

Alignment Optimization

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Load Packages

```
library(mirt)          # for IRT analyses
```

Loading required package: stats4

Loading required package: lattice

```
# install.packages("remotes")
# remotes::install_github("marklhc/pinsearch")
library(pinsearch)    # for noninvariance effect size
library(ggplot2)      # for plotting
library(umx)          # for analyzing harmonized scores
```

Loading required package: OpenMx

To take full advantage of multiple cores, use:

```
mxOption(key='Number of Threads', value=parallel::detectCores()) #now
Sys.setenv(OMP_NUM_THREADS=parallel::detectCores()) #before library(OpenMx)
```

For an overview type '?umx'

Attaching package: 'umx'

The following object is masked from 'package:stats':

loadings

Import Data

Data can be downloaded from <https://github.com/jmk7cj/SEM-mnlfa>.

```
data <- read.csv(here::here("Kush_etal_example/SEM-mnlfa", "data.csv"))
# Define grouping variable
data$group <- data$study_id
# Sort data by group
data <- data[order(data$group), ]
head(data)
```

	id	study_id	sex	race	T_TOCA_breakrule	T_TOCA_harmpro	T_TOCA_breakthg
1	FT001	FAST	male	black	0	NA	0
2	FT002	FAST	male	white	1	NA	0
3	FT003	FAST	female	white	0	NA	0
4	FT004	FAST	female	white	1	NA	0
5	FT005	FAST	male	black	1	NA	0
6	FT006	FAST	male	black	1	NA	1
	T_TOCA_takeprop	T_TOCA_fight	T_TOCA_lies	T_TOCA_yell	T_TOCA_stub	T_TOCA_teas	
1	0	0	0	0	0	0	
2	1	1	0	1	1	1	
3	0	0	0	1	0	0	
4	0	1	1	0	1	0	
5	0	1	0	1	1	1	
6	1	1	1	1	1	1	

```

      hs group
1    1  FAST
2    1  FAST
3    1  FAST
4    0  FAST
5    1  FAST
6    1  FAST

```

```

# Sample sizes
# Study 1 = FAST; Study 2 = LIFT; Study 3 = PIRC1;
# Study 4 = PIRC2; Study 5 = SAFE
table(data$study_id)

```

```

FAST  LIFT PIRC1 PIRC2  SAFE
817   451  1884   639   157

```

```

item_names <- names(data)[5:13] # save item names

```

Configural Model

Run separate mirts (2PL)

```

groups <- unique(data$group)
# Initialize output
# a. number of observations per group and item
item_n <- matrix(NA,
  nrow = length(groups),
  ncol = length(item_names),
  dimnames = list(groups, item_names))
# b. Matrix of loadings and intercepts
lambda <- nu <- item_n
# c. Loop over groups, fitting a 2PL model to each
for (g in seq_along(groups)) {
  datg <- data[data$group == groups[g], item_names]
  # Drop completely missing items
  item_n[g, ] <- colSums(!is.na(datg))
  itemsg <- which(colSums(!is.na(datg)) > 0)
  fitg <- mirt(datg[names(itemsg)], model = 1,
    itemtype = "2PL")
}

```

```

coefg <- coef(fitg, simplify = TRUE)
lambda[g, names(itemsg)] <- coefg$items[, "a1"]
nu[g, names(itemsg)] <- coefg$items[, "d"]
}

```

Warning: data contains response patterns with only NAs

```

Iteration: 1, Log-Lik: -2701.839, Max-Change: 1.34190
Iteration: 2, Log-Lik: -2463.453, Max-Change: 0.72218
Iteration: 3, Log-Lik: -2412.010, Max-Change: 0.43110
Iteration: 4, Log-Lik: -2392.083, Max-Change: 0.30417
Iteration: 5, Log-Lik: -2383.941, Max-Change: 0.20276
Iteration: 6, Log-Lik: -2380.398, Max-Change: 0.13371
Iteration: 7, Log-Lik: -2377.699, Max-Change: 0.05533
Iteration: 8, Log-Lik: -2377.254, Max-Change: 0.04833
Iteration: 9, Log-Lik: -2376.981, Max-Change: 0.02805
Iteration: 10, Log-Lik: -2376.733, Max-Change: 0.02276
Iteration: 11, Log-Lik: -2376.618, Max-Change: 0.01234
Iteration: 12, Log-Lik: -2376.534, Max-Change: 0.00950
Iteration: 13, Log-Lik: -2376.384, Max-Change: 0.01048
Iteration: 14, Log-Lik: -2376.358, Max-Change: 0.00530
Iteration: 15, Log-Lik: -2376.338, Max-Change: 0.00516
Iteration: 16, Log-Lik: -2376.279, Max-Change: 0.00127
Iteration: 17, Log-Lik: -2376.278, Max-Change: 0.00099
Iteration: 18, Log-Lik: -2376.278, Max-Change: 0.00087
Iteration: 19, Log-Lik: -2376.276, Max-Change: 0.00021
Iteration: 20, Log-Lik: -2376.276, Max-Change: 0.00021
Iteration: 21, Log-Lik: -2376.276, Max-Change: 0.00021
Iteration: 22, Log-Lik: -2376.275, Max-Change: 0.00072
Iteration: 23, Log-Lik: -2376.275, Max-Change: 0.00052
Iteration: 24, Log-Lik: -2376.275, Max-Change: 0.00069
Iteration: 25, Log-Lik: -2376.275, Max-Change: 0.00021
Iteration: 26, Log-Lik: -2376.275, Max-Change: 0.00050
Iteration: 27, Log-Lik: -2376.275, Max-Change: 0.00016
Iteration: 28, Log-Lik: -2376.275, Max-Change: 0.00015
Iteration: 29, Log-Lik: -2376.275, Max-Change: 0.00049
Iteration: 30, Log-Lik: -2376.275, Max-Change: 0.00012
Iteration: 31, Log-Lik: -2376.275, Max-Change: 0.00010
Iteration: 32, Log-Lik: -2376.275, Max-Change: 0.00036
Iteration: 33, Log-Lik: -2376.275, Max-Change: 0.00048

```

Iteration: 34, Log-Lik: -2376.275, Max-Change: 0.00015
Iteration: 35, Log-Lik: -2376.275, Max-Change: 0.00040
Iteration: 36, Log-Lik: -2376.275, Max-Change: 0.00012
Iteration: 37, Log-Lik: -2376.275, Max-Change: 0.00011
Iteration: 38, Log-Lik: -2376.275, Max-Change: 0.00036
Iteration: 39, Log-Lik: -2376.275, Max-Change: 0.00009

Warning: data contains response patterns with only NAs

Iteration: 1, Log-Lik: -866.529, Max-Change: 1.38272
Iteration: 2, Log-Lik: -773.763, Max-Change: 0.93477
Iteration: 3, Log-Lik: -752.996, Max-Change: 0.66499
Iteration: 4, Log-Lik: -745.988, Max-Change: 0.47619
Iteration: 5, Log-Lik: -743.015, Max-Change: 0.32199
Iteration: 6, Log-Lik: -741.639, Max-Change: 0.24704
Iteration: 7, Log-Lik: -740.333, Max-Change: 0.11340
Iteration: 8, Log-Lik: -740.227, Max-Change: 0.09039
Iteration: 9, Log-Lik: -740.160, Max-Change: 0.06572
Iteration: 10, Log-Lik: -740.057, Max-Change: 0.02925
Iteration: 11, Log-Lik: -740.039, Max-Change: 0.01853
Iteration: 12, Log-Lik: -740.024, Max-Change: 0.01111
Iteration: 13, Log-Lik: -739.997, Max-Change: 0.01445
Iteration: 14, Log-Lik: -739.990, Max-Change: 0.00956
Iteration: 15, Log-Lik: -739.985, Max-Change: 0.00817
Iteration: 16, Log-Lik: -739.971, Max-Change: 0.00957
Iteration: 17, Log-Lik: -739.969, Max-Change: 0.00546
Iteration: 18, Log-Lik: -739.967, Max-Change: 0.00476
Iteration: 19, Log-Lik: -739.962, Max-Change: 0.00526
Iteration: 20, Log-Lik: -739.961, Max-Change: 0.00331
Iteration: 21, Log-Lik: -739.961, Max-Change: 0.00284
Iteration: 22, Log-Lik: -739.959, Max-Change: 0.00239
Iteration: 23, Log-Lik: -739.959, Max-Change: 0.00165
Iteration: 24, Log-Lik: -739.959, Max-Change: 0.00140
Iteration: 25, Log-Lik: -739.959, Max-Change: 0.00106
Iteration: 26, Log-Lik: -739.958, Max-Change: 0.00096
Iteration: 27, Log-Lik: -739.958, Max-Change: 0.00091
Iteration: 28, Log-Lik: -739.958, Max-Change: 0.00027
Iteration: 29, Log-Lik: -739.958, Max-Change: 0.00030
Iteration: 30, Log-Lik: -739.958, Max-Change: 0.00029
Iteration: 31, Log-Lik: -739.958, Max-Change: 0.00023
Iteration: 32, Log-Lik: -739.958, Max-Change: 0.00021

Iteration: 33, Log-Lik: -739.958, Max-Change: 0.00021
Iteration: 34, Log-Lik: -739.958, Max-Change: 0.00084
Iteration: 35, Log-Lik: -739.958, Max-Change: 0.00078
Iteration: 36, Log-Lik: -739.958, Max-Change: 0.00063
Iteration: 37, Log-Lik: -739.958, Max-Change: 0.00028
Iteration: 38, Log-Lik: -739.958, Max-Change: 0.00065
Iteration: 39, Log-Lik: -739.958, Max-Change: 0.00020
Iteration: 40, Log-Lik: -739.958, Max-Change: 0.00015
Iteration: 41, Log-Lik: -739.958, Max-Change: 0.00056
Iteration: 42, Log-Lik: -739.958, Max-Change: 0.00012
Iteration: 43, Log-Lik: -739.958, Max-Change: 0.00012
Iteration: 44, Log-Lik: -739.958, Max-Change: 0.00052
Iteration: 45, Log-Lik: -739.958, Max-Change: 0.00011
Iteration: 46, Log-Lik: -739.958, Max-Change: 0.00053
Iteration: 47, Log-Lik: -739.958, Max-Change: 0.00014
Iteration: 48, Log-Lik: -739.958, Max-Change: 0.00047
Iteration: 49, Log-Lik: -739.958, Max-Change: 0.00017
Iteration: 50, Log-Lik: -739.958, Max-Change: 0.00046
Iteration: 51, Log-Lik: -739.958, Max-Change: 0.00012
Iteration: 52, Log-Lik: -739.958, Max-Change: 0.00009

Warning: data contains response patterns with only NAs

Iteration: 1, Log-Lik: -8223.527, Max-Change: 0.97353
Iteration: 2, Log-Lik: -7401.627, Max-Change: 0.66823
Iteration: 3, Log-Lik: -7226.007, Max-Change: 0.49820
Iteration: 4, Log-Lik: -7163.834, Max-Change: 0.29288
Iteration: 5, Log-Lik: -7141.293, Max-Change: 0.19222
Iteration: 6, Log-Lik: -7131.656, Max-Change: 0.14900
Iteration: 7, Log-Lik: -7127.390, Max-Change: 0.10689
Iteration: 8, Log-Lik: -7125.368, Max-Change: 0.06951
Iteration: 9, Log-Lik: -7124.388, Max-Change: 0.05193
Iteration: 10, Log-Lik: -7123.488, Max-Change: 0.01236
Iteration: 11, Log-Lik: -7123.434, Max-Change: 0.00858
Iteration: 12, Log-Lik: -7123.407, Max-Change: 0.00632
Iteration: 13, Log-Lik: -7123.363, Max-Change: 0.00362
Iteration: 14, Log-Lik: -7123.356, Max-Change: 0.00262
Iteration: 15, Log-Lik: -7123.351, Max-Change: 0.00243
Iteration: 16, Log-Lik: -7123.332, Max-Change: 0.00084
Iteration: 17, Log-Lik: -7123.332, Max-Change: 0.00112
Iteration: 18, Log-Lik: -7123.331, Max-Change: 0.00076

Iteration: 19, Log-Lik: -7123.331, Max-Change: 0.00052
 Iteration: 20, Log-Lik: -7123.331, Max-Change: 0.00039
 Iteration: 21, Log-Lik: -7123.331, Max-Change: 0.00172
 Iteration: 22, Log-Lik: -7123.330, Max-Change: 0.00113
 Iteration: 23, Log-Lik: -7123.330, Max-Change: 0.00068
 Iteration: 24, Log-Lik: -7123.330, Max-Change: 0.00061
 Iteration: 25, Log-Lik: -7123.329, Max-Change: 0.00050
 Iteration: 26, Log-Lik: -7123.329, Max-Change: 0.00040
 Iteration: 27, Log-Lik: -7123.329, Max-Change: 0.00038
 Iteration: 28, Log-Lik: -7123.329, Max-Change: 0.00030
 Iteration: 29, Log-Lik: -7123.329, Max-Change: 0.00040
 Iteration: 30, Log-Lik: -7123.329, Max-Change: 0.00023
 Iteration: 31, Log-Lik: -7123.329, Max-Change: 0.00025
 Iteration: 32, Log-Lik: -7123.329, Max-Change: 0.00018
 Iteration: 33, Log-Lik: -7123.329, Max-Change: 0.00020
 Iteration: 34, Log-Lik: -7123.329, Max-Change: 0.00014
 Iteration: 35, Log-Lik: -7123.329, Max-Change: 0.00018
 Iteration: 36, Log-Lik: -7123.329, Max-Change: 0.00011
 Iteration: 37, Log-Lik: -7123.329, Max-Change: 0.00012
 Iteration: 38, Log-Lik: -7123.329, Max-Change: 0.00009

Iteration: 1, Log-Lik: -2691.206, Max-Change: 1.37907
 Iteration: 2, Log-Lik: -2384.546, Max-Change: 0.88232
 Iteration: 3, Log-Lik: -2318.976, Max-Change: 0.53559
 Iteration: 4, Log-Lik: -2296.037, Max-Change: 0.40448
 Iteration: 5, Log-Lik: -2286.737, Max-Change: 0.28161
 Iteration: 6, Log-Lik: -2282.220, Max-Change: 0.20677
 Iteration: 7, Log-Lik: -2278.645, Max-Change: 0.07540
 Iteration: 8, Log-Lik: -2278.051, Max-Change: 0.05908
 Iteration: 9, Log-Lik: -2277.675, Max-Change: 0.04316
 Iteration: 10, Log-Lik: -2277.059, Max-Change: 0.01590
 Iteration: 11, Log-Lik: -2276.930, Max-Change: 0.01428
 Iteration: 12, Log-Lik: -2276.825, Max-Change: 0.01334
 Iteration: 13, Log-Lik: -2276.480, Max-Change: 0.01694
 Iteration: 14, Log-Lik: -2276.450, Max-Change: 0.01265
 Iteration: 15, Log-Lik: -2276.426, Max-Change: 0.01023
 Iteration: 16, Log-Lik: -2276.342, Max-Change: 0.00477
 Iteration: 17, Log-Lik: -2276.335, Max-Change: 0.00608
 Iteration: 18, Log-Lik: -2276.330, Max-Change: 0.00386
 Iteration: 19, Log-Lik: -2276.327, Max-Change: 0.00341
 Iteration: 20, Log-Lik: -2276.322, Max-Change: 0.00234
 Iteration: 21, Log-Lik: -2276.319, Max-Change: 0.00230
 Iteration: 22, Log-Lik: -2276.306, Max-Change: 0.00122

Iteration: 23, Log-Lik: -2276.306, Max-Change: 0.00192
Iteration: 24, Log-Lik: -2276.305, Max-Change: 0.00097
Iteration: 25, Log-Lik: -2276.304, Max-Change: 0.00160
Iteration: 26, Log-Lik: -2276.304, Max-Change: 0.00066
Iteration: 27, Log-Lik: -2276.304, Max-Change: 0.00066
Iteration: 28, Log-Lik: -2276.303, Max-Change: 0.00103
Iteration: 29, Log-Lik: -2276.302, Max-Change: 0.00077
Iteration: 30, Log-Lik: -2276.302, Max-Change: 0.00085
Iteration: 31, Log-Lik: -2276.302, Max-Change: 0.00034
Iteration: 32, Log-Lik: -2276.302, Max-Change: 0.00012
Iteration: 33, Log-Lik: -2276.302, Max-Change: 0.00011
Iteration: 34, Log-Lik: -2276.302, Max-Change: 0.00013
Iteration: 35, Log-Lik: -2276.302, Max-Change: 0.00012
Iteration: 36, Log-Lik: -2276.302, Max-Change: 0.00012
Iteration: 37, Log-Lik: -2276.302, Max-Change: 0.00045
Iteration: 38, Log-Lik: -2276.302, Max-Change: 0.00046
Iteration: 39, Log-Lik: -2276.302, Max-Change: 0.00034
Iteration: 40, Log-Lik: -2276.302, Max-Change: 0.00011
Iteration: 41, Log-Lik: -2276.302, Max-Change: 0.00039
Iteration: 42, Log-Lik: -2276.302, Max-Change: 0.00008

Warning: data contains response patterns with only NAs

Iteration: 1, Log-Lik: -744.931, Max-Change: 1.23308
Iteration: 2, Log-Lik: -655.005, Max-Change: 0.95430
Iteration: 3, Log-Lik: -634.533, Max-Change: 0.80731
Iteration: 4, Log-Lik: -627.163, Max-Change: 0.58697
Iteration: 5, Log-Lik: -624.106, Max-Change: 0.43579
Iteration: 6, Log-Lik: -622.719, Max-Change: 0.33565
Iteration: 7, Log-Lik: -621.452, Max-Change: 0.10091
Iteration: 8, Log-Lik: -621.418, Max-Change: 0.07328
Iteration: 9, Log-Lik: -621.401, Max-Change: 0.05263
Iteration: 10, Log-Lik: -621.382, Max-Change: 0.01659
Iteration: 11, Log-Lik: -621.380, Max-Change: 0.01216
Iteration: 12, Log-Lik: -621.379, Max-Change: 0.00886
Iteration: 13, Log-Lik: -621.376, Max-Change: 0.00277
Iteration: 14, Log-Lik: -621.376, Max-Change: 0.00283
Iteration: 15, Log-Lik: -621.376, Max-Change: 0.00297
Iteration: 16, Log-Lik: -621.374, Max-Change: 0.00142
Iteration: 17, Log-Lik: -621.374, Max-Change: 0.00160
Iteration: 18, Log-Lik: -621.374, Max-Change: 0.00157

Iteration: 19, Log-Lik: -621.373, Max-Change: 0.00101
 Iteration: 20, Log-Lik: -621.373, Max-Change: 0.00084
 Iteration: 21, Log-Lik: -621.373, Max-Change: 0.00077
 Iteration: 22, Log-Lik: -621.373, Max-Change: 0.00079
 Iteration: 23, Log-Lik: -621.373, Max-Change: 0.00050
 Iteration: 24, Log-Lik: -621.373, Max-Change: 0.00022
 Iteration: 25, Log-Lik: -621.373, Max-Change: 0.00025
 Iteration: 26, Log-Lik: -621.373, Max-Change: 0.00025
 Iteration: 27, Log-Lik: -621.373, Max-Change: 0.00025
 Iteration: 28, Log-Lik: -621.373, Max-Change: 0.00022
 Iteration: 29, Log-Lik: -621.373, Max-Change: 0.00020
 Iteration: 30, Log-Lik: -621.373, Max-Change: 0.00019
 Iteration: 31, Log-Lik: -621.373, Max-Change: 0.00013
 Iteration: 32, Log-Lik: -621.373, Max-Change: 0.00013
 Iteration: 33, Log-Lik: -621.373, Max-Change: 0.00013
 Iteration: 34, Log-Lik: -621.373, Max-Change: 0.00011
 Iteration: 35, Log-Lik: -621.373, Max-Change: 0.00010
 Iteration: 36, Log-Lik: -621.373, Max-Change: 0.00010

Configural Parameters

```
knitr::kable(t(lambda), digits = 2)
```

Table 1: Configural Loadings

	FAST	LIFT	PIRC1	PIRC2	SAFE
T_TOCA_breakrule	3.82	2.57	3.27	2.65	2.94
T_TOCA_harmpro	NA	5.28	1.91	4.27	4.88
T_TOCA_breakthg	2.86	3.15	3.77	4.69	4.34
T_TOCA_takeprop	3.12	2.81	3.86	4.19	6.19
T_TOCA_fight	2.22	4.08	2.86	4.28	3.51
T_TOCA_lies	2.65	2.68	3.20	3.56	3.15
T_TOCA_yell	3.02	5.34	2.71	2.75	3.84
T_TOCA_stub	3.06	2.48	2.50	1.92	2.02
T_TOCA_texas	2.44	2.14	3.01	2.76	2.74

```
knitr::kable(t(nu), digits = 2)
```

Table 2: Configural Intercepts

	FAST	LIFT	PIRC1	PIRC2	SAFE
T_TOCA_breakrule	5.28	0.19	1.06	0.16	1.71
T_TOCA_harmpro	NA	-5.15	-0.85	-3.83	-2.08
T_TOCA_breakthg	0.33	-3.36	-2.01	-5.15	-1.85
T_TOCA_takeprop	1.53	-2.54	-0.93	-3.37	-0.71
T_TOCA_fight	2.53	-1.41	-1.22	-2.51	-0.34
T_TOCA_lies	2.01	-2.10	-0.60	-2.83	0.58
T_TOCA_yell	3.13	-1.86	-0.11	-1.38	-0.18
T_TOCA_stub	4.08	1.99	0.91	-1.08	0.27
T_TOCA_tetas	2.75	0.51	0.34	-0.98	0.12

Alignment Optimization

```
aligned <- sirt::invariance.alignment(
  lambda = lambda,
  nu = nu,
  wgt = sqrt(item_n),
  optimizer = "nlminb",
  eps = .0001,
  control = list(maxit = 10000)
)
```

```
knitr::kable(t(aligned$lambda.aligned), digits = 2)
```

Table 3: Aligned Loadings

	FAST	LIFT	PIRC1	PIRC2	SAFE
T_TOCA_breakrule	3.82	1.55	2.64	1.97	1.93
T_TOCA_harmpro	NA	3.18	1.55	3.18	3.20
T_TOCA_breakthg	2.86	1.90	3.04	3.49	2.85
T_TOCA_takeprop	3.12	1.70	3.12	3.12	4.07
T_TOCA_fight	2.22	2.47	2.31	3.19	2.31
T_TOCA_lies	2.65	1.62	2.59	2.65	2.07
T_TOCA_yell	3.02	3.22	2.19	2.05	2.52
T_TOCA_stub	3.06	1.50	2.02	1.43	1.33
T_TOCA_tetas	2.44	1.29	2.43	2.06	1.80

```
knitr::kable(t(aligned$nu.aligned), digits = 2)
```

Table 4: Aligned Intercepts

	FAST	LIFT	PIRC1	PIRC2	SAFE
T_TOCA_breakrule	5.28	2.87	3.15	3.26	3.18
T_TOCA_harmpro	NA	0.37	0.37	1.17	0.37
T_TOCA_breakthg	0.33	-0.06	0.39	0.35	0.33
T_TOCA_takeprop	1.53	0.40	1.53	1.53	2.41
T_TOCA_fight	2.53	2.86	0.61	2.50	1.43
T_TOCA_lies	2.01	0.71	1.45	1.34	2.16
T_TOCA_yell	3.13	3.72	1.62	1.84	1.74
T_TOCA_stub	4.08	4.59	2.50	1.17	1.29
T_TOCA_treas	2.75	2.75	2.26	2.26	1.50

```
knitr::kable(t(aligned$pars), digits = 2)
```

Table 5: Aligned Latent means and variances

	FAST	LIFT	PIRC1	PIRC2	SAFE
alpha0	0	-1.73	-0.79	-1.57	-0.77
psi0	1	1.66	1.24	1.34	1.52

Quality: Effect size of noninvariance

```
# Note: we need to do this twice as `pinsearch::fmacs()`
# does not handle NAs in the input, so we need to
# treat item 2 (not in Study 1) separately
# pooled SD
vars <- apply(data[item_names],
  MARGIN = 2,
  FUN = \(x) tapply(x,
    INDEX = data$study_id,
    FUN = var, na.rm = TRUE
  )
)
item_n2 <- ifelse(item_n > 0, item_n, NA)
```

```

pooled_sd <- sqrt(
  colSums(vars * (item_n2 - 1), na.rm = TRUE) /
  colSums(item_n2 - 1, na.rm = TRUE)
)

# a. Items 1, 3-9
thres1 <- aligned$nu.aligned[, c(1, 3:9)]
colnames(thres1) <- seq_len(ncol(thres1))
# Noninvariance effect size for items 1, 3-9
f1 <- pinsearch::fmacs_ordered(
  thresholds = thres1,
  loadings = aligned$lambda.aligned[, c(1, 3:9)],
  link = "logit",
  pooled_item_sd = pooled_sd[c(1, 3:9)]
)

# b. Item 2
thres2 <- aligned$nu.aligned[-1, 2, drop = FALSE]
colnames(thres2) <- 1
f2 <- pinsearch::fmacs_ordered(
  thresholds = thres2,
  loadings = aligned$lambda.aligned[-1, 2, drop = FALSE],
  link = "logit",
  pooled_item_sd = pooled_sd[2]
)

# Combine to form one effect size vector
f_effsize <- cbind(f1[1, 1, drop = FALSE], f2, f1[1, -1, drop = FALSE])
knitr::kable(t(f_effsize), digits = 2)

```

	fmacs
T_TOCA_breakrule	0.11
T_TOCA_harmpro	0.14
T_TOCA_breakthg	0.09
T_TOCA_takeprop	0.14
T_TOCA_fight	0.25
T_TOCA_lies	0.16
T_TOCA_yell	0.14
T_TOCA_stub	0.29
T_TOCA_tetas	0.18

The magnitude of noninvariance effect size is not trivial (with fmacs > .1) for most items.

Compute Harmonized Factor Scores

```
# Refit the configural model, with latent means and variances
# from the aligned parameters
aligned_pars <- aligned$pars
# Define a syntax template to allow latent means and
# variances to be substituted by the aligned values
mod <- "
  f = 1-[I]
  START = (GROUP, MEAN_1, [mu]), (GROUP, COV_11, [sigma2])
"
# Substitute into the aligned means and variances
fsg <- NULL # place holder for factor scores
for (g in seq_along(groups)) {
  datg <- data[data$group == groups[g], item_names]
  # Drop completely missing items
  itemsg <- which(item_n[g, ] > 0)
  modg <- gsub("\\[I\\]", replacement = length(itemsg), mod)
  modg <- gsub("\\[mu\\]", replacement = aligned_pars[g, 1], modg)
  # Note: aligned psi is latent SD, but `mirt`
  # needs latent variance (i.e., SD^2)
  modg <- gsub("\\[sigma2\\]", replacement = aligned_pars[g, 2]^2, modg)
  fitg <- mirt(datg[names(itemsg)], model = modg,
               itemtype = "2PL")
  fsg <- rbind(
    fsg,
    data.frame(
      fscores(fitg,
              method = "EAP", full.scores.SE = TRUE
            ),
      group = groups[g],
      mean_eta = aligned_pars[g, 1],
      sd_eta = aligned_pars[g, 2]
    )
  )
}
```

Warning: data contains response patterns with only NAs

Iteration: 1, Log-Lik: -2701.839, Max-Change: 1.34190

Iteration: 2, Log-Lik: -2463.453, Max-Change: 0.72218
Iteration: 3, Log-Lik: -2412.010, Max-Change: 0.43110
Iteration: 4, Log-Lik: -2392.083, Max-Change: 0.30417
Iteration: 5, Log-Lik: -2383.941, Max-Change: 0.20276
Iteration: 6, Log-Lik: -2380.398, Max-Change: 0.13371
Iteration: 7, Log-Lik: -2377.699, Max-Change: 0.05533
Iteration: 8, Log-Lik: -2377.254, Max-Change: 0.04833
Iteration: 9, Log-Lik: -2376.981, Max-Change: 0.02805
Iteration: 10, Log-Lik: -2376.733, Max-Change: 0.02276
Iteration: 11, Log-Lik: -2376.618, Max-Change: 0.01234
Iteration: 12, Log-Lik: -2376.534, Max-Change: 0.00950
Iteration: 13, Log-Lik: -2376.384, Max-Change: 0.01048
Iteration: 14, Log-Lik: -2376.358, Max-Change: 0.00530
Iteration: 15, Log-Lik: -2376.338, Max-Change: 0.00516
Iteration: 16, Log-Lik: -2376.279, Max-Change: 0.00127
Iteration: 17, Log-Lik: -2376.278, Max-Change: 0.00099
Iteration: 18, Log-Lik: -2376.278, Max-Change: 0.00087
Iteration: 19, Log-Lik: -2376.276, Max-Change: 0.00021
Iteration: 20, Log-Lik: -2376.276, Max-Change: 0.00021
Iteration: 21, Log-Lik: -2376.276, Max-Change: 0.00021
Iteration: 22, Log-Lik: -2376.275, Max-Change: 0.00072
Iteration: 23, Log-Lik: -2376.275, Max-Change: 0.00052
Iteration: 24, Log-Lik: -2376.275, Max-Change: 0.00069
Iteration: 25, Log-Lik: -2376.275, Max-Change: 0.00021
Iteration: 26, Log-Lik: -2376.275, Max-Change: 0.00050
Iteration: 27, Log-Lik: -2376.275, Max-Change: 0.00016
Iteration: 28, Log-Lik: -2376.275, Max-Change: 0.00015
Iteration: 29, Log-Lik: -2376.275, Max-Change: 0.00049
Iteration: 30, Log-Lik: -2376.275, Max-Change: 0.00012
Iteration: 31, Log-Lik: -2376.275, Max-Change: 0.00010
Iteration: 32, Log-Lik: -2376.275, Max-Change: 0.00036
Iteration: 33, Log-Lik: -2376.275, Max-Change: 0.00048
Iteration: 34, Log-Lik: -2376.275, Max-Change: 0.00015
Iteration: 35, Log-Lik: -2376.275, Max-Change: 0.00040
Iteration: 36, Log-Lik: -2376.275, Max-Change: 0.00012
Iteration: 37, Log-Lik: -2376.275, Max-Change: 0.00011
Iteration: 38, Log-Lik: -2376.275, Max-Change: 0.00036
Iteration: 39, Log-Lik: -2376.275, Max-Change: 0.00009

Warning: data contains response patterns with only NAs

Iteration: 1, Log-Lik: -866.533, Max-Change: 1.23966
Iteration: 2, Log-Lik: -793.682, Max-Change: 0.48395
Iteration: 3, Log-Lik: -774.644, Max-Change: 0.37382
Iteration: 4, Log-Lik: -764.558, Max-Change: 0.31631
Iteration: 5, Log-Lik: -758.253, Max-Change: 0.29618
Iteration: 6, Log-Lik: -753.946, Max-Change: 0.26431
Iteration: 7, Log-Lik: -743.133, Max-Change: 0.15807
Iteration: 8, Log-Lik: -742.484, Max-Change: 0.09342
Iteration: 9, Log-Lik: -742.031, Max-Change: 0.08349
Iteration: 10, Log-Lik: -740.559, Max-Change: 0.06006
Iteration: 11, Log-Lik: -740.450, Max-Change: 0.04979
Iteration: 12, Log-Lik: -740.364, Max-Change: 0.04585
Iteration: 13, Log-Lik: -740.051, Max-Change: 0.01748
Iteration: 14, Log-Lik: -740.035, Max-Change: 0.01589
Iteration: 15, Log-Lik: -740.023, Max-Change: 0.01487
Iteration: 16, Log-Lik: -739.978, Max-Change: 0.01004
Iteration: 17, Log-Lik: -739.976, Max-Change: 0.00802
Iteration: 18, Log-Lik: -739.974, Max-Change: 0.00723
Iteration: 19, Log-Lik: -739.969, Max-Change: 0.00750
Iteration: 20, Log-Lik: -739.968, Max-Change: 0.00373
Iteration: 21, Log-Lik: -739.967, Max-Change: 0.00264
Iteration: 22, Log-Lik: -739.967, Max-Change: 0.00255
Iteration: 23, Log-Lik: -739.966, Max-Change: 0.00286
Iteration: 24, Log-Lik: -739.966, Max-Change: 0.00216
Iteration: 25, Log-Lik: -739.966, Max-Change: 0.00115
Iteration: 26, Log-Lik: -739.966, Max-Change: 0.00124
Iteration: 27, Log-Lik: -739.966, Max-Change: 0.00265
Iteration: 28, Log-Lik: -739.965, Max-Change: 0.00307
Iteration: 29, Log-Lik: -739.965, Max-Change: 0.00066
Iteration: 30, Log-Lik: -739.965, Max-Change: 0.00068
Iteration: 31, Log-Lik: -739.965, Max-Change: 0.00133
Iteration: 32, Log-Lik: -739.965, Max-Change: 0.00194
Iteration: 33, Log-Lik: -739.965, Max-Change: 0.00044
Iteration: 34, Log-Lik: -739.965, Max-Change: 0.00040
Iteration: 35, Log-Lik: -739.965, Max-Change: 0.00011
Iteration: 36, Log-Lik: -739.965, Max-Change: 0.00070
Iteration: 37, Log-Lik: -739.965, Max-Change: 0.00011
Iteration: 38, Log-Lik: -739.965, Max-Change: 0.00079
Iteration: 39, Log-Lik: -739.965, Max-Change: 0.00015
Iteration: 40, Log-Lik: -739.965, Max-Change: 0.00012
Iteration: 41, Log-Lik: -739.965, Max-Change: 0.00041
Iteration: 42, Log-Lik: -739.964, Max-Change: 0.00053
Iteration: 43, Log-Lik: -739.964, Max-Change: 0.00118

Iteration: 44, Log-Lik: -739.964, Max-Change: 0.00008

Warning: data contains response patterns with only NAs

Iteration: 1, Log-Lik: -8039.426, Max-Change: 0.88834
Iteration: 2, Log-Lik: -7405.722, Max-Change: 0.64594
Iteration: 3, Log-Lik: -7251.327, Max-Change: 0.32330
Iteration: 4, Log-Lik: -7195.496, Max-Change: 0.30969
Iteration: 5, Log-Lik: -7167.166, Max-Change: 0.19065
Iteration: 6, Log-Lik: -7152.648, Max-Change: 0.12190
Iteration: 7, Log-Lik: -7144.845, Max-Change: 0.08947
Iteration: 8, Log-Lik: -7139.814, Max-Change: 0.07054
Iteration: 9, Log-Lik: -7136.303, Max-Change: 0.06529
Iteration: 10, Log-Lik: -7131.126, Max-Change: 0.05762
Iteration: 11, Log-Lik: -7129.591, Max-Change: 0.03820
Iteration: 12, Log-Lik: -7128.463, Max-Change: 0.03142
Iteration: 13, Log-Lik: -7126.882, Max-Change: 0.02838
Iteration: 14, Log-Lik: -7126.301, Max-Change: 0.02486
Iteration: 15, Log-Lik: -7125.768, Max-Change: 0.02110
Iteration: 16, Log-Lik: -7123.971, Max-Change: 0.01329
Iteration: 17, Log-Lik: -7123.853, Max-Change: 0.01148
Iteration: 18, Log-Lik: -7123.756, Max-Change: 0.00949
Iteration: 19, Log-Lik: -7123.463, Max-Change: 0.00631
Iteration: 20, Log-Lik: -7123.437, Max-Change: 0.00481
Iteration: 21, Log-Lik: -7123.418, Max-Change: 0.00424
Iteration: 22, Log-Lik: -7123.376, Max-Change: 0.00645
Iteration: 23, Log-Lik: -7123.366, Max-Change: 0.00298
Iteration: 24, Log-Lik: -7123.360, Max-Change: 0.00248
Iteration: 25, Log-Lik: -7123.339, Max-Change: 0.00146
Iteration: 26, Log-Lik: -7123.338, Max-Change: 0.00083
Iteration: 27, Log-Lik: -7123.336, Max-Change: 0.00123
Iteration: 28, Log-Lik: -7123.335, Max-Change: 0.00135
Iteration: 29, Log-Lik: -7123.334, Max-Change: 0.00114
Iteration: 30, Log-Lik: -7123.333, Max-Change: 0.00087
Iteration: 31, Log-Lik: -7123.332, Max-Change: 0.00091
Iteration: 32, Log-Lik: -7123.331, Max-Change: 0.00064
Iteration: 33, Log-Lik: -7123.331, Max-Change: 0.00024
Iteration: 34, Log-Lik: -7123.331, Max-Change: 0.00019
Iteration: 35, Log-Lik: -7123.331, Max-Change: 0.00012
Iteration: 36, Log-Lik: -7123.331, Max-Change: 0.00060
Iteration: 37, Log-Lik: -7123.330, Max-Change: 0.00060

Iteration: 38, Log-Lik: -7123.330, Max-Change: 0.00013
Iteration: 39, Log-Lik: -7123.330, Max-Change: 0.00044
Iteration: 40, Log-Lik: -7123.330, Max-Change: 0.00050
Iteration: 41, Log-Lik: -7123.330, Max-Change: 0.00023
Iteration: 42, Log-Lik: -7123.330, Max-Change: 0.00045
Iteration: 43, Log-Lik: -7123.330, Max-Change: 0.00026
Iteration: 44, Log-Lik: -7123.330, Max-Change: 0.00039
Iteration: 45, Log-Lik: -7123.330, Max-Change: 0.00017
Iteration: 46, Log-Lik: -7123.330, Max-Change: 0.00014
Iteration: 47, Log-Lik: -7123.330, Max-Change: 0.00034
Iteration: 48, Log-Lik: -7123.330, Max-Change: 0.00010

Iteration: 1, Log-Lik: -2752.067, Max-Change: 1.00488
Iteration: 2, Log-Lik: -2474.889, Max-Change: 0.48329
Iteration: 3, Log-Lik: -2397.638, Max-Change: 0.34021
Iteration: 4, Log-Lik: -2359.032, Max-Change: 0.26210
Iteration: 5, Log-Lik: -2336.196, Max-Change: 0.24034
Iteration: 6, Log-Lik: -2321.843, Max-Change: 0.23482
Iteration: 7, Log-Lik: -2311.923, Max-Change: 0.19653
Iteration: 8, Log-Lik: -2304.379, Max-Change: 0.18105
Iteration: 9, Log-Lik: -2298.886, Max-Change: 0.16132
Iteration: 10, Log-Lik: -2294.630, Max-Change: 0.14412
Iteration: 11, Log-Lik: -2291.261, Max-Change: 0.12338
Iteration: 12, Log-Lik: -2288.576, Max-Change: 0.11155
Iteration: 13, Log-Lik: -2278.957, Max-Change: 0.04534
Iteration: 14, Log-Lik: -2278.516, Max-Change: 0.04511
Iteration: 15, Log-Lik: -2278.145, Max-Change: 0.04019
Iteration: 16, Log-Lik: -2276.731, Max-Change: 0.02166
Iteration: 17, Log-Lik: -2276.661, Max-Change: 0.01864
Iteration: 18, Log-Lik: -2276.602, Max-Change: 0.01639
Iteration: 19, Log-Lik: -2276.376, Max-Change: 0.00847
Iteration: 20, Log-Lik: -2276.364, Max-Change: 0.00721
Iteration: 21, Log-Lik: -2276.354, Max-Change: 0.00619
Iteration: 22, Log-Lik: -2276.317, Max-Change: 0.00438
Iteration: 23, Log-Lik: -2276.315, Max-Change: 0.00320
Iteration: 24, Log-Lik: -2276.313, Max-Change: 0.00356
Iteration: 25, Log-Lik: -2276.310, Max-Change: 0.00266
Iteration: 26, Log-Lik: -2276.309, Max-Change: 0.00142
Iteration: 27, Log-Lik: -2276.308, Max-Change: 0.00153
Iteration: 28, Log-Lik: -2276.308, Max-Change: 0.00136
Iteration: 29, Log-Lik: -2276.307, Max-Change: 0.00171
Iteration: 30, Log-Lik: -2276.307, Max-Change: 0.00223
Iteration: 31, Log-Lik: -2276.306, Max-Change: 0.00193

Iteration: 32, Log-Lik: -2276.306, Max-Change: 0.00066
Iteration: 33, Log-Lik: -2276.306, Max-Change: 0.00053
Iteration: 34, Log-Lik: -2276.305, Max-Change: 0.00063
Iteration: 35, Log-Lik: -2276.305, Max-Change: 0.00087
Iteration: 36, Log-Lik: -2276.305, Max-Change: 0.00048
Iteration: 37, Log-Lik: -2276.305, Max-Change: 0.00020
Iteration: 38, Log-Lik: -2276.305, Max-Change: 0.00020
Iteration: 39, Log-Lik: -2276.305, Max-Change: 0.00019
Iteration: 40, Log-Lik: -2276.305, Max-Change: 0.00019
Iteration: 41, Log-Lik: -2276.305, Max-Change: 0.00018
Iteration: 42, Log-Lik: -2276.305, Max-Change: 0.00018
Iteration: 43, Log-Lik: -2276.305, Max-Change: 0.00018
Iteration: 44, Log-Lik: -2276.305, Max-Change: 0.00016
Iteration: 45, Log-Lik: -2276.305, Max-Change: 0.00016
Iteration: 46, Log-Lik: -2276.305, Max-Change: 0.00017
Iteration: 47, Log-Lik: -2276.305, Max-Change: 0.00018
Iteration: 48, Log-Lik: -2276.305, Max-Change: 0.00020
Iteration: 49, Log-Lik: -2276.305, Max-Change: 0.00020
Iteration: 50, Log-Lik: -2276.305, Max-Change: 0.00021
Iteration: 51, Log-Lik: -2276.305, Max-Change: 0.00028
Iteration: 52, Log-Lik: -2276.305, Max-Change: 0.00019
Iteration: 53, Log-Lik: -2276.305, Max-Change: 0.00024
Iteration: 54, Log-Lik: -2276.305, Max-Change: 0.00024
Iteration: 55, Log-Lik: -2276.305, Max-Change: 0.00020
Iteration: 56, Log-Lik: -2276.305, Max-Change: 0.00020
Iteration: 57, Log-Lik: -2276.305, Max-Change: 0.00028
Iteration: 58, Log-Lik: -2276.305, Max-Change: 0.00017
Iteration: 59, Log-Lik: -2276.305, Max-Change: 0.00023
Iteration: 60, Log-Lik: -2276.305, Max-Change: 0.00021
Iteration: 61, Log-Lik: -2276.305, Max-Change: 0.00018
Iteration: 62, Log-Lik: -2276.305, Max-Change: 0.00018
Iteration: 63, Log-Lik: -2276.305, Max-Change: 0.00025
Iteration: 64, Log-Lik: -2276.305, Max-Change: 0.00016
Iteration: 65, Log-Lik: -2276.305, Max-Change: 0.00021
Iteration: 66, Log-Lik: -2276.305, Max-Change: 0.00019
Iteration: 67, Log-Lik: -2276.305, Max-Change: 0.00017
Iteration: 68, Log-Lik: -2276.305, Max-Change: 0.00017
Iteration: 69, Log-Lik: -2276.305, Max-Change: 0.00023
Iteration: 70, Log-Lik: -2276.305, Max-Change: 0.00014
Iteration: 71, Log-Lik: -2276.305, Max-Change: 0.00019
Iteration: 72, Log-Lik: -2276.305, Max-Change: 0.00018
Iteration: 73, Log-Lik: -2276.305, Max-Change: 0.00015
Iteration: 74, Log-Lik: -2276.305, Max-Change: 0.00015

Iteration: 75, Log-Lik: -2276.305, Max-Change: 0.00021
Iteration: 76, Log-Lik: -2276.305, Max-Change: 0.00013
Iteration: 77, Log-Lik: -2276.305, Max-Change: 0.00017
Iteration: 78, Log-Lik: -2276.305, Max-Change: 0.00016
Iteration: 79, Log-Lik: -2276.305, Max-Change: 0.00014
Iteration: 80, Log-Lik: -2276.305, Max-Change: 0.00014
Iteration: 81, Log-Lik: -2276.305, Max-Change: 0.00020
Iteration: 82, Log-Lik: -2276.305, Max-Change: 0.00012
Iteration: 83, Log-Lik: -2276.305, Max-Change: 0.00016
Iteration: 84, Log-Lik: -2276.305, Max-Change: 0.00015
Iteration: 85, Log-Lik: -2276.305, Max-Change: 0.00013
Iteration: 86, Log-Lik: -2276.305, Max-Change: 0.00013
Iteration: 87, Log-Lik: -2276.305, Max-Change: 0.00018
Iteration: 88, Log-Lik: -2276.305, Max-Change: 0.00011
Iteration: 89, Log-Lik: -2276.305, Max-Change: 0.00015
Iteration: 90, Log-Lik: -2276.305, Max-Change: 0.00013
Iteration: 91, Log-Lik: -2276.305, Max-Change: 0.00012
Iteration: 92, Log-Lik: -2276.305, Max-Change: 0.00011
Iteration: 93, Log-Lik: -2276.305, Max-Change: 0.00017
Iteration: 94, Log-Lik: -2276.305, Max-Change: 0.00010

Warning: data contains response patterns with only NAs

Iteration: 1, Log-Lik: -699.738, Max-Change: 0.87083
Iteration: 2, Log-Lik: -648.553, Max-Change: 0.59220
Iteration: 3, Log-Lik: -635.400, Max-Change: 0.45063
Iteration: 4, Log-Lik: -629.899, Max-Change: 0.34148
Iteration: 5, Log-Lik: -627.161, Max-Change: 0.24548
Iteration: 6, Log-Lik: -625.633, Max-Change: 0.18007
Iteration: 7, Log-Lik: -623.326, Max-Change: 0.09093
Iteration: 8, Log-Lik: -623.054, Max-Change: 0.07756
Iteration: 9, Log-Lik: -622.829, Max-Change: 0.06992
Iteration: 10, Log-Lik: -621.889, Max-Change: 0.03803
Iteration: 11, Log-Lik: -621.822, Max-Change: 0.03216
Iteration: 12, Log-Lik: -621.765, Max-Change: 0.03277
Iteration: 13, Log-Lik: -621.518, Max-Change: 0.01871
Iteration: 14, Log-Lik: -621.499, Max-Change: 0.01904
Iteration: 15, Log-Lik: -621.483, Max-Change: 0.01717
Iteration: 16, Log-Lik: -621.429, Max-Change: 0.01341
Iteration: 17, Log-Lik: -621.421, Max-Change: 0.00826
Iteration: 18, Log-Lik: -621.415, Max-Change: 0.00868

Iteration: 19, Log-Lik: -621.394, Max-Change: 0.00978
Iteration: 20, Log-Lik: -621.392, Max-Change: 0.00619
Iteration: 21, Log-Lik: -621.391, Max-Change: 0.00526
Iteration: 22, Log-Lik: -621.388, Max-Change: 0.00662
Iteration: 23, Log-Lik: -621.387, Max-Change: 0.00375
Iteration: 24, Log-Lik: -621.387, Max-Change: 0.00404
Iteration: 25, Log-Lik: -621.384, Max-Change: 0.00349
Iteration: 26, Log-Lik: -621.384, Max-Change: 0.00269
Iteration: 27, Log-Lik: -621.384, Max-Change: 0.00264
Iteration: 28, Log-Lik: -621.382, Max-Change: 0.00157
Iteration: 29, Log-Lik: -621.382, Max-Change: 0.00138
Iteration: 30, Log-Lik: -621.382, Max-Change: 0.00141
Iteration: 31, Log-Lik: -621.381, Max-Change: 0.00088
Iteration: 32, Log-Lik: -621.381, Max-Change: 0.00087
Iteration: 33, Log-Lik: -621.381, Max-Change: 0.00065
Iteration: 34, Log-Lik: -621.381, Max-Change: 0.00080
Iteration: 35, Log-Lik: -621.381, Max-Change: 0.00031
Iteration: 36, Log-Lik: -621.381, Max-Change: 0.00034
Iteration: 37, Log-Lik: -621.381, Max-Change: 0.00032
Iteration: 38, Log-Lik: -621.381, Max-Change: 0.00031
Iteration: 39, Log-Lik: -621.381, Max-Change: 0.00030
Iteration: 40, Log-Lik: -621.381, Max-Change: 0.00029
Iteration: 41, Log-Lik: -621.381, Max-Change: 0.00027
Iteration: 42, Log-Lik: -621.381, Max-Change: 0.00026
Iteration: 43, Log-Lik: -621.381, Max-Change: 0.00025
Iteration: 44, Log-Lik: -621.381, Max-Change: 0.00025
Iteration: 45, Log-Lik: -621.381, Max-Change: 0.00023
Iteration: 46, Log-Lik: -621.381, Max-Change: 0.00023
Iteration: 47, Log-Lik: -621.381, Max-Change: 0.00022
Iteration: 48, Log-Lik: -621.381, Max-Change: 0.00022
Iteration: 49, Log-Lik: -621.381, Max-Change: 0.00020
Iteration: 50, Log-Lik: -621.381, Max-Change: 0.00020
Iteration: 51, Log-Lik: -621.381, Max-Change: 0.00019
Iteration: 52, Log-Lik: -621.381, Max-Change: 0.00019
Iteration: 53, Log-Lik: -621.381, Max-Change: 0.00018
Iteration: 54, Log-Lik: -621.381, Max-Change: 0.00018
Iteration: 55, Log-Lik: -621.381, Max-Change: 0.00017
Iteration: 56, Log-Lik: -621.381, Max-Change: 0.00017
Iteration: 57, Log-Lik: -621.381, Max-Change: 0.00016
Iteration: 58, Log-Lik: -621.381, Max-Change: 0.00016
Iteration: 59, Log-Lik: -621.381, Max-Change: 0.00015
Iteration: 60, Log-Lik: -621.381, Max-Change: 0.00015
Iteration: 61, Log-Lik: -621.381, Max-Change: 0.00015

```

Iteration: 62, Log-Lik: -621.381, Max-Change: 0.00015
Iteration: 63, Log-Lik: -621.381, Max-Change: 0.00014
Iteration: 64, Log-Lik: -621.381, Max-Change: 0.00014
Iteration: 65, Log-Lik: -621.381, Max-Change: 0.00013
Iteration: 66, Log-Lik: -621.381, Max-Change: 0.00013
Iteration: 67, Log-Lik: -621.381, Max-Change: 0.00013
Iteration: 68, Log-Lik: -621.381, Max-Change: 0.00013
Iteration: 69, Log-Lik: -621.381, Max-Change: 0.00012
Iteration: 70, Log-Lik: -621.381, Max-Change: 0.00012
Iteration: 71, Log-Lik: -621.381, Max-Change: 0.00012
Iteration: 72, Log-Lik: -621.381, Max-Change: 0.00011
Iteration: 73, Log-Lik: -621.381, Max-Change: 0.00011
Iteration: 74, Log-Lik: -621.381, Max-Change: 0.00011
Iteration: 75, Log-Lik: -621.381, Max-Change: 0.00011
Iteration: 76, Log-Lik: -621.381, Max-Change: 0.00010
Iteration: 77, Log-Lik: -621.381, Max-Change: 0.00010
Iteration: 78, Log-Lik: -621.381, Max-Change: 0.00010
Iteration: 79, Log-Lik: -621.381, Max-Change: 0.00010

```

```

# Save factor scores
saveRDS(fsg, file = here::here("Kush_etal_example", "fsg.RDS"))

```

Reliability

```

# Average reliability
rel_avg <- var(fsg[, 1], na.rm = TRUE) /
  (var(fsg[, 1], na.rm = TRUE) + mean(fsg[, 2]^2, na.rm = TRUE))
rel_avg

```

```
[1] 0.8364975
```

```

# Group-specific reliability
relg <- tapply(fsg, INDEX = fsg$group,
  FUN = \(x) var(x[, 1], na.rm = TRUE) /
    (var(x[, 1], na.rm = TRUE) + mean(x[, 2]^2, na.rm = TRUE)))
relg

```

	FAST	LIFT	PIRC1	PIRC2	SAFE
	0.7439284	0.8468285	0.8158746	0.7938676	0.8416167

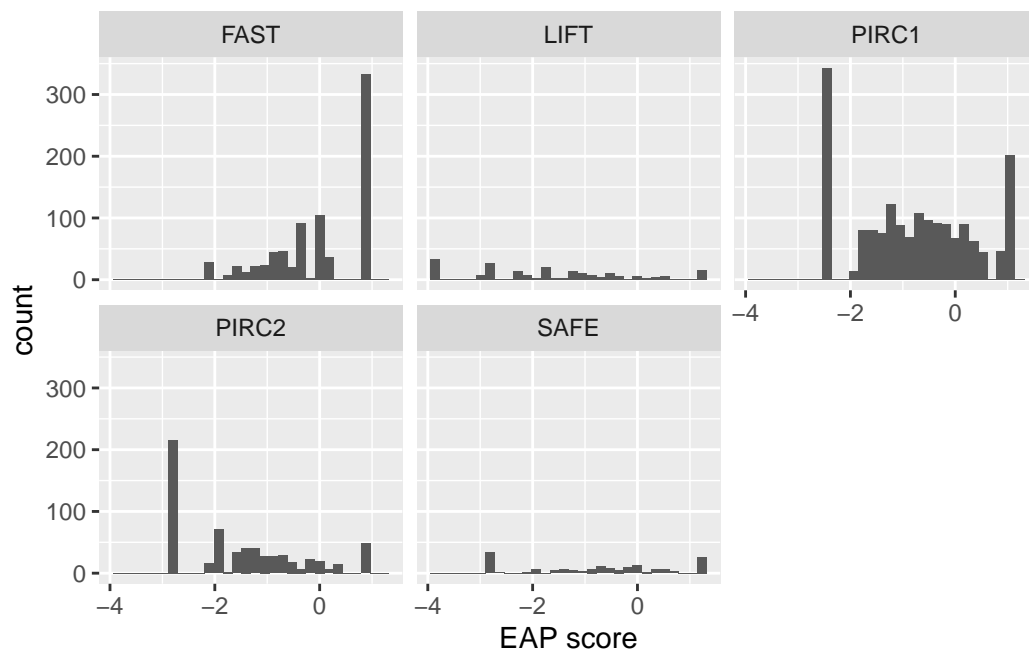
Analyses of Harmonized Scores

Descriptives

```
ggplot(fsg, aes(x = f)) +  
  geom_histogram() +  
  facet_wrap(~ group) +  
  labs(x = "EAP score")
```

``stat_bin()`` using ``bins = 30``. Pick better value with ``binwidth``.

Warning: Removed 386 rows containing non-finite outside the scale range (``stat_bin()``).



Factor score regression (no reliability adjustment)

Using EAP scores to predict high school completion with probit regression

```
# Merged with other variables
# Warning: the code requires data to be sorted by study_id before analyses
data <- cbind(data, fsg)
# GLM (probit regression); interaction not significant
m_int <- glm(hs ~ f * study_id, data = data, family = binomial("probit"))
anova(m_int)
```

Analysis of Deviance Table

Model: binomial, link: probit

Response: hs

Terms added sequentially (first to last)

	Df	Deviance	Resid. Df	Resid. Dev	Pr(>Chi)
NULL			3561	3809.5	
f	1	95.93	3560	3713.5	<2e-16 ***
study_id	4	363.52	3556	3350.0	<2e-16 ***
f:study_id	4	3.06	3552	3347.0	0.5471

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
m_main <- glm(hs ~ f + study_id, data = data, family = binomial("probit"))
anova(m_main)
```

Analysis of Deviance Table

Model: binomial, link: probit

Response: hs

Terms added sequentially (first to last)

	Df	Deviance	Resid. Df	Resid. Dev	Pr(>Chi)
NULL			3561	3809.5	
f	1	95.93	3560	3713.5	< 2.2e-16 ***
study_id	4	363.52	3556	3350.0	< 2.2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
summary(m_main)
```

Call:

```
glm(formula = hs ~ f + study_id, family = binomial("probit"),  
     data = data)
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	0.60831	0.04764	12.768	< 2e-16 ***
f	-0.15622	0.02250	-6.942	3.87e-12 ***
study_idLIFT	0.23214	0.12593	1.843	0.0653 .
study_idPIRC1	0.07322	0.06059	1.208	0.2269
study_idPIRC2	0.62058	0.09386	6.612	3.80e-11 ***
study_idSAFE	-1.81915	0.13649	-13.328	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 3809.5 on 3561 degrees of freedom
Residual deviance: 3350.0 on 3556 degrees of freedom
(386 observations deleted due to missingness)
AIC: 3362

Number of Fisher Scoring iterations: 5

Using umx (without unreliability adjustment)

```
# Recode discrete variable to be used with OpenMx  
data$hs2 <- mxFactor(data$hs, levels = c(0, 1))  
# Create dummy variables for Study ID  
data <- cbind(data, model.matrix(~ study_id, data = data)[, -1])  
# Pooled analysis without adjusting for unreliability  
# TOCA-R -> high school completion  
probitreg_umx <- umxRAM("  
  hs2 ~ f + study_idLIFT + study_idPIRC1 + study_idPIRC2 + study_idSAFE  
  hs2 ~ 0 * 1  
  hs2 ~~ 1 * hs2  
",  
  data = data,
```



```
    tryHard = "ordinal" # try hard for convergence
  )
```

Polite note: Variance of variable(s) NULL is < 0.1.

You might want to express the variable in smaller units, e.g. multiply to use cm instead of m.
Alternatively `umx_scale()` for data already in long-format, or `umx_scale_wide_twin_data` for wide-format.

object 'threshMat' created to handle: 1 binary variables: 'hs2'

1 trait(s) are binary: 'hs2'

For these, you you MUST fix the mean and variance of the latent traits driving each variable.
See `?umxThresholdMatrix`

Using deviation-based model: Thresholds will be in 'threshMat' based on deviations in 'deviations'

Running m1 with 26 parameters

Beginning initial fit attempt

Running m1 with 26 parameters

Lowest minimum so far: 21783.2738419408

Beginning fit attempt 1 of at maximum 10 extra tries

Running m1 with 26 parameters

Fit attempt generated errors

Beginning fit attempt 2 of at maximum 10 extra tries

Running m1 with 26 parameters

Fit attempt generated errors

Beginning fit attempt 3 of at maximum 10 extra tries

Running m1 with 26 parameters

Fit attempt generated errors

Beginning fit attempt 4 of at maximum 10 extra tries

Running m1 with 26 parameters

Fit attempt generated errors

Beginning fit attempt 5 of at maximum 10 extra tries

Running m1 with 26 parameters

Fit attempt generated errors

Beginning fit attempt 6 of at maximum 10 extra tries

Running m1 with 26 parameters

Fit attempt generated errors

Beginning fit attempt 7 of at maximum 10 extra tries

Running m1 with 26 parameters

Fit attempt generated errors

Beginning fit attempt 8 of at maximum 10 extra tries

Running m1 with 26 parameters

Fit attempt generated errors

Beginning fit attempt 9 of at maximum 10 extra tries

Running m1 with 26 parameters

Fit attempt generated errors

Beginning fit attempt 10 of at maximum 10 extra tries

Running m1 with 26 parameters

Fit attempt generated errors

Retry limit reached

Solution found

Final run, for Hessian and/or standard errors and/or confidence intervals

Running m1 with 26 parameters

Solution found! Final fit=21783.274 (started at 27940.691) (11 attempt(s): 1 valid, 10 erro

Start values from best fit:

-0.151829728361806,0.312103303403434,0.123663396514519,0.59719771468182,-1.59082705147643,1.5

?umxSummary options: std=T|F', digits=, report= 'html', filter= 'NS' & more

[1] “Warning calling mxRefModels: mxRefModels can’t handle all designs, including twin, and WLS <https://github.com/OpenMx/OpenMx/issues/184>”

Table 7: Parameter loadings for model ‘m1’

	name	Estimate	SE	type
7	f_with_study_idLIFT	-0.098	0.009	Manifest Cov
8	f_with_study_idPIRC1	0.032	0.011	Manifest Cov
9	f_with_study_idPIRC2	-0.116	0.008	Manifest Cov
10	f_with_study_idSAFE	0.004	0.004	Manifest Cov
13	study_idLIFT_with_study_idPIRC1	-0.055	0.003	Manifest Cov
14	study_idLIFT_with_study_idPIRC2	-0.018	0.002	Manifest Cov
15	study_idLIFT_with_study_idSAFE	-0.005	0.001	Manifest Cov
17	study_idPIRC1_with_study_idPIRC2	-0.077	0.003	Manifest Cov
18	study_idPIRC1_with_study_idSAFE	-0.019	0.002	Manifest Cov
20	study_idPIRC2_with_study_idSAFE	-0.006	0.001	Manifest Cov
1	f_to_hs2	-0.152	0.022	Manifest path
2	study_idLIFT_to_hs2	0.312	0.096	Manifest path
3	study_idPIRC1_to_hs2	0.124	0.064	Manifest path
4	study_idPIRC2_to_hs2	0.597	0.088	Manifest path
5	study_idSAFE_to_hs2	-1.591	0.115	Manifest path
22	one_to_f	-0.853	0.021	Mean
23	one_to_study_idLIFT	0.114	0.005	Mean
24	one_to_study_idPIRC1	0.477	0.008	Mean
25	one_to_study_idPIRC2	0.161	0.006	Mean
26	one_to_study_idSAFE	0.041	0.003	Mean
6	f_with_f	1.575	0.038	Residual
11	hs2_with_hs2	1.000	0	Residual
12	study_idLIFT_with_study_idLIFT	0.101	0.002	Residual
16	study_idPIRC1_with_study_idPIRC1	0.249	0.006	Residual
19	study_idPIRC2_with_study_idPIRC2	0.136	0.003	Residual
21	study_idSAFE_with_study_idSAFE	0.038	0.001	Residual

Model Fit: $\chi^2(1) = \text{NA}$, $p = \text{NA}$; CFI = NA; TLI = NA; RMSEA = NA

Algebra 'threshMat' = -0.589CI95[-0.686, -0.492]. $p\text{-value} < 0.001$

```
# plot(probitreg_umx) # path diagram
```

Two-Stage Path Analysis

Adjusting for measurement error

```
# Define loading and error variances
# Loading = 1 - SE^2 / latent variance
data$rel_f <- 1 - data$SE_f^2 / data$sd_eta^2
# Error variance = SE^2 * reliability
data$ev_f <- data$SE_f^2 * data$rel_f
# Drop rows with missing rel_f
data2 <- data[!is.na(data$rel_f), ]
# Incorporate measurement model for factor scores
probitreg_2spa <- umxRAM(
  "2spa",
  # Main effects of EAP scores and study indicators
  umxPath(c("eta", "study_idLIFT", "study_idPIRC1",
            "study_idPIRC2", "study_idSAFE"),
          to = "hs2"),
  # Loading = reliability (as definition variable)
  umxPath("eta", to = "f", labels = "data.rel_f", free = FALSE),
  # Error variance (as definition variable)
  umxPath(var = "f", labels = "data.ev_f", free = FALSE),
  # Covariances of predictors
  umxPath(unique.pairs = c("eta", "study_idLIFT", "study_idPIRC1",
                           "study_idPIRC2", "study_idSAFE")),
  # Means of predictors
  umxPath(means = c("eta", "study_idLIFT", "study_idPIRC1",
                    "study_idPIRC2", "study_idSAFE")),
  # For model identification: Fix latent variate
  # to be standard normal
  umxPath(v1m0 = "hs2"),
  data = data2,
  tryHard = "ordinal"
)
```

A latent variable 'eta' was created.

Polite note: Variance of variable(s) NULL is < 0.1.

You might want to express the variable in smaller units, e.g. multiply to use cm instead of m.
Alternatively `umx_scale()` for data already in long-format, or `umx_scale_wide_twin_data` for wide-format.

object 'threshMat' created to handle: 1 binary variables: 'hs2'

1 trait(s) are binary: 'hs2'

For these, you you MUST fix the mean and variance of the latent traits driving each variable.
See `?umxThresholdMatrix`

Using deviation-based model: Thresholds will be in 'threshMat' based on deviations in 'deviations'

Running 2spa with 26 parameters

Beginning initial fit attempt

Running 2spa with 26 parameters

Lowest minimum so far: 20052.804644306

Beginning fit attempt 1 of at maximum 10 extra tries

Running 2spa with 26 parameters

Fit attempt generated errors

Beginning fit attempt 2 of at maximum 10 extra tries

Running 2spa with 26 parameters

Fit attempt generated errors

Beginning fit attempt 3 of at maximum 10 extra tries

Running 2spa with 26 parameters

Fit attempt generated errors

Beginning fit attempt 4 of at maximum 10 extra tries

Running 2spa with 26 parameters

Fit attempt generated errors

Beginning fit attempt 5 of at maximum 10 extra tries

Running 2spa with 26 parameters

Fit attempt generated errors

Beginning fit attempt 6 of at maximum 10 extra tries

Running 2spa with 26 parameters

Fit attempt generated errors

Beginning fit attempt 7 of at maximum 10 extra tries

Running 2spa with 26 parameters

Fit attempt generated errors

Beginning fit attempt 8 of at maximum 10 extra tries

Running 2spa with 26 parameters

Fit attempt generated errors

Beginning fit attempt 9 of at maximum 10 extra tries

Running 2spa with 26 parameters

Beginning fit attempt 10 of at maximum 10 extra tries

Running 2spa with 26 parameters

Fit attempt generated errors

Retry limit reached

Solution found

Final run, for Hessian and/or standard errors and/or confidence intervals

Running 2spa with 26 parameters

Solution found! Final fit=20052.805 (started at 27663.782) (11 attempt(s): 2 valid, 9 errors)

Start values from best fit:

0.200683342605373,0.0441315007148669,0.575825950140818,-1.84038923943269,-0.145034588723763,0.000000000000000

?umxSummary options: std=T|F', digits=, report= 'html', filter= 'NS' & more

[1] “Warning calling mxRefModels: mxRefModels can’t handle all designs, including twin, and WLS <https://github.com/OpenMx/OpenMx/issues/184>”

Warning in mxStandardizeRAMpaths(model, SE = SE): 'model' (or one of its submodels) contains definition variables; interpret results of mxStandardizeRAMpaths() cautiously

Table 8: Parameter loadings for model ‘2spa’

	name	Estimate	SE	type
5	data.rel_f	0.782	0	Factor loading
6	eta_to_hs2	-0.145	0.022	Factor loading
23	eta_with_eta	2.273	0.072	Factor Variance
13	eta_with_study_idLIFT	-0.056	0.006	Latent-Manifest Cov
17	eta_with_study_idPIRC1	-0.002	0.014	Latent-Manifest Cov
20	eta_with_study_idPIRC2	-0.180	0.011	Latent-Manifest Cov
22	eta_with_study_idSAFE	0.004	0.006	Latent-Manifest Cov
10	study_idLIFT_with_study_idPIRC1	-0.028	0.002	Manifest Cov
11	study_idLIFT_with_study_idPIRC2	-0.010	0.001	Manifest Cov
12	study_idLIFT_with_study_idSAFE	-0.002	0.001	Manifest Cov
15	study_idPIRC1_with_study_idPIRC2	-0.089	0.004	Manifest Cov
16	study_idPIRC1_with_study_idSAFE	-0.022	0.002	Manifest Cov
19	study_idPIRC2_with_study_idSAFE	-0.008	0.001	Manifest Cov
1	study_idLIFT_to_hs2	0.201	0.128	Manifest path
2	study_idPIRC1_to_hs2	0.044	0.062	Manifest path
3	study_idPIRC2_to_hs2	0.576	0.097	Manifest path
4	study_idSAFE_to_hs2	-1.840	0.137	Manifest path
24	one_to_study_idLIFT	0.056	0.004	Mean
25	one_to_study_idPIRC1	0.497	0.008	Mean
26	one_to_study_idPIRC2	0.179	0.006	Mean
27	one_to_study_idSAFE	0.044	0.003	Mean
28	one_to_eta	-0.970	0.027	Mean
7	data.ev_f	0.170	0	Residual
8	hs2_with_hs2	1.000	0	Residual
9	study_idLIFT_with_study_idLIFT	0.053	0.001	Residual
14	study_idPIRC1_with_study_idPIRC1	0.250	0.006	Residual
18	study_idPIRC2_with_study_idPIRC2	0.147	0.003	Residual
21	study_idSAFE_with_study_idSAFE	0.042	0.001	Residual

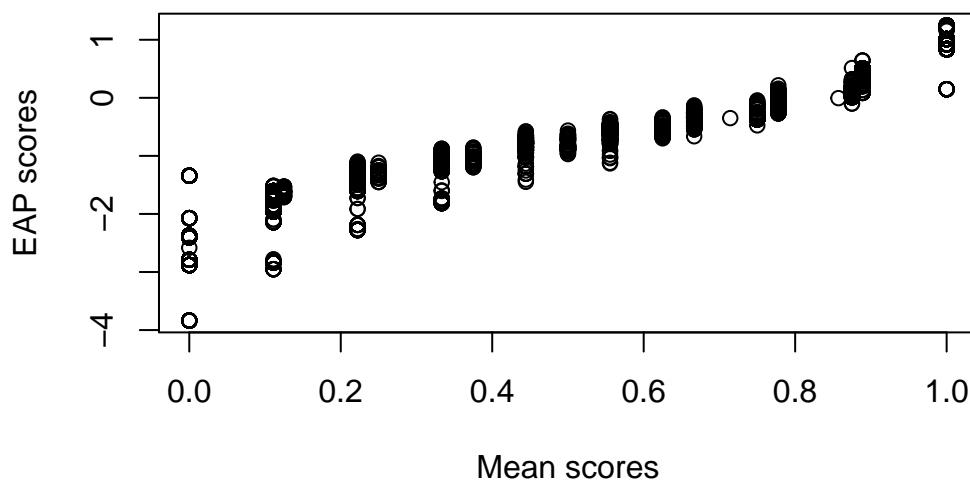
Model Fit: $\chi^2(1) = \text{NA}$, $p = \text{NA}$; CFI = NA; TLI = NA; RMSEA = NA

Algebra'threshMat' = -0.622CI95[-0.716, -0.528]. $p\text{-value} < 0.001$

Sensitivity Analysis

Using sum/mean scores

```
data$toca_mean <- rowMeans(data[item_names], na.rm = TRUE)
# Scatterplot
plot(x = data$toca_mean, y = fsg[, 1], xlab = "Mean scores", ylab = "EAP scores")
```



```
cor(data$toca_mean, fsg[, 1], use = "complete")
```

```
[1] 0.9655881
```

Probit regression with mean scores; note that the scale of the predictors are different, so the coefficients are not comparable.

```
# GLM (probit regression); interaction not significant
m_int_meanscore <- glm(hs ~ toca_mean * study_id, data = data, family = binomial("probit"))
anova(m_int_meanscore)
```

Analysis of Deviance Table

Model: binomial, link: probit

Response: hs

Terms added sequentially (first to last)

		Df	Deviance	Resid. Df	Resid. Dev	Pr(>Chi)
NULL				3561	3809.5	
toca_mean	1	83.12		3560	3726.4	<2e-16 ***
study_id	4	370.75		3556	3355.6	<2e-16 ***
toca_mean:study_id	4	2.28		3552	3353.3	0.6842

Signif. codes:	0	'***'	0.001	'**'	0.01	'*' 0.05 '.' 0.1 ' ' 1

```
m_main_meanscore <- glm(hs ~ toca_mean + study_id, data = data, family = binomial("probit"))
anova(m_main_meanscore)
```

Analysis of Deviance Table

Model: binomial, link: probit

Response: hs

Terms added sequentially (first to last)

		Df	Deviance	Resid. Df	Resid. Dev	Pr(>Chi)
NULL				3561	3809.5	
toca_mean	1	83.12		3560	3726.4	< 2.2e-16 ***
study_id	4	370.75		3556	3355.6	< 2.2e-16 ***

Signif. codes:	0	'***'	0.001	'**'	0.01	'*' 0.05 '.' 0.1 ' ' 1

```
summary(m_main_meanscore)
```

```
Call:
glm(formula = hs ~ toca_mean + study_id, family = binomial("probit"),
    data = data)
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	0.97192	0.07425	13.090	< 2e-16 ***
toca_mean	-0.47136	0.07211	-6.537	6.29e-11 ***
study_idLIFT	0.30165	0.12372	2.438	0.0148 *
study_idPIRC1	0.04913	0.06200	0.792	0.4281
study_idPIRC2	0.63601	0.09383	6.778	1.22e-11 ***
study_idSAFE	-1.81877	0.13620	-13.354	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 3809.5 on 3561 degrees of freedom
Residual deviance: 3355.6 on 3556 degrees of freedom
(386 observations deleted due to missingness)
AIC: 3367.6

Number of Fisher Scoring iterations: 5