |
|8  |hs2_with_hs2                     |    1.000|0     |Residual            |
|9  |study_idLIFT_with_study_idLIFT   |    0.053|0.001 |Residual            |
|14 |study_idPIRC1_with_study_idPIRC1 |    0.250|0.006 |Residual            |
|18 |study_idPIRC2_with_study_idPIRC2 |    0.147|0.003 |Residual            |
|21 |study_idSAFE_with_study_idSAFE   |    0.042|0.001 |Residual            |
```

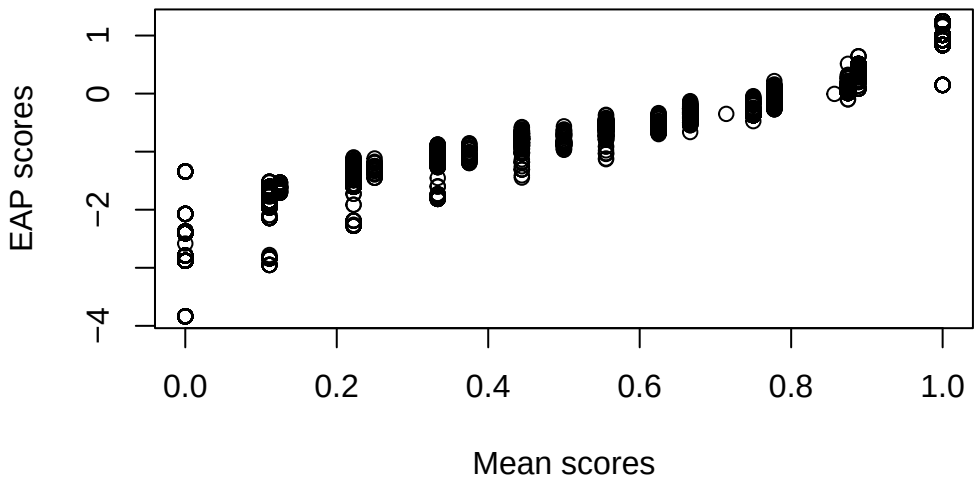
```
Model Fit:  ²(1) = NA, p = NA; CFI = NA; TLI = NA; RMSEA = NA
```

```
Algebra'threshMat' = -0.594CI95[-0.688, -0.499]. p-value < 0.001
```

Sensitivity Analysis

Using sum/mean scores

```r
data$toca_mean <- rowMeans(data[item_names], na.rm = TRUE)
# Scatterplot
plot(x = data$toca_mean, y = fsg[, 1], xlab = "Mean scores", ylab = "EAP scores")
```




```
cor(data$toca_mean, fsg[, 1], use = "complete")
```

```
[1] 0.9655881
```

Probit regression with mean scores; note that the scale of the predictors are different, so the coefficients are not comparable.

```
# GLM (probit regression); interaction not significant
m_int_meanscore <- glm(hs ~ toca_mean * study_id, data = data, family = binomial("probit"))
anova(m_int_meanscore)
```

```
Analysis of Deviance Table

Model: binomial, link: probit

Response: hs

Terms added sequentially (first to last)

                  Df Deviance Resid. Df Resid. Dev Pr(>Chi)
NULL                                3561     3809.5
```

```
toca_mean           1    83.12      3560     3726.4   <2e-16 ***
study_id            4   370.75      3556     3355.6   <2e-16 ***
toca_mean:study_id  4     2.28      3552     3353.3   0.6842
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
m_main_meanscore <- glm(hs ~ toca_mean + study_id, data = data, family = binomial("probit"))
anova(m_main_meanscore)
```

```
Analysis of Deviance Table

Model: binomial, link: probit

Response: hs

Terms added sequentially (first to last)

          Df Deviance Resid. Df Resid. Dev  Pr(>Chi)
NULL                       3561     3809.5
toca_mean  1    83.12      3560     3726.4 < 2.2e-16 ***
study_id   4   370.75      3556     3355.6 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(m_main_meanscore)
```

```
Call:
glm(formula = hs ~ toca_mean + study_id, family = binomial("probit"),
    data = data)

Coefficients:
              Estimate Std. Error z value Pr(>|z|)
(Intercept)    0.97192    0.07425  13.090  < 2e-16 ***
toca_mean     -0.47136    0.07211  -6.537 6.29e-11 ***
study_idLIFT   0.30165    0.12372   2.438   0.0148 *
study_idPIRC1  0.04913    0.06200   0.792   0.4281
study_idPIRC2  0.63601    0.09383   6.778 1.22e-11 ***
study_idSAFE  -1.81877    0.13620 -13.354  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 3809.5  on 3561  degrees of freedom
Residual deviance: 3355.6  on 3556  degrees of freedom
  (386 observations deleted due to missingness)
AIC: 3367.6

Number of Fisher Scoring iterations: 5
```