MLM for Experimental Data

PSYC 575

September 15, 2020 (updated: 19 September 2020)

Experimental Designs

- Within-subjects/mixed designs
  - Random assignment at level 1
- Between-subjects
  - Cluster-randomized trial
    - Treatment at level 2
  - Multisite trial
    - Treatment at level 1
  - See example in chapter 11 of Snijders and Bosker (2012)

Learning Objectives

- Identify the correct levels with experimental studies
- Describe designs with crossed random levels
- Assign variables to appropriate levels
  - And tell which variables can have random slopes at which levels
- Compute a version of effect size (*d*) for experimental data

Changes in Driving Scenes

- Example 1 from Hoffman & Rovine (2007)
  - Originally from Hoffman & Atchley (2001)
- Flicker paradigm: https://coglab.cengage.com/labs/change_detection.shtml

Changes in Driving Scenes

- 153 persons
  - Younger (n = 96), M_{age} = 19.7 years (SD = 2.3);
  - Older (n = 57), M_{age} = 75.7 years (SD = 5.4)
- 51 scenes/items
  - Meaningfulness (0-5): meaningfulness to driving of the change
  - Salience (0-5): how visually conspicuous the change was within the scene
- Original plan: 2 (age group) × 2 (meaning) × 2 (salience) split-plot ANOVA

Data

- While RM-ANOVA uses the wide format, MLM requires the long format
  - Each unique response is in its own row
- See RStudio

Issues With ANOVA

- Unbalanced data/Missing responses
  - NA when change not detected within 60s
  - ANOVA uses listwise deletion: individual record is discarded when an individual has 1+ missing response

Issues of ANOVA

- ANOVA may require discretization of meaning and salience
- E.g., 0-2 for low salience, 3-5 for high salience
  - May hurt statistical power

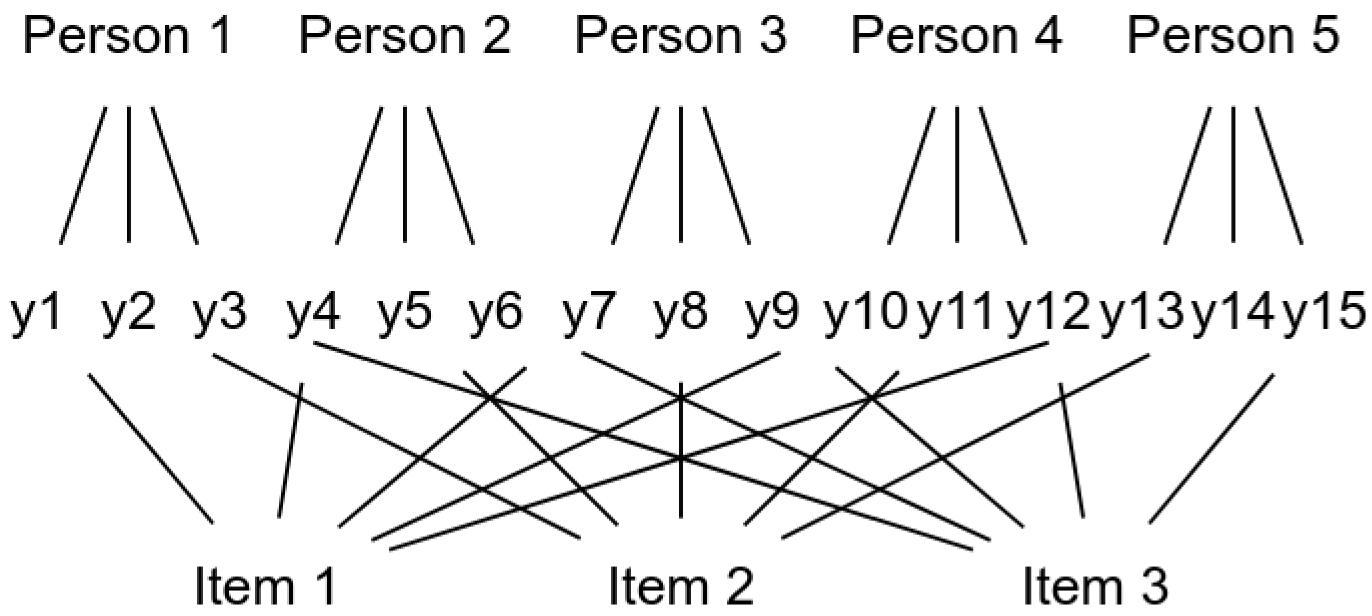
What is the Data Structure?

- Each response represents a person answering an item
- Responses nested within persons?
- Responses nested within items?

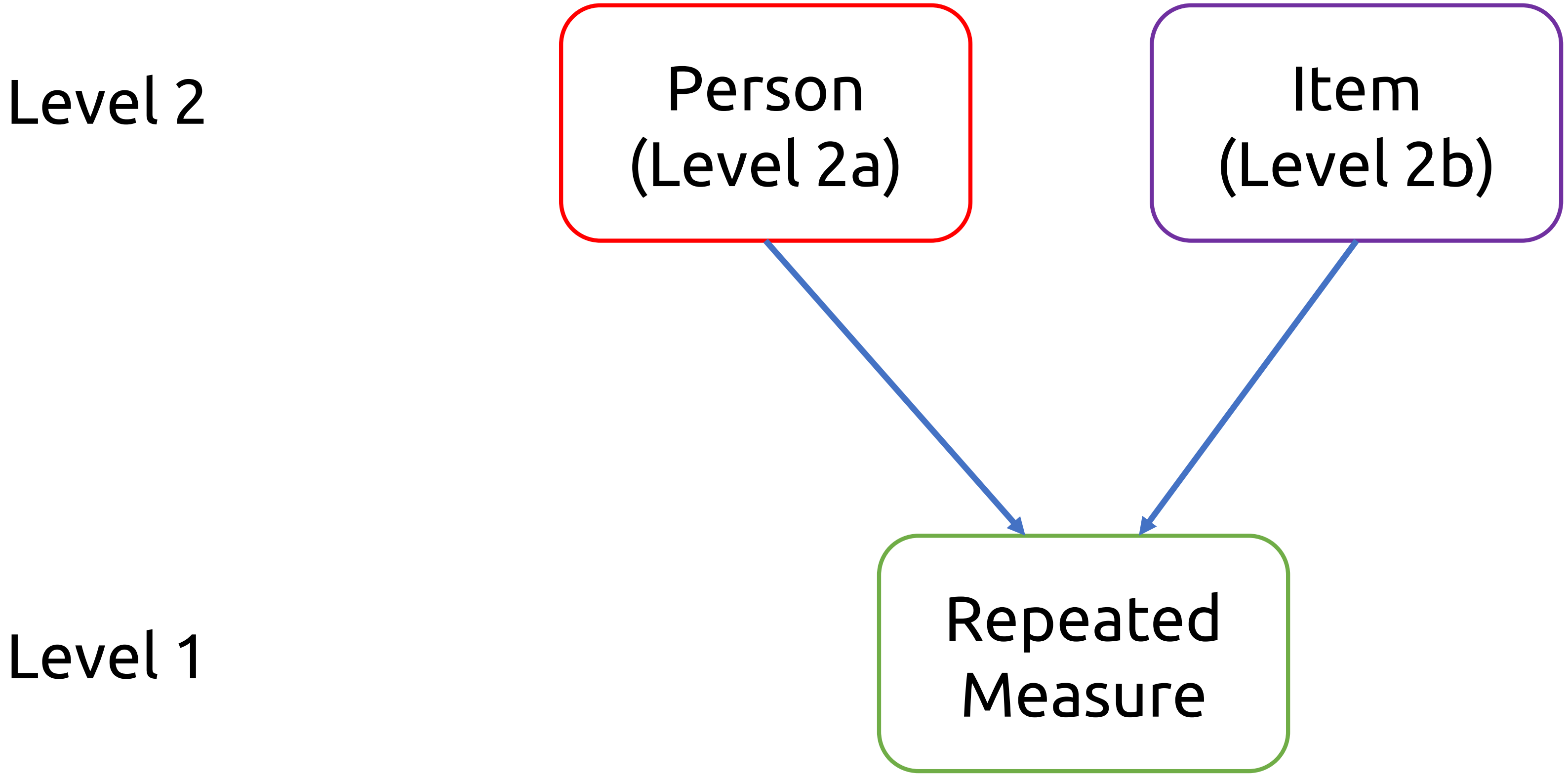
Crossed Levels

Crossed Levels

- Cross-classified model

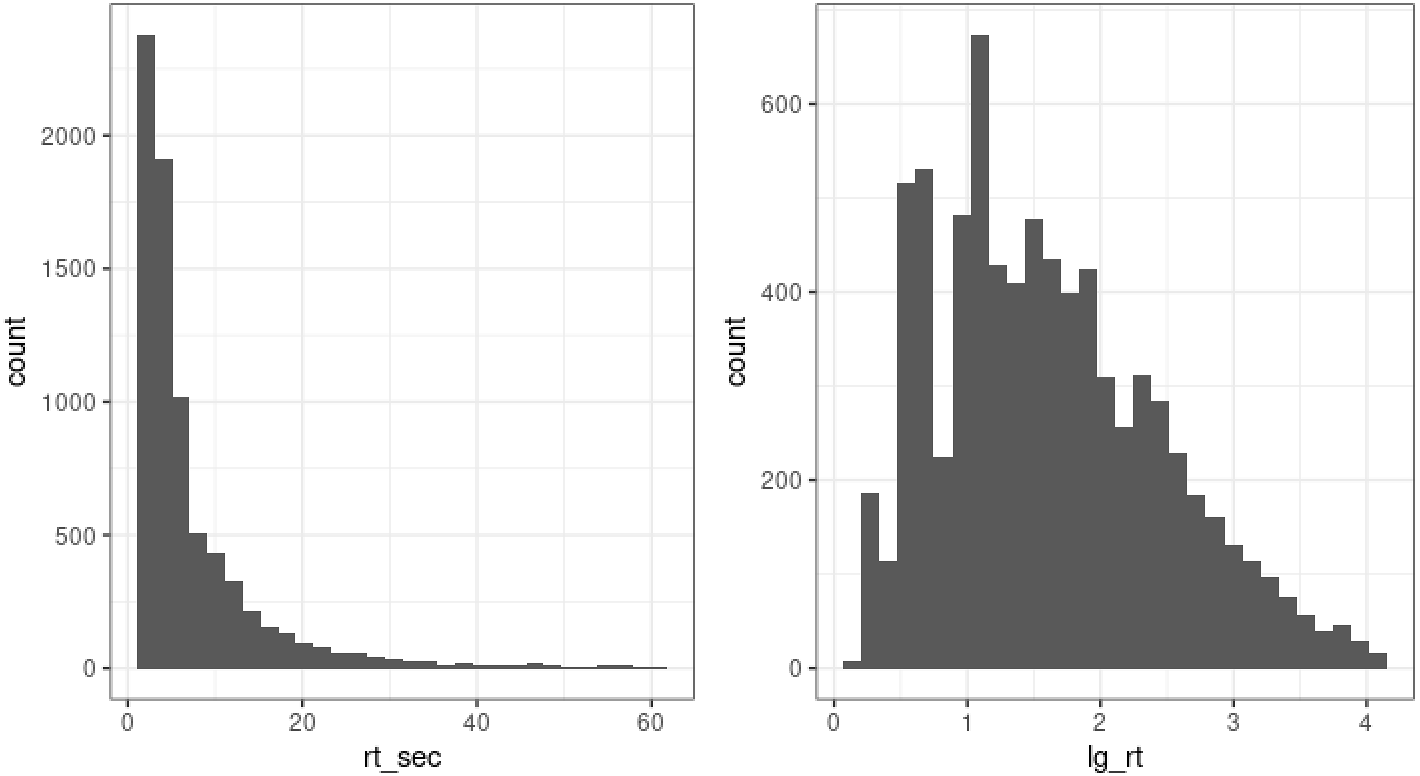


Crossed Levels at Level 2



Pre-Processing

- Log transformation of response time



Unconditional Model

- Repeated Measure (Within-cell) level (Lv 1)
  - $\text{lg\_rt}_{i(j,\,k)} = \beta_{0(j,\,k)} + e_{ijk}$
- Between-cell (Person × Item) level
  - $\beta_{0(j,\,k)} = \gamma_{00} + u_{0j} + v_{0k}$

Intraclass Correlation

- Person level (Lv 2a) random effect: $u_{0j} \sim N(0, \tau^2_{u_0})$
  - ICC(person) = $\frac{\tau^2_{u_0}}{\tau^2_{u_0}+\tau^2_{v_0}+\sigma^2}$
- Item level (Lv 2b) random effect: $v_{0k} \sim N(0, \tau^2_{v_0})$
  - ICC(item) = $\frac{\tau^2_{v_0}}{\tau^2_{u_0}+\tau^2_{v_0}+\sigma^2}$
- ICC(person + item) = $\frac{\tau^2_{u_0}+\tau^2_{v_0}}{\tau^2_{u_0}+\tau^2_{v_0}+\sigma^2}$

Intraclass Correlation

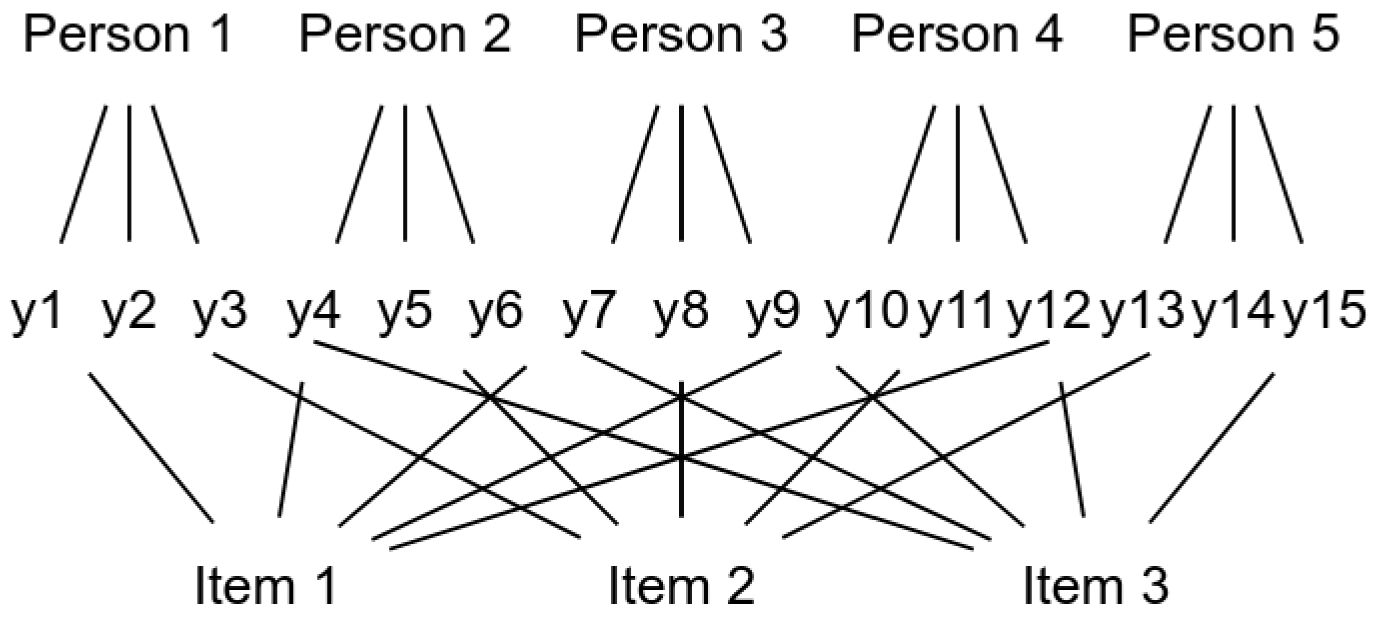
```
## Formula: lg_rt ~ (1 | id) + (1 | Item)
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  id       (Intercept) 0.1803   0.4246
##  Item     (Intercept) 0.1259   0.3549
##  Residual             0.3899   0.6244
## Number of obs: 7646, groups:  id, 153; Item, 51
```

- ICC(person) = 0.26
- ICC(item) = 0.18
- ICC(person + item) = 0.44

Varying Slopes With Crossed Levels

Rule for Random Slopes

- A predictor can have random slopes at a level above or at a crossed level



Varying Slopes Across Persons

- Repeated Measure level (Lv 1)
  - $\text{lg\_rt}_{i(j,k)} = \beta_{0(j,k)} + e_{ijk}$
- Between-cell (Person × Item) level
  - $\beta_{0(j,k)} = \gamma_{00} + \beta_{1j}\,\text{meaning}_{ik} + u_{0j} + v_{0k}$
- Person level (Lv 2a)
  - $\beta_{1j} = \gamma_{10} + u_{1j}$

Any predictors at the repeated measure level or at the item level can have random slopes across persons

Varying Slopes Across Items

- Repeated Measure level (Lv 1)
  - $\mathrm{lg\_rt}_{i(j,\,k)} = \beta_{0(j,\,k)} + e_{ijk}$
- Between-cell (Person × Item) level
  - $\beta_{0(j,\,k)} = \gamma_{00} + \beta_{4k}\,\mathrm{oldage}_{ij} + u_{0j} + v_{0k}$
- Item level (Lv 2b)
  - $\beta_{4k} = \gamma_{40} + v_{4k}$

Any predictors at the repeated measure level or at the person level can have random slopes across items

Hypothesized Model

- Repeated Measure level (Lv 1)
  - $\text{lg\_rt}_{i(j,\,k)} = \beta_{0(j,\,k)} + e_{ijk}$
- Between-cell (Person × Item) level
  - $\beta_{0(j,\,k)} = \gamma_{00} + \beta_{1j}\,\text{meaning}_{ik} + \beta_{2j}\,\text{salience}_{ik} + \beta_{3j}\,\text{meaning}_{ik} \times \text{salience}_{ik}$
    $+ \beta_{4k}\,\text{oldage}_{ij} + u_{0j} + v_{0k}$
- Person level (Lv 2a) random slopes
  - $\beta_{1j} = \gamma_{10} + \gamma_{11}\,\text{oldage}_{ij} + u_{1j}$
  - $\beta_{2j} = \gamma_{20} + \gamma_{21}\,\text{oldage}_{ij} + u_{2j}$
  - $\beta_{3j} = \gamma_{30} + \gamma_{31}\,\text{oldage}_{ij} + u_{3j}$

Hypothesized Model

- Item level (Lv 2b) random slopes
  - $\beta_{4k} = \gamma_{40} + v_{4k}$

Hypothesized Model

- $$\begin{aligned}
\text{lg_rt}_{i(j,\,k)} = {} & \gamma_{00} && \text{Grand intercept} \\
& + \gamma_{10}\,\text{meaning}_{ik} + \gamma_{20}\,\text{salience}_{ik} && \text{Item-level main and interaction effects} \\
& + \gamma_{30}\,\text{meaning}_{ik} \times \text{salience}_{ik} && \\
& + \gamma_{40}\,\text{oldage}_{ij} && \text{Person-level main effect} \\
& + \gamma_{11}\,\text{meaning}_{ik} \times \text{oldage}_{ij} && \text{Person} \times \text{item cross-level interaction} \\
& + \gamma_{21}\,\text{salience}_{ik} \times \text{oldage}_{ij} && \\
& + \gamma_{31}\,\text{meaning}_{ik} \times \text{salience}_{ik} \times \text{oldage}_{ij} && \\
& + u_{0j} + u_{1j} \times \text{meaning}_{ik} + u_{2j} \times \text{salience}_{ik} && \text{Item-level random intercepts and slopes} \\
& + u_{3j} \times \text{meaning}_{ik} \times \text{salience}_{ik} && \\
& + v_{0k} + v_{4k} \times \text{oldage}_{ij} && \text{Person-level random intercepts and slopes} \\
& + e_{ijk} && \text{Within-cell deviation}
\end{aligned}$$

Notes

- Because of counterbalancing
  - Person-level variables have no item-level variance
  - Item-level variables have no person-level variance
- Therefore, no need for cluster-mean centering

```
Formula: c_sal ~ (1 | id)
   Data: driving_dat
REML criterion at convergence:
23545.51
Random effects:
 Groups   Name        Std.Dev.
 id       (Intercept) 0.000
 Residual             1.094
```

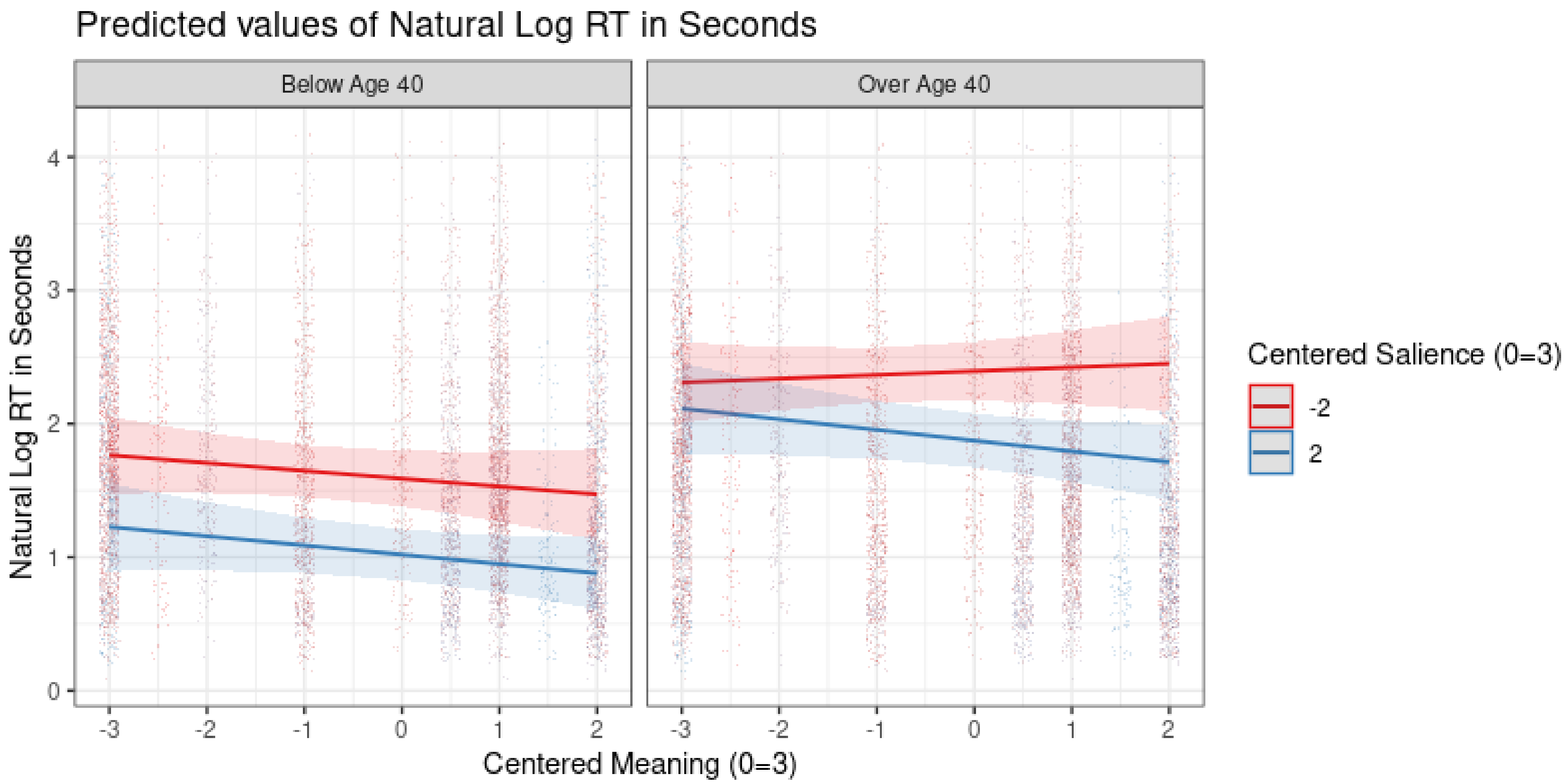
Notes

- To more easily interpret the interactions, we want to grand-mean center meaning and salience
  - They were centered to 3.0 in the data

Notes

- By testing random slopes one by one, the final model includes
  - Random slopes of `c_sal` (across persons)
  - Random slopes of `oldage` (across items)
- All two-way and three-way interactions were found not significant

Three-Way Interaction Plot



Effect Size

Effect Size (*d*)

- *d* = Treatment effect / [some estimate of population *SD*]
- Still an active area of research; no consensus how to standardize
- In my opinion, we should think about what is the natural standard deviation in the population without intervention
- One option: $\sqrt{\tau_{u_0}^2 + \sigma^2}$ from the unconditional model[1]
  - For salience (from 0 to 5), $d = \frac{(-0.13)\times 5}{\sqrt{0.18+0.39}} = -0.87$
  - See R code

[1]: See more discussion in Judd et al. (2017, doi: 10.1146/annurev-psych-122414-033702)

Effect Size (*d*)

- For cluster-randomized trials (random assignment at school level), see Lai (2020)[1]

[1]: https://doi.org/10.1080/00273171.2020.1746902

Alternative Models

- See R code for log-normal model that directly models the distribution of response time