SHORELINE MODELING AND EROSION PREDICTION

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ABSTRACT

This paper presents a shoreline-erosion prediction model of Lake Erie that can forecast shoreline changes from annual to 10-year increments. It was developed by using historical bluffline data of years 1973, 1990, 1994, and 2000 at Lake Erie provided by NOAA and local government agencies. The relationships among these historical shorelines are analyzed using a least-squares method. Erosion rates are then derived from shoreline changes. In addition, other influential factors such as changes in terrain and water-levels are also considered in the model.

INTRODUCTION

In coastal regions the natural forces that cause shoreline changes are embodied in waves, currents, wind and other factors. The backward movement of land due to actions of these forces is termed erosion. The loss of coastal land properties, submergence of beaches and structures near the shore are direct consequences of these coastal erosion processes. Most of the coastal regions in the United States are suffering from erosion. The direct financial and resource loss caused by this natural process makes it a significant problem that needs to be researched. In order to increase the possibility of overcoming the effect of the erosion process, attempts must be made to predict its occurrence and make research results publicly available. Shoreline changes can be used as a good indicator of occurrence of coastal erosion. Accurate information is required regarding the past and present movement of the shoreline in order to take preventive actions against loss of infrastructure along the shores. One of the most important factors in any shoreline change analysis is the consistency of the shoreline model applied from one coastal region to another. All the natural forces responsible for shoreline movement are the functions of space and time because the intensities of these activities change according to geographic location and seasonal variations. The main challenge in shoreline prediction modeling has been to create models with sophisticated spatio-temporal numerical analysis which can generate testable predictions about the functioning of a coastal erosion system (Fletcher et al., 2003).

In the presence of modern GIS technology, these models will have greater reliability, accuracy, and analyzing/visualization capability. In the past, errors in the process of identifying shoreline positions and digitization, as well as the absence of spatio-temporal tools for analyzing the trend of shoreline changes, were among the potential causes for restricting the ability of models to provide defensible shoreline change rate evaluations. There is certainly a need for revision of such prediction models due to the evolution of technologies and the increasing need for such models.

The first requirement is to choose a shoreline indicator for coastal mapping purposes. The primary requirement in choosing an indicator should be its easy identification in the coastal area and on aerial photographs. Morton and Speed (1998) and Pajak and Leatherman (2002) mentioned some of these shoreline indicators including bluff edges, vegetation lines, high water marks, beach crust, dune crust, beach scarp, etc. Due to the scope of the research project, the top bluff edge is chosen in this research as the shoreline indicator based on its visibility on aerial photographs.

In shoreline change modeling, various mathematical models have been proposed. An empirical model is used in bluffline modeling which does not involve sand transportation (Ali, 2003). In this empirical model shorelines are identified in increasing order of time (from past to future) and then the relationship between the time and the shoreline position changes is analyzed by using a numerical method. Bluffline modeling would become unstable in the presence of sand transportation due to its impermanency as sand can be transported back into the water. Moreover, the lack of involvement of sand transportation in the empirical model makes it easier to implement. However, it is reasonable to assume that by modeling the shoreline position changes we are taking the underlying coastal erosion processes into account because all of the changes in the geometry of the shoreline are the end result of erosion phenomena. The Division of Geological Survey in the Ohio Department of Natural Resources (ODNR) proposed and implemented a technique for creating the correspondence between the available shoreline indicators by creating transects at a master shoreline. Here, the master shoreline implies a shoreline with a good quality. Once these transects are created, then the rates of shoreline change are derived on these transects. By using these change rates, the future shoreline position can be derived. Another approach, the End Point rate (EPR) method, presented by Liu (1998) and Galgano and Douglas (2000), is based on an empirical equation which shows that the future position of a shoreline can be derived by a linear relationship between past shoreline positions and time. The change rate (m) and intercept (c) involved in this model are derived by a line ($y = mx+c$) extracted from the points on the earliest and latest available shorelines (y and x represent the shoreline position and time respectively). This model may not be applied widely because of the absence of positional quality information and due to undesirable results for short periods, for instance, less than eighty years (Galgano and Douglas, 2000).

Models described in the above paragraph assume the determination of future shorelines is based on modeling points on past shorelines. The shoreline change prediction based on some chosen points from shoreline depictions cannot be fully justified because these points do not guarantee the continuity of the shoreline. A shoreline is a continuous and dynamic feature; a model for shoreline change analysis must maintain the continuity of the shoreline. In this regard, Liu (1998) and Li et al. (2001) presented a method for shoreline change modeling and analysis by using a dynamic segmentation concept. This approach preserves the continuity of the shoreline by dividing the shoreline into various line segments. These line segments are continuous due to the fact that the end location of the first segment would be the same as the start location of the second segment, and so on. This paper implements the dynamic segmentation approach by modeling the changes with line segments of available past shorelines.

ANALYTICAL MODEL: ASSUMPTIONS AND METHODLOGY

The shape of a bluffline with a reasonable length is generally irregular and cannot be expressed using analytical functions. To apply any numerical modeling approach in the analysis of the bluffline change, its geometry should be expressed under some assumptions. We divide a bluffline A_1A_{n+1} into a finite number (n) of segments $\overset{\frown}{A_1A_2}$, $\overset{\frown}{A_2A_3}$, $\overset{\frown}{A_3A_4}$, ..., $\overset{\frown}{A_nA_{n+1}}$ which are defined by their starting and ending points $A_1\,(x_1, y_1)$, $A_2\,(x_2, y_2)$, $A_3\,(x_3, y_3)$, $A_4\,(x_4, y_4)$, ..., $A_n\,(x_n, y_n)$, $A_{n+1}(x_{n+1}, y_{n+1})$. Moreover, these segments with small spatial extents can be approximated to linear segments $\overline{A_1A_2}$, $\overline{A_2A_3}$, $\overline{A_3A_4}$, ..., $\overline{A_nA_{n+1}}$ (Figure 1).

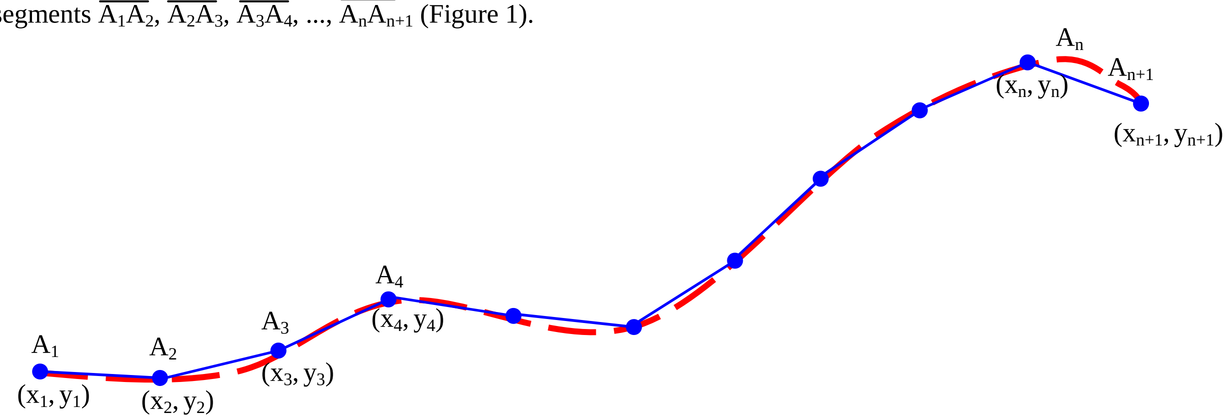


Figure 1. Linear approximation of the bluffline

In implementing the linear approximation on all the available blufflines of different times, a standardized parameterization method is used (Schmidley, 1996). According to this method each of the curves is normalized onto the interval of [0, 1] by considering their length equal to 1. These normalized curves are then divided into the same number of line segments. The ending points of these line segments are defined by their normalized coordinates. The same division of each bluffline will ensure the line segments of a bluffline have the same constant length. The graphical representation for this procedure is shown in Figure 2, where the lengths of two blufflines A_1A_{n+1} (time t =k) and $A_1'A_{n+1}'$ (time t =k+m) are normalized to 1 and equal-length line segments A_1A_2, $A_{p-1}A_p$, ..., A_nA_{n+1} and $A_1'A_2'$, $A_{p-1}'A_p'$, ..., $A_n'A_{n+1}'$ are created on both normalized blufflines. The next step is to derive correspondences among blufflines. Setting up a correspondence between two blufflines is equivalent to deriving the correlation between every pair of divided line segments on both blufflines, for instance, A_1A_2 and $A_1' A_2'$, or $A_{p-1}A_p$ and $A_{p-1}' A_p'$. Thus, the bluffline change analysis is reduced to a problem of identifying parameters which are responsible for the change of the same line segment on different blufflines and establishing a method to predict the variability of these parameters in temporal dimension. For example, modeling the change of a pair of the line segments S_m and S_m' ($A_{p-1}A_p$ and $A_{p-1}'A_p'$) on different blufflines A_1A_{n+1} (time t =k) and $A_1'A_{n+1}'$ (time t =k+m) can be treated as a subproblem for the entire bluffline change analysis (Figure 2).

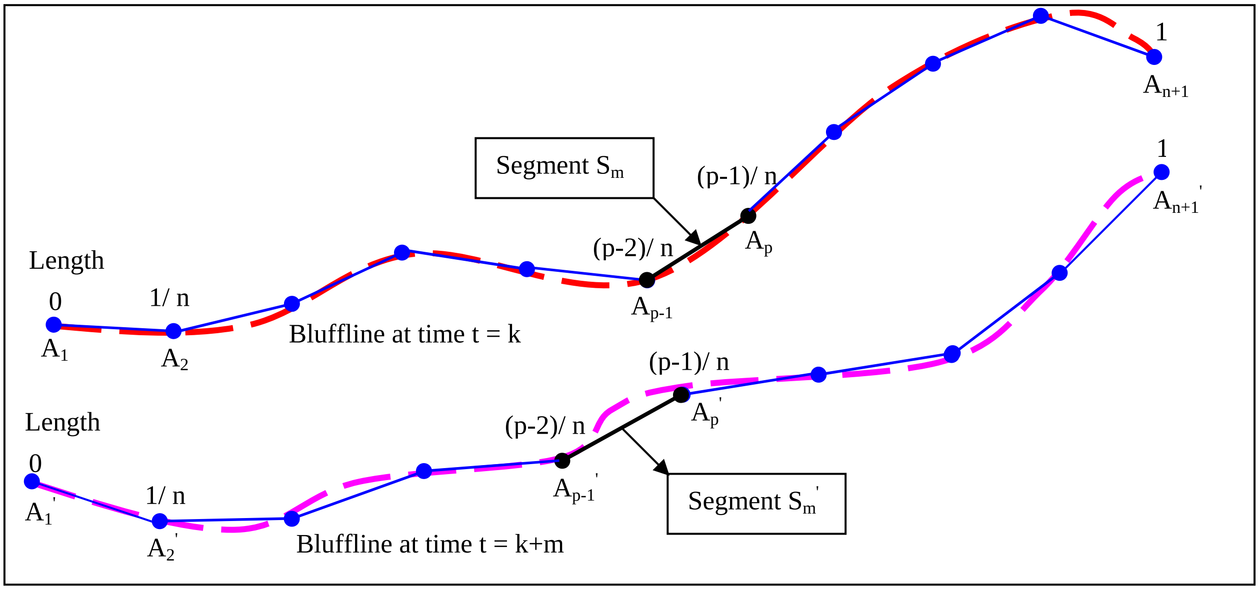


Figure 2. Concept of basic unit model in the analysis of bluffline change

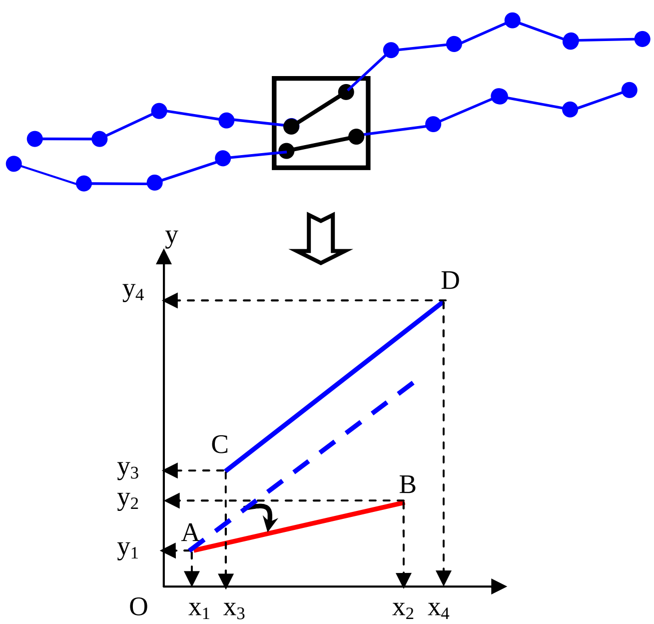


Figure 3. Transformation of one line segment to another line segment (Ali, 2003)

Identification of Parameters of Line Segments

The transformation from one line segment to another line segment includes translation, rotation and scale changes. Three basic parameters can be identified as translation (ΔT), rotation (ΔR) and scale change (ΔS). In two-dimensional space, translation is composed of two subchanges (Δx and Δy), i.e., change in the x direction and change in the y direction. This transformation is shown in Figure 3. Suppose that we want to transform a line segment CD into another line segment AB. The line segment CD is represented by its ending points $C(x_3, y_3)$ and $D(x_4, y_4)$ and the line segment AB is represented by $A(x_1, y_1)$ and $B(x_2, y_2)$ (Figure 3).

The parameters of translation (ΔT) in x and y direction, rotation (ΔR) and scale change (ΔS) are unknown, but they can be derived from the coordinates of ending points of every pair of corresponding line segments (A_1, A_2,....., A_{p-1}, A_p, ..., A_n, A_{n+1} and similarly A_1', A_2',...., A_{p-1}', A_p', ..., A_n',A_{n+1}') from two different blufflines. The word "corresponding" is of special importance here. It alludes to the fact that we are deriving these parameters for every set of line segments with consistency as described earlier in this section. The equations involved in deriving the transformation parameters are as follows.

$$\begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \end{bmatrix} = (\Delta S) * (\Delta R) \begin{bmatrix} x_3 \\ y_3 \\ x_4 \\ y_4 \end{bmatrix} + \Delta T$$

Where $\Delta T = \begin{bmatrix} x_1 - x_3 \\ y_1 - y_3 \\ x_1 - x_3 \\ y_1 - y_3 \end{bmatrix}$, $\Delta R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix}$ and $\Delta S = \left[\dfrac{Length(AB)}{Length(CD)}\right] = \left[\dfrac{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{\sqrt{(x_3 - x_4)^2 + (y_3 - y_4)^2}}\right]$

A Least-Squares Approach: Relationship between Parameters and Time

Once the transformation parameters are known for every pair of corresponding line segments, change of these parameters needs to be studied in the temporal dimension. This analysis will allow us to predict the trend of these parameters. More pairs of blufflines will be helpful to create a prediction model with a better accuracy. For example, three historical blufflines (at time t_1, t_2, and t_3, where $t_3 > t_2 > t_1$) would make three different pairs of blufflines (t_1-t_2, t_2-t_3, t_3-t_1). For each identical line segment, therefore, three sets of transformation parameters can be derived. Based on these three sets of parameters, an analytical model can be set up to predict the future position of the line segment.

The continuity property of future blufflines should also be retained by the model. In this case, once the first point of a future bluffline is known, it will enable us to derive the Δx and Δy for the first segment of the future bluffline. Since every segment of the bluffline is connected, we can calculate the Δx and Δy for the remaining segments. Therefore, Δx and Δy are unnecessary to be modeled in transformation parameter analysis. Only two unknowns (rotation and scale change) remain unresolved in this model. To derive the position of future blufflines from available historical blufflines, a least-squares approach is used to determine rotation and scale change parameters.

In general, for m number of blufflines (at time t_1, t_2, t_3, …t_m) the calculation of theta and scale change can be expressed as per the following polynomials.

$$\Delta\theta_{1-2} = a_1 (t_2-t_1)^{k-2} + a_2 (t_2-t_1)^{k-3} + a_3 (t_2-t_1)^{k-4} + \ldots\ldots\ldots\ldots + a_{k-1}$$
$$\Delta\theta_{2-3} = a_1 (t_3-t_2)^{k-2} + a_2 (t_3-t_2)^{k-3} + a_3 (t_3-t_2)^{k-4} + \ldots\ldots\ldots\ldots + a_{k-1}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

Where $k = m(m-1)/2$, is the total number of pairs that can be selected out of m blufflines. For a general interval of time ($t_p - t_q$), $\Delta\theta_{q-p}$ can be expressed as follows.

$$\Delta\,\theta_{q-p} = \sum_{i=1}^{k} a_{i-1}(t_q - t_p)^{k-i-1} \tag{1}$$

Similarly, scale change modeling can be written by using the following polynomial.

$$\Delta\,S_{q-p} = \sum_{i=1}^{k} b_{i-1}(t_q - t_p)^{k-i-1} \tag{2}$$

The number of unknowns (a_i , b_i) depends on the available number (m) of blufflines. The greater number of unknowns in these equations will increase the order of the polynomial and will definitely increase the accuracy in determining the values of these unknowns.

First, transformation parameters ($\Delta\,\theta_{q-p}$, ΔS_{q-p}) between all the available pairs of blufflines are calculated, and then a numerical relationship between transformation parameters and time are established based on the above equations. The coefficients of the above polynomials can be calculated by using the least-squares approach as explained below. For calculation of a_i (for i=1 to k-1), equation (1) can be replaced by the following matrix form.

$$\begin{bmatrix} \Delta\theta_{1-2} \\ \Delta\theta_{2-3} \\ \ldots \\ \Delta\theta_{q-p} \\ \ldots \end{bmatrix} = \begin{bmatrix} (t_2 - t_1)^{k-2} & (t_2 - t_1)^{k-3} & \ldots & \ldots & 1 \\ (t_3 - t_2)^{k-2} & (t_3 - t_2)^{k-3} & \ldots & \ldots & 1 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ (t_q - t_p)^{k-2} & (t_q - t_p)^{k-3} & \ldots & \ldots & 1 \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \ldots \\ a_{k-2} \\ a_{k-1} \end{bmatrix}$$

Which corresponds to $\quad Y_{k\times 1} \quad = \quad A_{k\times (k-1)} \quad X_{(k-1)\times 1}$

By least-squares method, the solution is $X = (A^tA)^{-1}A^tY$ which is obtained by minimizing the following function.

$$\sum_{i=1}^{k} \Delta\theta_i^{\,2} = (AX - Y)^t (AX - Y)$$

Similar calculations can be performed for scale change analysis. Now, in order to predict θ, ΔS for the future bluffline, we use equations (1) and (2) with known values of coefficients and time intervals (between the latest available bluffline and future bluffline). In the knowledge of Δx, Δy and estimated $\Delta\,\theta$, ΔS, the future bluffline can easily be derived.

EXPERIMENTAL RESULTS

The study area for this research is located at Painesville, Ohio, a fifteen-kilometer coastal region along the southern coast of Lake Erie. This area of Lake Erie is chosen due to two main reasons: 1) The availability of the historical blufflines in this region provides appropriate information for the analysis; 2) The Division of Geological Survey in the Ohio Department Natural Resources (ODNR) considers this region as a more vulnerable area for study due to severe coastal changes of the region (Zuzek, 2003). The bluffline, a shoreline indicator, is used in this entire experiment.

Historic blufflines of the years 1973, 1990, 1994, and 2000 in this region are used in this research. The blufflines of years 1973 and 1990 were digitized from the Lake Erie Coastal Erosion Area maps, Ohio Coastal Management Program published by the Lake Erie Geology Group of the Division of Geological Survey at the Ohio Department of Natural Resources in 1997. The bluffline of 1994 was interpreted from United State Geological Survey (USGS) DOQQ (taken in 1994) downloaded from the website (ftp://ftp.geodata.gis.state.oh.us) of Ohio Geographically Referenced Information Program (OGRIP). The bluffline of year 2000 was also derived from the orthophotos of the same year provided by the Lake County GIS Department. All these bluffline datasets were projected into the same horizontal coordinate system: State Plane, Ohio North - NAD 1983. In the analysis, a predicted bluffline of year 2000 was derived based three blufflines of years 1973, 1990, and 1994. The bluffline of year 2000 is kept as a reference to evaluate the least-squares prediction model.

During the prediction process, three pairs of blufflines were available (Figure 4). The scale and rotation parameters were derived between each of the pairs using the procedure described in the previous section. Δ x and Δ y for each line segment were determined by the initial locations of the blufflines and derived based on the continuous property of blufflines.

In order to determine the trends of scale and rotation for future use, their dependencies over time were predicted by setting up the polynomial (Equation 3). In this research problem we have blufflines of three years (1973, 1990 and 1994) which allows us to use the first-order polynomial in deriving the relationship of scale and rotation change with time. The coefficients involved (a_1, a_2 and b_1, b_2) in the following equations are solved by the least-squares method based on the calculated values of the rotation and scale parameters for each pair of bluff-lines.

$$\theta = a_1 \Delta t + a_2 \text{ and } S = b_1 \Delta t + b_2 \tag{3}$$

$$\begin{bmatrix} \theta_{73-90} \\ \theta_{73-94} \\ \theta_{90-94} \end{bmatrix} = \begin{bmatrix} (1990-1973) & 1 \\ (1994-1973) & 1 \\ (1994-1990) & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \text{ and } \begin{bmatrix} S_{73-90} \\ S_{73-94} \\ S_{90-94} \end{bmatrix} = \begin{bmatrix} (1990-1973) & 1 \\ (1994-1973) & 1 \\ (1994-1990) & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Comparing the above matrixes with the general form of observation equation (Y = AX) and applying the least-squares method will provide the solution for the coefficients a_1, a_2 and b_1, b_2 as follows.

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = [(A^t A)^{-1} A^t] \begin{bmatrix} \theta_{73-90} \\ \theta_{73-94} \\ \theta_{90-94} \end{bmatrix} \text{ and } \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = [(A^t A)^{-1} A^t] \begin{bmatrix} S_{73-90} \\ S_{73-94} \\ S_{90-94} \end{bmatrix} \text{ where, } A = \begin{bmatrix} 17 & 1 \\ 21 & 1 \\ 4 & 1 \end{bmatrix}$$

After determination of the coefficients, substituting the time difference (Δ t =2000-1994) between the unknown bluffline (of year 2000) and latest known bluffline (of year 1994) into equation 3 allows us to calculate the rotation and scale change for every line segment of the bluffline of year 2000 from the latest available bluffline (of year 1994). Once the values of Δ x and Δ y for the first line segment are determined, the position of the bluffline of year 2000 can be derived.

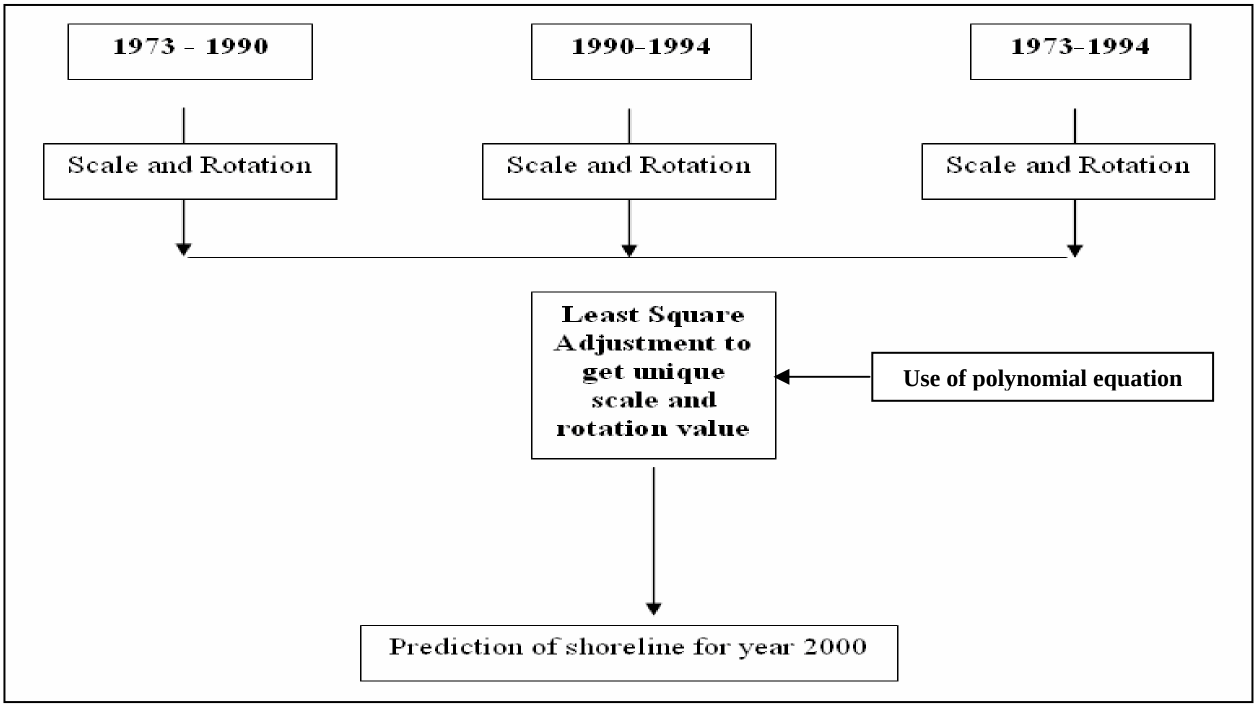


Figure 4. Flow chart of the experimental model

Figure 5 shows the comparison results of the predicted bluffline of year 2000 and the digitized bluffline from the orthophoto of year 2000 of the entire study area. One more bluffline of year 2000 (dark blue colored in Figure 5 and Figure 6) is derived by using the erosion rates published by the Lake Erie Geology Group of the Division of Geological Survey at the ODNR in 1997 (Coastal Erosion Area Tabulated Data) and is called erosion-based bluffline. The region A in Figure 5 is enlarged and shown in Figure 6.

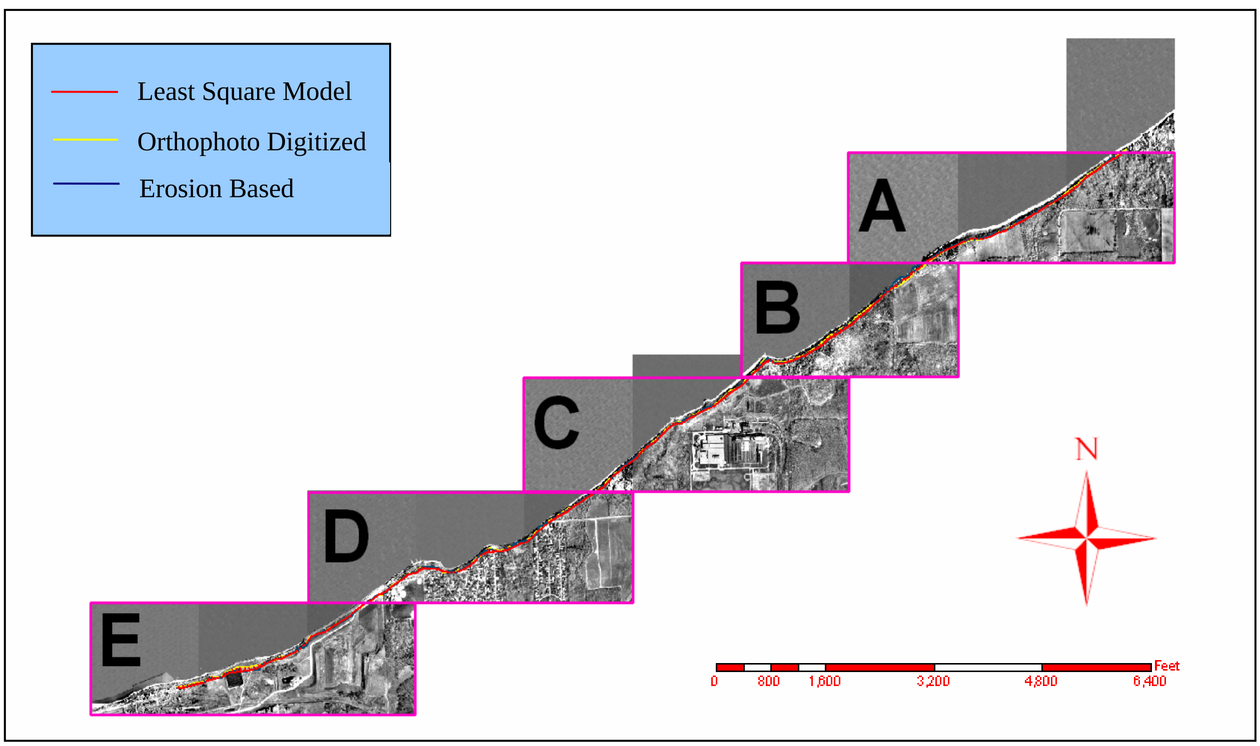


Figure 5. Study area and blufflines

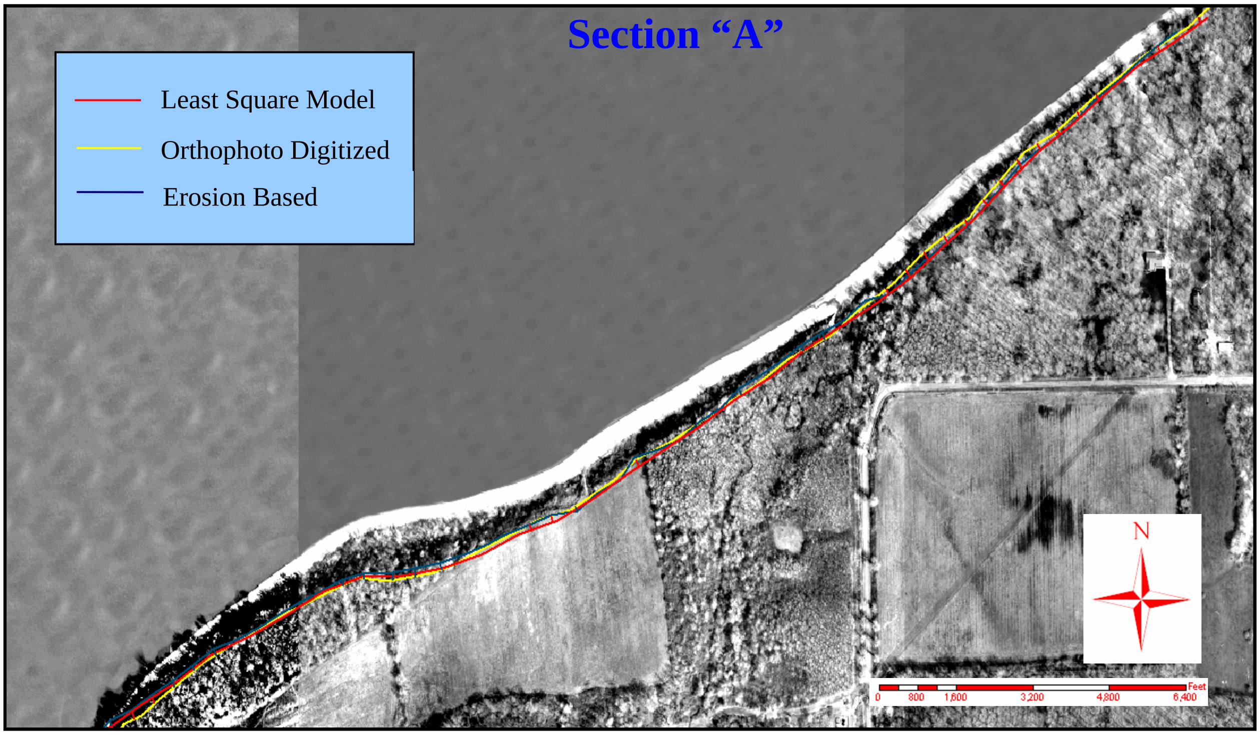


Figure 6. Enlarged view of region A

Error Analysis

The evaluation of the predicted bluffline of year 2000 was done by comparing it to the orthophoto-digitized bluffline of year 2000. As a comparison, the erosion-based bluffline of year 2000 is also compared to the bluffline from orthophoto (Figure 7). The idea behind these two sets of comparisons was to present the error analysis between the bluffline based on the traditional method of erosion rate and the predicted bluffline from the least-squares model.

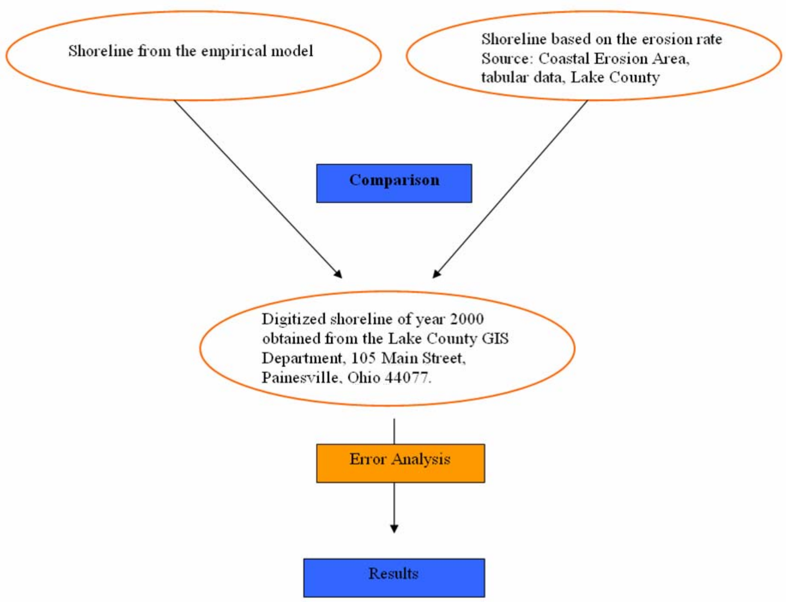


Figure 7. Error analysis flow chart for the bluffline of year 2000

Now, the main concern was to choose the methods which could compare the two linear datasets after having the desired datasets. In spatial data quality, the comparison of the two-point datasets for the positional quality information is fully defined but the estimation for the positional quality of any linear dataset with respect to the reference dataset is still a topic of ongoing research.

Table 1. Error analysis results from transects method

Bluffline pairs	Average error	Maximum error	Minimum error
Predicted vs. orthophoto digitized	4.99 m	23.86 m	0.00 m
Erosion rate- based vs. orthophoto digitized	4.94 m	40.54 m	0.00 m

Two methods are used in the accuracy analysis of the prediction model: i) In the first method, a finite number of transects were digitized from the coastal area erosion maps produced by the Division of Geological Survey of ODNR. Along these digitized transects, intersection points of transects and blufflines represent the bluffline movement. Once distances (movements of points from reference bluffline to erosion based and least-squares model-based blufflines) were measured, the average errors were derived by dividing the sum of those distances by the number of transects. This approach approximately estimated the accuracy of the predicted bluffline and erosion-rate-based bluffline as compared to the digitized bluffline. The accuracy of this comparison method depends upon the number of transects taken into consideration. A denser network of transects will ensure a more exhaustive comparison between the blufflines. The results showed comparable values of average and same minimum errors between the two datasets along transects (Table 1). However, while comparing the maximum errors for two datasets we found that the bluffline predicted by the least squares model is more accurate in comparison to the traditional erosion rate based bluffline with reference to the digitized bluffline from the orthophoto.

ii) The second error analysis approach was based on the metrics developed by Ali (2003) for the positional quality assessment of the linear features. He developed four quality metrics (distortion factor, generalization factor, bias factor, and fuzziness factor) and an automated interface for calculating these metrics in order to access the positional quality of any linear dataset quantitatively. For the error analysis of this research, these four quality metrics were evaluated for two pairs of blufflines and the results are then compared. An overview of these quality metrics and calculation of these metrics are shown in Figure 8. Based on the four metrics, Ali further extended this method to produce a single quantitative measure for estimating the positional accuracy of linear datasets. This overall quality factor, composed of three quality metrics, can be obtained by the following equation. Fuzziness factor alone is also a measure of spatial data quality.

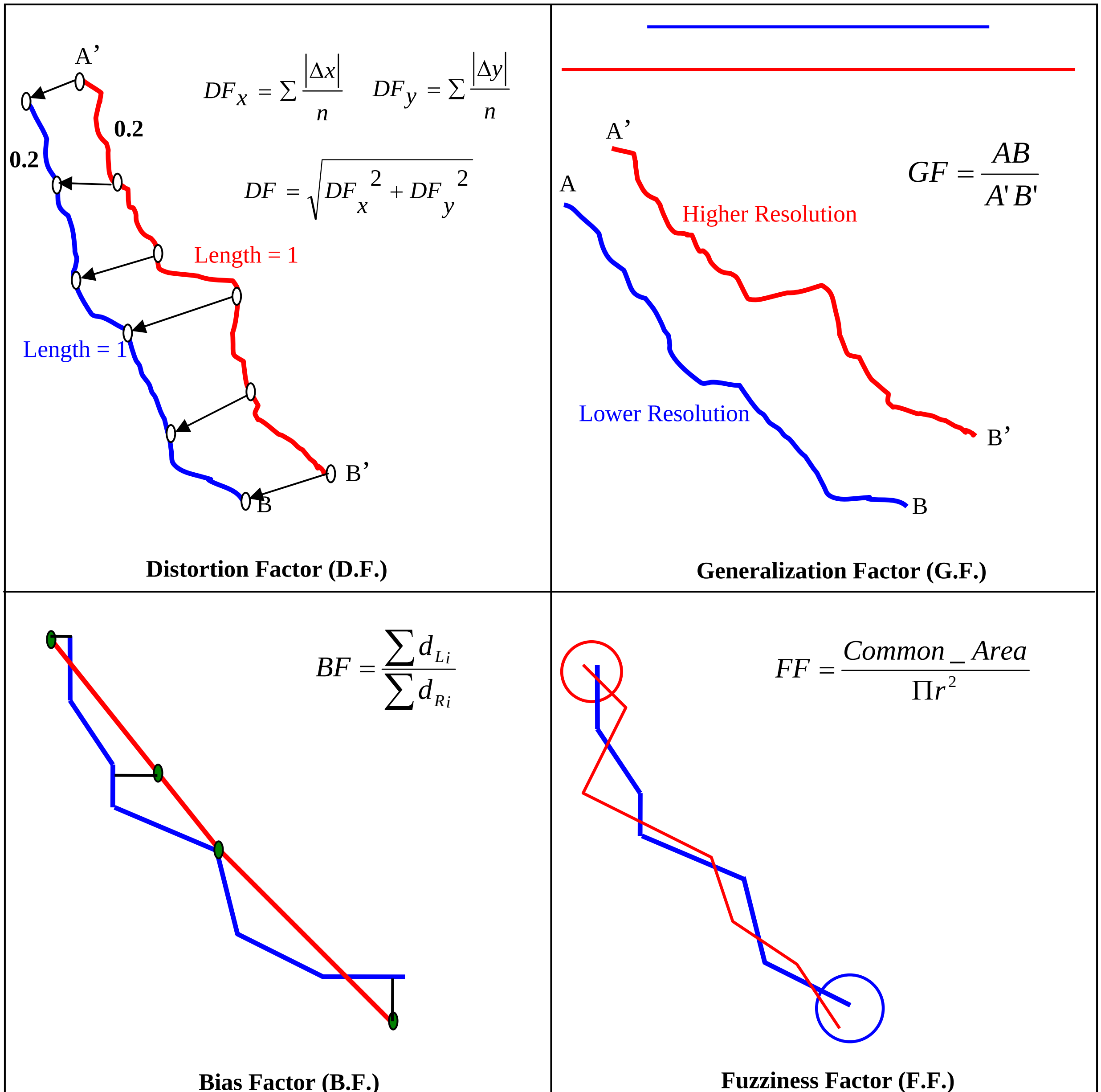
$$\text{Overall Quality Factor } = \{(0.33 \times G.F.) + (0.34 \times D.F.) + (0.33 \times B.F.)\}$$

Based on the values of these metrics, the results are more accurate for the Erosion-rate-based bluffline in compared to the bluffline predicted by the least-squares model.

DISCUSSION

The demarcation of shoreline indicator from the aerial photographs, as well as errors in surveying and the digitization process can be major sources of prediction errors of blufflines. These types of errors can be characterized as random errors. The seasonal changes and storm-inducing erosion can be classified as systematic error sources in the presented research.

Another reason for the lower accuracy in the prediction model experiment could be the sudden curvature change in the region which leads to inaccuracy as the prediction model is based on a linear equation. The least-squares model can be improved by employing a high-order polynomial if more numbers of shorelines of previous years are available. We believe that the use of a higher-order polynomial will lead to a better result as it can take into account the curved nature of the shoreline.



Factor	Digitized vs. Predicted	Digitized vs. Erosion-based
Distortion Factor	0.467	0.794
Generalization Factor	1.032	0.999
Bias Factor	4.329	1.618
Fuzziness Factor	0.000673	3.44×10^{-5}
Overall Quality Factor	1.928	1.134

Figure 8. Error analysis using quantitative matrices (Ali, 2003)

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